

Exotic Five-brane

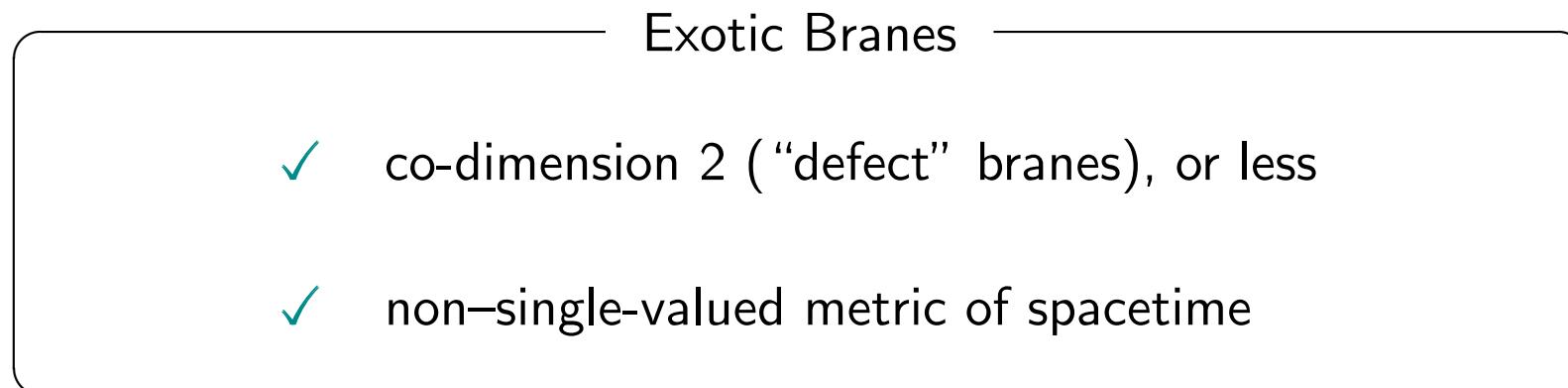
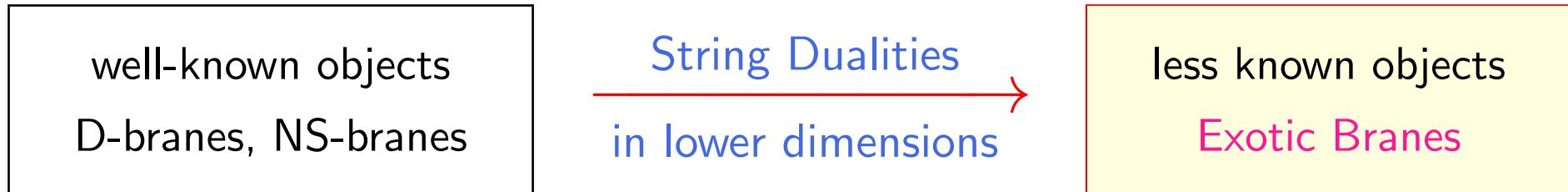
Tetsuji KIMURA

Rikkyo University

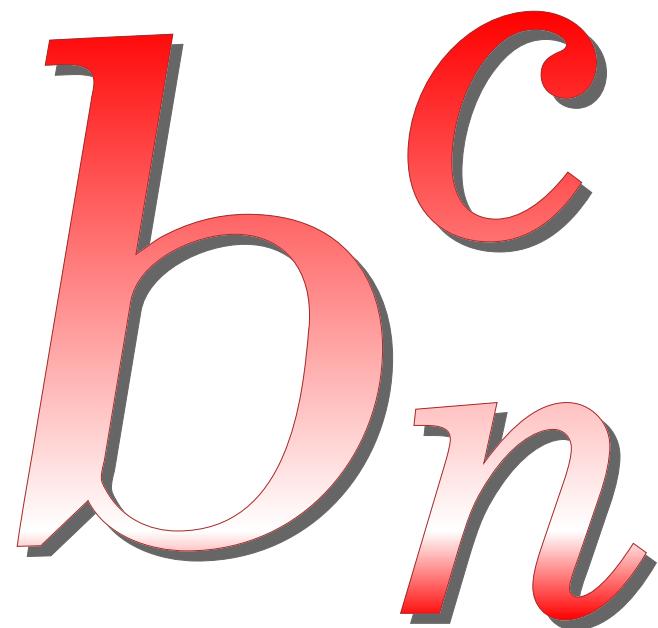
based on [arXiv:1304.4061](#), [1305.4439](#), [1310.6163](#), [1402.5580](#), and more

with Shin SASAKI and Masaya YATA





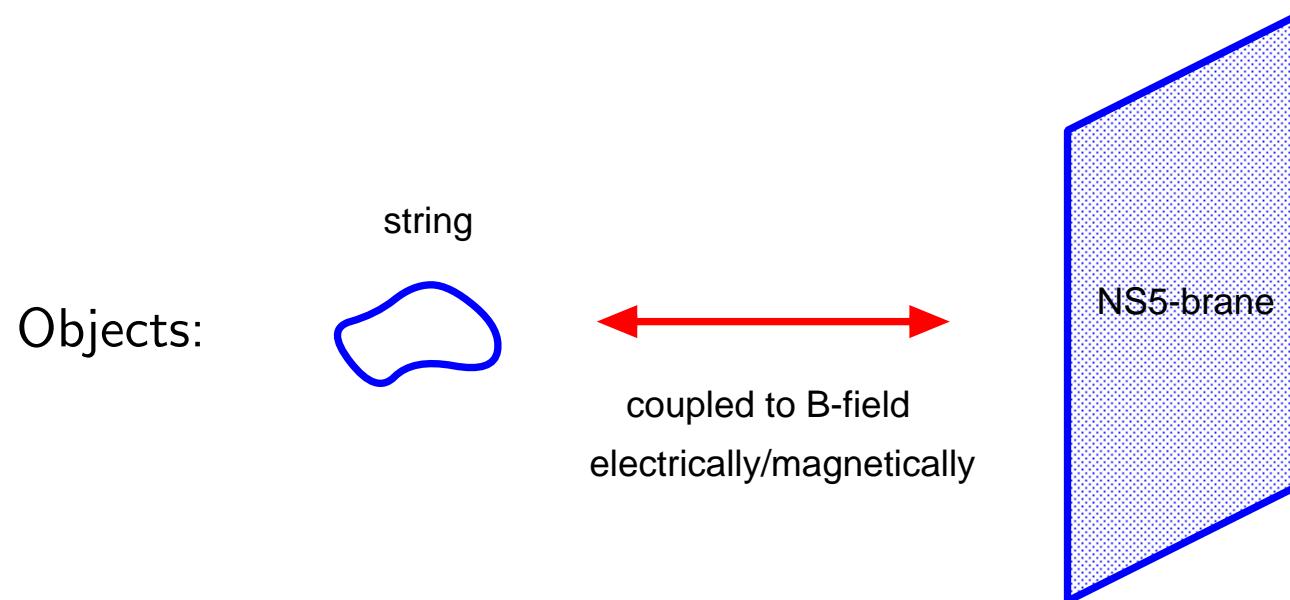
(Exotic) Branes are labelled as b_n^c -branes.



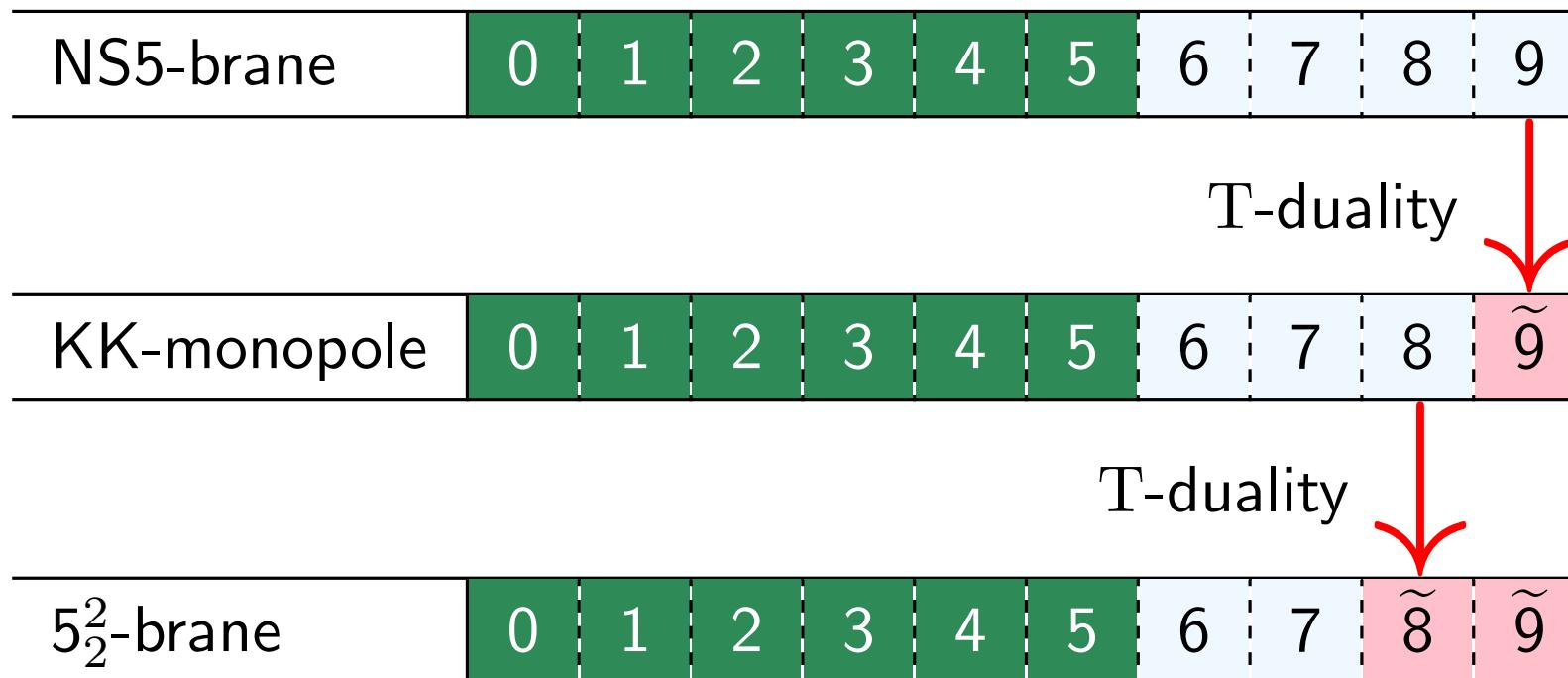
- b : spatial dimensions
- c : # of isometry directions
- n : mass of brane $\sim g_s^{-n}$

$5\frac{1}{2}$ -brane

We begin with NS5-brane.



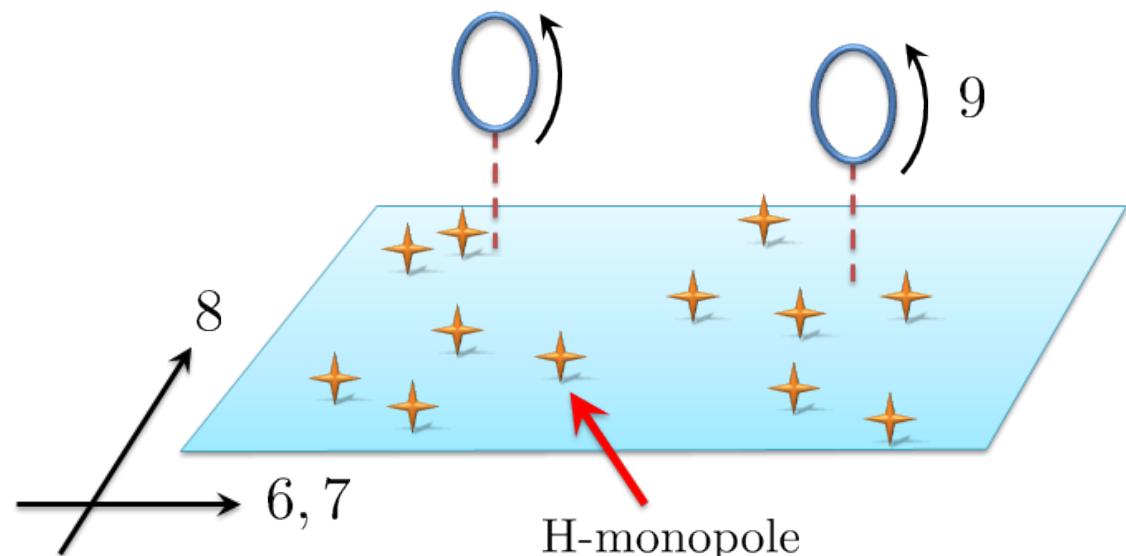
Exotic 5_2^2 -brane is constructed from NS5-brane.



