Gauged Linear Sigma Model of Exotic Five-brane

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based on arXiv:1304.4061, 1305.4439, 1310.6163
佐々木伸氏(北里大学)との共同研究

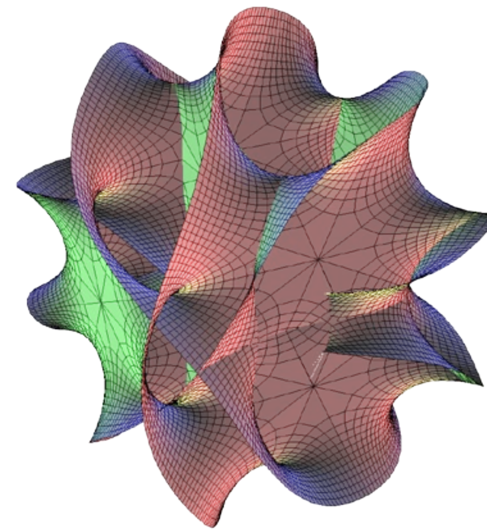
動機とまとめ

ヘテロティック弦理論において

仮定 $d\varphi = 0, \;\; H_3 = 0$ の下で

「４次元平坦時空に超対称ゲージ理論が出現すべし」とすると、

コンパクト化される6次元空間は Calabi-Yau 空間となる。



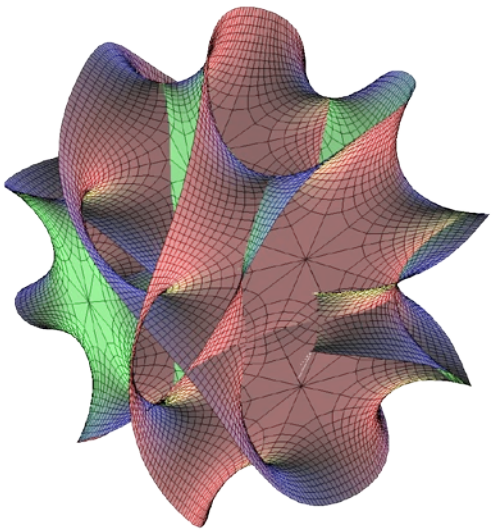
６次元 Calabi-Yau 空間

ヘテロティック弦理論において

仮定 $d\varphi = 0, \quad H_3 = 0$ の下で

「4次元平坦時空に超対称ゲージ理論が出現すべし」とすると、

コンパクト化される6次元空間は Calabi-Yau 空間となる。



6次元 Calabi-Yau 空間

この仮定は必要か？

仮定を外しても
4次元時空に超対称ゲージ理論が出現するとき、
6次元空間はどこまで一般化できるか？

ヘテロティック弦理論において

特別な仮定をせず

「４次元平坦時空に超対称ゲージ理論が出現すべし」とする。

コンパクト化される空間は…

ヘテロティック弦理論において

特別な仮定をせず

「4次元平坦時空に超対称ゲージ理論が出現すべし」とする。

コンパクト化される空間は…

P. Yi and TK, hep-th/0605247

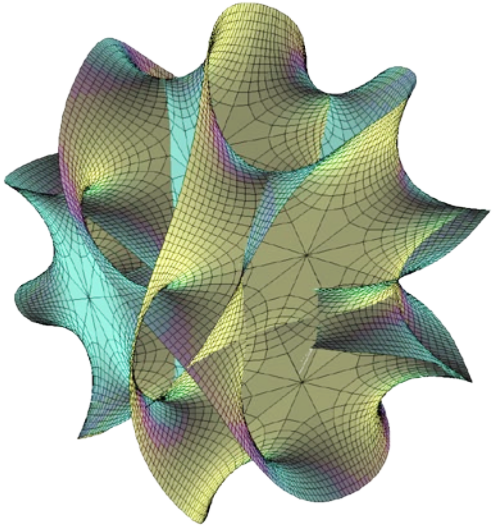
現在の研究の起点のひとつ

滑らかでコンパクトならば、

必ず $d\varphi = 0, H_3 = 0$ となり、Calabi-Yau 空間。

「滑らか」「コンパクト」のいずれかを緩めたら、

H_3 がトーションとなり、Calabi-Yau空間が変形。



６次元 非 Calabi-Yau 空間
(トーションあり)

テンソル場 B_{MN} が6次元空間に直接寄与する新しい幾何学

N.J. Hitchin "Generalized Geometry"

計量 G_{MN} ： 点粒子や弦の重心の運動量モード

テンソル場 B_{MN} ： 弦の巻き付きモード

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Generalized Geometry は、弦の T-duality を自然に含む

geometry associated with G_{MN}	Conventional Geometry (manifold) $O(6)$ symmetry
geometry associated with G_{MN}, B_{MN}	Generalized Geometry $O(6,6)$ T-duality symmetry

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弦の巻き付きが
時空構造に組み込まれる

C. Albertsson, R.A. Reid-Edwards and TK, arXiv:0806.1783

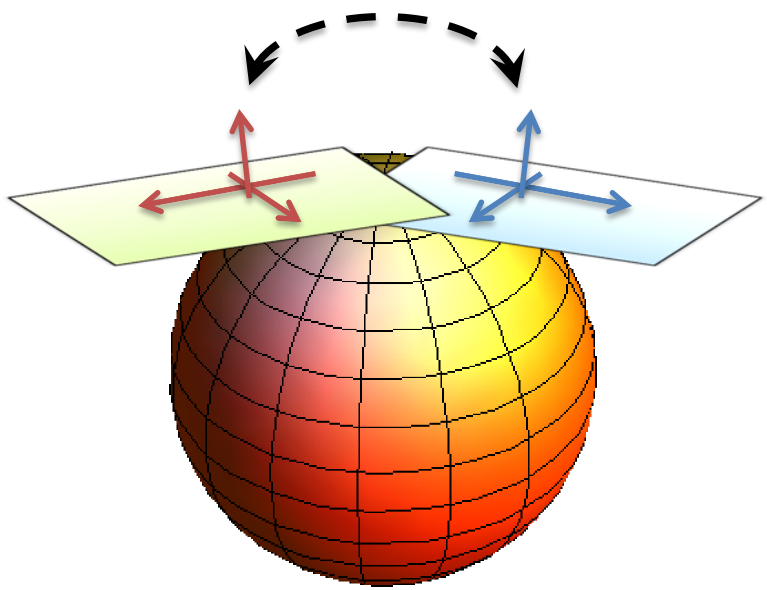
TK, arXiv:0810.0937

M. Hatsuda and TK, arXiv:1203.5499

Generalized Geometry は非常に一般的

具体性に欠ける

物理の探究に使いづらい



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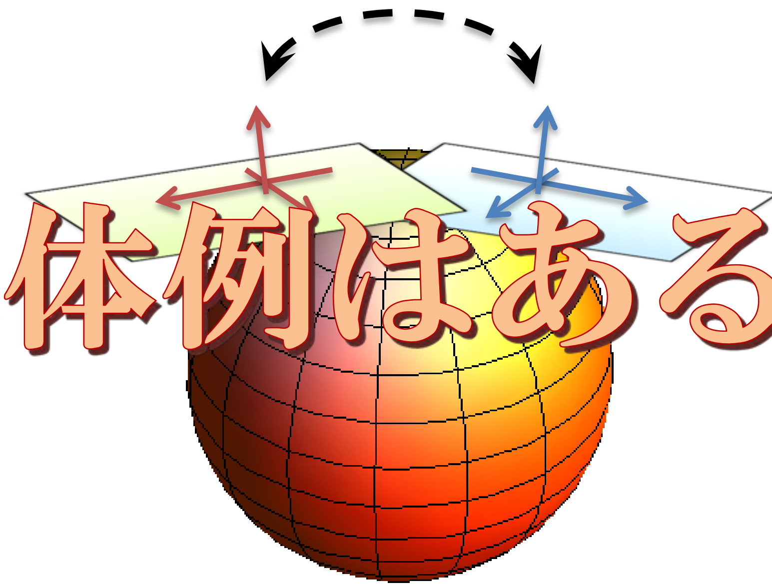


