Supersymmetric Domain Walls

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Contents

登場人物紹介

p-branes :　D 次元時空に存在する、空間方向に p 次元広がった物体

$p \leq D-4$: standard branes
$p = D-3$: defect branes
$p = D-2$: Domain Walls

張力は $T_p \sim (g_s)^{+\alpha}$ で特徴付けられる $(l_s = 1)$

$\alpha = 0$: fundamental
$\alpha = -1$: Dirichlet
$\alpha = -2$: solitonic

$S_{p\text{-brane}} = T_p \int$ (時空を這う部分の体積) $+$ (時空上の場と brane 上の場の結合)

Dirac-Born-Infeld type　　Wess-Zumino type

良く知っている例：$D = 10$ IIA/IIB 型超弦理論

$D=10$	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$
$\alpha=0$		$\mathrm{F1}_{\mathrm{IIA/IIB}}$								
$\alpha=-1$	$\mathrm{D0}_{\mathrm{IIA}}$	$\mathrm{D1}_{\mathrm{IIB}}$	$\mathrm{D2}_{\mathrm{IIA}}$	$\mathrm{D3}_{\mathrm{IIB}}$	$\mathrm{D4}_{\mathrm{IIA}}$	$\mathrm{D5}_{\mathrm{IIB}}$	$\mathrm{D6}_{\mathrm{IIA}}$	$(\mathrm{D7})_{\mathrm{IIB}}$	$(\mathrm{D8})_{\mathrm{IIA}}$	$(\mathrm{D9})_{\mathrm{IIB}}$
$\alpha=-2$						$\mathrm{NS5}_{\mathrm{IIA/IIB}}$				

良く知っている例：$D = 10$ IIA/IIB 型超弦理論

$D=10$	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$
$\alpha=0$		$\mathrm{F1}_{\mathrm{IIA/IIB}}$								
$\alpha=-1$	$\mathrm{D0}_{\mathrm{IIA}}$	$\mathrm{D1}_{\mathrm{IIB}}$	$\mathrm{D2}_{\mathrm{IIA}}$	$\mathrm{D3}_{\mathrm{IIB}}$	$\mathrm{D4}_{\mathrm{IIA}}$	$\mathrm{D5}_{\mathrm{IIB}}$	$\mathrm{D6}_{\mathrm{IIA}}$	$(\mathrm{D7})_{\mathrm{IIB}}$	$(\mathrm{D8})_{\mathrm{IIA}}$	$(\mathrm{D9})_{\mathrm{IIB}}$
$\alpha=-2$						$\mathrm{NS5}_{\mathrm{IIA/IIB}}$				

$p \leq 6$ については、**力学的な**テンソル場の sources である:

$$\mathrm{F1} \sim B_{(2)}, \quad \mathrm{NS5} \sim B_{(6)}, \quad \mathrm{D}p \sim C_{(p+1)}\ (p \leq 3), \quad \mathrm{D}p' \sim C_{(p'+1)}\ (p' > 4)$$

$$\mathrm{d}B_{(2)} = *_{10}\mathrm{d}B_{(6)}, \quad \mathrm{d}C_{(p+1)} = *_{10}\mathrm{d}C_{(7-p)} \equiv *_{10}\mathrm{d}C_{(p'+1)}$$

Dp	=	standard branes ($p \leq 6$)	$\sim$	RR potentials $C_{(p+1)}$
D7	=	defect branes	$\sim$	scalar fields ($+\alpha$)
D8	=	Domain Walls	$\sim$	Romans' mass (**変形パラメータ**)
D9	=	spacetime-filling branes	$\sim$	I 型超弦理論へ

動機

低次元超重力理論における Domain Walls **の役割は何か？**

(10 **次元では** D8-brane $\sim$ Romans mass **だった**)

極大超重力理論 $=$ 32**個**(**最大**)**の超対称生成子を持つ重力理論**

重力場・グラビティーノ・ベクトルゲージ場・テンソルゲージ場・スピノル場・スカラー場

超対称性によって、スカラー場が住む空間が coset space G_0/H **に定まっている**

D	U-duality G_0	R-対称性 H	$\dim(G_0/H)$	T-duality
11	1	1	0	1
IIA	$\mathbb{R}^+$	1	1	1
IIB	$SL(2,\mathbb{R})$	$SO(2)$	2	1
9	$GL(2,\mathbb{R})$	$SO(2)$	3	$SO(1,1)$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	7	$SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$
7	$SL(5,\mathbb{R})$	$Sp(2)$	14	$SL(4,\mathbb{R})$
6	$SO(5,5)$	$Sp(2) \times Sp(2)$	25	$SO(4,4)$
5	$E_{6(6)}$	$USp(8)$	42	$SO(5,5)$
4	$E_{7(7)}$	$SU(8)$	70	$SO(6,6)$
3	$E_{8(8)}$	$SO(16)$	128	$SO(7,7)$

Domain Walls **は極大超重力理論にどのような変形をもたらすか？**

- D8-brane in 10-dim.

Ramond-Ramond potential $C_{(9)}$ **の源**

$*_{10}\mathrm{d}C_{(9)} = m$ **(定数) を与え、**IIA **型超重力理論を変形する**

$\rightarrow$ Romans' massive IIA SUGRA

Domain Walls **は極大超重力理論にどのような変形をもたらすか？**

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- $(D-2)$-branes in D-dim.

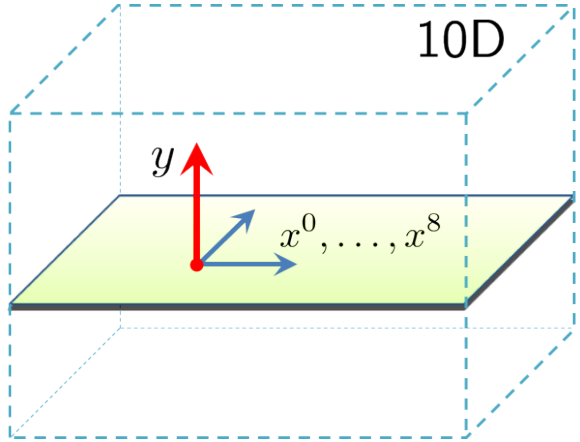
  各次元にいくつの SUSY Domain Walls **が存在するのか？**

  超重力理論における Domain Walls **の役割は何か？**

分析 1

SUSY Domain Walls と Wess-Zumino 項

7 次元理論における Domain Walls



D8-brane **解**

$$ds_{10}^2 = H^{\frac{9}{8}}(y)\,dy^2 + H^{\frac{1}{8}}(y)\,ds_9^2 \qquad \textbf{計量}$$

$$e^{\phi} = H^{\frac{5}{4}}(y) \qquad \text{dilaton}$$

$$C_{012\cdots 8} = \pm\frac{1}{H(y)}, \quad m = \pm\partial_y H(y) \qquad \text{RR potential (Romans mass)}$$

"mass" $m \neq 0$ **のため、**RR potentials **のゲージ変換が変更される**

$$\delta B_2 = d\Sigma_1, \quad \delta C_1 = -m\Sigma_1$$

$$\to \mathcal{F}_2 \equiv dC_1 + mB_2 : \qquad \text{Stückelberg pairing}$$

gauging	C_1	B_2	C_3	C_5	B_6	C_7
m	eaten	massive	massless	massless	eaten	massive

D8-brane に関する場とゲージ変換　(D8 からの back reaction がないとして)

$$\begin{aligned} \delta C_9 &= d\lambda_8 + H_3 \wedge \lambda_6 && : \text{RR tensor in bulk} \\ \delta B_2 &= d\Sigma_1 && : \text{NSNS tensor in bulk} \\ \delta X &= 0 && : \text{transverse scalar} \\ \delta b_\mu &= d\Sigma_0 - \Sigma_1 && : \text{D8-brane 上にのみ住む「ゲージ場」} \end{aligned}$$

この D8-brane 上には SUSY 多重項が乗っている

$\{X, b_\mu; \psi\}$: on-shell ($8_{\text{boson}} + 8_{\text{fermion}}$) 自由度

これらを用いて、ゲージ不変でSUSY不変な相互作用項を構成する： Wess-Zumino 項

$$\mathscr{L}_{\text{WZ}} = C_9 + C_7 \wedge \mathcal{F}_2 + \ldots = \left(C \wedge e^{\mathcal{F}_2}\right)_9$$

$$\mathcal{F}_2 = db_1 + B_2 \,, \quad H_3 = dB_2$$

(注意) 本当は $m \neq 0$ で具体的に記述したいが…

D 次元理論における "Wess-Zumino" 項

$$\mathscr{L}_{\mathrm{WZ}} \equiv (A \wedge \mathrm{e}^{\mathcal{F}})_{D-1}$$

A : D 次元時空を伝播するテンソル場 (超重力理論)