NS5-branes (smeared), or H-monopoles

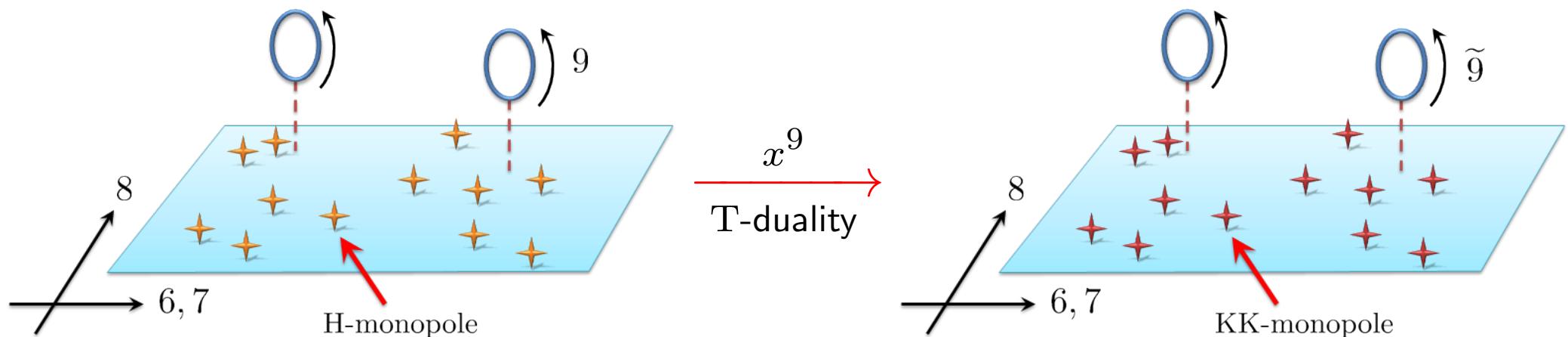
- co-dim. 3 ($\mathbb{R}^3 \times S^1$, $\vec{r} \in \mathbb{R}^3$)
- $ds^2 = dx_{012345}^2 + H(x) \left[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2 \right]$
- $H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}$, $H_{mnp} = \varepsilon_{mnp}{}^q \partial_q \log H(x)$, $\Phi = \frac{1}{2} \log H(x)$

$$dx_{012345}^2 = -dt^2 + (dx^1)^2 + \dots + (dx^5)^2$$



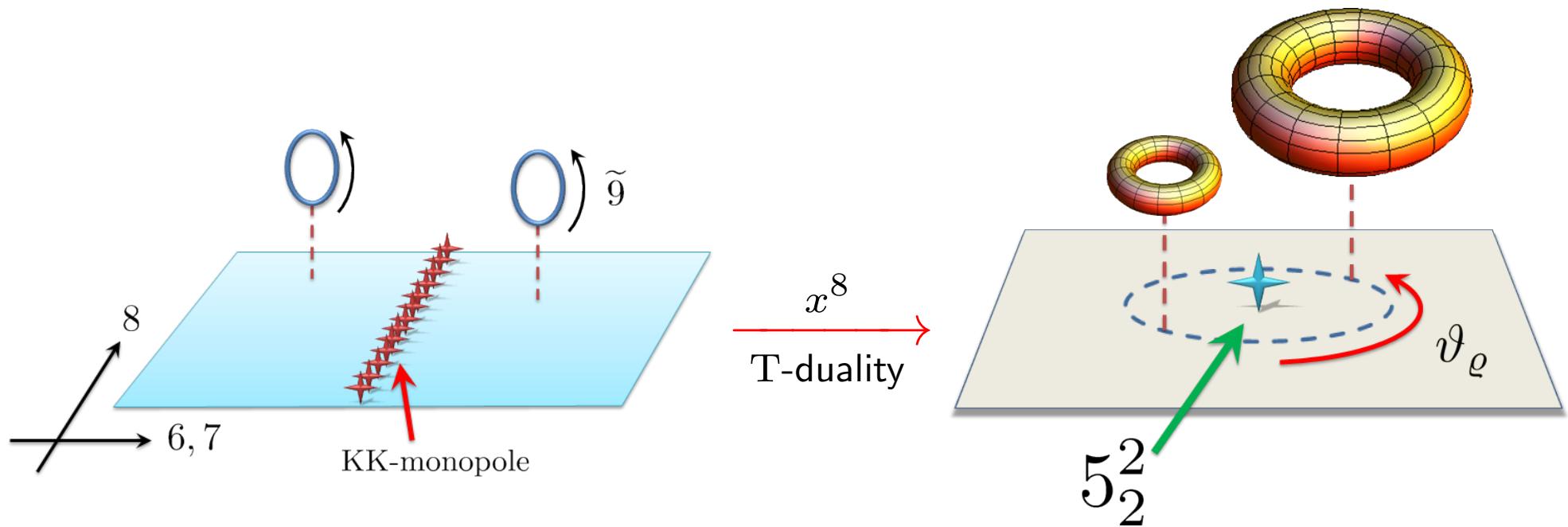
KK-monopoles

- co-dim. 3 ($\mathbb{R}^3 \times \widetilde{S}^1$: Taub-NUT space, $\vec{r} \in \mathbb{R}^3$)
- $ds^2 = dx_{012345}^2 + H(x) \left[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 \right] + \frac{1}{H(x)} (d\tilde{x}^9 + \omega)^2$
- $H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}$, $H_{mnp} = 0 = \Phi$



5₂²-brane

- co-dim. 2 ($\mathbb{R}^2 \times T^2$)
- $H(x) = h + \sigma \log \left(\frac{\mu}{\varrho} \right)$, $(\varrho, \vartheta_\varrho) \in \mathbb{R}^2$



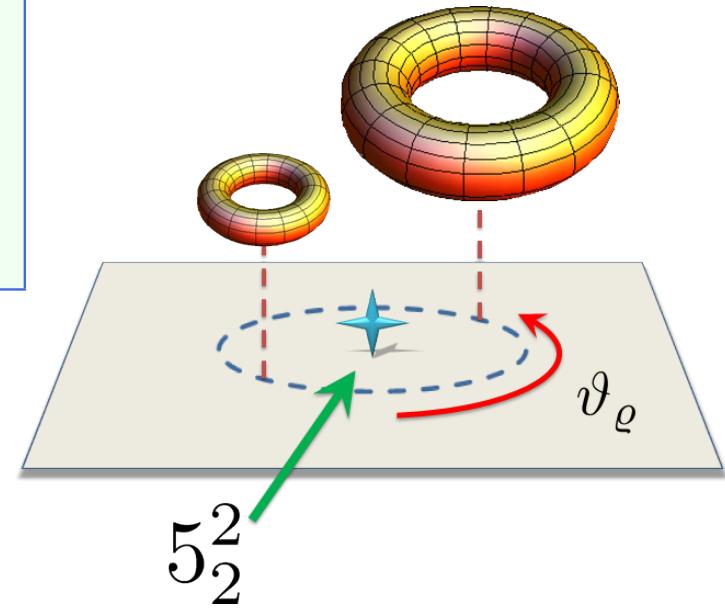
$$ds^2 = dx_{012345}^2 + H[d\varrho^2 + \varrho^2(d\vartheta_\varrho)^2] + \frac{H}{K}[(d\tilde{x}^8)^2 + (d\tilde{x}^9)^2]$$

$$B_{89} = -\frac{\sigma \vartheta_\varrho}{K}, \quad e^{2\Phi} = \frac{H}{K}, \quad K = H^2 + (\sigma \vartheta_\varrho)^2$$

$$H = h + \sigma \log \left(\frac{\mu}{\varrho} \right)$$

$$\vartheta_\varrho = 0 : G_{88} = G_{99} = \frac{1}{H}$$

$$\vartheta_\varrho = 2\pi : G_{88} = G_{99} = \frac{H}{H^2 + (2\pi\sigma)^2}$$



Exotic 5_2^2 -brane has been analyzed in spacetime (SUGRA) picture.

Ready to study **STRING THEORY** pictures!

Exotic 5_2^2 -brane has been analyzed in spacetime (SUGRA) picture.

Ready to study **STRING THEORY** pictures!

string theory pictures

- 👉 string worldsheet picture: NLSM, GLSM
- 👉 five-brane's worldvolume picture

Contents

- GLSM for (exotic) five-branes: 1st version

Sasaki and TK: 1304.4061, 1305.4439, 1310.6163

- GLSM for (exotic) five-branes: 2nd version

Yata and TK: 1402.5580 and more

- Worldvolume theories

Sasaki, Yata and TK: to appear

GLSM: 1st ver.

- ✓ Duality transformation of neutral chiral superfield in D-term and **F-term**
- ✓ Exotic structure in NLSM is carried by non-dynamical field in GLSM

$$\mathcal{L}_{\text{NS5}} = + \int d^4\theta \frac{1}{g^2} \left(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \right)$$

GLSM for NS5-brane

D. Tong [hep-th/0204186](#)

$\mathcal{N} = (4, 4)$	$\mathcal{N} = (2, 2)$	role
neutral HM	chiral $\Psi = X^6 + iX^8 + \dots$ twisted chiral $\Theta = X^7 + iX^9 + \dots$	spacetime coord.

$$\begin{aligned}
\mathcal{L}_{\text{NS5}} = & \int d^4\theta \left\{ \frac{1}{e^2} \left(-\bar{\Sigma}\Sigma + \bar{\Phi}\Phi \right) \right. \\
& \left. \right\} + \int d^4\theta \frac{1}{g^2} \left(-\bar{\Theta}\Theta + \bar{\Psi}\Psi \right) \\
& + \int d^2\theta \left(\quad + (s - \Psi)\Phi \right) + (\text{h.c.}) \\
& + \int d^2\tilde{\theta} (t - \Theta)\Sigma + (\text{h.c.})
\end{aligned}$$

GLSM for NS5-brane

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neutral HM	chiral $\Psi = X^6 + iX^8 + \dots$ twisted chiral $\Theta = X^7 + iX^9 + \dots$	spacetime coord.
VMs	twisted chiral $\Sigma = \bar{D}_+ D_- V$ chiral Φ	gauging isometry
FI parameters	$s = s^6 + i s^8$	$t = t^7 + i t^9$

$$\begin{aligned}
\mathcal{L}_{\text{NS5}} = & \int d^4\theta \left\{ \frac{1}{e^2} \left(-\bar{\Sigma}\Sigma + \bar{\Phi}\Phi \right) + \bar{Q} e^{-2V} Q + \bar{\tilde{Q}} e^{+2V} \tilde{Q} \right\} + \int d^4\theta \frac{1}{g^2} \left(-\bar{\Theta}\Theta + \bar{\Psi}\Psi \right) \\
& + \int d^2\theta \left(\tilde{Q}\Phi Q + (s - \Psi)\Phi \right) + (\text{h.c.}) \\
& + \int d^2\bar{\theta} (t - \Theta)\Sigma + (\text{h.c.})
\end{aligned}$$

GLSM for NS5-brane

D. Tong [hep-th/0204186](#)

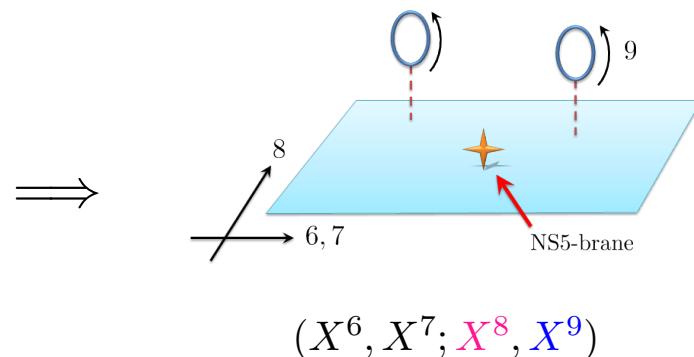
$\mathcal{N} = (4, 4)$	$\mathcal{N} = (2, 2)$	role
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VMs	twisted chiral $\Sigma = \bar{D}_+ D_- V$	gauging isometry
charged HM	chiral Q (-)	curving geometry
FI parameters	$s = s^6 + i s^8$	position of five-branes

