

Exotic Five-branes

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arXiv:[1304.4061](#) [1305.4439](#) [1310.6163](#) [1402.5580](#) [1404.5442](#) [1406.0087](#) [1410.8403](#) [1411.3457](#)

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2. I focus on the exotic 5_2^2 -brane. The exotic 5_2^2 -brane can be obtained from NS5-brane via T-duality.
Though its supergravity description is not globally well-defined, I can construct it from the perspective of 2D supersymmetric gauge theory, called the gauged linear sigma model (GLSM).
In particular, I show that the “exotic” structure of the 5_2^2 -brane indeed originates from the string winding modes.

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Though its supergravity description is not globally well-defined, I can construct it from the perspective of 2D supersymmetric gauge theory, called the gauged linear sigma model (GLSM).
In particular, I show that the “exotic” structure of the 5_2^2 -brane indeed originates from the string winding modes.
3. I try to discuss the importance of the stringy spacetime picture beyond the Einstein gravity.

Introduction

well-known objects
D-branes, NS-branes

String Dualities

in lower dimensions

well-known objects
D-branes, NS-branes

String Dualities
in lower dimensions

less known objects
Exotic Branes

well-known objects
D-branes, NS-branes

String Dualities
in lower dimensions

less known objects
Exotic Branes

- ✓ co-dimension 2 (“defect” branes), or less
- ✓ non-single-valued metric of spacetime
- ✓ monodromy from string dualities

(Exotic) Branes are labelled as b_n^c -branes.

