理化学研究所 橋本数理物理学研究室

セミナー (2012年5月07日)

Flux Compactifications, and Gauged Supergravities

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以下の流れでお話をします

- 動機
  - 超重力理論と高次元理論
- 疑問
  - ゲージ対称性
- 幾何の拡張
- 低次元超重力理論
  - 非可換ゲージ化
- 期待
- 課題

動 機

低次元の超重力理論は、
すべて高次元のストリング理論に帰着されるのか？
を (ただ純粋に) 知りたい。

Gauged supergravity :　超重力理論に (非) 可換ゲージ場が結合した理論

(狭義では R 対称性をゲージ化した超重力理論を意味するが、最近はこちらを意味する)

10D ヘテロティック弦 $\xrightarrow[\mathcal{M}_6]{\text{コンパクト化}}$ 4D $\mathcal{N}=1$ 超対称カイラルゲージ理論

✓ 10D = 4D Minkowski + 6D internal space $\mathcal{M}_6$:

$$\mathrm{d}s^2_{\text{10D}} = \underbrace{\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu}_{\text{4D}} + \underbrace{g_{mn}(x,y)\,\mathrm{d}y^m\,\mathrm{d}y^n}_{\mathcal{M}_6}$$

✓ 4D $\mathcal{N}=1$ vacuum :

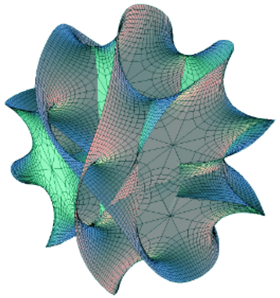
$$^{\exists}\eta = \begin{pmatrix} 0 \\ 0 \\ 0 \\ * \end{pmatrix} \text{ on } \mathcal{M}_6 \qquad \text{with} \quad 0 = \delta\psi_m = \nabla_m \eta + \ldots$$

✓ Trivial background on $\mathcal{M}_6$: $H_3 = 0 = \mathrm{d}\phi$

✓ Anomaly cancellation condition : $0 = \mathrm{Tr}\{R_2 \wedge R_2 - F_2 \wedge F_2\}$

コンパクト空間 $\mathcal{M}_6$ = Calabi-Yau 多様体

Calabi-Yau 多様体 $\mathcal{M}_{\mathrm{CY}}$



Ricci 平坦な Kähler 多様体

トーションなし

ホロノミー群は $SU(3) \subset SU(4) \sim SO(6)$

Levi-Civita 接続の共変微分について共変定数な 2 形式 (J) と正則 3 形式 (Ω)：

$$\mathrm{d}J \ = \ \nabla_{[m}J_{np]} \ = \ 0 \qquad \mathrm{d}\Omega \ = \ \nabla_{[m}\Omega_{npq]} \ = \ 0$$

NS-NS 場の展開：

jump

$$\phi(x,y) = \varphi(x)$$

$$g_{\mathfrak{m}\bar{\mathfrak{n}}}(x,y) = \mathrm{i}v^a(x)\,(\omega_a)_{\mathfrak{m}\bar{\mathfrak{n}}}(y), \quad g_{\mathfrak{m}\mathfrak{n}}(x,y) = \mathrm{i}\,\bar{z}^{\bar{\jmath}}(x)\left(\frac{(\overline{\chi}_{\bar{\jmath}})_{\mathfrak{m}\bar{\mathfrak{p}}\bar{\mathfrak{q}}}\Omega^{\bar{\mathfrak{p}}\bar{\mathfrak{q}}}{}_{\mathfrak{n}}}{||\Omega||^2}\right)(y)$$

$$B_2(x,y) = B_2(x) + b^a(x)\omega_a(y)$$

$$\mathfrak{t}^a(x) \equiv b^a(x) + \mathrm{i}v^a(x)$$

R-R 場の展開：

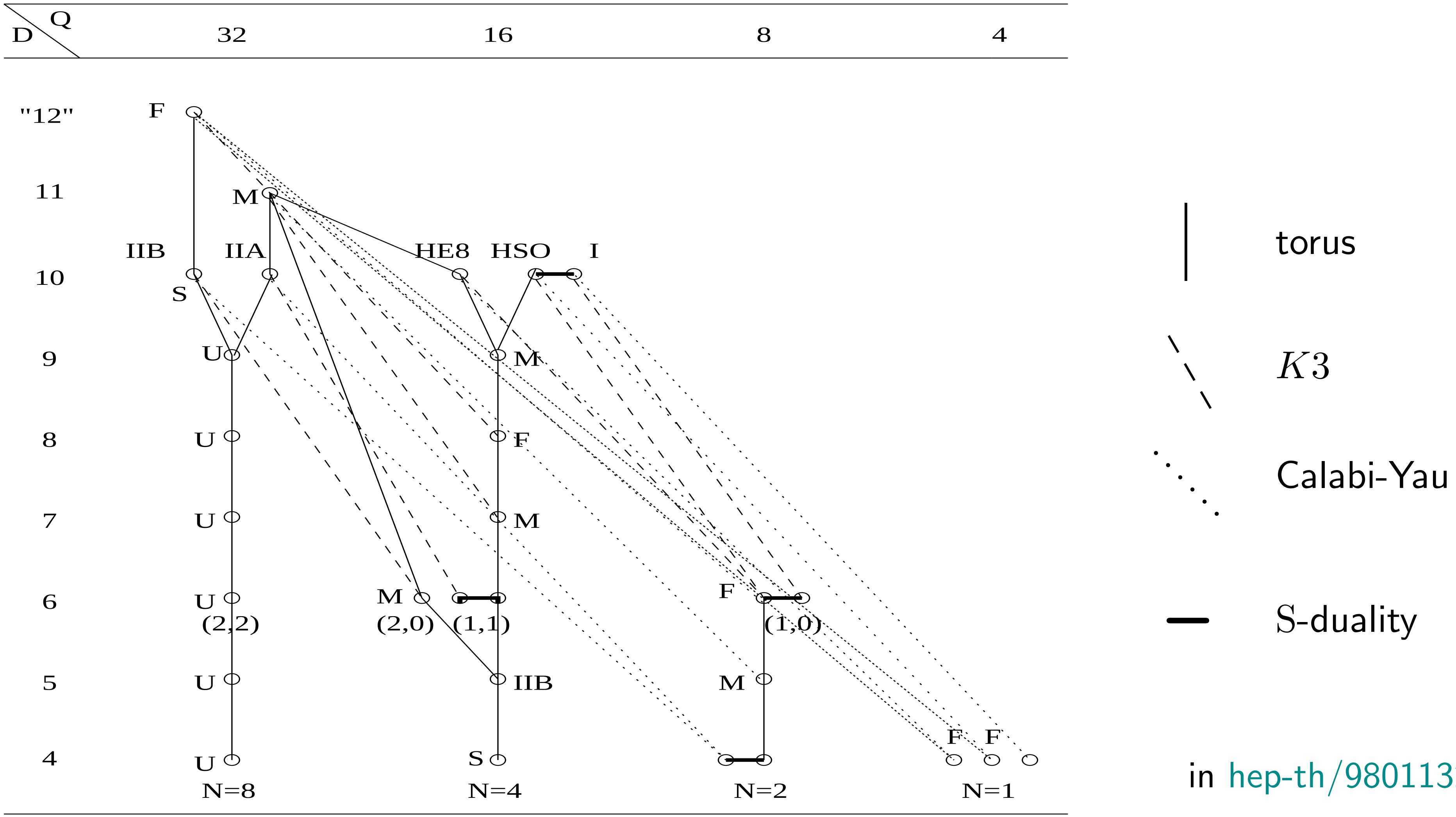
$$C_1(x,y) = A_1^0(x)$$

$$C_3(x,y) = A_1^a(x)\wedge\omega_a(y) + \xi^I(x)\alpha_I(y) - \widetilde{\xi}_I(x)\beta^I(y)$$

コホモロジー	基底	自由度
$H^{(1,1)}$	ω_a	$a = 1,\ldots,h^{(1,1)}$
$H^{(0)}\oplus H^{(1,1)}$	$\omega_\Lambda = (1,\omega_a)$	$\Lambda = 0,1,\ldots,h^{(1,1)}$
$H^{(2,2)}\oplus H^{(6)}$	$\widetilde{\omega}^\Lambda = (\widetilde{\omega}^a, \frac{\text{vol.}}{\lvert\text{vol.}\rvert})$	
$H^{(2,1)}$	χ_i	$i = 1,\ldots,h^{(2,1)}$
$H^{(3)}$	(α_I,β^I)	$I = 0,1,\ldots,h^{(2,1)}$

$$d\omega_\Lambda = 0 = d\widetilde{\omega}^\Lambda$$

$$d\alpha_I = 0 = d\beta^I$$



in hep-th/9801132

「ストリング理論の有効理論」の地図はこれで十分か？ $\rightarrow$ No!

余分なゼロ質量場が登場 / type II 弦からのゲージ群は基本的に abelian

non-CY manifold $\mathcal{M}_6$

Ricci 2-form がゼロのまま、トーションを許す

($SU(3)$-structure manifold)

$\mathrm{d}J \neq 0$ and/or $\mathrm{d}\Omega \neq 0$

CY からのズレ：

$$\mathrm{d}J = \frac{3}{2} \mathrm{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

$\mathcal{W}_1 \,,\ \mathcal{W}_2 \,,\ \mathcal{W}_3 \,,\ \mathcal{W}_4 \,,\ \mathcal{W}_5$: intrinsic torsion classes

分類 $G_2/SU(3)$ cosets

- Heterotic string on $SU(3)$-structure manifolds

  if $\mathrm{d}H = 0$: smooth $\mathcal{M}_6$ with fluxes is reduced to $\mathcal{M}_{\mathrm{CY}}$ without fluxes

  Piljin Yi and TK, hep-th/0605247

- Index theorem on torsionful manifolds

$$\text{relation among} \begin{cases} \omega - \frac{1}{3}H & \text{(Dirac eq.)} \\ \omega - H & \text{(SUSY variation)} \\ \omega + H & \text{(invariant polynomial)} \end{cases}$$

  TK, arXiv:0704.2111

- Fluxes, α' corrections and SUSY vacua

  hep-th/0605247, arXiv:0810.0937

- Intersecting five-branes in heterotic string

  S. Mizoguchi and TK, arxiv:0905.2185, arxiv:0912.1334

Calabi-Yau や

トーションを持つ「多様体」では

せいぜいabelianゲージ相互作用しか

実現しない (type II strings の場合)

Kaluza-Klein ベクトル場 (NSNS-sector 由来)： truncated out (32-SUSY $\rightarrow$ 8-SUSY)

