

On Static Charged Black Holes in Type IIA on a Nearly-Kahler Coset

based on arXiv:1108.1113, arXiv:1203.1544

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以下の流れでお話をします

- **動機**

  ストリング理論のコンパクト化と AdS ブラックホール

- **AdS ブラックホール**

  nearly-Kähler coset space $G_2/SU(3)$

  AdS 真空

  静的条件と解

動 機

ストリング理論のコンパクト化において

Reissner-Nordström AdS ブラックホール解を考えたい

Reissner-Nordström AdS ブラックホール解を考えたい

理由

- ハイパー多重項がある $\mathcal{N} = 2$ gauged SUGRA の探索
- 「AdS/CMT」の設定 (?):

荷電粒子が AdS ブラックホールに落ちていく過程をみる

Einstein + $\Lambda_{\text{c.c.}}$ + Maxwell + charged matters

この設定(に近いもの)をストリング理論で与えるには？

フラックスコンパクト化を用いたシナリオで考えたい

コンパクト化された幾何学の一般化を世に知らしめる

- 2005年から「広報」活動

  Flux compactications in heterotic string

  Index theorem of torsionful manifolds

  AdS vacua via generalized geometries in type IIA string

  D-branes in doubled geometries

  Intersecting five-branes, etc.

- なかなか浸透しない

  抽象的すぎる？

  ありがたみが見えない？

  この頃ようやく日本の方々も「torsion」「nongeometric backgrounds」を触っているが...

設定を超重力理論に埋め込むときの障害

- **宇宙項 vs 物質場 vs 超対称性**

  物質場がある系で負の宇宙「定数」をそのまま超対称作用積分に導入することはできない

- **4D 宇宙項 vs コンパクト化**

  4D で負の宇宙項を持つ真空解を与えるには超重力をゲージ化すべし

  ゲージ化するにはフラックスコンパクト化すべし

  フラックスを乗せるにはトーションを用意すべし

non-CY manifold $\mathcal{M}_6$

Ricci 2-form がゼロのまま、トーションを許す

($SU(3)$-structure manifold)

$\mathrm{d}J \neq 0$ **and/or** $\mathrm{d}\Omega \neq 0$

CY からのズレ：

$$\mathrm{d}J = \frac{3}{2}\,\mathrm{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

$\mathcal{W}_1 \,,\ \mathcal{W}_2 \,,\ \mathcal{W}_3 \,,\ \mathcal{W}_4 \,,\ \mathcal{W}_5$ **: intrinsic torsion classes**

10次元II型作用 (democratic formulation) $S_{\text{II}}^{(10D)} = S_{\text{NS}} + \widetilde{S}_{\text{R}}$:

$$S_{\text{NS}} + \widetilde{S}_{\text{R}} = \frac{1}{2}\int e^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\} - \frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$

"自己双対 $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})$" と "場の方程式 $(\mathrm{d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow (\mathrm{d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

↓ **SU(3)-structure manifolds**

4次元 $\mathcal{N}=2$ ゲージ化された超重力理論

ポテンシャル項が現れる (期待値が宇宙項になる)

(可換ゲージ群に限られる)

AdS ブラックホール

in 4D $\mathcal{N}=2$ gauged SUGRA with B-field

from massive type IIA on a nearly-Kähler coset space $G_2/SU(3)$

D. Cassani and A.K. Kashani-Poor [arXiv:0901.4251]

✓ **NSNS-sector : torsion and H-flux**

✓ **RR-sector : 2-, 4-form and Romans' mass (0-form)**

$$\mathrm{d}J = \frac{3}{2}\,\mathrm{Im}(\overline{\mathcal{W}}_1\Omega)\,, \quad \mathrm{d}\Omega = \mathcal{W}_1\, J \wedge J$$

$$\mathrm{d}\omega_\Lambda = e_\Lambda\,\alpha\,, \quad \mathrm{d}\alpha = 0\,, \quad \mathrm{d}\beta = e_\Lambda\,\widetilde{\omega}^\Lambda\,, \quad \mathrm{d}\widetilde{\omega}^\Lambda = 0$$

$$e_\Lambda m_{\mathbf{R}}^\Lambda = 0$$

✓ **1 vector multiplet with cubic prepotential** $\mathcal{F} = \dfrac{X^1 X^1 X^1}{X^0}$

✓ **1 universal hypermultiplet (no other HMs) $\rightarrow$ hyper-tensor multiplet**

4D multiplets	from NSNS	from RR	fermions
supergravity multiplet	$g_{\mu\nu}$	A_μ^0	ψ_μ^i
1 vector multiplet	$\mathfrak{t}$	A_μ^1	λ_i
1 hyper-tensor multiplet	φ, $B_{\mu\nu}$	ξ^0, $\widetilde{\xi}_0$	ζ^α