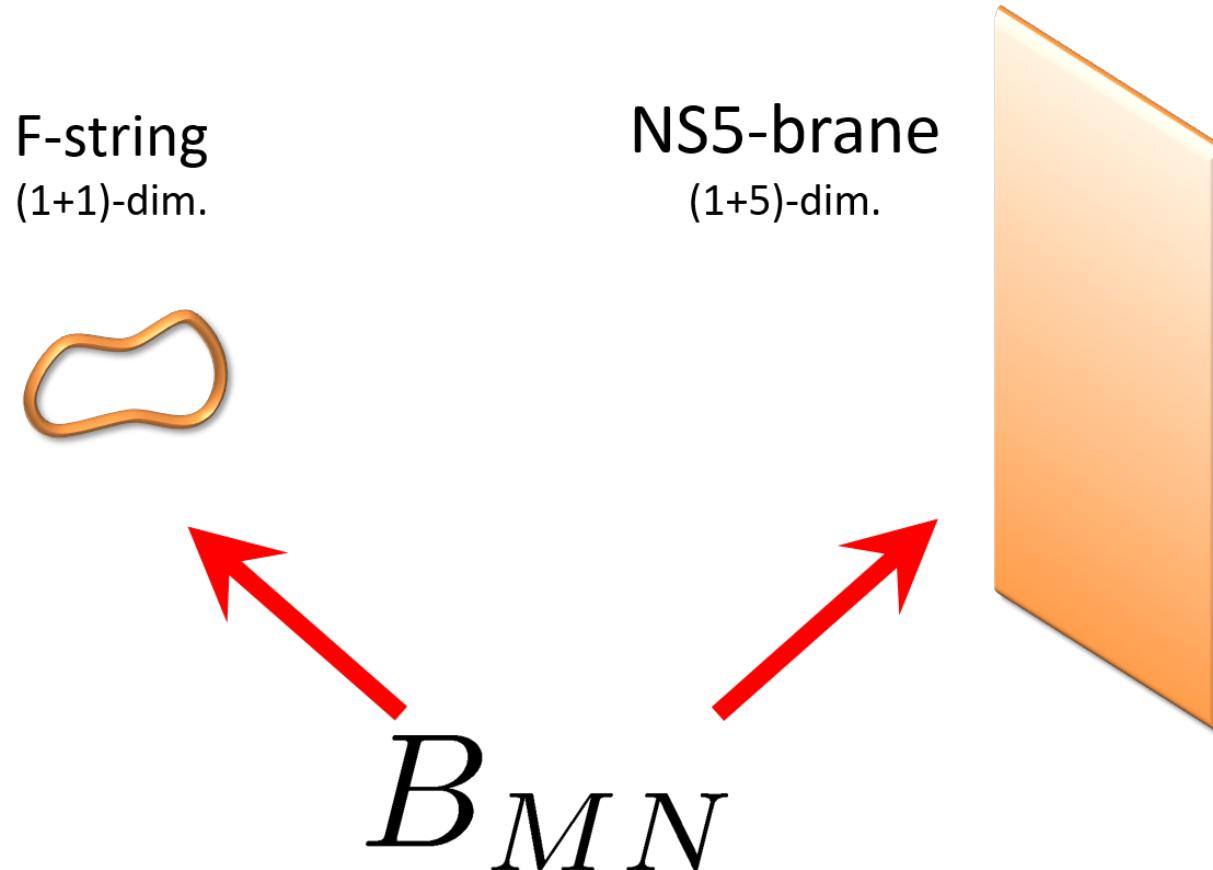
b_n^c

b : spatial dimensions

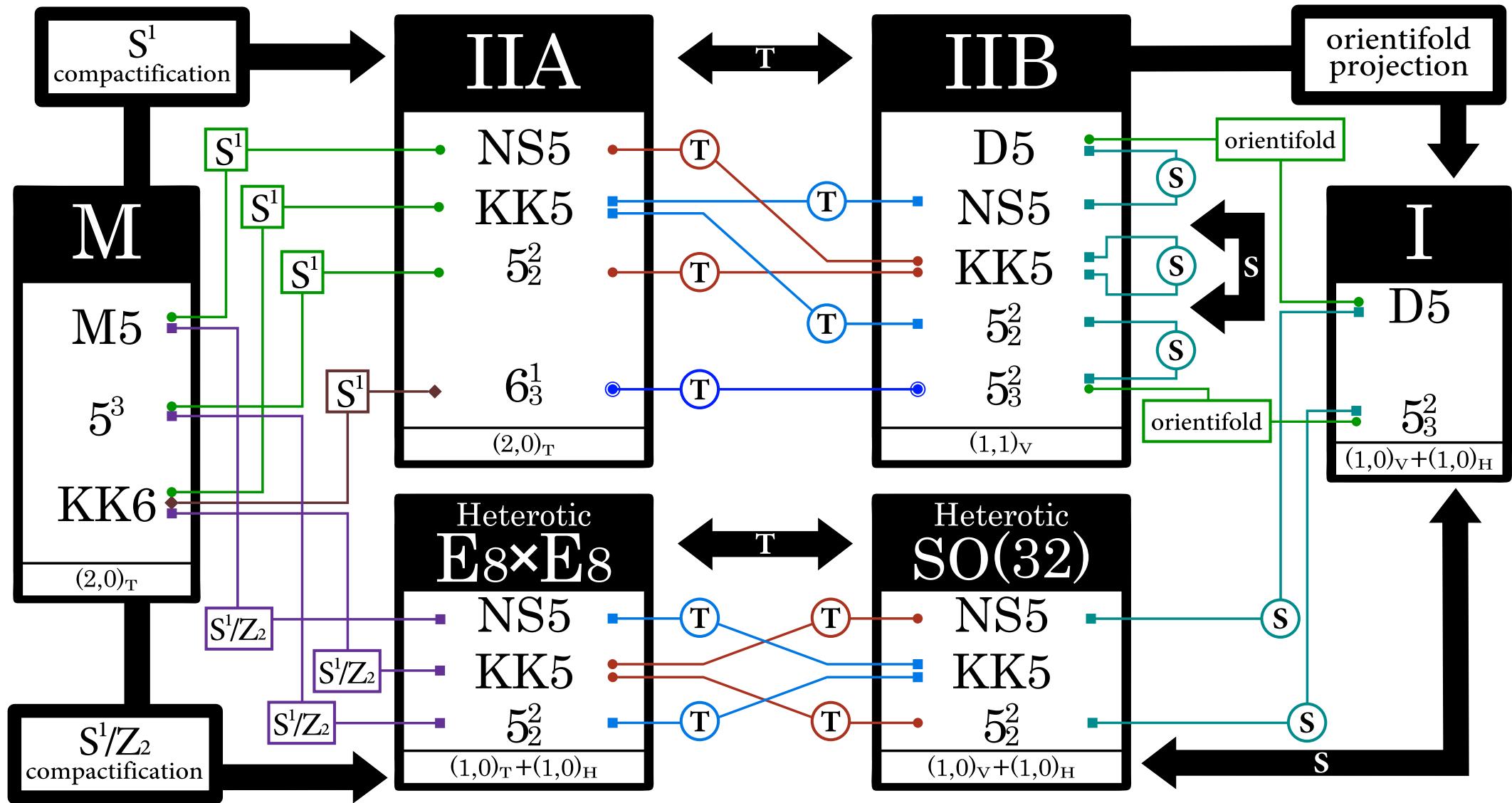
c : # of isometry directions

n : tension $\sim g_s^{-n}$

An example of five-branes in string theory



F-string = 1-brane, NS5-brane = 5₂-brane
(cf. D-string = 1₁-brane, D5-brane = 5₁-brane)



