Three Exotics

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$$\Theta_M{}^\alpha \qquad Q^{ab}{}_c \qquad b_n^c$$

embedding tensors | (non)geometric fluxes | exotic branes

疑 問

超重力理論は

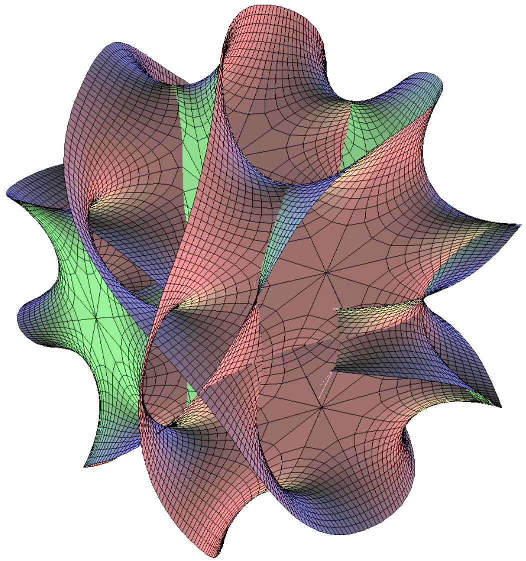
超弦理論を起源とするか？

極大超重力理論

D	U-duality G_0	R-symmetry H	$\dim(G_0/H)$	T-duality
11	1	1	0	1
IIA	$\mathbb{R}^+$	1	1	1
IIB	$SL(2,\mathbb{R})$	$SO(2)$	2	1
9	$GL(2,\mathbb{R})$	$SO(2)$	3	$SO(1,1)$
8	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$	$SO(3)\times SO(2)$	7	$SL(2,\mathbb{R})\times SL(2,\mathbb{R})$
7	$SL(5,\mathbb{R})$	$Sp(2)$	14	$SL(4,\mathbb{R})$
6	$SO(5,5)$	$Sp(2)\times Sp(2)$	25	$SO(4,4)$
5	$E_{6(6)}$	$USp(8)$	42	$SO(5,5)$
4	$E_{7(7)}$	$SU(8)$	70	$SO(6,6)$
3	$E_{8(8)}$	$SO(16)$	128	$SO(7,7)$

11次元理論をトーラスコンパクト化すれば実現できる

32個より少ない超対称生成子を持つ超重力理論



特殊ホロノミー群多様体によるコンパクト化で実現できる

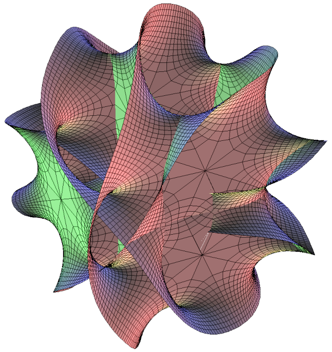
しかし…

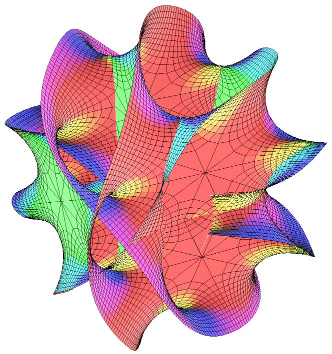
ゲージ場が物質場と結合していない

その一方で

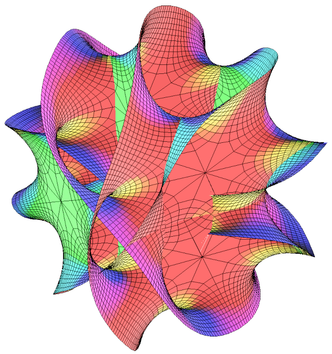
ゲージ場と物質場が結合した低次元超重力理論は構成できる

(超重力理論の変形)





$$+\ H_{abc}$$

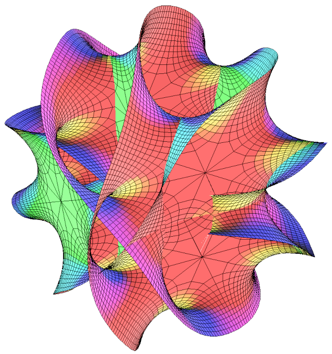
 $+ H_{abc}$

$$Q^{ab}{}_{c}$$

(non)geometric fluxes

$$\Theta_M{}^{\alpha}$$

embedding tensors

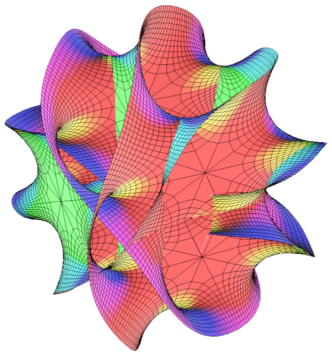
 $+ H_{abc}$

$\Theta_M{}^\alpha$ $Q^{ab}{}_c$

embedding tensors (non)geometric fluxes

b_n^c

exotic branes



$$+ \; H_{abc}$$

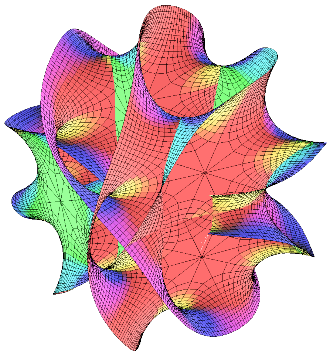
$$Q^{ab}{}_{c} \qquad b_n^c$$

(non)geometric fluxes　　exotic branes

$$\Theta_M{}^\alpha \qquad\qquad\qquad b^c_n$$

embedding tensors

exotic branes

 $+ H_{abc}$

$$\Theta_M{}^{\alpha} \qquad Q^{ab}{}_c \qquad b_n^c$$

embedding tensors (non)geometric fluxes exotic branes

期 待

全ての超重力理論は

超弦理論を起源とできるのではないか？

Contents

- あらすじ
- $Q^{ab}{}_{c}$ ： (Non)geometric Fluxes

  フラックスコンパクト化

- $\Theta_M{}^{\alpha}$ ： Embedding Tensors

  超重力理論の変形

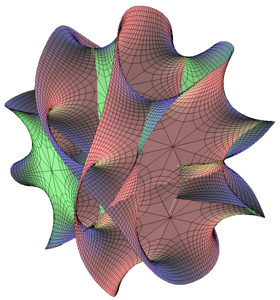
- b_n^c ： Exotic Branes

  変形の弦理論起源

- なすべきこと

$$Q^{ab}_{c}$$

Calabi-Yau 3-fold



Ricci **平坦な** Kähler **多様体**

トーションなし

ホロノミー群 $SU(3) \subset SU(4) \sim SO(6)$

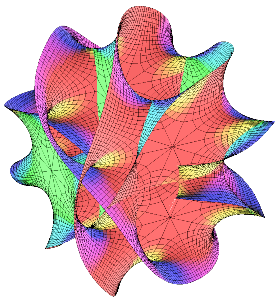
$$\mathrm{d}s^2_{\text{10D}} \;=\; \eta_{\mu\nu}(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + g_{mn}(x,y)\,\mathrm{d}y^m\,\mathrm{d}y^n$$