ベクトル多重項のスカラー場 (NSNS-sector 由来) と

ゲージ場(RR-sector 由来) はベクトル多重項と相互作用できない

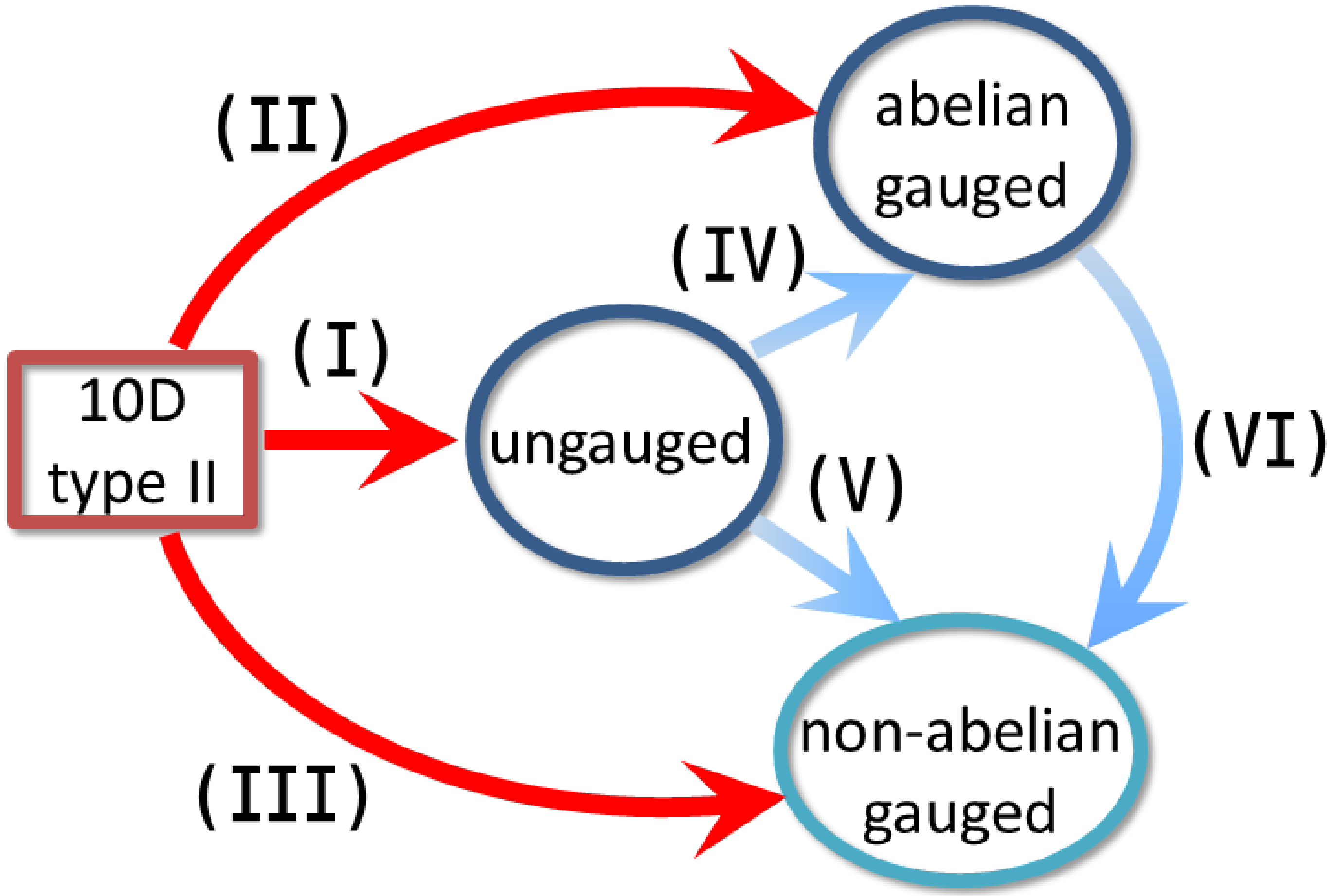
疑 問

Non-abelian ゲージ相互作用を
コンパクト化の機構で実現するには？

ヘテロティック弦から non-abelian ゲージ場が登場するなら、

type II 弦からも non-abelian ゲージ場が結合する重力理論が登場すべき

(heterotic/type II duality in lower-dimensions)



(I)	Calabi-Yau
(II)	$SU(3)$-structure manifolds
(III)	??
(IV),(V),(VI)	??

幾何 の 拡張

低次元超重力理論の高次元起源

Generalized Geometry and Doubled Formalism

低次元超重力理論の高次元起源

4D $\mathcal{N} = 8$ SUGRA with $CSO(p, q, r)$ gauge symmetry, etc. jump

$\downarrow$

ストリング理論を “拡張された” 幾何でフラックスコンパクト化

(Non)geometric String Backgrounds

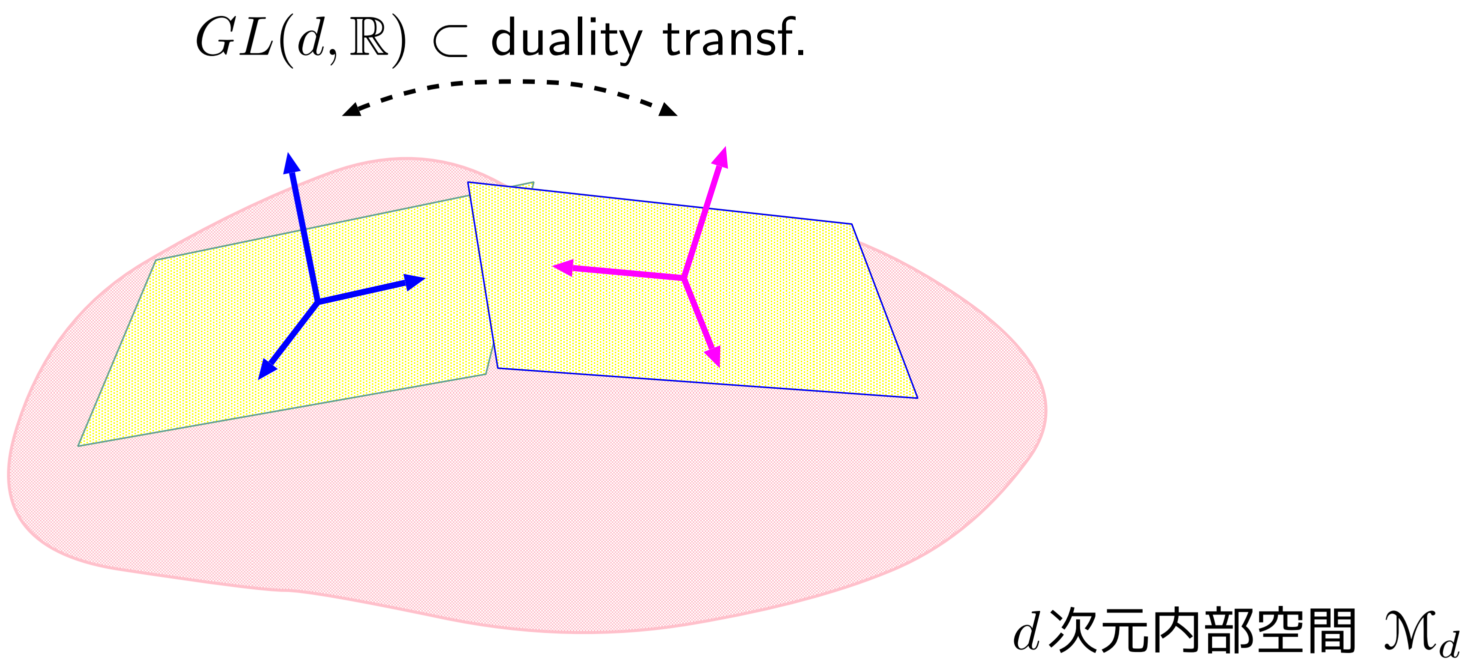
全ての超重力理論を導出できる可能性 (?)

(Non)geometric String Backgrounds とは?

構造群 $=$ 「Diffeo群 $(GL(d,\mathbb{R}))$ $\subset$ 双対変換群 $(O(d,d)$, U-双対変換)」

↑

弦理論の双対性に起因



多様体・幾何の拡張

拡張された幾何

N.J. Hitchin

Generalized Geometries and Doubled Formalism

math/0209099 hep-th/0406102

C.M. Hull

幾何を記述する「要素」として、計量 g_{mn} 以外の場を組み込む

geometry associated with g_{mn}	Conventional geometry (manifold) $O(6)$ global symmetry
geometry associated with g_{mn}, B_{mn}	Generalized geometry $O(6,6)$ T-duality symmetry
geometry associated with g_{mn}, B_{mn}, $C_{(p)}$	Exceptional generalized geometry $E_{7(7)}$ U-duality symmetry

almost complex structure $J_m{}^n$ on $T\mathcal{M}_6$ s.t.

$$J_m{}^n : \ T\mathcal{M}_6 \ \longrightarrow \ T\mathcal{M}_6$$

$$J^2 \ = \ -\mathbb{1}_6$$

$$^{\exists} \ O(6) \text{ invariant metric } \eta, \text{ s.t. } \ J^{\mathrm{T}} \eta J \ = \ \eta$$

Structure group on $T\mathcal{M}_6$:

η_{mn}	$GL(6)$	$\dashrightarrow$	$O(6)$
$\eta_{mn} \, , \ \varepsilon_{m_1 \cdots m_6}$	$O(6)$	$\dashrightarrow$	$SO(6)$
$\eta_{mn} \, , \ \varepsilon_{m_1 \cdots m_6} \, , \ J_{mn}$	$SO(6)$	$\dashrightarrow$	$U(3)$
$\eta_{mn} \, , \ \varepsilon_{m_1 \cdots m_6} \, , \ J_{mn} \, , \ \Omega_{mnp}$	$U(3)$	$\dashrightarrow$	$SU(3)$

Connection between geometry and physics :

$$J_{mn} \ = \ \mp 2 \mathrm{i} \, \eta_{\pm}^{\dagger} \, \gamma_{mn} \, \eta_{\pm} \, , \qquad \Omega_{mnp} \ = \ -2 \mathrm{i} \, \eta_{-}^{\dagger} \, \gamma_{mnp} \, \eta_{+}$$

a generalized almost complex structure ${\mathcal{J}_\Pi}^\Sigma$ on $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$ s.t.

$$
\begin{aligned}
{\mathcal{J}_\Pi}^\Sigma : \ T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \ &\longrightarrow \ T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \\
\mathcal{J}^2 \ &= \ -\mathbb{1}_{12} \\
{}^\exists \ O(6,6) \text{ invariant metric } L, \text{ s.t. } \ \mathcal{J}^{\mathrm{T}} L \mathcal{J} \ &= \ L
\end{aligned}
$$

Structure group on $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$:

L	$GL(12)$	$\dashrightarrow$	$O(6,6)$
$\mathcal{J}^2 = -\mathbb{1}_{12}$	$O(6,6)$	$\dashrightarrow$	$U(3,3)$
$\mathcal{J}_1$, $\mathcal{J}_2$	$U_1(3,3) \cap U_2(3,3)$	$\dashrightarrow$	$U(3) \times U(3)$
integrable $\mathcal{J}_{1,2}$	$U(3) \times U(3)$	$\dashrightarrow$	$SU(3) \times SU(3)$

$$
\mathcal{J}_{\pm \Pi\Sigma} \ = \ \big\langle \mathrm{Re}\Phi_\pm, \Gamma_{\Pi\Sigma} \mathrm{Re}\Phi_\pm \big\rangle
$$

$$
\Phi_+ \ = \ \frac{1}{4} \sum_{k=0}^{6} \frac{1}{k!} \eta_+^\dagger \gamma_{m_k \cdots m_1} \eta_+ \gamma^{m_1 \cdots m_k} \ \sim \ \mathrm{e}^{-\mathrm{i}J} \, , \qquad \Phi_- \ = \ \frac{1}{4} \sum_{k=0}^{6} \frac{1}{k!} \eta_-^\dagger \gamma_{m_k \cdots m_1} \eta_+ \gamma^{m_1 \cdots m_k} \ \sim \ -\mathrm{i}\Omega
$$

generalized geometry $SU(3) \times SU(3)$ を $E_{7(7)}$ 対称性に拡大：

$T\mathcal{M}_6$	$\oplus$	$T^*\mathcal{M}_6$
momentum		winding
(GCT)		(B_2 gauge)
$\mathbf{6}$		$\mathbf{6}$