$\mathcal{F}$: Domain Walls 上を伝播するテンソル場

A, $\mathcal{F}$ は共に D 次元理論の U-duality G_0 に対して変換する

1. どの表現に従う $(A, \mathcal{F})$ の組が SUSY 多重項を持ち得るか
2. SUSY 多重項が乗る Domain Walls はいくつあるか

D	U-duality G_0	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
IIA	$\mathbb{R}^+$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	–	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}$
IIB	$SL(2,\mathbb{R})$	–	$\mathbf{2}$	–	$\mathbf{1}$	–	$\mathbf{2}$	–	$\mathbf{3}$	–	$\mathbf{4}\oplus\mathbf{2}$
9	$GL(2,\mathbb{R})$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{3}\oplus\mathbf{1}$	$\mathbf{3}\oplus\mathbf{2}$	$\mathbf{4}\oplus\mathbf{2}\oplus\mathbf{2}$	–
8	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $\oplus(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $\oplus(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $\oplus(\mathbf{3},\mathbf{3})$ $\oplus(\mathbf{3},\mathbf{1})$ $\oplus(\mathbf{3},\mathbf{1})$	–	–
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}}\oplus\overline{\mathbf{15}}$	$\mathbf{70}$ $\oplus\mathbf{45}$ $\oplus\mathbf{5}$	–	–	–
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\mathbf{320}$ $\oplus\overline{\mathbf{126}}$ $\oplus\mathbf{10}$	–	–	–	–
5	$E_{6(6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\overline{\mathbf{1728}}$ $\oplus\overline{\mathbf{27}}$	–	–	–	–	–
4	$E_{7(7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{8645}$ $\oplus\mathbf{133}$	–	–	–	–	–	–
3	$E_{8(8)}$	$\mathbf{248}$	$\mathbf{3875}\oplus\mathbf{1}$	$\mathbf{147250}$ $\oplus\mathbf{3875}$ $\oplus\mathbf{248}$	–	–	–	–	–	–	–

F.Riccioni, D.Steele and P.West, arXiv:0906.1177

D	U-duality G_0	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
IIA	$\mathbb{R}^+$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	–	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1} \oplus \mathbf{1}$
IIB	$SL(2,\mathbb{R})$	–	$\mathbf{2}$	–	$\mathbf{1}$	–	$\mathbf{2}$	–	$\mathbf{3}$	–	$\mathbf{4} \oplus \mathbf{2}$
9	$GL(2,\mathbb{R})$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{3} \oplus \mathbf{1}$	$\mathbf{3} \oplus \mathbf{2}$	$\mathbf{4} \oplus \mathbf{2} \oplus \mathbf{2}$	–
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $\oplus(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $\oplus(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $\oplus(\mathbf{3},\mathbf{3})$ $\oplus(\mathbf{3},\mathbf{1})$ $\oplus(\mathbf{3},\mathbf{1})$	–	–
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}} \oplus \overline{\mathbf{15}}$	$\mathbf{70}$ $\oplus\mathbf{45}$ $\oplus\mathbf{5}$	–	–	–
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\mathbf{320}$ $\oplus\overline{\mathbf{126}}$ $\oplus\mathbf{10}$	–	–	–	–
5	$E_{6(6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\overline{\mathbf{1728}}$ $\oplus\overline{\mathbf{27}}$	–	–	–	–	–
4	$E_{7(7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{8645}$ $\oplus\mathbf{133}$	–	–	–	–	–	–
3	$E_{8(8)}$	$\mathbf{248}$	$\mathbf{3875} \oplus \mathbf{1}$	$\mathbf{147250}$ $\oplus\mathbf{3875}$ $\oplus\mathbf{248}$	–	–	–	–	–	–	–

Domain walls **に結合するテンソル場**: $(D-1)$-forms

F.Riccioni, D.Steele and P.West, arXiv:0906.1177

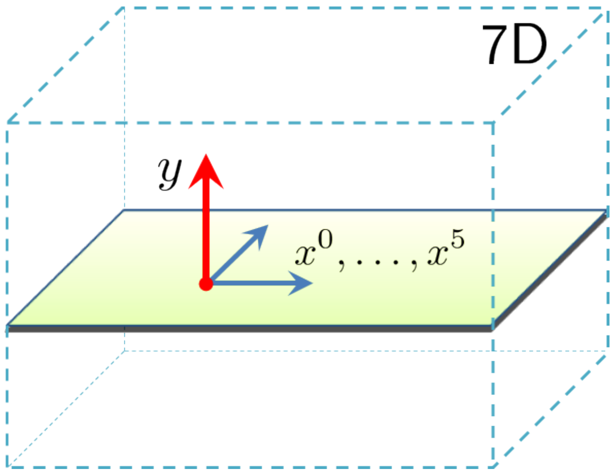
D	U-duality G_0	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
IIA	$\mathbb{R}^+$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	–	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}$
IIB	$SL(2,\mathbb{R})$	–	$\mathbf{2}$	–	$\mathbf{1}$	–	$\mathbf{2}$	–	$\mathbf{3}$	–	$\mathbf{4}\oplus\mathbf{2}$
9	$GL(2,\mathbb{R})$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{3}\oplus\mathbf{1}$	$\mathbf{3}\oplus\mathbf{2}$	$\mathbf{4}\oplus\mathbf{2}\oplus\mathbf{2}$	–
8	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $\oplus(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $\oplus(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $\oplus(\mathbf{3},\mathbf{3})$ $\oplus(\mathbf{3},\mathbf{1})$ $\oplus(\mathbf{3},\mathbf{1})$	–	–
7	$SL(5)$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}}\oplus\overline{\mathbf{15}}$	$\mathbf{70}$ $\oplus\mathbf{45}$ $\oplus\mathbf{5}$	–	–	–
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\mathbf{320}$ $\oplus\overline{\mathbf{126}}$ $\oplus\mathbf{10}$	–	–	–	–
5	$E_{6(6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\overline{\mathbf{1728}}$ $\oplus\overline{\mathbf{27}}$	–	–	–	–	–
4	$E_{7(7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{8645}$ $\oplus\mathbf{133}$	–	–	–	–	–	–
3	$E_{8(8)}$	$\mathbf{248}$	$\mathbf{3875}\oplus\mathbf{1}$	$\mathbf{147250}$ $\oplus\mathbf{3875}$ $\oplus\mathbf{248}$	–	–	–	–	–	–	–

7 次元理論で議論しよう

F.Riccioni, D.Steele and P.West, arXiv:0906.1177

7 次元理論での Domain Walls = 5-branes

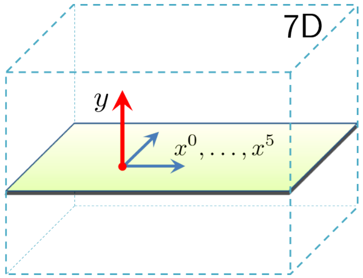
7D テンソル場 A		$G_0 = SL(5,\mathbb{R})$ の表現
$A_{1,[MN]}$	1-form	$\overline{\mathbf{10}}$
A_2^M	2-form	$\mathbf{5}$
$A_{3,M}$	3-form	$\overline{\mathbf{5}}$
$A_4^{[MN]}$	4-form	$\mathbf{10}$
$A_{5,M}{}^N$	5-form	$\mathbf{24}$ (adjoint)
$A_{6,(MN)} \oplus A_6^{[MN],P}$	6-forms	$\overline{\mathbf{15}} \oplus \overline{\mathbf{40}}$



$(M, N = 1, \ldots, 5$ of $SL(5,\mathbb{R}))$

$\overline{\mathbf{15}}$ 表現：

$$\mathscr{L}_{\text{WZ}}^{\overline{\mathbf{15}}} \sim A_{6,(MN)} + A_{5,(M}{}^{P} \wedge \mathcal{F}_{1,N)P} + A_{3,(M} \wedge \mathcal{F}_{3,N)} + \ldots$$

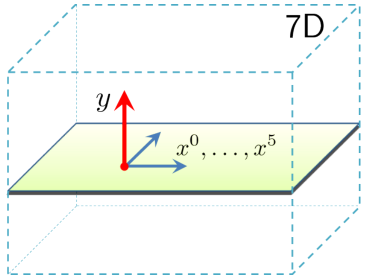


$\overline{\mathbf{15}}$ 表現：

$$\mathscr{L}_{\mathrm{WZ}}^{\overline{\mathbf{15}}} \sim A_{6,(MN)} + A_{5,(M}{}^{P} \wedge \mathcal{F}_{1,N)P} + A_{3,(M} \wedge \mathcal{F}_{3,N)} + \ldots$$

5-brane上の場：

$\mathcal{F}_{3,N} = \mathrm{d}b_{2,N} + A_{3,N}$	自己双対テンソル場
$\mathcal{F}_{1,NP} = \mathrm{d}b_{0,NP} + A_{1,NP}$	スカラー場
X^y	スカラー場

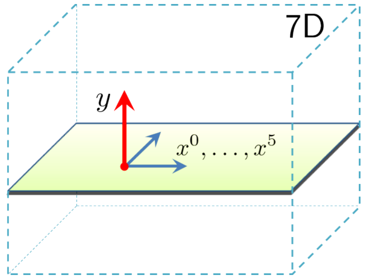


$\overline{\mathbf{15}}$ 表現：

$$\mathscr{L}_{\mathsf{WZ}}^{\overline{\mathbf{15}}} \sim A_{6,(MN)} + A_{5,(M}{}^{P} \wedge \mathcal{F}_{1,N)P} + A_{3,(M} \wedge \mathcal{F}_{3,N)} + \ldots$$

5-brane上の場：

$$\begin{cases} \mathcal{F}_{3,N} = \mathrm{d}b_{2,N} + A_{3,N} & \text{自己双対テンソル場} \\ \mathcal{F}_{1,NP} = \mathrm{d}b_{0,NP} + A_{1,NP} & \text{スカラー場} \\ X^{y} & \text{スカラー場} \end{cases}$$



- $M = N(=1)$ case :

$b_{2,N=1}$:	${}_4C_2/2 = 3$
$b_{0,[N=1,P]}$:	$1 \times 4 = 4$
X^{y} :	1

$\leftarrow$ $8_{\text{boson}} + 8_{\text{fermion}} \leftarrow \frac{1}{2}$-SUSY!

