

Gauged Linear Sigma Model for **Exotic** Five-brane

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[arXiv:1304.4061](https://arxiv.org/abs/1304.4061) [accepted by NPB]

[arXiv:1305.4439](https://arxiv.org/abs/1305.4439) [JHEP 08 (2013) 126]

佐々木伸氏との共同研究

良く知る物体
D-branes, NS-branes

string dualities
→
in D -dimensions

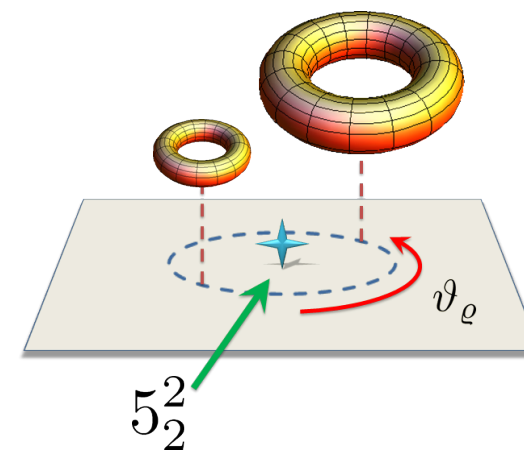
馴染みの薄い物体
exotic branes

特徴 :

- ✓ 漸近的振る舞いや質量に赤外発散が入る
(co-dimension ≤ 2)
- ✓ 計量などが一価関数にならない
(非自明なモノドロミー)

5_2^2 -brane

NS5-brane	0	1	2	3	4	5	6	7	8	9
	T-dual along x^9									
KK-monopole	0	1	2	3	4	5	6	7	8	$\tilde{9}$
	T-dual along x^8									
5_2^2 -brane	0	1	2	3	4	5	6	7	$\tilde{8}$	$\tilde{9}$



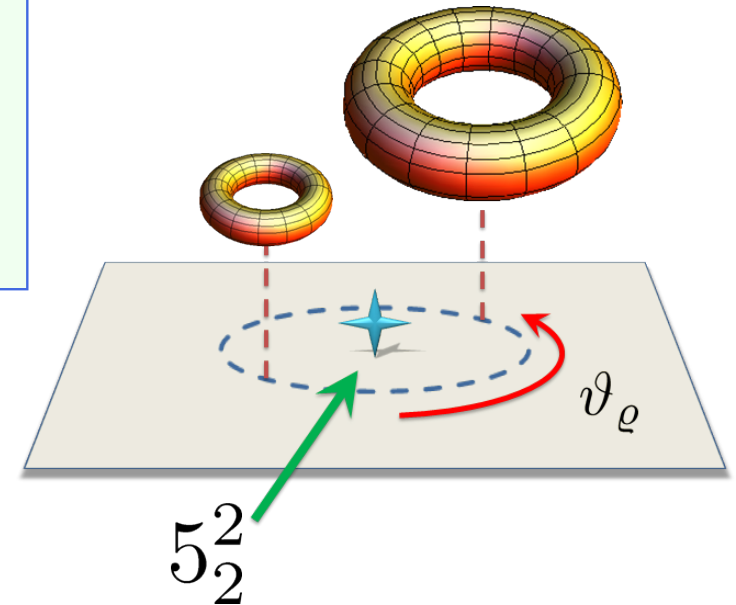
$$ds^2 = dx_{012345}^2 + H [d\varrho^2 + \varrho^2 d\vartheta_\varrho^2] + \frac{H}{K} [(d\tilde{x}^8)^2 + (d\tilde{x}^9)^2]$$

$$B_{89} = -\frac{\sigma \vartheta_\varrho}{K}, \quad e^{2\Phi} = \frac{H}{K}, \quad K = H^2 + (\sigma \vartheta_\varrho)^2$$

$$H = h + \sigma \log\left(\frac{\mu}{\varrho}\right)$$

$$\vartheta_\varrho = 0 \quad : \quad G_{\tilde{8}\tilde{8}} = G_{\tilde{9}\tilde{9}} = \frac{1}{H}$$

$$\vartheta_\varrho = 2\pi \quad : \quad G_{\tilde{8}\tilde{8}} = G_{\tilde{9}\tilde{9}} = \frac{H}{H^2 + (2\pi\sigma)^2}$$



Locally geometric, **しかし** Globally **non-geometric** (non-single-valued metric)

フラックスコンパクト化に於ける **T-fold** の具体例

J. de Boer and M. Shigemori [arXiv:1209.6056](https://arxiv.org/abs/1209.6056)




C. Hull [hep-th/0406102](https://arxiv.org/abs/hep-th/0406102)

これまでは主に、点粒子をプローブとした 超重力理論 での記述であった

弦をプローブとした **Worldsheet理論** で解析する

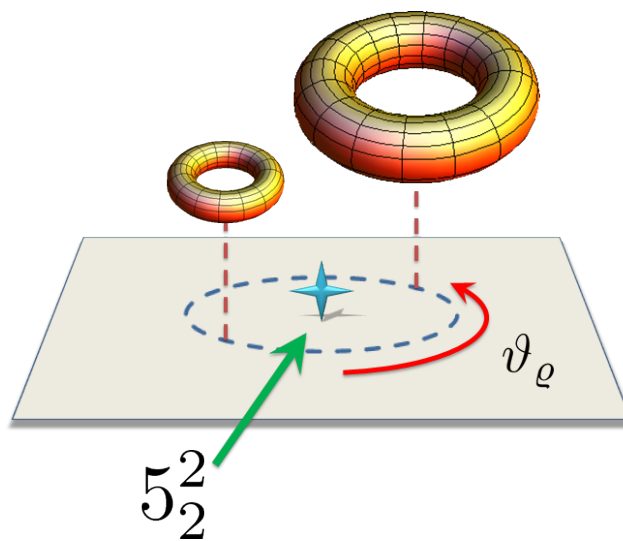
(弦理論特有のエキゾチックブレンは弦理論で理解すべし)

Worldsheet理論

-  nonlinear sigma model (NLSM)
-  conformal field theory (CFT)
-  gauged linear sigma model (GLSM)

GLSM (UltraViolet) \iff NLSM/CFT (InfraRed)

弦をプローブとして 5_2^2 -brane を記述する



$$\begin{aligned}
 \mathcal{L}_H = & \sum_{a=1}^k \int d^4\theta \left\{ \frac{1}{e_a^2} \left(-\bar{\Sigma}_a \Sigma_a + \bar{\Phi}_a \Phi_a \right) + \bar{Q}_a e^{2V_a} Q_a + \bar{\tilde{Q}}_a e^{-2V_a} \tilde{Q}_a \right\} + \int d^4\theta \frac{1}{g^2} \left(-\bar{\Theta} \Theta + \bar{\Psi} \Psi \right) \\
 & + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\theta \left(\tilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \right) + (\text{h.c.}) \right\} \\
 & + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\tilde{\theta} (t_a - \Theta) \Sigma_a + (\text{h.c.}) \right\}
 \end{aligned}$$

GLSM for H-monopoles (NS5)

 D. Tong [hep-th/0204186](https://arxiv.org/abs/hep-th/0204186)

$\mathcal{N} = (4, 4)$	$\mathcal{N} = (2, 2)$		役割
neutral HM	chiral $\Psi = \frac{1}{\sqrt{2}}(X^6 + iX^8) + \dots$	twisted chiral $\Theta = \frac{1}{\sqrt{2}}(X^7 + iX^9) + \dots$	時空座標
VMs	twisted chiral $\Sigma_a = \frac{1}{\sqrt{2}}\bar{D}_+ D_- V_a$	chiral Φ_a	isometry のゲージ化
charged HMs	chiral $Q_a (+)$	chiral $\tilde{Q}_a (-)$	時空を曲げる
FI parameters	$s_a = s_{6,a} + i s_{8,a}$	$t_a = t_{7,a} + i t_{9,a}$	five-branes の位置

$$\begin{aligned}
\mathcal{L}_H = & \sum_{a=1}^k \int d^4\theta \left\{ \frac{1}{e_a^2} \left(-\bar{\Sigma}_a \Sigma_a + \bar{\Phi}_a \Phi_a \right) + \bar{Q}_a e^{2V_a} Q_a + \bar{\tilde{Q}}_a e^{-2V_a} \tilde{Q}_a \right\} + \int d^4\theta \frac{1}{g^2} \left(-\bar{\Theta} \Theta + \bar{\Psi} \Psi \right) \\
& + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\theta \left(\tilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \right) + (\text{h.c.}) \right\} \\
& + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\tilde{\theta} (t_a - \Theta) \Sigma_a + (\text{h.c.}) \right\}
\end{aligned}$$

GLSM for H-monopoles (NS5)

D. Tong [hep-th/0204186](https://arxiv.org/abs/hep-th/0204186)

Ψ と Θ について duality (Roček-Verlinde) 変換

$$\begin{aligned}
-\frac{1}{g^2}(\Theta + \bar{\Theta}) &= (\Gamma + \bar{\Gamma}) + 2 \sum_a V_a \\
-\frac{1}{g^2}(\Psi + \bar{\Psi}) &= (\Xi + \bar{\Xi}) - \sqrt{2} \sum_a (C_a + \bar{C}_a)
\end{aligned}$$