A_μ^0 : from RR one-form C_1

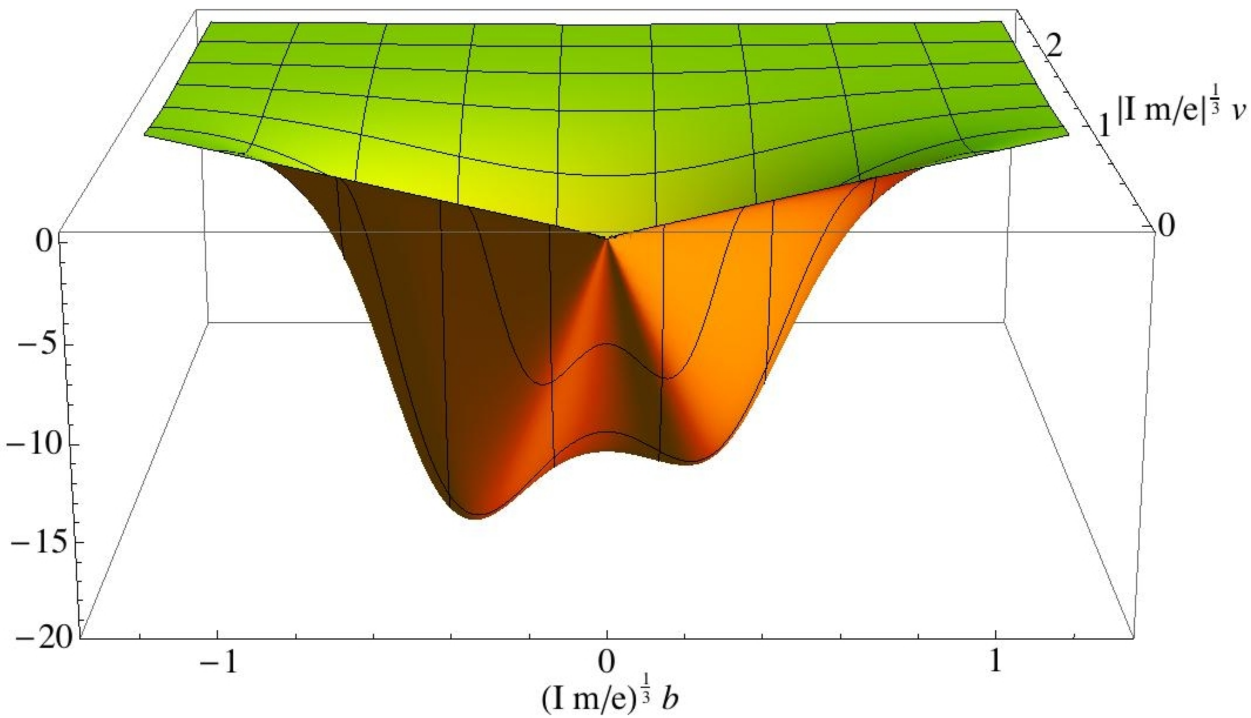
$\mathfrak{t}$: from complexified "Kähler" modulus $\mathfrak{t} = X^1/X^0 = b + \mathrm{i}v$

A_μ^1 : from RR three-form C_3

ξ^0, $\widetilde{\xi}_0$: from RR three-form C_3

Expansion Lagrangian

$F_2^\Lambda = \mathrm{d}A_1^\Lambda + m_{\mathbf{R}}^\Lambda B_2$: Stückelberg-type deformation



in arXiv:0901.4251

ひとつの $\mathcal{N}=1$ AdS 真空 と ふたつの $\mathcal{N}=0$ AdS 真空 Appendix

[NOTE]

type IIA 弦理論から 4D $\mathcal{N}=1$ AdS vacua を導出するには Romans' mass が必要

D. Lüst and D. Tsimpis, hep-th/0412250

Reissner-Nordström AdS ブラックホールの下、物質場の配位を調べる：

$$ds^2 = -e^{2A(r)}dt^2 + e^{-2A(r)}dr^2 + r^2 d\Omega^2$$

$$e^{2A(r)} = 1 - \frac{2\eta}{r} + \frac{\mathcal{Z}^2}{r^2} + \frac{r^2}{\ell^2}\,, \quad \mathcal{Z}^2 = Q^2 + P^2\,, \quad \Lambda_{\text{c.c.}} = -\frac{3}{\ell^2}$$

vector fields A_μ^Λ の電荷・磁荷： $p^\Lambda = \int F_2^\Lambda\,, \quad q_\Lambda = \int \widetilde{F}_{2\Lambda}$

$$\downarrow$$

$$F_{\theta\phi}^\Lambda \equiv f^\Lambda(\theta,\phi)\sin\theta\,, \quad F_{tr}^\Lambda \equiv \frac{e^{-2C(r)}}{r^2} g^\Lambda(\theta,\phi)$$

時間依存性がない解を探そうとすると、

A_μ^Λ, $B_{\mu\nu}$, $g_{\mu\nu}$ の運動方程式から「すべての場の共変定数性」が示される：

$$0 = \partial_\mu \mathrm{t} = \partial_\mu \varphi = D_\mu \xi^0 = D_\mu \widetilde{\xi}_0 = \partial_{[\mu} B_{\nu\rho]} = F_{\mu\nu}^\Lambda$$

Appendix

ブラックホール電荷 $\mathscr{Z}^2 = Q^2 + P^2$ **は**

次の様に記述される：

$$\mathscr{Z}^2 \;=\; -\frac{1}{2}\Big[p^{\Lambda}\mu_{\Lambda\Sigma}\,p^{\Sigma} + (q_{\Lambda} - \nu_{\Lambda\Gamma}\,p^{\Gamma})(\mu^{-1})^{\Lambda\Sigma}(q_{\Sigma} - \nu_{\Sigma\Delta}\,p^{\Delta})\Big]$$

$$e^{-1}\mathscr{L} = \tfrac{1}{2}R + \tfrac{1}{2}\mu_{\Lambda\Sigma}F^{\Lambda} \wedge * F^{\Sigma} + \tfrac{1}{2}\nu_{\Lambda\Sigma}F^{\Lambda} \wedge F^{\Sigma} + \ldots$$

一方で、ゲージ場と電荷 $(p^{\Lambda}, q_{\Lambda})$ **の関係は**

次の様に与えられる：

$$0 \;=\; F^{\Lambda}_{\theta\phi} \;=\; p^{\Lambda}\sin\theta\,, \qquad 0 \;=\; F^{\Lambda}_{tr} \;=\; -\frac{1}{r^2}(\mu^{-1})^{\Lambda\Sigma}(q_{\Sigma} - \nu_{\Sigma\Gamma}p^{\Gamma})$$

つまり、

$$p^{\Lambda} \;=\; 0 \;=\; q_{\Lambda} \quad\rightarrow\quad \mathscr{Z}^2 \;=\; 0$$