各 $M = 1, \ldots, 5$ において成立

$\overline{\mathbf{15}}$ 表現：

$$\mathscr{L}_{\text{WZ}}^{\overline{\mathbf{15}}} \sim A_{6,(MN)} + A_{5,(M}{}^{P} \wedge \mathcal{F}_{1,N)P} + A_{3,(M} \wedge \mathcal{F}_{3,N)} + \dots$$

5-brane上の場：

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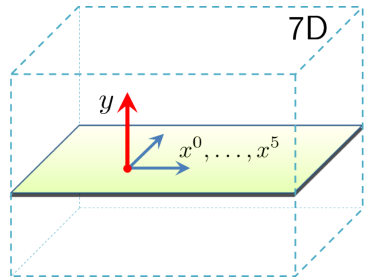


- $M = N(= 1)$ case :

$$\begin{array}{rl} b_{2,N=1}: & {}_4C_2/2 = 3 \\ b_{0,[N=1,P]}: & 1 \times 4 = 4 \\ X^y: & 1 \end{array}$$

$\leftarrow$ $8_{\text{boson}} + 8_{\text{fermion}} \leftarrow \frac{1}{2}$-SUSY!

各 $M = 1, \dots, 5$ において成立

- $M \neq N$ case :

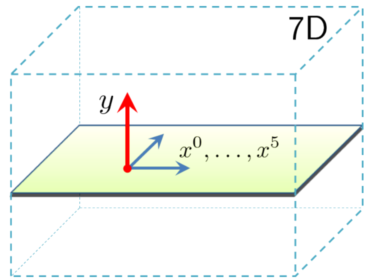
boson **自由度の和が**4**の倍数にならない** $\rightarrow$ SUSY **多重項を組めない**

$\overline{\mathbf{15}}$ 表現：

$$\mathscr{L}_{\rm WZ}^{\overline{\mathbf{15}}} \sim A_{6,(MN)} + A_{5,(M}{}^{P} \wedge \mathcal{F}_{1,N)P} + A_{3,(M} \wedge \mathcal{F}_{3,N)} + \dots$$

5-brane **上の場：**

$$\begin{cases} \mathcal{F}_{3,N} = \mathrm{d}b_{2,N} + A_{3,N} & \text{自己双対テンソル場} \\ \mathcal{F}_{1,NP} = \mathrm{d}b_{0,NP} + A_{1,NP} & \text{スカラー場} \\ X^{y} & \text{スカラー場} \end{cases}$$



- $M = N(=1)$ case： SUSY **多重項を持つ** 5-branes **は 5 通り**

$b_{2,N=1}$	:	${}_4C_2/2 = 3$
$b_{0,[N=1,P]}$	:	$1 \times 4 = 4$
X^y	:	1

$\leftarrow$ $8_{\rm boson} + 8_{\rm fermion} \leftarrow \frac{1}{2}$-SUSY!

各 $M = 1, \dots, 5$ **において成立**

$5 < 15$

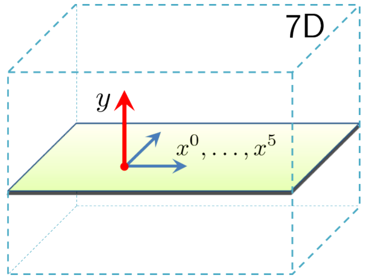
Elementary SUSY DWs

- $M \neq N$ case： SUSY **多重項を持つ** 5-branes **はない**

boson **自由度の和が** 4 **の倍数にならない** $\rightarrow$ SUSY **多重項を組めない**

$\overline{\mathbf{40}}$ 表現：

$$\mathscr{L}^{\overline{\mathbf{40}}}_{\mathrm{WZ}} \sim A_6^{[MN],P} + A_{5,Q}{}^{P} \wedge \mathcal{F}_{1,RS}\, \epsilon^{MNQRS} + A_4^{[MN]} \wedge \mathcal{F}_2^{P} - (\cdots)^{[MNP]} + \ldots$$

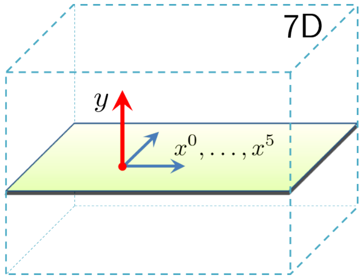


$\overline{\mathbf{40}}$ 表現：

$$\mathscr{L}^{\overline{\mathbf{40}}}_{\mathrm{WZ}} \sim A_6^{[MN],P} + A_{5,Q}{}^{P} \wedge \mathcal{F}_{1,RS}\, \epsilon^{MNQRS} + A_4^{[MN]} \wedge \mathcal{F}_2^P - (\cdots)^{[MNP]} + \ldots$$

5-brane上の場：

$$\begin{cases} \mathcal{F}_2^P = \mathrm{d}b_1^P + A_2^P & \text{ベクトル場} \\ \mathcal{F}_{1,RS} = \mathrm{d}b_{0,RS} + A_{1,RS} & \text{スカラー場} \\ X^y & \text{スカラー場} \end{cases}$$

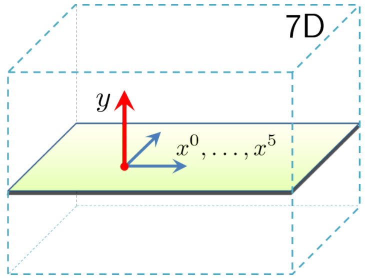


$\overline{\mathbf{40}}$ 表現：

$$\mathscr{L}_{\rm WZ}^{\overline{\mathbf{40}}} \sim A_6^{[MN],P} + A_{5,Q}{}^P \wedge \mathcal{F}_{1,RS}\, \epsilon^{MNQRS} + A_4^{[MN]} \wedge \mathcal{F}_2^P - (\cdots)^{[MNP]} + \ldots$$

5-brane上の場：

$$\begin{cases} \mathcal{F}_2^P = {\rm d}b_1^P + A_2^P & \text{ベクトル場} \\ \mathcal{F}_{1,RS} = {\rm d}b_{0,RS} + A_{1,RS} & \text{スカラー場} \\ X^y & \text{スカラー場} \end{cases}$$



- $P = M (= 1 \neq N)$ case：

- $P \neq M \neq N$ case：

$\overline{\mathbf{40}}$ 表現：

$$\mathscr{L}_{\mathsf{WZ}}^{\overline{\mathbf{40}}} \sim A_6^{[MN],P} + A_{5,Q}{}^{P} \wedge \mathcal{F}_{1,RS}\, \epsilon^{MNQRS} + A_4^{[MN]} \wedge \mathcal{F}_2^{P} - (\cdots)^{[MNP]} + \ldots$$

5-brane上の場：

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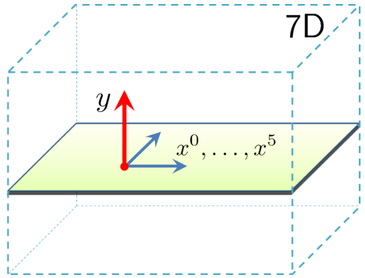


- $P = M (= 1 \neq N)$ case：

b_1^P	:	4
$b_{0,[RS]}$	:	$1 \times 3 = 3$
X^y	:	1

$\leftarrow$ $8_{\mathsf{boson}} + 8_{\mathsf{fermion}} \leftarrow \frac{1}{2}$-SUSY!