S. Sasaki, M. Yata and TK arXiv:1404.5442

b^c_n

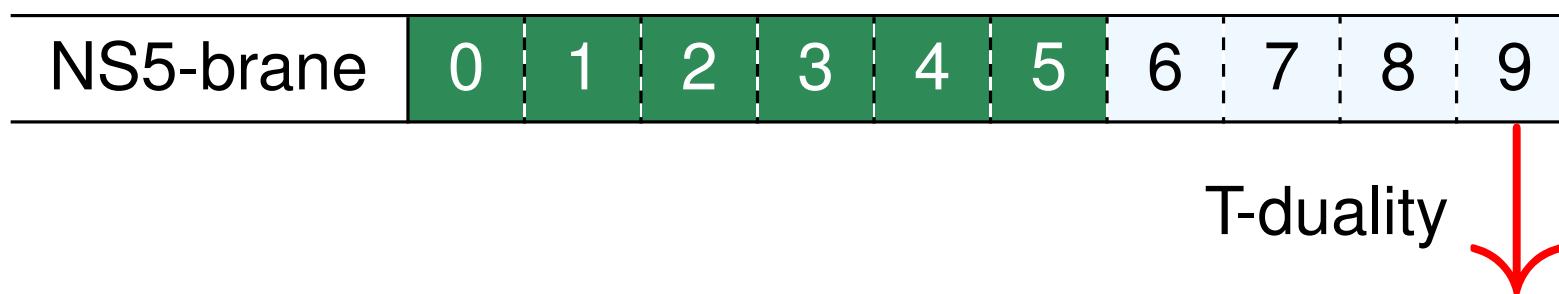
We would like to see a deeper insight of exotic branes
from various viewpoints !

Exotic 5_2^2 -brane

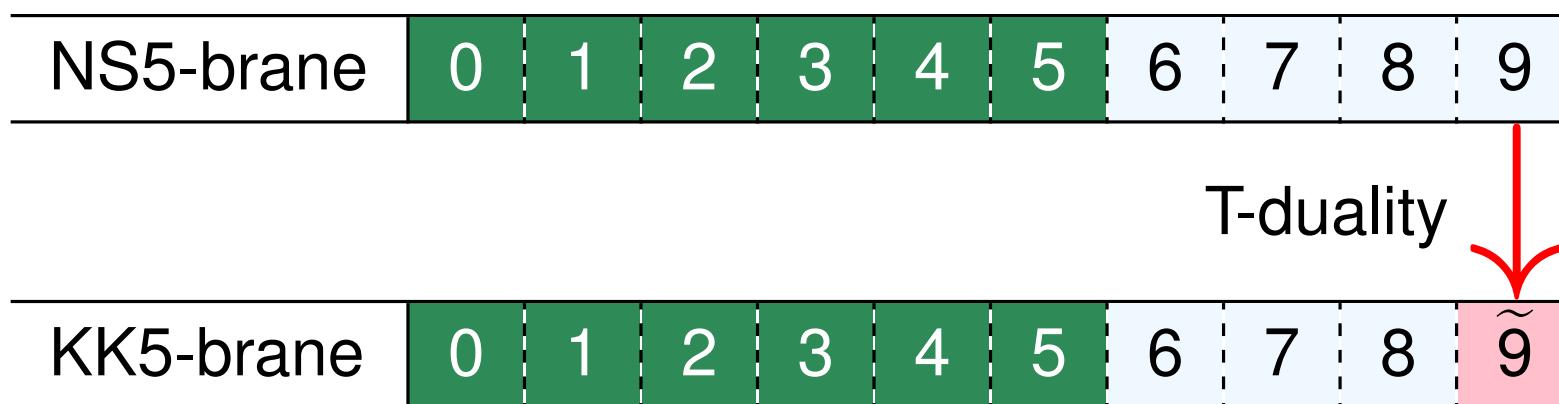
Exotic 5_2^2 -brane is constructed from [NS5-brane](#).