結果

Massive type IIA **on** $G_2/SU(3)$ **から得られる**

4D $\mathcal{N}=2$ **gauged SUGRA with B-field は、**

Reissner-Nordström AdS ブラックホール解を持ち得ない ($\mathcal{Z}^2=0$ **に退化)。**

ブラックホール質量・電荷・宇宙項などをコンパクト化の情報で記述するために、
もう少し非自明なブラックホール解を考える必要がある？

おわりに

以下の流れでお話をしました

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- **AdS ブラックホール**

  nearly-Kähler coset space $G_2/SU(3)$

  AdS 真空

  静的条件と共変定数解

Appendix

4D $\mathcal{N} = 2$ gauged SUGRA

Coset spaces

Gauged SUGRA from type IIA on $G_2/SU(3)$

Analysis on static AdS black holes

10D type IIA action $S^{(10D)}_{IIA} = S_{NS} + \widetilde{S}_R = S_{NS} + S_R + S_{CS}$: (democratic form)

$$S_{NS} = \frac{1}{2}\int e^{-2\phi}\Big\{R * \mathbb{1} + 4d\phi \wedge *d\phi - \frac{1}{2}H_3 \wedge *H_3\Big\}, \quad \widetilde{S}_R = -\frac{1}{8}\int [\mathbf{F} \wedge *\mathbf{F}]_{10}$$

with "constraint $\mathbf{F} = \lambda(*\mathbf{F})$" and "EoM (Bianchi) $(d + H\wedge) * \mathbf{F} = 0 \Leftrightarrow (d - H\wedge)\mathbf{F} = 0$"

↓ $SU(3)$-structure with $m^\Lambda_R = 0$

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^I \equiv (\xi^I, \widetilde{\xi}_I)^T$):

$$S^{(4D)} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}R + \frac{1}{4}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^\Sigma_{\rho\sigma} - g_{a\bar{b}}\,\partial_\mu t^a\partial^\mu \bar{t}^{\bar{b}} - g_{i\bar{\jmath}}\,\partial_\mu z^i\partial^\mu \bar{z}^{\bar{\jmath}}$$
$$-\partial_\mu\varphi\partial^\mu\varphi + \frac{e^{2\varphi}}{2}(\mathbb{M}_H)_{IJ}D_\mu\xi^I D^\mu\xi^J - \frac{e^{2\varphi}}{4}\big(D_\mu a - \xi^I(\mathbb{C}_H)_{IJ}D_\mu\xi^J\big)^2 - V(t, \bar{t}, q)\Big]$$

- $(e_\Lambda{}^I, e_{\Lambda I})$: geometric flux charges & $e_{R\Lambda}$: RR-flux charges ⟵ non-CY data
  (with constraints $e_\Lambda{}^I e_{\Sigma I} - e_{\Lambda I}e_\Sigma{}^I = 0$)
- $t^a \in$ SKG$_V$ and $z^i \in$ SKG$_H \subset \mathcal{HM}$ are ungauged (in general)
- $D_\mu\xi^I = \partial_\mu\xi^I - e_\Lambda{}^I A^\Lambda_\mu$ & $D_\mu\widetilde{\xi}_I = \partial_\mu\widetilde{\xi}_I - e_{\Lambda I}A^\Lambda_\mu$
- $D_\mu a = \partial_\mu a - (2e_{R\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_\Lambda{}^I)A^\Lambda_\mu$
- $V(t, \bar{t}, q)$: scalar potential

D. Cassani, arXiv:0804.0595

Non-vanishing $m_{\mathbf{R}}^{\Lambda}$ dualizes the axion field a in standard SUGRA to B-field.

4D gauged action is different from the standard one:

$$\begin{aligned} S^{(\mathbf{4D})} \;=\; \int \Big[& \frac{1}{2} R(*\mathbb{1}) + \frac{1}{2} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F_2^{\Lambda} \wedge * F_2^{\Sigma} + \frac{1}{2} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F_2^{\Lambda} \wedge F_2^{\Sigma} - g_{a\bar{b}}\, \mathrm{d}\mathfrak{t}^a \wedge * \mathrm{d}\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}}\, \mathrm{d}z^i \wedge * \mathrm{d}\bar{z}^{\bar{\jmath}} \\ & - \mathrm{d}\varphi \wedge * \mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} H_3 \wedge * H_3 - \frac{\mathrm{e}^{2\varphi}}{2} (\mathbb{M}_{\mathbf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{I}} \wedge * D\xi^{\boldsymbol{J}} - V(*\mathbb{1}) \\ & + \frac{1}{2} \mathrm{d}B \wedge \Big[\xi^{\boldsymbol{I}} (\mathbb{C}_{\mathbf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{J}} + \big(2 e_{\mathbf{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_{\Lambda}{}^I \big) A_1^{\Lambda} \Big] - \frac{1}{2} m_{\mathbf{R}}^{\Lambda} e_{\mathbf{R}\Lambda} B_2 \wedge B_2 \Big] \end{aligned}$$

Constraints among flux charges:

$$e_{\Lambda}{}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^I \;=\; 0, \quad m_{\mathbf{R}}^{\Lambda} e_{\Lambda}{}^I \;=\; 0 \;=\; m_{\mathbf{R}}^{\Lambda} e_{\Lambda I}$$

Scalar potential from (non)geometric flux compactifications:

$$V = \mathrm{g}^2 \Big[4 h_{uv} k^u \overline{k}^v + \sum_{x=1}^{3} \Big(g^{a\overline{b}} D_a \mathcal{P}_x D_{\overline{b}} \overline{\mathcal{P}}_x - 3 |\mathcal{P}_x|^2 \Big) \Big] = \ldots \equiv V_{\mathbf{NS}} + V_{\mathbf{R}} \quad \textbf{(abelian: } k_\Lambda^a = 0\textbf{)}$$

$$V_{\mathbf{NS}} = g^{a\overline{b}} D_a \mathcal{P}_+ D_{\overline{b}} \overline{\mathcal{P}}_+ + g^{i\overline{j}} D_i \mathcal{P}_+ D_{\overline{j}} \overline{\mathcal{P}}_+ - 2 |\mathcal{P}_+|^2$$