良い具体例はあるのか？

エキゾチック
ブレーン
特に 5^2_2-brane

木村哲士

5_2^2-brane は

NS5-brane を T-duality 変換することで得られる

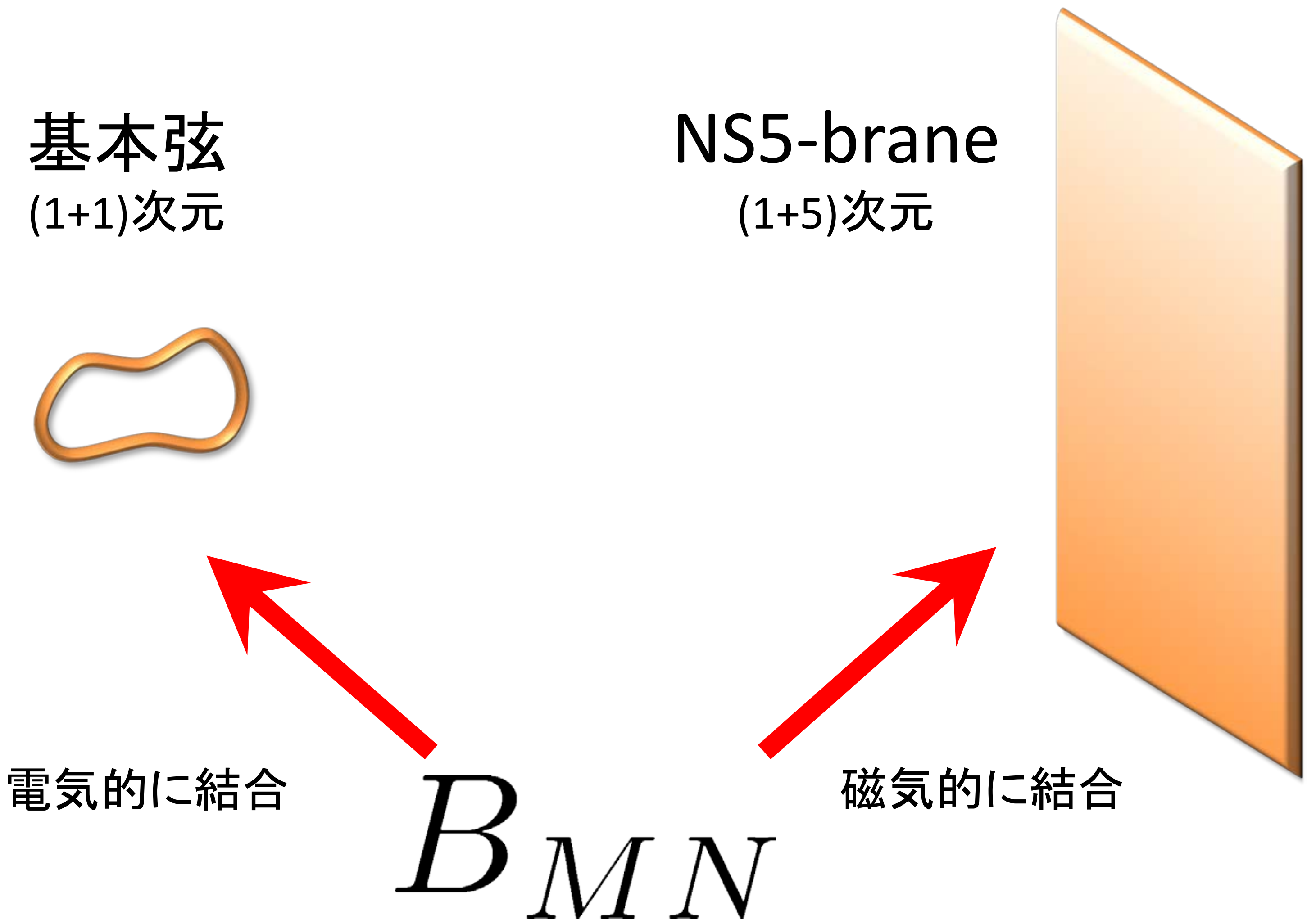
奇妙な物体である

5_2^2-brane 周囲の時空計量が

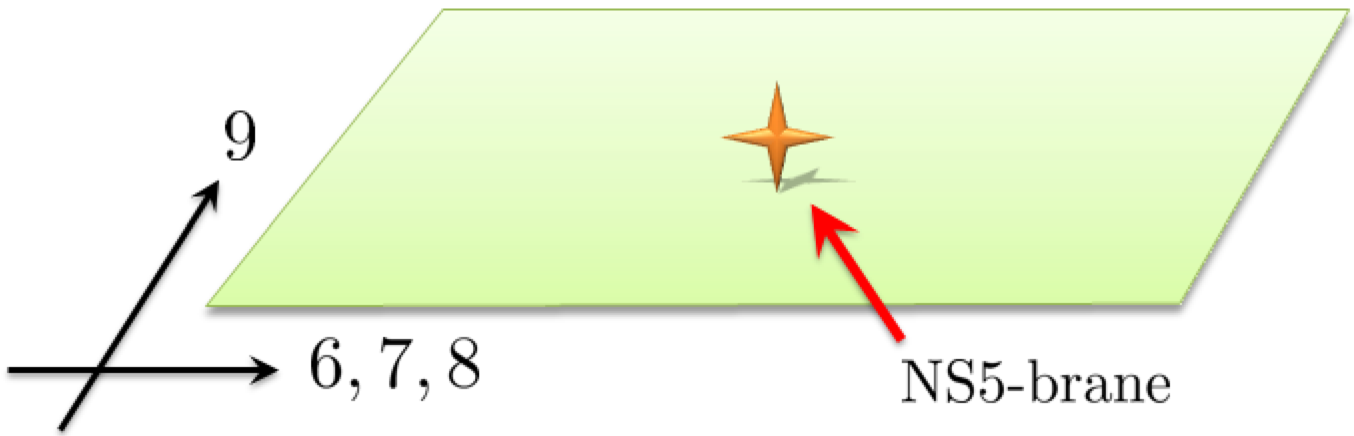
一価関数で表現できない

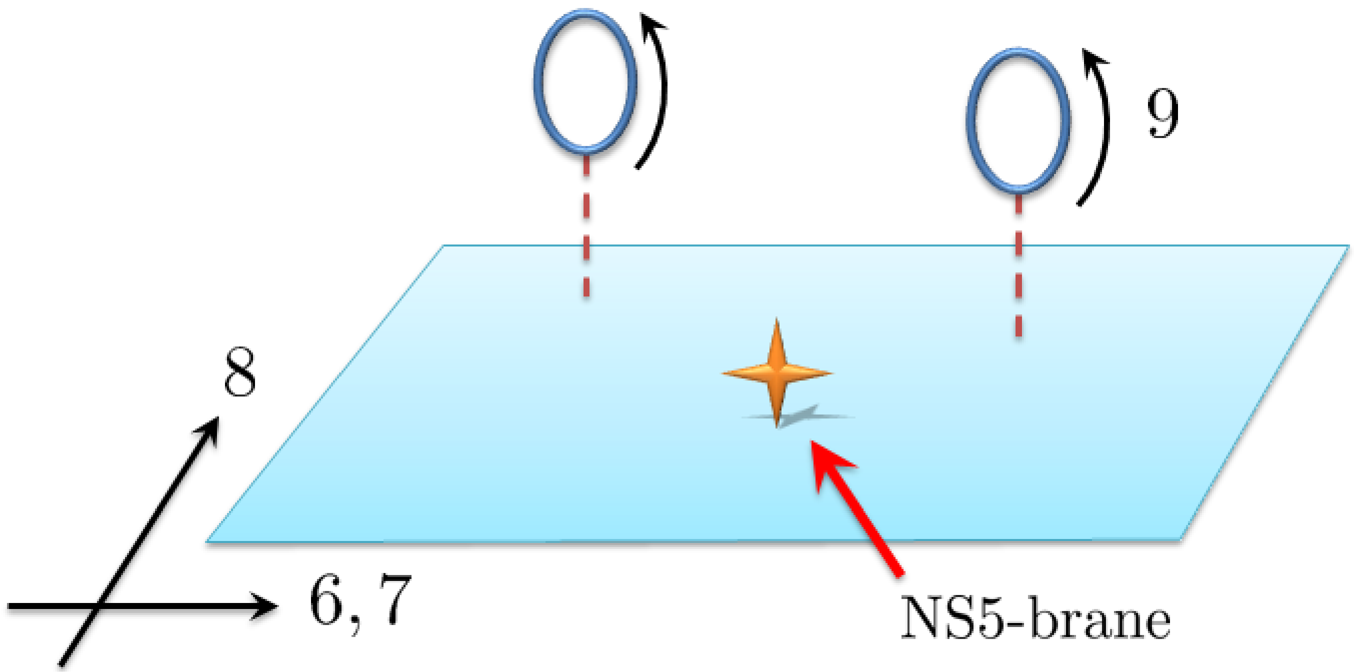
弦の巻き付き自由度が時空構造を深化させている

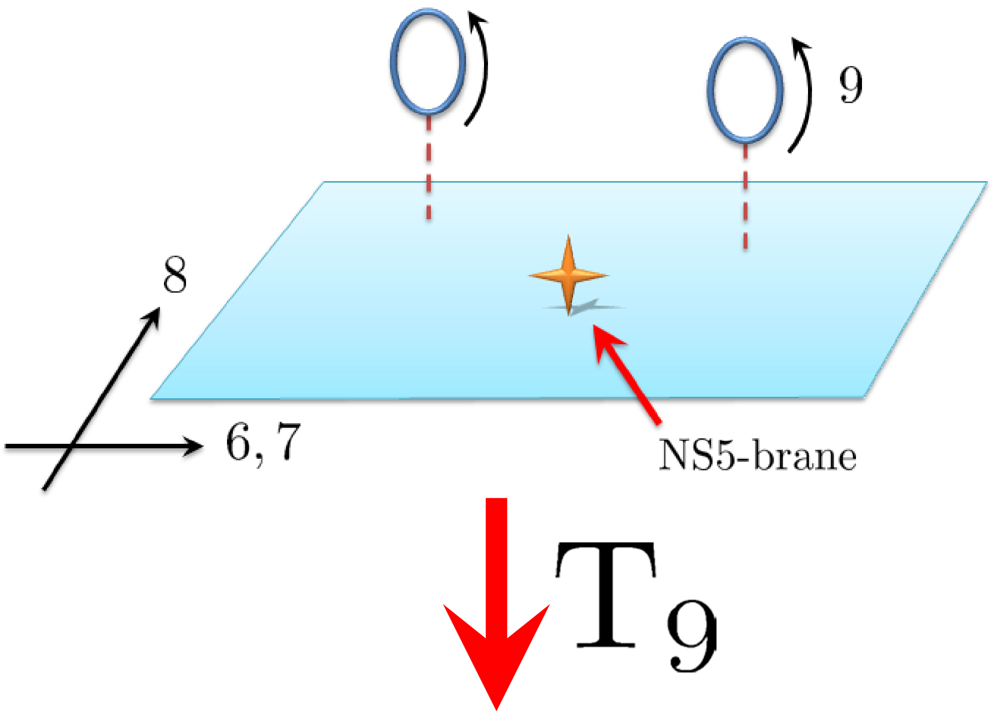
Generalized Geometry の具体例！

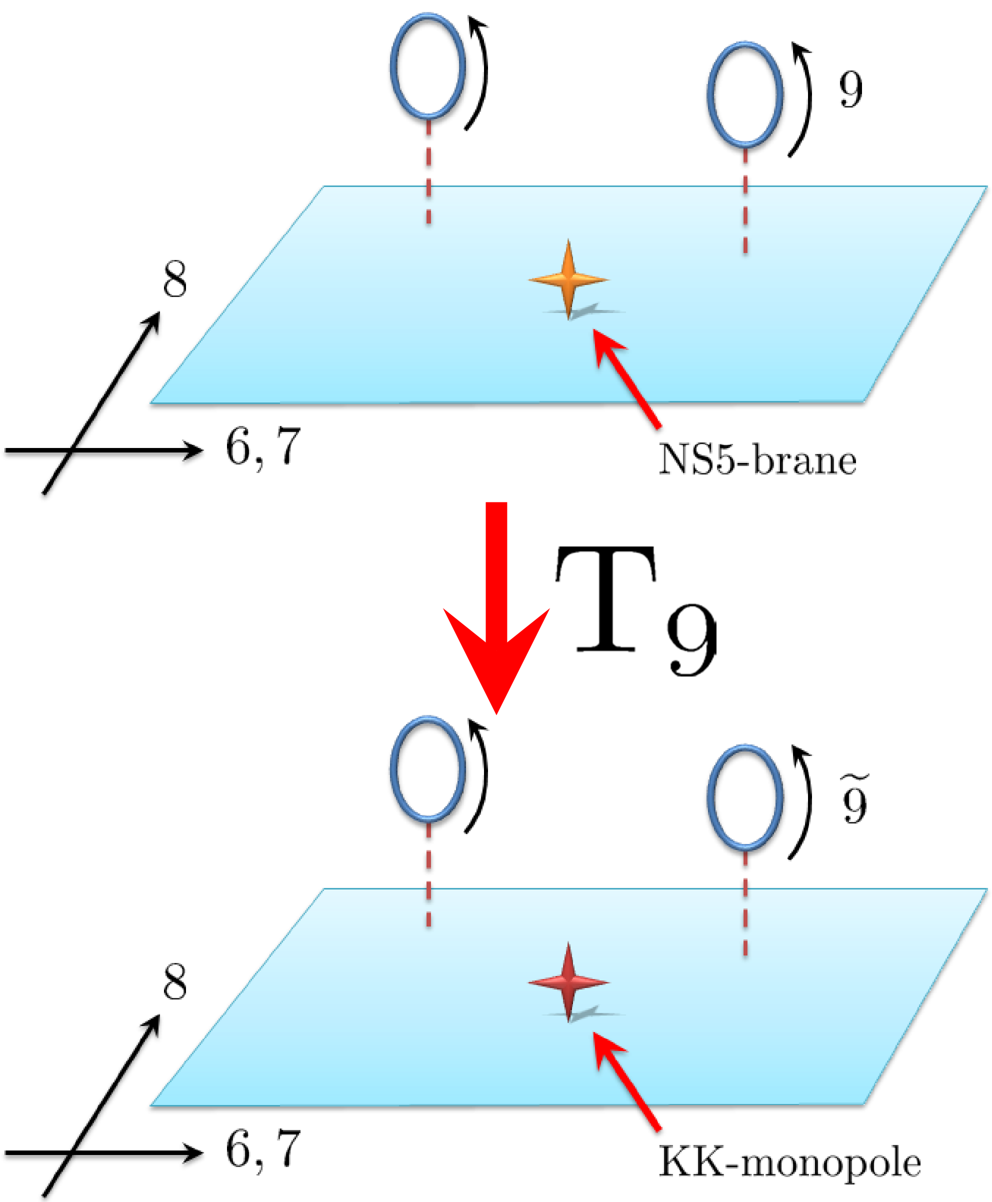

基本弦
(1+1)次元
NS5-brane
(1+5)次元
電気的に結合
B_{MN}
磁気的に結合

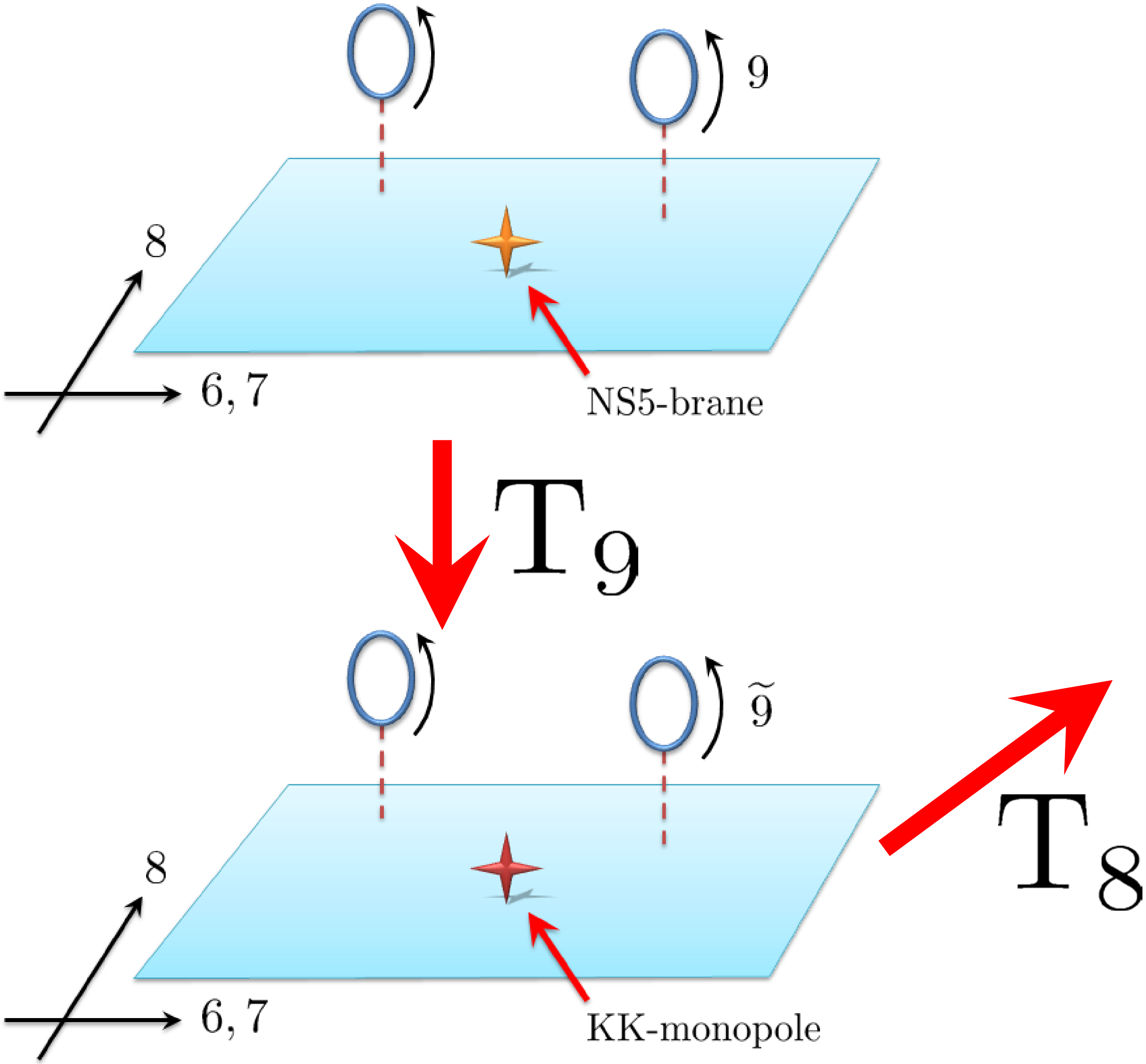
NS5-brane に垂直な4方向に着目する




9
8
6, 7
NS5-brane


9
8
6, 7
NS5-brane
T9


9
8
6, 7
NS5-brane
T_9
$\tilde{9}$
8
6, 7
KK-monopole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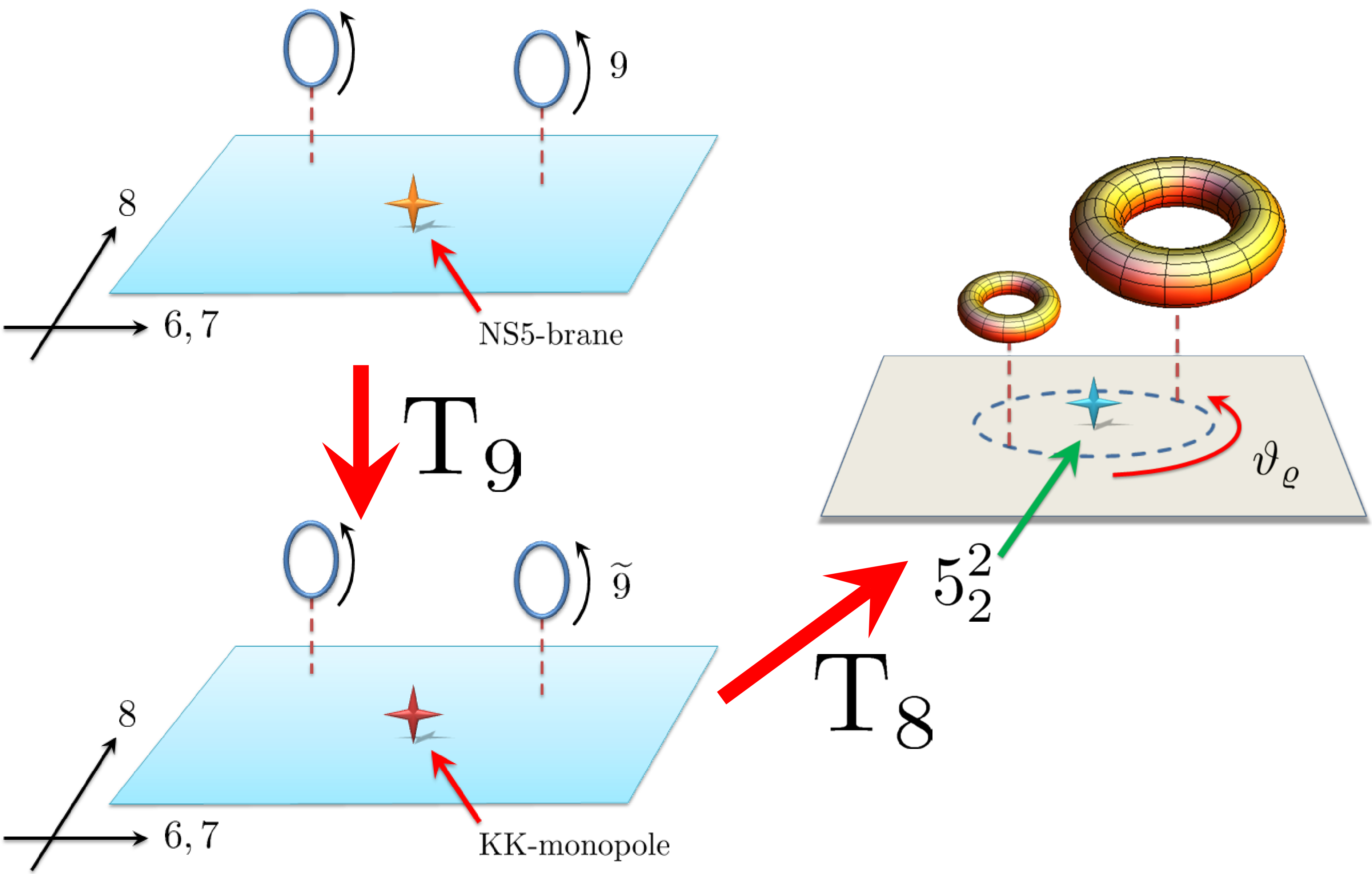
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8
6, 7
KK-monopole
T_8
5^2_2
ϑ_ϱ

木村哲士

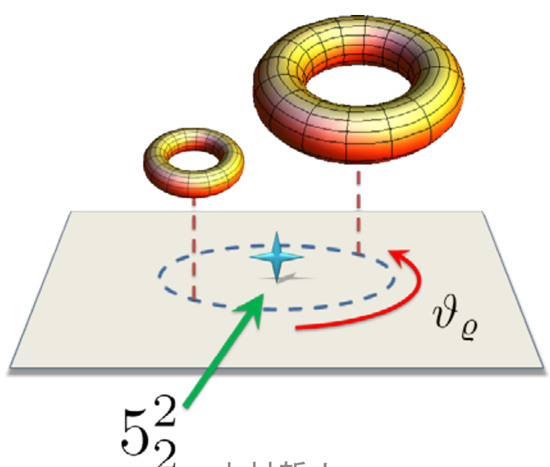
$$\mathrm{d}s^2 = \mathrm{d}x^2_{012345} + H\Big[\mathrm{d}\varrho^2 + \varrho^2(\mathrm{d}\vartheta_\varrho)^2\Big] + \frac{H}{K}\Big[(\mathrm{d}y^8)^2 + (\mathrm{d}y^9)^2\Big]$$

$$B_{89} = -\frac{\sigma\vartheta_\varrho}{K}\,, \qquad e^{2\Phi} = \frac{H}{K}$$

$$H = \frac{1}{g^2} + \sigma \log\frac{\Lambda}{\varrho}\,, \qquad K = H^2 + (\sigma\vartheta_\varrho)^2$$

5^2_2-brane の周りを一周しても元の位置に戻って来ない！

T-duality 変換 (計量 G_{MN} と反対称テンソル場 B_{MN} の混合) に起因



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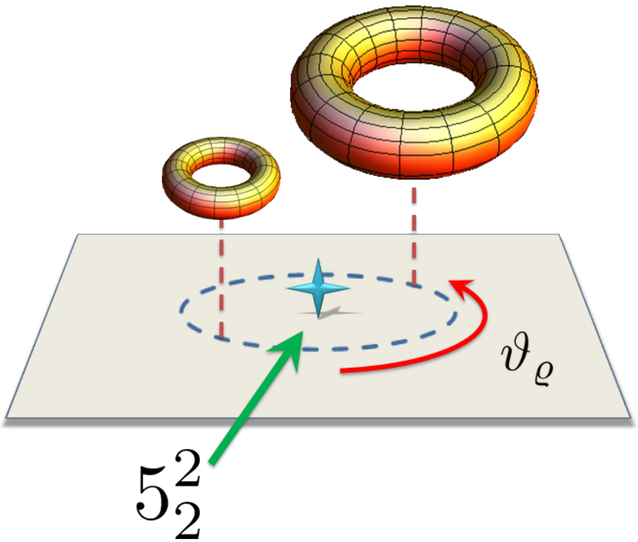
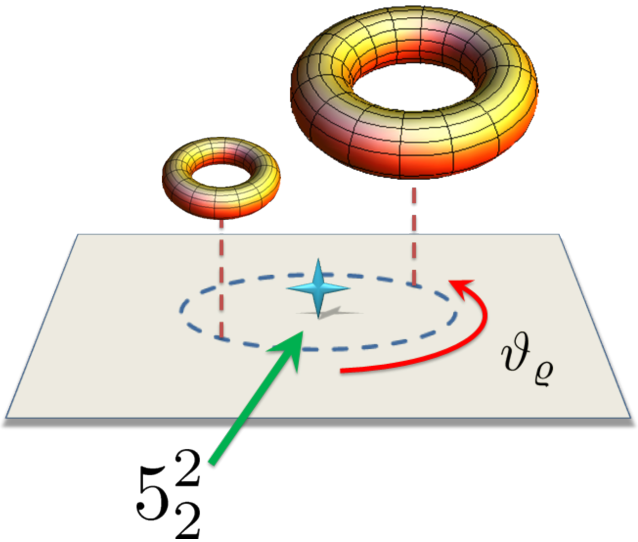


これまでは 5^2_2-brane を

超重力理論の枠内でしか考察されなかった

しかし「計量の多価性」は弦の巻き付きに起因

超重力理論は超弦理論の点粒子極限なので
弦の巻き付きが自然には扱えていない



これまでは 5^2_2-brane を

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弦理論特有の性質は
弦理論の言葉で追究すべき！

超重力理論は超弦理論の点粒子極限なので
弦の巻き付きが自然には扱えていない

2D Gauged Linear Sigma Model (GLSM) as String Worldsheet Theory

S. Sasaki and TK

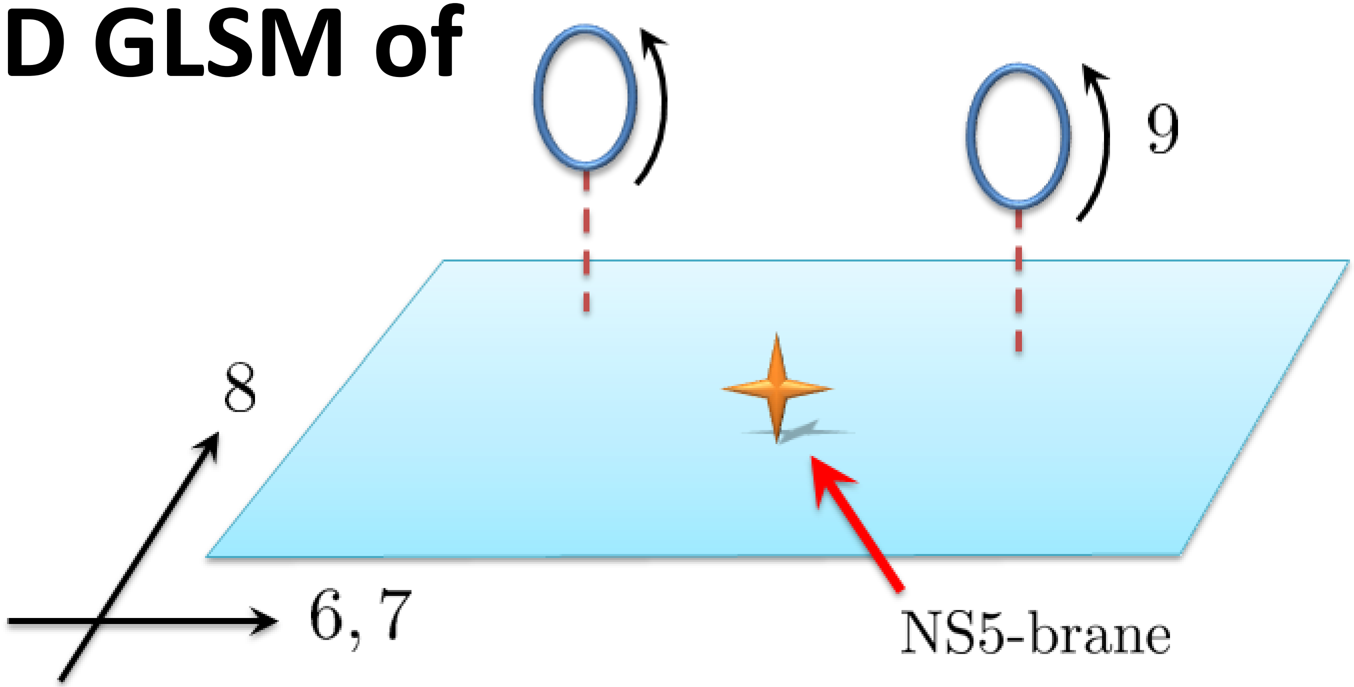
arXiv:1304.4061, 1305.4439, 1310.6361

非線形で多価な時空計量の性質を

２次元の線形な超対称ゲージ理論で記述する

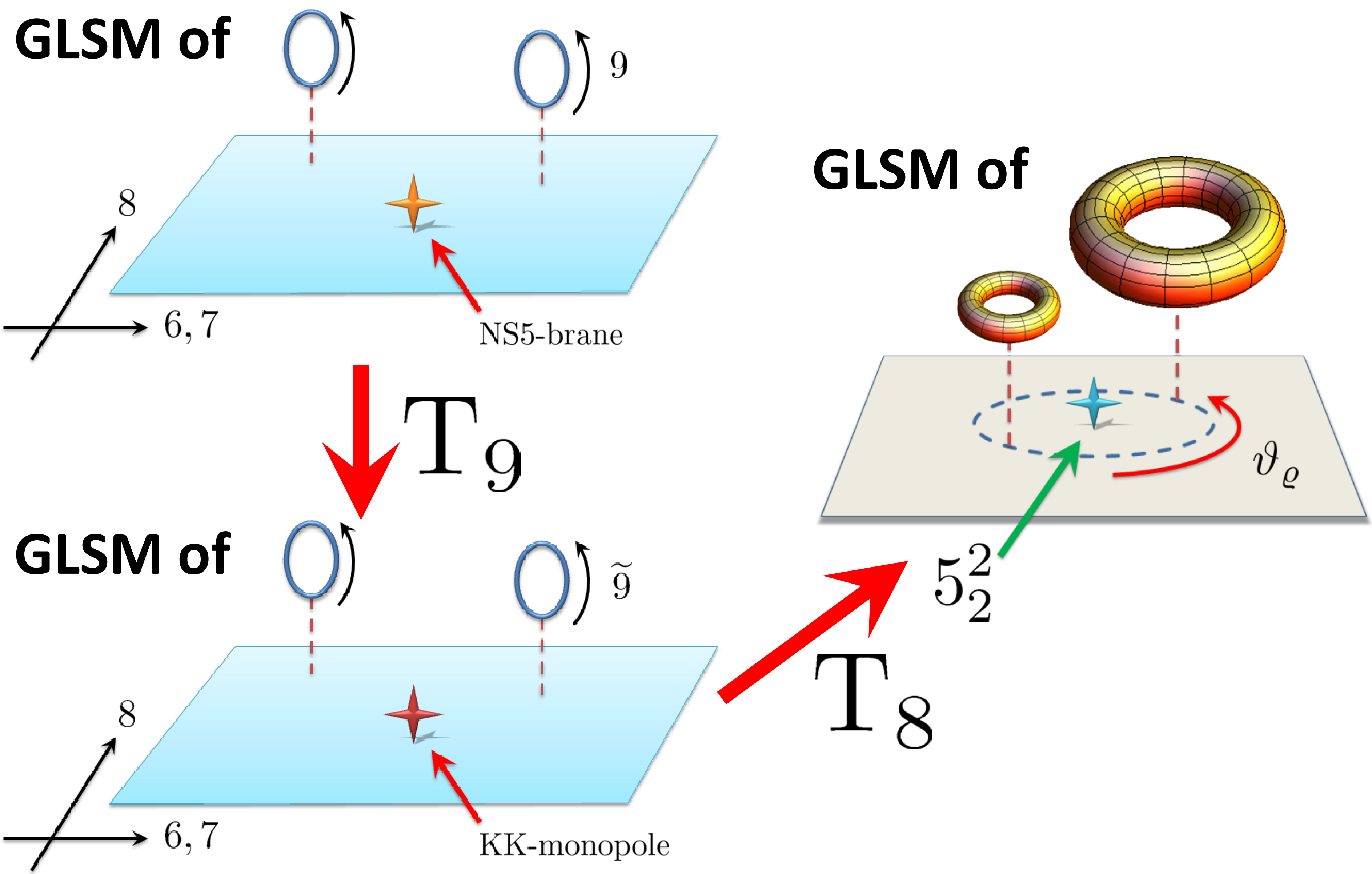
次の模型を起点とする

2D GLSM of



by D. Tong

T-duality 変換 ＝ Legendre 変換 in 2D


GLSM of
9
8
6, 7
NS5-brane
T9
GLSM of
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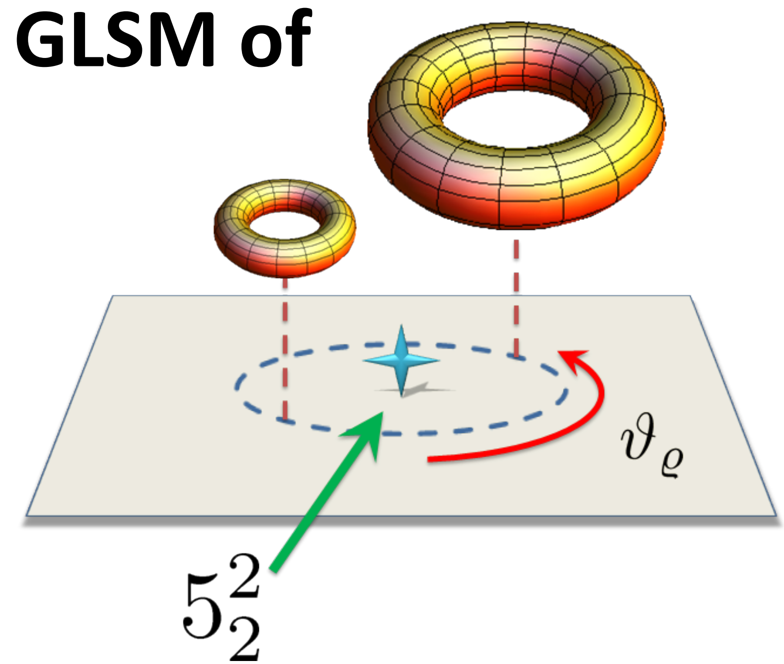
KK-monopole
GLSM of
ϑ_ϱ
5^2_2
T8