4D　　CY

Levi-Civita **接続の共変微分について共変定数な** 2 **形式** (J) **と正則** 3 **形式** (Ω) **:**

$$\mathrm{d}J = \nabla_{[m}J_{np]} = 0 \qquad \mathrm{d}\Omega = \nabla_{[m}\Omega_{npq]} = 0$$

non-CY 3-fold



Ricci 2-form **はゼロ**

トーションを許す (non-Kähler)

$\mathrm{d}J \neq 0$ and/or $\mathrm{d}\Omega \neq 0$

CY **からのズレ：**

$$\mathrm{d}J = \frac{3}{2}\,\mathrm{Im}(\overline{\mathcal{W}}_1\Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3\,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

閉形式でない $(\mathrm{d}J, \mathrm{d}\Omega)$ **を、基底形式の外微分の性質に翻訳する：**

$$\mathcal{D} \equiv \mathrm{d} - H^{\mathsf{fl}} \wedge - f \cdot - Q \cdot - R \llcorner \qquad (H = H^{\mathsf{fl}} + \mathrm{d}B)$$

$$\mathcal{D} \begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix} \sim \begin{pmatrix} e_\Lambda{}^I & m^{\Lambda I} \\ e_{\Lambda I} & m^\Lambda{}_I \end{pmatrix} \begin{pmatrix} \widetilde{\omega}^\Lambda \\ \omega_\Lambda \end{pmatrix}$$

$e_0{}^I, e_{0I}$: H-flux charges ($H^{\mathsf{fl}} = -e_0{}^I \alpha_I + e_{0I} \beta^I$)

$e_a{}^I, e_{aI}$: geometric flux charges (**トーション**)

$m^{\Lambda I}, m^\Lambda{}_I$: Non-geometric flux charges ($e_\Lambda{}^I, e_{\Lambda I}$ **の "磁気的" 双対**)

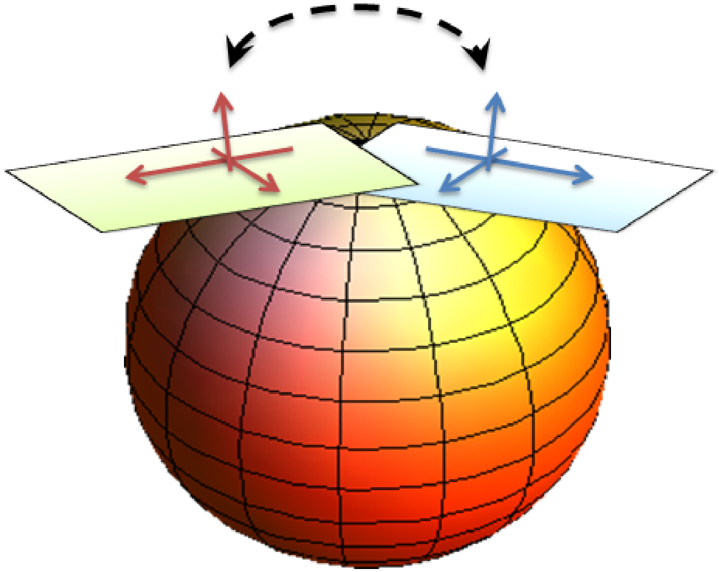
Non-geometric structure **という概念の出現**

構造群 $=$ 「Diffeo**群** $(GL(d,\mathbb{R}))$ $\subset$ **双対変換群**$(O(d,d)$, U-**双対変換**$)$」

↑

弦理論の双対性に起因

$GL(d,\mathbb{R}) \subset$ duality transf.



Generalized Geometry

Doubled Geometry

幾何を記述する「要素」として、計量 g_{mn} 以外の場を組み込む

$\mathcal{M}_6$	geometry associated with g_{mn}	Conventional geometry (manifold) $O(6)$ global symmetry
	geometry associated with g_{mn}, B_{mn}	Generalized geometry $O(6,6)$ T-duality symmetry
	geometry associated with g_{mn}, B_{mn}, $C_{(p)}$	Exceptional generalized geometry $E_{7(7)}$ U-duality symmetry

✓ **共変微分化(例)：**

$$\nabla_\mu q^u = \partial_\mu q^u + \mathrm{g}\, k^u_\Lambda\, A^\Lambda_\mu + \mathrm{g}\, k^{u\Lambda}\, A_{\mu\Lambda}$$

$$k_\Lambda = -\big[2\, e_{\mathsf{R}\Lambda} + e_\Lambda{}^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}}\xi)_{\boldsymbol{I}}\big] \frac{\partial}{\partial a} - e_\Lambda{}^{\boldsymbol{I}} \frac{\partial}{\partial \xi^{\boldsymbol{I}}}$$

$$k^\Lambda = -\big[2\, m^\Lambda_{\mathsf{R}} + m^{\Lambda \boldsymbol{I}} (\mathbb{C}_{\mathsf{H}}\xi)_{\boldsymbol{I}}\big] \frac{\partial}{\partial a} + m^{\Lambda \boldsymbol{I}} \frac{\partial}{\partial \xi^{\boldsymbol{I}}}$$

✓ (RR fluxes m^Λ_{R} **を導入して)スカラー場からテンソル場への双対変換：**

$$-h_{uv}\, \partial_\mu q^u\, \partial^\mu q^v \quad \longrightarrow \quad -\mathcal{M}_{AB}\, H^A_{\mu\nu\rho}\, H^{\mu\nu\rho B}$$

$$\Theta_M^{\alpha}$$

理論には最初から自由なゲージ場 A_μ^M が含まれている

理論が持つ大域的対称性 G_0 をゲージ対称性に格上げする

ゲージ化可能な全てを構築したい

$$T_M \equiv \Theta_M{}^\alpha t_\alpha \qquad \begin{cases} t_\alpha & \in \quad \mathrm{Lie}\, G_0 \qquad \text{global} \\ T_M & \in \quad \mathrm{Lie}\, G \qquad \text{gauge} \end{cases}$$

$$\partial_\mu \longrightarrow \mathcal{D}_\mu \equiv \partial_\mu - \mathrm{g} A_\mu^M T_M$$