$$= -2\, \mathrm{g}^2 \mathrm{e}^{2\varphi} \Big[\overline{\Pi}_{\mathbf{H}}^{\mathrm{T}} \widetilde{Q}^{\mathrm{T}} \mathbb{M}_{\mathbf{V}} \widetilde{Q}\, \Pi_{\mathbf{H}} + \overline{\Pi}_{\mathbf{V}}^{\mathrm{T}} Q\, \mathbb{M}_{\mathbf{H}} Q^{\mathrm{T}} \Pi_{\mathbf{V}} + 4 \overline{\Pi}_{\mathbf{H}}^{\mathrm{T}} \mathbb{C}_{\mathbf{H}}^{\mathrm{T}} Q^{\mathrm{T}} (\Pi_{\mathbf{V}} \overline{\Pi}_{\mathbf{V}}^{\mathrm{T}} + \overline{\Pi}_{\mathbf{V}} \Pi_{\mathbf{V}}^{\mathrm{T}}) Q\, \mathbb{C}_{\mathbf{H}} \Pi_{\mathbf{H}} \Big]$$

$$V_{\mathbf{R}} = g^{a\overline{b}} D_a \mathcal{P}_3 D_{\overline{b}} \overline{\mathcal{P}}_3 + |\mathcal{P}_3|^2$$

$$= -\frac{1}{2} \mathrm{g}^2 \mathrm{e}^{4\varphi} \big(e_{\mathbf{R}\Lambda} - e_{\Lambda I} \xi^I + e_\Lambda{}^I \widetilde{\xi}_I \big) (\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma} \big(e_{\mathbf{R}\Sigma} - e_{\Sigma I} \xi^I + e_\Sigma{}^I \widetilde{\xi}_I \big)$$

$$\Pi_{\mathbf{V}} = \mathrm{e}^{\mathcal{K}_{\mathbf{V}}/2} (X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$$
$$\mathrm{t}^a = X^a / X^0$$
$$a = 1, \ldots, n_{\mathbf{V}}$$

SKG$_{\mathbf{V}}$ of vector-moduli

$$\mathcal{P}_+ \equiv \mathcal{P}_1 + \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\, \Pi_{\mathbf{V}}^{\mathrm{T}} Q\, \mathbb{C}_{\mathbf{H}} \Pi_{\mathbf{H}}$$
$$\mathcal{P}_- \equiv \mathcal{P}_1 - \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\, \Pi_{\mathbf{V}}^{\mathrm{T}} Q\, \mathbb{C}_{\mathbf{H}} \overline{\Pi}_{\mathbf{H}}$$
$$\mathcal{P}_3 = \mathrm{e}^{2\varphi}\, \Pi_{\mathbf{V}}^{\mathrm{T}} \mathbb{C}_{\mathbf{V}} (c_{\mathbf{R}} + \widetilde{Q} \xi)$$

$$\Pi_{\mathbf{H}} = \mathrm{e}^{\mathcal{K}_{\mathbf{H}}/2} (Z^I, \mathcal{G}_I)^{\mathrm{T}}$$
$$z^i = Z^i / Z^0$$
$$i = 1, \ldots, n_{\mathbf{H}}$$

SKG$_{\mathbf{H}}$ of hyper-moduli

$$\mathbb{C}_{\mathbf{V,H}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} ; \quad Q = \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix} , \quad \widetilde{Q} = \mathbb{C}_{\mathbf{H}}^{\mathrm{T}} Q\, \mathbb{C}_{\mathbf{V}} \quad c_{\mathbf{R}} = \begin{pmatrix} m_{\mathbf{R}}^\Lambda \\ e_{\mathbf{R}\Lambda} \end{pmatrix}$$

Cassani et.al., arXiv:0804.0595, arXiv:0911.2708 App.top

D. Cassani and A.K. Kashani-Poor, arXiv:0901.4251 App.top

$\mathcal{M}_6$	$\frac{G_2}{SU(3)} = S^6$	$\frac{Sp(2)}{S(U(2)\times U(1))} = \mathbb{C}P^3$	$\frac{SU(3)}{U(1)\times U(1)} = \mathbb{F}(1,2;3)$
$\mathcal{SM}$ = **SKG$_\mathrm{V}$**	$\frac{SU(1,1)}{U(1)}$: $\mathfrak{t}^3$	$\left(\frac{SU(1,1)}{U(1)}\right)^2$: $\mathfrak{st}^2$	$\left(\frac{SU(1,1)}{U(1)}\right)^3$: $\mathfrak{stu}$
$\mathcal{HM}$ = **SQG**	$\frac{SU(2,1)}{U(2)}$: **UHM**	$\frac{SU(2,1)}{U(2)}$: **UHM**	$\frac{SU(2,1)}{U(2)}$: **UHM**
SKG$_\mathrm{H}$ $\subset \mathcal{HM}$	—	—	—
matters	**1 VM + 1 UHM**	**2 VM + 1 UHM**	**3 VM + 1 UHM**

Each SKG$_\mathrm{V}$ has a cubic prepotential: $\mathcal{F} = \frac{1}{3!} d_{abc} \frac{X^a X^b X^c}{X^0}$

nilmanifolds and solvmanifolds: M. Graña, R. Minasian, M. Petrini and A. Tomasiello, hep-th/0609124

coset spaces with $SU(3)$- or $SU(2)$-structure: P. Koerber, D. Lüst and D. Tsimpis, arXiv:0804.0614

a pair of $SU(3)$-structures with $(m^{\Lambda I}, m^{\Lambda}{}_I)$: D. Gaiotto and A. Tomasiello, arXiv:0904.3959

10D type IIA on $\dfrac{G_2}{SU(3)}$ with fluxes

$\downarrow$

4D $\mathcal{N}=2$ abelian gauged SUGRA with B-field ($\Lambda = 0, 1$ and $\xi^0 \equiv (\xi^0, \widetilde{\xi}_0)^{\rm T}$)

$$S = \int \Big[\frac{1}{2} R\, (*1) + \frac{1}{2} \mu_{\Lambda\Sigma}\, F^{\Lambda} \wedge *F^{\Sigma} + \frac{1}{2} \nu_{\Lambda\Sigma}\, F^{\Lambda} \wedge F^{\Sigma} - g_{\mathfrak{t}\bar{\mathfrak{t}}}\, \mathrm{d}\mathfrak{t} \wedge *\mathrm{d}\bar{\mathfrak{t}}$$
$$- \mathrm{d}\varphi \wedge *\mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} \mathrm{d}B \wedge *\mathrm{d}B - \frac{\mathrm{e}^{2\varphi}}{2} \Big(D\xi^0 \wedge *D\xi^0 + D\widetilde{\xi}_0 \wedge *D\widetilde{\xi}_0 \Big) + \mathrm{d}B \wedge \xi^0\, \mathrm{d}\widetilde{\xi}_0$$
$$+ \mathrm{d}B \wedge \big(e_{\mathbf{R}\Lambda} - e_{\Lambda 0}\, \xi^0 \big) A^{\Lambda} - \frac{1}{2} m_{\mathbf{R}}^{\Lambda}\, e_{\mathbf{R}\Lambda}\, B \wedge B - V\, (*1) \Big]$$