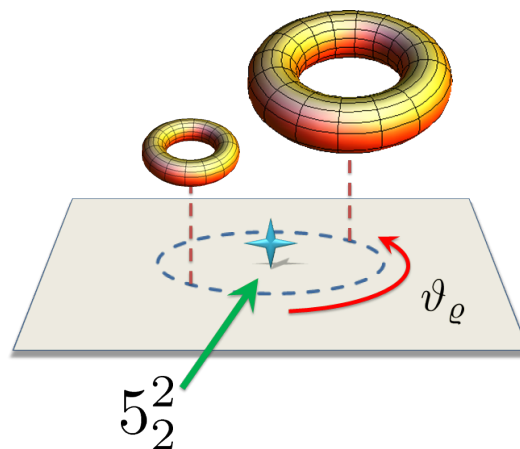
$$\Sigma_a = \frac{1}{\sqrt{2}} \bar{D}_+ D_- V_a \quad \text{そして} \quad \Phi_a = \bar{D}_+ \bar{D}_- C_a$$

$$\begin{aligned}
 \mathcal{L}_{\text{new}} = & \sum_{a=1}^k \int d^4\theta \left\{ \frac{1}{e_a^2} \left(-\bar{\Sigma}_a \Sigma_a + \bar{\Phi}_a \Phi_a \right) + \bar{Q}_a e^{2V_a} Q_a + \bar{\tilde{Q}}_a e^{-2V_a} \tilde{Q}_a \right\} \\
 & + \int d^4\theta \frac{g^2}{2} \left(\Gamma + \bar{\Gamma} + 2 \sum_{a=1}^k V_a \right)^2 + \int d^4\theta \left\{ -\frac{g^2}{2} \left(\Xi + \bar{\Xi} - \sqrt{2} \sum_{a=1}^k (C_a + \bar{C}_a) \right)^2 \right\} \\
 & + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\theta \left(\tilde{Q}_a \Phi_a Q_a + s_a \Phi_a \right) + \sqrt{2} \int d^2\tilde{\theta} t_a \Sigma_a + (\text{h.c.}) \right\} \\
 & - \sqrt{2} \int d^4\theta \left(\dot{\Psi} - \bar{\dot{\Psi}} \right) \sum_{a=1}^k (C_a - \bar{C}_a) - \sqrt{2} \varepsilon^{mn} \sum_{a=1}^k \partial_m (X^9 A_{n,a})
 \end{aligned}$$

GLSM for exotic 5_2^2 -brane : **NEW!**

Steps to NLSM for 5_2^2 -brane [geometry](#) [details](#) [superfields](#)

1. SUSY 真空を探す $\mathcal{L}^{\text{potential}} = 0$
2. charged HMs (Q_a, \tilde{Q}_a) の拘束条件を解く
3. IR 極限下で VM (V_a, Φ_a) を積分
- ★4. T-dual 座標場 X^8 を積分



$$(X^6, X^7; Y^8, Y^9)$$

Step 1.,2.,3. :

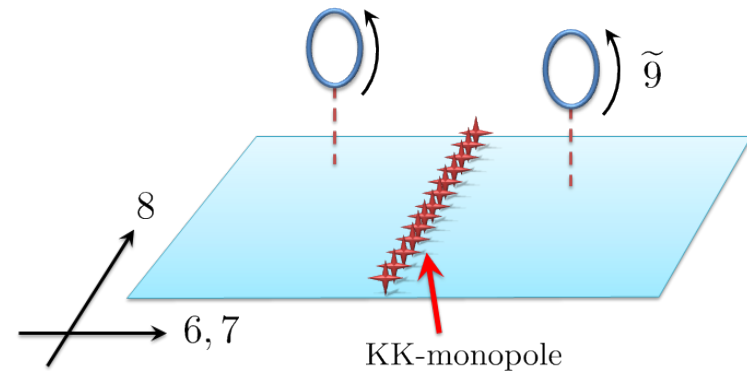
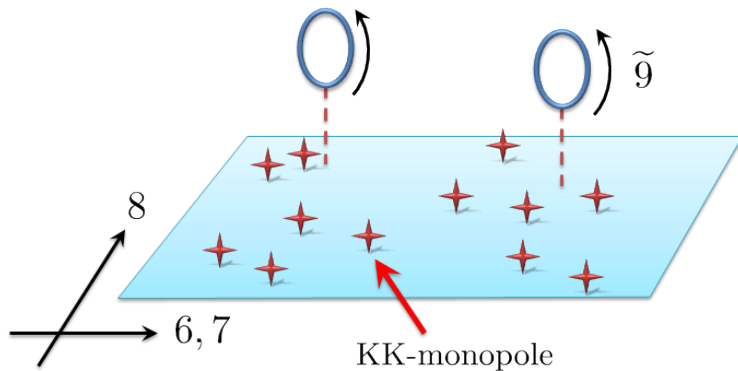
$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{2}H \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] - \frac{1}{2H} (\partial_m Y^9)^2 \\
 & - \frac{(\Omega_8)^2}{2H} (\partial_m X^8)^2 + \frac{\Omega_8}{H} (\partial_m Y^9) (\partial^m X^8) \\
 & - \frac{(\Omega_6)^2}{2H} (\partial_m X^6)^2 - \frac{\Omega_6 \Omega_8}{H} (\partial_m X^6) (\partial^m X^8) + \frac{\Omega_6}{H} (\partial_m Y^9) (\partial^m X^6) \\
 & - \sqrt{2} \varepsilon^{mn} \sum_a \partial_m ((X^9 - t_{9,a}) A_{n,a}) + \varepsilon^{mn} (\partial_m X^8) (\partial_n Y^8)
 \end{aligned}$$

$$\begin{aligned}
 H = \frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2} R_a}, \quad \Omega_6 = \sum_a \frac{X^8 - s_{8,a}}{\sqrt{2} R_a (R_a + (X^7 - t_{7,a}))}, \quad \Omega_8 = - \sum_a \frac{X^6 - s_{6,a}}{\sqrt{2} R_a (R_a + (X^7 - t_{7,a}))} \\
 A_{m,a} = -\frac{1}{2R_a H} (\partial_m Y^9 - \Omega_i \partial_m X^i) - \frac{1}{\sqrt{2}} \Omega_{i,a} \partial_m X^i
 \end{aligned}$$

Step 4. : $s_{6,a} = 0 = t_{7,a} = t_{9,a}$ and $s_{8,a} = 2\pi\mathcal{R}_8 a$ with $k \rightarrow \infty$

main

$$\rightarrow \begin{cases} H \rightarrow h_0 + \sigma \log(\mu/\varrho) & : \text{co-dim. 2} \quad \varrho^2 = (X^6)^2 + (X^7)^2 \\ \Omega_6 \rightarrow 0 & : \text{isometry along } X^8 \\ \Omega_8 \rightarrow \sigma \arctan(X^7/X^6) \equiv \sigma \vartheta_\varrho & : \text{"non-single-valued" metric} \end{cases}$$

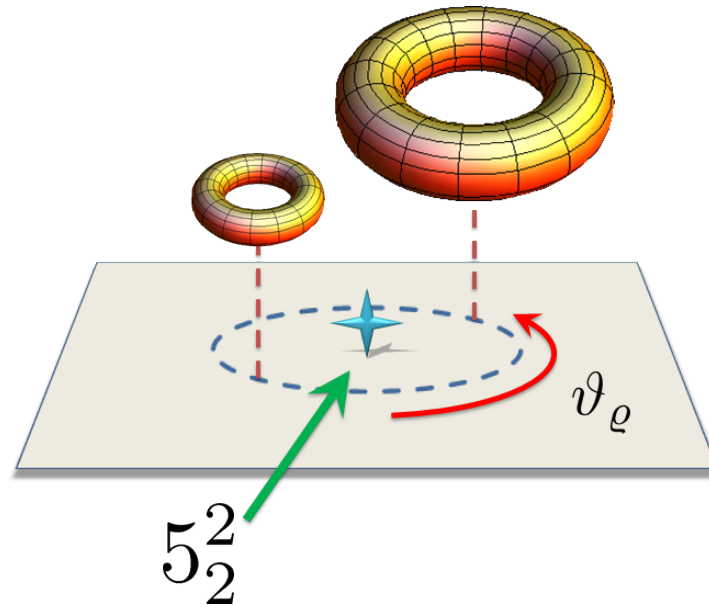


X^8 を積分 : $\partial_m X^8 = \frac{H}{K} \left[\frac{\sigma \vartheta_\varrho}{H} (\partial_m Y^9) + \varepsilon_{mn} (\partial^n Y^8) \right]$

$$K = H^2 + (\sigma \vartheta_\varrho)^2$$

$$\mathcal{L}_{\text{NLSM}} = -\frac{1}{2}H \left[(\partial_m \varrho)^2 + \varrho^2 (\partial_m \vartheta_\varrho)^2 \right] - \frac{1}{2K} \left[(\partial_m Y^8)^2 + (\partial_m Y^9)^2 \right]$$

$$- \frac{\sigma \vartheta_\varrho}{K} \varepsilon^{mn} (\partial_m Y^8) (\partial_n Y^9) - \sqrt{2} \varepsilon^{mn} \sum_a \partial_m (X^9 A_{n,a})$$



We established GLSM for exotic five-brane!

- ✓ [co-dim. 2] [2 つの isometry] [一価関数でない計量] を実現
- ✓ この GLSM の最初の応用：量子補正 ← 佐々木氏の講演

この $\mathcal{N} = (4, 4)$ GLSM はまだまだ多くの情報を持つだろう

- ✓ defect NS5-brane と 5_2^2 -brane の束縛状態 de Boer and Shigemori
- ✓ modular invariance, dipole description ? T. Kikuchi, T. Okada and Y. Sakatani
- ✓ より本質的な GLSM がある (!!)