一見「通常の」手段に見えるが…

$$[T_M, T_N] = -T_{MN}{}^P T_P$$

$$T_M = \Theta_M{}^\alpha t_\alpha$$

構造定数 $T_{MN}{}^P$ の対称部分が非自明でも良い！ただし

$$T_{(MN)}{}^P \Theta_P{}^\alpha = 0$$

こんな事すると $\mathcal{F}_2 = \mathrm{d}A + A \wedge A$ が共変でなくなるが、

共変性を回復させるためにテンソル補助場を導入

p-form potentials

Hodge-dual $\Updownarrow$ in D-dim

$(D-p-2)$-form potentials

(例) $D=4$ 電磁双対 $\begin{cases} \text{電場} \longleftrightarrow \text{磁場} \\ \text{スカラー場} \longleftrightarrow \text{テンソル場} \end{cases}$

${\Theta_M}^\alpha$ の配位を指定すれば定まる

(テンソル補助場が力学場に)

原理的に全ての可能なゲージ化(電気的だけでなく磁気的なものも)が構成できる

超対称性 :　$\Theta_M{}^\alpha$ **の自由度** $\dim G \times \dim G_0$ **に制限が課される**

$$\left(\mathcal{D}_\mu = \partial_\mu - g A_\mu^M \, \Theta_M{}^\alpha \, t_\alpha\right)$$

D	U-duality G_0	constraints on $R(M) \otimes R(\alpha)$
9	$GL(2)$	$(2 \oplus 1) \otimes (3 \oplus 1) = \cancel{1} \oplus \boxed{2} \oplus \cancel{2} \oplus \boxed{3} \oplus \cancel{4}$
8	$SL(3) \otimes SL(2)$	$(3,2) \otimes [(8,1) \oplus (1,3)] = \boxed{(3,2)} \oplus \cancel{(3,2)} \oplus \cancel{(3,4)} \oplus \boxed{(6,2)} \oplus \cancel{(15,2)}$
7	$SL(5)$	$10 \otimes 24 = \cancel{10} \oplus \boxed{15} \oplus \boxed{40} \oplus \cancel{175}$
6	$SO(5,5)$	$16 \otimes 45 = \cancel{16} \oplus \boxed{144} \oplus \cancel{560}$
5	$E_{6(6)}$	$27 \otimes 78 = \cancel{27} \oplus \boxed{351} \oplus \cancel{1728}$
4	$E_{7(7)}$	$56 \otimes 133 = \cancel{56} \oplus \boxed{912} \oplus \cancel{6480}$
3	$E_{8(8)}$	$248 \otimes 248 = \boxed{1} \oplus \cancel{248} \oplus \boxed{3875} \oplus \cancel{27000} \oplus \cancel{30380}$

F.Riccioni, D.Steele and P.West, arXiv:0906.1177

✓ 共変微分化：

$$\nabla_\mu \phi^A = \partial_\mu \phi^A - \mathrm{g}\, \mathscr{K}^A{}_\Sigma\, A^\Sigma_\mu - \mathrm{g}\, \mathscr{K}^{A\Sigma}\, A_{\mu\Sigma}$$

$$\mathscr{K}_\Sigma = \Theta_\Sigma{}^{\mathsf{m}}\, (t_{\mathsf{m}})^\alpha{}_\beta\, B_a{}^\beta\, (\mathcal{U}^{-1})^{Aa}_\alpha\, \frac{\partial}{\partial \phi^A}$$

$$\mathscr{K}^\Sigma = \Theta^{\Sigma\mathsf{m}}\, (t_{\mathsf{m}})^\alpha{}_\beta\, B_a{}^\beta\, (\mathcal{U}^{-1})^{Aa}_\alpha\, \frac{\partial}{\partial \phi^A}$$

✓ ($\Theta_M{}^{\mathsf{m}}$ を指定して) スカラー場からテンソル場への双対変換：

$$-\mathcal{G}_{AB}\, \nabla_\mu \phi^A\, \nabla^\mu \phi^B \quad \longrightarrow \quad -\mathcal{M}_{\mathsf{mn}}\, H^{\mathsf{m}}_{\mu\nu\rho}\, H^{\mu\nu\rho\mathsf{n}}$$

$\Theta^{\Sigma m}$:   nongeometric flux charges **が起源** (?)

テンソル場が結合する超重力理論

$\Updownarrow$

Nongeometric flux compactifications

work in progress...

D	32-SUSY	16-SUSY	8-SUSY
9	arXiv:1105.1760	(unknown)	–
8	arXiv:1203.6562	(unknown)	–
7	hep-th/0506237	(unknown)	–
6	arXiv:0712.4277	**(unknown)**	arXiv:1012.1818
5	hep-th/0412173	hep-th/0702084	(unknown)
4	arXiv:0705.2101	hep-th/0602024	**arXiv:1107.3305**
3	hep-th/0103032	arXiv:0806.2584	arXiv:0807.2841

$$b_n^c$$

ある適当な方向をコンパクト化すると「風変わりな」物体が登場する:

M-theory on $S^1(R_s)$	mass/tension ($l_s \equiv 1$)	type IIA
longitudinal M2	1	F1
transverse M2	$\frac{1}{g_s}$	D2
longitudinal M5	$\frac{1}{g_s}$	D4
transverse M5	$\frac{1}{g_s^2}$	NS5
longitudinal KK6	$\frac{R_{TN}^2}{g_s^2}$	KK5
KK6 with $R_{TN} = R_s$	$\frac{1}{g_s}$	D6
transverse KK6	$\frac{R_{TN}^2}{g_s^3}$	6^1_3