- $g_{\mu\nu}$, $\mathfrak{t}$, $B_{\mu\nu}$, φ; $(e_{\Lambda}{}^0, e_{\Lambda 0})$: **NS-NS sector**
- A_{μ}^{Λ}, ξ^0, $\widetilde{\xi}_0$; $(m_{\mathbf{R}}^{\Lambda}, e_{\mathbf{R}\Lambda})$: **R-R sector**
- **GM :** $(g_{\mu\nu}, A_{\mu}^0)$, **VM :** $(A_{\mu}^a, \mathfrak{t})$, **UHM $\to$ TM :** $(\varphi, B_{\mu\nu}, \xi^0, \widetilde{\xi}_0)$
- $D\xi^0 = \mathrm{d}\xi^0 - e_{\Lambda}{}^0 A_1^{\Lambda}$, $D\widetilde{\xi}_0 = \mathrm{d}\widetilde{\xi}_0 - e_{\Lambda 0} A_1^{\Lambda}$
- $F_2^{\Sigma} = \mathrm{d}A_1^{\Sigma} + m_{\mathbf{R}}^{\Sigma} B_2$
- $V(\mathfrak{t}, \varphi, \xi^0) = V_{\mathbf{NS}}(\mathfrak{t}, \varphi) + V_{\mathbf{R}}(\mathfrak{t}, \varphi, \xi^0)$

Precise data on $\dfrac{G_2}{SU(3)}$:

$e_{10} \neq 0$, $m_{\mathbf{R}}^0 \neq 0$, $e_{\mathbf{R}0} \neq 0$

$e_{\Lambda}{}^0 = 0 = e_{00}$

$m_{\mathbf{R}}^1 = 0 = e_{\mathbf{R}1}$

$\mu_{\Lambda\Sigma} \equiv {\rm Im}\mathcal{N}_{\Lambda\Sigma}$, $\nu_{\Lambda\Sigma} \equiv {\rm Re}\mathcal{N}_{\Lambda\Sigma}$

D. Cassani, arXiv:0804.0595 Main

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} &= \frac{1}{4} g_{\mu\nu}\, \mu_{\Lambda\Sigma} F^{\Lambda}_{\rho\sigma} F^{\Sigma\rho\sigma} - \mu_{\Lambda\Sigma} F^{\Lambda}_{\mu\rho} F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma} - g_{\mu\nu}\, g_{\mathrm{t}\bar{\mathrm{t}}} \partial_\rho \mathrm{t} \partial^\rho \bar{\mathrm{t}} + 2 g_{\mathrm{t}\bar{\mathrm{t}}}\, \partial_\mu \mathrm{t} \partial_\nu \bar{\mathrm{t}} \\
&\quad - g_{\mu\nu}\, \partial_\rho \varphi \partial^\rho \varphi + 2 \partial_\mu \varphi \partial_\nu \varphi - \frac{\mathrm{e}^{-4\varphi}}{24} g_{\mu\nu}\, H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} + \frac{\mathrm{e}^{-4\varphi}}{4} H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma} \qquad (\delta g_{\mu\nu}) \\
&\quad - \frac{\mathrm{e}^{2\varphi}}{2} g_{\mu\nu} \Big(D_\rho \xi^0 D^\rho \xi^0 + D_\rho \widetilde{\xi}_0 D^\rho \widetilde{\xi}_0 \Big) + \mathrm{e}^{2\varphi} \Big(D_\mu \xi^0 D_\nu \xi^0 + D_\mu \widetilde{\xi}_0 D_\nu \widetilde{\xi}_0 \Big) - g_{\mu\nu} V \,,
\end{aligned}
$$

$$
0 = \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, \mu_{\Lambda\Sigma} F^{\Sigma\mu\sigma} \Big) - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu \Big(\nu_{\Lambda\Sigma} F^{\Sigma}_{\nu\rho} \Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu B_{\nu\rho} (e_{\mathbf{R}\Lambda} - \xi^0 e_{\Lambda 0}) - \mathrm{e}^{2\varphi} Q_{\Lambda\mathbf{0}} D^\sigma \xi^{\mathbf{0}} \,, \qquad (\delta A^{\Lambda}_{\mu})
$$

$$
0 = \frac{1}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g_{\mathrm{t}\bar{\mathrm{t}}}\, g^{\mu\nu} \partial_\nu \bar{\mathrm{t}} \Big) + \frac{1}{4} \partial_{\mathrm{t}} (\mu_{\Lambda\Sigma}) F^{\Lambda}_{\mu\nu} F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}} \partial_{\mathrm{t}} (\nu_{\Lambda\Sigma}) F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} - \partial_{\mathrm{t}} g_{\mathrm{t}\bar{\mathrm{t}}}\, \partial_\mu \mathrm{t} \partial^\mu \bar{\mathrm{t}} - \partial_{\mathrm{t}} V \,, \qquad (\delta \mathrm{t})
$$

$$
0 = \frac{2}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \varphi \Big) + \frac{\mathrm{e}^{4\varphi}}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} - \mathrm{e}^{2\varphi} \Big(D_\mu \xi^0 D^\mu \xi^0 + D_\mu \widetilde{\xi}_0 D^\mu \widetilde{\xi}_0 \Big) - \partial_\varphi V \,, \qquad (\delta \varphi)
$$