理解できたこと

✓ 計量 G_{MN} とテンソル場 B_{MN} の振る舞いを
2Dゲージ場と荷電カイラル超場で記述

⇨ 2D場の理論として自然な振舞いで記述できた

✓ 5^2_2-brane に対する弦理論のインスタントン補正が
2Dゲージ場のボーテックス配位で記述できた



理解できたこと

✓ 計量 G_{MN} とテンソル場 B_{MN} の振る舞いを
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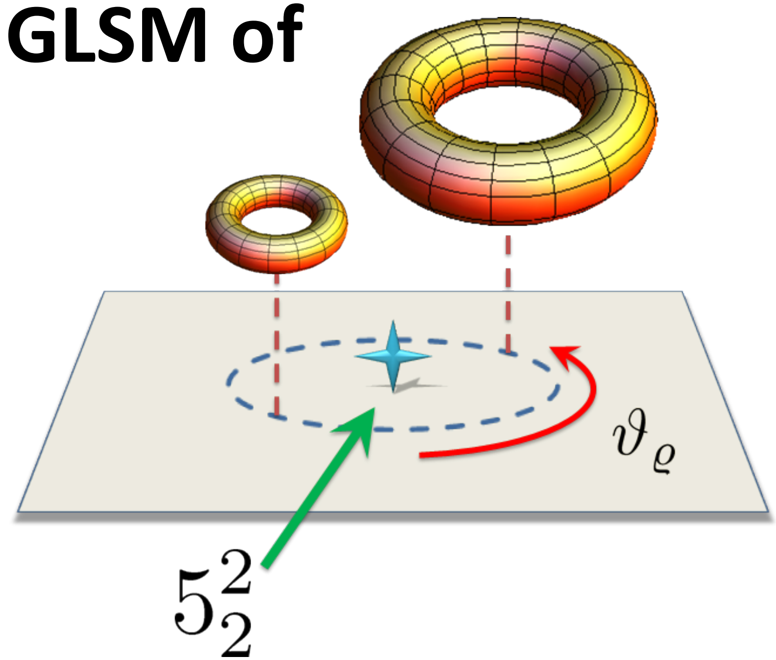
GLSM of



エキゾチックブレーンの理解が大きく前進!!

Contents

- GLSM of exotic 5^2_2-brane
- Quantum corrections
- Summary

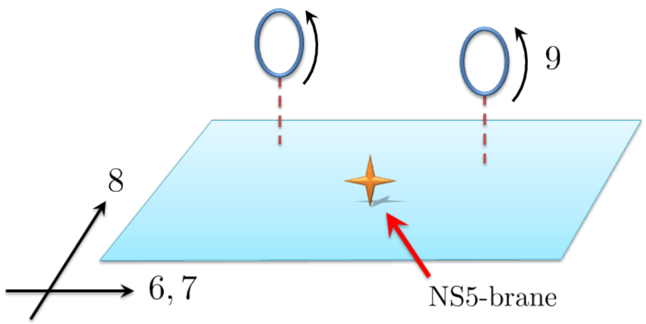
GLSM of 5^2_2-brane

Focus only on 4-transverse directions of five-branes.

S. Sasaki and TK arXiv:1304.4061

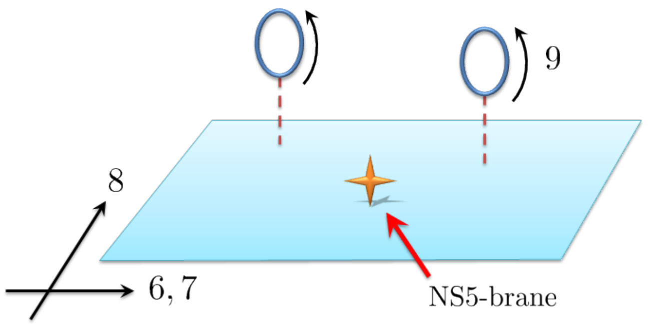
GLSM of 5^2_2-brane is constructed from GLSM of NS5-brane, as in SUGRA.

NS5



GLSM of 5^2_2-brane is constructed from GLSM of NS5-brane, as in SUGRA.

$$\mathrm{NS5} \xrightarrow{\mathrm{T}_9}$$



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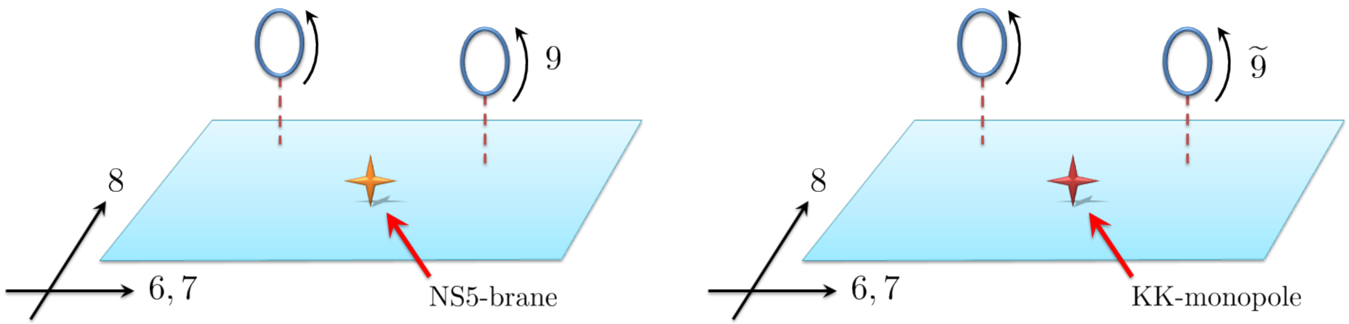
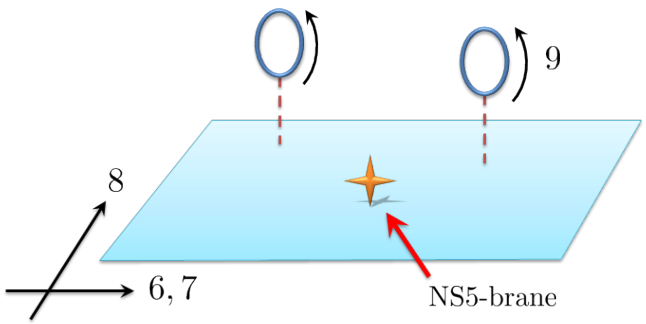
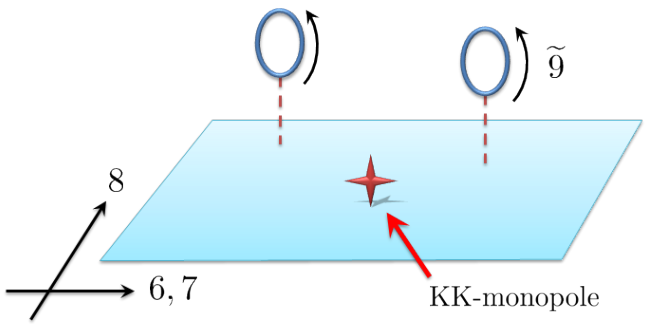
$$\mathrm{NS5} \xrightarrow{\ \mathrm{T}_9\ } \mathrm{KKM}$$



GLSM of 5^2_2-brane is constructed from GLSM of NS5-brane, as in SUGRA.

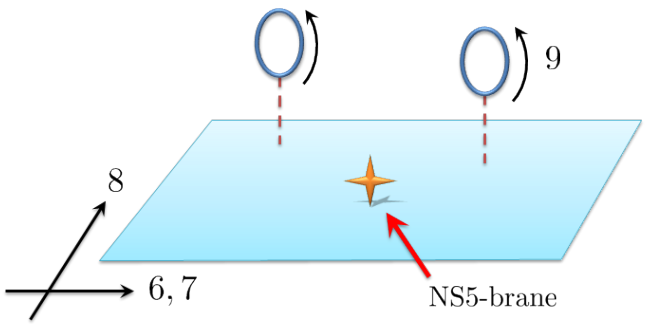
$$\mathrm{NS5} \xrightarrow{T_9} \mathrm{KKM} \xrightarrow{T_8}$$

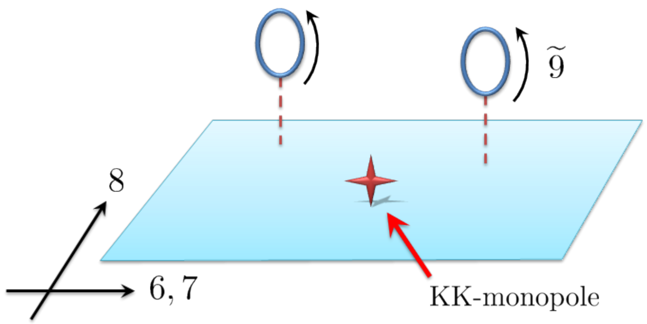


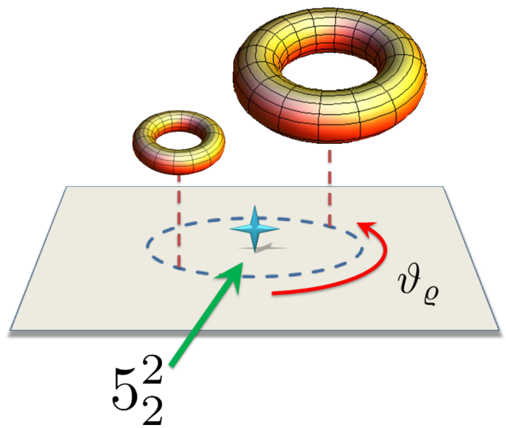


GLSM of 5^2_2-brane is constructed from GLSM of NS5-brane, as in SUGRA.

$$\mathrm{NS5} \xrightarrow{T_9} \mathrm{KKM} \xrightarrow{T_8} 5^2_2$$







$$\mathscr{L}_{\text{NS5}} = + \int d^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

GLSM of NS5-brane

D. Tong hep-th/0204186

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$	role
neutral HM	chiral $\Psi = X^6 + \mathrm{i}\, X^8 + \ldots$ twisted chiral $\Theta = X^7 + \mathrm{i}\, X^9 + \ldots$	spacetime coord.

$$\mathscr{L}_{\text{NS5}} = \int d^4\theta \left\{ \frac{1}{e^2}\Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) \qquad \right\} + \int d^4\theta \, \frac{1}{g^2}\Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \, \Big(\qquad + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \int d^2\widetilde{\theta} \, (t - \Theta)\Sigma + (\text{h.c.})$$

GLSM of NS5-brane

D. Tong hep-th/0204186

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		role
neutral HM	chiral $\Psi = X^6 + \mathrm{i}\, X^8 + \ldots$	twisted chiral $\Theta = X^7 + \mathrm{i}\, X^9 + \ldots$	spacetime coord.
VMs	twisted chiral $\Sigma = \overline{D}_+ D_- V$	chiral Φ	gauging isometry
FI parameters	$s = s^6 + \mathrm{i}\, s^8$	$t = t^7 + \mathrm{i}\, t^9$	position of five-branes

$$\mathscr{L}_{\mathsf{NS5}} = \int d^4\theta \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \Big\} + \int d^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \int d^2\widetilde{\theta} \, (t - \Theta)\Sigma + (\text{h.c.})$$

GLSM of NS5-brane

D. Tong hep-th/0204186

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		role
neutral HM	chiral $\Psi = X^6 + i\, X^8 + \ldots$	twisted chiral $\Theta = X^7 + i\, X^9 + \ldots$	spacetime coord.
VMs	twisted chiral $\Sigma = \overline{D}_+ D_- V$	chiral Φ	gauging isometry
charged HM	chiral Q $(+)$	chiral $\widetilde{Q}$ $(-)$	curving geometry
FI parameters	$s = s^6 + i\, s^8$	$t = t^7 + i\, t^9$	position of five-branes

Bosonic Lagrangian after integrating-out auxiliary fields :

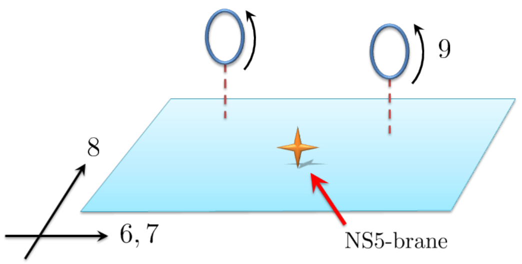
$$\mathscr{L}^{\text{kin}}_{\text{NS5}} = \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big]$$
$$-\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2\Big] - (X^9 - t^9)\, F_{01}$$

$$\mathscr{L}^{\text{pot}}_{\text{NS5}} = -2\big(|\sigma|^2 + |\phi|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big)$$
$$-\frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2$$

Steps to NLSM of NS5-brane

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, X^9)$

$$
\begin{aligned}
\mathscr{L}^{\text{NLSM}}_{\text{NS5}} &= -\frac{1}{2}\Big(\frac{1}{g^2} + \frac{1}{r}\Big)\Big[(\partial_m \vec{X})^2 + (\partial_m X^9)^2\Big] + \varepsilon^{mn}\,\Omega_i\,\partial_m X^i\,\partial_n X^9 \\
&= -\frac{1}{2} G_{IJ}\,\partial_m X^I\,\partial^m X^J + \frac{1}{2} B_{IJ}\,\varepsilon^{mn} \partial_m X^I\,\partial_n X^J
\end{aligned}
$$

Target space geometry is...

$$
\begin{aligned}
G_{IJ} &= H\,\delta_{IJ}\,, \qquad & B_{i9} &= \Omega_i \\
H &= \frac{1}{g^2} + \frac{1}{r}\,, \qquad & \Omega_i &= ...(skip)
\end{aligned}
$$

$$\mathscr{L}_{\text{NS5}} = \int \mathrm{d}^4\theta \left\{ \frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\,\mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\,\mathrm{e}^{-2V}\widetilde{Q} \right\} + \int \mathrm{d}^4\theta\, \frac{1}{g^2}\Big(-\overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int \mathrm{d}^2\theta\, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \int \mathrm{d}^2\widetilde{\theta}\, (t - \Theta)\Sigma + (\text{h.c.})$$

T-duality transformation to KK-monopole

$$\mathscr{L}_{\text{NS5}} = \int d^4\theta \left\{ \frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \right\} + \int d^4\theta\, \frac{1}{g^2}\Big(-\overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \left(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \right) + (\text{h.c.})$$

$$+ \int d^2\widetilde{\theta}\, (t - \Theta)\Sigma + (\text{h.c.})$$

T-duality transformation to KK-monopole

(free) string	sign flip (parity) in right-mover	momentum $\leftrightarrow$ winding
spacetime	Buscher rule	$(G_{IJ}, B_{IJ}) \to (G'_{IJ}, B'_{IJ})$
SUSY sigma model	Roček-Verlinde formula	chiral $\leftrightarrow$ twisted chiral

$$\mathscr{L}_{\text{NS5}} = \int d^4\theta \left\{ \frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \right\} + \int d^4\theta\, \frac{1}{g^2}\Big(-\overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \left(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \right) + \text{(h.c.)}$$

$$+ \int d^2\widetilde{\theta}\, (t - \Theta)\Sigma + \text{(h.c.)}$$

T-duality transformation to KK-monopole

Duality (Roček-Verlinde) transformation

$$-\frac{1}{g^2}(\Theta + \overline{\Theta}) = (\Gamma + \overline{\Gamma}) + V$$

$$\pm(\partial_0 \pm \partial_1) X^9 = -g^2 (D_0 \pm D_1) Y^9$$

$$D_m Y^9 = \partial_m Y^9 + A_m$$

$$
\begin{aligned}
\mathscr{L}_{\mathsf{KK}} \;=\; & \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\, \mathrm{e}^{-2V} \widetilde{Q} \Big\} + \int \mathrm{d}^4\theta \, \Big\{ \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2} \overline{\Psi}\Psi \Big\} \\
& + \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.}) \\
& + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, t\, \Sigma + (\text{h.c.}) \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)
\end{aligned}
$$

GLSM of KK-monopole

Bosonic Lagrangian after integrating-out auxiliary fields :

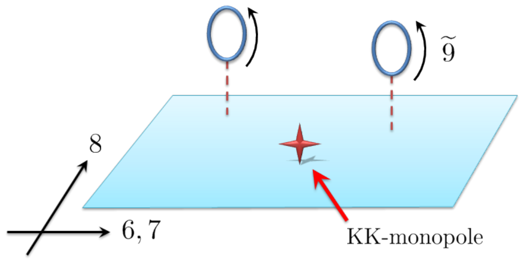
$$\mathscr{L}^{\text{kin}}_{\text{KK}} = \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big]$$
$$- \frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] - \frac{g^2}{2}(D_m Y^9)^2 - \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)$$

$$\mathscr{L}^{\text{pot}}_{\text{KK}} = -2\big(|\sigma|^2 + |\phi|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big)$$
$$- \frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2$$

Steps to NLSM of KK-monopole

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, Y^9)$

$$\mathscr{L}_{\text{KK}}^{\text{NLSM}} = -\frac{1}{2}H\Big[(\partial_m \vec{X})^2\Big] - \frac{1}{2}H^{-1}(\partial_m Y^9 - \Omega_i \, \partial_m X^i)^2$$

$$= -\frac{1}{2}G_{IJ}\,\partial_m X^I \,\partial^m X^J + \frac{1}{2}B_{IJ}\,\varepsilon^{mn}\partial_m X^I \,\partial_n X^J$$

Target space geometry is...

$$G_{ij} = H\,\delta_{ij}\,, \qquad G_{99} = H^{-1}\,, \qquad B_{IJ} = 0$$

$$H = \frac{1}{g^2} + \frac{1}{r}\,, \qquad \Omega_i = ...(skip)$$

$$\mathscr{L}_{\mathsf{KK}} = \int d^4\theta \, \Big\{ \frac{1}{e^2} \Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \Big\} + \int d^4\theta \, \Big\{ \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2} \overline{\Psi}\Psi \Big\}$$

$$+ \int d^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \Big\{ \int d^2\widetilde{\theta} \, t\, \Sigma + (\text{h.c.}) \Big\} - \varepsilon^{mn} \, \partial_m (X^9 A_n)$$

T-duality transformation to 5^2_2-brane!

$$\mathscr{L}_{\mathsf{KK}} = \int \mathrm{d}^4\theta \Big\{ \frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi\Big) + \overline{Q}\,\mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\,\mathrm{e}^{-2V}\widetilde{Q}\Big\} + \int \mathrm{d}^4\theta \Big\{ \frac{g^2}{2}\Big(\Gamma + \overline{\Gamma} + V\Big)^2 + \frac{1}{g^2}\overline{\Psi}\Psi\Big\}$$

$$+ \int \mathrm{d}^2\theta \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi\Big) + (\text{h.c.})$$

$$+ \Big\{ \int \mathrm{d}^2\widetilde{\theta}\, t\,\Sigma + (\text{h.c.})\Big\} - \varepsilon^{mn}\,\partial_m(X^9 A_n)$$

T-duality transformation to 5^2_2-brane!