Bosonic Lagrangian after integrating-out auxiliary fields :

$$\begin{aligned}\mathcal{L}_{\text{NS5}}^{\text{kin}} &= \frac{1}{e^2} \left[\frac{1}{2} (F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2 \right] - \left[|D_m q|^2 + |D_m \tilde{q}|^2 \right] \\ &\quad - \frac{1}{2g^2} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2 \right] - (X^9 - t^9) F_{01} \\ \mathcal{L}_{\text{NS5}}^{\text{pot}} &= -2(|\sigma|^2 + |\phi|^2) (|q|^2 + |\tilde{q}|^2 + g^2) \\ &\quad - \frac{e^2}{2} \left(|q|^2 - |\tilde{q}|^2 - (X^7 - t^7) \right)^2 - e^2 \left| q\tilde{q} - (X^6 - s^6) - i(X^8 - s^8) \right|^2\end{aligned}$$

Steps to NLSM for NS5-brane

1. SUSY vacua $\mathcal{L}^{\text{pot}} = 0$
2. solve constraints on (q, \tilde{q})
3. IR limit $e \rightarrow \infty$, and integrate out A_m



$$\mathcal{L}_{\text{NS5}}^{\text{NLSM}} = -\frac{1}{2} \left(\frac{1}{g^2} + \frac{1}{r} \right) \left[(\partial_m \vec{X})^2 + (\partial_m X^9)^2 \right] + \varepsilon^{mn} \Omega_i \partial_m X^i \partial_n X^9$$

Target space geometry is...

$$G_{IJ} = H \delta_{IJ} \quad H = \frac{1}{g^2} + \frac{1}{r}$$

$$B_{i9} = \Omega_i \quad \nabla_i H = (\nabla \times \Omega)_i$$

T-duality transformations

(free) string	sign flip (parity) in right-mover	momentum \leftrightarrow winding
spacetime	Buscher rule	$(G_{IJ}, B_{IJ}) \rightarrow (G'_{IJ}, B'_{IJ})$
SUSY sigma model	Roček-Verlinde formula	chiral \leftrightarrow twisted chiral

$$\begin{aligned}
\mathcal{L}_{\text{NS5}} = & \int d^4\theta \left\{ \frac{1}{e^2} \left(-\bar{\Sigma}\Sigma + \bar{\Phi}\Phi \right) + \bar{Q} e^{-2V} Q + \bar{\tilde{Q}} e^{+2V} \tilde{Q} \right\} + \int d^4\theta \frac{1}{g^2} \left(-\bar{\Theta}\Theta + \bar{\Psi}\Psi \right) \\
& + \int d^2\theta \left(\tilde{Q}\Phi Q + (s - \Psi)\Phi \right) + (\text{h.c.}) \\
& + \int d^2\bar{\theta} (t - \Theta)\Sigma + (\text{h.c.})
\end{aligned}$$

T-duality transformations to 5_2^2 -brane

Duality (Roček-Verlinde) transformations

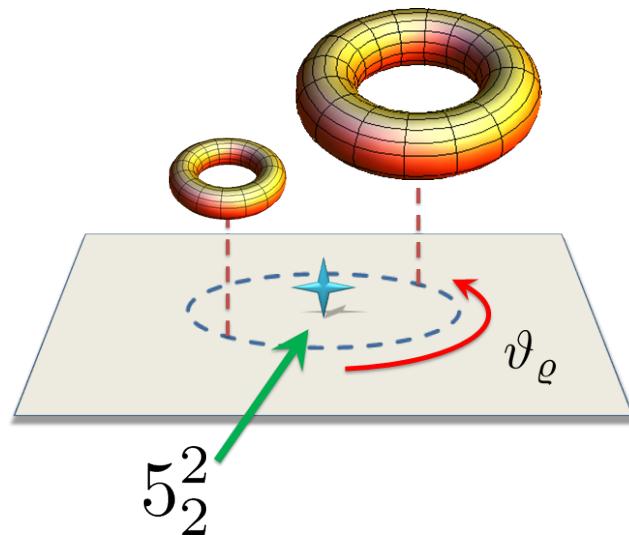
$$\begin{aligned}
-\frac{1}{g^2}(\Theta + \bar{\Theta}) &= (\Gamma + \bar{\Gamma}) + V & \Sigma &= \bar{D}_+ D_- V \\
-\frac{1}{g^2}(\Psi + \bar{\Psi}) &= (\Xi + \bar{\Xi}) - (C + \bar{C}) & \Phi &= \bar{D}_+ \bar{D}_- C
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_E = & \int d^4\theta \left\{ \frac{1}{e^2} \left(-\bar{\Sigma}\Sigma + \bar{\Phi}\Phi \right) + \bar{Q} e^{-2V} Q + \bar{\tilde{Q}} e^{+2V} \tilde{Q} \right\} \\
& + \int d^4\theta \frac{g^2}{2} \left\{ \left(\Gamma + \bar{\Gamma} + V \right)^2 - \left(\Xi + \bar{\Xi} - (C + \bar{C}) \right)^2 \right\} - \int d^4\theta (\Psi - \bar{\Psi})(C - \bar{C}) \\
& + \left\{ \int d^2\theta \left(\tilde{Q}\Phi Q + s\Phi \right) + (\text{h.c.}) \right\} \\
& + \left\{ \int d^2\tilde{\theta} t\Sigma + (\text{h.c.}) \right\} - \varepsilon^{mn} \partial_m (\textcolor{blue}{X}^9 A_n)
\end{aligned}$$

GLSM for exotic 5_2^2 -brane

S. Sasaki and TK arXiv:1304.4061

$$\begin{aligned}\mathcal{L}_{\text{exotic}} = & -\frac{H}{2} \left[(\partial_m \varrho)^2 + \varrho^2 (\partial_m \vartheta_\varrho)^2 \right] - \frac{H}{2K} \left[(\partial_m \tilde{X}^8)^2 + (\partial_m \tilde{X}^9)^2 \right] \\ & - \frac{\sigma \vartheta_\varrho}{K} \varepsilon^{mn} (\partial_m \tilde{X}^8) (\partial_n \tilde{X}^9) - \varepsilon^{mn} \partial_m ((\textcolor{blue}{X}^9 - t^9) A_n)\end{aligned}$$



We obtained GLSM/NLSM for **Exotic Five-brane!**

Remark 1:

Quantum corrections to the geometry can be traced by Vortices in GLSM:

$$\mathcal{L}_{\text{topological}}^{\text{GLSM}} = (\textcolor{blue}{X}^9 - t) F_{01}$$

$$H = h + \sigma \log \left(\frac{\mu}{\varrho} \right) \quad \rightarrow \quad h + \sigma \log \left(\frac{\mu}{\varrho} \right) + \sum_{n \neq 0} K_0(|n| \varrho) \exp(in \textcolor{blue}{X}^9)$$

	physical coordinate in NS5-brane	Tong
$\textcolor{blue}{X}^9$ is ...	dual coordinate in KKM	Tong, Harvey-Jensen
	dual coordinate in 5_2^2 -brane	Sasaki-TK 1305.4439

Remark 2:

5_2^2 -brane geometry has **two** isometries (8- and 9-th)

Possible to gauge both directions?

[Sasaki-TK 1310.6163](#) as a trial

Not completely understood yet...

We want to analyze 5_2^2 -brane from a different viewpoint



Another GLSM

GLSM: 2nd ver.

- ✓ Toric structure is manifest
- ✓ Duality transformation of **Charged** chiral superfield in D-term and **F-term**
- ✓ Kähler potential for the system of “metric + B-field”

A_N-type ALE Space

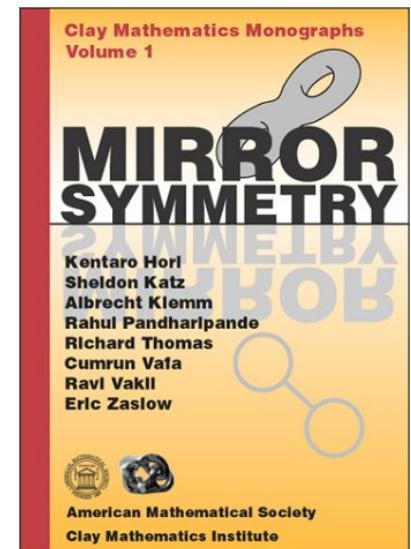
- ✓ hyper-Kähler
- ✓ transverse space of $N + 1$ parallel KKM s
- ✓ toric

- 👉 T-duality along an isometry direction
 - $N + 1$ parallel NS5-branes with B-field
- 👉 large- N limit + T-dualize along the singularity points
 - **exotic** five-brane with B-field

Explicit form of GLSM for A_N -type ALE space associated with **toric** data?

field contents	A_1	A_2	A_3	A_4	\dots	A_N	A_{N+1}	A_{N+2}
$U(1)_1 : V_1$	+1	-2	+1			...		
$U(1)_2 : V_2$		+1	-2	+1				
$U(1)_3 : V_3$			+1	-2				
\vdots					:			
$U(1)_{N-1} : V_{N-1}$						-2	+1	
$U(1)_N : V_N$						+1	-2	+1

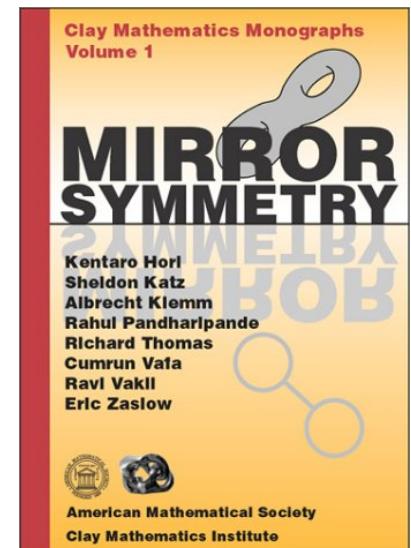
in



Explicit form of GLSM for A_N -type ALE space associated with **toric** data?

field contents	A_1	A_2	A_3	A_4	\dots	A_N	A_{N+1}	A_{N+2}
$U(1)_1 : V_1$	+1	-2	+1			...		
$U(1)_2 : V_2$		+1	-2	+1				
$U(1)_3 : V_3$			+1	-2				
\vdots					:			
$U(1)_{N-1} : V_{N-1}$						-2	+1	
$U(1)_N : V_N$						+1	-2	+1

in



Neither [INSPIRE](#) nor [Google](#) tells us a good answer...

OK! Construct it by ourselves!

Notice:

D-term associated with toric data is **not enough** to derive correct NLSM

F-term is inevitable to fix the complex structure of the geometry

Notice:

D-term associated with toric data is **not enough** to derive correct NLSM

F-term is inevitable to fix the complex structure of the geometry

$\mathcal{N} = (4, 4)$ SUSY ($SU(2)_R$ symmetry) helps everything!