Thanks

Appendix

10D string theory = D -dim spacetime \otimes compact space \mathcal{M}_d

\mathcal{M}_d	geometry associated with G_{mn}	Conventional geometry (manifold) $O(d)$ global symmetry [Calabi-Yau, etc]	ordinary compactifications
	geometry associated with G_{mn}, B_{mn}	Generalized geometry $O(d, d; \mathbb{Z})$ T-duality symmetry [T-fold]	flux compactifications
	geometry associated with $G_{mn}, \tau = C_{(0)} + i e^{-\Phi}$	Generalized geometry $SL(2, \mathbb{Z})$ S-duality symmetry [S-fold]	F-theory
	geometry associated with $G_{mn}, B_{mn}, \Phi, C_{(p)}$	Generalized geometry $E_{d+1(d+1)}(\mathbb{Z})$ U-duality symmetry [U-fold]	compactifications with non-abelian gauge

M-theory on $S^1(R_s)$	mass/tension ($l_s \equiv 1$)	type IIA
longitudinal M2	1	F1
transverse M2	$\frac{1}{g_s}$	D2
longitudinal M5	$\frac{1}{g_s}$	D4
transverse M5	$\frac{1}{g_s^2}$	NS5
longitudinal KK6	$\frac{R_{\text{TN}}^2}{g_s^2}$	KK5
KK6 with $R_{\text{TN}} = R_s$	$\frac{1}{g_s}$	D6
transverse KK6	$\frac{R_{\text{TN}}^2}{g_s^3}$	6_3^1

0	1	2	3	4	5	6	7	8	9	M
✓	✓	✓	✓	✓	✓	✓	S^1			\mathbb{R}^3
KK6 $\rightarrow 6_3^1$							Taub-NUT			

$$b_n^c : M = \frac{(R_1 \cdots R_c)^2}{g_s^n}$$

for review: N. Obers and B. Pioline [hep-th/9809039](https://arxiv.org/abs/hep-th/9809039)

b : spatial dimensions

c : # of isometry directions

n : mass of brane $\sim g_s^{-n}$

NS5-brane : 5_2 -brane

KK-monopole : 5_2^1 -brane

more : 5_2^2 -brane , etc

an instructive discussion : J. de Boer and M. Shigemori [arXiv:1209.6056](https://arxiv.org/abs/1209.6056)

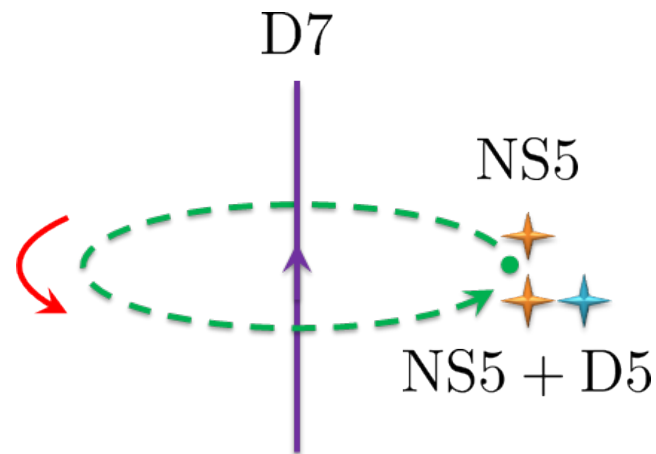
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2, \mathbb{Z})$ monodromy charge q

$$q : (C_2, B_2) \rightarrow (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

$$(0, \text{NS5}) \rightarrow (\text{D5}, \text{NS5})$$

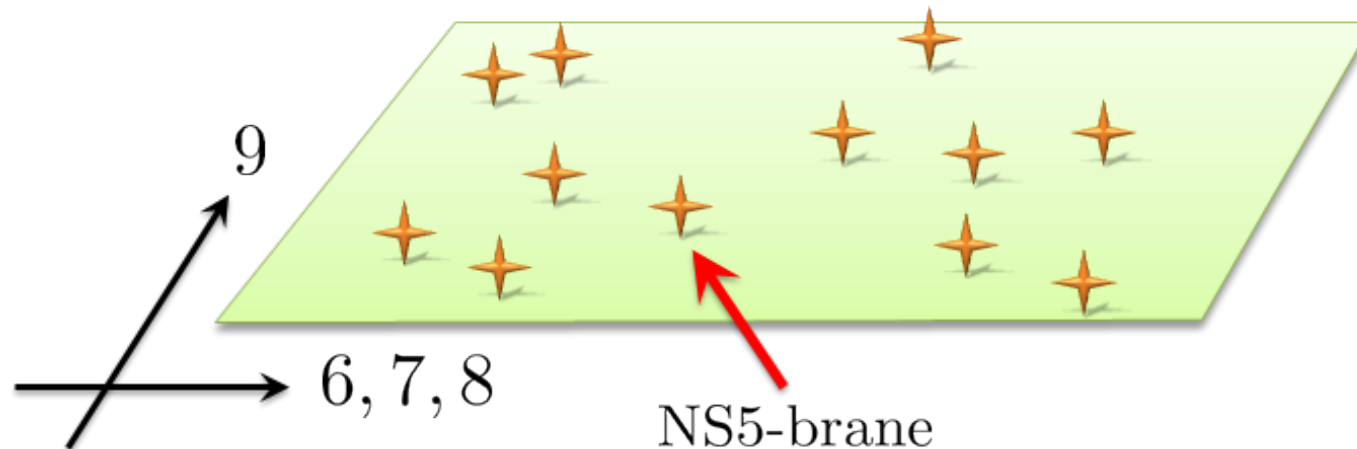


an instructive discussion : J. de Boer and M. Shigemori [arXiv:1209.6056](https://arxiv.org/abs/1209.6056)

NS5-branes

- co-dim. 4 (\mathbb{R}^4 , $\vec{x} \in \mathbb{R}^4$)
- $ds^2 = dx_{012345}^2 + H(x) \left[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2 \right]$
- $H(x) = 1 + \sum_p \frac{Q}{|\vec{x} - \vec{x}_p|^2}$, $H_{mnp} = \varepsilon_{mnp}{}^q \partial_q \log H(x)$, $\Phi = \frac{1}{2} \log H(x)$

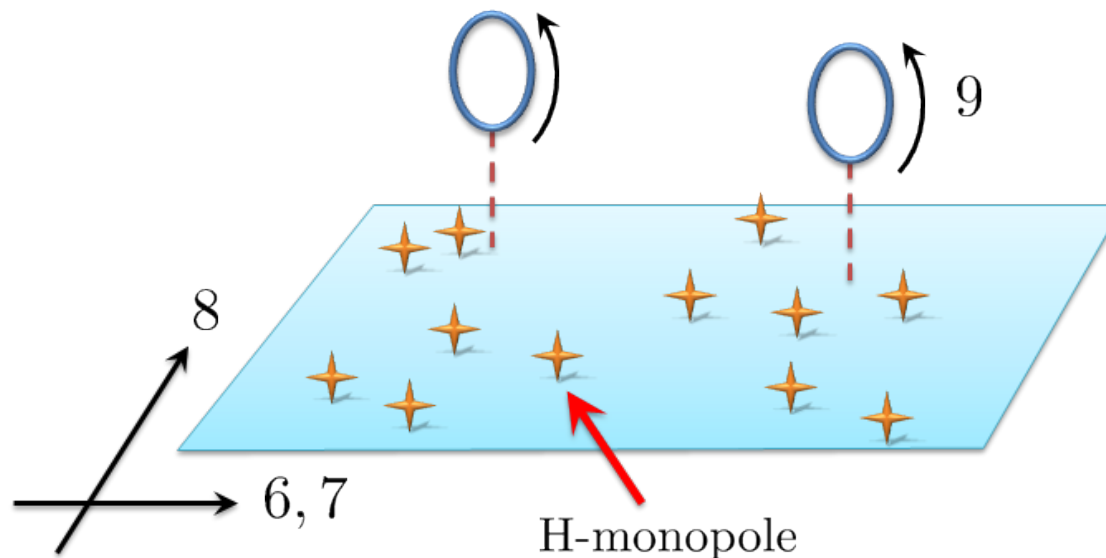
$$dx_{012345}^2 = -dt^2 + (dx^1)^2 + \dots + (dx^5)^2$$



NS5-branes (smeared), or H-monopoles

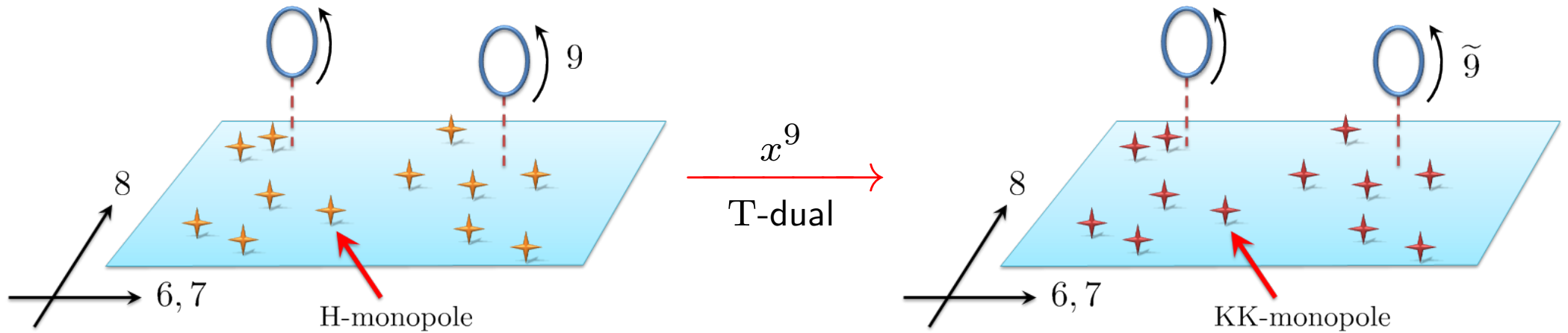
- co-dim. 3 ($\mathbb{R}^3 \times S^1$, $\vec{r} \in \mathbb{R}^3$)
- $ds^2 = dx_{012345}^2 + H(x) \left[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2 \right]$ NLSM
- $H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}$, $H_{mnp} = \varepsilon_{mnp}{}^q \partial_q \log H(x)$, $\Phi = \frac{1}{2} \log H(x)$