各 M において成立 $({}_5C_2 = 20)$

- $P \neq M \neq N$ case：

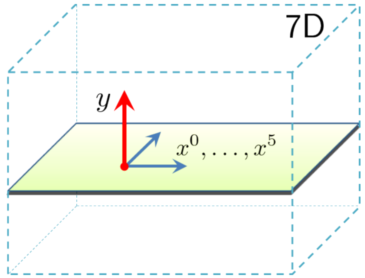
boson **自由度の和が4の倍数にならない** $\rightarrow$ SUSY **多重項を組めない**

$\overline{\mathbf{40}}$ 表現：

$$\mathscr{L}_{\mathrm{WZ}}^{\overline{\mathbf{40}}} \sim A_6^{[MN],P} + A_{5,Q}{}^{P} \wedge \mathcal{F}_{1,RS}\,\epsilon^{MNQRS} + A_4^{[MN]} \wedge \mathcal{F}_2^{P} - (\cdots)^{[MNP]} + \dots$$

5-brane 上の場：

$$\begin{cases} \mathcal{F}_2^P = \mathrm{d}b_1^P + A_2^P & \text{ベクトル場} \\ \mathcal{F}_{1,RS} = \mathrm{d}b_{0,RS} + A_{1,RS} & \text{スカラー場} \\ X^y & \text{スカラー場} \end{cases}$$



- $P = M (= 1 \neq N)$ case： SUSY 多重項を持つ 5-branes は 20 通り

b_1^P :	4
$b_{0,[RS]}$:	$1 \times 3 = 3$
X^y :	1

$\leftarrow$ $8_{\text{boson}} + 8_{\text{fermion}} \leftarrow \frac{1}{2}$-SUSY!

各 M において成立 $({}_5C_2 = 20)$

$20 < 40$

Elementary SUSY DWs

- $P \neq M \neq N$ case： SUSY 多重項を持つ 5-branes はない

boson 自由度の和が 4 の倍数にならない $\rightarrow$ SUSY 多重項を組めない

				fundamental	Dirichlet	solitonic			(brane's tension) $\sim (g_s)^{+\alpha}$
D	U	T	# of EDWs	$\alpha = 0$	$\alpha = -1$	$\alpha = -2$	$\alpha = -3$	$\alpha = -4$	$\alpha = -5$
IIA	$\mathbb{R}^+$	1	1		1				
9	$GL(2,\mathbb{R})$	$SO(1,1)$	$2 \subset \mathbf{3}_{\mathrm{U}}$		1	–	1		
8	$SL(3,\mathbb{R})$ $\times SL(2,\mathbb{R})$	$SL(2,\mathbb{R})$ $\times SL(2,\mathbb{R})$	$6 \subset (\mathbf{6},\mathbf{2})_{\mathrm{U}}$		$(\mathbf{1},\mathbf{2})_{\mathrm{T}}$	–	$4 \subset (\mathbf{3},\mathbf{2})_{\mathrm{T}}$		
7	$SL(5,\mathbb{R})$	$SL(4,\mathbb{R})$	$20 \subset \overline{\mathbf{40}}_{\mathrm{U}}$		$\mathbf{4}_{\mathrm{T}}$	$4 \subset \mathbf{10}_{\mathrm{T}}$	$12 \subset \overline{\mathbf{20}}_{\mathrm{T}}$		
			$5 \subset \overline{\mathbf{15}}_{\mathrm{U}}$			$4 \subset \overline{\mathbf{10}}_{\mathrm{T}}$	–	$\mathbf{1}_{\mathrm{T}}$	
6	$SO(5,5)$	$SO(4,4)$	$80 \subset \overline{\mathbf{144}}_{\mathrm{U}}$		$\mathbf{8}_{\mathrm{S}}\vert_{\mathrm{T}}$	$32 \subset \mathbf{56}_{\mathrm{C}}\vert_{\mathrm{T}}$	$32 \subset \mathbf{56}_{\mathrm{S}}\vert_{\mathrm{T}}$	$\mathbf{8}_{\mathrm{C}}\vert_{\mathrm{T}}$	
5	$E_{6(6)}$	$SO(5,5)$	$216 \subset \mathbf{351}_{\mathrm{U}}$		$\overline{\mathbf{16}}_{\mathrm{T}}$	$80 \subset \mathbf{120}_{\mathrm{T}}$	$80 \subset \mathbf{144}_{\mathrm{T}}$	$40 \subset \mathbf{45}_{\mathrm{T}}$	
4	$E_{7(7)}$	$SO(6,6)$	$576 \subset \mathbf{912}_{\mathrm{U}}$		$\mathbf{32}_{\mathrm{T}}$	$160 \subset \mathbf{220}_{\mathrm{T}}$	$192 \subset \mathbf{352}_{\mathrm{T}}$	$160 \subset \mathbf{220}_{\mathrm{T}}$	$\mathbf{32}_{\mathrm{T}}$
3	$E_{8(8)}$	$SO(7,7)$	$2160 \subset \mathbf{3875}_{\mathrm{U}}$	$\mathbf{1}_{\mathrm{T}}$	$\overline{\mathbf{64}}_{\mathrm{T}}$	$280 \subset \mathbf{364}_{\mathrm{T}}$	$448 \subset \mathbf{832}_{\mathrm{T}}$	$560 \subset \mathbf{1001}_{\mathrm{T}}$ $14 \subset \mathbf{104}_{\mathrm{T}}$	$448 \subset \overline{\mathbf{832}}_{\mathrm{T}}$
$(\alpha \leq -6)$							$280 \subset \mathbf{364}_{\mathrm{T},-6}$	$\mathbf{64}_{\mathrm{T},-7}$	$\mathbf{1}_{\mathrm{T},-8}$

$$\alpha' = -\alpha - \frac{4(D-1)}{D-2}$$

$D = 3, 4, 6$ の時のみ S-dual branes あり

by D-dim. S-duality $(g'_{\mu\nu})_S = \mathrm{e}^{-8\phi/(D-2)}(g_{\mu\nu})_S$

$$\left((g_s)^{\alpha} \int \mathrm{d}^{D-2}x \, [\mathrm{NG}(g_{\mu\nu})] = (g_s)^{\alpha'} \int \mathrm{d}^{D-2}x \, [\mathrm{NG}(g'_{\mu\nu})] \right)$$

	fundamental	Dirichlet	solitonic				$\alpha' = -\alpha - \frac{4(D-1)}{D-2}$ via S-duality		
D	$\alpha=0$	$\alpha=-1$	$\alpha=-2$	$\alpha=-3$	$\alpha=-4$	$\alpha=-5$	$\alpha=-6$	$\alpha=-7$	$\alpha=-8$
IIA		C_9 [D8]							
9		C_8 [D7]		$E_{9,1,1}$ $[7_3^{(0,1)}]$					
8		C_7 [D6]		$E_{9,2,1}$ $[6_3^{(1,1)}]$					
7		C_6 [D5]	D_6 [NS5] $D_{7,1}$ [KK5] $D_{8,2}$ $[5_2^2]$	$E_{9,3,1}$ $[5_3^{(2,1)}]$	$F_{9,3}$ $[5_4^3]$				
6		C_5 [D4]		$E_{9,4,1}$ $[4_3^{(3,1)}]$	$F_{9,4,1}$ $[4_4^{(3,1)}]$				
5		C_4 [D3]		$E_{9,5,1}$ $[3_3^{(4,1)}]$	$F_{9,5,2}$ $[3_4^{(3,2)}]$				
4		C_3 [D2]		$E_{9,6,1}$ $[2_3^{(5,1)}]$	$F_{9,6,3}$ $[2_4^{(3,3)}]$	$G_{9,6,2m}$ $G_{9,6,2m+1}$			
3	B_2 [F1]	C_2 [D1]		$E_{9,7,1}$ $[1_3^{(6,1)}]$	$F_{9,7,4}$ $[1_4^{(3,4)}]$ $F_{9,7,1,1}$ $[1_4^{(6,0,1)}]$	$G_{9,7,2m,1}$ $G_{9,7,2m+1,1}$	$H_{9,7,4+n,n}$	$(\mathrm{S}_3(C_2))$	$(\mathrm{S}_3(B_2))$

A_{D-T,I_1+I_2,I_2}-forms : "mixed-symmetry tensors" $\leftrightarrow$ $p_\alpha^{(I_1,I_2)}$-branes

$T + p + \sum_i I_i = D-1$ with $T=1$: transverse, p : spatial, I_i : isometry directions

E.A. Bergshoeff et al, arXiv:1108.5067, arXiv:1210.1422

$Z_{(a)}$: a-form central charge

D	R-対称性 H	$Z_{(1)}$	$Z_{(2)}$	# of EDWs	縮退度
9	$SO(2)$		$\mathbf{1}$	2	2
8	$SO(3) \times SO(2)$		$(\mathbf{1}, \mathbf{2})$	6	3
7	$Sp(2)$		$\mathbf{5} + \mathbf{1}$	$20 + 5$	$4_{(\mathrm{V})}$, $5_{(\mathrm{T})}$
6	$Sp(2) \times Sp(2)$		$(\mathbf{4}, \mathbf{4})$	80	5
5	$USp(8)$		$\mathbf{36}$	216	6
4	$SU(8)$		$\mathbf{36}^{+} + \overline{\mathbf{36}}^{-}$	576	8
3	$SO(16)$	$\mathbf{135}$		2160	16

$$\#\text{ of EDWs} = \begin{cases} \{(10-D)+1\} \cdot (\#\text{ of } Z_{(2)}) & 5 \leq D \leq 9 \\ 8 & D = 4 \\ 16 & D = 3 \end{cases}$$