- chiral multiplets $A_i \rightarrow \mathcal{N} = (4, 4)$ HMs (A_i, B_i) as doublets
- vector multiplets $V_a \rightarrow \mathcal{N} = (4, 4)$ VMs (V_a, Φ_a) as doublets
- F-term “ $-\alpha\Phi_a A_a B_a$ ” as a pair of “ $|A_a|^2 e^{2\alpha V_a} + |B_a|^2 e^{-2\alpha V_a}$ ”

$\mathcal{N} = (4, 4)$	(A_1, B_1)	(A_2, B_2)	(A_3, B_3)	(A_4, B_4)	$\dots (A_K, B_K) \dots$	(A_{N+1}, B_{N+1})	(A_{N+2}, B_{N+2})
(V_1, Φ_1)	$(+1, -1)$	$(-2, +2)$	$(+1, -1)$			\dots	
(V_2, Φ_2)		$(+1, -1)$	$(-2, +2)$	$(+1, -1)$			
(V_3, Φ_3)			$(+1, -1)$	$(-2, +2)$			
\vdots					\vdots		
(V_{N-1}, Φ_{N-1})						$(+1, -1)$	
(V_N, Φ_N)						$(-2, +2)$	$(+1, -1)$
(V_0, Φ_0)				$(-\alpha, +\alpha)$			

$$\begin{aligned}
\mathcal{L} = & \int d^4\theta \sum_a \frac{1}{e_a^2} \left(-|\Sigma_a|^2 + |\Phi_a|^2 \right) + \int d^4\theta \sum_i \left(|A_i|^2 e^{2\sum_a Q_i^a V_a} + |B_i|^2 e^{-2\sum_a Q_i^a V_a} \right) \\
& + \int d^2\theta \sum_{a,i} \left(-Q_i^a \Phi_a A_i B_i \right) + \int d^2\tilde{\theta} \sum_a \left(-t_a \Sigma_a \right) + (\text{h.c.})
\end{aligned}$$

Eguchi-Hanson (A₁-ALE):

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$$\begin{aligned}
\mathcal{L} = & \int d^4\theta \frac{1}{e_1^2} \left(-|\Sigma_1|^2 + |\Phi_1|^2 \right) + \int d^4\theta \frac{1}{e_0^2} \left(-|\Sigma_0|^2 + |\Phi_0|^2 \right) \\
& + \int d^4\theta \left(|A_1|^2 e^{2V_1} + |A_2|^2 e^{-4V_1-2\alpha V_0} + |A_3|^2 e^{2V_1} \right) \\
& + \int d^4\theta \left(|B_1|^2 e^{-2V_1} + |B_2|^2 e^{4V_1+2\alpha V_0} + |B_3|^2 e^{-2V_1} \right) \\
& - \int d^2\theta \left(\Phi_1(A_1 B_1 - 2A_2 B_2 + A_3 B_3) - \alpha \Phi_0(A_2 B_2) \right) - \int d^2\tilde{\theta} t_1 \Sigma_1 + (\text{h.c.})
\end{aligned}$$

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& + \int d^4\theta \left(|A_1|^2 e^{2V_1} + |A_2|^2 e^{-4V_1-2\alpha V_0} + |A_3|^2 e^{2V_1} \right) \\
& + \int d^4\theta \left(|B_1|^2 e^{-2V_1} + |B_2|^2 e^{4V_1+2\alpha V_0} + |B_3|^2 e^{-2V_1} \right) \\
& - \int d^2\theta \left(\Phi_1(A_1 B_1 - 2A_2 B_2 + A_3 B_3) - \alpha \Phi_0(A_2 B_2) \right) - \int d^2\tilde{\theta} t_1 \Sigma_1 + (\text{h.c.})
\end{aligned}$$

Integrate out $\{\Sigma_a, \Phi_a\}$ in the IR limit $e_a \rightarrow \infty$



$$\mathcal{L}_{\text{IR}} = \int d^4\theta \left\{ \sqrt{(t_1)^2 + 4(|A_1|^2 + |A_3|^2)^2} - t_1 \log \frac{t_1 + \sqrt{(t_1)^2 + 4(|A_1|^2 + |A_3|^2)^2}}{2(|A_1|^2 + |A_3|^2)} \right\}$$

with parametrization: $A_1 = \frac{r}{\sqrt{2}} \cos \frac{\vartheta}{2} e^{\frac{i}{2}(\psi+\varphi)}$, $A_2 = 0$, $A_3 = \frac{r}{\sqrt{2}} \sin \frac{\vartheta}{2} e^{\frac{i}{2}(\psi-\varphi)}$

Eguchi-Hanson (A₁-ALE):

$$\begin{aligned}
\mathcal{L} = & \int d^4\theta \frac{1}{e_1^2} \left(-|\Sigma_1|^2 + |\Phi_1|^2 \right) + \int d^4\theta \frac{1}{e_0^2} \left(-|\Sigma_0|^2 + |\Phi_0|^2 \right) \\
& + \int d^4\theta \left(|A_1|^2 e^{2V_1} + |A_2|^2 e^{-4V_1-2\alpha V_0} + |A_3|^2 e^{2V_1} \right) \\
& + \int d^4\theta \left(|B_1|^2 e^{-2V_1} + |B_2|^2 e^{4V_1+2\alpha V_0} + |B_3|^2 e^{-2V_1} \right) \\
& - \int d^2\theta \left(\Phi_1(A_1 B_1 - 2A_2 B_2 + A_3 B_3) - \alpha \Phi_0(A_2 B_2) \right) - \int d^2\tilde{\theta} t_1 \Sigma_1 + (\text{h.c.})
\end{aligned}$$

T-duality along φ -direction

Duality transformations of $\{A_1, A_3\}$ in D-term and **F-term**

$$\Phi_1 = \overline{D}_+ \overline{D}_- C$$

Eguchi-Hanson (A₁-ALE):

$$\begin{aligned}
\mathcal{L} = & - \int d^4\theta \left((Y_1 + \bar{Y}_1) \log(Y_1 + \bar{Y}_1) + (Y_3 + \bar{Y}_3) \log(Y_1 + \bar{Y}_3) \right) \\
& - \int d^4\theta \left((Y_1 + \bar{Y}_1) - (Y_3 + \bar{Y}_3) - t_1 \right) \log \left((Y_1 + \bar{Y}_1) - (Y_3 + \bar{Y}_3) - t_1 \right) \\
& + \int d^4\theta \mathcal{F}(C) \\
& + \dots
\end{aligned}$$

$$|A_1|^2 e^{2V_1} = +(Y_1 + \bar{Y}_1) + T_1(C)$$

$$|A_3|^2 e^{2V_1} = -(Y_3 + \bar{Y}_3) + T_3(C)$$

GLSM for 5_2^2 -brane:

GLSM for 5_2^2 -brane:

sorry, under construction

Worldvolume

- ✓ covariant form of gauged isometries and Wess-Zumino terms
- ✓ type IIB, IIA, and Heterotics

Worldvolume action of 5_2^2 -brane could be obtained from

D5-brane's action via S- and T-dualities (IIB)

KK6-brane action via reduction and T-duality (IIB)

M5-brane action via reduction and T-dualities (IIA)

Different point: **Two** isometries along transverse directions

How to describe its worldvolume theory with isometries?

Worldvolume action of $5\frac{1}{2}$ -brane could be obtained from

D5-brane's action via S- and T-dualities (IIB)

KK6-brane action via reduction and T-duality (IIB)

M5-brane action via reduction and T-dualities (IIA)

Different point: **Two** isometries along transverse directions

How to describe its worldvolume theory with isometries?

Already Known!

Bergshoeff et al hep-th/9706117

Worldvolume action with WZ-term is **gauged** with respect to isometries

ex) gravity sector of KK6-brane action with gauged isometry in M-theory:

$$\begin{aligned}
 \mathcal{L}_{\text{KK6}}^{\text{M}} &= -\frac{1}{2} T_{\text{KK6}}^{\text{M}} \sqrt{-\gamma} \left(k^{\frac{4}{7}} \gamma^{ab} D_a X^\mu D_b X^\nu g_{\mu\nu} - 5 \right) \\
 &= -T_{\text{KK6}}^{\text{M}} k^2 \sqrt{-\det(D_a X^\mu D_b X^\nu g_{\mu\nu})} \\
 &= -T_{\text{KK6}}^{\text{M}} k^2 \sqrt{-\det(\partial_a X^\mu \partial_b X^\nu \Pi_{\mu\nu})}
 \end{aligned}$$

$$D_a X^\mu = \partial_a X^\mu + C_a k^\mu \quad k^\mu : \text{Killing vector}$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

- ✓ Construct the worldvolume action with WZ-term in a covariant way:

$$\text{KK6} \xrightarrow{\text{reduction}} \text{KK5 in IIA} \xrightarrow{\text{T-duality}} 5_2^2 \text{ in IIB}$$

$$\text{M5} \xrightarrow{\text{reduction}} \text{NS5 in IIA} \xrightarrow{\text{T-dualities}} 5_2^2 \text{ in IIA}$$

- ✓ 5_2^2 -branes in heterotic theories can be obtained by truncations:

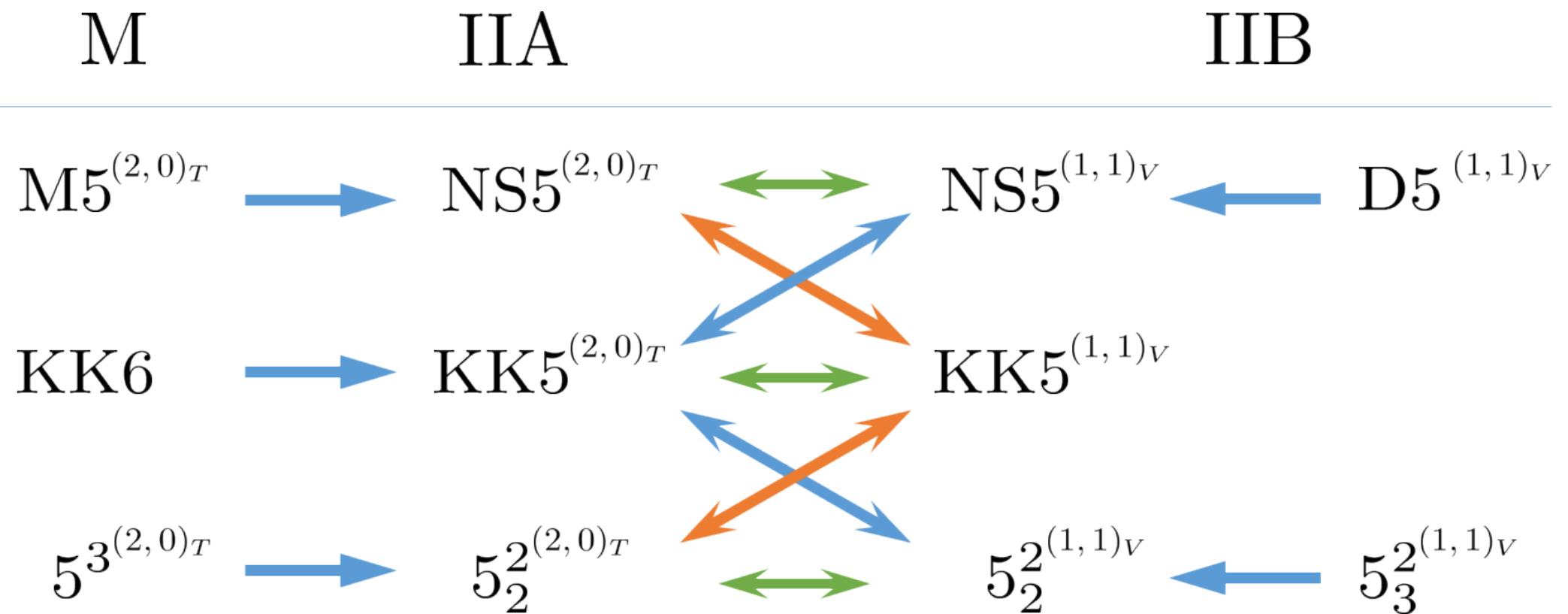
$$5_2^2 \text{ in IIB} \xrightarrow[\mathcal{N}=(1,1) \rightarrow \mathcal{N}=(1,0)]{\text{RR} = 0} 5_2^2 \text{ in HSO}$$