Duality (Roček-Verlinde) transformation

$$-\frac{1}{g^2}\big(\Psi + \overline{\Psi}\big) = \big(\Xi + \overline{\Xi}\big) - \big(C + \overline{C}\big)$$

$$\Phi = \overline{D}_+ \overline{D}_- C \qquad \text{cf. } \Sigma = \overline{D}_+ D_- V$$

$$
\begin{aligned}
\mathscr{L}_{\mathsf{E}} \;=\; & \int \mathrm{d}^4\theta \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\, \mathrm{e}^{-2V} \widetilde{Q} \Big\} \\
& + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big\{ \Big(\Gamma + \overline{\Gamma} + V \Big)^2 - \Big(\Xi + \overline{\Xi} - (C + \overline{C}) \Big)^2 \Big\} \\
& + \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}\Phi Q + s\Phi \Big) + (\text{h.c.}) \Big\} - \int \mathrm{d}^4\theta \, (\Psi - \overline{\Psi})(C - \overline{C}) \\
& + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, t\, \Sigma + (\text{h.c.}) \Big\} - \varepsilon^{mn} \partial_m (X^9 A_n)
\end{aligned}
$$

GLSM of exotic 5^2_2-brane

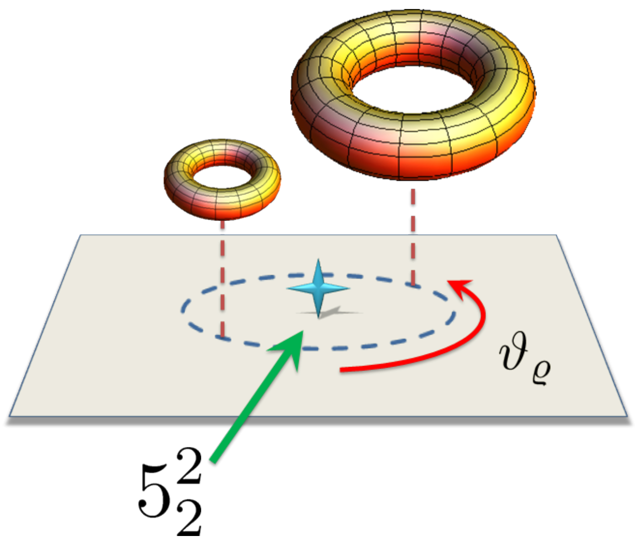
S. Sasaki and TK arXiv:1304.4061

Steps to NLSM of 5^2_2-brane

1. search SUSY vacua $\mathscr{L}^{\text{potential}} = 0$
2. solve constraints of charged HM $(Q, \widetilde{Q})$
3. integrate out VM (V, Φ) in IR

★4. integrate s^8, and solve EOM for T-dual field X^8

$$\mathscr{L}_{\text{exotic}} = -\frac{H}{2}\Big[(\partial_m \varrho)^2 + \varrho^2(\partial_m \vartheta_\varrho)^2\Big] - \frac{H}{2K}\Big[(\partial_m Y^8)^2 + (\partial_m Y^9)^2\Big] - \frac{\sigma\vartheta_\varrho}{K}\,\varepsilon^{mn}(\partial_m Y^8)(\partial_n Y^9) - \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)$$



We obtained GLSM/NLSM of Exotic Five-brane!

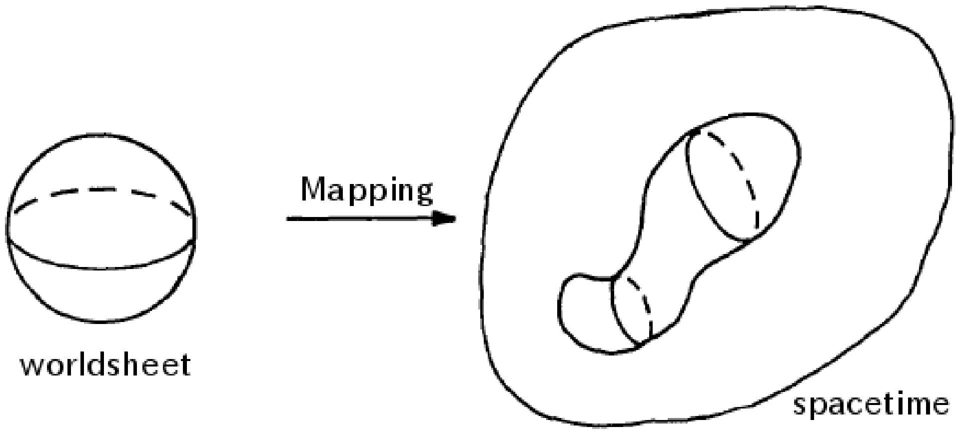
Quantum Corrections

S. Sasaki and TK arXiv:1305.4439

String Worldsheet Instanton Corrections

↓

Deform target space geometry by momentum and/or winding effects



GLSM is a powerful tool in this stage :

Worldsheet instantons in NLSM can be captured by
vortex solution in gauge theory

- NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{r} : \text{ radius of } X^9$$

$g \to 0$ case	general r
radius of X^9	∞
KK-modes	light
winding modes	heavy

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of X^9	∞	0
KK-modes	light	heavy
winding modes	heavy	light

- NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{r} \ : \ \text{radius of } X^9$$

$g \to 0$ case	general r
radius of X^9	∞
KK-modes	light
winding modes	heavy

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of X^9	∞	0
KK-modes	light	heavy
winding modes	heavy	light

GLSM of NS5-brane has $X^9 F_{01}$

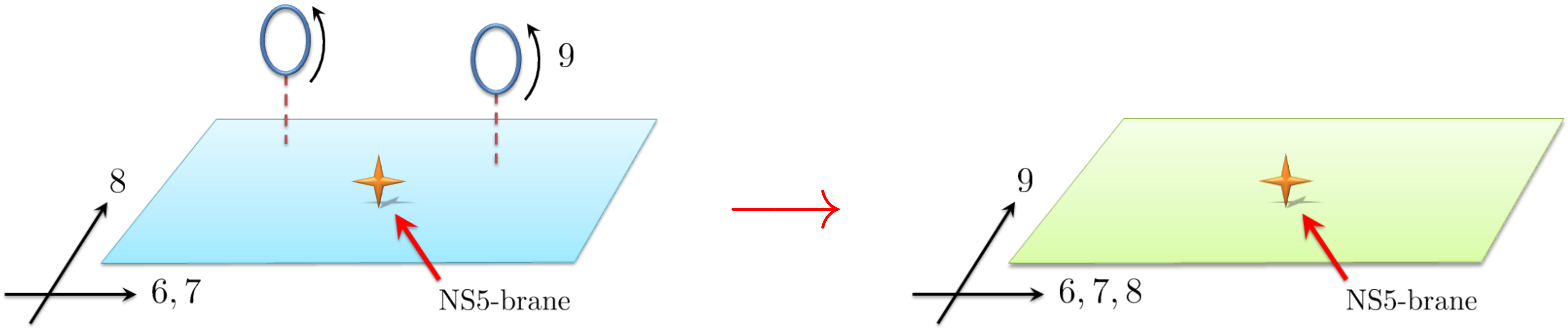
$\downarrow$

KK-mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to NS5-brane :

$X^9 F_{01}$ in GLSM

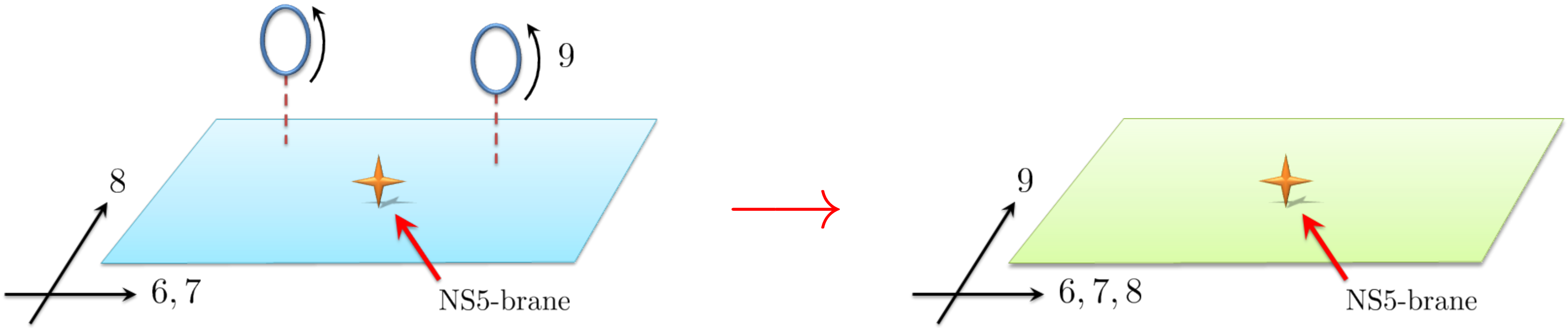
$\rightarrow$ unfolding effect on compactified circle X^9



Worldsheet instanton corrections to NS5-brane :

$X^9 F_{01}$ in GLSM

$\rightarrow$ unfolding effect on compactified circle X^9



$$H = \frac{1}{g^2} + \frac{1}{r} \rightarrow \frac{1}{g^2} + \frac{1}{r} \sum_{n=1}^{\infty} e^{-nr} \left[e^{+i\,nX^9} + e^{-i\,nX^9} \right]$$
$$= \frac{1}{g^2} + \frac{1}{r} \frac{\sinh(r)}{\cosh(r) - \cos(X^9)}$$

D. Tong hep-th/0204186

- KK-monopole

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{r}\right)^{-1} \text{ : radius of } Y^9$$

$g \to 0$ case	general r
radius of Y^9	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of Y^9	0	∞
KK-modes	heavy	light
winding modes	light	heavy

- KK-monopole

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{r}\right)^{-1} \ : \text{ radius of } Y^9$$

$g \to 0$ case	general r
radius of Y^9	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of Y^9	0	∞
KK-modes	heavy	light
winding modes	light	heavy

GLSM of KK-monopole has $\varepsilon^{mn}\partial_m(X^9 A_n)$

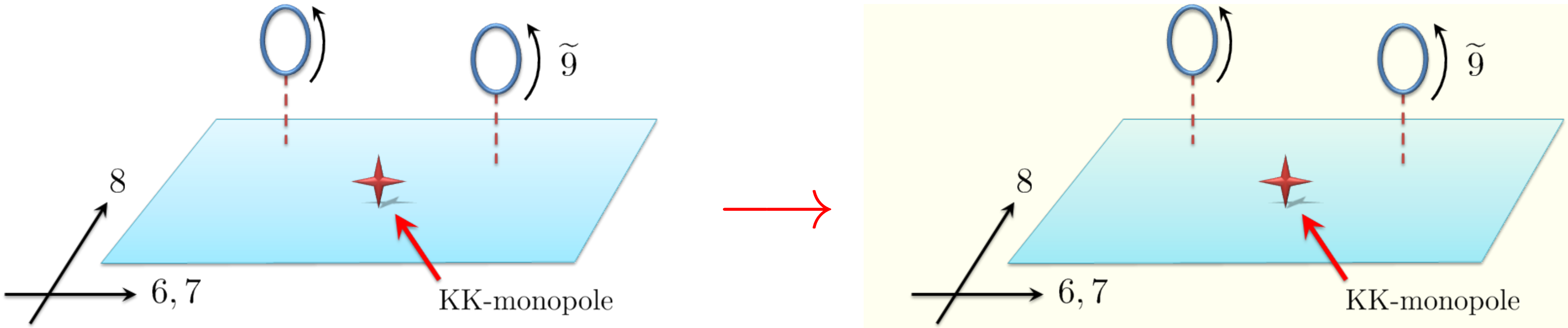
$\downarrow$

Winding mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to KK-monopole :

$\varepsilon^{mn}\,\partial_m(X^9 A_n)$ in GLSM

$\to$ string winding modes along X^9



$$H \to \frac{1}{g^2} + \frac{1}{r}\frac{\sinh(r)}{\cosh(r) - \cos(X^9)}$$

+

string winding along
dual circle X^9

J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097

- 5^2_2-brane

$$\frac{H}{K} : \text{ radius of } Y^9 \qquad H = \frac{1}{g^2} + \sigma \log\left(\frac{\Lambda}{\varrho}\right), \; K = H^2 + (\sigma \vartheta_{\varrho})^2$$

$g \to 0$ case	general ϱ
radius of Y^9	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\varrho \to 0$	$\varrho \to \Lambda$	
radius of Y^9	0	0	∞ at $\vartheta_{\varrho} = 0$
KK-modes	heavy	heavy	light at $\vartheta_{\varrho} = 0$
winding modes	light	light	heavy at $\vartheta_{\varrho} = 0$

- 5^2_2-brane

$$\frac{H}{K} : \text{ radius of } Y^9 \qquad H = \frac{1}{g^2} + \sigma \log\left(\frac{\Lambda}{\varrho}\right), \; K = H^2 + (\sigma \vartheta_\varrho)^2$$

$g \to 0$ case	general ϱ
radius of Y^9	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\varrho \to 0$	$\varrho \to \Lambda$	
radius of Y^9	0	0	∞ at $\vartheta_\varrho = 0$
KK-modes	heavy	heavy	light at $\vartheta_\varrho = 0$
winding modes	light	light	heavy at $\vartheta_\varrho = 0$

GLSM of 5^2_2 with one gauged isometry has $\varepsilon^{mn} \partial_m (X^9 A_n)$

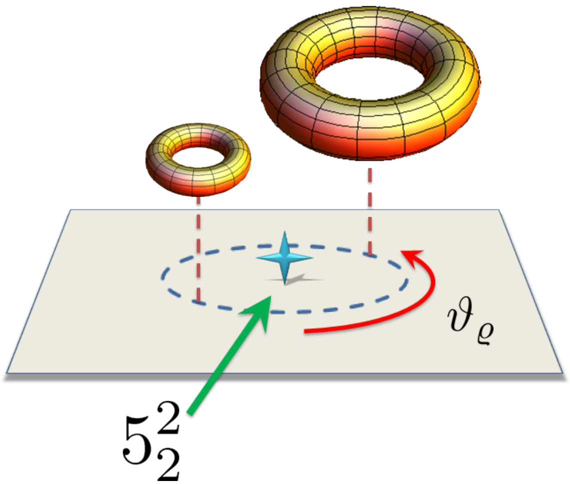
$\downarrow$

Winding mode corrections can be captured by vortex solution in gauge theory

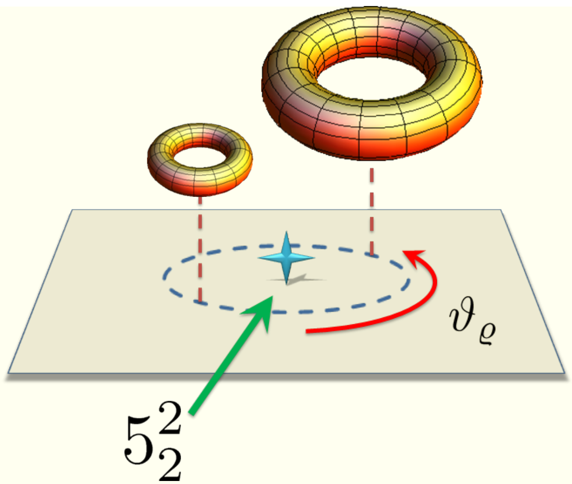
Worldsheet instanton corrections to 5^2_2-brane with one gauged isometry :

$$\varepsilon^{mn}\,\partial_m(X^9 A_n) \text{ in GLSM}$$

$$\to \text{ string winding modes along } X^9$$



$\longrightarrow$



$+$

string winding along dual circle X^9

$$H \;\to\; h_0 + \sigma \log(\mu/\varrho) + \sigma \sum_{n\neq 0} \exp(\mathrm{i}\, n\, X^9) K_0(|n|\varrho)$$

S. Sasaki and TK arXiv:1305.4439

SUMMARY

We established GLSM for exotic five-brane!

- ✓ co-dim. 2 ; isometry ; non–single-valued metric
- ✓ First application : worldsheet instanton corrections along X^9-direction

We hope this $\mathcal{N} = (4,4)$ GLSM tells us more and more !

- ✓ Another instantons : string winding modes along X^8-direction ?
- ✓ modular invariance, dipole description ? T. Kikuchi, T. Okada and Y. Sakatani
- ✓ more exotic as "multiple states of NS5 + 5^2_2" de Boer and Shigemori

Thanks

Appendix

10D string theory $=$ D-dim spacetime $\otimes$ compact space $\mathcal{M}_d$

$\mathcal{M}_d$	geometry associated with G_{mn}	Conventional geometry (manifold) $O(d)$ global symmetry [Calabi-Yau, etc]	ordinary compactifications
	geometry associated with G_{mn}, B_{mn}	Generalized geometry $O(d,d;\mathbb{Z})$ T-duality symmetry [T-fold]	flux compactifications
	geometry associated with G_{mn}, $\tau = C_{(0)} + \mathrm{i}\,\mathrm{e}^{-\Phi}$	Generalized geometry $SL(2,\mathbb{Z})$ S-duality symmetry [S-fold]	F-theory
	geometry associated with G_{mn}, B_{mn}, Φ, $C_{(p)}$	Generalized geometry $E_{d+1(d+1)}(\mathbb{Z})$ U-duality symmetry [U-fold]	compactifications with non-abelian gauge

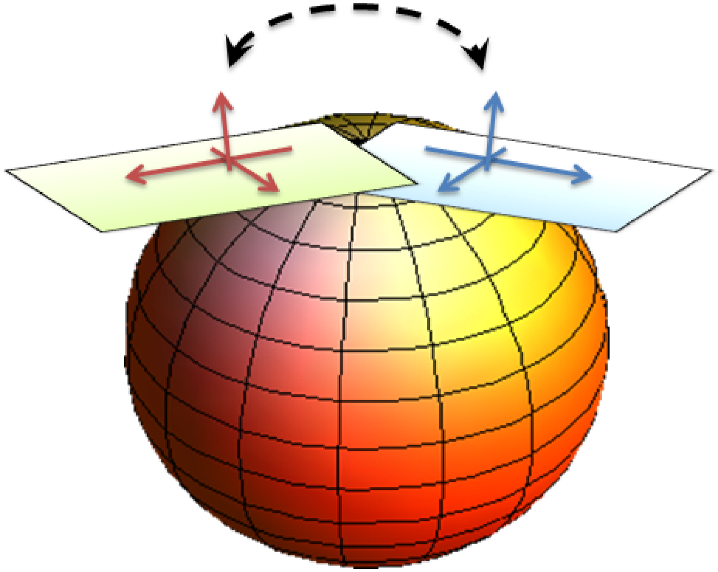
Non-geometric structure

structure groups $=$ diffeom. $(GL(d,\mathbb{R}))$ $\cup$ String duality groups

$\uparrow$

T-duality, U-duality, etc.

$GL(d,\mathbb{R}) \cup$ duality transf.