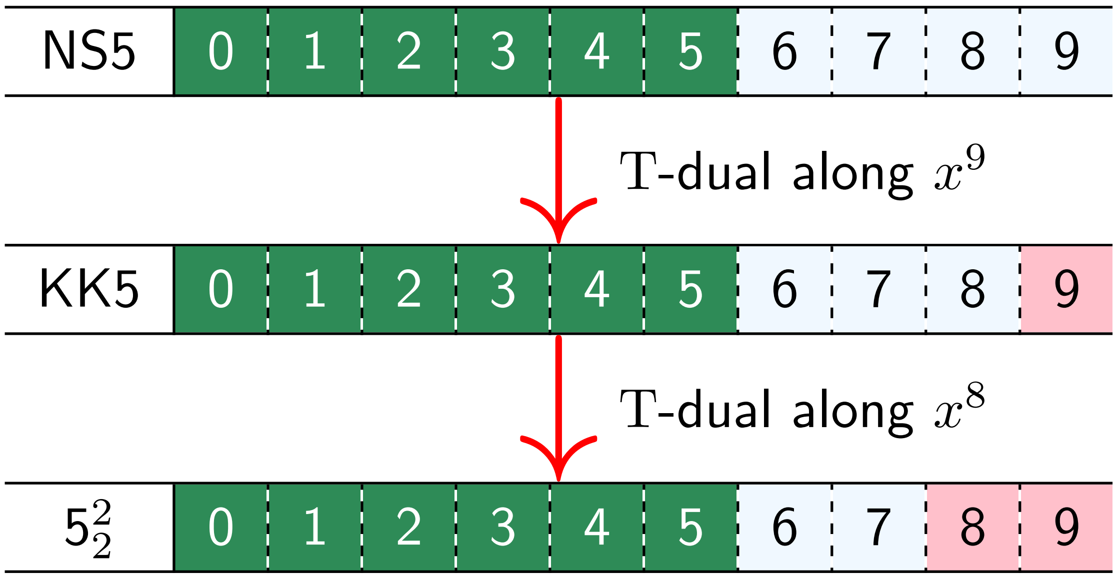
0	1	2	3	4	5	6	7	8	9	M
✓	✓	✓	✓	✓	✓	✓	S^1	$\mathbb{R}^3$		
KK6 $\to 6^1_3$							Taub-NUT			

$$b_n^c \,:\, M = \frac{(R_1 \cdots R_c)^2}{g_s^n}$$

for review: N. Obers and B. Pioline, hep-th/9809039

5^2_2-brane

$$M = \frac{(R_8 R_9)^2}{g_s^2}$$

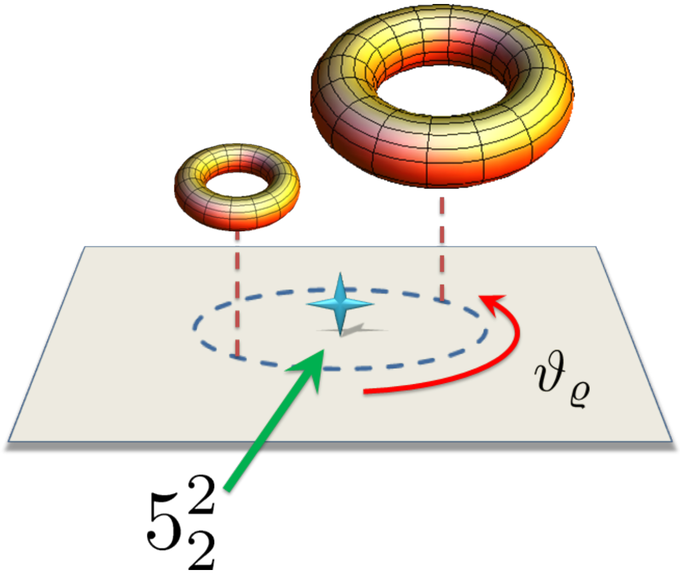


風変わりな物体は「風変わりな振る舞い」をする

$$
\begin{aligned}
\mathrm{d}s^2 &= -\mathrm{d}t^2 + \mathrm{d}x^2_{12345} + H(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2) + \frac{H}{K}\mathrm{d}x^2_{89} \\
B_{89} &= -\frac{\theta\,\sigma}{K}\,, \quad \mathrm{e}^{2\Phi} = \frac{H}{K}\,, \quad K \equiv H^2 + \sigma^2\theta^2 \\
H(r) &= h + \sigma\log\left(\frac{\mu}{r}\right), \quad \sigma \equiv \frac{R_8 R_9}{2\pi\alpha'}
\end{aligned}
$$

$$\theta = 0 \quad : \quad G_{88} = G_{99} = H^{-1}$$

$$\theta = 2\pi \quad : \quad G_{88} = G_{99} = \frac{H}{H^2 + (2\pi\sigma)^2}$$



Globally nongeometric : θ-**方向の座標張り替えで** fiber T^{89} **が** single-valued **でない**

Locally geometric : **いたるところで、局所座標系が張れる**

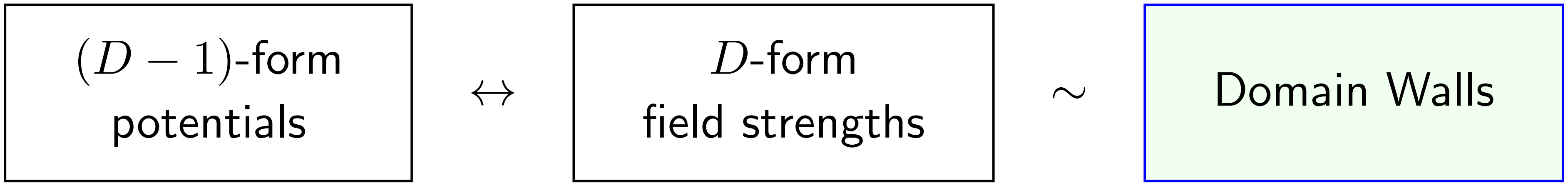
(non)geometric flux $Q^{ab}{}_c$ **を体現する** T-fold