NS5-brane	0	1	2	3	4	5	6	7	8	9
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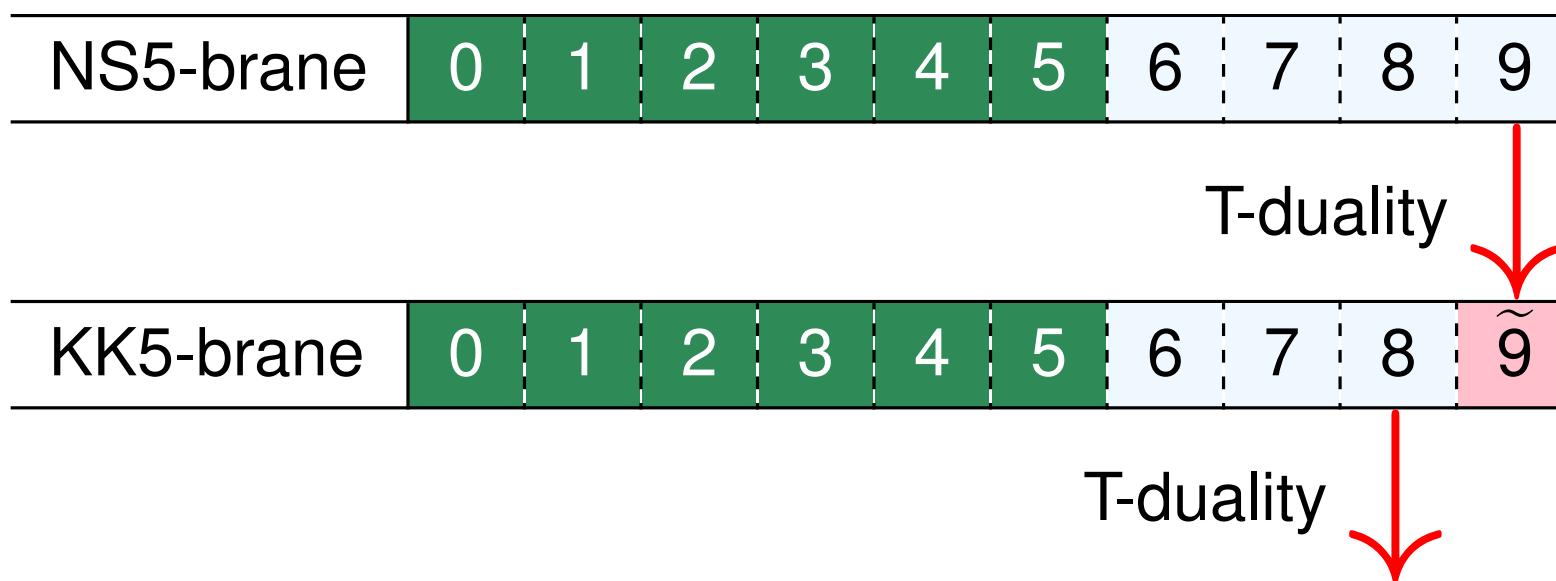
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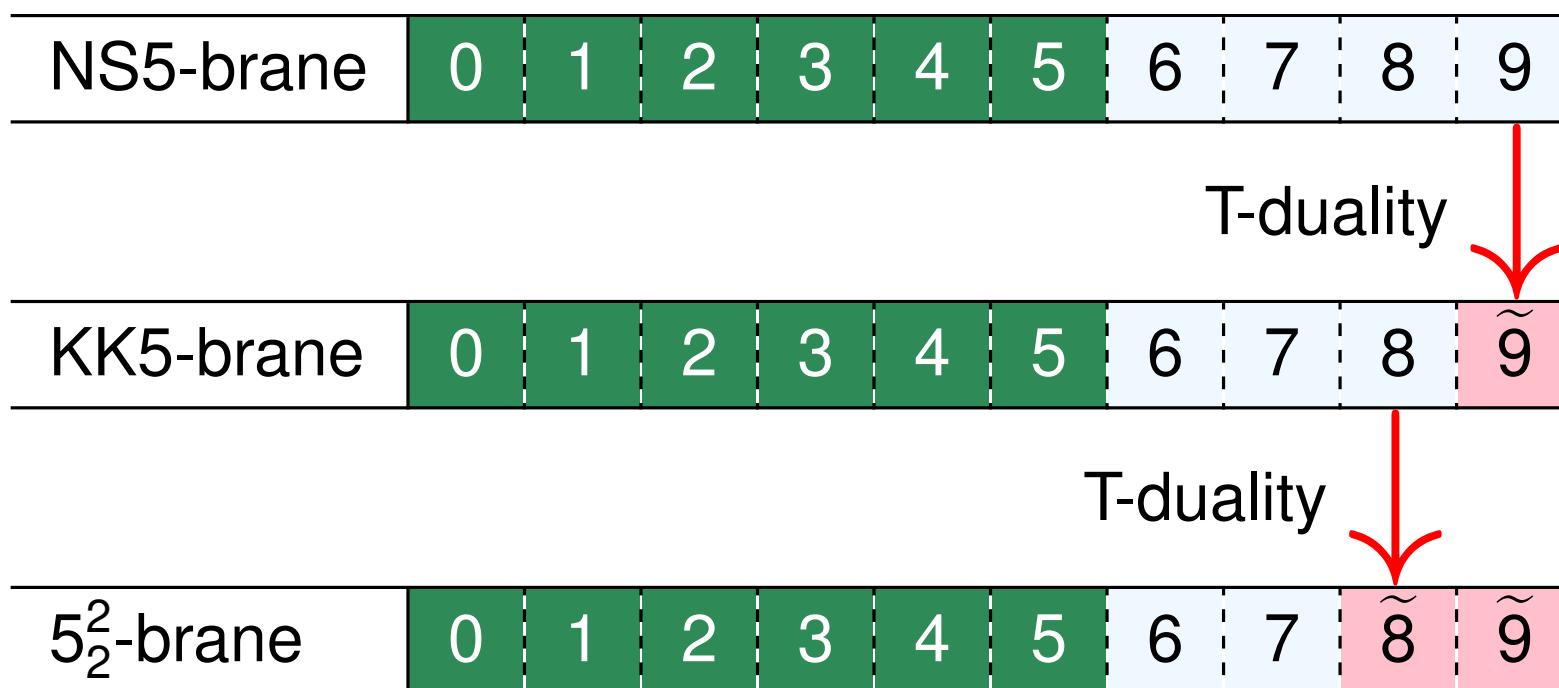
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Exotic 5_2^2 -brane is constructed from **NS5-brane**.