Generalized Geometry	(N. Hitchin)
Doubled Geometry	(C. Hull)

5^2_2-brane is a concrete example (T-fold)

Exotic brane shows us a new insight of stringy spacetime

M-theory on $S^1(R_s)$	mass/tension ($l_{\text{s}} \equiv 1$)	type IIA
longitudinal M2	1	F1
transverse M2	$\frac{1}{g_{\text{s}}}$	D2
longitudinal M5	$\frac{1}{g_{\text{s}}}$	D4
transverse M5	$\frac{1}{g_{\text{s}}^2}$	NS5
longitudinal KK6	$\frac{R_{\text{TN}}^2}{g_{\text{s}}^2}$	KK5
KK6 with $R_{\text{TN}} = R_{\text{s}}$	$\frac{1}{g_{\text{s}}}$	D6
transverse KK6	$\frac{R_{\text{TN}}^2}{g_{\text{s}}^3}$	6^1_3

0	1	2	3	4	5	6	7	8	9	M
✓	✓	✓	✓	✓	✓	✓	S^1	$\mathbb{R}^3$		
KK6 $\to$ 6^1_3							Taub-NUT			

$$b^c_n \, : \, M = \frac{(R_1 \cdots R_c)^2}{g_{\text{s}}^n}$$

for review: N. Obers and B. Pioline hep-th/9809039

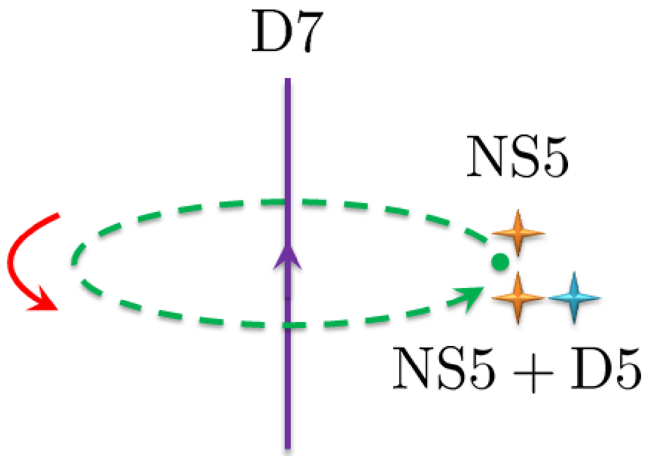
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2,\mathbb{Z})$ monodromy charge q

$$q : (C_2, B_2) \to (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

$$(0, \text{NS5}) \to (\text{D5}, \text{NS5})$$

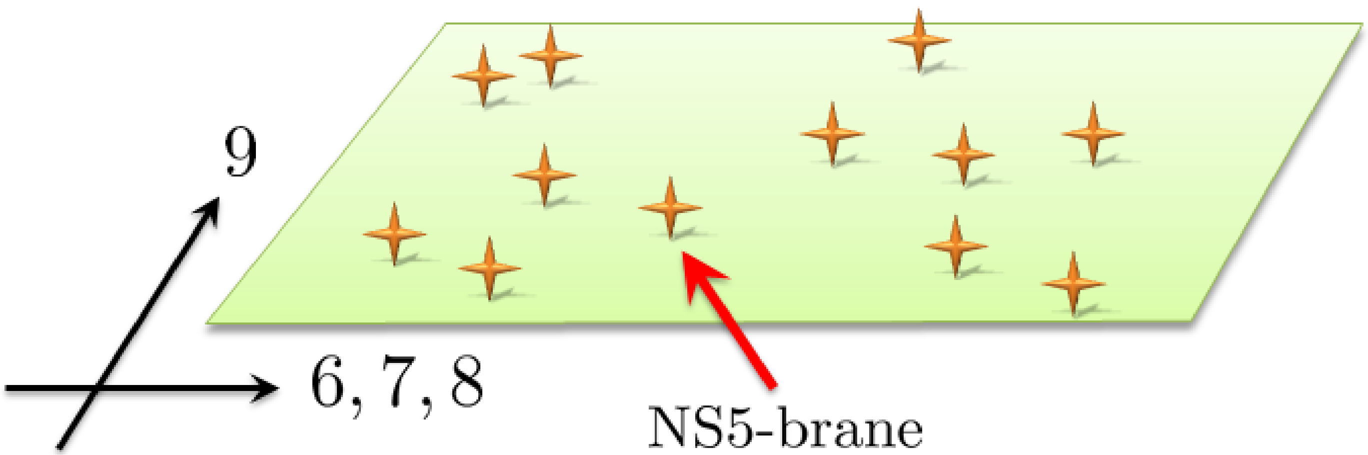


an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

NS5-branes

- co-dim. 4 $(\mathbb{R}^4\,, \quad \vec{\mathrm{x}} \in \mathbb{R}^4)$
- $\mathrm{d}s^2 \;=\; \mathrm{d}x^2_{012345} + H(x) \Big[(\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2 + (\mathrm{d}x^9)^2 \Big]$
- $H(x) = 1 + \sum_p \frac{Q}{|\vec{\mathrm{x}} - \vec{\mathrm{x}}_p|^2} \,, \qquad H_{mnp} = \varepsilon_{mnp}{}^q \, \partial_q \, \log H(x) \,, \qquad \Phi = \frac{1}{2} \log H(x)$

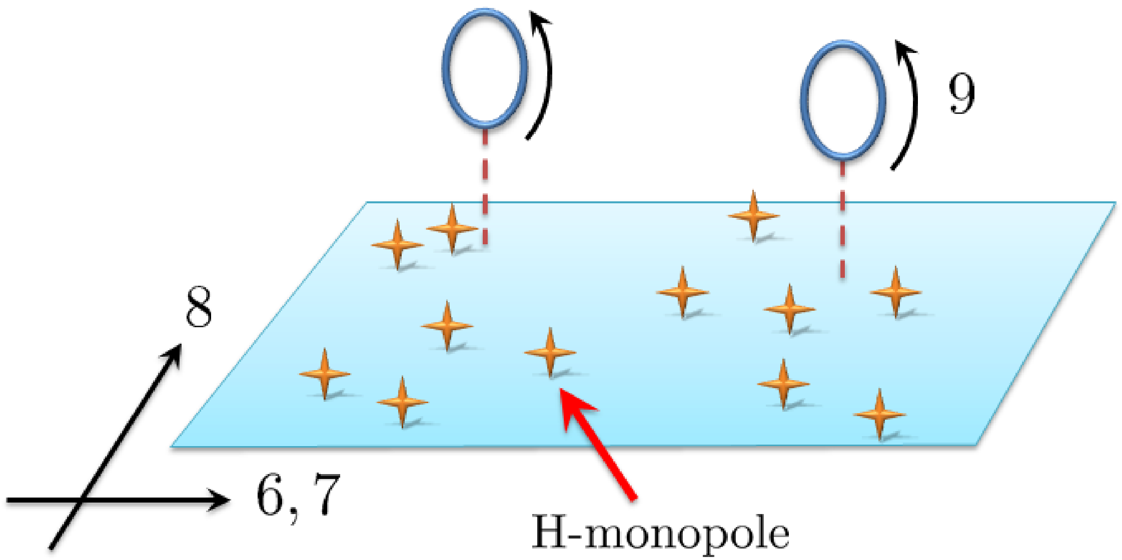
$\mathrm{d}x^2_{012345} = -\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + \ldots + (\mathrm{d}x^5)^2$



NS5-branes (smeared), or H-monopoles

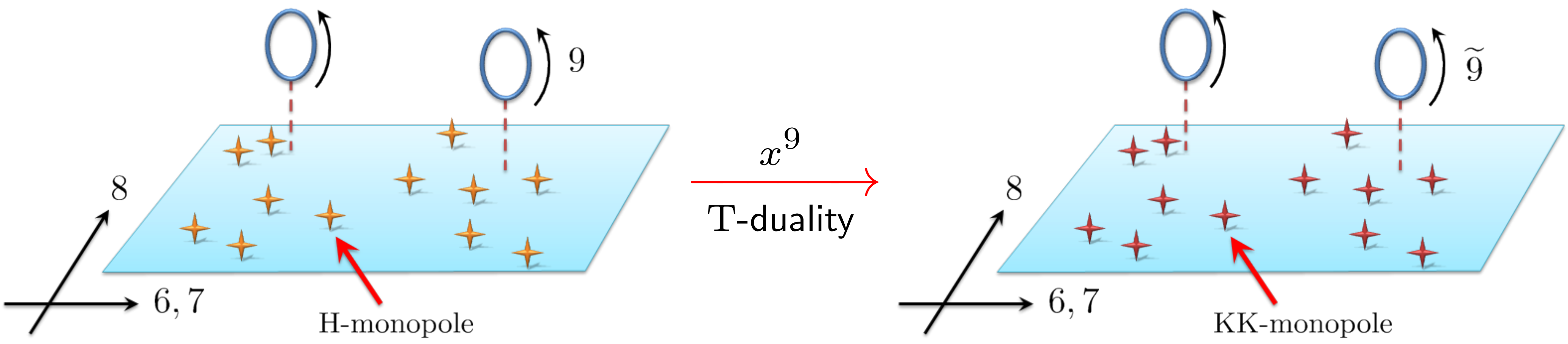
- co-dim. 3 ($\mathbb{R}^3 \times S^1\,, \quad \vec{r} \in \mathbb{R}^3$)
- $\mathrm{d}s^2 \;=\; \mathrm{d}x^2_{012345} + H(x)\Big[(\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2 + (\mathrm{d}x^9)^2\Big]$
- $H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}\,, \qquad H_{mnp} = \varepsilon_{mnp}{}^q\,\partial_q \log H(x)\,, \qquad \Phi = \frac{1}{2}\log H(x)$

$\mathrm{d}x^2_{012345} = -\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + \ldots + (\mathrm{d}x^5)^2$



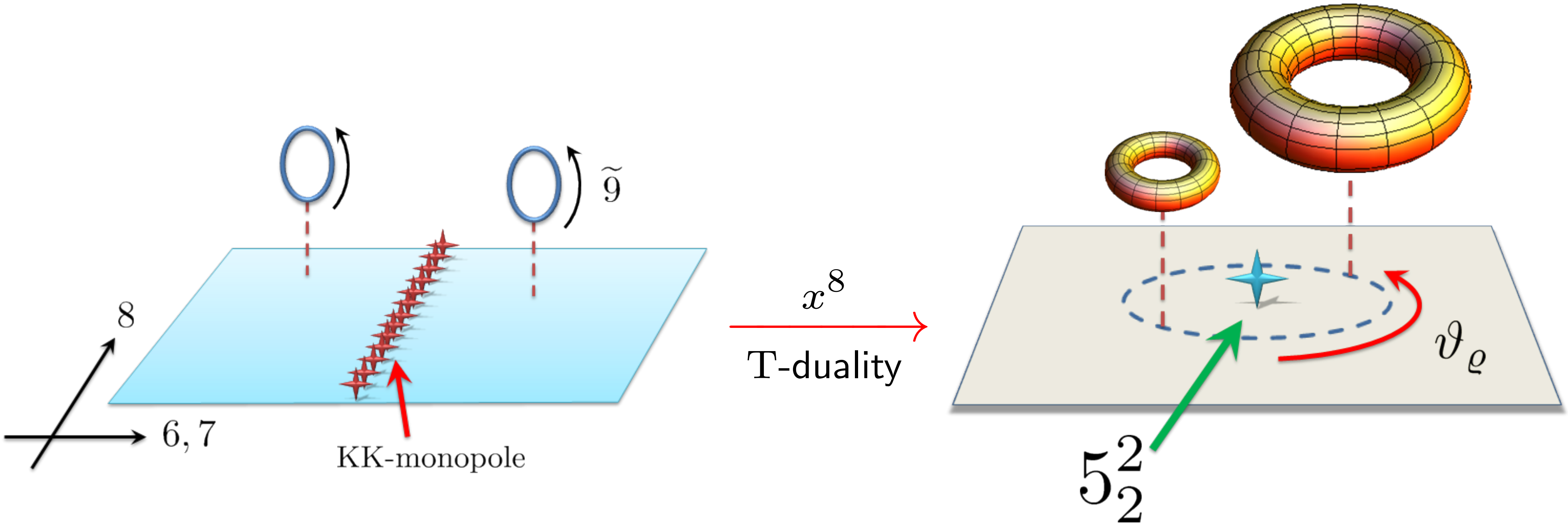
KK-monopoles

- co-dim. 3 ($\mathbb{R}^3 \times \widetilde{S}^1$: Taub-NUT space, $\vec{r} \in \mathbb{R}^3$)
- $$\mathrm{d}s^2 \;=\; \mathrm{d}x^2_{012345} + H(x)\Big[(\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2\Big] + \frac{1}{H(x)}\big(\mathrm{d}\widetilde{x}^9 + \omega\big)^2$$
- $$H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}\,, \qquad H_{mnp} = 0 = \Phi$$



5^2_2-brane

- co-dim. 2 $(\mathbb{R}^2 \times T^2)$
- $H(x) = h + \sigma \log\left(\frac{\mu}{\varrho}\right) \ , \quad (\varrho, \vartheta_\varrho) \in \mathbb{R}^2$



Gauged Linear Sigma Model (GLSM) is quite powerful !

in $\mathcal{N} = (2, 2)$ case :

- ✓ CY/LG (geometry/CFT) correspondence [phases]
  - ← E. Witten hep-th/9301042
- ✓ mirror symmetry [T-duality]
  - ← K. Hori and C. Vafa hep-th/0002222
- ✓ quantum Kähler moduli space [instantons]
  - ← N. Doroud et al arXiv:1206.2606; H. Jockers et al arXiv:1208.6244

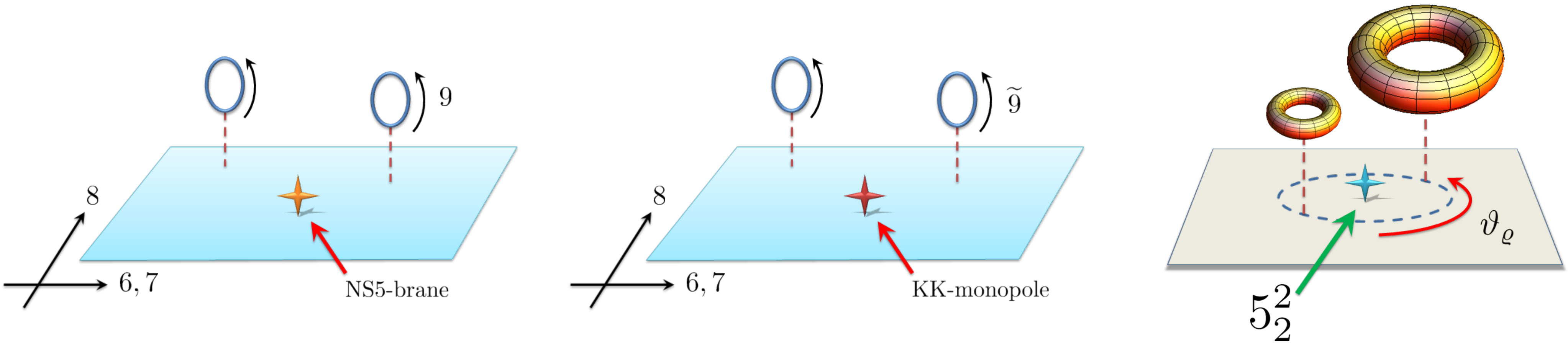
- T-duality transformation is represented as

(free) string	sign flip (parity) in right-mover	momentum $\leftrightarrow$ winding
spacetime	Buscher rule	$(G_{IJ}, B_{IJ}) \to (G'_{IJ}, B'_{IJ})$
SUSY sigma model	Roček-Verlinde formula	chiral $\leftrightarrow$ twisted chiral

- NS5-brane(s) and KK-monopole(s) can also be described by

IR limit of $\mathcal{N} = (4,4)$ GLSM

D. Tong hep-th/0204186; J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097



from KK-monopoles to 5^2_2-brane...

1. logarithmic function (co-dim. 2)
2. isometry, T-duality along x^8
3. nongeometric coordinates, non–single-valued metric

LESSON 1 : GLSM of NS5-brane

Bosonic Lagrangian after integrating-out auxiliary fields :

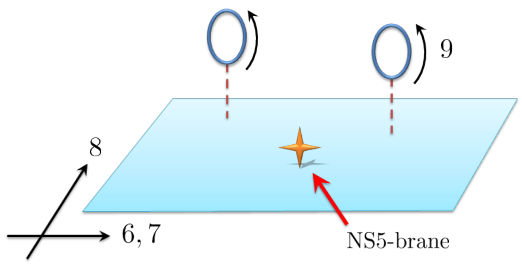
$$\begin{aligned}\mathscr{L}^{\mathsf{kin}}_{\mathsf{NS5}} &= \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big] \\ &\quad -\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2\Big] - (X^9 - t^9)\, F_{01}\end{aligned}$$

$$\begin{aligned}\mathscr{L}^{\mathsf{pot}}_{\mathsf{NS5}} &= -2\big(|\sigma|^2 + |\phi|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big) \\ &\quad -\frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2\end{aligned}$$

Steps to NLSM of NS5-brane

1. SUSY vacua $\mathscr{L}^{\mathsf{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, X^9)$

1. SUSY vacua

$$\sigma = 0 = \phi \, , \qquad |q|^2 - |\widetilde{q}|^2 = X^7 - t^7 \, , \qquad q\widetilde{q} = (X^6 - s^6) + \mathrm{i}(X^8 - s^8)$$

2. solve constraints on $(q, \widetilde{q})$

$$q = -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\alpha}\sqrt{r + (X^7 - t^7)} \, , \qquad \widetilde{q} = \mathrm{i}\,\mathrm{e}^{+\mathrm{i}\alpha_a}\frac{(X^6 - s^6) + \mathrm{i}(X^8 - s^8)}{\sqrt{r + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \widetilde{q}|^2 = \frac{1}{2r}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] + r\Big(\partial_m \alpha - A_m - \Omega_i\,\partial_m X^i\Big)^2$$

$$r = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2 + (X^8 - s^8)^2}$$

$$\Omega_i\,\partial_m X^i = \frac{-(X^6 - s^6)\partial_m X^8 + (X^8 - s^8)\partial_m X^6}{r(r + (X^7 - t^7))}$$

3. IR limit $e \to \infty$, and integrate out A_m

$$A_m = -\Omega_i\,\partial_m X^i - \frac{1}{r}\,\varepsilon_{mn}\,\partial^n X^9$$

$$\Longrightarrow \quad \mathscr{L}^{\mathsf{NLSM}}_{\mathsf{NS5}} \; = \; -\frac{1}{2}\Big(\frac{1}{g^2} + \frac{1}{r}\Big)\Big[(\partial_m \vec{X})^2 + (\partial_m X^9)^2\Big] + \varepsilon^{mn}\,\Omega_i\,\partial_m X^i\,\partial_n X^9$$

LESSON 2 : $\mathbb{T}$-duality

$\Theta \to \Gamma$:

$$\begin{aligned}\mathscr{L}_{\mathsf{H}} \ni \mathscr{L}_{\Theta} &= \int \mathrm{d}^4\theta \Big(-\frac{1}{g^2}\overline{\Theta}\Theta \Big) + \Big\{ \int \mathrm{d}^2\widetilde{\theta}\, (-\Theta)\Sigma + (\mathsf{h.c.}) \Big\} \\ &= \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2}(\Theta + \overline{\Theta})^2 - (\Theta + \overline{\Theta})V \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)\end{aligned}$$

$$\downarrow$$

$$\mathscr{L}_{B\Gamma} = \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2}B^2 - BV - B(\Gamma + \overline{\Gamma}) \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

real $\overline{B} = B$; chiral $\overline{D}_{\pm}\Gamma = 0$

$\Theta \to \Gamma$:

$$\mathscr{L}_{B\Gamma} \;=\; \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2}B^2 - BV - (\Gamma + \overline{\Gamma})B \Big\} - \varepsilon^{mn}\,\partial_m(X^9 A_n)$$

Integrating out Γ, $\overline{\Gamma}$: $\to$ GLSM of NS5-brane $\qquad B = \Theta + \overline{\Theta}$

or, Integrating out B : $\to$ GLSM of KK-monopole $\qquad \frac{1}{g^2}B = -(\Gamma + \overline{\Gamma}) - V$

duality relation : $\qquad \Theta = X^7 + \mathrm{i}X^9 + \ldots, \quad \Gamma = Y^7 + \mathrm{i}Y^9 + \ldots$

$$\Theta + \overline{\Theta} \;=\; -g^2(\Gamma + \overline{\Gamma}) - g^2 V \quad \to \quad \boxed{\begin{aligned} X^7 &= -g^2\,Y^7 \\ \pm(\partial_0 \pm \partial_1)X^9 &= -g^2(D_0 \pm D_1)Y^9 \\ D_m Y^9 &= \partial_m Y^9 + A_m \end{aligned}}$$

$$\mathscr{L}_{\mathsf{KK}} = \int d^4\theta \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \Big\} + \int d^4\theta \Big\{ \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2} \overline{\Psi}\Psi \Big\}$$

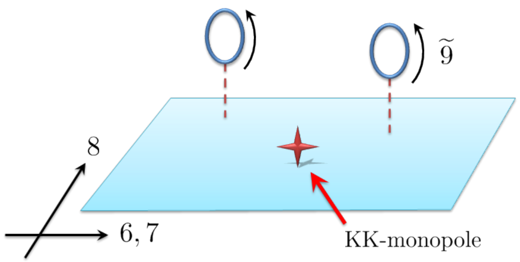
$$+ \int d^2\theta \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \Big\{ \int d^2\widetilde{\theta}\, t\, \Sigma + (\text{h.c.}) \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