$$dx_{012345}^2 = -dt^2 + (dx^1)^2 + \dots + (dx^5)^2$$



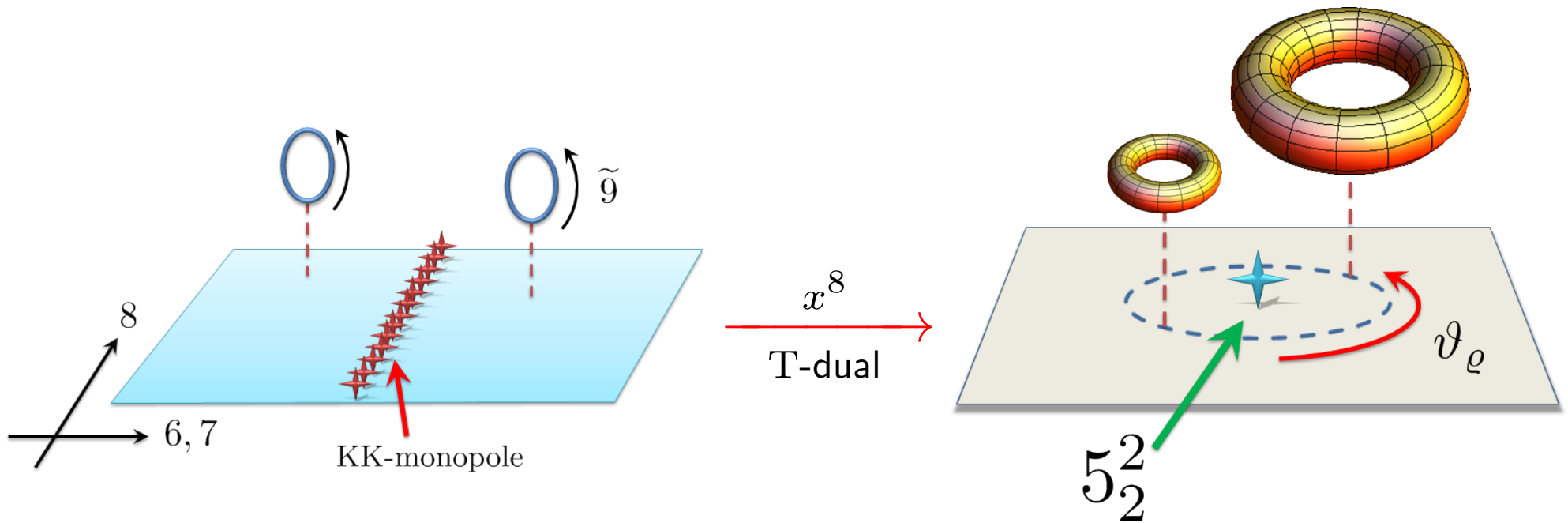
KK-monopoles

- co-dim. 3 ($\mathbb{R}^3 \times \tilde{S}^1$: Taub-NUT space, $\vec{r} \in \mathbb{R}^3$)
- $ds^2 = dx_{012345}^2 + H(x) \left[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 \right] + \frac{1}{H(x)} (d\tilde{x}^9 + \omega)^2$ NLSM
- $H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}$, $H_{mnp} = 0 = \Phi$



5₂²-brane

- co-dim. 2 ($\mathbb{R}^2 \times T^2$)
- $H(x) = h + \sigma \log \left(\frac{\mu}{\varrho} \right)$, $(\varrho, \vartheta_\varrho) \in \mathbb{R}^2$



- T-duality transformation is represented as

(free) string	sign flip (parity) in right-mover	momentum \leftrightarrow winding
spacetime	Buscher rule	$(G_{mn}, B_{mn}) \rightarrow (G'_{mn}, B'_{mn})$
SUSY sigma model	Roček-Verlinde formula	chiral \leftrightarrow twisted chiral

Gauged Linear Sigma Model (GLSM) はとても強力 !

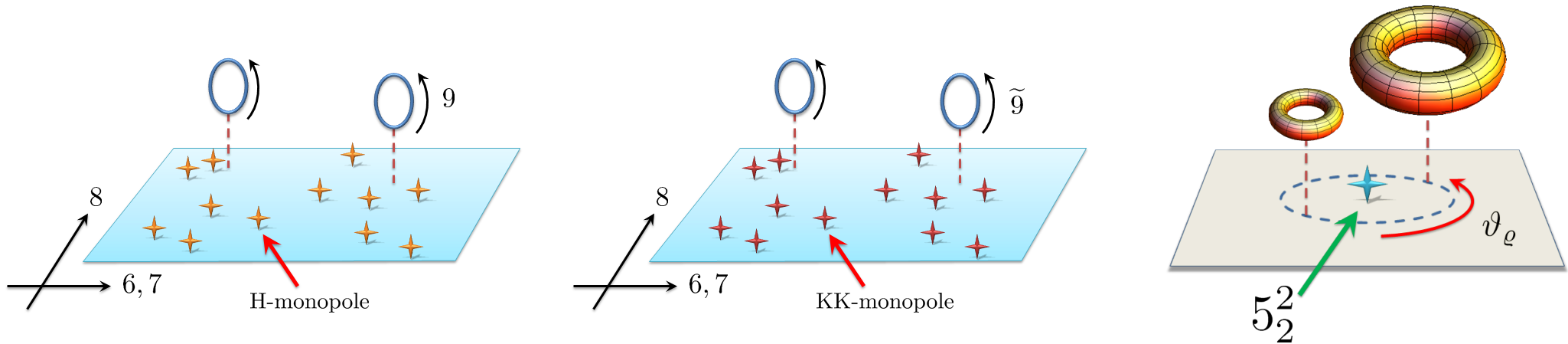
$\mathcal{N} = (2, 2)$ 理論の場合 :

- ✓ CY/LG (geometry/CFT) 対応 [phases]
 ← E. Witten [hep-th/9301042](https://arxiv.org/abs/hep-th/9301042)
- ✓ ミラー対称性 [T-duality]
 ← K. Hori and C. Vafa [hep-th/0002222](https://arxiv.org/abs/hep-th/0002222)
- ✓ 量子 Kähler moduli 空間 [instantons]
 ← N. Doroud et al [arXiv:1206.2606](https://arxiv.org/abs/1206.2606); H. Jockers et al [arXiv:1208.6244](https://arxiv.org/abs/1208.6244)

- 弦をプローブとした NS5-branes や KK-monopoles について :

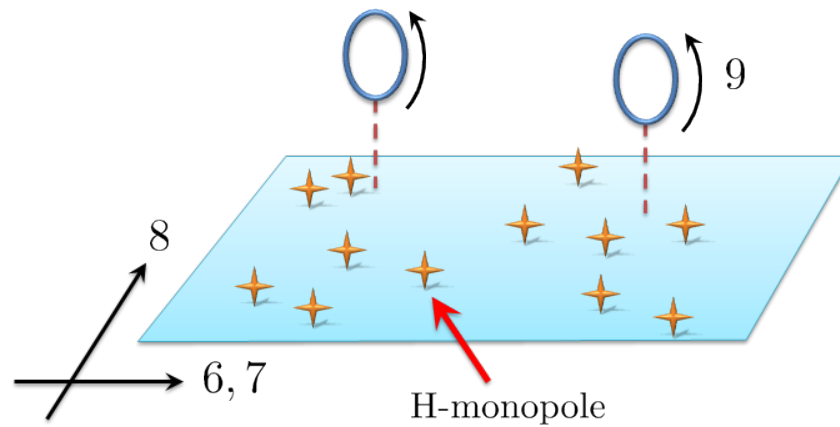
$\mathcal{N} = (4, 4)$ GLSM の IR 極限で実現

D. Tong [hep-th/0204186](#); J. Harvey and S. Jensen [hep-th/0507204](#); K. Okuyama [hep-th/0508097](#)



- 5_2^2 -brane の記述には
1. 調和関数が対数関数 (co-dim. 2)
 2. 2つめの isometry の存在
 3. 時空計量が一価関数ではない

LESSON 1 : GLSM for H-monopoles



- $\mathcal{N} = (4, 4)$ for H-monopoles by $\mathcal{N} = (2, 2)$ language :

superfields

$\mathcal{N} = (4, 4)$	$\mathcal{N} = (2, 2)$	役割
neutral HM	chiral $\Psi = \frac{1}{\sqrt{2}}(X^6 + iX^8) + \dots$ twisted chiral $\Theta = \frac{1}{\sqrt{2}}(X^7 + iX^9) + \dots$	時空座標
VM	twisted chiral $\Sigma_a = \frac{1}{\sqrt{2}}\bar{D}_+ D_- V_a$ chiral Φ_a	isometry のゲージ化
charged HMs	chiral $Q_a (+)$ chiral $\tilde{Q}_a (-)$	時空を曲げる
FI parameters	$s_a = s_{6,a} + i s_{8,a}$ $t_a = t_{7,a} + i t_{9,a}$	five-branes の位置

$$\begin{aligned}
 \mathcal{L}_H = & \sum_{a=1}^k \int d^4\theta \left\{ \frac{1}{e_a^2} \left(-\bar{\Sigma}_a \Sigma_a + \bar{\Phi}_a \Phi_a \right) + \bar{Q}_a e^{2V_a} Q_a + \bar{\tilde{Q}}_a e^{-2V_a} \tilde{Q}_a \right\} + \int d^4\theta \frac{1}{g^2} \left(-\bar{\Theta}\Theta + \bar{\Psi}\Psi \right) \\
 & + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\theta \left(\tilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \right) + (\text{h.c.}) \right\} \\
 & + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\tilde{\theta} (t_a - \Theta) \Sigma_a + (\text{h.c.}) \right\}
 \end{aligned}$$