$$
\begin{aligned}
0 &= \frac{1}{\sqrt{-g}} \partial_\mu \Big(\mathrm{e}^{-4\varphi} \sqrt{-g} H^{\mu\rho\sigma} \Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} \Big[D_\mu \xi^{\mathbf{0}} (\mathbb{C}_{\mathbf{H}})_{\mathbf{00}} D_\nu \xi^{\mathbf{0}} + (e_{\mathbf{R}\Lambda} - \xi^0 e_{\Lambda 0}) F^{\Lambda}_{\mu\nu} \Big] \qquad (\delta B_{\mu\nu}) \\
&\quad + 2 m^{\Lambda}_{\mathbf{R}} \mu_{\Lambda\Sigma} F^{\Sigma\rho\sigma} - \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} m^{\Lambda}_{\mathbf{R}} \nu_{\Lambda\Sigma} F^{\Sigma}_{\mu\nu} \,,
\end{aligned}
$$

$$
0 = -\frac{2}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g}\, \mathrm{e}^{2\varphi} g^{\mu\nu} D_\nu \xi^{\mathbf{0}} \Big) + \frac{\partial V}{\partial \xi^{\mathbf{0}}} - \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}} \partial_\mu B_{\nu\rho} D_\sigma \xi^{\mathbf{0}} (\mathbb{C}_{\mathbf{H}})_{\mathbf{00}} \,. \qquad (\delta \xi^{\mathbf{0}})
$$

Vacuum I : $\mathcal{N} = 1$

$$t_* = -\frac{\pm 1 + \mathrm{i}\sqrt{15}}{2} \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{\mathbf{R}0}}{m^0_{\mathbf{R}}} \right| \right]^{1/3} , \quad \xi^0_* = -\frac{2}{5} \left[\frac{2\sqrt{3}\, m^0_{\mathbf{R}} (e_{\mathbf{R}0})^2}{5\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) = \frac{4}{3} \left[\frac{\sqrt{5}\, e_{10}}{\sqrt{3}\, m^0_{\mathbf{R}} (e_{\mathbf{R}0})^2} \right]^{1/3}$$

$$V_* = -\frac{5\sqrt{5}}{2} \left[\frac{5\,(e_{10})^4}{2\sqrt{3}\, |m^0_{\mathbf{R}} (e_{\mathbf{R}0})^5|} \right]^{1/3} \equiv \Lambda^{\mathrm{I}}_{\mathbf{c.c.}} < 0$$

Vacuum II : $\mathcal{N} = 0$

$$t_* = \big(\pm 1 - \mathrm{i}\sqrt{3} \big) \left[\frac{3}{5\,(e_{10})^2} \left| \frac{e_{\mathbf{R}0}}{m^0_{\mathbf{R}}} \right| \right]^{1/3} , \quad \xi^0_* = \left[\frac{9\, m^0_{\mathbf{R}} (e_{\mathbf{R}0})^2}{25\, e_{10}} \right]^{1/3} , \quad \exp(\varphi_*) = \frac{2}{3} \left[\frac{25\, e_{10}}{\sqrt{3}\, m^0_{\mathbf{R}} (e_{\mathbf{R}0})^2} \right]^{1/3}$$

$$V_* = -\frac{80}{27} \left[\frac{25\,(e_{10})^4}{\sqrt{3}\, |m^0_{\mathbf{R}} (e_{\mathbf{R}0})^5|} \right]^{1/3} \equiv \Lambda^{\mathrm{II}}_{\mathbf{c.c.}} < 0$$

Vacuum III : $\mathcal{N} = 0$

$$t_* = -\mathrm{i} \left[\frac{12}{\sqrt{5}\,(e_{10})^2} \left| \frac{e_{\mathbf{R}0}}{m^0_{\mathbf{R}}} \right| \right]^{1/3} , \quad \xi^0_* = 0 \,, \quad \exp(\varphi_*) = \sqrt{5} \left[\frac{5\, e_{10}}{18\, m^0_{\mathbf{R}} (e_{\mathbf{R}0})^2} \right]^{1/3}$$

$$V_* = -\frac{25\sqrt{5}}{6} \left[\frac{5\,(e_{10})^4}{18\, |m^0_{\mathbf{R}} (e_{\mathbf{R}0})^5|} \right]^{1/3} \equiv \Lambda^{\mathrm{III}}_{\mathbf{c.c.}} < 0$$

Note: $m^0_{\mathbf{R}} > 0$; $\widetilde{\xi}_0$ **is not fixed ;** $\Lambda^{\mathrm{II}}_{\mathbf{c.c.}} < \Lambda^{\mathrm{I}}_{\mathbf{c.c.}} < \Lambda^{\mathrm{III}}_{\mathbf{c.c.}}$

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静的：

$$0 \;=\; \partial_t(\text{任意の場})$$

計量：

$$\mathrm{d}s^2 \;=\; -\mathrm{e}^{2A(r)}\mathrm{d}t^2 + \mathrm{e}^{-2A(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)} r^2 \big(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\big)$$

電荷・磁荷：

$$p^{\Lambda} \;=\; \frac{1}{4\pi}\int F_2^{\Lambda}\,, \qquad q_{\Lambda} \;=\; \frac{1}{4\pi}\int \widetilde{F}_{\Lambda 2} \quad \textbf{with} \;\; \widetilde{F}_{\mu\nu\Lambda} \;=\; \frac{\sqrt{-g}}{2}\epsilon_{\mu\nu\rho\sigma}\frac{\partial\mathscr{L}}{\partial F_{\mu\nu}^{\Lambda}}$$

$$F_{\theta\phi}^{\Lambda} \;\equiv\; f^{\Lambda}(\theta,\phi)\sin\theta\,, \qquad F_{tr}^{\Lambda} \;\equiv\; \frac{\mathrm{e}^{-2C}}{r^2}g^{\Lambda}(\theta,\phi)$$