(注意) standard branes **と** central charges **とは「1対1」対応**

D	$\#$ of $(D-1)$-forms	$\#$ of EDWs	$\#$ of non-EDWs
9	$\mathbf{3} \oplus \mathbf{2}$	$2+0$	$1+2$
8	$(\mathbf{6},\mathbf{2}) \oplus (\overline{\mathbf{3}},\mathbf{2})$	$6+0$	$6+6$
7	$\overline{\mathbf{40}} \oplus \overline{\mathbf{15}}$	$20+5$	$20+10$
6	$\mathbf{144}$	80	64
5	$\mathbf{351}$	216	135
4	$\mathbf{912}$	576	336
3	$\mathbf{3875} \oplus \mathbf{1}$	$2160+0$	$1715+1$

Elementary SUSY DWs (EDWs) **と結合しない** $(D-1)$-forms **は、**

EDWs **の束縛状態な** $(D-2)$-branes **と結合する (詳細省略)**

複数の EDWs **が同じ** $\frac{1}{2}$-SUSY **条件の時** $\rightarrow$ threshold bound states of EDWs

複数の EDWs **が異なる** $\frac{1}{2}$-SUSY **条件の時** $\rightarrow$ non-threshold bound states of EDWs

分析 2

変形 (ゲージ化) された極大超重力理論

Embedding Tensor Formalism

極大超重力理論

重力場・グラビティーノ・ベクトルゲージ場・テンソルゲージ場・スピノル場・スカラー場

超対称性によって、スカラー場が住む空間が coset space G_0/H に定まっている

D	U-duality G_0	R-対称性 H	$\dim(G_0/H)$	T-duality
11	1	1	0	1
IIA	$\mathbb{R}^+$	1	1	1
IIB	$SL(2,\mathbb{R})$	$SO(2)$	2	1
9	$GL(2,\mathbb{R})$	$SO(2)$	3	$SO(1,1)$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	7	$SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$
7	$SL(5,\mathbb{R})$	$Sp(2)$	14	$SL(4,\mathbb{R})$
6	$SO(5,5)$	$Sp(2) \times Sp(2)$	25	$SO(4,4)$
5	$E_{6(6)}$	$USp(8)$	42	$SO(5,5)$
4	$E_{7(7)}$	$SU(8)$	70	$SO(6,6)$
3	$E_{8(8)}$	$SO(16)$	128	$SO(7,7)$

ゲージ場はスカラー場やスピノル場と結合していない

ゲージ化： embedding tensor $\Theta_M{}^\alpha$ **を導入して、共変微分化**

$$T_M \;\equiv\; \Theta_M{}^\alpha\, t_\alpha \qquad \begin{cases} t_\alpha \;\in\; \mathrm{Lie}\, G_0 & \text{global} \\ T_M \;\in\; \mathrm{Lie}\, G & \text{local} \end{cases}$$

$$\partial_\mu \;\longrightarrow\; \mathcal{D}_\mu \;\equiv\; \partial_\mu - \mathrm{g} A_\mu^M\, T_M$$

ゲージ化： embedding tensor $\Theta_M{}^\alpha$ **を導入して、共変微分化**

$$T_M \equiv \Theta_M{}^\alpha t_\alpha \qquad \begin{cases} t_\alpha \in \text{Lie}\, G_0 & \text{global} \\ T_M \in \text{Lie}\, G & \text{local} \end{cases}$$

$$\partial_\mu \longrightarrow \mathcal{D}_\mu \equiv \partial_\mu - g A_\mu^M T_M$$

$$[T_M, T_N] = -T_{MN}{}^P T_P\,, \qquad T_{MN}{}^P \equiv \Theta_M{}^\alpha (t_\alpha)_N{}^P$$

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] \equiv -g \mathcal{F}_{\mu\nu}^M T_M$$

$$\mathcal{F}_{\mu\nu}^M \equiv \partial_\mu A_\nu^M - \partial_\nu A_\mu^M + g T_{[NP]}{}^M A_\mu^N A_\nu^P$$

ゲージ群： $0 = f_{\alpha\beta}{}^\gamma \Theta_M{}^\alpha \Theta_N{}^\beta + (t_\alpha)_N{}^P \Theta_M{}^\alpha \Theta_P{}^\gamma$

$$T_{(MN)}{}^{P}\Theta_{P}{}^{\alpha} \;=\; 0$$

$[T_M, T_N] = -T_{MN}{}^{P}T_P$ でも $T_{(MN)}{}^{P} \;=\; 0$ である必要はない

$$\delta\mathcal{F}^{M}_{\mu\nu} \;=\; 2\mathcal{D}_{[\mu}\delta A^{M}_{\nu]} - 2\mathrm{g}\,T_{(PQ)}{}^{M}\,A^{P}_{[\mu}\delta A^{Q}_{\nu]}$$

ゲージ変換 $\delta A^{M}_{\mu} = \mathcal{D}_{\mu}\Lambda^{M}$ に対して共変ではない

tensor gauge fields $B^{(NP)}_{\mu\nu}$ を導入して共変化$\rightarrow$ “Stückelberg pairing”

$$\mathcal{H}^{M}_{\mu\nu} \;\equiv\; \mathcal{F}^{M}_{\mu\nu} + \mathrm{g}\,T_{(NP)}{}^{M}B^{(NP)}_{\mu\nu}$$

超対称性 :　$\Theta_M{}^\alpha$ **の自由度** $\dim G \times \dim G_0$ **に制限が課される**

$$\left(\mathcal{D}_\mu = \partial_\mu - \mathrm{g} A_\mu^M \, \Theta_M{}^\alpha \, t_\alpha\right)$$

添字 M **は極大超重力理論にあるゲージ場の数** $\leftarrow G_0$ **のある表現に属す**

超対称性： $\Theta_M{}^\alpha$ **の自由度** $\dim G \times \dim G_0$ **に制限が課される**

$$\left(\mathcal{D}_\mu = \partial_\mu - \mathrm{g} A_\mu^M \, \Theta_M{}^\alpha \, t_\alpha\right)$$

添字 M **は極大超重力理論にあるゲージ場の数** $\leftarrow G_0$ **のある表現に属す**

D	U-duality G_0	constraints on $R(M) \otimes R(\alpha)$
9	$GL(2,\mathbb{R})$	$(2 \oplus 1) \otimes (3 \oplus 1) = \cancel{1} \oplus \boxed{2} \oplus \cancel{2} \oplus \boxed{3} \oplus \cancel{4}$
8	$SL(3,\mathbb{R}) \otimes SL(2,\mathbb{R})$	$(3,2) \otimes [(1,3) \oplus (8,1)] = \boxed{(3,2)} \oplus \cancel{(3,2)} \oplus \cancel{(3,4)} \oplus \boxed{(6,2)} \oplus \cancel{(15,2)}$
7	$SL(5,\mathbb{R})$	$10 \otimes 24 = \cancel{10} \oplus \boxed{15} \oplus \boxed{40} \oplus \cancel{175}$
6	$SO(5,5)$	$16 \otimes 45 = \cancel{16} \oplus \boxed{144} \oplus \cancel{560}$
5	$E_{6(6)}$	$27 \otimes 78 = \cancel{27} \oplus \boxed{351} \oplus \cancel{1728}$
4	$E_{7(7)}$	$56 \otimes 133 = \cancel{56} \oplus \boxed{912} \oplus \cancel{6480}$
3	$E_{8(8)}$	$248 \otimes 248 = \boxed{1} \oplus \cancel{248} \oplus \boxed{3875} \oplus \cancel{27000} \oplus \cancel{30380}$

F.Riccioni, D.Steele and P.West, arXiv:0906.1177

$\Theta_M{}^\alpha$ は D 次元理論の $(D-1)$-form 場の表現に等しい

D	U-duality G_0	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
IIA	$\mathbb{R}^+$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	–	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}$
IIB	$SL(2,\mathbb{R})$	–	$\mathbf{2}$	–	$\mathbf{1}$	–	$\mathbf{2}$	–	$\mathbf{3}$	–	$\mathbf{4}\oplus\mathbf{2}$
9	$GL(2,\mathbb{R})$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{3}\oplus\mathbf{1}$	$\mathbf{3}\oplus\mathbf{2}$	$\mathbf{4}\oplus\mathbf{2}\oplus\mathbf{2}$	–
8	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $\oplus(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $\oplus(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $\oplus(\mathbf{3},\mathbf{3})$ $\oplus(\mathbf{3},\mathbf{1})$ $\oplus(\mathbf{3},\mathbf{1})$	–	–
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}}\oplus\overline{\mathbf{15}}$	$\mathbf{70}$ $\oplus\mathbf{45}$ $\oplus\mathbf{5}$	–	–	–
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\mathbf{320}$ $\oplus\overline{\mathbf{126}}$ $\oplus\mathbf{10}$	–	–	–	–
5	$E_{6(6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\overline{\mathbf{1728}}$ $\oplus\overline{\mathbf{27}}$	–	–	–	–	–
4	$E_{7(7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{8645}$ $\oplus\mathbf{133}$	–	–	–	–	–	–
3	$E_{8(8)}$	$\mathbf{248}$	$\mathbf{3875}\oplus\mathbf{1}$	$\mathbf{147250}$ $\oplus\mathbf{3875}$ $\oplus\mathbf{248}$	–	–	–	–	–	–	–