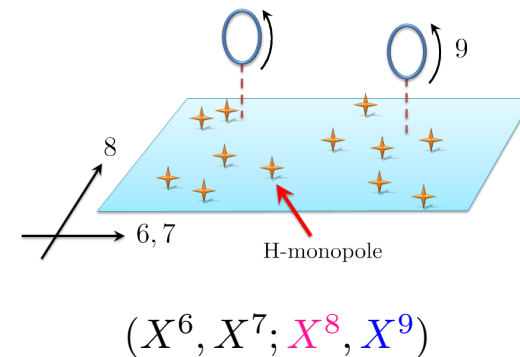
Bosonic Lagrangian after integrating-out auxiliary fields :

$$\mathcal{L}_H^{\text{kin}} = \sum_a \frac{1}{e_a^2} \left[\frac{1}{2} (F_{01,a})^2 - |\partial_m \sigma_a|^2 - |\partial_m \phi_a|^2 \right] - \sum_a \left[|D_m q_a|^2 + |D_m \tilde{q}_a|^2 \right] \\ - \frac{1}{2g^2} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2 \right] - \sqrt{2} \sum_a (X^9 - t_{9,a}) F_{01,a}$$

$$\mathcal{L}_H^{\text{pot}} = -2 \sum_a (|\sigma_a|^2 + |\phi_a|^2) (|q_a|^2 + |\tilde{q}_a|^2 + g^2) \\ - \sum_a \frac{e_a^2}{2} \left(|q_a|^2 - |\tilde{q}_a|^2 - \sqrt{2} (X^7 - t_{7,a}) \right)^2 - \sum_a e_a^2 \left| \sqrt{2} q_a \tilde{q}_a - (X^6 - s_{6,a}) - i(X^8 - s_{8,a}) \right|^2$$

Steps to NLSM for H-monopoles [geometry](#) [details](#)

1. SUSY vacua $\mathcal{L}^{\text{pot}} = 0$
2. solve constraints on (q_a, \tilde{q}_a)
3. IR limit $e_a \rightarrow \infty$, and integrate out $A_{m,a}$



1. SUSY vacua

main

$$\sigma_a = 0 = \phi_a, \quad |q_a|^2 - |\tilde{q}_a|^2 = \sqrt{2}(X^7 - t_{7,a}), \quad \sqrt{2} q_a \tilde{q}_a = (X^6 - s_{6,a}) + i(X^8 - s_{8,a})$$

 2. solve constraints on (q_a, \tilde{q}_a)

$$q_a = -\frac{i}{2^{1/4}} e^{-i\alpha_a} \sqrt{R_a + (X^7 - t_{7,a})}, \quad \tilde{q}_a = \frac{i}{2^{1/4}} e^{+i\alpha_a} \frac{(X^6 - s_{6,a}) + i(X^8 - s_{8,a})}{\sqrt{R_a + (X^7 - t_{7,a})}}$$

$$|D_m q_a|^2 + |D_m \tilde{q}_a|^2 = \frac{1}{2\sqrt{2}R_a} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] + \sqrt{2} R_a \left(\partial_m \alpha_a - A_{m,a} - \frac{1}{\sqrt{2}} \Omega_{i,a} \partial_m X^i \right)^2$$

$$R_a = \sqrt{(X^6 - s_{6,a})^2 + (X^7 - t_{7,a})^2 + (X^8 - s_{8,a})^2}$$

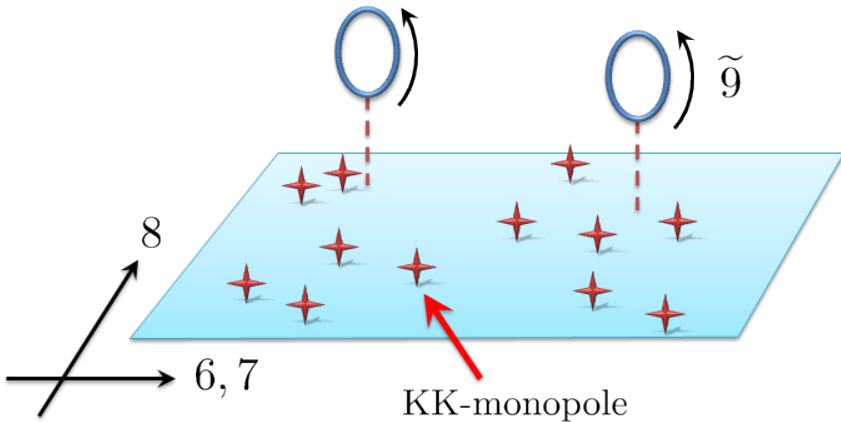
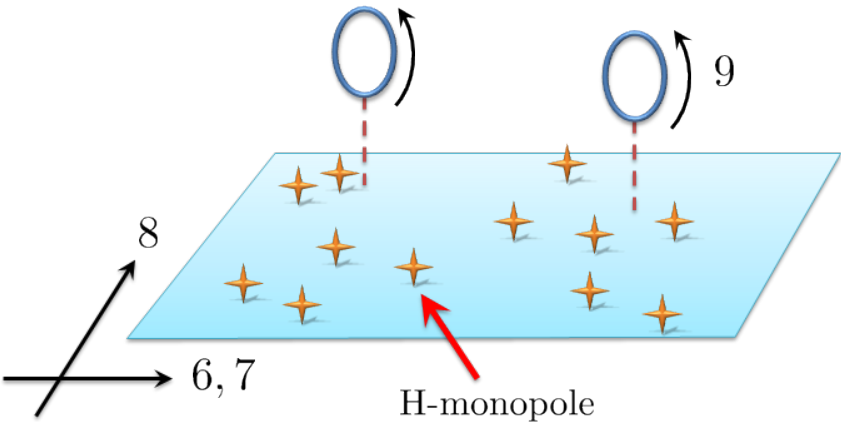
$$\Omega_{i,a} \partial_m X^i = \frac{-(X^6 - s_{6,a}) \partial_m X^8 + (X^8 - s_{8,a}) \partial_m X^6}{\sqrt{2} R_a (R_a + (X^7 - t_{7,a}))}$$

 3. IR limit $e_a \rightarrow \infty$, and integrate out $A_{m,a}$

$$A_{m,a} = -\frac{1}{\sqrt{2}} \Omega_{i,a} \partial_m X^i - \frac{1}{2R_a} \varepsilon^{mn} \partial^n X^9, \quad \Omega_i = \sum_a \Omega_{i,a}$$

$$\Rightarrow \mathcal{L}_H^{\text{NLMS}} = -\frac{1}{2} \left(\frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2}R_a} \right) \left[(\partial_m \vec{X})^2 + (\partial_m X^9)^2 \right] + \varepsilon^{mn} \Omega_i \partial_m X^i \partial_n X^9$$

LESSON 2 : T-duality



$\Theta \rightarrow \Gamma$:

$$\begin{aligned} \mathcal{L}_H \ni \mathcal{L}_\Theta &= \int d^4\theta \left(-\frac{1}{g^2} \bar{\Theta} \Theta \right) + \sum_a \left\{ \sqrt{2} \int d^2\tilde{\theta} (-\Theta) \Sigma_a + (\text{h.c.}) \right\} \\ &= \int d^4\theta \left\{ -\frac{1}{2g^2} (\Theta + \bar{\Theta})^2 - 2(\Theta + \bar{\Theta}) \sum_a V_a \right\} - \sqrt{2} \varepsilon^{mn} \sum_a \partial_m (X^9 A_{n,a}) \end{aligned}$$

↓

$$\mathcal{L}_{B\Gamma} = \int d^4\theta \left\{ -\frac{1}{2g^2} B^2 - 2B \sum_a V_a - B(\Gamma + \bar{\Gamma}) \right\} - \sqrt{2} \varepsilon^{mn} \sum_a \partial_m (X^9 A_{n,a})$$

real $\bar{B} = B$; chiral $\bar{D}_\pm \Gamma = 0$

$\Theta \rightarrow \Gamma$:

$$\mathcal{L}_{B\Gamma} = \int d^4\theta \left\{ -\frac{1}{2g^2}B^2 - 2B \sum_a V_a - (\Gamma + \bar{\Gamma})B \right\} - \sqrt{2} \varepsilon^{mn} \sum_a \partial_m (X^9 A_{n,a})$$

Integrating out $\Gamma, \bar{\Gamma}$: \rightarrow GLSM for H-monopoles

$$B = \Theta + \bar{\Theta}$$

or, Integrating out B : \rightarrow GLSM for KK-monopoles

$$\frac{1}{g^2}B = -(\Gamma + \bar{\Gamma}) - 2 \sum_a V_a$$

duality relation :

$$\Theta = \frac{1}{\sqrt{2}}(X^7 + iX^9) + \dots, \quad \Gamma = \frac{1}{\sqrt{2}}(Y^7 + iY^9) + \dots$$

$$\Theta + \bar{\Theta} = -g^2(\Gamma + \bar{\Gamma}) - 2g^2 \sum_a V_a \rightarrow$$