↓

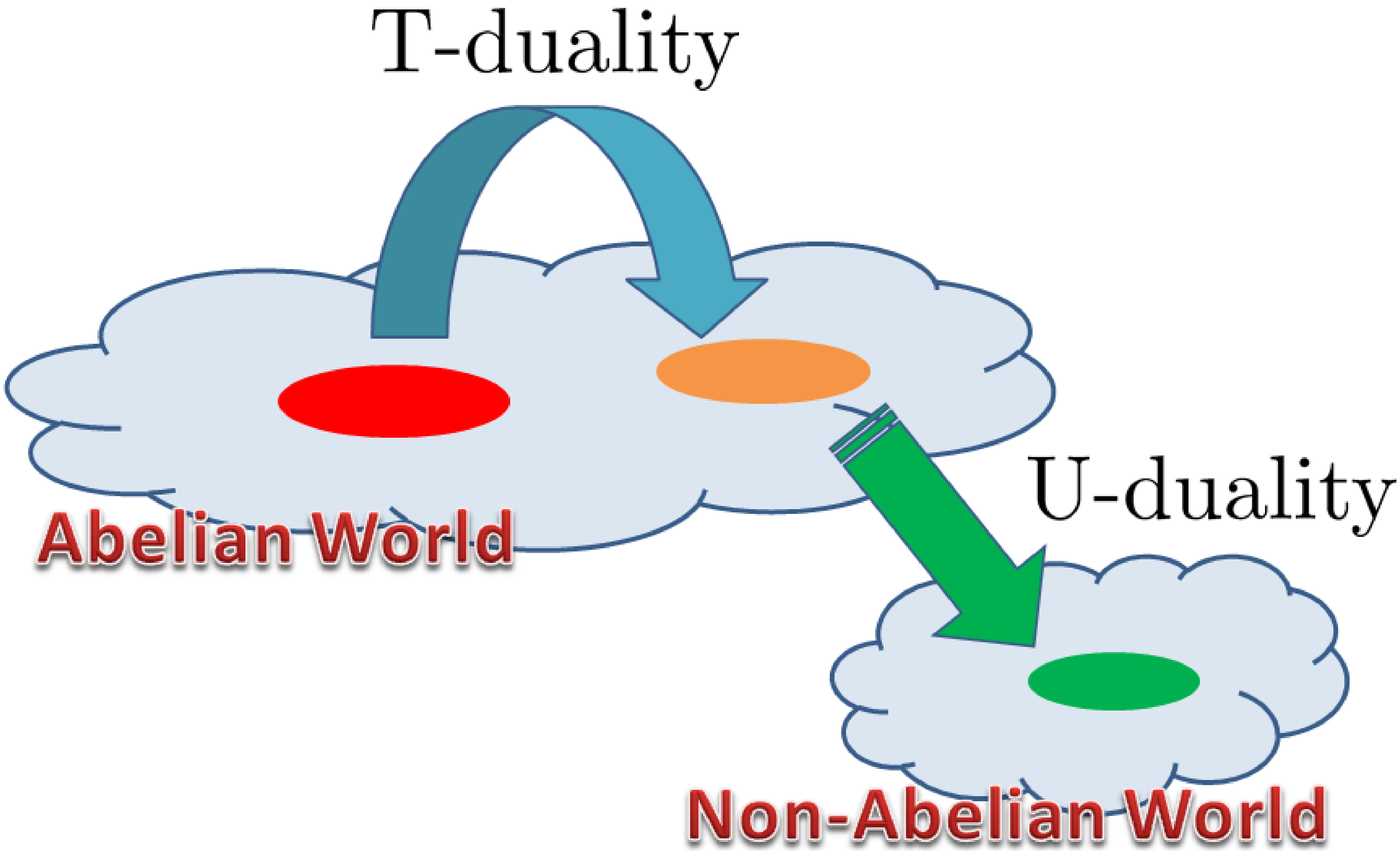
$T\mathcal{M}_6$	$\oplus$	$T^*\mathcal{M}_6$	$\oplus$	$\Lambda^5 T^*\mathcal{M}_6$	$\oplus$	$\Lambda^5 T\mathcal{M}_6$	$\oplus$	$\Lambda^{\text{even}} T^*\mathcal{M}_6$
momentum		winding		NS5		KK5		D0, D2, D4, D6
(GCT)		(B_2 gauge)		(B_6 gauge)		(GCT of dual vielbein)		($C_{(p)}, C_{(8-p)}$ gauges)
$\mathbf{6}$		$\mathbf{6}$		$\mathbf{6}$		$\mathbf{6}$		$\mathbf{1} + \mathbf{15} + \mathbf{15} + \mathbf{1}$

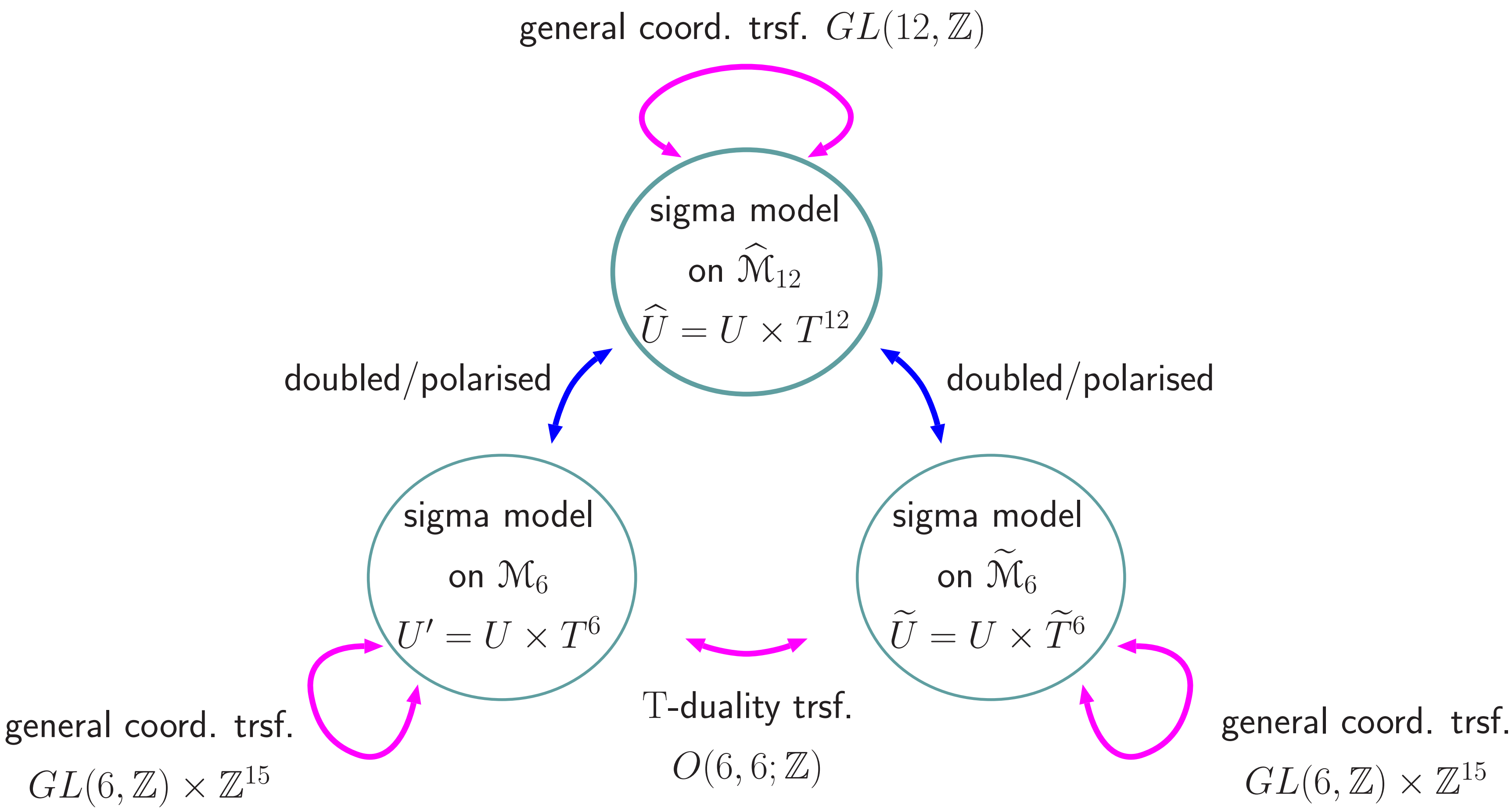
$\mathrm{d}B_6 = *_{10}\mathrm{d}B_2$; $\mathrm{d}C_{(8-p)} = *_{10}\mathrm{d}C_{(p)}$ $(p = 1, 3)$ in type IIA

$$\mathbf{6} + \mathbf{6} = \mathbf{12} : \quad O(6,6) \text{ の基本表現}$$

$$\mathbf{6} + \mathbf{6} + \mathbf{6} + \mathbf{6} + (\mathbf{1} + \mathbf{15} + \mathbf{15} + \mathbf{1}) = \mathbf{56} : \quad E_{7(7)} \text{ の基本表現}$$

hep-th/0701203, arXiv:0904.2333, arXiv:1007.5509, arXiv:1202.0770, etc. jump


T-duality
U-duality
Abelian World
Non-Abelian World



generalized geometry とは異なり、geometry 自身を「倍に広げる」: $\mathcal{M}_6 \to \widehat{\mathcal{M}}_{12}$

Sigma model action on a doubled space $\widehat{\mathcal{M}}_{12}$ with scalar moduli matrix $\mathcal{M}_{IJ}$:

$$S = \int_{\Sigma} \frac{1}{4} \mathcal{M}_{MN} \, d\mathbb{Y}^M \wedge * d\mathbb{Y}^N$$

$$\mathcal{M}_{MN} = \begin{pmatrix} g_{mn} - B_{mp} \, g^{pq} \, B_{qn} & B_{mp} \, g^{pn} \\ -g^{mp} \, B_{qn} & g^{mn} \end{pmatrix} \qquad \mathbb{Y}^M = \begin{pmatrix} Y^m \\ \widetilde{Y}_m \end{pmatrix}$$

$\mathcal{M}_{MN}$ takes value in coset $\dfrac{O(6,6)}{O(6) \times O(6)}$

- $O(6,6)$ global symmetry by $g \in O(6,6)$: $\mathbb{Y}^M \ \to \ \mathbb{Y}'^M = g^M{}_N \mathbb{Y}^N$
  - fractional transformation on $M_{mn} = g_{mn} + B_{mn}$:

$$g = \begin{pmatrix} A & \beta \\ \Theta & A^{-\mathrm{T}} \end{pmatrix} : \qquad M \to \big(A^{-\mathrm{T}} M + \Theta\big) \big(\beta M + A\big)^{-1}$$

Θ : B場のゲージ変換 $B \to B + \Theta \ (\Theta = d\theta)$

A : g_{mn} の一般座標変換

β : g_{mn} と B_{mn} が混ざる ←-- T-duality

generalized geometry でも、generalized vector $V = (v, \xi)^{\mathrm{T}}$ について上と同様の変換が内蔵される $\qquad (v \in T\mathcal{M}_6 \, , \ \xi \in T^*\mathcal{M}_6)$

compact spaces $\mathcal{M}_6, \widehat{\mathcal{M}}_{12}$ を開いたものが double field theory

低次元超重力理論

そもそも、低次元超重力理論はどれだけ構成できるのか？

ungauged SUGRA $\rightarrow$ gauged SUGRA

そもそも低次元超重力理論はどれだけ構成できるのか？

B. de Wit

H. Samtleben

M. Trigiante

and many guys..