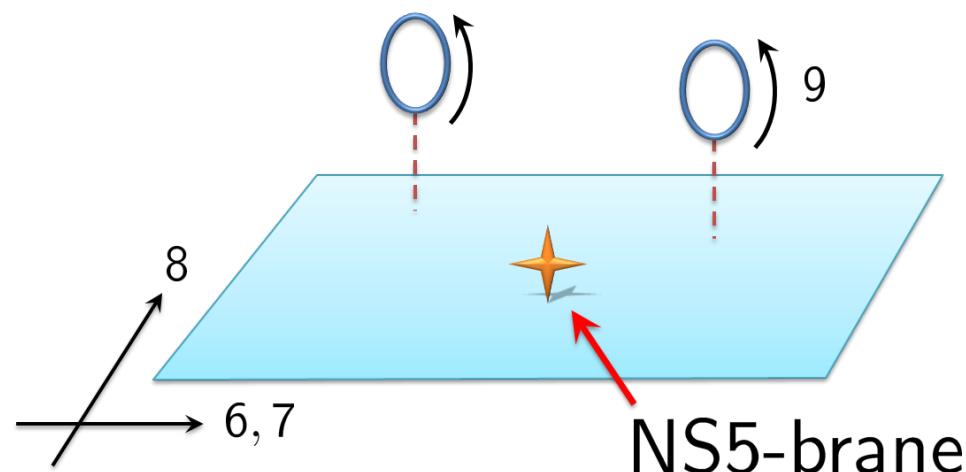


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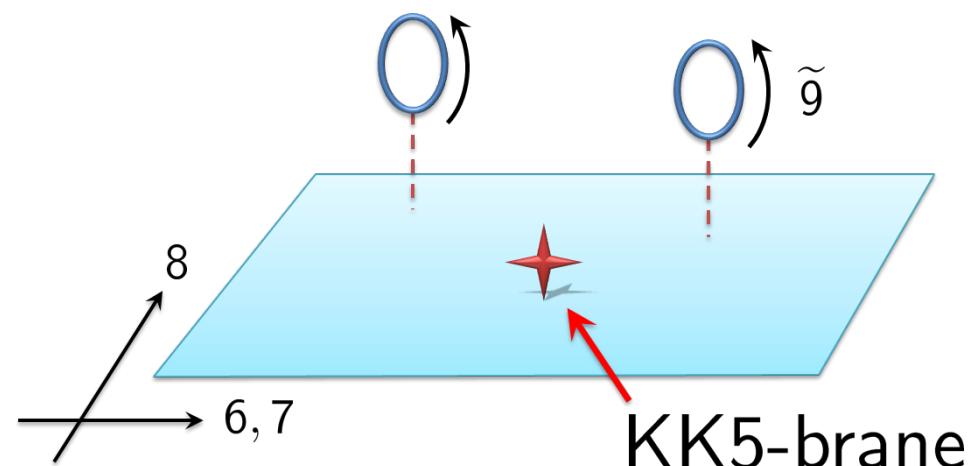
NS5-brane (smeared along 9th-direction)

- co-dim. 3 ($\mathbb{R}^3 \times S^1$, $\vec{x} \in \mathbb{R}^3$)
- $ds^2 = dx_{012345}^2 + H \left[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2 \right]$
- $H = 1 + \frac{1}{|\vec{x}|}$
- $H_{mnp} = \epsilon_{mnp}{}^q \partial_q \log H$, $e^{2\Phi} = H$



KK5-brane (Taub-NUT space)

- co-dim. 3 ($\mathbb{R}^3 \times \widetilde{S}^1$, $\vec{x} \in \mathbb{R}^3$)
- $ds^2 = dx_{012345}^2 + H[(dx^6)^2 + (dx^7)^2 + (dx^8)^2] + \frac{1}{H}(d\tilde{x}^9 - \vec{V} \cdot d\vec{x})^2$
- $H = 1 + \frac{1}{|\vec{x}|}$, $\vec{\nabla}H = \vec{\nabla} \times \vec{V}$
- $H_{mnp} = 0 = \Phi$