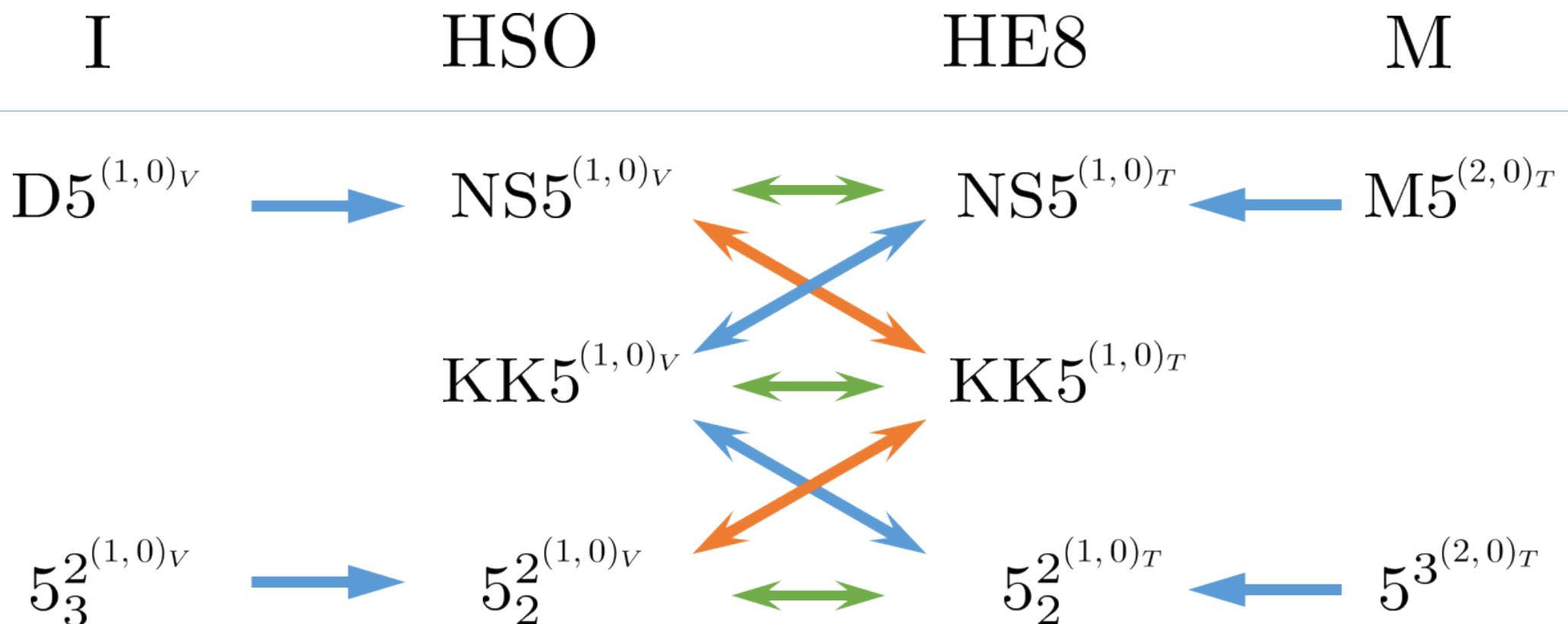
$$5_2^2 \text{ in IIA} \xrightarrow[\mathcal{N}=(2,0) \rightarrow \mathcal{N}=(1,0)]{\text{RR} = 0} 5_2^2 \text{ in HE8}$$

$$\begin{aligned}
S_{5_2^2}^{\text{IIB}} = & -T_{5_2^2} \int d^6 \xi e^{-2\phi} (\det h_{IJ}) \sqrt{1 + e^{+2\phi} (\det h_{IJ})^{-1} (i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B))^2} \\
& \times \sqrt{-\det \left[\partial_a X^\mu \partial_b X^\nu \Pi_{\mu\nu}^{(2)} + \frac{K_a^{(1)} K_b^{(1)}}{(k_2)^2} - (k_2)^2 \frac{K_a^{(2)} K_b^{(2)} + K_a^{(3)} K_b^{(3)}}{(\det h_{IJ})} + F_{ab} \right]} \\
& - \mu_5 \int_{\mathcal{M}_6} \left[P[i_{k_1} i_{k_2} B^{(8,2)}] - \frac{1}{2} P[\check{B} \wedge \check{C}^{(2)} \wedge \check{C}^{(2)}] + P[\check{C}^{(4)} - \check{C}^{(2)} \wedge \check{B}] \wedge \check{F} \right. \\
& \left. - P[\check{B}] \wedge \check{F} \wedge \check{F} - \frac{i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B)}{(i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B))^2 + e^{-2\phi} (\det h_{IJ})} \check{F} \wedge \check{F} \wedge \check{F} \right]
\end{aligned}$$

$$h_{IJ} = k_I^\mu k_J^\nu (g_{\mu\nu} + B_{\mu\nu})$$

For details, please wait for our coming paper!!





SUMMARY

We established GLSM for exotic five-brane!

- ✓ Dualization of neutral chiral superfields in D-term and F-term (1st ver.)
- ✓ Dualization of Charged chiral superfields in D-term and F-term (2nd ver.)
- ✓ Gauge covariant isometries in worldvolume theories

We hope our five-branes' theories tell us more and more !

- ✓ modular invariance, dipole description ? T. Kikuchi, T. Okada and Y. Sakatani
- ✓ more exotic as “multiple states of NS5 + 5_2^2 ” de Boer and Shigemori

Thanks

APPENDIX

$$10\text{D string theory} = D\text{-dim spacetime} \otimes \text{compact space } \mathcal{M}_d$$

geometry associated with G_{mn}	Conventional geometry (manifold) $O(d)$ global symmetry [Calabi-Yau, etc]	ordinary compactifications
geometry associated with G_{mn}, B_{mn}	Generalized geometry $O(d, d; \mathbb{Z})$ T-duality symmetry [T-fold]	flux compactifications
geometry associated with $G_{mn}, \tau = C_{(0)} + i e^{-\Phi}$	Generalized geometry $SL(2, \mathbb{Z})$ S-duality symmetry [S-fold]	F-theory
geometry associated with $G_{mn}, B_{mn}, \Phi, C_{(p)}$	Generalized geometry $E_{d+1(d+1)}(\mathbb{Z})$ U-duality symmetry [U-fold]	compactifications with non-abelian gauge

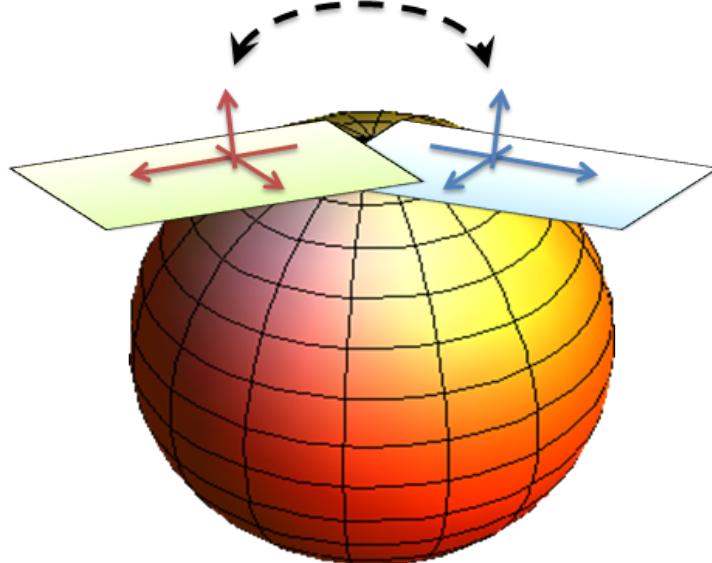
Non-geometric structure

structure groups = diffeom. ($GL(d, \mathbb{R})$) \cup String duality groups



T-duality, U-duality, etc.

$GL(d, \mathbb{R}) \cup$ duality transf.



Generalized Geometry (N. Hitchin)
Doubled Geometry (C. Hull)

5_2^2 -brane is a **concrete** example (T-fold)

Exotic brane shows us a new insight of stringy spacetime

M-theory on $S^1(R_s)$ mass/tension ($l_s \equiv 1$) type IIA

longitudinal M2	1	F1
transverse M2	$\frac{1}{g_s}$	D2
longitudinal M5	$\frac{1}{g_s}$	D4
transverse M5	$\frac{1}{g_s^2}$	NS5
longitudinal KK6	$\frac{R_{\text{TN}}^2}{g_s^2}$	KK5
KK6 with $R_{\text{TN}} = R_s$	$\frac{1}{g_s}$	D6
transverse KK6	$\frac{R_{\text{TN}}^2}{g_s^3}$	6_3^1

0	1	2	3	4	5	6	7	8	9	M
✓	✓	✓	✓	✓	✓	✓	S^1	\mathbb{R}^3		
KK6 $\rightarrow 6_3^1$						Taub-NUT				

$$b_n^c : M = \frac{(R_1 \cdots R_c)^2}{g_s^n}$$

for review: N. Obers and B. Pioline [hep-th/9809039](https://arxiv.org/abs/hep-th/9809039)

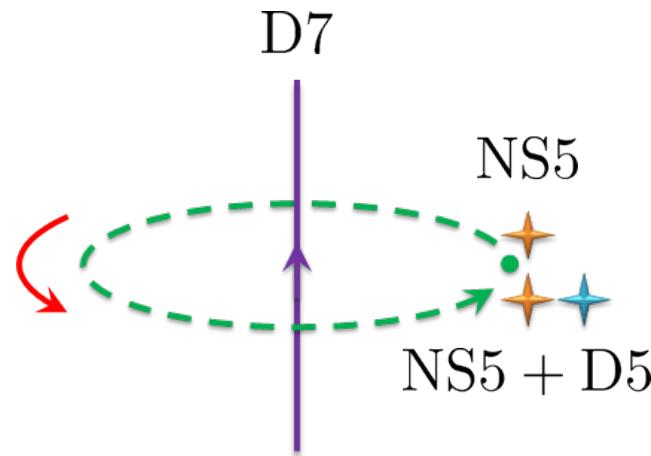
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2, \mathbb{Z})$ monodromy charge q

$$q : (C_2, B_2) \rightarrow (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

$$(0, \text{NS5}) \rightarrow (\text{D5}, \text{NS5})$$



an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

LESSON 1 : GLSM for NS5-brane

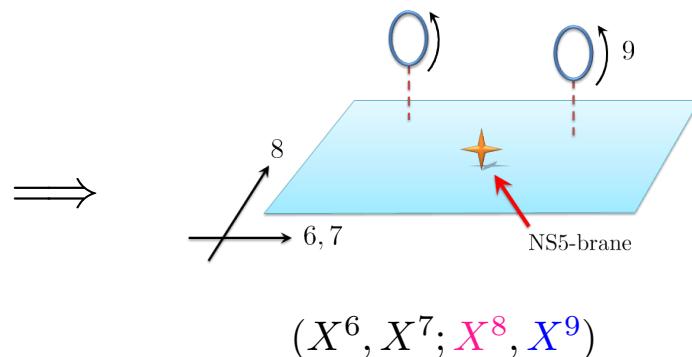
Bosonic Lagrangian after integrating-out auxiliary fields :

$$\begin{aligned}\mathcal{L}_{\text{NS5}}^{\text{kin}} &= \frac{1}{e^2} \left[\frac{1}{2} (F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2 \right] - \left[|D_m q|^2 + |D_m \tilde{q}|^2 \right] \\ &\quad - \frac{1}{2g^2} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2 \right] - (X^9 - t^9) F_{01}\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\text{NS5}}^{\text{pot}} &= -2(|\sigma|^2 + |\phi|^2) (|q|^2 + |\tilde{q}|^2 + g^2) \\ &\quad - \frac{e^2}{2} \left(|q|^2 - |\tilde{q}|^2 - (X^7 - t^7) \right)^2 - e^2 \left| q\tilde{q} - (X^6 - s^6) - i(X^8 - s^8) \right|^2\end{aligned}$$

Steps to NLSM for NS5-brane

1. SUSY vacua $\mathcal{L}^{\text{pot}} = 0$
2. solve constraints on (q, \tilde{q})
3. IR limit $e \rightarrow \infty$, and integrate out A_m



1. SUSY vacua

$$\sigma = 0 = \phi , \quad |q|^2 - |\tilde{q}|^2 = X^7 - t^7 , \quad q\tilde{q} = (X^6 - s^6) + i(X^8 - s^8)$$

2. solve constraints on (q, \tilde{q})

$$q = -i e^{-i\alpha} \sqrt{r + (X^7 - t^7)} , \quad \tilde{q} = i e^{+i\alpha_a} \frac{(X^6 - s^6) + i(X^8 - s^8)}{\sqrt{r + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \tilde{q}|^2 = \frac{1}{2r} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] + r \left(\partial_m \alpha + A_m - \Omega_i \partial_m X^i \right)^2$$

$$r = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2 + (X^8 - s^8)^2}$$

$$\Omega_i \partial_m X^i = \frac{-(X^6 - s^6) \partial_m X^8 + (X^8 - s^8) \partial_m X^6}{r(r + (X^7 - t^7))}$$

3. IR limit $e \rightarrow \infty$, and integrate out A_m

$$A_m = \Omega_i \partial_m X^i + \frac{1}{r} \varepsilon_{mn} \partial^n X^9$$

$$\implies \mathcal{L}_{\text{NS5}}^{\text{NLSM}} = -\frac{1}{2} \left(\frac{1}{g^2} + \frac{1}{r} \right) \left[(\partial_m \vec{X})^2 + (\partial_m X^9)^2 \right] + \varepsilon^{mn} \Omega_i \partial_m X^i \partial_n X^9$$