Embedding Tensor Formalism from hep-th/0212239

ゲージ対称性 : $X_M = \Theta_M{}^{\alpha} t_\alpha$: 大域的対称性

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - g A_\mu^M X_M$$

# SUSY	9D	8D	7D	6D	5D	4D	3D
32	arXiv:1105.1760	arXiv:1110.2886	hep-th/0506237	arXiv:0712.4277	hep-th/0412173	arXiv:0705.2101	hep-th/0103032
16						hep-th/0602024	arXiv:0806.2584
8						arXiv:1107.3305	(arXiv:0807.2841)

スカラー場が住む空間の isometry group G の部分群 G_0 を使って

理論に元々内蔵されているベクトル場 A_μ^M を非可換ゲージ場に格上げ：

$$\text{(ゲージ対称性 } G_0) \quad X_M = \Theta_M{}^\alpha t_\alpha \quad \text{(大域的対称性 } G)$$

$$[X_M, X_N] = -T_{MN}{}^P X_P \qquad (T_{MN}{}^P \equiv \Theta_M{}^\alpha (t_\alpha)_N{}^P)$$

$$\delta A_\mu^M = \partial_\mu \Lambda^M + \underline{g A_\mu^P T_{PQ}{}^M \Lambda^Q} \equiv D_\mu \Lambda^M$$

full covariance を実現するためにテンソル補助場を導入：

$$\mathcal{F}_{\mu\nu} \equiv -g^{-1}[D_\mu, D_\nu]^M = \partial_\mu A_\nu^M - \partial_\nu A_\mu^M + g T_{[NP]}{}^M A_\mu^N A_\nu^P$$

$$\delta \mathcal{F}_{\mu\nu}^M = g \Lambda^P T_{NP}{}^M \mathcal{F}_{\mu\nu}^M - 2g T_{(PQ)}{}^M A_{[\mu}^P \delta A_{\nu]}^Q \qquad \text{not covariant!}$$

$$\downarrow$$

$$\delta A_\mu^M = D_\mu \Lambda^M \underline{- g T_{(PQ)}{}^M \Xi_\mu^{(PQ)}}, \qquad \mathcal{H}_{\mu\nu}^M \equiv \mathcal{F}_{\mu\nu}{}^M + g T_{(PQ)}{}^M B_{\mu\nu}^{(PQ)}$$

期待

Embedding tensor $\Theta_M{}^\alpha$ が

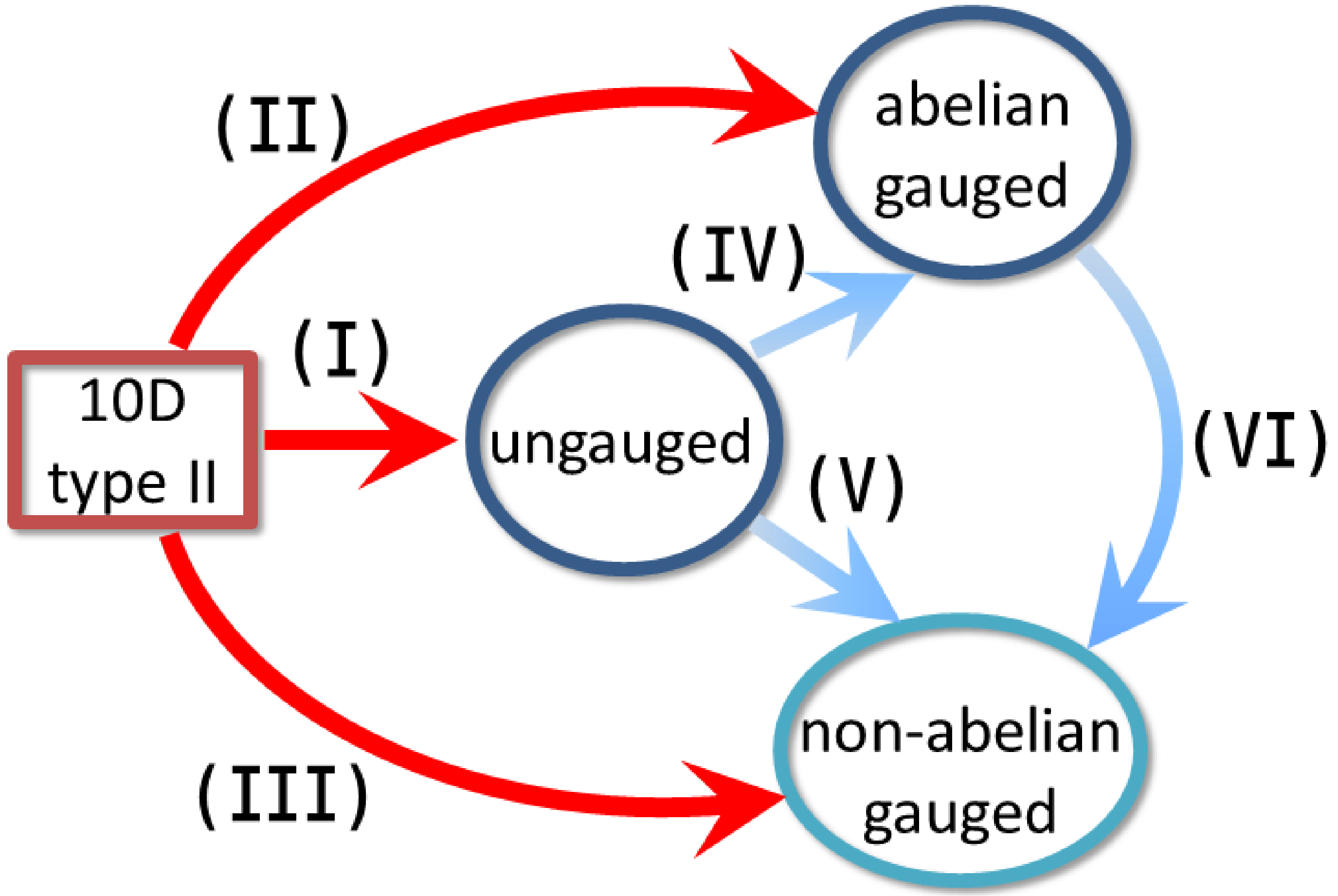
Exceptional generalized geometry の $E_{7(7)}$ の表現を適切に分解する

➪ Flux compactifications の (non)geometric fluxes と一致する (?)

➪ Non-abelianity の幾何学起源を教えてくれる (!?)

►► Heterotic string compactifications との双対の理解が進む

真空解やブラックホール解などの分類に対して多方面から解析する手段を与える



(I)	Calabi-Yau
(II)	$SU(3)$-structure manifolds / generalized geometries
(III)	doubled formalism / exceptional generalized geometries
(IV),(V),(VI)	embedding tensor formalism

課題

大きな課題「non-abelianity の幾何学起源を追究」するためのデータ収集

まず、実際に計算できるものは？

- フラックスコンパクト化で登場する超重力理論の非自明な解を模索

漸近的に AdS な時空におけるブラックホール解 arXiv:1108.1113, arXiv:1203.1544

$$F_{2}^{\Lambda} \;=\; \mathrm{d}A_{1}^{\Lambda} + m^{\Lambda} B_{2}$$

- Embedding tensor formalism におけるテンソル補助場の役割

electric frame から magnetic frame に双対変換すると

補助場から力学的場になる（？） (work in progress??)

$$B_{\mu\nu}^{(PQ)} \text{ in } \begin{cases} \text{vector multiplets} & \longrightarrow \quad \text{vector-tensor multiplets?} \\ \text{hypermultiplets} & \longrightarrow \quad \text{hyper-tensor multiplets?} \end{cases}$$

arXiv:1107.3305

hep-th/9710212

hep-th/0606148

AdSブラックホール

in 4D $\mathcal{N} = 2$ gauged SUGRA with B-field

from massive type IIA on a nearly-Kähler coset

TK, arXiv:1108.1113, arXiv:1203.1544

Reissner-Nordström AdS ブラックホール解を考えたい

理由

- フラックスコンパクト化での重力解の理解
- ハイパー多重項がある $\mathcal{N} = 2$ gauged SUGRA の探索
- 「AdS/CMT 対応」の設定を小耳に挟む：

荷電粒子が AdS ブラックホールに落ちていく過程をみる

Einstein + $\Lambda_{\text{c.c.}}$ + Maxwell + charged matters

これをストリング理論の枠内で見ることはできるのか？

設定を超重力理論に埋め込むときの障害

- 宇宙項 vs 物質場 vs 超対称性 (4D)

  物質場がある系で負の宇宙「定数」をそのまま超対称作用積分に導入することはできない

- 宇宙項 vs コンパクト化 (10D)

  負の宇宙項を持つ真空解を与えるには Romans' mass が 必要 (type IIA)

- Maxwell 場 vs フラックス (4D)

  field strength は Romans' mass のために変形

AdS/CMT の起源を type IIA ストリング理論に求めないならば、
上記の障害は気にならない (?)

D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

- ✓ NSNS-sector : torsion and H-flux
- ✓ RR-sector : 2-, 4-form and Romans' mass (0-form)

$$\mathrm{d}J = \frac{3}{2}\,\mathrm{Im}(\overline{\mathcal{W}}_1 \Omega)\,, \quad \mathrm{d}\Omega = \mathcal{W}_1\, J \wedge J$$

$$\mathrm{d}\omega_\Lambda = e_\Lambda\, \alpha\,, \quad \mathrm{d}\alpha = 0\,, \quad \mathrm{d}\beta = e_\Lambda\, \widetilde{\omega}^\Lambda\,, \quad \mathrm{d}\widetilde{\omega}^\Lambda = 0$$

jump

$$e_\Lambda m_{\mathsf{R}}^\Lambda = 0$$

- ✓ 1 vector multiplet with cubic prepotential $\mathcal{F} = \dfrac{X^1 X^1 X^1}{X^0}$
- ✓ 1 universal hypermultiplet (no other HMs) → hyper-tensor multiplet

---

4D multiplets	from NSNS	from RR	fermions
supergravity multiplet	$g_{\mu\nu}$	A_μ^0	ψ_μ^i
1 vector multiplet	$\mathfrak{t}$	A_μ^1	λ_i
1 hyper-tensor multiplet	φ, $B_{\mu\nu}$	ξ, $\widetilde{\xi}$	ζ^α

A_μ^0 : from RR one-form C_1

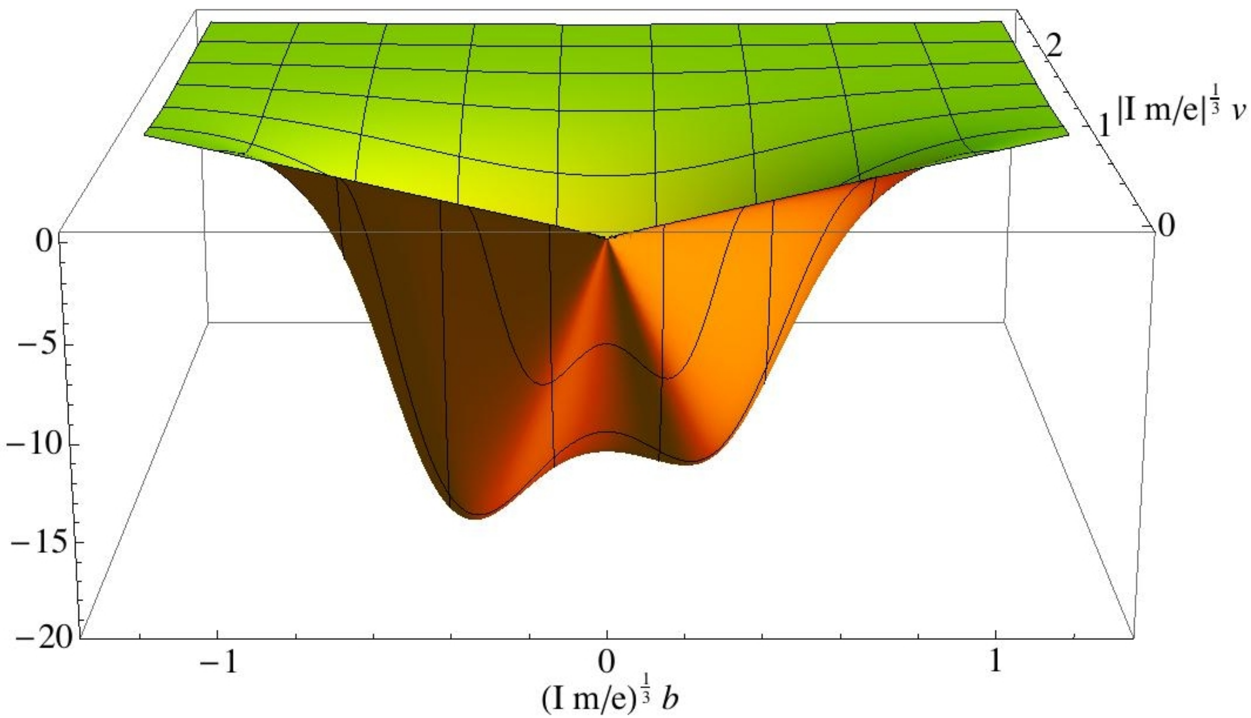
$\mathfrak{t}$: from complexified "Kähler" modulus $\mathfrak{t} = X^1/X^0 = b + \mathrm{i}v$