Equation of motion for A_μ^Λ $(F_{\mu\nu}^\Lambda = 2\partial_{[\mu} A_{\nu]}^\Lambda + m_{\mathbf{R}}^\Lambda B_{\mu\nu})$ **:**

$$0 = \frac{\epsilon^{\sigma\mu\nu\rho}}{2\sqrt{-g}} \partial_\mu \widetilde{F}_{\Lambda\nu\rho} - \frac{\epsilon^{\sigma\mu\nu\rho}}{2\sqrt{-g}} \partial_\mu B_{\nu\rho} (e_{\mathbf{R}\Lambda} - e_{\Lambda 0} \xi^0) - \mathrm{e}^{2\varphi} e_{\Lambda 0} D^\sigma \widetilde{\xi}_0$$

$$\begin{aligned}
H_{r\theta\phi} &= 0 \,, \qquad H_{\theta\phi t} = 0 \,, \\
H_{\phi tr} &= \Big(\partial_\phi B_{tr} + \partial_r B_{\phi t} \Big) = \frac{1}{m_{\mathbf{R}}^\Sigma e_{\mathbf{R}\Sigma}} \frac{\mathrm{e}^{-2C}}{r^2} \partial_\phi \Big[e_{\mathbf{R}\Lambda} g^\Lambda(\theta,\phi) \Big] \,, \\
0 &= \partial_r \Big[\nu_{\Lambda\Sigma} f^\Sigma(\theta,\phi) - \mu_{\Lambda\Sigma} \, g^\Sigma(\theta,\phi) \Big] \,, \\
0 &= \partial_{\phi,\theta} \Big[(m_{\mathbf{R}}^\Lambda \mu_{\Lambda\Sigma}) f^\Sigma(\theta,\phi) + \big(m_{\mathbf{R}}^\Lambda \nu_{\Lambda\Sigma} - e_{\mathbf{R}\Sigma} \big) g^\Sigma(\theta,\phi) \Big] \,.
\end{aligned}$$

$\rightarrow$

$$\begin{aligned}
0 &= D_t \widetilde{\xi}_0 = -e_{\Lambda 0} A_t^\Lambda \quad \rightarrow \quad e_{\Lambda 0} F_{tr}^\Lambda = 0 \quad \rightarrow \quad e_{\Lambda 0} g^\Lambda(\theta,\phi) = 0 \,, \\
0 &= D_r \widetilde{\xi}_0 = \partial_r \widetilde{\xi}_0 - e_{\Lambda 0} A_r^\Lambda \,, \\
0 &= \frac{\mathrm{e}^{-2C}}{r^2} \partial_\phi \Big[\mu_{\Lambda\Sigma} f^\Sigma(\theta,\phi) + \Big(\nu_{\Lambda\Sigma} - \frac{e_{\mathbf{R}\Lambda} - e_{\Lambda 0}\xi^0}{m_{\mathbf{R}}^\Gamma e_{\mathbf{R}\Gamma}} e_{\mathbf{R}\Sigma} \Big) g^\Sigma(\theta,\phi) \Big] + e_{\Lambda 0} \, \mathrm{e}^{2\varphi} \sin\theta \, D_\theta \widetilde{\xi}_0 \,, \\
0 &= \frac{\mathrm{e}^{-2C}}{r^2} \partial_\theta \Big[\mu_{\Lambda\Sigma} f^\Sigma(\theta,\phi) + \Big(\nu_{\Lambda\Sigma} - \frac{e_{\mathbf{R}\Lambda} - e_{\Lambda 0}\xi^0}{m_{\mathbf{R}}^\Gamma e_{\mathbf{R}\Gamma}} e_{\mathbf{R}\Sigma} \Big) g^\Sigma(\theta,\phi) \Big] - e_{\Lambda 0} \frac{\mathrm{e}^{2\varphi}}{\sin\theta} D_\phi \widetilde{\xi}_0 \,.
\end{aligned}$$

Equation of motion for $B_{\mu\nu}$:

$$
\begin{aligned}
0 \;&=\; \frac{1}{\sqrt{-g}} \partial_\mu \Big(\mathrm{e}^{-4\varphi} \sqrt{-g} H^{\mu\rho\sigma} \Big) + \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} \Big[D_\mu \xi^0 D_\nu \widetilde{\xi}_0 - D_\mu \widetilde{\xi}_0 D_\nu \xi^0 + (e_{\mathbf{R}\Lambda} - e_{\Lambda 0} \xi^0) F_{\mu\nu}^\Lambda \Big] \\
&\quad + 2 m_{\mathbf{R}}^\Lambda \mu_{\Lambda\Sigma} F^{\Sigma\rho\sigma} - \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} m_{\mathbf{R}}^\Lambda \nu_{\Lambda\Sigma} F_{\mu\nu}^\Sigma
\end{aligned}
$$

$$
\rightarrow \quad
\boxed{
\begin{aligned}
0 \;&=\; - \frac{1}{m_{\mathbf{R}}^\Sigma e_{\mathbf{R}\Sigma}} \frac{\mathrm{e}^{-2C}}{r^2} \partial_\theta \Big[\mathrm{e}^{-4\varphi} \sin\theta \, \partial_\theta \Big(e_{\mathbf{R}\Lambda} g^\Lambda(\theta,\phi) \Big) \Big] - \frac{1}{m_{\mathbf{R}}^\Sigma e_{\mathbf{R}\Sigma}} \frac{\mathrm{e}^{-2C}}{r^2 \sin\theta} \partial_\phi \Big[\mathrm{e}^{-4\varphi} \partial_\phi \Big(e_{\mathbf{R}\Lambda} g^\Lambda(\theta,\phi) \Big) \Big] \\
&\quad + 2 \Big[\Big(m_{\mathbf{R}}^\Lambda \nu_{\Lambda\Sigma} - (e_{\mathbf{R}\Sigma} - e_{\Sigma 0} \, \xi^0) \Big) f^\Sigma(\theta,\phi) - (m_{\mathbf{R}}^\Lambda \mu_{\Lambda\Sigma}) g^\Sigma(\theta,\phi) \Big] \sin\theta \\
&\quad - 2 \Big(D_\theta \xi^0 D_\phi \widetilde{\xi}_0 - D_\theta \widetilde{\xi}_0 D_\phi \xi^0 \Big) \,, \\
0 \;&=\; \frac{\sin\theta}{m_{\mathbf{R}}^\Sigma e_{\mathbf{R}\Sigma}} \partial_\theta \Big(e_{\mathbf{R}\Lambda} g^\Lambda(\theta,\phi) \Big) \partial_r \Big(\frac{\mathrm{e}^{-4\varphi - 2C}}{r^2} \Big) + 2 D_r \xi^0 D_\phi \widetilde{\xi}_0 \,, \\
0 \;&=\; \frac{1}{m_{\mathbf{R}}^\Sigma e_{\mathbf{R}\Sigma}} \frac{1}{\sin\theta} \partial_\phi \Big(e_{\mathbf{R}\Lambda} g^\Lambda(\theta,\phi) \Big) \partial_r \Big(\frac{\mathrm{e}^{-4\varphi - 2C}}{r^2} \Big) - 2 D_r \xi^0 D_\theta \widetilde{\xi}_0 \,, \\
0 \;&=\; \frac{2\mathrm{e}^{-4C}}{r^4 \sin\theta} \Big[(m_{\mathbf{R}}^\Lambda \mu_{\Lambda\Sigma}) f^\Sigma(\theta,\phi) + (m_{\mathbf{R}}^\Lambda \nu_{\Lambda\Sigma} - e_{\mathbf{R}\Sigma}) g^\Sigma(\theta,\phi) \Big]
\end{aligned}
}
$$