このような振る舞いをする exotic branes **の出現は**

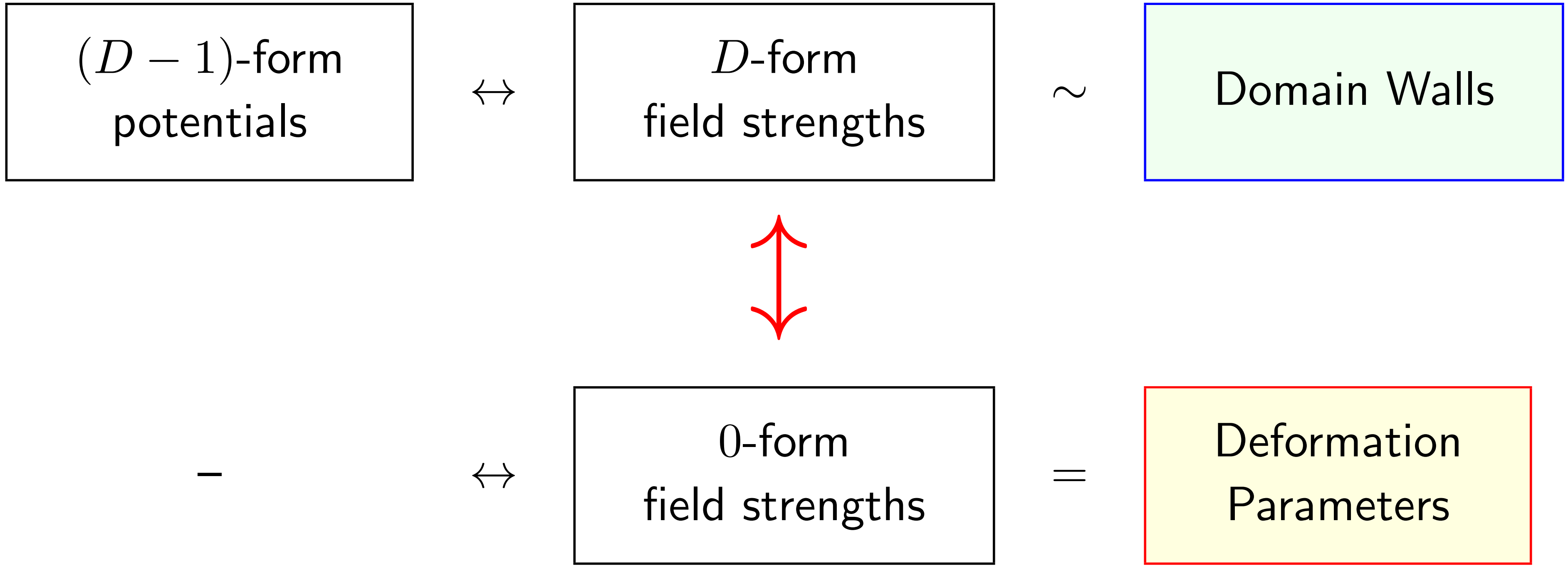
D-**次元時空の** co-dim. 2, 1 **な物体で顕著**

co-dim. 2	:	Defect Branes	$\leftarrow (D-2)$-form potentials
co-dim. 1	:	Domain Walls	$\leftarrow (D-1)$-form potentials

これらは D-**次元超重力理論にも「出現」する**



$(D-1)$-form potentials
$\leftrightarrow$
D-form field strengths
$\sim$
Domain Walls



(D − 1)-form
potentials
↔
D-form
field strengths
∼
Domain Walls
–
↔
0-form
field strengths
=
Deformation
Parameters

- D8-brane in 10-dim.

  RR potential C_9 の源

  $*_{10}\mathrm{d}C_9 = m$ (定数) を与え、IIA 型超重力理論を変形する

  $\rightarrow$ Romans' massive IIA SUGRA

- $(D-2)$-branes in D-dim.

  各次元にいくつの SUSY Domain Walls が存在するのか？

  Domain Walls が超重力理論をどのように変形するか？

  そもそも Domain Walls の弦理論起源は全て理解できているか？

$(D-1)$-form potentials **は** DWs **に結合する**

$+$

$\Theta_M{}^\alpha$ **は** D **次元理論の** $(D-1)$-form potentials **の表現に等しい**

D	U-duality G_0	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
IIA	$\mathbb{R}^+$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	–	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1} \oplus \mathbf{1}$
IIB	$SL(2,\mathbb{R})$	–	$\mathbf{2}$	–	$\mathbf{1}$	–	$\mathbf{2}$	–	$\mathbf{3}$	–	$\mathbf{4} \oplus \mathbf{2}$
9	$GL(2,\mathbb{R})$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{3} \oplus \mathbf{1}$	$\mathbf{3} \oplus \mathbf{2}$	$\mathbf{4} \oplus \mathbf{2} \oplus \mathbf{2}$	–
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}}, \mathbf{2})$	$(\mathbf{3}, \mathbf{1})$	$(\mathbf{1}, \mathbf{2})$	$(\overline{\mathbf{3}}, \mathbf{1})$	$(\mathbf{3}, \mathbf{2})$	$(\mathbf{8}, \mathbf{1})$ $\oplus(\mathbf{1}, \mathbf{3})$	$(\mathbf{6}, \mathbf{2})$ $\oplus(\overline{\mathbf{3}}, \mathbf{2})$	$(\mathbf{15}, \mathbf{1})$ $\oplus(\mathbf{3}, \mathbf{3})$ $\oplus(\mathbf{3}, \mathbf{1})$ $\oplus(\mathbf{3}, \mathbf{1})$	–	–
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}} \oplus \overline{\mathbf{15}}$	$\mathbf{70}$ $\oplus\mathbf{45}$ $\oplus\mathbf{5}$	–	–	–
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\mathbf{320}$ $\oplus\overline{\mathbf{126}}$ $\oplus\mathbf{10}$	–	–	–	–
5	$E_{6(6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\overline{\mathbf{1728}}$ $\oplus\overline{\mathbf{27}}$	–	–	–	–	–
4	$E_{7(7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{8645}$ $\oplus\mathbf{133}$	–	–	–	–	–	–
3	$E_{8(8)}$	$\mathbf{248}$	$\mathbf{3875} \oplus \mathbf{1}$	$\mathbf{147250}$ $\oplus\mathbf{3875}$ $\oplus\mathbf{248}$	–	–	–	–	–	–	–

$(D-1)$-forms： Embedding Tensors $\Theta_M{}^\alpha$ の表現はこれと一致

F.Riccioni, D.Steele and P.West, arXiv:0906.1177

of (Elementary SUSY) DWs

D	$n=0$ (fundamental)	$n=-1$ (Dirichlet)	$n=-2$ (solitonic)	$n=-3$	$n=-4$	$n=-5$	#
IIA		1					1
9		1	–	1			2
8		$(\mathbf{1},\mathbf{2})_{\mathrm{T}}$	–	$4 \subset (\mathbf{3},\mathbf{2})_{\mathrm{T}}$			6
7		$\mathbf{4}_{\mathrm{T}}$	$4 \subset \mathbf{10}_{\mathrm{T}}$	$12 \subset \overline{\mathbf{20}}_{\mathrm{T}}$			25
			$4 \subset \overline{\mathbf{10}}_{\mathrm{T}}$	–	$\mathbf{1}_{\mathrm{T}}$		
6		$\mathbf{8}_{\mathrm{S}}\vert_{\mathrm{T}}$	$32 \subset \mathbf{56}_{\mathrm{C}}\vert_{\mathrm{T}}$	$32 \subset \mathbf{56}_{\mathrm{S}}\vert_{\mathrm{T}}$	$\mathbf{8}_{\mathrm{C}}\vert_{\mathrm{T}}$		80
5		$\overline{\mathbf{16}}_{\mathrm{T}}$	$80 \subset \mathbf{120}_{\mathrm{T}}$	$80 \subset \mathbf{144}_{\mathrm{T}}$	$40 \subset \mathbf{45}_{\mathrm{T}}$		216
4		$\mathbf{32}_{\mathrm{T}}$	$160 \subset \mathbf{220}_{\mathrm{T}}$	$192 \subset \mathbf{352}_{\mathrm{T}}$	$160 \subset \mathbf{220}_{\mathrm{T}}$	$\mathbf{32}_{\mathrm{T}}$	576
3	$\mathbf{1}_{\mathrm{T}}$	$\overline{\mathbf{64}}_{\mathrm{T}}$	$280 \subset \mathbf{364}_{\mathrm{T}}$	$448 \subset \mathbf{832}_{\mathrm{T}}$	$560 \subset \mathbf{1001}_{\mathrm{T}}$ $14 \subset \mathbf{104}_{\mathrm{T}}$	$448 \subset \overline{\mathbf{832}}_{\mathrm{T}}$	2160
$(\alpha \leq -6)$				$280 \subset \mathbf{364}_{\mathrm{T},-6}$	$\mathbf{64}_{\mathrm{T},-7}$	$\mathbf{1}_{\mathrm{T},-8}$	