5_2^2 -brane : co-dim. 2

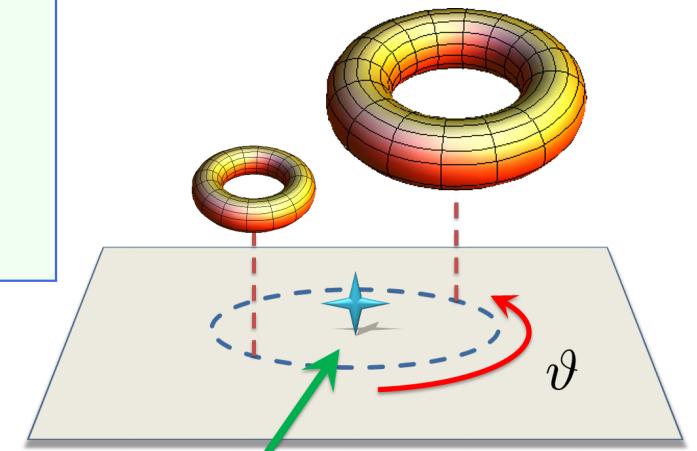
$$ds^2 = dx_{012345}^2 + H[\mathrm{d}\varrho^2 + \varrho^2(\mathrm{d}\vartheta)^2] + \frac{H}{K}[(\mathrm{d}\tilde{x}^8)^2 + (\mathrm{d}\tilde{x}^9)^2]$$

$$B_{89} = -\frac{\sigma \vartheta}{K}, \quad e^{2\Phi} = \frac{H}{K}, \quad K = H^2 + (\sigma \vartheta)^2$$

$$H = 1 + \sigma \log \frac{\Lambda}{\varrho}$$

$$\vartheta = 0 : G_{88} = G_{99} = \frac{1}{H}$$

$$\vartheta = 2\pi : G_{88} = G_{99} = \frac{H}{H^2 + (2\pi\sigma)^2}$$



We never remove the shift $2\pi\sigma$ by $\left\{ \begin{array}{l} \text{coordinate transformations,} \\ \text{B-field gauge transformation.} \end{array} \right.$

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Should we abandon it ??

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Should we abandon it ??

No!

“Supergravity limit” is not globally well-defined.

ex.) D7-branes

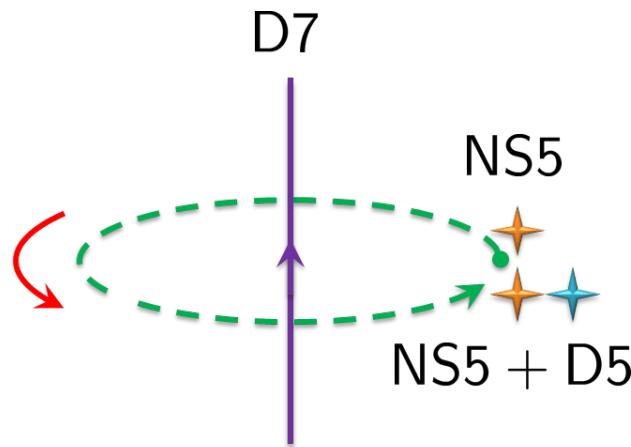
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2, \mathbb{Z})$ monodromy charge q

$$q : (C_2, B_2) \rightarrow (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

$$(0, \text{NS5}) \rightarrow (\text{D5}, \text{NS5})$$



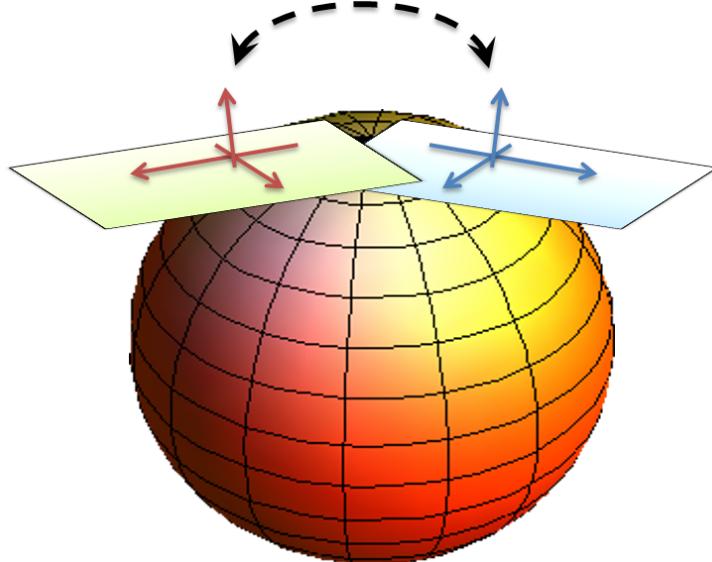
an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

Non-geometric structure

structure groups = diffeom. ($GL(d, \mathbb{R})$) + String duality groups

↑
T-duality, U-duality, etc.