LESSON 2 : T-duality

$\Theta \rightarrow \Gamma :$

$$\begin{aligned}\mathcal{L}_H \ni \mathcal{L}_\Theta &= \int d^4\theta \left(-\frac{1}{g^2} \bar{\Theta} \Theta \right) + \left\{ \int d^2\tilde{\theta} (-\Theta) \Sigma + (\text{h.c.}) \right\} \\ &= \int d^4\theta \left\{ -\frac{1}{2g^2} (\Theta + \bar{\Theta})^2 - (\Theta + \bar{\Theta}) V \right\} - \varepsilon^{mn} \partial_m (X^9 A_n)\end{aligned}$$

↓

$$\mathcal{L}_{B\Gamma} = \int d^4\theta \left\{ -\frac{1}{2g^2} \bar{B}^2 - \bar{B} V - \bar{B} (\Gamma + \bar{\Gamma}) \right\} - \varepsilon^{mn} \partial_m (X^9 A_n)$$

real $\bar{B} = B$; chiral $\bar{D}_\pm \Gamma = 0$

$\Theta \rightarrow \Gamma :$

$$\mathcal{L}_{B\Gamma} = \int d^4\theta \left\{ -\frac{1}{2g^2}B^2 - BV - (\Gamma + \bar{\Gamma})B \right\} - \varepsilon^{mn} \partial_m (X^9 A_n)$$

Integrating out $\Gamma, \bar{\Gamma} :$ → GLSM for NS5-brane

$$B = \Theta + \bar{\Theta}$$

or, Integrating out $B :$ → GLSM for KK-monopole

$$\frac{1}{g^2}B = -(\Gamma + \bar{\Gamma}) - V$$

duality relation :

$$\Theta = X^7 + iX^9 + \dots, \quad \Gamma = \tilde{X}^7 + i\tilde{X}^9 + \dots$$

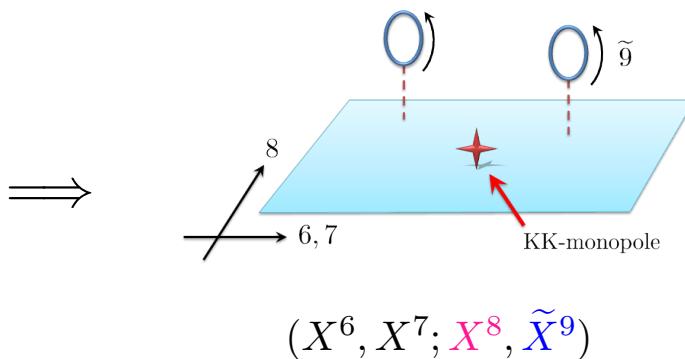
$$\Theta + \bar{\Theta} = -g^2(\Gamma + \bar{\Gamma}) - g^2V \rightarrow$$

$X^7 = -g^2 \tilde{X}^7$
$\pm(\partial_0 \pm \partial_1)X^9 = -g^2(D_0 \pm D_1)\tilde{X}^9$
$D_m \tilde{X}^9 = \partial_m \tilde{X}^9 + A_m$

$$\begin{aligned}
\mathcal{L}_{KK} = & \int d^4\theta \left\{ \frac{1}{e^2} \left(-\bar{\Sigma}\Sigma + \bar{\Phi}\Phi \right) + \bar{Q} e^{-2V} Q + \bar{\tilde{Q}} e^{+2V} \tilde{Q} \right\} + \int d^4\theta \left\{ \frac{g^2}{2} \left(\Gamma + \bar{\Gamma} + V \right)^2 + \frac{1}{g^2} \bar{\Psi}\Psi \right\} \\
& + \int d^2\theta \left(\tilde{Q}\Phi Q + (s - \Psi)\Phi \right) + (\text{h.c.}) \\
& + \left\{ \int d^2\tilde{\theta} t\Sigma + (\text{h.c.}) \right\} - \varepsilon^{mn} \partial_m (\textcolor{blue}{X}^9 A_n)
\end{aligned}$$

Steps to NLSM for KK-monopole

1. SUSY vacua $\mathcal{L}^{\text{pot}} = 0$
2. solve constraints on (q, \tilde{q})
3. IR limit $e \rightarrow \infty$, and integrate out A_m



1. SUSY vacua

$$\sigma = 0 = \phi , \quad |q|^2 - |\tilde{q}|^2 = X^7 - t^7 , \quad q\tilde{q} = (X^6 - s^6) + i(X^8 - s^8)$$

2. solve constraints on (q_a, \tilde{q}_a)

$$q = -i e^{-i\alpha} \sqrt{r + (X^7 - t^7)} , \quad \tilde{q} = i e^{+i\alpha} \frac{(X^6 - s^6) + i(X^8 - s^8)}{\sqrt{r + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \tilde{q}|^2 = \frac{1}{2r} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] + r \left(\partial_m \alpha + A_m - \Omega_i \partial_m X^i \right)^2$$

3. IR limit $e \rightarrow \infty$, and integrate out A_m

$$A_m = \frac{1}{rH} (\partial_m \tilde{X}^9 - \Omega_i \partial_m X^i) + \Omega_i \partial_m X^i , \quad H = \frac{1}{g^2} + \frac{1}{r}$$

$$\mathcal{L}_{\text{KK}}^{\text{NLSM}} = -\frac{1}{2} H (\partial_m \vec{X})^2 - \frac{1}{2} H^{-1} (\partial_m \tilde{X}^9 - \Omega_i \partial_m X^i)^2 - \varepsilon^{mn} \partial_m ((X^9 - t^9) A_n)$$

LESSON 3 : 5_2^2 -brane

$\Psi \rightarrow \Xi :$

$$\begin{aligned}\mathcal{L}_{KK} \ni \mathcal{L}_\Psi &= \int d^4\theta \frac{1}{g^2} \bar{\Psi} \Psi + \left\{ \int d^2\theta (-\Psi) \Phi + (\text{h.c.}) \right\} \\ &= \int d^4\theta \left\{ \frac{a}{g^2} (\Psi + \bar{\Psi})^2 - (\Psi + \bar{\Psi})(C + \bar{C}) \right\} \\ &\quad + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (\Psi - \bar{\Psi})^2 - (\Psi - \bar{\Psi})(C - \bar{C}) \right\}\end{aligned}$$

↓

$$\begin{aligned}\mathcal{L}_{RSX\Xi} &= \int d^4\theta \left\{ \frac{a}{g^2} R^2 - \textcolor{blue}{R}(C + \bar{C}) + \textcolor{blue}{R}(\Xi_1 + \bar{\Xi}_1) + \textcolor{blue}{R}(\textcolor{teal}{X} + \bar{X}) \right\} \\ &\quad + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (\textcolor{red}{i}S)^2 - (\textcolor{red}{i}S)(C - \bar{C}) + \textcolor{red}{i}S(\Xi_2 - \bar{\Xi}_2) + \textcolor{red}{i}S(\textcolor{teal}{X} - \bar{X}) \right\}\end{aligned}$$

$$\bar{R} = R, \quad \bar{S} = S, \quad \bar{D}_+ \Xi_{1,2} = 0 = D_- \Xi_{1,2}, \quad \bar{D}_\pm X = 0, \quad \Phi_a = \bar{D}_+ \bar{D}_- C_a$$

$\Psi \rightarrow \Xi :$

$$\begin{aligned}\widetilde{\mathcal{L}} &= \int d^4\theta \left\{ \frac{a}{g^2} R^2 - R(C + \bar{C}) + R(\Xi_1 + \bar{\Xi}_1) + R(X + \bar{X}) \right\} \\ &\quad + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - (\mathrm{i}S)(C - \bar{C}) + \mathrm{i}S(\Xi_2 - \bar{\Xi}_2) + \mathrm{i}S(X - \bar{X}) \right\}\end{aligned}$$

Integrating out $\Xi_1, \Xi_2, X : \rightarrow$ GLSM for KK-monopole

or, Integrating out $R, \Xi_2 : \rightarrow$ GLSM for 5_2^2 -brane

$$\frac{2a}{g^2} R = -(\Xi_1 + \bar{\Xi}_1) + (C + \bar{C})$$

duality relation at $a = \frac{1}{2} :$

$$\Psi = X^6 + \mathrm{i}X^8 + \dots$$

$$\Psi + \bar{\Psi} = -g^2(\Xi_1 + \bar{\Xi}_1) + g^2(C + \bar{C})$$

$X^6 \sim$ real part of Ξ

$\partial X^8 \sim \partial(\text{imaginary part of } \Xi) + \text{"gauge" fields in } C_a$

$$\begin{aligned}
\mathcal{L}_E = & \int d^4\theta \left\{ \frac{1}{e^2} \left(-\bar{\Sigma}\Sigma + \bar{\Phi}\Phi \right) + \bar{Q} e^{-2V} Q + \bar{\tilde{Q}} e^{+2V} \tilde{Q} \right\} \\
& + \int d^4\theta \frac{g^2}{2} \left\{ \left(\Gamma + \bar{\Gamma} + V \right)^2 - \left(\Xi + \bar{\Xi} - (C + \bar{C}) \right)^2 \right\} \\
& + \left\{ \int d^2\theta \left(\tilde{Q}\Phi Q + s\Phi \right) + (\text{h.c.}) \right\} - \int d^4\theta (\Psi - \bar{\Psi})(C - \bar{C}) \\
& + \left\{ \int d^2\tilde{\theta} t\Sigma + (\text{h.c.}) \right\} - \varepsilon^{mn} \partial_m (\textcolor{blue}{X}^9 A_n)
\end{aligned}$$

$\mathcal{N} = (4, 4)$	$\mathcal{N} = (2, 2)$		role
neutral HM	twisted chiral $\Xi = X^6 + i\tilde{X}^8 + \dots$	chiral $\Gamma = X^7 + i\tilde{X}^9 + \dots$	spacetime coord.
VMs	twisted chiral $\Sigma = \bar{D}_+ D_- V$	chiral $\Phi = \bar{D}_+ \bar{D}_- C$	gauging isometry
charged HM	chiral Q (-)	chiral \tilde{Q} (+)	curving geometry
FI parameters	$s = s^6 + i s^8$	$t = t^7 + i t^9$	position of five-branes

$$\begin{aligned}
\mathcal{L}_{\text{E}}^{\text{kin}} &= \frac{1}{e^2} \left[\frac{1}{2} (F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2 \right] - \left[|D_m q|^2 + |D_m \tilde{q}|^2 \right] \\
&\quad - \frac{1}{2g^2} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 \right] - \frac{g^2}{2} \left[(\partial_m \tilde{X}^8)^2 + (D_m \tilde{X}^9)^2 \right] - (X^9 - t^9) F_{01} \\
\mathcal{L}_{\text{E}}^{\text{pot}} &= -2(|\sigma|^2 + 4|M_c|^2)(|q|^2 + |\tilde{q}|^2 + g^2) \\
&\quad - \frac{e^2}{2} \left(|q|^2 - |\tilde{q}|^2 - (X^7 - t^7) \right)^2 - e^2 \left| q\tilde{q} - (X^6 - s^6) - i(\tilde{X}^8 - s^8) \right|^2 \\
&\quad + \frac{g^2}{2} (A_{c=} + \bar{A}_{c=})(B_{c\pm} + \bar{B}_{c\pm})
\end{aligned}$$

$$\begin{aligned}
(\partial_0 + \partial_1) X^8 &= -g^2 (\partial_0 + \partial_1) \tilde{X}^8 + g^2 (B_{c\pm} + \bar{B}_{c\pm}) \\
(\partial_0 - \partial_1) X^8 &= +g^2 (\partial_0 - \partial_1) \tilde{X}^8 + g^2 (A_{c=} + \bar{A}_{c=}) \\
+\frac{g^2}{2} (A_{c=} + \bar{A}_{c=})(B_{c\pm} + \bar{B}_{c\pm}) &= -\frac{1}{2g^2} (\partial_m X^8)^2 + \frac{g^2}{2} (\partial_m \tilde{X}^8)^2 + \varepsilon^{mn} (\partial_m X^8) (\partial_n \tilde{X}^8)
\end{aligned}$$

Steps to NLSM for 5_2^2 -brane

1. search SUSY vacua $\mathcal{L}^{\text{potential}} = 0$
2. solve constraints of charged HM (Q, \tilde{Q})
3. integrate out VM (V, Φ) in IR
- ★4. integrate s^8 , and solve EOM for X^8 in $\Psi - \bar{\Psi}$

Step 1.,2.,3. :