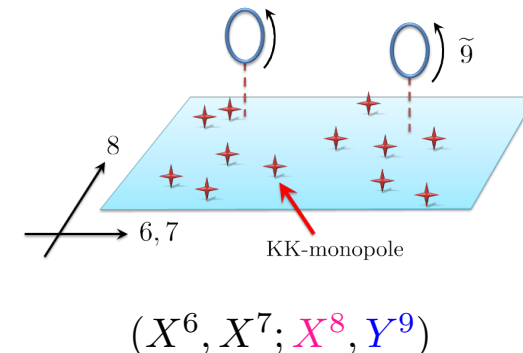
$$\begin{aligned} X^7 &= -g^2 Y^7 \\ \pm(\partial_0 \pm \partial_1)X^9 &= -g^2(D_0 \pm D_1)Y^9 \\ D_m Y^9 &= \partial_m Y^9 + \sqrt{2} \sum_a A_{m,a} \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{\text{KK}} = & \sum_{a=1}^k \int d^4\theta \left\{ \frac{1}{e_a^2} \left(-\bar{\Sigma}_a \Sigma_a + \bar{\Phi}_a \Phi_a \right) + \bar{Q}_a e^{2V_a} Q_a + \bar{\tilde{Q}}_a e^{-2V_a} \tilde{Q}_a \right\} \\
 & + \int d^4\theta \frac{g^2}{2} \left(\Gamma + \bar{\Gamma} + 2 \sum_{a=1}^k V_a \right)^2 + \int d^4\theta \frac{1}{g^2} \bar{\Psi} \Psi \\
 & + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\theta \left(\tilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \right) + (\text{h.c.}) \right\} \\
 & + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\tilde{\theta} t_a \Sigma_a + (\text{h.c.}) \right\} - \sqrt{2} \varepsilon^{mn} \sum_{a=1}^k \partial_m (X^9 A_{n,a})
 \end{aligned}$$

Steps to NLSM for KK-monopoles

[geometry](#)
[details](#)
[superfields](#)

1. SUSY vacua $\mathcal{L}^{\text{pot}} = 0$
2. solve constraints on (q_a, \tilde{q}_a)
3. IR limit $e_a \rightarrow \infty$, and integrate out $A_{m,a}$

 \Rightarrow


1. SUSY vacua

main

$$\sigma_a = 0 = \phi_a, \quad |q_a|^2 - |\tilde{q}_a|^2 = \sqrt{2}(X^7 - t_{7,a}), \quad \sqrt{2} q_a \tilde{q}_a = (X^6 - s_{6,a}) + i(X^8 - s_{8,a})$$

 2. solve constraints on (q_a, \tilde{q}_a)

$$q_a = -\frac{i}{2^{1/4}} e^{-i\alpha_a} \sqrt{R_a + (X^7 - t_{7,a})}, \quad \tilde{q}_a = \frac{i}{2^{1/4}} e^{+i\alpha_a} \frac{(X^6 - s_{6,a}) + i(X^8 - s_{8,a})}{\sqrt{R_a + (X^7 - t_{7,a})}}$$

$$|D_m q_a|^2 + |D_m \tilde{q}_a|^2 = \frac{1}{2\sqrt{2}R_a} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] + \sqrt{2} R_a \left(\partial_m \alpha_a - A_{m,a} - \frac{1}{\sqrt{2}} \Omega_{i,a} \partial_m X^i \right)^2$$

$$R_a = \sqrt{(X^6 - s_{6,a})^2 + (X^7 - t_{7,a})^2 + (X^8 - s_{8,a})^2}$$

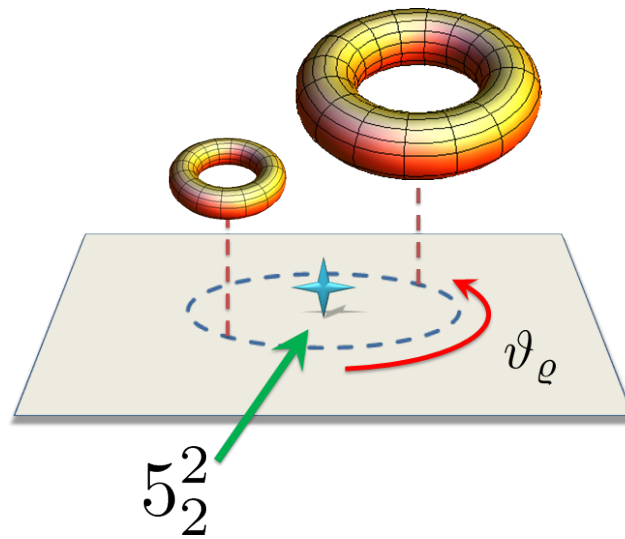
$$\Omega_{i,a} \partial_m X^i = \frac{-(X^6 - s_{6,a}) \partial_m X^8 + (X^8 - s_{8,a}) \partial_m X^6}{\sqrt{2} R_a (R_a + (X^7 - t_{7,a}))}$$

 3. IR limit $e_a \rightarrow \infty$, and integrate out $A_{m,a}$

$$A_{m,a} = -\frac{1}{2R_a H} (\partial_m Y^9 - \Omega_i \partial_m X^i) - \frac{1}{\sqrt{2}} \Omega_{i,a} \partial_m X^i, \quad \Omega_i = \sum_a \Omega_{i,a}, \quad H = \frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2}R_a}$$

$$\mathcal{L}_{\text{KK}}^{\text{NLMSM}} = -\frac{1}{2} H (\partial_m \vec{X})^2 - \frac{1}{2} H^{-1} (\partial_m Y^9 - \Omega_i \partial_m X^i)^2 - \sqrt{2} \varepsilon^{mnp} \sum_a \partial_m ((X^9 - t_{9,a}) \dot{A}_{n,a})$$

MAIN : $5\frac{2}{2}$ -brane



T-duality : NS5 \rightarrow KKM \rightarrow 5_2^2 -brane

technique 1

F-terms ありでの Roček-Verlinde formula



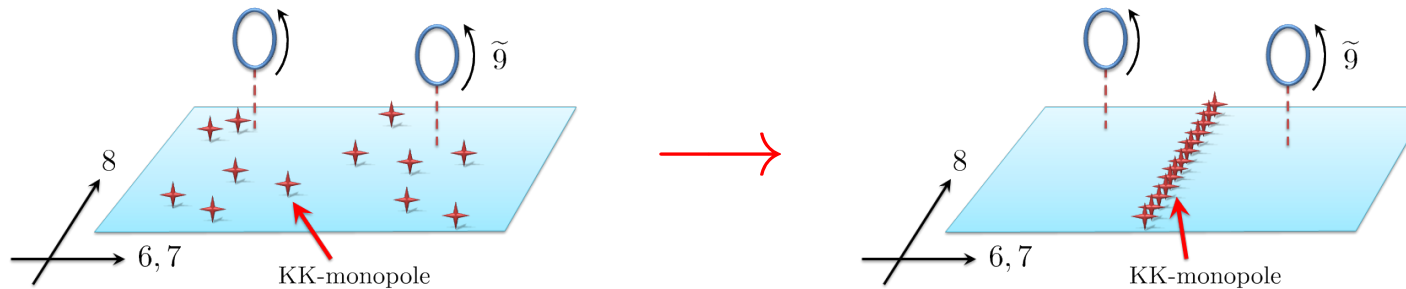
F-terms を D-terms に書き換える

technique 2

2 つめの isometry



KKM を無限個並べる



$\Psi \rightarrow \Xi$:

$$\begin{aligned}
 \mathcal{L}_{\text{KK}} \ni \mathcal{L}_{\Psi} &= \int d^4\theta \frac{1}{g^2} \bar{\Psi} \Psi + \sum_a \left\{ \sqrt{2} \int d^2\theta (-\Psi) \Phi_a + (\text{h.c.}) \right\} \\
 &= \int d^4\theta \left\{ \frac{a}{g^2} (\Psi + \bar{\Psi})^2 - \sqrt{2} (\Psi + \bar{\Psi}) \sum_a (C_a + \bar{C}_a) \right\} \\
 &\quad + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (\Psi - \bar{\Psi})^2 - \sqrt{2} (\Psi - \bar{\Psi}) \sum_a (C_a - \bar{C}_a) \right\}
 \end{aligned}$$

↓

$$\begin{aligned}
 \mathcal{L}_{RSX\Xi} &= \int d^4\theta \left\{ \frac{a}{g^2} R^2 - \sqrt{2} R \sum_a (C_a + \bar{C}_a) + R(\Xi_1 + \bar{\Xi}_1) + R(X + \bar{X}) \right\} \\
 &\quad + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (iS)^2 - \sqrt{2} (iS) \sum_a (C_a - \bar{C}_a) + iS(\Xi_2 - \bar{\Xi}_2) + iS(X - \bar{X}) \right\}
 \end{aligned}$$

$$\bar{R} = R, \quad \bar{S} = S, \quad \bar{D}_+ \Xi_{1,2} = 0 = D_- \Xi_{1,2}, \quad \bar{D}_{\pm} X = 0, \quad \Phi_a = \bar{D}_+ \bar{D}_- C_a$$

$\Psi \rightarrow \Xi$:

$$\begin{aligned} \widetilde{\mathcal{L}} = & \int d^4\theta \left\{ \frac{a}{g^2} R^2 - \sqrt{2} R \sum_a (C_a + \bar{C}_a) + R(\Xi_1 + \bar{\Xi}_1) + R(X + \bar{X}) \right\} \\ & + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (iS)^2 - \sqrt{2} (iS) \sum_a (C_a - \bar{C}_a) + iS(\Xi_2 - \bar{\Xi}_2) + iS(X - \bar{X}) \right\} \end{aligned}$$