$GL(d, \mathbb{R}) \cup$ duality transf.



Generalized Geometry (N. Hitchin)
 Doubled Geometry (C. Hull)

$5\frac{1}{2}$ -brane is a **concrete** example (T-fold).
 (D7 is an example of S-fold.)

Exotic brane shows us a new insight of stringy spacetime

Exotic 5_2^2 -brane has been analyzed in spacetime (SUGRA) picture.

Ready to study **string worldsheet** picture!

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Ready to study **string worldsheet** picture!



nonlinear sigma model (NLSM)



conformal field theory (CFT)



gauged linear sigma model (GLSM)

Quantum aspects of NLSM/CFT can be traced by **Gauge fields** in GLSM

Indeed, GLSM is the UV completion of string worldsheet theory

GLSM

Gauged Linear Sigma Model (2D SUSY gauge theory)

S. Sasaki and TK arXiv:1304.4061, 1305.4439, 1310.6163

M. Yata and TK arXiv:1402.5580, 1406.0087

GLSM is a powerful tool to explore non-trivial configurations in string theory :
(as the UV completion of string worldsheet theory)



Kähler and Calabi-Yau spaces
 $\mathcal{N} = (2, 2)$ SUSY



ALE, ALF spaces (hyper-Kähler) and NS5-branes
 $\mathcal{N} = (4, 4)$ SUSY

In particular, it is important to study T-duality of their configurations.
(related to stringy “nongeometric” backgrounds)

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In particular, it is important to study T-duality of their configurations.
(related to stringy “nongeometric” backgrounds)

T-duality transformations

(free) string	sign flip (parity) in right-mover	momentum \leftrightarrow winding
spacetime	Buscher rule	$(G_{IJ}, B_{IJ}) \leftrightarrow (G'_{IJ}, B'_{IJ})$
SUSY sigma model	Roček-Verlinde formula	chiral \leftrightarrow twisted chiral

Duality transformations with **F**-term

in $\mathcal{N} = (2, 2)$ framework

Duality : chiral Ψ \longleftrightarrow twisted chiral Y

Target space geometry is T-dualized under the above transformation.

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- dualize **neutral** chiral in D-term (ex. torus)

$$\int d^4\theta |\Psi|^2 \leftrightarrow \int d^4\theta \left[\frac{1}{2}R^2 - R(Y + \bar{Y}) \right] \leftrightarrow - \int d^4\theta |Y|^2$$

- dualize **charged** chiral in D-term (ex. projective space)

$$\int d^4\theta |\Psi|^2 e^{2V} \leftrightarrow \int d^4\theta \left[e^{2V+R} - R(Y + \bar{Y}) \right] \leftrightarrow - \int d^4\theta (Y + \bar{Y}) \left[\log(Y + \bar{Y}) - 2V \right]$$

These are well established.

“Global symmetry” $\Psi + \alpha$ (or $e^{i\alpha} \Psi$) is preserved in D-term, but broken in F-term.

— Make sense? —

- ▶ dualize neutral / charged chiral in D-term and F-term

They can be interpreted as T-duality transformations
under conversion from F-term to D-term with trick(es).

- Dualize **neutral** chiral in D-term and **F**-term

$$\mathcal{L}_1 = \int d^4\theta |\Psi|^2 + \left\{ \int d^2\theta \Psi \mathcal{W}(\Phi) + (\text{h.c.}) \right\}$$

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1. Convert **F**-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int d^2\theta \Psi \mathcal{W} + (\text{h.c.}) = \int d^4\theta \left[(\Psi + \overline{\Psi})(C + \overline{C}) + (\Psi - \overline{\Psi})(C - \overline{C}) \right]$$

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2. Replace $\Psi \pm \overline{\Psi}$ to auxiliary fields **R** and **iS** :

$$\mathcal{L}_2 = \int d^4\theta \left[\frac{1}{2} R^2 + R(C + \overline{C}) + iS(C - \overline{C}) - R(Y + \overline{Y}) - iS(\Upsilon - \overline{\Upsilon}) \right]$$

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3. Integrating out R and Υ , we obtain the “dual” system :

$$\mathcal{L}_3 = \int d^4\theta \left[-\frac{1}{2} \left((Y + \overline{Y}) + (C + \overline{C}) \right)^2 + (\Psi - \overline{\Psi})(C - \overline{C}) \right]$$

Instead, integrate out Y and $\Upsilon \rightarrow \mathcal{L}_1$ appears

$\Psi - \bar{\Psi}$ still remains **after** the transformation.

The existence is rather important to complete the Duality transformations.

$\Psi - \bar{\Psi}$ appears as an auxiliary field, which **must be removed** finally.