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}H \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] - \frac{1}{2H}(\partial_m \tilde{X}^9)^2 \\
& - \frac{(\Omega_8)^2}{2H}(\partial_m \textcolor{red}{X}^8)^2 + \frac{\Omega_8}{H}(\partial_m \textcolor{red}{X}^8)(\partial^m \tilde{X}^9) \\
& - \frac{(\Omega_6)^2}{2H}(\partial_m X^6)^2 - \frac{\Omega_6 \Omega_8}{H}(\partial_m X^6)(\partial^m \textcolor{red}{X}^8) + \frac{\Omega_6}{H}(\partial_m X^6)(\partial^m \tilde{X}^9) \\
& - \varepsilon^{mn} \partial_m ((X^9 - t^9) A_n) + \varepsilon^{mn} (\partial_m X^8)(\partial_n \tilde{X}^8)
\end{aligned}$$

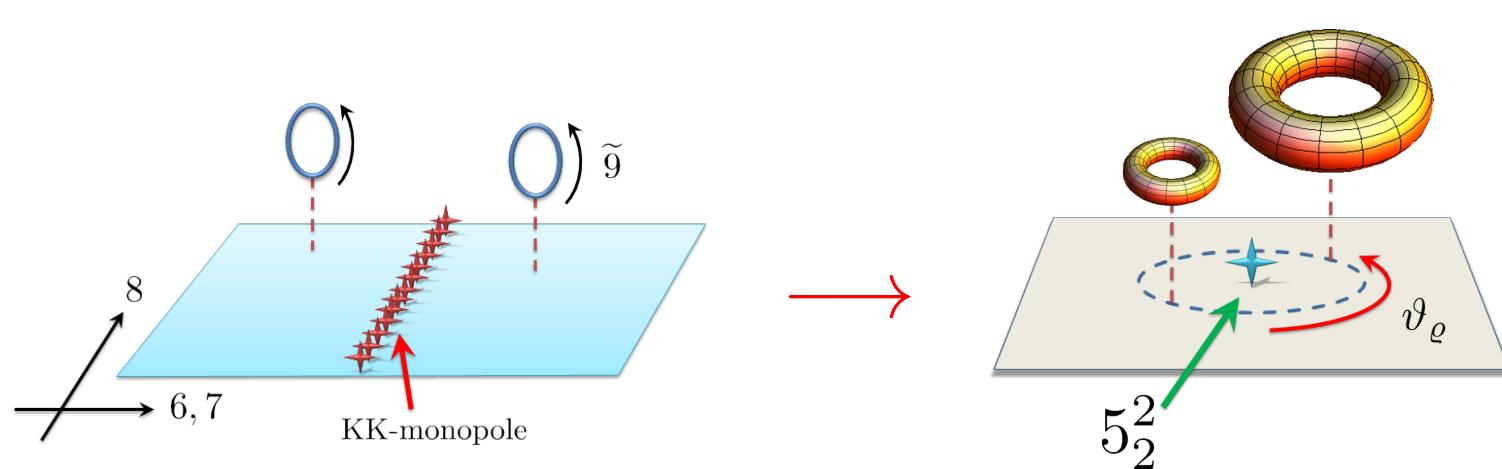
$$H = \frac{1}{g^2} + \frac{1}{r}, \quad \Omega_6 = \frac{X^8 - s^8}{r(r + (X^7 - t^7))}, \quad \Omega_8 = -\frac{X^6 - s^6}{r(r + (X^7 - t^7))}$$

$$A_m = \frac{1}{rH} (\partial_m \tilde{X}^9 - \Omega_i \partial_m X^i) + \Omega_i \partial_m X^i$$

Step 4. : $s^8 = 2\pi \mathcal{R}_8 s \xrightarrow{\text{integral of } s}$ emerge isometry

$$\left\{ \begin{array}{l} H \rightarrow h_0 + \sigma \log(\mu/\varrho) \\ \Omega_6 \rightarrow 0 \\ \Omega_8 \rightarrow \sigma \arctan \left(\frac{X^7 - t^7}{X^6 - s^6} \right) \equiv \sigma \vartheta_\varrho \end{array} \right. \begin{array}{l} : \text{co-dim. 2} \quad \varrho = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2} \\ : \text{isometry along } X^8 \\ : \text{"non-single-valued" metric} \end{array}$$

EOM for X^8 : $\partial_m X^8 = \frac{H}{K} \left[\frac{\sigma \vartheta_\varrho}{H} (\partial_m \tilde{X}^9) + \varepsilon_{mn} (\partial^n \tilde{X}^8) \right]$ $K = H^2 + (\sigma \vartheta_\varrho)^2$



GLSM is a powerful tool, also in this stage :

Worldsheet instantons in NLSM can be captured by
vortex solution in gauge theory

ex.) GLSM for NS5-brane :

Take the configuration : $\phi = 0 = \sigma$ with $g^2 \rightarrow 0$ and finite e^2

$$\begin{aligned}\mathcal{L}_E &= \frac{1}{2e^2}(F_{12})^2 + |D_m q|^2 + \frac{e^2}{2}(|q|^2 - \zeta)^2 + i \textcolor{blue}{X}^9 F_{12} \\ F_{12} &= \mp e^2(|q|^2 - \zeta), \quad 0 = (D_1 \pm i D_2)q\end{aligned}$$

Abrikosov-Nielsen-Olesen vortex eq.

$$\text{then, } S_E = \frac{1}{2\pi} \int d^2x \mathcal{L}_E = \zeta |n| - i \textcolor{blue}{X}^9 n \quad \textcolor{teal}{n} = -\frac{1}{2\pi} \int d^2x F_{12}$$

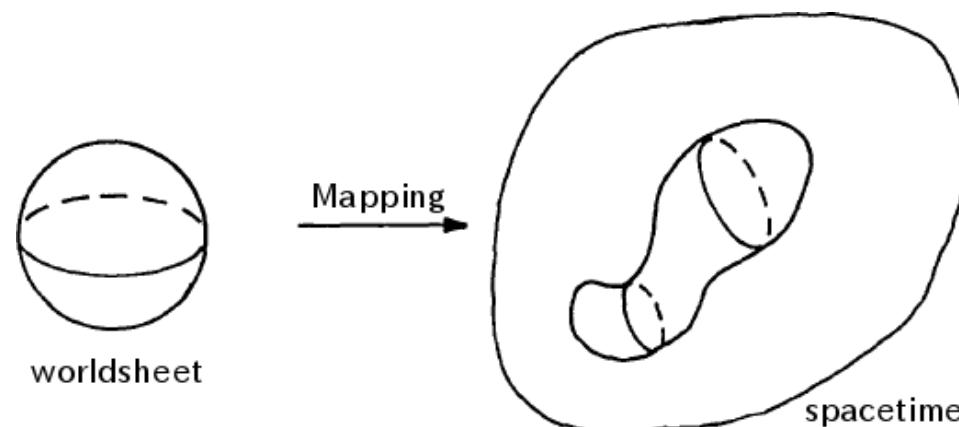
QUANTUM CORRECTIONS

S. Sasaki and TK arXiv:1305.4439

String Worldsheet Instanton Corrections



Deform target space geometry by momentum and/or winding effects



GLSM is a powerful tool in this stage :

Worldsheet instantons in NLSM can be captured by
vortex solution in gauge theory

► NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{r} : \text{radius of } X^9$$

$g \rightarrow 0$ case	general r	$g \rightarrow \infty$ case	$r \rightarrow 0$	$r \rightarrow \infty$
radius of X^9	∞	radius of X^9	∞	0
KK-modes	light	KK-modes	light	heavy
winding modes	heavy	winding modes	heavy	light

► NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{r} : \text{radius of } X^9$$

$g \rightarrow 0$ case	general r	$g \rightarrow \infty$ case	$r \rightarrow 0$	$r \rightarrow \infty$
radius of X^9	∞	radius of X^9	∞	0
KK-modes	light	KK-modes	light	heavy
winding modes	heavy	winding modes	heavy	light

GLSM for NS5-brane has $X^9 F_{01}$

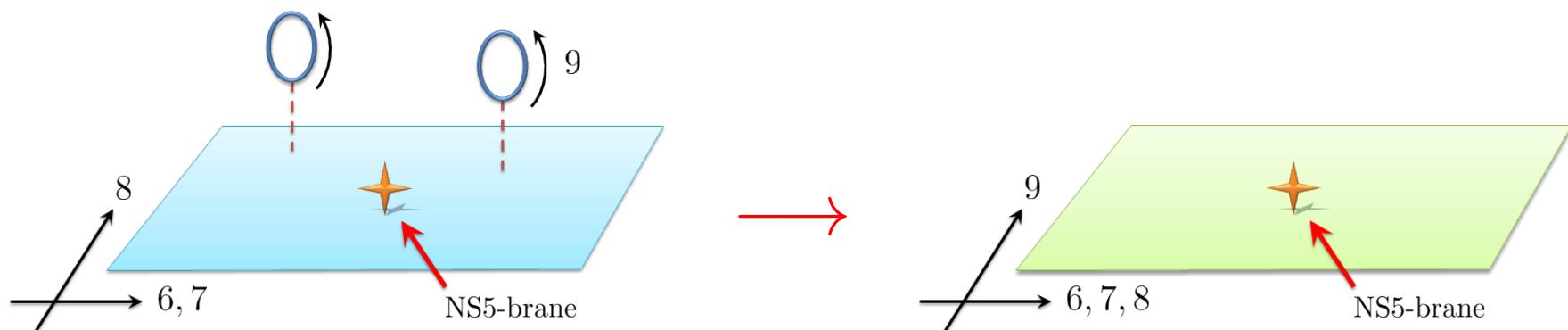


KK-mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to NS5-brane :

$$X^9 F_{01} \text{ in GLSM}$$

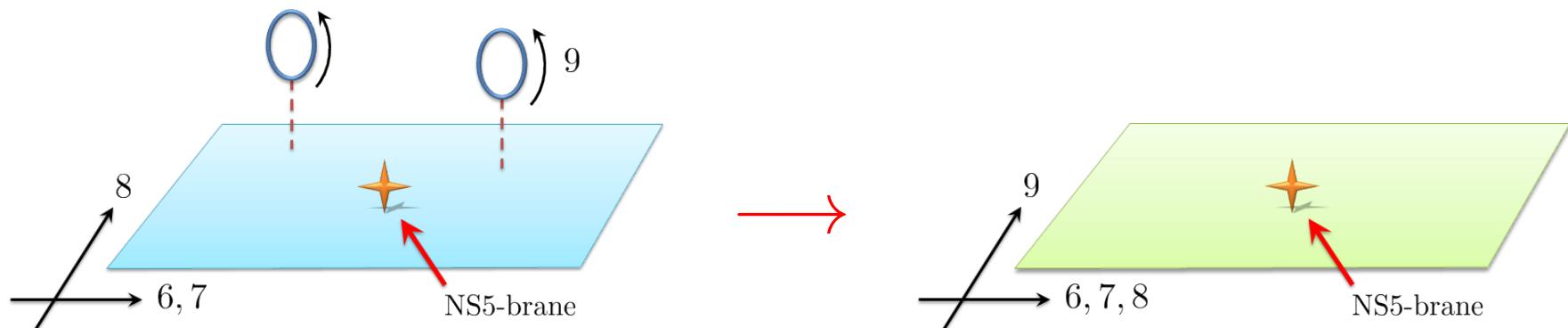
→ unfolding effect on compactified circle X^9



Worldsheet instanton corrections to NS5-brane :

$$X^9 F_{01} \text{ in GLSM}$$

→ unfolding effect on compactified circle X^9



$$\begin{aligned} H &= \frac{1}{g^2} + \frac{1}{r} \rightarrow \frac{1}{g^2} + \frac{1}{r} \sum_{n=1}^{\infty} e^{-nr} [e^{+i n X^9} + e^{-i n X^9}] \\ &= \frac{1}{g^2} + \frac{1}{r} \frac{\sinh(r)}{\cosh(r) - \cos(X^9)} \end{aligned}$$

D. Tong hep-th/0204186

► KK-monopole

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{r} \right)^{-1} : \text{radius of } \tilde{X}^9$$

$g \rightarrow 0$ case	general r	$g \rightarrow \infty$ case	$r \rightarrow 0$	$r \rightarrow \infty$
radius of \tilde{X}^9	0	radius of \tilde{X}^9	0	∞
KK-modes	heavy	KK-modes	heavy	light
winding modes	light	winding modes	light	heavy

► KK-monopole

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GLSM for KK-monopole has $\varepsilon^{mn} \partial_m (\tilde{X}^9 A_n)$