brane's tension $\sim g_{\mathrm{s}}^{+n}$

E.A. Bergshoeff et al, arXiv:1108.5067, arXiv:1210.1422

D	$n=0$	$n=-1$	$n=-2$	$n=-3$	$n=-4$	$n \leq -5$
IIA		D8 $[C_9]$				
9		D7 $[C_8]$		$7_3^{(0,1)}$ $[E_{9,1,1}]$		
8		D6 $[C_7]$		$6_3^{(1,1)}$ $[E_{9,2,1}]$		
7		D5 $[C_6]$	NS5 $[D_6]$ KKM $[D_{7,1}]$ 5_2^2 $[D_{8,2}]$	$5_3^{(2,1)}$ $[E_{9,3,1}]$	5_4^3 $[F_{9,3}]$	
6		D4 $[C_5]$		$4_3^{(3,1)}$ $[E_{9,4,1}]$	$4_4^{(3,1)}$ $[F_{9,4,1}]$	
5		D3 $[C_4]$		$3_3^{(4,1)}$ $[E_{9,5,1}]$	$3_4^{(3,2)}$ $[F_{9,5,2}]$	
4		D2 $[C_3]$		$2_3^{(5,1)}$ $[E_{9,6,1}]$	$2_4^{(3,3)}$ $[F_{9,6,3}]$	$\cdots$
3	F1 $[B_2]$	D1 $[C_2]$		$1_3^{(6,1)}$ $[E_{9,7,1}]$	$1_4^{(3,4)}$ $[F_{9,7,4}]$ $1_4^{(6,0,1)}$ $[F_{9,7,1,1}]$	$\cdots$

A_{D-T,I_1+I_2,I_2}-forms : "mixed-symmetry fields" $\leftrightarrow$ $b_n^{(I_1,I_2)}$-branes

$T + b + \sum_i I_i = D - 1$ with $T = 1$: transverse, b : spatial, I_i : isometry directions

Exotic branes b_n^c とは一体何か

まだほとんど分かってない

(様々なところで重要な役割を果たすと期待される)

J. de Boer and M. Shigemori, arXiv:1004.2521 and arXiv:1209.6056

なすべきこと

期 待

全ての超重力理論は

超弦理論を起源とできるのではないか？

期 待

全ての超重力理論は

超弦理論を起源とできるのではないか？

$$\Theta_M{}^\alpha \qquad Q^{ab}{}_c \qquad b_n^c$$

embedding tensors (non)geometric fluxes exotic branes

極大超重力理論においては、対応関係がある程度把握できている

${Q^{ab}}_c$	vs	${\Theta_M}^\alpha$	: G. Dall'Agata et al, arXiv:0712.1026
${\Theta_M}^\alpha$	vs	b^c_n	: E. Bergshoeff et al, arXiv:1206.5697

超対称性が低い場合に

対応関係を追究する → 超弦理論起源の完備に向けて

現在： 4D $\mathcal{N} = 2$ 理論での ${Q^{ab}}_c$ vs ${\Theta_M}^\alpha$ を追跡中

(注意) 超対称性が低くなると、大域的対称性に対する縛りが弱くなる

Flux Compactifications on $SU(3) \times SU(3)$ generalized geometry

vs

Embedding Tensor Formalism in 4D $\mathcal{N} = 2$ theory

双方で記述される gauged supergravity **の相互作用項を比較して**

$Q^{ab}{}_{c}$ vs $\Theta_{M}{}^{\alpha}$ **を直接関連付ける**

一見「単純作業」だが、超重力理論なので、計算がいちいち入り組んでいる

さらに、embedding tensor formalism **側は共形超重力理論で記述されている**

「rigid special Kähler vs local special Kähler」 「hyper-Kähler cone vs quaternonic Kähler」 etc.

おしまい

Appendix

NS-NS 場の展開：

$$\phi(x,y) = \varphi(x)$$

$$g_{\mathfrak{m}\bar{\mathfrak{n}}}(x,y) = \mathrm{i}v^a(x)\,(\omega_a)_{\mathfrak{m}\bar{\mathfrak{n}}}(y), \quad g_{\mathfrak{m}\mathfrak{n}}(x,y) = \mathrm{i}\,\bar{z}^{\bar{\jmath}}(x)\left(\frac{(\overline{\chi}_{\bar{\jmath}})_{\mathfrak{m}\bar{\mathfrak{p}}\bar{\mathfrak{q}}}\Omega^{\bar{\mathfrak{p}}\bar{\mathfrak{q}}}{}_{\mathfrak{n}}}{||\Omega||^2}\right)(y)$$

$$B_2(x,y) = B_2(x) + b^a(x)\omega_a(y)$$

$$\mathfrak{t}^a(x) \equiv b^a(x) + \mathrm{i}v^a(x)$$

R-R 場の展開：

$$C_1(x,y) = A_1^0(x)$$

$$C_3(x,y) = A_1^a(x)\wedge\omega_a(y) + \xi^I(x)\alpha_I(y) - \widetilde{\xi}_I(x)\beta^I(y)$$