A_μ^1 : from RR three-form C_3

ξ, $\widetilde{\xi}$: from RR three-form C_3

jump

$$F_2^\Lambda = \mathrm{d}A_1^\Lambda + m_{\mathsf{R}}^\Lambda B_2$$: Stückelberg-type deformation



in arXiv:0901.4251

ひとつの $\mathcal{N}=1$ AdS 真空　と　ふたつの $\mathcal{N}=0$ AdS 真空　jump

$$\Lambda_{\text{c.c.}}^{\mathcal{N}=0} < \Lambda_{\text{c.c.}}^{\mathcal{N}=1} < \Lambda_{\text{c.c.}}^{\mathcal{N}=0}$$

[NOTE] type IIA 弦理論から 4D $\mathcal{N}=1$ AdS vacua を導出するには Romans' mass が必要

D. Lüst and D. Tsimpis, hep-th/0412250

Reissner-Nordström AdS ブラックホールの下、物質場の配位を調べる：

$$\mathrm{d}s^2 \;=\; -\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$\mathrm{e}^{2A(r)} \;=\; 1 - \frac{2\eta}{r} + \frac{Z^2}{r^2} + \frac{r^2}{\ell^2}\,, \quad Z^2 \;=\; Q^2 + P^2\,, \quad \Lambda_{\text{c.c.}} \;=\; -\frac{3}{\ell^2}$$

vector fields A_μ^Λ の電荷・磁荷： $p^\Lambda \;=\; \int F_2^\Lambda\,, \quad q_\Lambda \;=\; \int \widetilde{F}_{2\Lambda}$

$$\downarrow$$

$$F_{\theta\phi}^\Lambda \;\equiv\; f^\Lambda(\theta,\phi)\sin\theta\,, \quad F_{tr}^\Lambda \;\equiv\; \frac{\mathrm{e}^{-2C(r)}}{r^2}g^\Lambda(\theta,\phi)$$

時間依存性がない解を探そうとすると、

A_μ^Λ, $B_{\mu\nu}$, $g_{\mu\nu}$ の運動方程式から「すべての場の共変定数性」が示される：

$$0 \;=\; \partial_\mu \mathfrak{t} \;=\partial_\mu\varphi \;=\; D_\mu\xi \;=\; D_\mu\widetilde{\xi} \;=\; \partial_{[\mu}B_{\nu\rho]} \;=\; F_{\mu\nu}^\Lambda$$

結果

Massive type IIA on $G_2/SU(3)$ から得られる

4D $\mathcal{N}=2$ gauged SUGRA with B-field は、

Reissner-Nordström AdS ブラックホール解を持ち得ない ($Z^2=0$ に退化)。

ブラックホール質量・電荷・宇宙項などをコンパクト化の情報で記述するために、
もう少し非自明なブラックホール解を考える必要がある？

(そろそろ違うテーマをやりたい...)

おわりに

以下の流れでお話をしました

- 動機

  超重力理論の高次元起源

- 疑問

  type II 弦と non-abelian gauge symmetries

- 幾何の拡張

  generalized geometry, doubled formalism

- 低次元超重力理論

  embedding tensor formalism によるゲージ化

- 期待

  embedding tensor がコンパクト化で果たす役割

- 課題

  例) AdS-BH 解, embedding tensor とテンソル補助場

Embedding tensor formalism in 4D local $\mathcal{N} = 2$ [arXiv:1107.3305] は

$$\text{Conformal supergravity with} \begin{cases} \text{off-shell Weyl multiplet} \\ \text{off-shell vector multiplets} \\ \text{on-shell hypermultiplets} \end{cases}$$

具体的記述

で記述されている (他は Poincaré supergravity)

勉強ノートは適宜更新中

身につけるべき技術

Lie群の表現の分解や積 は、計算機プログラム

LiE: a computer algebra package for Lie group computations

http://www-math.univ-poitiers.fr/~maavl/LiE/

が使われているが、自分には敷居が高い (どうやって使うの?)

cf. R. Slansky, Phys. Rept. 79 (1981) 1

$E_{7(7)}$	$\mathbf{56} \times \mathbf{133}$	$=$	$\mathbf{56} + \mathbf{912} + \mathbf{6480}$
	$(\mathbf{912} \times \mathbf{912})_{\rm S}$	$=$	$\mathbf{133} + \mathbf{8645} + \mathbf{1463} + \mathbf{152152} + \mathbf{253935}$
	$\mathbf{56} \times \mathbf{912}$	$=$	$\mathbf{133} + \mathbf{8645} + \mathbf{1539} + \mathbf{40755}$
$E_{7(7)} \to SU(8)$	$\mathbf{56}$	$=$	$\mathbf{28} + \overline{\mathbf{28}}$
	$\mathbf{133}$	$=$	$\mathbf{63} + \mathbf{70}$
	$\mathbf{912}$	$=$	$\mathbf{36} + 420 + \overline{\mathbf{36}} + \overline{\mathbf{420}}$
	$\mathbf{8645}$	$=$	$\mathbf{64} + \mathbf{378} + \overline{\mathbf{378}} + \mathbf{945} + \overline{\mathbf{945}} + \mathbf{2352} + \mathbf{3584}$
$E_{7(7)} \to SL(2) \times SO(6,6)$	$\mathbf{56}$	$=$	$(\mathbf{2},\mathbf{12}) + (\mathbf{1},\mathbf{32})$
	$\mathbf{133}$	$=$	$(\mathbf{1},\mathbf{66}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{32'})$
	$\mathbf{912}$	$=$	$(\mathbf{2},\mathbf{220}) + (\mathbf{2},\mathbf{12}) + (\mathbf{1},\mathbf{352'}) + (\mathbf{3},\mathbf{32})$
	$\mathbf{8645}$	$=$	$(\mathbf{1},\mathbf{66}) + (\mathbf{1},\mathbf{2079}) + (\mathbf{3},\mathbf{66}) + (\mathbf{3},\mathbf{495}) + (\mathbf{3},\mathbf{1}) + (\mathbf{4},\mathbf{462'}) + (\mathbf{2},\mathbf{32}) + (\mathbf{2},\mathbf{352}) + (\mathbf{2},\mathbf{1728'}) + (\mathbf{4},\mathbf{32'})$

Appendix

10D type IIA action $S^{(10D)}_{\rm IIA} = S_{\sf NS} + S_{\sf R} + S_{\sf CS}$:

$$\begin{aligned} S_{\sf NS} &= \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{ \widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3 \Big\} \\ S_{\sf R} + S_{\sf CS} &= -\frac{1}{4}\int \Big\{ \widehat{F}_2 \wedge *\widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \wedge *(\widehat{F}_4 - \widehat{C}_1 \wedge \widehat{F}_3) \Big\} - \frac{1}{4}\int \widehat{B}_2 \wedge \widehat{F}_4 \wedge \widehat{F}_4 \end{aligned}$$

$\downarrow$

4D $\mathcal{N}=2$ ungauged SUGRA: Neither gauge couplings, Nor scalar potential

$$S^{(4D)} = \int \Big\{ \frac{1}{2} R * \mathbb{1} - G_{a\bar{b}}\, \mathrm{d}\mathfrak{t}^a \wedge *\mathrm{d}\bar{\mathfrak{t}}^{\bar{b}} - h_{uv}\, \mathrm{d}q^u \wedge *\mathrm{d}q^v + \frac{1}{2}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F_2^\Lambda \wedge *F_2^\Sigma + \frac{1}{2}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F_2^\Lambda \wedge F_2^\Sigma \Big\}$$

gravitational multiplet	$g_{\mu\nu}\,,\ A_1^0$		
vector multiplet (VM)	$A_1^a\,,\ \mathfrak{t}^a\,,\ \bar{\mathfrak{t}}^{\bar{b}}$	$\mathfrak{t}^a \in \mathsf{SKG_V}$	
hypermultiplet (HM)	$z^i\,,\ \bar{z}^{\bar{\jmath}}\,,\ \xi^i\,,\ \widetilde{\xi}_j$	$z^i \in \mathsf{SKG_H}$	
universal hypermultiplet (UHM)	$\varphi\,,\ a\,,\ \xi^0\,,\ \widetilde{\xi}_0$	$a \leftrightarrow B_2$	(Hodge dual)

$\mathcal{HM}$ = Special QG

$$\{q^u\} = \{z^i, \bar{z}^{\bar{\jmath}}\} + \{\xi^i, \widetilde{\xi}_j\} + \{\varphi, a, \xi^0, \widetilde{\xi}_0\} = \{z^i, \bar{z}^{\bar{\jmath}}\} + \{\varphi\} + \{a, \xi^I, \widetilde{\xi}_J\}$$

$4n_{\sf H} + 4$ = $2n_{\sf H}(\mathsf{SKG_H})$ + $2n_{\sf H}$ + 4 (UHM) = $\mathsf{SKG_H}$ + … + "Heisenberg"

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$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}(\overline{\mathcal{W}}_1\Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

complex	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	Calabi-Yau	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\mathrm{Im}\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

jump App.top

閉形式でない $(\mathrm{d}J, \mathrm{d}\Omega)$ を、基底形式の外微分の性質に翻訳する：

NS-NS

$$\mathrm{d}\begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix} \sim \begin{pmatrix} e_\Lambda{}^I & m^{\Lambda I} \\ e_{\Lambda I} & m^\Lambda{}_I \end{pmatrix} \begin{pmatrix} \widetilde{\omega}^\Lambda \\ \omega_\Lambda \end{pmatrix}$$

Σ_- $\quad Q^{\mathrm{T}}$ $\quad \Sigma_+$

$e_0{}^I, e_{0I}$:	H-flux charges $(H^{\mathrm{fl}} = -e_0{}^I \alpha_I + e_{0I} \beta^I)$
$e_a{}^I, e_{aI}$:	geometric flux charges (トーション)
$m^{\Lambda I}, m^\Lambda{}_I$:	nongeometric flux charges ($e_\Lambda{}^I, e_{\Lambda I}$ の "磁気的" 双対)

R-R

$$\widehat{\mathbf{F}} \equiv \widehat{F}_0 + \widehat{F}_2 + \ldots + \widehat{F}_{10} \equiv \mathrm{e}^{\widehat{B}} \widehat{\mathbf{G}}$$

(自己双対条件 $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})\,, \quad \lambda(\widehat{F}_{(k)}) \equiv (-)^{[\frac{k+1}{2}]} \widehat{F}_{(k)}$)

$$\begin{aligned} \frac{1}{\sqrt{2}} \widehat{\mathbf{G}} \;&=\; (G_0^\Lambda + G_2^\Lambda + G_4^\Lambda)\, \omega_\Lambda - (\widetilde{G}_{0\Lambda} + \widetilde{G}_{2\Lambda} + \widetilde{G}_{4\Lambda})\, \widetilde{\omega}^\Lambda \\ &\quad + (G_1^I + G_3^I)\, \alpha_I - (\widetilde{G}_{1I} + \widetilde{G}_{3I})\, \beta^I \end{aligned}$$

$$G_0^\Lambda \equiv p^\Lambda\,, \qquad \widetilde{G}_{0\Lambda} \equiv q_\Lambda - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_\Lambda{}^I$$

$c \equiv (p^\Lambda, q_\Lambda)^{\mathrm{T}}$: R-R flux charges

(p^0: Romans' mass)

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10D type IIA action $S^{(10D)}_{\rm IIA} = S_{\rm NS} + \widetilde{S}_{\rm R} = S_{\rm NS} + S_{\rm R} + S_{\rm CS}$: (democratic form)

$$S_{\rm NS} \;=\; \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\}\,, \qquad \widetilde{S}_{\rm R} \;=\; -\frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$

with "constraint $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})$" and "EoM (Bianchi) $(\mathrm{d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow (\mathrm{d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