Steps to NLSM of KK-monopole

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, Y^9)$

1. SUSY vacua

$$\sigma = 0 = \phi \,, \qquad |q|^2 - |\widetilde{q}|^2 = X^7 - t^7 \,, \qquad q\widetilde{q} = (X^6 - s^6) + \mathrm{i}(X^8 - s^8)$$

2. solve constraints on $(q_a, \widetilde{q}_a)$

$$q = -\mathrm{i}\, \mathrm{e}^{-\mathrm{i}\alpha} \sqrt{r + (X^7 - t^7)} \,, \qquad \widetilde{q} = \mathrm{i}\, \mathrm{e}^{+\mathrm{i}\alpha} \frac{(X^6 - s^6) + \mathrm{i}(X^8 - s^8)}{\sqrt{r + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \widetilde{q}|^2 = \frac{1}{2r}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] + r\Big(\partial_m \alpha - A_m - \Omega_i\, \partial_m X^i\Big)^2$$

3. IR limit $e \to \infty$, and integrate out A_m

$$A_m = -\frac{1}{rH}\big(\partial_m Y^9 - \Omega_i\, \partial_m X^i\big) - \Omega_i\, \partial_m X^i \,, \qquad H = \frac{1}{g^2} + \frac{1}{r}$$

$$\mathscr{L}^{\mathsf{NLSM}}_{\mathsf{KK}} \;=\; -\frac{1}{2} H (\partial_m \vec{X})^2 - \frac{1}{2} H^{-1} (\partial_m Y^9 - \Omega_i\, \partial_m X^i)^2 - \varepsilon^{mn}\, \partial_m((X^9 - t^9) A_n)$$

LESSON 3 : 5^2_2-brane

$\Psi \to \Xi$:

$$\begin{aligned}
\mathscr{L}_{\mathsf{KK}} \ni \mathscr{L}_{\Psi} &= \int \mathrm{d}^4\theta \, \frac{1}{g^2} \overline{\Psi} \Psi + \Big\{ \int \mathrm{d}^2\theta \, (-\Psi)\Phi + (\text{h.c.}) \Big\} \\
&= \int \mathrm{d}^4\theta \, \Big\{ \frac{a}{g^2} (\Psi + \overline{\Psi})^2 - (\Psi + \overline{\Psi})(C + \overline{C}) \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\Psi - \overline{\Psi})^2 - (\Psi - \overline{\Psi})(C - \overline{C}) \Big\}
\end{aligned}$$

$$\downarrow$$

$$\begin{aligned}
\mathscr{L}_{RSX\Xi} &= \int \mathrm{d}^4\theta \, \Big\{ \frac{a}{g^2} R^2 - R(C + \overline{C}) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - (\mathrm{i}S)(C - \overline{C}) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}
\end{aligned}$$

$\overline{R} = R, \ \overline{S} = S, \ \overline{D}_+ \Xi_{1,2} = 0 = D_- \Xi_{1,2}, \ \overline{D}_{\pm} X = 0, \ \Phi_a = \overline{D}_+ \overline{D}_- C_a$

$\Psi \to \Xi$:

$$\widetilde{\mathscr{L}} = \int d^4\theta \Big\{ \frac{a}{g^2} R^2 - R(C + \overline{C}) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} + \int d^4\theta \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - (\mathrm{i}S)(C - \overline{C}) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}$$

Integrating out Ξ_1, Ξ_2, X : $\to$ GLSM of KK-monopole

or, Integrating out R, Ξ_2 : $\to$ GLSM of 5^2_2-brane

$$\frac{2a}{g^2} R = -(\Xi_1 + \overline{\Xi}_1) + (C + \overline{C})$$

duality relation at $a = \frac{1}{2}$:

$$\Psi = X^6 + \mathrm{i}\, X^8 + \ldots$$

$$\Psi + \overline{\Psi} = -g^2(\Xi_1 + \overline{\Xi}_1) + g^2(C + \overline{C})$$

$X^6 \sim$ real part of Ξ

$\partial X^8 \sim \partial$(imaginary part of Ξ) + "gauge" fields in C_a

$$\mathscr{L}_{\mathsf{E}} = \int \mathrm{d}^4\theta \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\,\mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\,\mathrm{e}^{-2V} \widetilde{Q} \Big\}$$
$$+ \int \mathrm{d}^4\theta\, \frac{g^2}{2} \Big\{ \Big(\Gamma + \overline{\Gamma} + V \Big)^2 - \Big(\Xi + \overline{\Xi} - (C + \overline{C}) \Big)^2 \Big\}$$
$$+ \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}\Phi Q + s\Phi \Big) + (\text{h.c.}) \Big\} - \int \mathrm{d}^4\theta\, (\Psi - \overline{\Psi})(C - \overline{C})$$
$$+ \Big\{ \int \mathrm{d}^2\widetilde{\theta}\, t\, \Sigma + (\text{h.c.}) \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		role
neutral HM	twisted chiral $\Xi = X^6 + \mathrm{i}\, Y^8 + \ldots$	chiral $\Gamma = X^7 + \mathrm{i}\, Y^9 + \ldots$	spacetime coord.
VMs	twisted chiral $\Sigma = \overline{D}_+ D_- V$	chiral $\Phi = \overline{D}_+ \overline{D}_- C$	gauging isometry
charged HM	chiral Q $(+)$	chiral $\widetilde{Q}$ $(-)$	curving geometry
FI parameters	$s = s^6 + \mathrm{i}\, s^8$	$t = t^7 + \mathrm{i}\, t^9$	position of five-branes

$$\mathscr{L}_{\mathsf{E}}^{\mathsf{kin}} = \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big]$$
$$-\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2\Big] - \frac{g^2}{2}\Big[(\partial_m Y^8)^2 + (D_m Y^9)^2\Big] - (X^9 - t^9)\, F_{01}$$

$$\mathscr{L}_{\mathsf{E}}^{\mathsf{pot}} = -2\big(|\sigma|^2 + 4|M_c|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big)$$
$$-\frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2$$
$$+ \frac{g^2}{2}(A_{c=} + \overline{A}_{c=})(B_{c+\!\!+} + \overline{B}_{c+\!\!+})$$

$$(\partial_0 + \partial_1)X^8 = -g^2(\partial_0 + \partial_1)Y^8 + g^2(B_{c+\!\!+} + \overline{B}_{c+\!\!+})$$

$$(\partial_0 - \partial_1)X^8 = +g^2(\partial_0 - \partial_1)Y^8 + g^2(A_{c=} + \overline{A}_{c=})$$

$$+\frac{g^2}{2}(A_{c=} + \overline{A}_{c=})(B_{c+\!\!+} + \overline{B}_{c+\!\!+}) = -\frac{1}{2g^2}(\partial_m X^8)^2 + \frac{g^2}{2}(\partial_m Y^8)^2 + \varepsilon^{mn}\,(\partial_m X^8)(\partial_n Y^8)$$

Step 1.,2.,3. :

$$\begin{aligned}\mathscr{L} \;=\;& -\frac{1}{2}H\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] - \frac{1}{2H}(\partial_m Y^9)^2 \\ & -\frac{(\Omega_8)^2}{2H}(\partial_m X^8)^2 + \frac{\Omega_8}{H}(\partial_m X^8)(\partial^m Y^9) \\ & -\frac{(\Omega_6)^2}{2H}(\partial_m X^6)^2 - \frac{\Omega_6 \Omega_8}{H}(\partial_m X^6)(\partial^m X^8) + \frac{\Omega_6}{H}(\partial_m X^6)(\partial^m Y^9) \\ & -\varepsilon^{mn}\partial_m((X^9 - t^9)A_n) + \varepsilon^{mn}(\partial_m X^8)(\partial_n Y^8)\end{aligned}$$

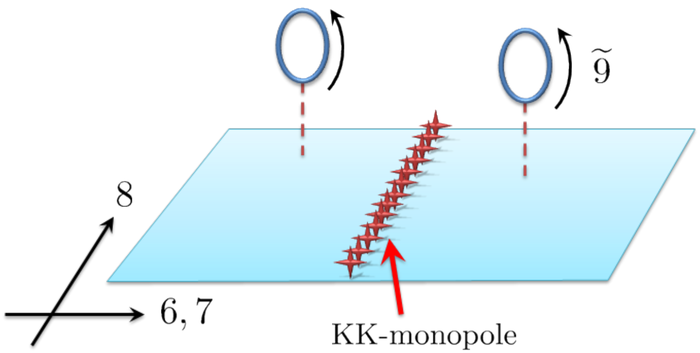
$$H \;=\; \frac{1}{g^2} + \frac{1}{r}, \qquad \Omega_6 \;=\; \frac{X^8 - s^8}{r(r + (X^7 - t^7))}, \quad \Omega_8 \;=\; -\frac{X^6 - s^6}{r(r + (X^7 - t^7))}$$

$$A_m = -\frac{1}{rH}(\partial_m Y^9 - \Omega_i\, \partial_m X^i) - \Omega_i\, \partial_m X^i$$

Step 4. : $s^8 = 2\pi \mathcal{R}_8\, s \xrightarrow{\text{integral of } s}$ emerge isometry

$$\begin{cases} H \to h_0 + \sigma \log(\mu/\varrho) & : \text{co-dim. 2} \qquad \varrho = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2} \\ \Omega_6 \to 0 & : \text{isometry along } X^8 \\ \Omega_8 \to \sigma \arctan\left(\dfrac{X^7 - t^7}{X^6 - s^6}\right) \equiv \sigma\, \vartheta_\varrho & : \text{“non–single-valued” metric} \end{cases}$$

EOM for X^8 : $\partial_m X^8 = \dfrac{H}{K}\Big[\dfrac{\sigma\vartheta_\varrho}{H}(\partial_m Y^9) + \varepsilon_{mn}(\partial^n Y^8)\Big]$ $\qquad K = H^2 + (\sigma\vartheta_\varrho)^2$



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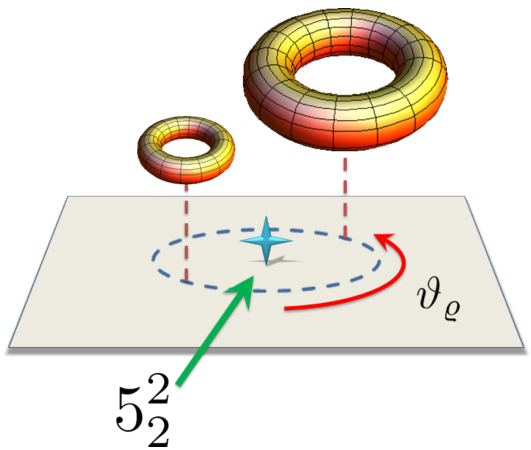


GLSM is a powerful tool, also in this stage :

Worldsheet instantons in NLSM can be captured by
vortex solution in gauge theory

ex.) GLSM of NS5-brane :

Take the configuration : $\phi = 0 = \sigma$ with $g^2 \to 0$ and finite e^2

$$\mathscr{L}_{\mathsf{E}} = \frac{1}{2e^2}(F_{12})^2 + |D_m q|^2 + \frac{e^2}{2}\big(|q|^2 - \zeta\big)^2 + \mathrm{i}\, X^9\, F_{12}$$

$$F_{12} = \mp e^2\big(|q|^2 - \zeta\big)\,, \qquad 0 = (D_1 \pm \mathrm{i} D_2) q$$

Abrikosov-Nielsen-Olesen vortex eq.

then, $S_{\mathsf{E}} = \frac{1}{2\pi}\int \mathrm{d}^2 x\, \mathscr{L}_{\mathsf{E}} = \zeta|n| - \mathrm{i}\, X^9\, n$ $\qquad n = -\frac{1}{2\pi}\int \mathrm{d}^2 x\, F_{12}$

- Duality transformation : $\Xi \to \Psi'$

$$\Xi + \overline{\Xi} - (C + \overline{C}) \;=\; -\frac{1}{g^2}(\Psi' + \overline{\Psi}' + V')$$

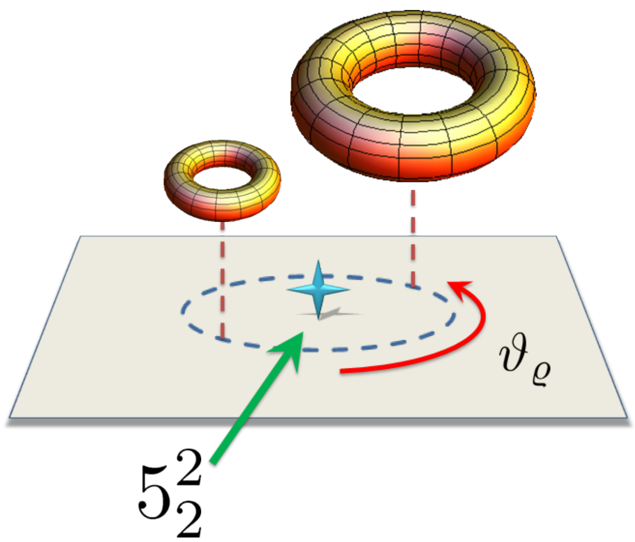
Remodeled GLSM of defect KK-monopole $\mathscr{L}_{\text{dKKM}}$

- Duality transformation : $\Gamma \to \Theta'$

$$\Gamma + \overline{\Gamma} + V \;=\; -\frac{1}{g^2}\Big[\Theta' + \overline{\Theta}' - (C' + \overline{C}')\Big]$$

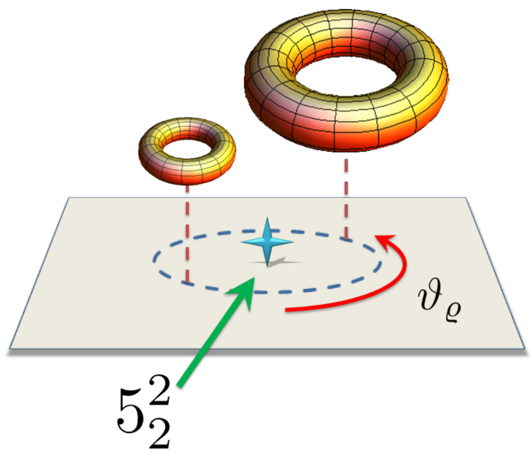
Remodeled GLSM of defect NS5-brane $\mathscr{L}_{\text{dNS5}}$

Extension ?



S. Sasaki and TK arXiv:1310.6163

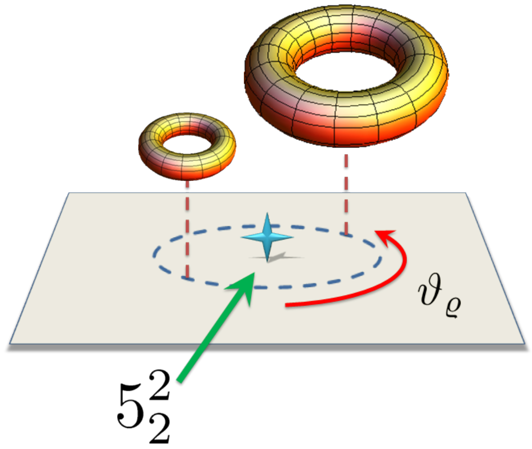
5^2_2-brane geometry has two isometries:



$$Y^9 \to Y^9 + \alpha \,, \qquad Y^8 \to Y^8 + \beta$$

5^2_2-brane geometry has two isometries:

$$\underbrace{Y^9 \;\to\; Y^9 + \alpha}_{\text{Gauged}}\,, \qquad \underbrace{Y^8 \;\to\; Y^8 + \beta}_{\text{Ungauged}}$$

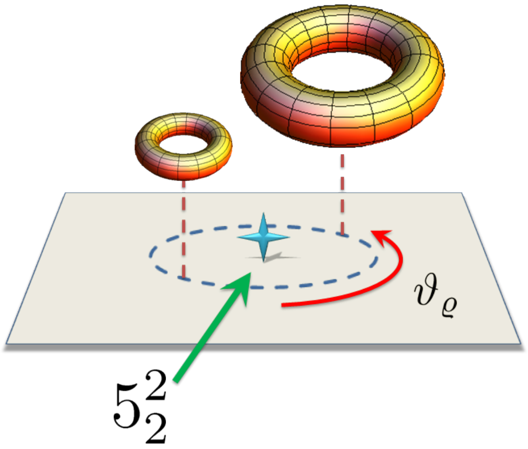


It is natural to think that the second isometry along Y^8 could be also gauged

because there are no difference between Y^9 and Y^8 in geometrical picture.

5^2_2-brane geometry has two isometries:

$$\underbrace{Y^9 \ \to \ Y^9 + \alpha}_{\text{Gauged}}, \qquad \underbrace{Y^8 \ \to \ Y^8 + \beta}_{\text{Ungauged}}$$



It is natural to think that the second isometry along Y^8 could be also gauged

because there are no difference between Y^9 and Y^8 in geometrical picture.

Extend the GLSM of Exotic Five-brane

$$\begin{aligned}\mathscr{L}_{\mathsf{E}} &= \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\, \mathrm{e}^{-2V} \widetilde{Q} \Big\} \\ &\quad + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big\{ \Big(\Gamma + \overline{\Gamma} + V \Big)^2 - \Big(\Xi + \overline{\Xi} - (C + \overline{C}) \Big)^2 \Big\} \\ &\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}\Phi Q + s\Phi \Big) + (\text{h.c.}) \Big\} - \int \mathrm{d}^4\theta \, (\Psi - \overline{\Psi})(C - \overline{C}) \\ &\quad + \Big\{ \int \mathrm{d}^2\widetilde{\theta}\, t\, \Sigma + (\text{h.c.}) \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)\end{aligned}$$

original GLSM of exotic 5^2_2-brane

Neutral HM $\Xi \ni Y^8$, $\Gamma \ni Y^9$ can be coupled to NEW multiplets $(\Sigma', \Phi'; Q', \widetilde{Q}')$

$$\begin{aligned}
\mathscr{L}_{\mathsf{E2}} \;=\; & \int \mathrm{d}^4\theta \left\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\, \mathrm{e}^{-2V} \widetilde{Q} \right\} \\
& + \int \mathrm{d}^4\theta\, \frac{g^2}{2} \Big\{ \Big(\Gamma + \overline{\Gamma} + V \Big)^2 - \Big(\Xi + \overline{\Xi} - (C + \overline{C}) \Big)^2 \Big\} \\
& + \Big\{ \int \mathrm{d}^2\theta\, \Big(\widetilde{Q}\Phi Q + s\Phi \Big) + (\mathsf{h.c.}) \Big\} - \int \mathrm{d}^4\theta\, (\Psi - \overline{\Psi})(C - \overline{C}) \\
& + \Big\{ \int \mathrm{d}^2\widetilde{\theta}\, t\, \Sigma + (\mathsf{h.c.}) \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n) + \mathscr{L}_{\mathsf{G}}
\end{aligned}$$

$\mathscr{L}_{\mathsf{G}}$ contains NEW multiplets $(\Sigma', \Phi'; Q', \widetilde{Q}')$ for second gauging $\Xi\Sigma'$

$$\begin{aligned}
\mathscr{L}_{\mathsf{E2}} \ &= \ \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma} \Sigma + \overline{\Phi} \Phi \Big) + \overline{Q} \, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}} \, \mathrm{e}^{-2V} \widetilde{Q} \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big\{ \Big(\Gamma + \overline{\Gamma} + V \Big)^2 - \Big(\Xi + \overline{\Xi} - (C + \overline{C}) \Big)^2 \Big\} \\
&\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q} \Phi Q + s \Phi \Big) + (\mathsf{h.c.}) \Big\} - \int \mathrm{d}^4\theta \, (\Psi - \overline{\Psi})(C - \overline{C}) \\
&\quad + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, t \, \Sigma + (\mathsf{h.c.}) \Big\} - \varepsilon^{mn} \, \partial_m (X^9 A_n) + \mathscr{L}_{\mathsf{G}} \\
\mathscr{L}_{\mathsf{G}} \ &= \ \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e'^2} \Big(- \overline{\Sigma}' \Sigma' + \overline{\Phi}' \Phi' \Big) \qquad \qquad \Big\} \\
&\quad + \int \mathrm{d}^2\theta \, \Big(\qquad + (s' - \Gamma) \Phi' \Big) + (\mathsf{h.c.}) \\
&\quad + \int \mathrm{d}^2\widetilde{\theta} \, (t' - \Xi) \, \Sigma' + (\mathsf{h.c.})
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_{\mathsf{E2}} &= \int d^4\theta \left\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \right\} \\
&\quad + \int d^4\theta\, \frac{g^2}{2} \left\{ \Big(\Gamma + \overline{\Gamma} + V \Big)^2 - \Big(\Xi + \overline{\Xi} - (C + \overline{C}) \Big)^2 \right\} \\
&\quad + \left\{ \int d^2\theta\, \Big(\widetilde{Q}\Phi Q + s\Phi \Big) + (\mathsf{h.c.}) \right\} - \int d^4\theta\, (\Psi - \overline{\Psi})(C - \overline{C}) \\
&\quad + \left\{ \int d^2\widetilde{\theta}\, t\, \Sigma + (\mathsf{h.c.}) \right\} - \varepsilon^{mn}\, \partial_m (X^9 A_n) + \mathscr{L}_{\mathsf{G}} \\
\mathscr{L}_{\mathsf{G}} &= \int d^4\theta \left\{ \frac{1}{e'^2} \Big(- \overline{\Sigma}'\Sigma' + \overline{\Phi}'\Phi' \Big) + \overline{Q}'\, e^{2V'} Q' + \overline{\widetilde{Q}}'\, e^{-2V'} \widetilde{Q}' \right\} \\
&\quad + \int d^2\theta\, \Big(\widetilde{Q}'\Phi' Q' + (s' - \Gamma)\Phi' \Big) + (\mathsf{h.c.}) \\
&\quad + \int d^2\widetilde{\theta}\, (t' - \Xi)\, \Sigma' + (\mathsf{h.c.})
\end{aligned}$$

Feature of $\mathscr{L}_{\text{E2}} = \mathscr{L}_{\text{E}} + \mathscr{L}_{\text{G}}$:

- Two gauged directions (as desired)

  $Y^9 \to Y^9 + \alpha, \quad Y^8 \to Y^8 + \beta$

- Broken SUSY : $\mathcal{N} = (4,4)$ to $\mathcal{N} = (2,2)$

  by two different $SU(2)_R$ symmetries associated with two VMs (Σ, Φ), (Σ', Φ')

Feature of $\mathscr{L}_{\mathsf{E2}} = \mathscr{L}_{\mathsf{E}} + \mathscr{L}_{\mathsf{G}}$:

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  $Y^9 \to Y^9 + \alpha, \quad Y^8 \to Y^8 + \beta$ Good!