コホモロジー	基底	自由度
$H^{(1,1)}$	ω_a	$a = 1,\ldots,h^{(1,1)}$
$H^{(0)}\oplus H^{(1,1)}$	$\omega_\Lambda = (1,\omega_a)$	$\Lambda = 0,1,\ldots,h^{(1,1)}$
$H^{(2,2)}\oplus H^{(6)}$	$\widetilde{\omega}^\Lambda = (\widetilde{\omega}^a, \frac{\mathrm{vol.}}{\lvert\mathrm{vol.}\rvert})$	
$H^{(2,1)}$	χ_i	$i = 1,\ldots,h^{(2,1)}$
$H^{(3)}$	(α_I,β^I)	$I = 0,1,\ldots,h^{(2,1)}$

$$\mathrm{d}\omega_\Lambda = 0 = \mathrm{d}\widetilde{\omega}^\Lambda$$

$$\mathrm{d}\alpha_I = 0 = \mathrm{d}\beta^I$$

10D type IIA action $S^{(10\text{D})}_{\text{IIA}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}$:

$$S_{\text{NS}} = \frac{1}{2}\int e^{-2\phi}\Big\{\widehat{R}*\mathbb{1} + 4\text{d}\phi\wedge*\text{d}\phi - \frac{1}{2}\widehat{H}_3\wedge*\widehat{H}_3\Big\}$$

$$S_{\text{R}} + S_{\text{CS}} = -\frac{1}{4}\int\Big\{\widehat{F}_2\wedge*\widehat{F}_2 + (\widehat{F}_4 - \widehat{C}_1\wedge\widehat{F}_3)\wedge*(\widehat{F}_4 - \widehat{C}_1\wedge\widehat{F}_3)\Big\} - \frac{1}{4}\int\widehat{B}_2\wedge\widehat{F}_4\wedge\widehat{F}_4$$

$$\downarrow$$

4D $\mathcal{N}=2$ ungauged SUGRA: Neither gauge couplings, Nor scalar potential

$$S^{(4\text{D})} = \int\Big\{\frac{1}{2}R*\mathbb{1} - G_{a\bar{b}}\,\text{d}t^a\wedge*\text{d}\bar{t}^{\bar{b}} - h_{uv}\,\text{d}q^u\wedge*\text{d}q^v + \frac{1}{2}\text{Im}\mathcal{N}_{\Lambda\Sigma}F_2^\Lambda\wedge*F_2^\Sigma + \frac{1}{2}\text{Re}\mathcal{N}_{\Lambda\Sigma}F_2^\Lambda\wedge F_2^\Sigma\Big\}$$

gravitational multiplet	$g_{\mu\nu}$, A_1^0		
vector multiplet (VM)	A_1^a, t^a, $\bar{t}^{\bar{b}}$	$t^a \in \text{SKG}_\text{V}$	
hypermultiplet (HM)	z^i, $\bar{z}^{\bar{j}}$, ξ^i, $\widetilde{\xi}_j$	$z^i \in \text{SKG}_\text{H}$	
universal hypermultiplet (UHM)	φ, a, ξ^0, $\widetilde{\xi}_0$	$a \leftrightarrow B_2$	(Hodge dual)

$\mathcal{HM}$ = Special QG

$$\underset{4n_\text{H}+4}{\{q^u\}} = \underset{2n_\text{H}(\text{SKG}_\text{H})}{\{z^i,\bar{z}^{\bar{j}}\}} + \underset{2n_\text{H}}{\{\xi^i,\widetilde{\xi}_j\}} + \underset{4\,(\text{UHM})}{\{\varphi,a,\xi^0,\widetilde{\xi}_0\}} = \underset{\text{SKG}_\text{H}}{\{z^i,\bar{z}^{\bar{j}}\}} + \{\varphi\} + \underset{\text{“Heisenberg”}}{\{a,\xi^I,\widetilde{\xi}_J\}}$$

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}(\overline{\mathcal{W}}_1\Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3\,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

complex	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	Calabi-Yau	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\mathrm{Im}\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

10D type IIA action $S^{(10D)}_{\mathrm{IIA}} = S_{\mathsf{NS}} + \widetilde{S}_{\mathsf{R}} = S_{\mathsf{NS}} + S_{\mathsf{R}} + S_{\mathsf{CS}}$: (democratic form)

$$S_{\mathsf{NS}} = \frac{1}{2}\int \mathrm{e}^{-2\phi}\Big\{\widehat{R} * \mathbb{1} + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}\widehat{H}_3 \wedge *\widehat{H}_3\Big\}, \quad \widetilde{S}_{\mathsf{R}} = -\frac{1}{8}\int \big[\widehat{\mathbf{F}} \wedge *\widehat{\mathbf{F}}\big]_{10}$$

with "constraint $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})$" and "EoM (Bianchi) $(\mathrm{d} + \widehat{H}\wedge) * \widehat{\mathbf{F}} = 0 \Leftrightarrow (\mathrm{d} - \widehat{H}\wedge)\widehat{\mathbf{F}} = 0$"

↓ non-CY with $SU(3)$-structure with $m^{\Lambda}_{\mathsf{R}} = 0$

4D $\mathcal{N} = 2$ abelian gauged SUGRA (with $\xi^{\boldsymbol{I}} \equiv (\xi^I, \widetilde{\xi}_I)^{\mathrm{T}}$):

$$S^{(4D)} = \int \mathrm{d}^4x\sqrt{-g}\,\Big[\frac{1}{2}R + \frac{1}{4}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} - \frac{\epsilon^{\mu\nu\rho\sigma}}{8\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} - g_{a\bar{b}}\,\partial_\mu \mathfrak{t}^a \partial^\mu \bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}}\,\partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}}$$
$$-\partial_\mu\varphi\partial^\mu\varphi + \frac{\mathrm{e}^{2\varphi}}{2}(\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}}D_\mu\xi^{\boldsymbol{I}}D^\mu\xi^{\boldsymbol{J}} - \frac{\mathrm{e}^{2\varphi}}{4}\big(D_\mu a - \xi^{\boldsymbol{I}}(\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}}D_\mu\xi^{\boldsymbol{J}}\big)^2 - V(\mathfrak{t},\bar{\mathfrak{t}},q)\Big]$$

- $({e_\Lambda}^I, e_{\Lambda I})$: geometric flux charges & $e_{\mathsf{R}\Lambda}$: RR-flux charges ⟵ non-CY data
  (with constraints ${e_\Lambda}^I e_{\Sigma I} - e_{\Lambda I}{e_\Sigma}^I = 0$)
- $\mathfrak{t}^a \in \mathsf{SKG_V}$ and $z^i \in \mathsf{SKG_H} \subset \mathcal{HM}$ are ungauged (in general)
- $D_\mu\xi^I = \partial_\mu\xi^I - {e_\Lambda}^I A^\Lambda_\mu$ & $D_\mu\widetilde{\xi}_I = \partial_\mu\widetilde{\xi}_I - e_{\Lambda I}A^\Lambda_\mu$
- $D_\mu a = \partial_\mu a - (2e_{\mathsf{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I {e_\Lambda}^I)A^\Lambda_\mu$
- $V(\mathfrak{t},\bar{\mathfrak{t}},q)$: scalar potential

D. Cassani, arXiv:0804.0595

Non-vanishing m_{R}^{Λ} dualizes the axion field a in standard SUGRA to B-field.