↓ non-CY with $SU(3)$-structure with $m^{\Lambda}_{\rm R} = 0$

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^{\boldsymbol{I}} \equiv (\xi^I, \widetilde{\xi}_I)^{\rm T}$):

$$S^{(4D)} = \int \mathrm{d}^4x\sqrt{-g}\,\Big[\frac{1}{2}R + \frac{1}{4}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_\mu \mathfrak{t}^a\partial^\mu\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{j}}\,\partial_\mu z^i\partial^\mu\bar{z}^{\bar{j}}$$
$$-\partial_\mu\varphi\partial^\mu\varphi + \frac{\mathrm{e}^{2\varphi}}{2}(\mathbb{M}_{\rm H})_{\boldsymbol{IJ}}D_\mu\xi^{\boldsymbol{I}}D^\mu\xi^{\boldsymbol{J}} - \frac{\mathrm{e}^{2\varphi}}{4}\big(D_\mu a - \xi^{\boldsymbol{I}}(\mathbb{C}_{\rm H})_{\boldsymbol{IJ}}D_\mu\xi^{\boldsymbol{J}}\big)^2 - V(\mathfrak{t},\bar{\mathfrak{t}},q)\Big]$$

- $({e_\Lambda}^I, e_{\Lambda I})$: geometric flux charges & $e_{{\rm R}\Lambda}$: RR-flux charges ⟵ non-CY data
  (with constraints ${e_\Lambda}^I e_{\Sigma I} - e_{\Lambda I}{e_\Sigma}^I = 0$)
- $\mathfrak{t}^a \in {\rm SKG}_{\rm V}$ and $z^i \in {\rm SKG}_{\rm H} \subset \mathcal{HM}$ are ungauged (in general)
- $D_\mu\xi^I = \partial_\mu\xi^I - {e_\Lambda}^I A^\Lambda_\mu$ & $D_\mu\widetilde{\xi}_I = \partial_\mu\widetilde{\xi}_I - e_{\Lambda I}A^\Lambda_\mu$
- $D_\mu a = \partial_\mu a - (2e_{{\rm R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I{e_\Lambda}^I)A^\Lambda_\mu$
- $V(\mathfrak{t},\bar{\mathfrak{t}},q)$: scalar potential

D. Cassani, arXiv:0804.0595

Non-vanishing m_{R}^{Λ} dualizes the axion field a in standard SUGRA to B-field.

4D gauged action is different from the standard one:

$$\begin{aligned} S^{(\mathsf{4D})} \ &= \int \Big[\frac{1}{2} R(*\mathbb{1}) + \frac{1}{2} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F_2^{\Lambda} \wedge *F_2^{\Sigma} + \frac{1}{2} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F_2^{\Lambda} \wedge F_2^{\Sigma} - g_{a\bar{b}} \, \mathrm{d}\mathfrak{t}^a \wedge *\mathrm{d}\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}} \, \mathrm{d}z^i \wedge *\mathrm{d}\bar{z}^{\bar{\jmath}} \\ &\qquad - \mathrm{d}\varphi \wedge *\mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} H_3 \wedge *H_3 - \frac{\mathrm{e}^{2\varphi}}{2} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{I}} \wedge *D\xi^{\boldsymbol{J}} - V(*\mathbb{1}) \\ &\qquad + \frac{1}{2} \mathrm{d}B \wedge \Big[\xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{J}} + \big(2e_{\mathsf{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_{\Lambda}{}^I \big) A_1^{\Lambda} \Big] - \frac{1}{2} m_{\mathsf{R}}^{\Lambda} e_{\mathsf{R}\Lambda} B_2 \wedge B_2 \Big] \end{aligned}$$

Constraints among flux charges:

$$e_{\Lambda}{}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^I \ = \ 0, \qquad m_{\mathsf{R}}^{\Lambda} e_{\Lambda}{}^I \ = \ 0 \ = \ m_{\mathsf{R}}^{\Lambda} e_{\Lambda I}$$

Scalar potential from (non)geometric flux compactifications:

$$V = \mathrm{g}^2\Big[4h_{uv}k^u\overline{k}^v + \sum_{x=1}^{3}\Big(g^{a\overline{b}}D_a\mathcal{P}_x D_{\overline{b}}\overline{\mathcal{P}}_x - 3|\mathcal{P}_x|^2\Big)\Big] = \ldots \equiv V_{\mathsf{NS}} + V_{\mathsf{R}} \qquad (\text{abelian: } k^a_\Lambda = 0)$$

$$V_{\mathsf{NS}} = g^{a\overline{b}}D_a\mathcal{P}_+D_{\overline{b}}\overline{\mathcal{P}}_+ + g^{i\overline{\jmath}}D_i\mathcal{P}_+D_{\overline{\jmath}}\overline{\mathcal{P}}_+ - 2|\mathcal{P}_+|^2$$

$$= -2\,\mathrm{g}^2\mathrm{e}^{2\varphi}\Big[\overline{\Pi}{}_{\mathsf{H}}^{\mathrm{T}}\,\widetilde{Q}^{\mathrm{T}}\,\mathbb{M}_{\mathsf{V}}\,\widetilde{Q}\,\Pi_{\mathsf{H}} + \overline{\Pi}{}_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{M}_{\mathsf{H}}\,Q^{\mathrm{T}}\,\Pi_{\mathsf{V}} + 4\overline{\Pi}{}_{\mathsf{H}}^{\mathrm{T}}\,\mathbb{C}_{\mathsf{H}}^{\mathrm{T}}\,Q^{\mathrm{T}}\,(\Pi_{\mathsf{V}}\overline{\Pi}{}_{\mathsf{V}}^{\mathrm{T}} + \overline{\Pi}_{\mathsf{V}}\Pi_{\mathsf{V}}^{\mathrm{T}})\,Q\,\mathbb{C}_{\mathsf{H}}\,\Pi_{\mathsf{H}}\Big]$$

$$V_{\mathsf{R}} = g^{a\overline{b}}D_a\mathcal{P}_3D_{\overline{b}}\overline{\mathcal{P}}_3 + |\mathcal{P}_3|^2$$

$$= -\frac{1}{2}\,\mathrm{g}^2\mathrm{e}^{4\varphi}\big(e_{\mathsf{R}\Lambda} - e_{\Lambda I}\xi^I + e_\Lambda{}^I\widetilde{\xi}_I\big)(\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma}\big(e_{\mathsf{R}\Sigma} - e_{\Sigma I}\xi^I + e_\Sigma{}^I\widetilde{\xi}_I\big)$$

$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$
$t^a = X^a/X^0$
$a = 1, \ldots, n_{\mathsf{V}}$
SKG$_{\mathsf{V}}$ of vector-moduli

$$\mathcal{P}_+ \equiv \mathcal{P}_1 + \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}\,\Pi_{\mathsf{H}}$$
$$\mathcal{P}_- \equiv \mathcal{P}_1 - \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}\,\overline{\Pi}_{\mathsf{H}}$$
$$\mathcal{P}_3 = \mathrm{e}^{2\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,\mathbb{C}_{\mathsf{V}}(c_{\mathsf{R}} + \widetilde{Q}\xi)$$

$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$
$z^i = Z^i/Z^0$
$i = 1, \ldots, n_{\mathsf{H}}$
SKG$_{\mathsf{H}}$ of hyper-moduli

$$\mathbb{C}_{\mathsf{V,H}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}; \quad Q = \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix}, \quad \widetilde{Q} = \mathbb{C}_{\mathsf{H}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{V}} \quad c_{\mathsf{R}} = \begin{pmatrix} m_{\mathsf{R}}^\Lambda \\ e_{\mathsf{R}\Lambda} \end{pmatrix}$$

Cassani et.al., arXiv:0804.0595, arXiv:0911.2708 App.top

D. Cassani and A.K. Kashani-Poor, arXiv:0901.4251 jump App.top

$\mathcal{M}_6$	$\frac{G_2}{SU(3)} = S^6$	$\frac{Sp(2)}{S(U(2)\times U(1))} = \mathbb{C}P^3$	$\frac{SU(3)}{U(1)\times U(1)} = \mathbb{F}(1,2;3)$
$\mathcal{SM} = \mathsf{SKG_V}$	$\frac{SU(1,1)}{U(1)}$: $\mathfrak{t}^3$	$\left(\frac{SU(1,1)}{U(1)}\right)^2$: $\mathfrak{st}^2$	$\left(\frac{SU(1,1)}{U(1)}\right)^3$: $\mathfrak{stu}$
$\mathcal{HM} = \mathsf{SQG}$	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM	$\frac{SU(2,1)}{U(2)}$: UHM
$\mathsf{SKG_H} \subset \mathcal{HM}$	—	—	—
matters	1 VM + 1 UHM	2 VM + 1 UHM	3 VM + 1 UHM

Each $\mathsf{SKG_V}$ has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello, hep-th/0609124

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis, arXiv:0804.0614

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello, arXiv:0904.3959

10D type IIA on $\dfrac{G_2}{SU(3)}$ with fluxes

$\downarrow$

4D $\mathcal{N}=2$ abelian gauged SUGRA with B-field $(\Lambda = 0,1$ and $\xi^{\mathbf{0}} \equiv (\xi^0, \widetilde{\xi}_0)^{\mathrm{T}})$

$$
\begin{aligned}
S \; = \int \Big[& \frac{1}{2} R\,(*1) + \frac{1}{2} \mu_{\Lambda\Sigma}\, F^{\Lambda} \wedge *F^{\Sigma} + \frac{1}{2} \nu_{\Lambda\Sigma}\, F^{\Lambda} \wedge F^{\Sigma} - g_{\mathfrak{t}\bar{\mathfrak{t}}}\, \mathrm{d}\mathfrak{t} \wedge *\mathrm{d}\bar{\mathfrak{t}} \\
& - \mathrm{d}\varphi \wedge *\mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} \mathrm{d}B \wedge *\mathrm{d}B - \frac{\mathrm{e}^{2\varphi}}{2} \Big(D\xi^0 \wedge *D\xi^0 + D\widetilde{\xi}_0 \wedge *D\widetilde{\xi}_0 \Big) + \mathrm{d}B \wedge \xi^0\, \mathrm{d}\widetilde{\xi}_0 \\
& + \mathrm{d}B \wedge \big(e_{\mathrm{R}\Lambda} - e_{\Lambda 0}\, \xi^0 \big) A^{\Lambda} - \frac{1}{2} m_{\mathrm{R}}^{\Lambda}\, e_{\mathrm{R}\Lambda}\, B \wedge B - V\,(*1) \Big]
\end{aligned}
$$

- $g_{\mu\nu}$, $\mathfrak{t}$, $B_{\mu\nu}$, φ; $(e_{\Lambda}{}^0, e_{\Lambda 0})$: NS-NS sector
- A_{μ}^{Λ}, ξ^0, $\widetilde{\xi}_0$; $(m_{\mathrm{R}}^{\Lambda}, e_{\mathrm{R}\Lambda})$: R-R sector
- GM : $(g_{\mu\nu}, A_{\mu}^0)$, VM : $(A_{\mu}^a, \mathfrak{t})$, UHM $\rightarrow$ TM : $(\varphi, B_{\mu\nu}, \xi^0, \widetilde{\xi}_0)$
- $D\xi^0 = \mathrm{d}\xi^0 - e_{\Lambda}{}^0 A_1^{\Lambda}$, $\quad D\widetilde{\xi}_0 = \mathrm{d}\widetilde{\xi}_0 - e_{\Lambda 0} A_1^{\Lambda}$
- $F_2^{\Sigma} = \mathrm{d}A_1^{\Sigma} + m_{\mathrm{R}}^{\Sigma} B_2$
- $V(\mathfrak{t}, \varphi, \xi^0) = V_{\mathrm{NS}}(\mathfrak{t}, \varphi) + V_{\mathrm{R}}(\mathfrak{t}, \varphi, \xi^0)$