$(D-1)$-forms： Embedding Tensors の表現はこれと一致

F.Riccioni, D.Steele and P.West, arXiv:0906.1177

$\Theta_M{}^\alpha$ は D 次元理論の $(D-1)$-form 場の表現に等しい

$(D-1)$-form 場は DWs に結合する

${\Theta_M}^\alpha$ は D 次元理論の $(D-1)$-form 場の表現に等しい

$(D-1)$-form 場は DWs に結合する

↓

Elementary SUSY DWs に相当する ${\Theta_M}^\alpha$ を選定して

極大超重力理論の変形の様子を理解する

- **ゲージ場：** $A_1,\ A_{1,a},\ A_{2,a},\ A_3$ $\qquad (a = 1, 2 \text{ of } GL(2, \mathbb{R}))$

- **ゲージ場：** $A_1,\ A_{1,a},\ A_{2,a},\ A_3$ $(a = 1, 2$ of $GL(2, \mathbb{R}))$
- embedding tensors：Θ^a in $\mathbf{2}$, Θ^{ab} in $\mathbf{3}$; with constraints

$$\Theta^a \Theta^{bc} \epsilon_{ab} = 0 \,, \qquad \Theta^{(a} \Theta^{bc)} = 0$$

- **ゲージ場：** $A_1,\ A_{1,a},\ A_{2,a},\ A_3$ $\quad (a = 1, 2$ of $GL(2,\mathbb{R}))$
- embedding tensors：Θ^a in $\mathbf{2}$, Θ^{ab} in $\mathbf{3}$; with constraints

$$\Theta^a \Theta^{bc} \epsilon_{ab} = 0 \,, \qquad \Theta^{(a} \Theta^{bc)} = 0$$

- Stückelberg pairing

$$\begin{aligned} \delta A_1 &= \mathrm{d}\lambda_0 - \Theta^a \lambda_{1,a} \\ \delta A_{1,a} &= \mathrm{d}\lambda_{0,a} - \epsilon_{ab}\Theta^{bc}\lambda_{1,c} \\ \delta A_{2,a} &= \mathrm{d}\lambda_{1,a} - \epsilon_{ab}\Theta^{b}\lambda_2 \\ \delta A_3 &= \mathrm{d}\lambda_2 \end{aligned} \quad \rightarrow \quad \begin{aligned} \mathcal{F}_2 &= \mathrm{d}A_1 + \Theta^a A_{2,a} \\ \mathcal{F}_{2,a} &= \mathrm{d}A_{1,a} + \epsilon_{ab}\Theta^{bc} A_{2,c} \\ \mathcal{F}_{3,a} &= \mathrm{d}A_{2,a} + \epsilon_{ab}\Theta^{b} A_3 \\ \mathcal{F}_4 &= \mathrm{d}A_3 \end{aligned}$$

- **ゲージ場：** $A_1,\ A_{1,a},\ A_{2,a},\ A_3$ $\quad(a = 1, 2$ of $GL(2,\mathbb{R}))$
- embedding tensors：Θ^a in $\mathbf{2}$, Θ^{ab} in $\mathbf{3}$; with constraints

$$\Theta^a \Theta^{bc} \epsilon_{ab} = 0\,, \qquad \Theta^{(a}\Theta^{bc)} = 0$$

- Stückelberg pairing

$$\begin{aligned} \delta A_1 &= \mathrm{d}\lambda_0 - \Theta^a \lambda_{1,a} \\ \delta A_{1,a} &= \mathrm{d}\lambda_{0,a} - \epsilon_{ab}\Theta^{bc}\lambda_{1,c} \\ \delta A_{2,a} &= \mathrm{d}\lambda_{1,a} - \epsilon_{ab}\Theta^{b}\lambda_2 \\ \delta A_3 &= \mathrm{d}\lambda_2 \end{aligned} \quad \rightarrow \quad \begin{aligned} \mathcal{F}_2 &= \mathrm{d}A_1 + \Theta^a A_{2,a} \\ \mathcal{F}_{2,a} &= \mathrm{d}A_{1,a} + \epsilon_{ab}\Theta^{bc} A_{2,c} \\ \mathcal{F}_{3,a} &= \mathrm{d}A_{2,a} + \epsilon_{ab}\Theta^{b} A_3 \\ \mathcal{F}_4 &= \mathrm{d}A_3 \end{aligned}$$

- Minimal Gauging **を探す** (Θ^a, Θ^{ab} **による最小の変形**)

Gauging	A_1	$A_{1,a=1}$	$A_{1,a=2}$	$A_{2,a=1}$	$A_{2,a=2}$	A_3	
$\Theta^1 = 1, \Theta^2 = 0, \Theta^{ab} = 0$	eaten	massless	massless	massive	eaten	massive	–
$\Theta^a = 0, \Theta^{11} = 1, \Theta^{22} = \pm 1$	massive	eaten	eaten	massive	massive	massless	–
$\Theta^a = 0, \Theta^{11} = 1, \Theta^{22} = 0$	massive	massless	eaten	massive	massless	massless	2**通り**

- **ゲージ場：** $A_{1,Ma}$, A_2^M, $A_{3,a}$ $(M = 1,2,3 \text{ of } SL(3,\mathbb{R}),\ a = 1,2 \text{ of } SL(2,\mathbb{R}))$

- **ゲージ場：** $A_{1,Ma}$, A_2^M, $A_{3,a}$ ($M = 1, 2, 3$ of $SL(3, \mathbb{R})$, $a = 1, 2$ of $SL(2, \mathbb{R})$)
- Minimal gauging **を与えるもの：** (Θ^{Ma} in $(\mathbf{3}, \mathbf{2})$ **に対応する** EDWs **はないので考えない**)

$$\Theta_{MN}{}^a \;=\; \{\Theta_{11}{}^1, \Theta_{11}{}^2, \Theta_{22}{}^1, \Theta_{22}{}^2, \Theta_{33}{}^1, \Theta_{33}{}^2\} \text{ in } (\overline{\mathbf{6}}, \mathbf{2})$$

：　6**通り**

- **ゲージ場：** $A_{1,Ma}$, A_2^M, $A_{3,a}$ ($M = 1, 2, 3$ of $SL(3,\mathbb{R})$, $a = 1, 2$ of $SL(2,\mathbb{R})$)
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$$\Theta_{MN}{}^a = \{\Theta_{11}{}^1, \Theta_{11}{}^2, \Theta_{22}{}^1, \Theta_{22}{}^2, \Theta_{33}{}^1, \Theta_{33}{}^2\} \text{ in } (\bar{\mathbf{6}}, \mathbf{2})$$
：6**通り**

- $\Theta_{MN}{}^1$ **に着目する** ($\Theta_{MN}{}^2 = 0$)

$$\Theta_{MN}{}^1 = \mathrm{diag}(1_p, -1_q, 0_r) \quad \text{with } p + q + r = 3$$

$$\downarrow$$

$$CSO(p, q, r) \text{ with } f^{MN}{}_P = \epsilon^{MNQ}\Theta_{PQ}$$

$$[T^1, T^2] = \Theta_{33}{}^1 T^3, \quad [T^2, T^3] = \Theta_{11}{}^1 T^1, \quad [T^3, T^1] = \Theta_{22}{}^1 T^2$$

Minimal：$\Theta_{22}{}^1 = \Theta_{33}{}^1 = 0 \rightarrow CSO(1, 0, 2)$ = Heisenberg algebra

Gauging	$A_{1,11}$	$A_{1,12}$	$A_{1,i1}$	$A_{1,i2}$	A_2^1	A_2^i	$A_{3,a}$
$\Theta_{11}{}^1 = 1$, others $= 0$	massless	eaten	massive	massless	massive	massless	massless

($i = 2, 3$)

- **ゲージ場：** $A_{1,MN},\ A_2^M$ $\quad (M = 1, 2, \ldots, 5 \text{ of } SL(5, \mathbb{R}))$

- **ゲージ場：** $A_{1,MN}$, A_2^M $\quad (M = 1, 2, \ldots, 5$ of $SL(5, \mathbb{R}))$

- embedding tensors $\Theta_{[MN],P} \equiv v_{[M} w_{N]P}$ in $\mathbf{40}$**：** 20**通り**

$$w_{NP} = \mathrm{diag}(1_p, -1_q, 0_r) \text{ with } p + q + r = 4$$
$$\rightarrow \text{minimal gauging} = CSO(1, 0, 3)$$