4D gauged action is different from the standard one:

$$\begin{aligned} S^{(\mathsf{4D})} \; = \; \int \Big[& \frac{1}{2} R(*\mathbb{1}) + \frac{1}{2} \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F_{\mathit{2}}^{\Lambda} \wedge * F_{\mathit{2}}^{\Sigma} + \frac{1}{2} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F_{\mathit{2}}^{\Lambda} \wedge F_{\mathit{2}}^{\Sigma} - g_{a\bar{b}}\, \mathrm{d}\mathfrak{t}^a \wedge * \mathrm{d}\bar{\mathfrak{t}}^{\bar{b}} - g_{i\bar{\jmath}}\, \mathrm{d}z^i \wedge * \mathrm{d}\bar{z}^{\bar{\jmath}} \\ & - \mathrm{d}\varphi \wedge * \mathrm{d}\varphi - \frac{\mathrm{e}^{-4\varphi}}{4} H_{\mathit{3}} \wedge * H_{\mathit{3}} - \frac{\mathrm{e}^{2\varphi}}{2} (\mathbb{M}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{I}} \wedge * D\xi^{\boldsymbol{J}} - V(*\mathbb{1}) \\ & + \frac{1}{2} \mathrm{d}B \wedge \Big[\xi^{\boldsymbol{I}} (\mathbb{C}_{\mathsf{H}})_{\boldsymbol{IJ}} D\xi^{\boldsymbol{J}} + \big(2 e_{\mathsf{R}\Lambda} - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_{\Lambda}{}^I \big) A_{\mathit{1}}^{\Lambda} \Big] - \frac{1}{2} m_{\mathsf{R}}^{\Lambda} e_{\mathsf{R}\Lambda} B_{\mathit{2}} \wedge B_{\mathit{2}} \Big] \end{aligned}$$

Constraints among flux charges:

$$e_{\Lambda}{}^I e_{\Sigma I} - e_{\Lambda I} e_{\Sigma}{}^I \; = \; 0, \qquad m_{\mathsf{R}}^{\Lambda} e_{\Lambda}{}^I \; = \; 0 \; = \; m_{\mathsf{R}}^{\Lambda} e_{\Lambda I}$$

Scalar potential from (non)geometric flux compactifications:

$$V = \mathrm{g}^2\Big[4h_{uv}k^u\overline{k}^v + \sum_{x=1}^{3}\Big(g^{a\overline{b}}D_a\mathcal{P}_xD_{\overline{b}}\overline{\mathcal{P}}_x - 3|\mathcal{P}_x|^2\Big)\Big] = \ldots \equiv V_{\mathsf{NS}} + V_{\mathsf{R}} \qquad (\text{abelian: } k^a_\Lambda = 0)$$

$$V_{\mathsf{NS}} = g^{a\overline{b}}D_a\mathcal{P}_+D_{\overline{b}}\overline{\mathcal{P}}_+ + g^{i\overline{\jmath}}D_i\mathcal{P}_+D_{\overline{\jmath}}\overline{\mathcal{P}}_+ - 2|\mathcal{P}_+|^2$$

$$= -2\,\mathrm{g}^2\mathrm{e}^{2\varphi}\Big[\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}}\,\widetilde{Q}^{\mathrm{T}}\,\mathbb{M}_{\mathsf{V}}\,\widetilde{Q}\,\Pi_{\mathsf{H}} + \overline{\Pi}_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{M}_{\mathsf{H}}\,Q^{\mathrm{T}}\,\Pi_{\mathsf{V}} + 4\overline{\Pi}_{\mathsf{H}}^{\mathrm{T}}\,\mathbb{C}_{\mathsf{H}}^{\mathrm{T}}\,Q^{\mathrm{T}}(\Pi_{\mathsf{V}}\overline{\Pi}_{\mathsf{V}}^{\mathrm{T}} + \overline{\Pi}_{\mathsf{V}}\Pi_{\mathsf{V}}^{\mathrm{T}})Q\,\mathbb{C}_{\mathsf{H}}\,\Pi_{\mathsf{H}}\Big]$$

$$V_{\mathsf{R}} = g^{a\overline{b}}D_a\mathcal{P}_3D_{\overline{b}}\overline{\mathcal{P}}_3 + |\mathcal{P}_3|^2$$

$$= -\frac{1}{2}\,\mathrm{g}^2\mathrm{e}^{4\varphi}\big(e_{\mathsf{R}\Lambda} - e_{\Lambda I}\xi^I + e_\Lambda{}^I\widetilde{\xi}_I\big)(\mathrm{Im}\mathcal{N})^{-1|\Lambda\Sigma}\big(e_{\mathsf{R}\Sigma} - e_{\Sigma I}\xi^I + e_\Sigma{}^I\widetilde{\xi}_I\big)$$

$\Pi_{\mathsf{V}} = \mathrm{e}^{\mathcal{K}_{\mathsf{V}}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$

$t^a = X^a/X^0$

$a = 1, \ldots, n_{\mathsf{V}}$

SKG$_{\mathsf{V}}$ of vector-moduli

$$\mathcal{P}_+ \equiv \mathcal{P}_1 + \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}\,\Pi_{\mathsf{H}}$$

$$\mathcal{P}_- \equiv \mathcal{P}_1 - \mathrm{i}\mathcal{P}_2 = 2\mathrm{e}^{\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{H}}\,\overline{\Pi}_{\mathsf{H}}$$

$$\mathcal{P}_3 = \mathrm{e}^{2\varphi}\,\Pi_{\mathsf{V}}^{\mathrm{T}}\,\mathbb{C}_{\mathsf{V}}(c_{\mathsf{R}} + \widetilde{Q}\xi)$$

$\Pi_{\mathsf{H}} = \mathrm{e}^{\mathcal{K}_{\mathsf{H}}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$

$z^i = Z^i/Z^0$

$i = 1, \ldots, n_{\mathsf{H}}$

SKG$_{\mathsf{H}}$ of hyper-moduli

$$\mathbb{C}_{\mathsf{V,H}} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}; \quad Q = \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix}, \quad \widetilde{Q} = \mathbb{C}_{\mathsf{H}}^{\mathrm{T}}\,Q\,\mathbb{C}_{\mathsf{V}} \quad c_{\mathsf{R}} = \begin{pmatrix} m_{\mathsf{R}}^\Lambda \\ e_{\mathsf{R}\Lambda} \end{pmatrix}$$

Cassani et.al., arXiv:0804.0595, arXiv:0911.2708