Precise data on $\dfrac{G_2}{SU(3)}$:

$$e_{10} \neq 0,\; m_{\mathrm{R}}^0 \neq 0,\; e_{\mathrm{R}0} \neq 0$$
$$e_{\Lambda}{}^0 = 0 = e_{00}$$
$$m_{\mathrm{R}}^1 = 0 = e_{\mathrm{R}1}$$

$\mu_{\Lambda\Sigma} \equiv \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}$, $\nu_{\Lambda\Sigma} \equiv \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}$

D. Cassani, arXiv:0804.0595

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{1}{4} g_{\mu\nu}\, \mu_{\Lambda\Sigma} F^{\Lambda}_{\rho\sigma} F^{\Sigma\rho\sigma} - \mu_{\Lambda\Sigma} F^{\Lambda}_{\mu\rho} F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma} - g_{\mu\nu}\, g_{\mathfrak{t}\bar{\mathfrak{t}}} \partial_\rho \mathfrak{t} \partial^\rho \bar{\mathfrak{t}} + 2 g_{\mathfrak{t}\bar{\mathfrak{t}}}\, \partial_\mu \mathfrak{t} \partial_\nu \bar{\mathfrak{t}}$$

$$- g_{\mu\nu}\, \partial_\rho \varphi \partial^\rho \varphi + 2 \partial_\mu \varphi \partial_\nu \varphi - \frac{\mathrm{e}^{-4\varphi}}{24} g_{\mu\nu}\, H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} + \frac{\mathrm{e}^{-4\varphi}}{4} H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma} \qquad (\delta g_{\mu\nu})$$

$$- \frac{\mathrm{e}^{2\varphi}}{2} g_{\mu\nu} \Big(D_\rho \xi^0 D^\rho \xi^0 + D_\rho \widetilde{\xi}_0 D^\rho \widetilde{\xi}_0 \Big) + \mathrm{e}^{2\varphi} \Big(D_\mu \xi^0 D_\nu \xi^0 + D_\mu \widetilde{\xi}_0 D_\nu \widetilde{\xi}_0 \Big) - g_{\mu\nu} V \,,$$

$$0 = \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, \mu_{\Lambda\Sigma} F^{\Sigma\mu\sigma} \Big) - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu \Big(\nu_{\Lambda\Sigma} F^{\Sigma}_{\nu\rho} \Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu B_{\nu\rho} (e_{\mathsf{R}\Lambda} - \xi^0 e_{\Lambda 0}) - \mathrm{e}^{2\varphi} Q_{\Lambda \mathbf{0}} D^\sigma \xi^{\mathbf{0}} \,, \qquad (\delta A^{\Lambda}_{\mu})$$

$$0 = \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g_{\mathfrak{t}\bar{\mathfrak{t}}}\, g^{\mu\nu} \partial_\nu \bar{\mathfrak{t}} \Big) + \frac{1}{4} \partial_{\mathfrak{t}} (\mu_{\Lambda\Sigma}) F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \partial_{\mathfrak{t}} (\nu_{\Lambda\Sigma}) F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} - \partial_{\mathfrak{t}} g_{\mathfrak{t}\bar{\mathfrak{t}}}\, \partial_\mu \mathfrak{t} \partial^\mu \bar{\mathfrak{t}} - \partial_{\mathfrak{t}} V \,, \qquad (\delta \mathfrak{t})$$

$$0 = \frac{2}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \varphi \Big) + \frac{\mathrm{e}^{4\varphi}}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} - \mathrm{e}^{2\varphi} \Big(D_\mu \xi^0 D^\mu \xi^0 + D_\mu \widetilde{\xi}_0 D^\mu \widetilde{\xi}_0 \Big) - \partial_\varphi V \,, \qquad (\delta \varphi)$$

$$0 = \frac{1}{\sqrt{-g}} \partial_\mu \Big(\mathrm{e}^{-4\varphi} \sqrt{-g} H^{\mu\rho\sigma} \Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} \Big[D_\mu \xi^{\mathbf{0}} (\mathbb{C}_{\mathsf{H}})_{\mathbf{00}} D_\nu \xi^{\mathbf{0}} + (e_{\mathsf{R}\Lambda} - \xi^0 e_{\Lambda 0}) F^{\Lambda}_{\mu\nu} \Big] \qquad (\delta B_{\mu\nu})$$

$$+ 2 m^{\Lambda}_{\mathsf{R}} \mu_{\Lambda\Sigma} F^{\Sigma\rho\sigma} - \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} m^{\Lambda}_{\mathsf{R}} \nu_{\Lambda\Sigma} F^{\Sigma}_{\mu\nu} \,,$$

$$0 = - \frac{2}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, \mathrm{e}^{2\varphi} g^{\mu\nu} D_\nu \xi^{\mathbf{0}} \Big) + \frac{\partial V}{\partial \xi^{\mathbf{0}}} - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu B_{\nu\rho} D_\sigma \xi^{\mathbf{0}} (\mathbb{C}_{\mathsf{H}})_{\mathbf{00}} \,. \qquad (\delta \xi^{\mathbf{0}})$$

Vacuum I : $\mathcal{N} = 1$

$$t_* = -\frac{\pm 1 + i\sqrt{15}}{2} \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = -\frac{2}{5} \left[\frac{2\sqrt{3}\, m_R^0 (e_{R0})^2}{5\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) = \frac{4}{3} \left[\frac{\sqrt{5}\, e_{10}}{\sqrt{3}\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{5\sqrt{5}}{2} \left[\frac{5\,(e_{10})^4}{2\sqrt{3}\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{I}} < 0$$

Vacuum II : $\mathcal{N} = 0$

$$t_* = (\pm 1 - i\sqrt{3}) \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = \left[\frac{9\, m_R^0 (e_{R0})^2}{25\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) = \frac{2}{3} \left[\frac{25\, e_{10}}{\sqrt{3}\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{80}{27} \left[\frac{25\,(e_{10})^4}{\sqrt{3}\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{II}} < 0$$

Vacuum III : $\mathcal{N} = 0$

$$t_* = -i \left[\frac{12}{\sqrt{5}\,(e_{10})^2} \left| \frac{e_{R0}}{m_R^0} \right| \right]^{1/3} , \quad \xi_*^0 = 0 , \quad \exp(\varphi_*) = \sqrt{5} \left[\frac{5\, e_{10}}{18\, m_R^0 (e_{R0})^2} \right]^{1/3}$$

$$V_* = -\frac{25\sqrt{5}}{6} \left[\frac{5\,(e_{10})^4}{18\, |m_R^0 (e_{R0})^5|} \right]^{1/3} \equiv \Lambda_{\text{c.c.}}^{\text{III}} < 0$$

Note: $m_R^0 > 0$; $\widetilde{\xi}_0$ is not fixed ; $\Lambda_{\text{c.c.}}^{\text{II}} < \Lambda_{\text{c.c.}}^{\text{I}} < \Lambda_{\text{c.c.}}^{\text{III}}$

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scalar field space : $\mathcal{M} = G/H$

D	maximal SUGRA G	maximal SUGRA H	half-maximal SUGRA G	half-maximal SUGRA H
9	$GL(2)$	$SO(2)$	$GL(1) \times SO(1, 1+n)$	$SO(1+n)$
8	$SL(2) \times SL(3)$	$SO(2) \times SO(3)$	$GL(1) \times SO(2, 2+n)$	$SO(2) \times SO(2+n)$
7	$SL(5)$	$SO(5)$	$GL(1) \times SO(3, 3+n)$	$SO(3) \times SO(3+n)$
6	$SO(5,5)$	$SO(5) \times SO(5)$	$GL(1) \times SO(4, 4+n)$	$SO(4) \times SO(4+n)$
5	$E_{6(6)}$	$USp(8)$	$GL(1) \times SO(5, 5+n)$	$SO(5) \times SO(5+n)$
4	$E_{7(7)}$	$SU(8)$	$SL(2) \times SO(6, 6+n)$	$SO(6) \times SO(6+n)$
	U-duality		S-duality $\times$ T-duality	

see, for instance, arXiv:0808.4076 App.top

Left-invariant one-form $E = g^{-1}\mathrm{d}g$ with $g \in G$:

$$\mathrm{d}E = -E \wedge E$$

or

$$\mathrm{d}E^A = -\frac{1}{2} f_{BC}{}^A \, E^B \wedge E^C \quad \text{with } E = E^A T_A \,, \quad [T_A, T_B] = f_{AB}{}^C \, T_C$$

This is written as

$$\mathrm{d}E^{AB} \equiv \theta_{CD} \, E^{AC} \wedge E^{DB} \quad \text{with } E^{AB} = E^C (T_C)^{AB}$$

$$\theta_{CD} = \text{diag.}(\underbrace{+1,\ldots,+1}_{p}, \underbrace{-1,\ldots,-1}_{q}, \underbrace{0,\ldots,0}_{r})$$

$$CSO(p,q,0) = SO(p,q)$$

$$CSO(p,q,1) = ISO(p,q)$$

$$CSO(p,q,r) = SO(p,q) \ltimes \mathbb{R}^{(p+q)r} \qquad \text{etc.}$$

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This terminology is intended by C.M. Hull, Phys. Lett. B148 (1984) 297

$\mathcal{N}=8$: $\dfrac{E_{7(7)}}{SU(8)}$

vectors	$\mathbf{56}$
scalars	$\mathbf{133}$
emb tens	$\mathbf{912}$

$\rightarrow$

$\mathcal{N}=2$: $\left(\dfrac{SL(2)}{SO(2)}\right)_{\mathrm{T}} \times \dfrac{G_{2(2)}}{SO(4)}$

vectors	$(\mathbf{4},\mathbf{1})$
scalars	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{14})$
emb tens	$(\mathbf{2},\mathbf{1})+(\mathbf{4},\mathbf{14})+(\mathbf{2},\mathbf{7})$

$\rightarrow$

$\mathcal{N}=2$: $\left(\dfrac{SL(2)}{SO(2)}\right)_{\mathrm{T}} \times \dfrac{SU(2,1)}{SU(2)\times U(1)_{\mathrm{U}}}$

vectors	$(\mathbf{4},\mathbf{1})$
scalars	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{8})$
emb tens	$(\mathbf{2},\mathbf{1})+(\mathbf{4},\mathbf{8})+(\mathbf{2},\mathbf{1})$

$\downarrow$

$\mathcal{N}=4$: $\left(\dfrac{SL(2)}{SO(2)}\right)_{\mathrm{S}} \times \dfrac{SO(6,6)}{SO(6)\times SO(6)}$

vectors	$(\mathbf{2},\mathbf{12})$
scalars	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{66})$
emb tens	$(\mathbf{2},\mathbf{12})+(\mathbf{2},\mathbf{220})$

$\rightarrow$

$\mathcal{N}=1$: $\prod_{\Phi=\mathrm{S,T,U}} \left(\dfrac{SL(2)}{SO(2)}\right)_{\Phi}$

vectors	—
scalars	$(\mathbf{3},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{3})$
emb tens	$(\mathbf{2},\mathbf{2},\mathbf{2})+(\mathbf{2},\mathbf{4},\mathbf{4})$

$\rightarrow$

$\mathcal{N}=1$: $\prod_{\Phi=\mathrm{S,T}} \left(\dfrac{SL(2)}{SO(2)}\right)_{\Phi} \times \mathbb{R}_{\mathrm{U}}$

vectors	—
scalars	$(\mathbf{1},\mathbf{1})+(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})$
emb tens	$(\mathbf{2},\mathbf{4})+(\mathbf{2},\mathbf{4})$

Starting from gauged maximal supergravity, one can move step by step downwards or towards the right by performing group-theoretical truncations.