Gauging	$A_{1,ij}$	$A_{1,12}$	$A_{1,1i}$	$A_{1,2i}$	A_2^1	A_2^2	A_2^i
$\Theta_{12,1} = 1$, others $= 0$	massive	eaten	massless	massless	massive	massless	massless

$(i = 3, 4, 5)$

- embedding tensors $\Theta^{(MN)}$ in $\mathbf{15}$**：** 5**通り**

$$\Theta^{MN} = \mathrm{diag}(1_p, -1_q, 0_r) \text{ with } p + q + r = 5$$
$$\rightarrow \text{minimal gauging} = CSO(1, 0, 4)$$

Gauging	$A_{1,1i}$	$A_{1,ij}$	A_2^1	A_2^i	$A_{3,1}$
$\Theta^{11} = 1$, others $= 0$	massive	massless	eaten	massless	massive

$(i = 2, 3, 4, 5)$

minimal gauging **される極大超重力理論は**

elementary SUSY Domain Walls **によって与えられる**

non-minimal gauging **な極大超重力理論は**

(non)-threshold bound states of EDWs **によって与えられる**

まとめ

D 次元時空の Domain Walls (DWs) は不思議な物体

- ✓ $\frac{1}{2}$-SUSY 多重項を持つ DWs (EDWs) の数は、U-duality のある表現の一部だけ
- ✓ non-EDWs (EDWs の束縛状態) が残りの部分を補完する
- ✓ Central charges との数勘定がズレている (ある一定のルールに従う)

EDWs は極大超重力理論を minimal gauging 変形する

Non-EDWs は極大超重力理論を non-minimal gauging 変形する

D	U	T	# of EDWs	$\alpha=0$	$\alpha=-1$	$\alpha=-2$	$\alpha=-3$	$\alpha=-4$	$\alpha=-5$
IIA	$\mathbb{R}^+$	1	1		1				
9	$GL(2,\mathbb{R})$	$SO(1,1)$	$2\subset\mathbf{3}_{\mathrm{U}}$		1	–	1		
8	$SL(3,\mathbb{R})$ $\times SL(2,\mathbb{R})$	$SL(2,\mathbb{R})$ $\times SL(2,\mathbb{R})$	$6\subset(\mathbf{6},\mathbf{2})_{\mathrm{U}}$		$(\mathbf{1},\mathbf{2})_{\mathrm{T}}$	–	$4\subset(\mathbf{3},\mathbf{2})_{\mathrm{T}}$		
7	$SL(5,\mathbb{R})$	$SL(4,\mathbb{R})$	$20\subset\overline{\mathbf{40}}_{\mathrm{U}}$		$\mathbf{4}_{\mathrm{T}}$	$4\subset\mathbf{10}_{\mathrm{T}}$	$12\subset\overline{\mathbf{20}}_{\mathrm{T}}$		
			$5\subset\overline{\mathbf{15}}_{\mathrm{U}}$			$4\subset\overline{\mathbf{10}}_{\mathrm{T}}$	–	$\mathbf{1}_{\mathrm{T}}$	
6	$SO(5,5)$	$SO(4,4)$	$80\subset\overline{\mathbf{144}}_{\mathrm{U}}$		$\mathbf{8}_{\mathrm{S}}\vert_{\mathrm{T}}$	$32\subset\mathbf{56}_{\mathrm{C}}\vert_{\mathrm{T}}$	$32\subset\mathbf{56}_{\mathrm{S}}\vert_{\mathrm{T}}$	$\mathbf{8}_{\mathrm{C}}\vert_{\mathrm{T}}$	
5	$E_{6(6)}$	$SO(5,5)$	$216\subset\mathbf{351}_{\mathrm{U}}$		$\overline{\mathbf{16}}_{\mathrm{T}}$	$80\subset\mathbf{120}_{\mathrm{T}}$	$80\subset\mathbf{144}_{\mathrm{T}}$	$40\subset\mathbf{45}_{\mathrm{T}}$	
4	$E_{7(7)}$	$SO(6,6)$	$576\subset\mathbf{912}_{\mathrm{U}}$		$\mathbf{32}_{\mathrm{T}}$	$160\subset\mathbf{220}_{\mathrm{T}}$	$192\subset\mathbf{352}_{\mathrm{T}}$	$160\subset\mathbf{220}_{\mathrm{T}}$	$\mathbf{32}_{\mathrm{T}}$
3	$E_{8(8)}$	$SO(7,7)$	$2160\subset\mathbf{3875}_{\mathrm{U}}$	$\mathbf{1}_{\mathrm{T}}$	$\overline{\mathbf{64}}_{\mathrm{T}}$	$280\subset\mathbf{364}_{\mathrm{T}}$	$448\subset\mathbf{832}_{\mathrm{T}}$	$560\subset\mathbf{1001}_{\mathrm{T}}$ $14\subset\mathbf{104}_{\mathrm{T}}$	$448\subset\overline{\mathbf{832}}_{\mathrm{T}}$
$(\alpha\leq-6)$							$280\subset\mathbf{364}_{\mathrm{T},-6}$	$\mathbf{64}_{\mathrm{T},-7}$	$\mathbf{1}_{\mathrm{T},-8}$

D	R-対称性 H	$Z_{(1)}$	$Z_{(2)}$	# of EDWs	縮退度
9	$SO(2)$		$\mathbf{1}$	2	2
8	$SO(3)\times SO(2)$		$(\mathbf{1},\mathbf{2})$	6	3
7	$Sp(2)$		$\mathbf{5}+\mathbf{1}$	$20+5$	$4_{(\mathrm{V})}$, $5_{(\mathrm{T})}$
6	$Sp(2)\times Sp(2)$		$(\mathbf{4},\mathbf{4})$	80	5
5	$USp(8)$		$\mathbf{36}$	216	6
4	$SU(8)$		$\mathbf{36}^{+}+\overline{\mathbf{36}}^{-}$	576	8
3	$SO(16)$	$\mathbf{135}$		2160	16

D	32-SUSY	16-SUSY	8-SUSY
9	arXiv:1105.1760	(unknown)	–
8	arXiv:1203.6562	(unknown)	–
7	hep-th/0506237	(unknown)	–
6	arXiv:0712.4277	(unknown)	arXiv:1012.1818
5	hep-th/0412173	hep-th/0702084	(unknown)
4	arXiv:0705.2101	hep-th/0602024	arXiv:1107.3305
3	hep-th/0103032	arXiv:0806.2584	arXiv:0807.2841

おしまい

Defect branes

U-duality G_0 における各 form fields の表現

D	U-duality G_0	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
IIA	$\mathbb{R}^+$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	–	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1} \oplus \mathbf{1}$
IIB	$SL(2,\mathbb{R})$	–	$\mathbf{2}$	–	$\mathbf{1}$	–	$\mathbf{2}$	–	$\mathbf{3}$	–	$\mathbf{4} \oplus \mathbf{2}$
9	$GL(2,\mathbb{R})$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{3} \oplus \mathbf{1}$	$\mathbf{3} \oplus \mathbf{2}$	$\mathbf{4} \oplus \mathbf{2} \oplus \mathbf{2}$	–
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2}) \oplus (\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1}) \oplus (\mathbf{3},\mathbf{3}) \oplus (\mathbf{3},\mathbf{1}) \oplus (\mathbf{3},\mathbf{1})$	–	–
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}} \oplus \overline{\mathbf{15}}$	$\mathbf{70} \oplus \mathbf{45} \oplus \mathbf{5}$	–	–	–
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\mathbf{320} \oplus \overline{\mathbf{126}} \oplus \mathbf{10}$	–	–	–	–
5	$E_{6(6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\overline{\mathbf{1728}} \oplus \overline{\mathbf{27}}$	–	–	–	–	–
4	$E_{7(7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{8645} \oplus \mathbf{133}$	–	–	–	–	–	–
3	$E_{8(8)}$	$\mathbf{248}$	$\mathbf{3875} \oplus \mathbf{1}$	$\mathbf{147250} \oplus \mathbf{3875} \oplus \mathbf{248}$	–	–	–	–	–	–	–

$(D-2)$-forms： U-duality group G_0 の随伴表現

F.Riccioni, D.Steele and P.West, arXiv:0906.1177

$$\mathscr{L}_i^{\mathsf{WZ}} \sim A_{8,i} + \overline{\mathcal{F}}_2 \Gamma_i A_6 + \dots$$

$A_{8,i}$: 8-forms in bulk

Γ_i : $SO(2,1) \sim SL(2,\mathbb{R})$ **ガンマ行列** $(i = +, -, 3)$

$\mathcal{F}_2 = (\mathrm{d}b_1, S(\mathrm{d}b_1))$: curvatures of DBI vector and its S-dual / spinor repr. of $SO(2,1)$