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Feature of $\mathscr{L}_{\mathsf{E2}} = \mathscr{L}_{\mathsf{E}} + \mathscr{L}_{\mathsf{G}}$:

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  $Y^9 \to Y^9 + \alpha, \quad Y^8 \to Y^8 + \beta$ Good!

- Broken SUSY : $\mathcal{N} = (4,4)$ to $\mathcal{N} = (2,2)$

  by two different $SU(2)_R$ symmetries associated with two VMs (Σ, Φ), (Σ', Φ')

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How is the NLSM in the IR limit?

Steps to NLSM

1. search SUSY vacua $\mathscr{L}^{\text{potential}} = 0$
2. solve constraints of charged HMs $(Q, \widetilde{Q})$, $(Q', \widetilde{Q}')$
3. integrate out VMs (Σ, Φ), (Σ', Φ') in IR

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★4. integrate s^8 and s'^9, and solve EOM for T-dual field X^8

$$\begin{aligned}
\mathscr{L}_{\text{NLSM}} \;=\; & -\frac{1}{2}\Big(H + \frac{1}{g^2}H'\Big)\Big[(\partial_m \varrho)^2 + \varrho^2(\partial_m \vartheta_\varrho)^2\Big] - \frac{1}{2}\Big(\frac{H}{K} + g^2 H'\Big)\Big[(\partial_m Y^8)^2 + (\partial_m Y^9)^2\Big] \\
& -\Big(\frac{\sigma \vartheta_\varrho}{K} + \sigma' \vartheta'_\varrho\Big)\,\varepsilon^{mn}(\partial_m Y^8)(\partial_n Y^9) - \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)
\end{aligned}$$

Spacetime background geometry :

$$G_{66} = G_{77} = H + \frac{1}{g^2} H' , \qquad G_{88} = G_{99} = \frac{H}{K} + g^2 H'$$

$$B_{89} = -\frac{\sigma \vartheta_{\varrho}}{K} - \sigma' \vartheta'_{\varrho}$$

$$H = \frac{1}{g^2} + \sigma \log \frac{\Lambda}{\varrho} , \qquad H' = \sigma' \log \frac{\Lambda'}{\varrho'} , \qquad K = H^2 + (\sigma \vartheta_{\varrho})^2$$

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This goes to the geometry of exotic 5^2_2-brane
if $\sigma' \to 0$ (i.e., $\widetilde{\mathcal{R}}_9 \to \infty$)

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What's happen in other region?

Spacetime background geometry :

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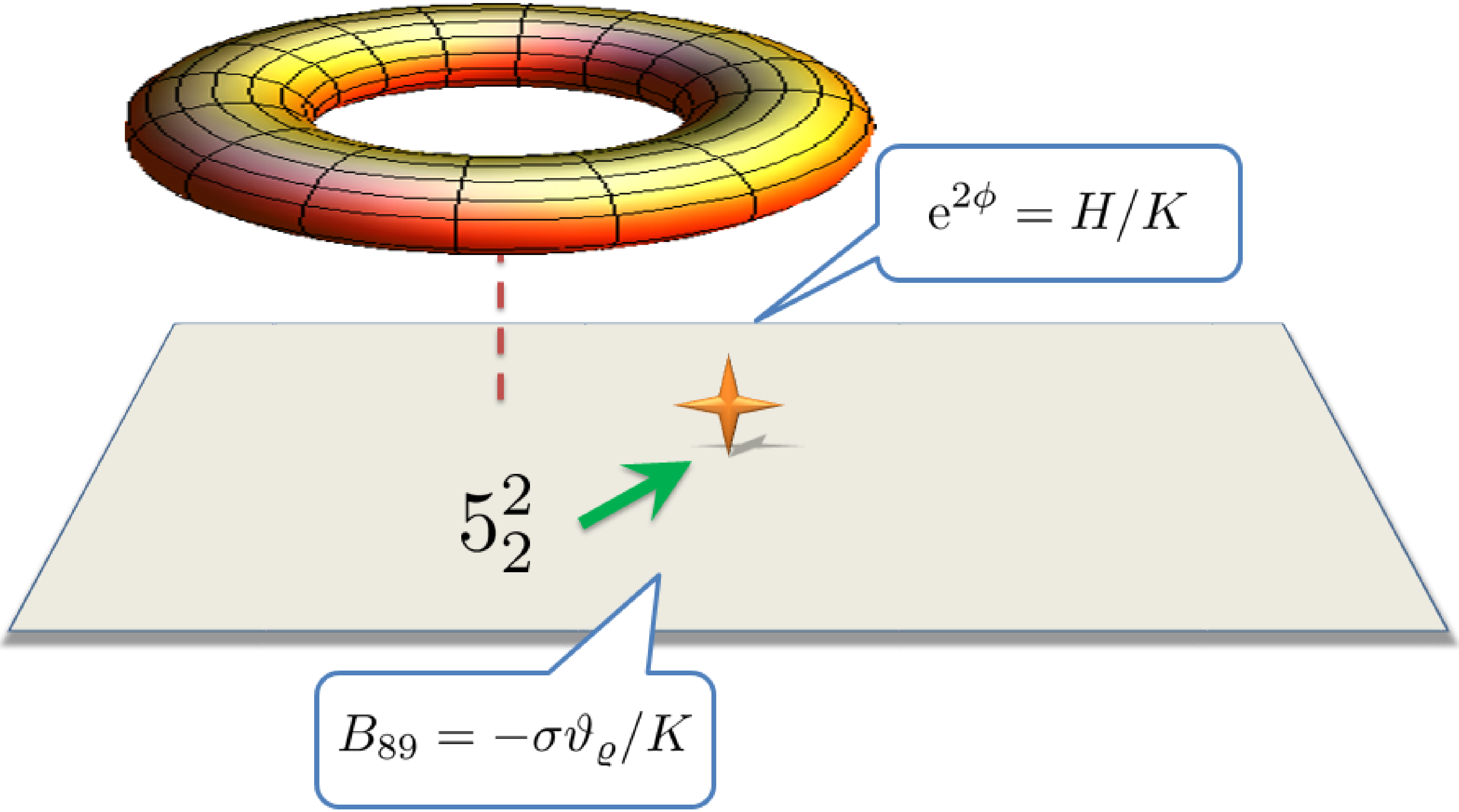
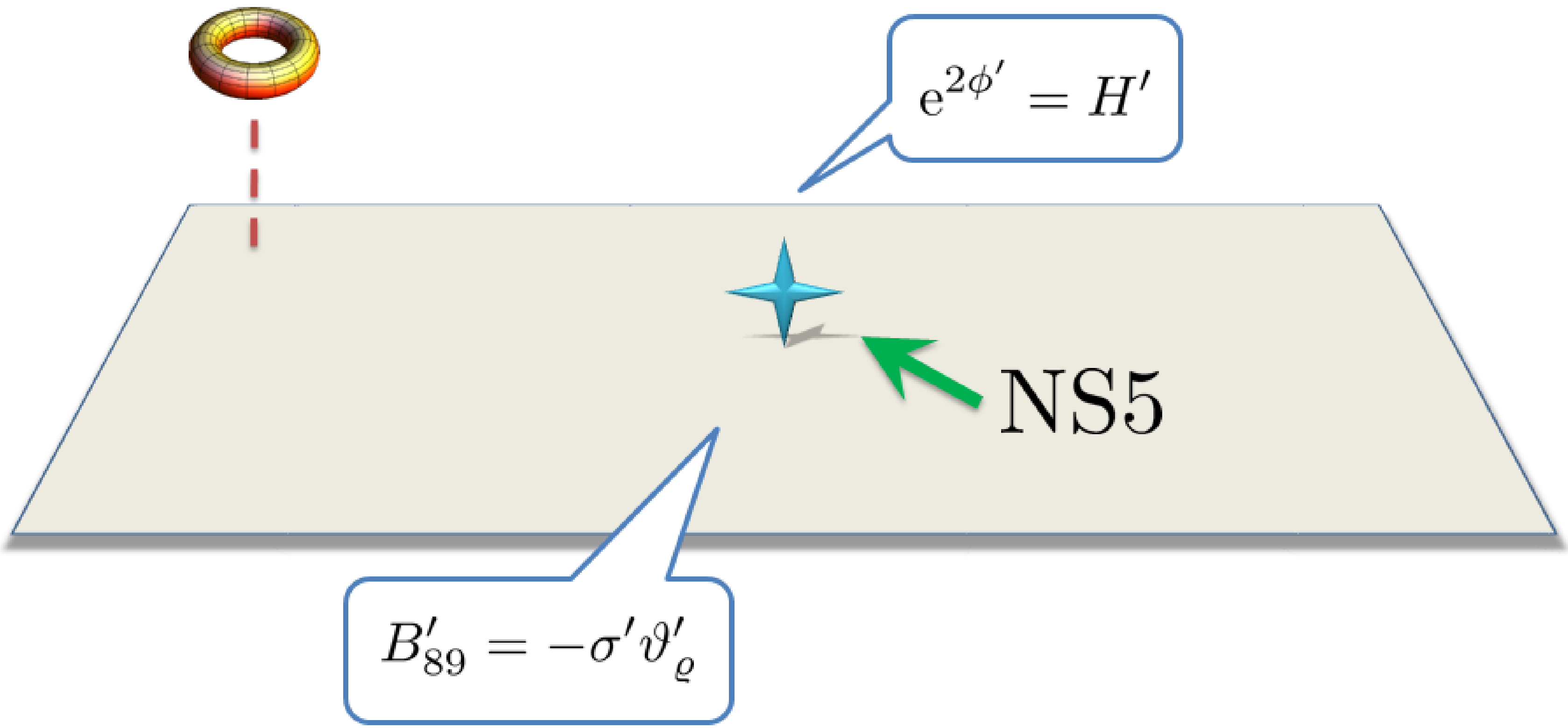
$$\sigma = \frac{1}{\pi \mathcal{R}_8} = \frac{\widetilde{\mathcal{R}}_8}{\pi \alpha'} , \qquad \sigma' = \frac{1}{\pi \widetilde{\mathcal{R}}_9}$$

It is not the correction to 5^2_2-brane geometry
but the emergence of another five-brane !

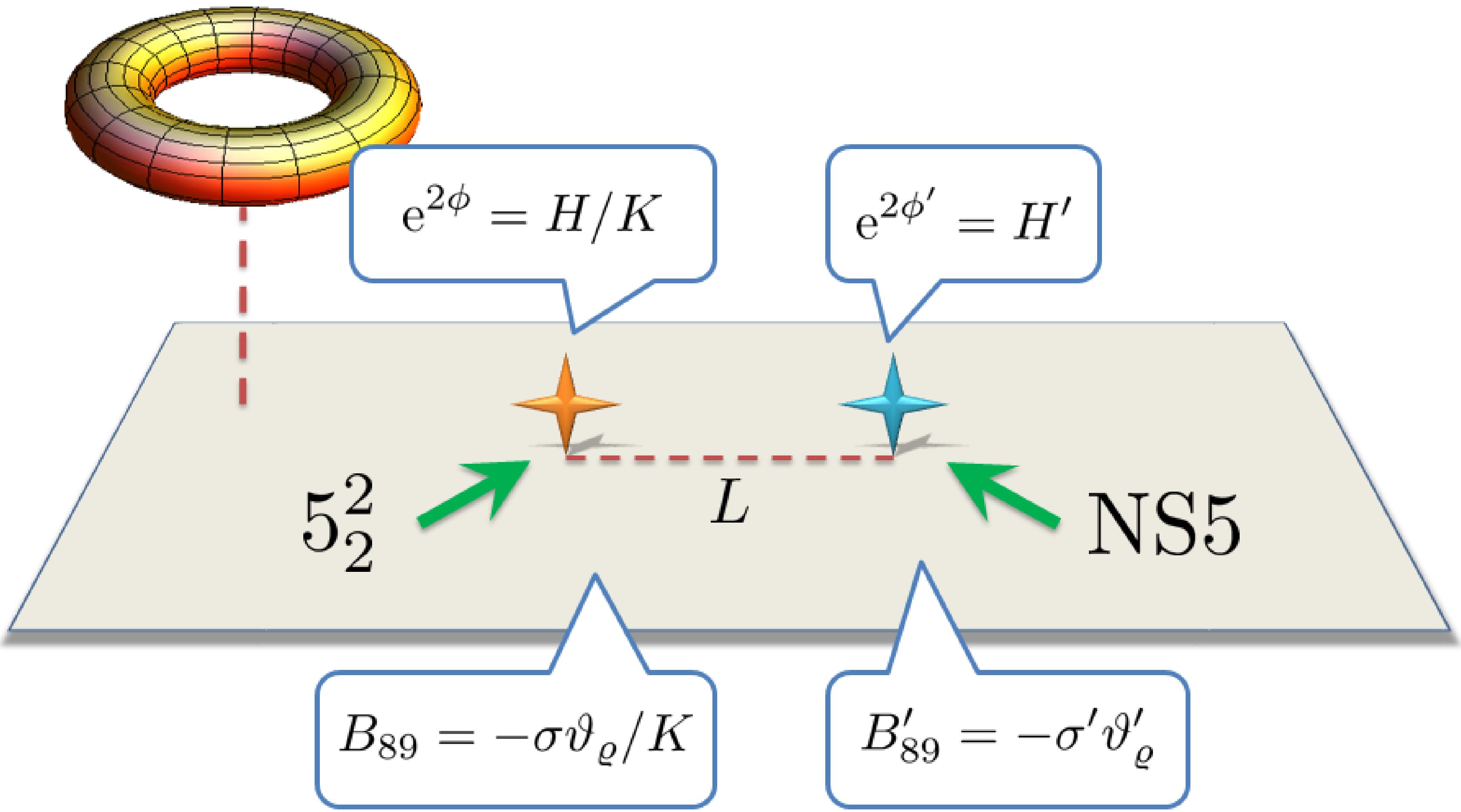
Small σ' limit : (i.e., $\widetilde{\mathcal{R}}_9 \to \infty$)



Large σ' limit : (i.e., $\widetilde{\mathcal{R}}_9 \to 0$)



General σ' : (finite $\widetilde{\mathcal{R}}_9$)



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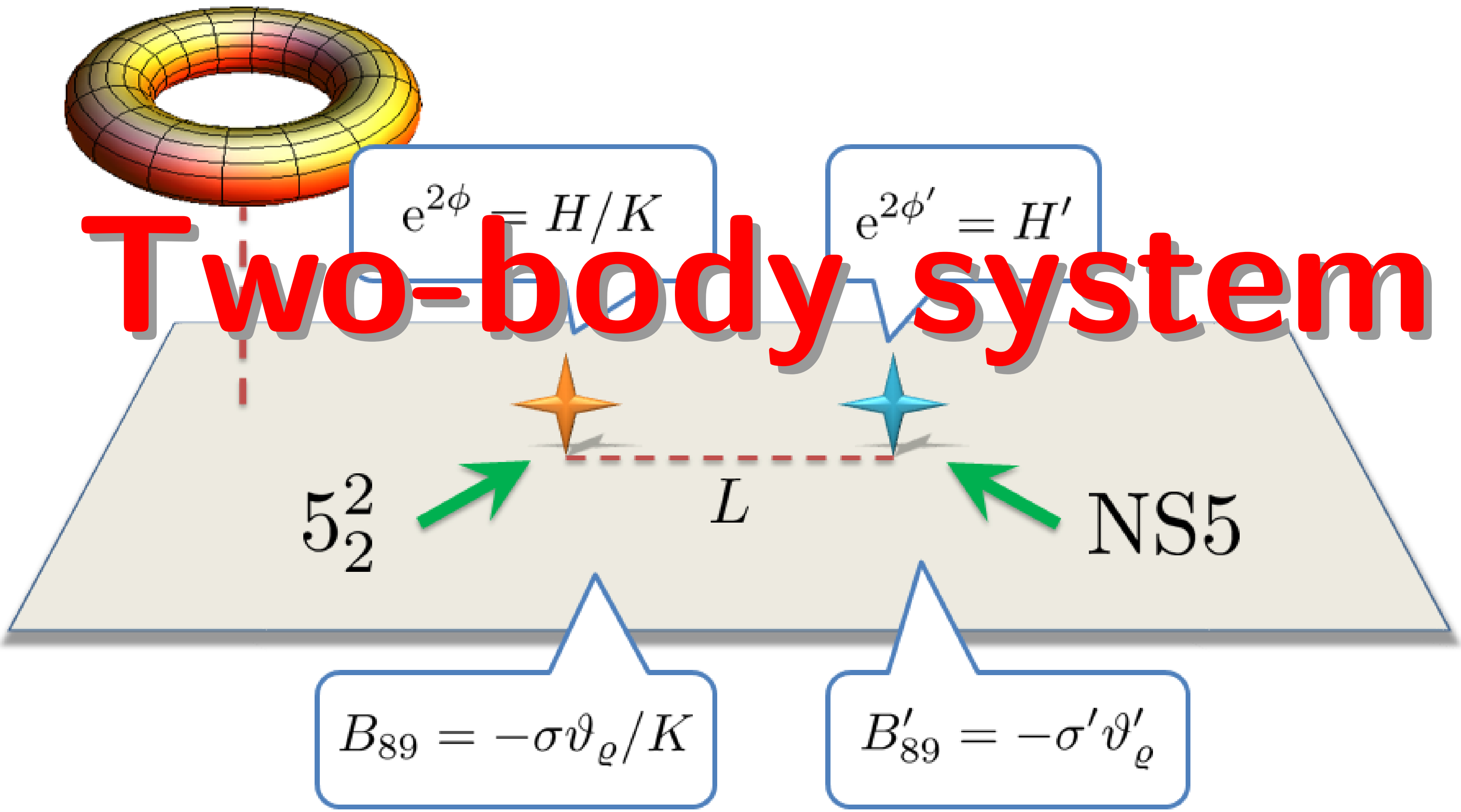