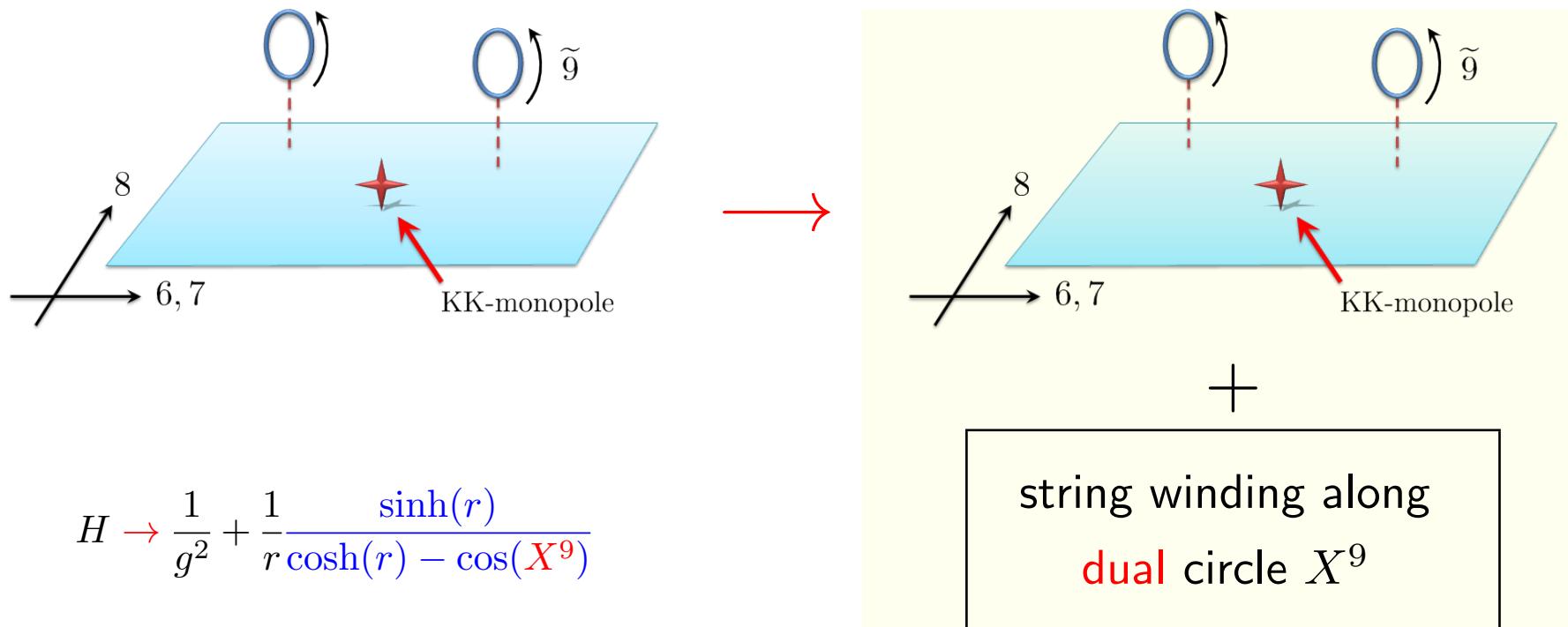


Winding mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to KK-monopole :

$$\varepsilon^{mn} \partial_m (\textcolor{blue}{X^9} A_n) \text{ in GLSM}$$

→ string **winding modes** along X^9



J. Harvey and S. Jensen [hep-th/0507204](#); K. Okuyama [hep-th/0508097](#)

► 5_2^2 -brane

$$\frac{H}{K} : \text{radius of } \tilde{X}^9$$

$$H = \frac{1}{g^2} + \sigma \log\left(\frac{\Lambda}{\varrho}\right), \quad K = H^2 + (\sigma \vartheta_\varrho)^2$$

$g \rightarrow 0$ case	general ϱ	$g \rightarrow \infty$ case	$\varrho \rightarrow 0$	$\varrho \rightarrow \Lambda$
radius of \tilde{X}^9	0	radius of \tilde{X}^9	0	∞ at $\vartheta_\varrho = 0$
KK-modes	heavy	KK-modes	heavy	light at $\vartheta_\varrho = 0$
winding modes	light	winding modes	light	heavy at $\vartheta_\varrho = 0$

► 5_2^2 -brane

$$\frac{H}{K} : \text{radius of } \tilde{X}^9$$

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$g \rightarrow 0$ case	general ϱ	$g \rightarrow \infty$ case	$\varrho \rightarrow 0$	$\varrho \rightarrow \Lambda$
radius of \tilde{X}^9	0	radius of \tilde{X}^9	0	∞ at $\vartheta_\varrho = 0$
KK-modes	heavy	KK-modes	heavy	light at $\vartheta_\varrho = 0$
winding modes	light	winding modes	light	heavy at $\vartheta_\varrho = 0$

GLSM for 5_2^2 with one gauged isometry has $\varepsilon^{mn} \partial_m (\tilde{X}^9 A_n)$

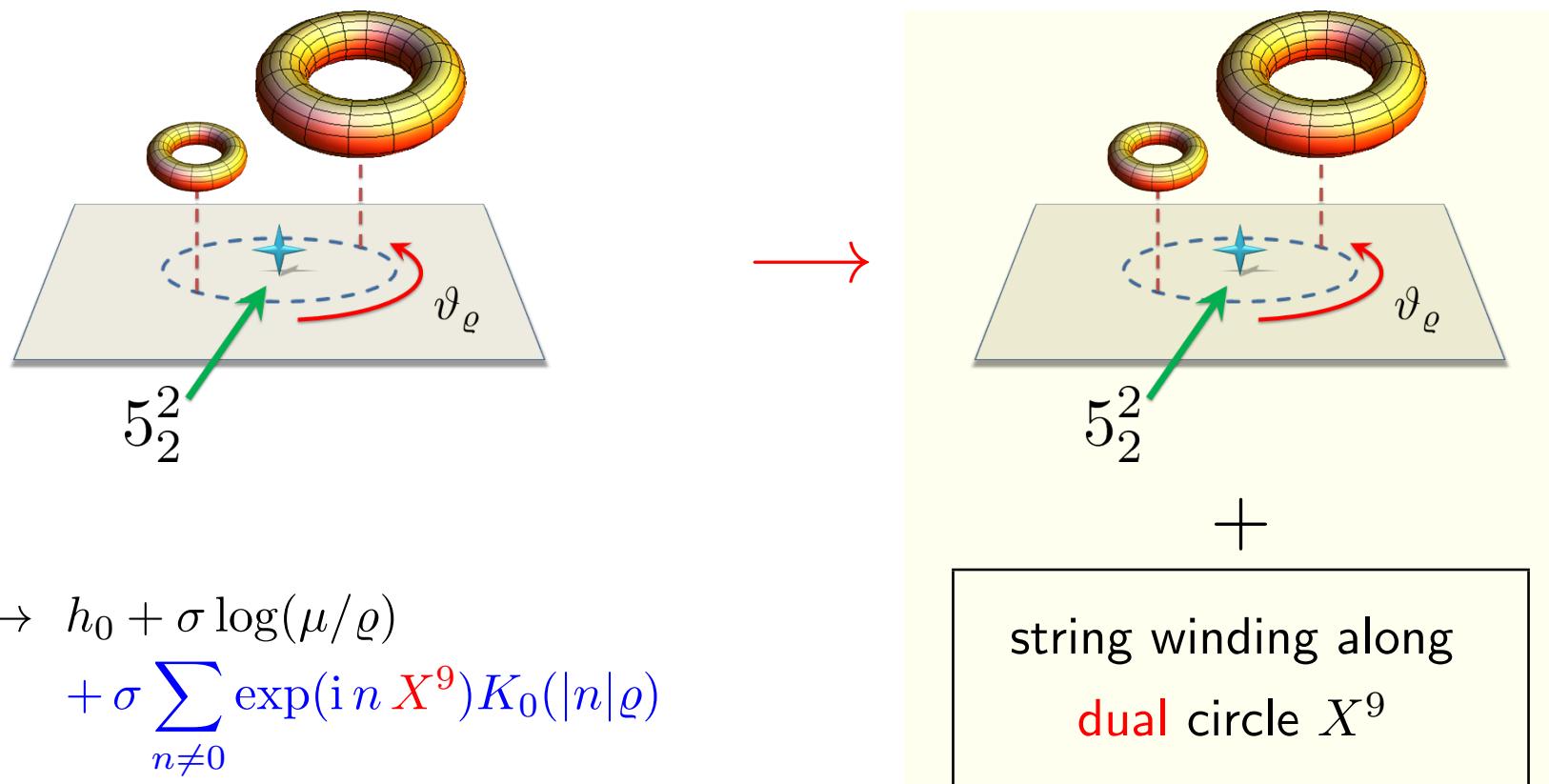


Winding mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to 5_2^2 -brane with **one** gauged isometry :

$$\varepsilon^{mn} \partial_m (\textcolor{blue}{X^9} A_n) \text{ in GLSM}$$

→ string **winding modes** along X^9



S. Sasaki and TK arXiv:1305.4439

$$\Psi = (X^6 + iX^8) + i\theta^+\chi_+ + i\theta^-\chi_- + i\theta^+\theta^-G + \dots$$

$$\Xi = (\tilde{X}^6 + i\tilde{X}^8) + i\theta^+\bar{\xi}_+ + i\bar{\theta}^-\xi_- + i\theta^+\bar{\theta}^-G_\Xi + \dots$$

$$\Theta = (X^7 + iX^9) + i\theta^+\bar{\tilde{\chi}}_+ + i\bar{\theta}^-\tilde{\chi}_- + i\theta^+\bar{\theta}^-\tilde{G} + \dots$$

$$\Gamma = (\tilde{X}^7 + i\tilde{X}^9) + i\theta^+\zeta_+ + i\theta^-\zeta_- + i\theta^+\theta^-G_\Gamma + \dots$$

$$Q_a = q + i\theta^+\psi_+ + i\theta^-\psi_- + i\theta^+\theta^-F + \dots$$

$$\tilde{Q}_a = \tilde{q} + i\theta^+\tilde{\psi}_+ + i\theta^-\tilde{\psi}_- + i\theta^+\theta^-\tilde{F} + \dots$$

$$\begin{aligned} V = & \theta^+\bar{\theta}^+(A_0 + A_1) + \theta^-\bar{\theta}^-(A_0 - A_1) - \theta^-\bar{\theta}^+\sigma - \theta^+\bar{\theta}^-\bar{\sigma} \\ & - i\theta^+\theta^-(\bar{\theta}^+\bar{\lambda}_+ + \bar{\theta}^-\bar{\lambda}_-) + i\bar{\theta}^+\bar{\theta}^-(\theta^+\lambda_+ + \theta^-\lambda_-) - \theta^+\theta^-\bar{\theta}^+\bar{\theta}^-D_V \end{aligned}$$

$$\Phi_a = \phi + i\theta^+\tilde{\lambda}_+ + i\theta^-\tilde{\lambda}_- + i\theta^+\theta^-D_\Phi + \dots = \overline{D}_+\overline{D}_-C$$

$$\begin{aligned} C = & \phi_c + i\theta^+\psi_{c+} + i\theta^-\psi_{c-} + i\bar{\theta}^+\chi_{c+} + i\bar{\theta}^-\chi_{c-} \\ & + i\theta^+\theta^-F_c + i\bar{\theta}^+\bar{\theta}^-M_c + i\theta^+\bar{\theta}^-G_c + i\bar{\theta}^+\theta^-N_c + \theta^-\bar{\theta}^-A_{c=} + \theta^+\bar{\theta}^+B_{c+} \\ & - i\theta^+\theta^-\bar{\theta}^+\zeta_{c+} - i\theta^+\theta^-\bar{\theta}^-\zeta_{c-} + i\bar{\theta}^+\bar{\theta}^-\theta^+\lambda_{c+} + i\bar{\theta}^+\bar{\theta}^-\theta^-\lambda_{c-} - \theta^+\theta^-\bar{\theta}^+\bar{\theta}^-D_c \end{aligned}$$

with $\overline{D}_\pm = -\frac{\partial}{\partial\bar{\theta}^\pm} + i\theta^\pm(\partial_0 \pm \partial_1)$, $D_\pm = \frac{\partial}{\partial\theta^\pm} - i\bar{\theta}^\pm(\partial_0 \pm \partial_1)$