The labels S, T and U are introduced in order to keep track of the different group factors along the truncations. G. Dibitetto, A. Guarino and D. Roest, arXiv:1202.0770

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$$\mathscr{L} = \mathscr{L}^{(1)}_{\text{kin}} + \mathscr{L}^{(2)}_{\text{kin}} + \mathscr{L}_{\text{aux}} + \mathscr{L}_{\text{conf}} + \mathscr{L}_{\text{H,conf}} + \mathscr{L}_{\text{top}} + \mathscr{L}_{\text{g}} + \mathscr{L}_{\text{g}^2}$$

$$e^{-1}\mathscr{L}^{(1)}_{\text{kin}} = -\mathrm{i}\Omega_{MN}\mathcal{D}_\mu X^M \mathcal{D}^\mu \overline{X}^N + \tfrac{\mathrm{i}}{4}\Omega_{MN}\Big[\overline{\Omega}^{iM}\not{\mathcal{D}}\Omega^N_i - \overline{\Omega}^M_i \not{\mathcal{D}}\Omega^{iN}\Big] - \tfrac{\mathrm{i}}{2}\Omega_{MN}\Big[\overline{\psi}^i_\mu \not{\mathcal{D}}\overline{X}^M\gamma^\mu\Omega^N_i - \overline{\psi}_{\mu i}\not{\mathcal{D}}X^M\gamma^\mu\Omega^{iN}\Big]$$

$$e^{-1}\mathscr{L}^{(2)}_{\text{kin}} = \tfrac{\mathrm{i}}{4}\Big[F_{\Lambda\Sigma}\mathcal{H}^{-\Lambda}_{\mu\nu}\mathcal{H}^{-\mu\nu\Sigma} - \overline{F}_{\Lambda\Sigma}\mathcal{H}^{+\Lambda}_{\mu\nu}\mathcal{H}^{+\mu\nu\Sigma}\Big] + \Big[\mathcal{O}^-_{\mu\nu\Lambda}\mathcal{H}^{-\mu\nu\Lambda} - N^{\Lambda\Sigma}\mathcal{O}^-_{\mu\nu\Lambda}\mathcal{O}^{-\mu\nu}_\Sigma + (\text{h.c.})\Big]$$

$$e^{-1}\mathscr{L}_{\text{aux}} = \tfrac18 N^{\Lambda\Sigma}\Big[N_{\Lambda\Gamma}Y^\Gamma_{ij} + \tfrac{\mathrm{i}}{2}\big(F_{\Lambda\Gamma\Pi}\overline{\Omega}^\Gamma_i\Omega^\Pi_j - \overline{F}_{\Lambda\Gamma\Pi}\overline{\Omega}^{k\Gamma}\Omega^{l\Pi}\varepsilon_{ik}\varepsilon_{jl}\big)\Big]\Big[N_{\Sigma\Xi}Y^{ij\Xi} - \tfrac{\mathrm{i}}{2}\big(\overline{F}_{\Sigma\Xi\Delta}\overline{\Omega}^{i\Xi}\Omega^{j\Delta} - F_{\Sigma\Xi\Delta}\overline{\Omega}^\Xi_m\Omega^\Delta_n\varepsilon^{im}\varepsilon^{jn}\big)\Big]$$

$$e^{-1}\mathscr{L}_{\text{conf}} = \tfrac16 K\Big[R + \Big(e^{-1}\varepsilon^{\mu\nu\rho\sigma}\overline{\psi}^i_\mu\gamma_\nu\mathcal{D}_\rho\psi_{\sigma i} - \overline{\psi}^i_\mu\psi^j_\nu T^{\mu\nu}{}_{ij} + (\text{h.c.})\Big)\Big] - K\Big[D + \tfrac12\overline{\psi}^i_\mu\gamma^\mu\chi_i + \tfrac12\overline{\psi}_{\mu i}\gamma^\mu\chi^i\Big]$$

$$-\Big[K_\Lambda\Big(\tfrac14 e^{-1}\varepsilon^{\mu\nu\rho\sigma}\overline{\psi}_{\mu i}\gamma_\nu\psi^i_\rho\mathcal{D}_\sigma X^\Lambda + \tfrac{1}{48}\overline{\psi}_{\mu i}\gamma^\mu\gamma_{\rho\sigma}\Omega^\Lambda_j T^{\rho\sigma ij}\Big) + (\text{h.c.})\Big] - \Big[K_\Lambda\Big(\tfrac13\overline{\Omega}^\Lambda_i\gamma^{\mu\nu}\mathcal{D}_\mu\psi^i_\nu - \overline{\Omega}^\Lambda_i\chi^i\Big) + (\text{h.c.})\Big]$$

$$e^{-1}\mathscr{L}_{\text{H,conf}} = \tfrac16\chi\Big[R + \Big(e^{-1}\varepsilon^{\mu\nu\rho\sigma}\overline{\psi}^i_\mu\gamma_\nu\mathcal{D}_\rho\psi_{\sigma i} - \tfrac14\overline{\psi}^i_\mu\psi^j_\nu T^{\mu\nu}{}_{ij} + (\text{h.c.})\Big)\Big] + \tfrac12\chi\Big[D + \tfrac12\overline{\psi}^i_\mu\gamma^\mu\chi_i + \tfrac12\overline{\psi}_{\mu i}\gamma^\mu\chi^i\Big]$$

$$-\tfrac12 G_{\overline{\alpha}\beta}\mathcal{D}_\mu A_i{}^\beta\mathcal{D}^\mu A^{i\overline{\alpha}} - G_{\overline{\alpha}\beta}\big(\overline{\zeta}^{\overline{\alpha}}\not{\mathcal{D}}\zeta^\beta + \overline{\zeta}^\beta\not{\mathcal{D}}\zeta^{\overline{\alpha}}\big) - \tfrac14 W_{\overline{\alpha}\beta\overline{\gamma}\delta}\,\overline{\zeta}^{\overline{\alpha}}\gamma_\mu\zeta^\beta\,\overline{\zeta}^{\overline{\gamma}}\gamma^\mu\zeta^\delta - \chi_A\Big[\gamma^A_{i\overline{\alpha}}\Big(\tfrac23\overline{\zeta}^{\overline{\alpha}}\gamma^{\mu\nu}\mathcal{D}_\mu\psi^i_\nu + \overline{\zeta}^{\overline{\alpha}}\chi^i - \tfrac16\overline{\zeta}^{\overline{\alpha}}\gamma_\mu\psi_{\nu i}T^{\mu\nu ij}\Big) + (\text{h.c.})\Big]$$

$$+\Big[\tfrac{1}{16}\overline{\Omega}_{\alpha\beta}\overline{\zeta}^\alpha\gamma^{\mu\nu}T_{\mu\nu ij}\varepsilon^{ij}\zeta^\beta - \tfrac12\overline{\zeta}^\alpha\gamma^\mu\gamma^\nu\psi_{\mu i}\big(\overline{\psi}^i_\nu G_{\alpha\overline{\beta}}\zeta^{\overline{\beta}} + \varepsilon^{ij}\overline{\Omega}_{\alpha\beta}\overline{\psi}_{\nu j}\zeta^\beta\big) + G_{\overline{\alpha}\beta}\overline{\zeta}^\beta\gamma^\mu\not{\mathcal{D}}A^{i\overline{\alpha}}\psi_{\mu i} - \tfrac14 e^{-1}\varepsilon^{\mu\nu\rho\sigma}G_{\overline{\alpha}\beta}\overline{\psi}^i_\mu\gamma_\nu\psi_{\rho j}A_i{}^\beta\mathcal{D}_\sigma A^{j\overline{\alpha}} + (\text{h.c.})\Big]$$

$$e^{-1}\mathscr{L}_{\text{top}} = \tfrac{\mathrm{i}}{8}\mathrm{g}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\big(\Theta^{\Lambda\mathsf{a}}B_{\mu\nu\mathsf{a}} + \Theta^{\Lambda\mathsf{m}}B_{\mu\nu\mathsf{m}}\big)\Big(2\partial_\rho W_{\sigma\Lambda} + \mathrm{g}T_{MN\Lambda}W^M_\rho W^N_\sigma - \tfrac14\mathrm{g}\Theta_\Lambda{}^{\mathsf{b}}B_{\rho\sigma\mathsf{b}} - \tfrac14\mathrm{g}\Theta_\Lambda{}^{\mathsf{n}}B_{\rho\sigma\mathsf{n}}\Big)$$

$$+\tfrac{\mathrm{i}}{3}\mathrm{g}e^{-1}\varepsilon^{\mu\nu\rho\sigma}T_{MN\Lambda}W^M_\mu W^N_\nu\Big(\partial_\rho W^\Lambda_\sigma + \tfrac14\mathrm{g}T_{PQ}{}^\Lambda W^P_\rho W^Q_\sigma\Big) + \tfrac{\mathrm{i}}{6}\mathrm{g}e^{-1}\varepsilon^{\mu\nu\rho\sigma}T_{MN}{}^\Lambda W^M_\mu W^N_\nu\Big(\partial_\rho W_{\sigma\Lambda} + \tfrac14\mathrm{g}T_{PQ\Lambda}W^P_\rho W^Q_\sigma\Big)$$

$$e^{-1}\mathscr{L}_{\text{g}} = -\tfrac12\mathrm{g}\Big[\mathrm{i}\Omega_{MQ}T_{PN}{}^Q\varepsilon^{ij}\overline{X}^N\overline{\Omega}^M_i\big(\Omega^P_j + \gamma^\mu\psi_{\mu j}X^P\big) + (\text{h.c.})\Big] + 2\mathrm{g}\Big[k_{AM}\gamma^A_{i\overline{\alpha}}\varepsilon^{ij}\overline{\zeta}^{\overline{\alpha}}\big(\Omega^M_j + \gamma^\mu\psi_{\mu j}X^M\big) + (\text{h.c.})\Big] + \mathrm{g}\Big[\mu^{ij}{}_M\overline{\psi}_{\mu i}\big(\gamma^\mu\Omega^M_j + \gamma^{\mu\nu}\psi_{\nu j}X^M\big) + (\text{h.c.})\Big]$$

$$+2\mathrm{g}\Big[\overline{X}^M T_M{}^\gamma{}_\alpha\overline{\Omega}_{\beta\gamma}\overline{\zeta}^\alpha\zeta^\beta + X^M T_M{}^{\overline{\gamma}}{}_{\overline{\alpha}}\Omega_{\overline{\beta}\overline{\gamma}}\overline{\zeta}^{\overline{\alpha}}\zeta^{\overline{\beta}}\Big] - \tfrac14\mathrm{g}\Big[F_{\Lambda\Sigma\Gamma}\mu^{ij\Lambda}\overline{\Omega}^\Sigma_i\Omega^\Gamma_j + \overline{F}_{\Lambda\Sigma\Gamma}\mu_{ij}{}^\Lambda\overline{\Omega}^{i\Sigma}\Omega^{j\Gamma}\Big] + \mathrm{g}Y^{ij\Lambda}\Big[\mu_{ij\Lambda} + \tfrac12(F_{\Lambda\Sigma} + \overline{F}_{\Lambda\Sigma})\mu_{ij}{}^\Sigma\Big]$$

$$e^{-1}\mathscr{L}_{\text{g}^2} = \mathrm{i}\mathrm{g}^2\Omega_{MN}\big(T_{PQ}{}^M X^P\overline{X}^Q\big)\big(T_{RS}{}^N\overline{X}^R X^S\big) - 2\mathrm{g}^2 k^A{}_M k^B{}_N g_{AB}X^M\overline{X}^N - \tfrac12\mathrm{g}^2 N_{\Lambda\Sigma}\,\mu_{ij}{}^\Lambda\,\mu^{ij\Sigma}$$

B. de Wit and M. van Zalk, arXiv:1107.3305 jump App.top

ヘテロティック弦におけるフラックスコンパクト化と真空構造

torsionful manifolds : hep-th/0605247

index theorem with torsion : arXiv:0704.2111

intersecting five-branes : arXiv:0905.2185, arXiv:0912.1334

type II 弦における一般化された幾何学を用いたコンパクト化と真空構造

doubled geometries, generalized geometries :

arXiv:0806.1783, arXiv:0810.0937, arXiv:1203.5499

フラックスコンパクト化と AdS 極限ブラックホール解

gauged SUGRA via flux compactifications :

arXiv:1005.4607, arXiv:1108.1113, arXiv:1203.1544

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