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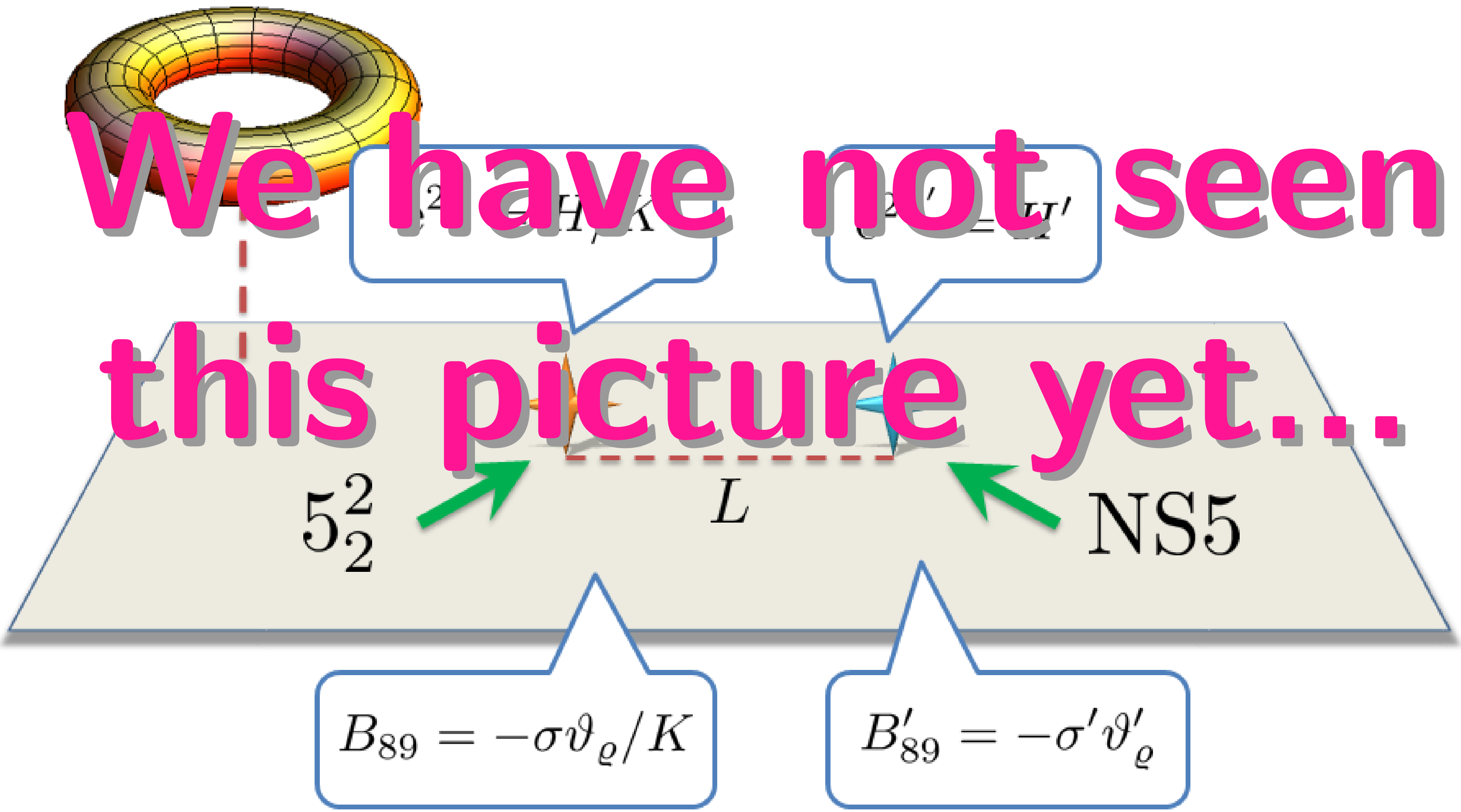

Signals from GLSM with two gauged isometries $\mathscr{L}_{\mathrm{E2}}$:

$\varepsilon^{mn}\,\partial_m(X^9 A_n)$: first gauging along dual coord. X^9

$Y^8 F'_{01}$: second gauging along physical coord. Y^8

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$\varepsilon^{mn}\,\partial_m(X^9 A_n)$: first gauging along dual coord. X^9

→ Winding corrections would be traced by A_n-vortices

$Y^8 F'_{01}$: second gauging along physical coord. Y^8

→ Momentum corrections would be traced by A'_m-vortices

- 5^2_2-brane with two gauged isometries

$$\frac{H}{K} + g^2 H' \ : \ \text{radius of } Y^8,\ Y^9 \qquad\qquad H' = \sigma' \log\left(\frac{\Lambda'}{\varrho}\right)$$

$g \to 0$ case	general ϱ
radius of Y^8, Y^9	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	general ϱ
radius of Y^8, Y^9	∞
KK-modes	light
winding modes	heavy

- 5^2_2-brane with two gauged isometries

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radius of Y^8, Y^9	∞
KK-modes	light
winding modes	heavy

$g \to 0$: Winding mode corrections along X^9 can be captured by A_m via $\varepsilon^{mn} \partial_m (X^9 A_n)$ in GLSM

$g \to \infty$: KK-mode corrections along Y^8 can be captured by A'_m via $Y^8 F'_{01}$ in GLSM

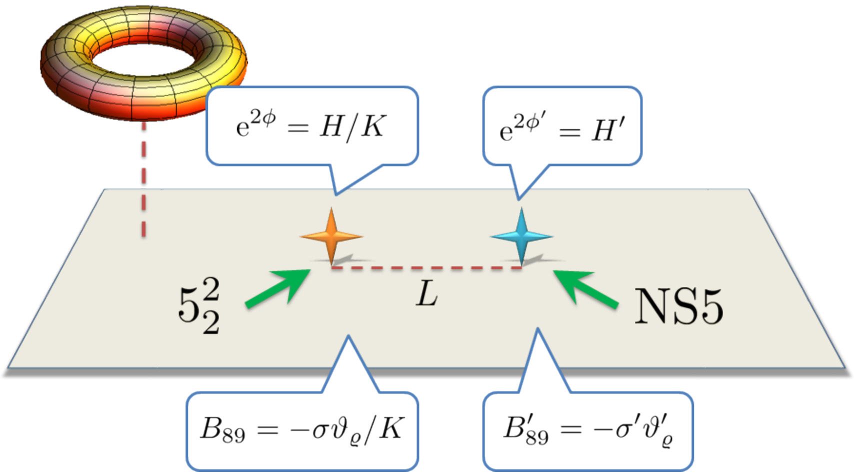
- 5^2_2-brane with two gauged isometries

$$\frac{H}{K} + g^2 H' \ : \ \text{radius of } Y^8,\ Y^9 \qquad H' = \sigma' \log\left(\frac{\Lambda'}{\varrho}\right)$$

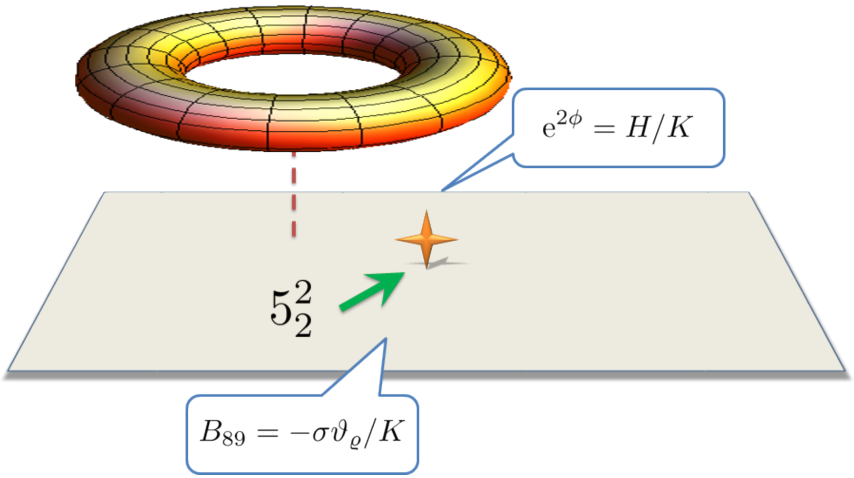
$g \to 0$ case	general ϱ	$g \to \infty$ case	general ϱ
radius of Y^8, Y^9	0	radius of Y^8, Y^9	∞
KK-modes	heavy	KK-modes	light
winding modes	light	winding modes	heavy

$g \to 0$: Winding mode corrections along X^9 can be captured by A_m via $\varepsilon^{mn} \partial_m (X^9 A_n)$ in GLSM

$g \to \infty$: However, corrections to Y^8 disappears in small σ' limit!

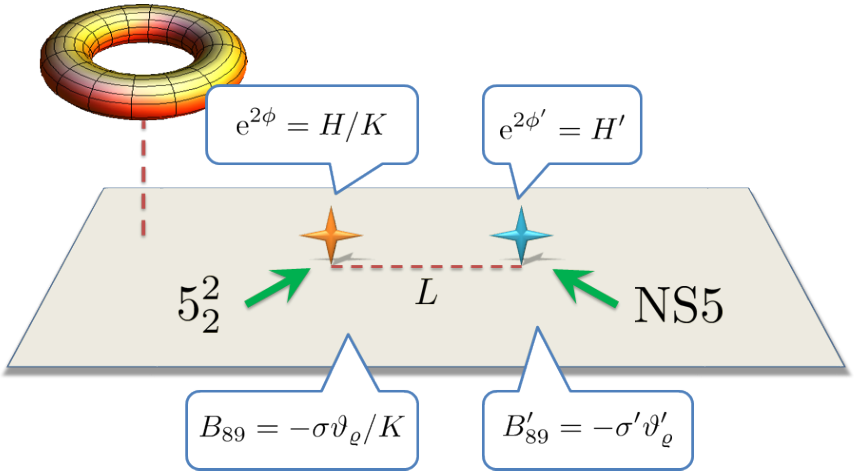


$\downarrow$ small σ' limit $\downarrow$

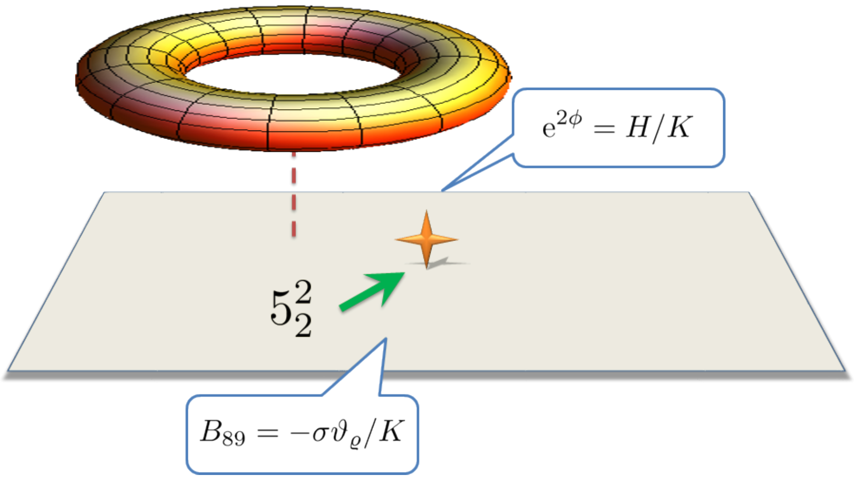


small σ' limit = large radius limit of Y^9

- disappearance of NS5-brane
- disappearance of Y^8-corrections



$\downarrow$ small σ' limit $\downarrow$



small σ' limit = large radius limit of Y^9

- disappearance of NS5-brane
- disappearance of Y^8-corrections

$\downarrow$

Y^8-corrections to NS5-brane,
rather than 5^2_2-brane !?

Current Interest

$$
\begin{aligned}
\mathscr{L}_{\mathsf{dKKM}} \ &= \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma} \Sigma + \overline{\Phi} \Phi \Big) + \overline{Q} \, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}} \, \mathrm{e}^{-2V} \widetilde{Q} \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2} \overline{\Psi} \Psi \Big\} \\
&\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q} \Phi Q + (s - \Psi) \Phi \Big) + (\mathsf{h.c.}) \Big\} + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, t \, \Sigma + (\mathsf{h.c.}) \Big\} \\
&\quad - \varepsilon^{mn} \, \partial_m (X^9 A_n) - \varepsilon^{mn} \, \partial_m (Y^8 A'_n) \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}' \Sigma' + \overline{\Phi}' \Phi' \Big) + \overline{Q}' \, \mathrm{e}^{2V'} Q' + \overline{\widetilde{Q}}' \, \mathrm{e}^{-2V'} \widetilde{Q}' \Big\} \\
&\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}' \Phi' Q' + (s' - \Gamma) \Phi' \Big) + (\mathsf{h.c.}) \Big\} + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, t' \, \Sigma' + (\mathsf{h.c.}) \Big\}
\end{aligned}
$$

$$
\Psi + \overline{\Psi} \ = \ \Psi' + \overline{\Psi}' + V' \ = \ -g^2 (\Xi + \overline{\Xi}') + g^2 (C + \overline{C})
$$

$$\mathscr{L}^{\text{NLSM}}_{\text{dKKM}} = -\frac{1}{2}\mathcal{A}\Big\{(\partial_m X^6)^2 + (\partial_m X^7)^2\Big\} - \frac{1}{2}\mathcal{B}^{-1}\big(\partial_m X^8 - \mathcal{C}\partial_m Y^9\big)^2 - \frac{1}{2}\mathcal{B}(\partial_m Y^9)^2$$
$$-\varepsilon^{mn}\,\partial_m(X^9 A_n) - \varepsilon^{mn}\,\partial_m(Y^8 A'_n)$$

$$G_{66} = G_{77} = \mathcal{A}\,, \qquad G_{88} = \mathcal{B}^{-1}\,, \qquad G_{99} = \mathcal{B} + \mathcal{C}^2\mathcal{B}^{-1}\,, \qquad G_{89} = -\mathcal{C}\mathcal{B}^{-1}$$

$$B_{IJ} = 0$$

$$\mathcal{A} = H + \frac{1}{g^2}H'\,, \qquad \mathcal{B} = \frac{H}{K} + g^2 H'\,, \qquad \mathcal{C} = \frac{\sigma\vartheta_\varrho}{K} + \sigma'\vartheta'_\varrho$$

$$
\begin{aligned}
\mathscr{L}_{\text{dNS5}} \;=\; & \int d^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \Big\} \\
& + \int d^4\theta \, \frac{1}{g^2} \Big\{ - \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big\} \\
& + \Big\{ \int d^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.}) \Big\} + \Big\{ \int d^2\widetilde{\theta} \, (t - \Theta)\Sigma + (\text{h.c.}) \Big\} \\
& - \int d^4\theta \, (\Gamma - \overline{\Gamma})(C' - \overline{C}') - \varepsilon^{mn} \, \partial_m (Y^8 A'_n) \\
& + \int d^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}'\Sigma' + \overline{\Phi}'\Phi' \Big) + \overline{Q}'\, e^{2V'} Q' + \overline{\widetilde{Q}}'\, e^{-2V'} \widetilde{Q}' \Big\} \\
& + \Big\{ \int d^2\theta \, \Big(\widetilde{Q}'\Phi' Q' + s'\Phi' \Big) + (\text{h.c.}) \Big\} + \Big\{ \int d^2\widetilde{\theta} \, t' \, \Sigma' + (\text{h.c.}) \Big\}
\end{aligned}
$$

$$
\Theta + \overline{\Theta} \;=\; \Theta' + \overline{\Theta}' - (C' + \overline{C}') \;=\; -g^2(\Gamma + \overline{\Gamma}) - g^2 V
$$

$$\mathscr{L}^{\mathsf{NLSM}}_{\mathsf{dNS5}} = -\frac{1}{2}\mathcal{A}\Big\{(\partial_m X^6)^2 + (\partial_m X^7)^2\Big\} - \frac{1}{2}\big(\mathcal{B}^{-1} + \mathcal{C}^2\mathcal{B}^{-1}\big)^{-1}\Big\{(\partial_m X^8)^2 + (\partial_m X^9)^2\Big\}$$

$$+\mathcal{C}\mathcal{B}^{-1}\big(\mathcal{B} + \mathcal{C}^2\mathcal{B}^{-1}\big)^{-1}\,\varepsilon^{mn}(\partial_m X^8)(\partial_n X^9)$$

$$-\varepsilon^{mn}\,\partial_m(Y^8 A'_n)$$

$$G_{66} = G_{77} = \mathcal{A}\,, \qquad G_{88} = G_{99} = \big(\mathcal{B} + \mathcal{C}^2\mathcal{B}^{-1}\big)^{-1}$$

$$B_{89} = \mathcal{C}\mathcal{B}^{-1}\big(\mathcal{B} + \mathcal{C}^2\mathcal{B}^{-1}\big)^{-1}$$

$$\mathcal{A} = H + \frac{1}{g^2}H'\,, \qquad \mathcal{B} = \frac{H}{K} + g^2 H'\,, \qquad \mathcal{C} = \frac{\sigma\vartheta_\varrho}{K} + \sigma'\vartheta'_\varrho$$

(p,q) 7-branes 系 $\overset{\text{記述の演繹}}{\Longrightarrow}$ (NS5, 5^2_2)-branes 系

7 次元簡約 ↓ ↓ 7 次元簡約

3 次元時空上の多粒子系 $\overset{SL(2,\mathbb{Z})_{\rm S} \longleftrightarrow SL(2,\mathbb{Z})_{\rm T}}{\longleftrightarrow}$ 3 次元時空上の多粒子系

記述の同等化

$$
\begin{aligned}
\Psi &= (X^6 + \mathrm{i} X^8) + \mathrm{i}\,\theta^+ \chi_+ + \mathrm{i}\,\theta^- \chi_- + \mathrm{i}\,\theta^+ \theta^- G + \ldots \\
\Xi &= (Y^6 + \mathrm{i}\,Y^8) + \mathrm{i}\,\theta^+ \overline{\xi}_+ + \mathrm{i}\,\overline{\theta}^- \xi_- + \mathrm{i}\,\theta^+ \overline{\theta}^- G_\Xi + \ldots \\
\Theta &= (X^7 + \mathrm{i} X^9) + \mathrm{i}\,\theta^+ \overline{\widetilde{\chi}}_+ + \mathrm{i}\,\overline{\theta}^- \widetilde{\chi}_- + \mathrm{i}\,\theta^+ \overline{\theta}^- \widetilde{G} + \ldots \\
\Gamma &= (Y^7 + \mathrm{i}\,Y^9) + \mathrm{i}\,\theta^+ \zeta_+ + \mathrm{i}\,\theta^- \zeta_- + \mathrm{i}\,\theta^+ \theta^- G_\Gamma + \ldots \\
Q_a &= q + \mathrm{i}\,\theta^+ \psi_+ + \mathrm{i}\,\theta^- \psi_- + \mathrm{i}\,\theta^+ \theta^- F + \ldots \\
\widetilde{Q}_a &= \widetilde{q} + \mathrm{i}\,\theta^+ \widetilde{\psi}_+ + \mathrm{i}\,\theta^- \widetilde{\psi}_- + \mathrm{i}\,\theta^+ \theta^- \widetilde{F} + \ldots \\
V &= \theta^+ \overline{\theta}^+ (A_0 + A_1) + \theta^- \overline{\theta}^- (A_0 - A_1) - \theta^- \overline{\theta}^+ \sigma - \theta^+ \overline{\theta}^- \overline{\sigma} \\
&\quad - \mathrm{i}\,\theta^+ \theta^- \big(\overline{\theta}^+ \overline{\lambda}_+ + \overline{\theta}^- \overline{\lambda}_- \big) + \mathrm{i}\,\overline{\theta}^+ \overline{\theta}^- \big(\theta^+ \lambda_+ + \theta^- \lambda_- \big) - \theta^+ \theta^- \overline{\theta}^+ \overline{\theta}^- D_V \\
\Phi_a &= \phi + \mathrm{i}\,\theta^+ \widetilde{\lambda}_+ + \mathrm{i}\,\theta^- \widetilde{\lambda}_- + \mathrm{i}\,\theta^+ \theta^- D_\Phi + \ldots \qquad = \overline{D}_+ \overline{D}_- C \\
C &= \phi_c + \mathrm{i}\,\theta^+ \psi_{c+} + \mathrm{i}\,\theta^- \psi_{c-} + \mathrm{i}\,\overline{\theta}^+ \chi_{c+} + \mathrm{i}\,\overline{\theta}^- \chi_{c-} \\
&\quad + \mathrm{i}\,\theta^+ \theta^- F_c + \mathrm{i}\,\overline{\theta}^+ \overline{\theta}^- M_c + \mathrm{i}\,\theta^+ \overline{\theta}^- G_c + \mathrm{i}\,\overline{\theta}^+ \theta^- N_c + \theta^- \overline{\theta}^- A_{c=} + \theta^+ \overline{\theta}^+ B_{c\#} \\
&\quad - \mathrm{i}\,\theta^+ \theta^- \overline{\theta}^+ \zeta_{c+} - \mathrm{i}\,\theta^+ \theta^- \overline{\theta}^- \zeta_{c-} + \mathrm{i}\,\overline{\theta}^+ \overline{\theta}^- \theta^+ \lambda_{c+} + \mathrm{i}\,\overline{\theta}^+ \overline{\theta}^- \theta^- \lambda_{c-} - \theta^+ \theta^- \overline{\theta}^+ \overline{\theta}^- D_c
\end{aligned}
$$

with $\overline{D}_\pm = -\dfrac{\partial}{\partial \overline{\theta}^\pm} + \mathrm{i}\,\theta^\pm (\partial_0 \pm \partial_1)$, $D_\pm = \dfrac{\partial}{\partial \theta^\pm} - \mathrm{i}\,\overline{\theta}^\pm (\partial_0 \pm \partial_1)$