$A_6 = (B_{(6)}, C_{(6)})$: 6-forms in bulk / spinor repr. of $SO(2,1)$

- $i = +$ **もしくは** $i = -$ **のとき**

  7-brane **上のベクトルかその** S-dual **を** project-out **できる**

- $i = 3$ **のとき**

  できない

Defect branes $\sim$ $(D-2)$-form potentials $\sim$ scalar fields

U-duality group G_0 の随伴表現を、T-duality group (と $\mathbb{R}^+$) で分解する

	$d = 10 - D$			fundamental	Dirichlet	solitonic		
	U	T		$\alpha = 0$	$\alpha = -1$	$\alpha = -2$	$\alpha = -3$	$\alpha = -4$
$D \geq 5$	$E_{d+1(d+1)}$	$SO(d,d)$	$\text{Adj}_{\vert\text{U}}$	–	$\text{spinor}_{\vert\text{T}}$	$(\text{Adj} + \text{singlet})_{\vert\text{T}}$	$\text{conj. spinor}_{\vert\text{T}}$	–
$D = 4$	$E_{7(7)}$	$SO(6,6)$	$\text{Adj}_{\vert\text{U}}$	$\text{singlet}_{\vert\text{T}}$	$\text{spinor}_{\vert\text{T}}$	$(\text{Adj} + \text{singlet})_{\vert\text{T}}$	$\text{conj. spinor}_{\vert\text{T}}$	$\text{singlet}_{\vert\text{T}}$
$D = 3$	$E_{8(8)}$	$SO(7,7)$	$\text{Adj}_{\vert\text{U}}$	$\text{vector}_{\vert\text{T}}$	$\text{spinor}_{\vert\text{T}}$	$(\text{Adj} + \text{singlet})_{\vert\text{T}}$	$\text{conj. spinor}_{\vert\text{T}}$	$\text{vector}_{\vert\text{T}}$

$$\alpha' = -\alpha - 4$$

by D-dim. S-duality $(g'_{\mu\nu})_S = \mathrm{e}^{-8\phi/(D-2)} (g_{\mu\nu})_S$

$$\left((g_s)^\alpha \int \mathrm{d}^{D-2}x \, [\mathrm{NG}(g_{\mu\nu})] = (g_s)^{\alpha'} \int \mathrm{d}^{D-2}x \, [\mathrm{NG}(g'_{\mu\nu})] \right)$$

E.A. Bergshoeff et al, arXiv:1009.4657, arXiv:1102.0934, arXiv:1108.5067 solitonic defect brane ($\alpha = -2$) **の全てが** supersymmetric **になるわけではない**

		fundamental	Dirichlet	solitonic	(brane's tension) $\sim (g_s)^{+\alpha}$	
D	# of SUSY defect branes	$\alpha = 0$	$\alpha = -1$	$\alpha = -2$	$\alpha = -3$	$\alpha = -4$
IIB	$2 \subset \mathbf{3}$		1	$-$	1	
9	$2 \subset \mathbf{3}_3$		1	$-$	1	
8	$6 \subset (\mathbf{8}, \mathbf{1})$		$(\mathbf{2}, \mathbf{1})$	$2 \subset (\mathbf{3}, \mathbf{1})$	$(\mathbf{2}, \mathbf{1})$	
	$2 \subset (\mathbf{1}, \mathbf{3})$			$2 \subset (\mathbf{1}, \mathbf{3})$		
7	$20 \subset \mathbf{24}$		$\overline{\mathbf{4}}$	$12 \subset \mathbf{15}$	$\mathbf{4}$	
6	$40 \subset \mathbf{45}$		$\mathbf{8}_{\mathrm{V}}$	$24 \subset \mathbf{28}$	$\mathbf{8}_{\mathrm{V}}$	
5	$72 \subset \mathbf{78}$		$\mathbf{16}$	$40 \subset \mathbf{45}$	$\overline{\mathbf{16}}$	
4	$126 \subset \mathbf{133}$	$\mathbf{1}$	$\mathbf{32}$	$60 \subset \mathbf{66}$	$\mathbf{32}$	$\mathbf{1}$
3	$240 \subset \mathbf{248}$	$\mathbf{14}$	$\mathbf{64}$	$84 \subset \mathbf{91}$	$\mathbf{64}$	$\mathbf{14}$

E.A. Bergshoeff et al, arXiv:1009.4657, arXiv:1102.0934, arXiv:1108.5067

	fundamental	Dirichlet	solitonic	S_D-dual of (Dirichlet)	S_D-dual of (fundamental)
D	$\alpha = 0$	$\alpha = -1$	$\alpha = -2$	$\alpha = -3$	$\alpha = -4$
IIB		C_8 [D7]		$E_8 = \mathrm{S}_{10}(C_8)$ [7_3]	
9		C_7 [D6]		$E_{8,1} = \mathrm{S}_9(C_7)$ [6_3^1]	
8		C_6 [D5]	D_6 [NS5] $D_{7,1}$ [KK5 $= 5_2^1$] $D_{8,2}$ [5_2^2]	$E_{8,2} = \mathrm{S}_8(C_7)$ [5_3^2]	
7		C_5 [D4]		$E_{8,3} = \mathrm{S}_7(C_5)$ [4_3^3]	
6		C_4 [D3]		$E_{8,4} = \mathrm{S}_6(C_4)$ [3_3^4]	
5		C_3 [D2]		$E_{8,5} = \mathrm{S}_5(C_3)$ [2_3^5]	
4	B_2 [F1]	C_2 [D1]		$E_{8,6} = \mathrm{S}_4(C_2)$ [1_3^6]	$F_{8,6} = \mathrm{S}_4(B_2)$ [1_4^6]
3	[P]	C_1 [D0]		$E_{8,7} = \mathrm{S}_3(C_1)$ [0_3^7]	$F_{8,7,1}$ [$0_4^{(6,1)}$]

$p_\alpha^{(I_1,I_2)}$-brane

$$A_{D-T,I_1+I_2,I_2} \leftrightarrow (T,p,I_1,I_2)_\alpha \quad \text{with} \quad T+p+\textstyle\sum_i I_i = D-1$$

$$\text{Mass}_{(T,p,I_1,I_2)_\alpha} = R_1 \cdots R_p (R_{p+1} \cdots R_{p+I_1})^2 (R_{p+I_1+1} \cdots R_{p+I_1+I_2})^3 (g_s)^\alpha$$

D	G_0/H	n_{P}	n_{D}	n_{S}
IIB	$SL(2,\mathbb{R})/SO(2)$	3	2	2
9	$SL(2,\mathbb{R})/SO(2) \times \mathbb{R}^+$	$3+1$	$2+0$	$2+1$
8	$SL(3,\mathbb{R})/SO(3) \times SL(2,\mathbb{R})/SO(2)$	$8+3$	$6+2$	$5+2$
7	$SL(5,\mathbb{R})/SO(5)$	24	20	14
6	$SO(5,5)/[SO(5) \times SO(5)]$	45	40	24
5	$E_{6(6)}/Sp(8)$	78	72	42
4	$E_{7(7)}/SU(8)$	133	126	70
3	$E_{8(8)}/SO(16)$	248	240	128

$n_{\mathsf{P}} = \dim G_0$: # of $(D-2)$-form potentials

$n_{\mathsf{D}} = \dim G_0 - \mathrm{rank}\, G_0$: # of SUSY defect branes ($\mathrm{rank}\, G_0 = \mathrm{rank}\, T + 1$)

$n_{\mathsf{S}} = \dim G_0 - \dim H$: # of coset scalars in D-dim. maximal SUGRA

$Z_{(a)}$: a-form central charge

D	R-対称性 H	$Z_{(0)}$	$Z_{(1)}$	$Z_{(2)}$	$Z_{(3)}$	n_{D}	縮退度
IIB	$SO(2)$				$\mathbf{1}$	2	2
9	$SO(2)$				$\mathbf{1}$	2	2
8	$SO(3) \times SO(2)$				$\mathbf{3} + \mathbf{1}$	$6 + 2$	2
7	$Sp(2)$				$\mathbf{10}$	20	2
6	$Sp(2) \times Sp(2)$				$(\mathbf{10}, \mathbf{1})^{+} + (\mathbf{1}, \mathbf{10})^{-}$	40	2
5	$USp(8)$			$\mathbf{36}$		72	2
4	$SU(8)$		$\mathbf{63}$			126	2
3	$SO(16)$	$\mathbf{120}$				240	2

$CSO(p,q,r)$ とは、とても粗っぽく言えば jump

$$\begin{aligned} CSO(p,q,0) &= SO(p,q) \\ CSO(p,q,1) &= ISO(p,q) \\ CSO(p,q,r) &\supset SO(p,q)\times U(1)^{\frac{r(r-1)}{2}} \quad \text{for } r\geq 2 \end{aligned}$$

C.M. Hull, PL 142B (1984) 39, PL 148B (1984) 297, NPB 253 (1985) 650

L. Andrianopoli et al, hep-th/0009048, etc.