Integrating out Ξ_1, Ξ_2, X : \rightarrow GLSM for KK-monopoles

or, Integrating out R, Ξ_2 : \rightarrow new GLSM

$$\frac{2a}{g^2} R = -(\Xi_1 + \bar{\Xi}_1) + \sqrt{2} \sum_a (C_a + \bar{C}_a)$$

duality relation at $a = \frac{1}{2}$:

$$\Psi = \frac{1}{\sqrt{2}}(X^6 + iX^8) + \dots$$

$$\Psi + \bar{\Psi} = -g^2(\Xi_1 + \bar{\Xi}_1) + \sqrt{2} g^2 \sum_a (C_a + \bar{C}_a)$$

$$\begin{aligned} X^6 & \sim \text{real part of } \Xi \\ \partial X^8 & \sim \partial(\text{imaginary part of } \Xi) + \text{“gauge” fields in } C_a \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{\text{new}} = & \sum_{a=1}^k \int d^4\theta \left\{ \frac{1}{e_a^2} \left(-\bar{\Sigma}_a \Sigma_a + \bar{\Phi}_a \Phi_a \right) + \bar{Q}_a e^{2V_a} Q_a + \bar{\tilde{Q}}_a e^{-2V_a} \tilde{Q}_a \right\} \\
 & + \int d^4\theta \frac{g^2}{2} \left(\Gamma + \bar{\Gamma} + 2 \sum_{a=1}^k V_a \right)^2 + \int d^4\theta \left\{ -\frac{g^2}{2} \left(\Xi + \bar{\Xi} - \sqrt{2} \sum_{a=1}^k (C_a + \bar{C}_a) \right)^2 \right\} \\
 & - \sqrt{2} \int d^4\theta (\dot{\Psi} - \bar{\dot{\Psi}}) \sum_{a=1}^k (C_a - \bar{C}_a) \\
 & + \sum_{a=1}^k \left\{ \sqrt{2} \int d^2\theta \left(\tilde{Q}_a \Phi_a Q_a + s_a \Phi_a \right) + \sqrt{2} \int d^2\tilde{\theta} t_a \Sigma_a + (\text{h.c.}) \right\} - \sqrt{2} \varepsilon^{mn} \sum_{a=1}^k \partial_m (X^9 A_{n,a})
 \end{aligned}$$

$\mathcal{N} = (4, 4)$	$\mathcal{N} = (2, 2)$		役割
neutral HM	twisted chiral $\Xi = \frac{1}{\sqrt{2}}(X^6 + iY^8) + \dots$	chiral $\Gamma = \frac{1}{\sqrt{2}}(X^7 + iY^9) + \dots$	時空座標
VMs	twisted chiral $\Sigma_a = \frac{1}{\sqrt{2}}\bar{D}_+ D_- V_a$	chiral $\Phi_a = \bar{D}_+ \bar{D}_- C_a$	isometry のゲージ化
charged HMs	chiral $Q_a (+)$	chiral $\tilde{Q}_a (-)$	時空を曲げる
FI parameters	$s_a = s_{6,a} + i s_{8,a}$	$t_a = t_{7,a} + i t_{9,a}$	five-branes の位置

$$\begin{aligned}
 \mathcal{L}_{\text{new}}^{\text{kin}} &= \sum_a \frac{1}{e_a^2} \left[\frac{1}{2} (F_{01,a})^2 - |\partial_m \sigma_a|^2 - |\partial_m \phi_a|^2 \right] - \sum_a \left[|D_m q_a|^2 + |D_m \tilde{q}_a|^2 \right] \\
 &\quad - \frac{1}{2g^2} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 \right] - \frac{g^2}{2} \left[(\partial_m Y^8)^2 + (D_m Y^9)^2 \right] - \sqrt{2} \sum_a (X^9 - t_{9,a}) F_{01,a} \\
 \mathcal{L}_{\text{new}}^{\text{pot}} &= -2g^2 \sum_{a,b} (\sigma_a \bar{\sigma}_b + 4M_{c,a} \bar{M}_{c,b}) - 2 \sum_a (|\sigma_a|^2 + 4|M_{c,a}|^2) (|q_a|^2 + |\tilde{q}_a|^2) \\
 &\quad - \sum_a \frac{e_a^2}{2} \left(|q_a|^2 - |\tilde{q}_a|^2 - \sqrt{2}(X^7 - t_{7,a}) \right)^2 - \sum_a e_a^2 \left| \sqrt{2} q_a \tilde{q}_a - (X^6 - s_{6,a}) - i(X^8 - s_{8,a}) \right|^2 \\
 &\quad + \frac{g^2}{2} \sum_{a,b} (A_{c=,a} + \bar{A}_{c=,a}) (B_{c\neq,b} + \bar{B}_{c\neq,b}) \\
 (\partial_0 + \partial_1) X^8 &= -g^2 (\partial_0 + \partial_1) Y^8 + g^2 \sum_a (B_{c\neq,a} + \bar{B}_{c\neq,a}) \\
 (\partial_0 - \partial_1) X^8 &= +g^2 (\partial_0 - \partial_1) Y^8 + g^2 \sum_a (A_{c=,a} + \bar{A}_{c=,a}) \\
 + \frac{g^2}{2} \sum_{a,b} (A_{c=,a} + \bar{A}_{c=,a}) (B_{c\neq,b} + \bar{B}_{c\neq,b}) &= -\frac{1}{2g^2} (\partial_m X^8)^2 + \frac{g^2}{2} (\partial_m Y^8)^2 + \varepsilon^{mn} (\partial_m X^8) (\partial_n Y^8)
 \end{aligned}$$

もしも $\int d^4\theta (\dot{\Psi} - \bar{\dot{\Psi}}) \sum_a (C_a - \bar{C}_a)$ がなかったら ...

- ☹ IR極限では chiral model になってしまう [conflict w/ $\mathcal{N} = (4, 4)$ SUSY]
- ☹ 時空計量が一価関数になってしまう [conflict w/ non-trivial monodromy]
- ☹ 時空上の B -場が登場しなくなってしまう [conflict w/ original NS5-brane]

この項が**存在する**ので

T-duality 変換を実行する前の座標場 X^8 がまだ系に存在する



積分して追い出す (可能)

一価関数でない時空上の (G, B) が導出される

GLSM is a powerful tool, also in this stage :

Worldsheet instantons in NLSM can be captured by soliton (vortex) solutions in gauge theory

Take the configuration : $\phi_a = 0 = \sigma_a$ with $g^2 \rightarrow 0$ and finite e_a^2

$$\mathcal{L}_E = \sum_{a=1}^k \left[\frac{1}{2e_a^2} (F_{12,a})^2 + |D_m q_a|^2 + \frac{e_a^2}{2} (|q_a|^2 - \sqrt{2} \zeta_a)^2 + i\sqrt{2} X^9 F_{12,a} \right]$$

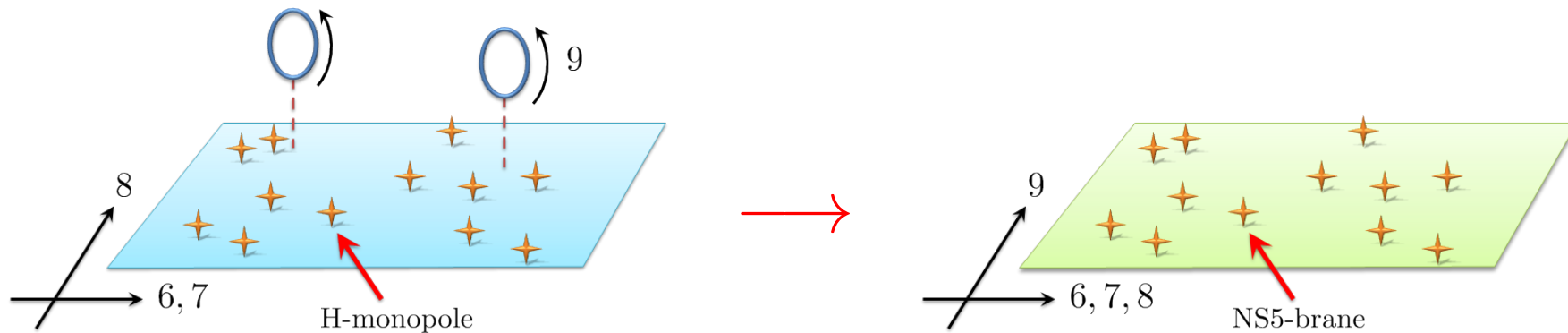
$$F_{12,a} = \mp e_a^2 (|q_a|^2 - \sqrt{2} \zeta_a), \quad 0 = (D_1 \pm iD_2)q_a$$

Abrikosov-Nielsen-Olesen vortex eq.

$$\text{then, } S_E = \frac{1}{2\pi} \int d^2x \mathcal{L}_E = \sqrt{2} \sum_{a=1}^k \left(\zeta_a |n_a| - i X^9 n_a \right) \quad n_a = -\frac{1}{2\pi} \int d^2x F_{12,a}$$

Worksheet instanton corrections to GLSM for H-monopoles :

unfolding effect on compactified circle x^9

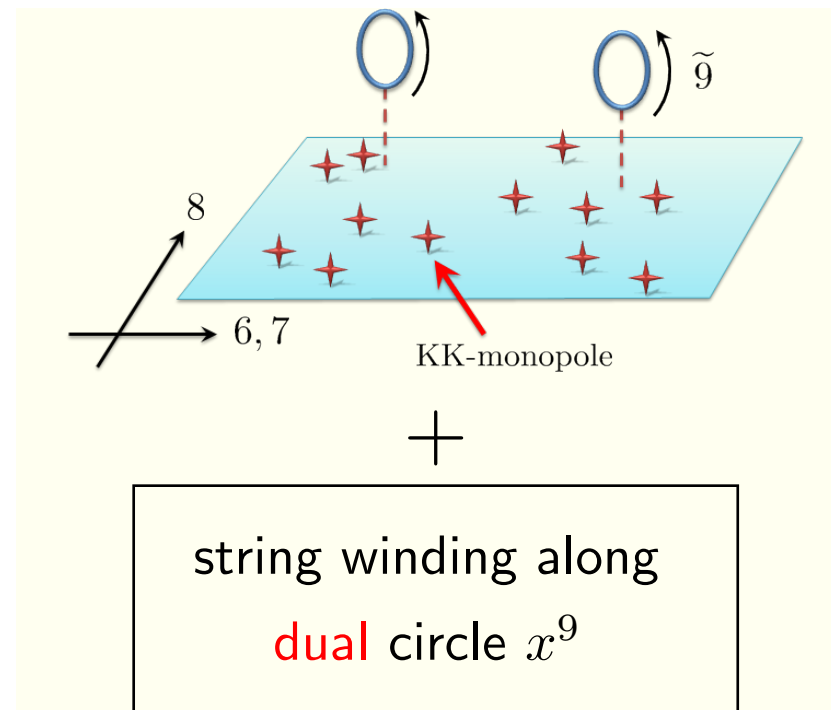
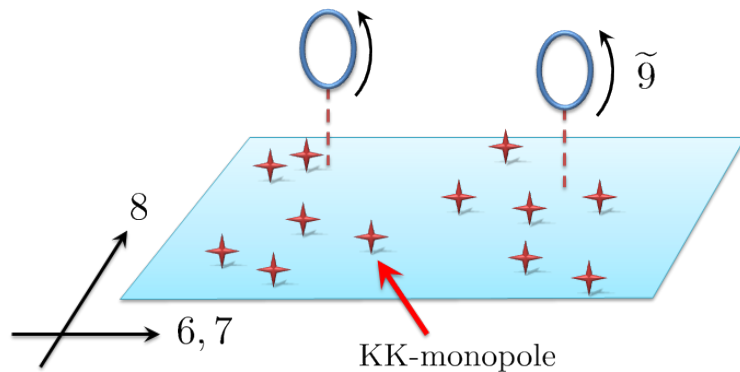