Equation of motion for $g_{\mu\nu}$:

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} \ =\ & \frac{1}{4} g_{\mu\nu}\, \mu_{\Lambda\Sigma} F^{\Lambda}_{\rho\sigma} F^{\Sigma\rho\sigma} - \mu_{\Lambda\Sigma} F^{\Lambda}_{\mu\rho} F^{\Sigma}_{\nu\sigma}\, g^{\rho\sigma} - g_{\mu\nu}\, g_{\mathfrak{t}\bar{\mathfrak{t}}} \partial_{\rho} \mathfrak{t} \partial^{\rho} \bar{\mathfrak{t}} + 2 g_{\mathfrak{t}\bar{\mathfrak{t}}}\, \partial_{\mu} \mathfrak{t} \partial_{\nu} \bar{\mathfrak{t}} \\
& - g_{\mu\nu}\, \partial_{\rho} \varphi \partial^{\rho} \varphi + 2 \partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{\mathrm{e}^{-4\varphi}}{24} g_{\mu\nu}\, H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} + \frac{\mathrm{e}^{-4\varphi}}{4} H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma} \\
& - \frac{\mathrm{e}^{2\varphi}}{2} g_{\mu\nu} \Big(D_{\rho} \xi^0 D^{\rho} \xi^0 + D_{\rho} \widetilde{\xi}_0 D^{\rho} \widetilde{\xi}_0 \Big) + \mathrm{e}^{2\varphi} \Big(D_{\mu} \xi^0 D_{\nu} \xi^0 + D_{\mu} \widetilde{\xi}_0 D_{\nu} \widetilde{\xi}_0 \Big) \\
& - g_{\mu\nu} V
\end{aligned}
$$

From now on we focus on

$$
\mathrm{e}^{2A(r)} \ =\ 1 - \frac{2\eta}{r} + \frac{\mathcal{Z}^2}{r^2} + \frac{r^2}{\ell^2}\, , \qquad \mathrm{e}^{2C(r)} \ =\ 1
$$

$$g^{tt}E_{tt} - g^{rr}E_{rr} \;=\; 0 \;=\; -2\mathrm{e}^{2A(r)}\Big[g_{\mathrm{t}\bar{\mathrm{t}}}|\partial_r \mathrm{t}|^2 + (\partial_r\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_r\xi^0)^2\Big]$$

$$g^{rr}E_{rr} + g^{\theta\theta}E_{\theta\theta} \;=\; \frac{6}{\ell^2} \;=\; -\frac{2}{r^2\sin^2\theta}\Big[g_{\mathrm{t}\bar{\mathrm{t}}}|\partial_\phi \mathrm{t}|^2 + (\partial_\phi\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\phi\xi^0)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\phi\widetilde{\xi}_0)^2\Big] - 2V$$

$$g^{rr}E_{rr} - g^{\theta\theta}E_{\theta\theta} \;=\; -\frac{2\mathcal{Z}^2}{r^4} \;=\; \frac{1}{r^4}\mu_{\Lambda\Sigma}\Big[f^\Lambda(\theta,\phi)f^\Sigma(\theta,\phi) + g^\Lambda g^\Sigma\Big]$$
$$-\frac{2}{r^2}\Big[g_{\mathrm{t}\bar{\mathrm{t}}}|\partial_\theta \mathrm{t}|^2 + (\partial_\theta\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\theta\xi^0)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\theta\widetilde{\xi}_0)^2\Big]$$

$$g^{\theta\theta}E_{\theta\theta} - g^{\phi\phi}E_{\phi\phi} \;=\; 0 \;=\; \frac{1}{r^2}\Big[g_{\mathrm{t}\bar{\mathrm{t}}}|\partial_\theta \mathrm{t}|^2 + (\partial_\theta\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\theta\xi^0)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\theta\widetilde{\xi}_0)^2\Big]$$
$$-\frac{1}{r^2\sin^2\theta}\Big[g_{\mathrm{t}\bar{\mathrm{t}}}|\partial_\phi \mathrm{t}|^2 + (\partial_\phi\varphi)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\phi\xi^0)^2 + \frac{\mathrm{e}^{2\varphi}}{2}(D_\phi\widetilde{\xi}_0)^2\Big]$$

$\rightarrow$

$$0 = \partial_r\mathrm{t} = \partial_\theta\mathrm{t} = \partial_\phi\mathrm{t}\,, \quad 0 = \partial_r\varphi = \partial_\theta\varphi = \partial_\phi\varphi$$
$$0 = D_r\xi^0 = D_\theta\xi^0 = D_\phi\xi^0\,, \quad 0 = D_\theta\widetilde{\xi}_0 = D_\phi\widetilde{\xi}_0$$
$$0 = H_{tr\theta} = H_{tr\phi}$$

and $f^\Lambda = p^\Lambda\,, \quad g^\Lambda = -(\mu^{-1})^{\Lambda\Sigma}\big(q_\Sigma - \nu_{\Sigma\Gamma}\,p^\Gamma\big)$

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