$$\begin{aligned}
 H &= \frac{1}{g^2} + \sum_{a=1}^k \frac{1}{\sqrt{2}R_a} \rightarrow \frac{1}{g^2} + \sum_{a=1}^k \frac{1}{\sqrt{2}R_a} \sum_{n_a=1}^{\infty} e^{-n_a R_a} \left[e^{+i n_a X^9} + e^{-i n_a X^9} \right] \\
 &= \frac{1}{g^2} + \sum_{a=1}^k \frac{1}{\sqrt{2}R_a} \frac{\sinh(R_a)}{\cosh(R_a) - \cos(X^9)}
 \end{aligned}$$

D. Tong [hep-th/0204186](https://arxiv.org/abs/hep-th/0204186)

Worldsheet instanton corrections to GLSM for KK-monopoles :

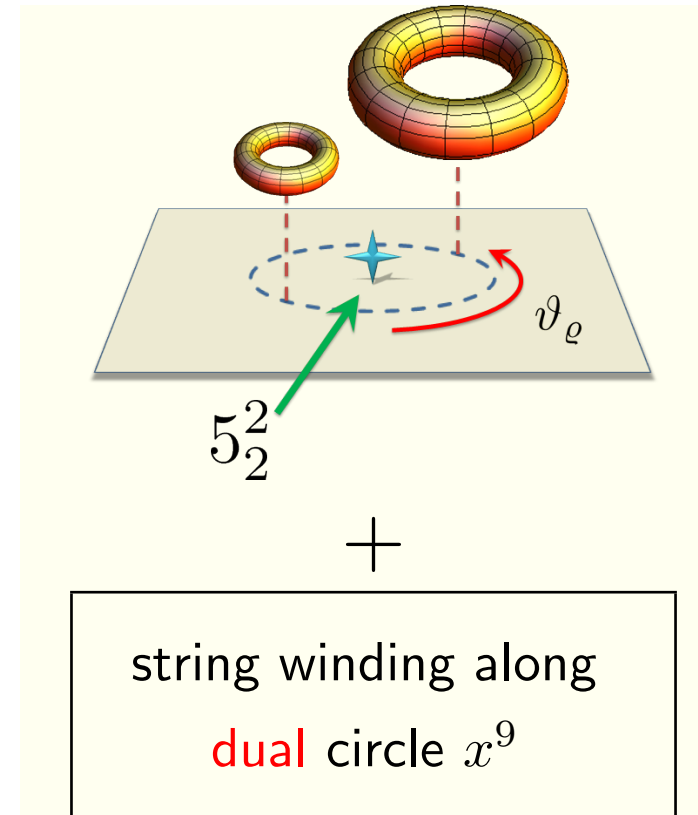
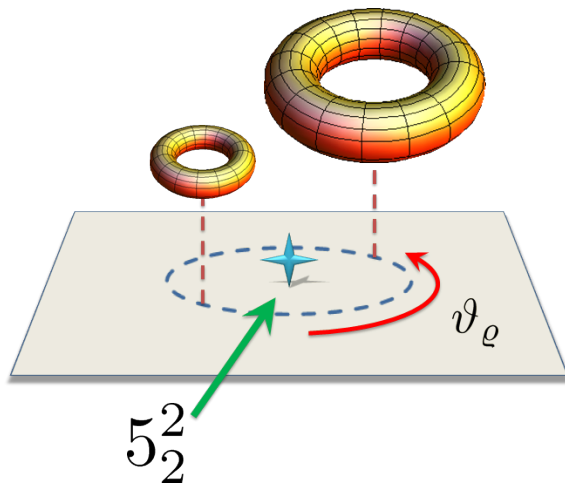
string **winding modes** along x^9



$$H \rightarrow \frac{1}{g^2} + \sum_{a=1}^k \frac{1}{\sqrt{2}R_a} \frac{\sinh(R_a)}{\cosh(R_a) - \cos(X^9)}$$

J. Harvey and S. Jensen [hep-th/0507204](https://arxiv.org/abs/hep-th/0507204); K. Okuyama [hep-th/0508097](https://arxiv.org/abs/hep-th/0508097)

Worldsheet instanton corrections to GLSM for 5_2^2 -brane :
 string **winding modes** along x^9



$$H \rightarrow h_0 + \sigma \log(\mu/\varrho) + \sigma \sum_{n \neq 0} \exp(i n X^9) K_0(|n|\varrho)$$

$$\Psi = \frac{1}{\sqrt{2}}(X^6 + iX^8) + i\sqrt{2}\theta^+\chi_+ + i\sqrt{2}\theta^-\chi_- + 2i\theta^+\theta^-G + \dots$$

$$\Xi = \frac{1}{\sqrt{2}}(Y^6 + iY^8) + i\sqrt{2}\theta^+\bar{\xi}_+ + i\sqrt{2}\bar{\theta}^-\xi_- + 2i\theta^+\bar{\theta}^-G_\Xi + \dots$$

$$\Theta = \frac{1}{\sqrt{2}}(X^7 + iX^9) + i\sqrt{2}\theta^+\bar{\chi}_+ + i\sqrt{2}\bar{\theta}^-\tilde{\chi}_- + 2i\theta^+\bar{\theta}^-\tilde{G} + \dots$$

$$\Gamma = \frac{1}{\sqrt{2}}(Y^7 + iY^9) + i\sqrt{2}\theta^+\zeta_+ + i\sqrt{2}\theta^-\zeta_- + 2i\theta^+\theta^-G_\Gamma + \dots$$

$$Q_a = q_a + i\sqrt{2}\theta^+\psi_{+,a} + i\sqrt{2}\theta^-\psi_{-,a} + 2i\theta^+\theta^-F_a + \dots$$

$$\tilde{Q}_a = \tilde{q}_a + i\sqrt{2}\theta^+\tilde{\psi}_{+,a} + i\sqrt{2}\theta^-\tilde{\psi}_{-,a} + 2i\theta^+\theta^-\tilde{F}_a + \dots$$

$$V_a = \theta^+\bar{\theta}^+(A_{0,a} + A_{1,a}) + \theta^-\bar{\theta}^-(A_{0,a} - A_{1,a}) - \sqrt{2}\theta^-\bar{\theta}^+\sigma_a - \sqrt{2}\theta^+\bar{\theta}^-\bar{\sigma}_a \\ - 2i\theta^+\theta^-(\bar{\theta}^+\bar{\lambda}_{+,a} + \bar{\theta}^-\bar{\lambda}_{-,a}) + 2i\bar{\theta}^+\bar{\theta}^-(\theta^+\lambda_{+,a} + \theta^-\lambda_{-,a}) - 2\theta^+\theta^-\bar{\theta}^+\bar{\theta}^-D_{V,a}$$

$$\Phi_a = \phi_a + i\sqrt{2}\theta^+\tilde{\lambda}_{+,a} + i\sqrt{2}\theta^-\tilde{\lambda}_{-,a} + 2i\theta^+\theta^-D_{\Phi,a} + \dots = \bar{D}_+\bar{D}_-C_a$$

$$C_a = \phi_{c,a} + i\sqrt{2}\theta^+\psi_{c+,a} + i\sqrt{2}\theta^-\psi_{c-,a} + i\sqrt{2}\bar{\theta}^+\chi_{c+,a} + i\sqrt{2}\bar{\theta}^-\chi_{c-,a} \\ + 2i\theta^+\theta^-F_{c,a} + 2i\bar{\theta}^+\bar{\theta}^-M_{c,a} + 2i\theta^+\bar{\theta}^-G_{c,a} + 2i\bar{\theta}^+\theta^-N_{c,a} + \theta^-\bar{\theta}^-A_{c=,a} + \theta^+\bar{\theta}^+B_{c++,a} \\ - 2i\theta^+\theta^-\bar{\theta}^+\zeta_{c+,a} - 2i\theta^+\theta^-\bar{\theta}^-\zeta_{c-,a} + 2i\bar{\theta}^+\bar{\theta}^-\theta^+\lambda_{c+,a} + 2i\bar{\theta}^+\bar{\theta}^-\theta^-\lambda_{c-,a} - 2\theta^+\theta^-\bar{\theta}^+\bar{\theta}^-D_{c,a}$$

$$\text{with } \bar{D}_\pm = -\frac{\partial}{\partial\bar{\theta}^\pm} + i\theta^\pm(\partial_0 \pm \partial_1), \quad D_\pm = \frac{\partial}{\partial\theta^\pm} - i\bar{\theta}^\pm(\partial_0 \pm \partial_1)$$