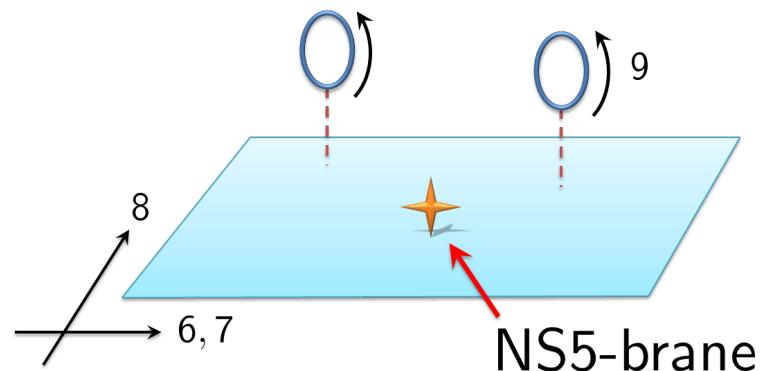
Indeed, the procedure “**integrating-out of $\Psi - \bar{\Psi}$** ” leads to
the correct involution of the dual fields in the system !

The charged chiral case is same as this.

GLSM for five-branes

NS5-branes, KK5-branes and exotic 5_2^2 -brane

NS5-brane

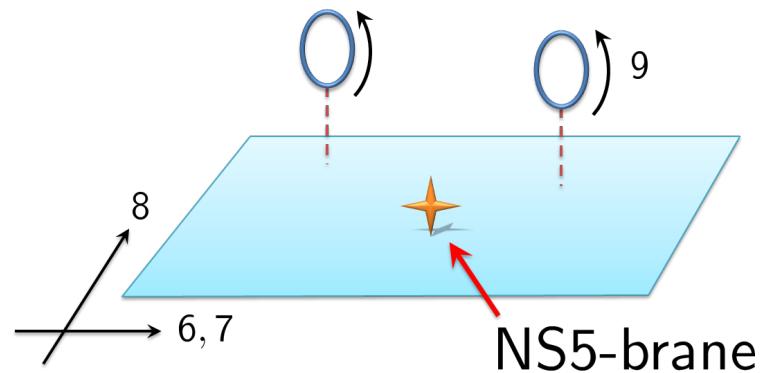


$$\begin{aligned}\mathcal{L} = & \int d^4\theta \left\{ \frac{1}{e^2}(|\Phi|^2 - |\Sigma|^2) + |Q|^2 e^{+2V} + |\tilde{Q}|^2 e^{-2V} \right\} \\ & + \frac{1}{g^2} \int d^4\theta \left\{ |\Psi|^2 - |\Theta|^2 \right\} \\ & + \left\{ \int d^2\theta \left(-\tilde{Q}\Phi Q + (s - \Psi)\Phi \right) + (\text{h.c.}) \right\} \\ & + \left\{ \int d^2\tilde{\theta} \left((t - \Theta)\Sigma \right) + (\text{h.c.}) \right\}\end{aligned}$$

$$X^6 + iX^8 \in \Psi \text{ (chiral)}, \quad X^7 + iX^9 \in \Theta \text{ (twisted chiral)}$$

D. Tong [hep-th/0204186](https://arxiv.org/abs/hep-th/0204186)

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Position is labelled by FI parameters (s, t)

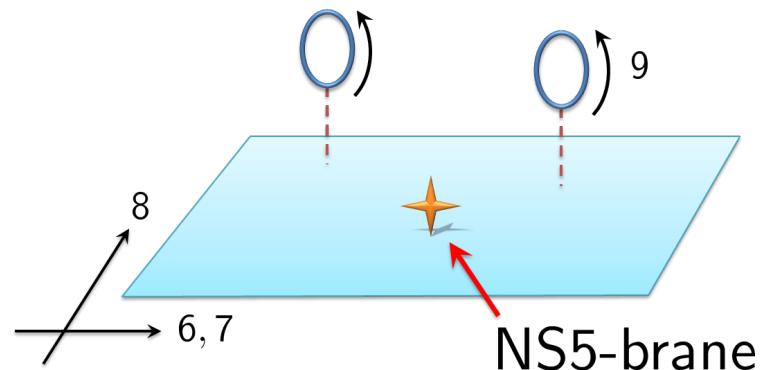
IR limit \rightarrow gauge multiplets are integrated out \rightarrow NLSM on NS5-brane geometry

$$\mathcal{L}_{\text{NS5}}^{\text{NLSM}} = -\frac{H}{2} \left[(\partial_m \vec{X})^2 + (\partial_m X^9)^2 \right] + \varepsilon^{mn} \Omega_i \partial_m X^i \partial_n X^9$$

Target space geometry is

$G_{IJ} = H \delta_{IJ}$	$B_{i9} = \Omega_i$
$H = \frac{1}{g^2} + \frac{1}{ \vec{X} }$	$\nabla_i H = (\nabla \times \Omega)_i$

NS5-brane

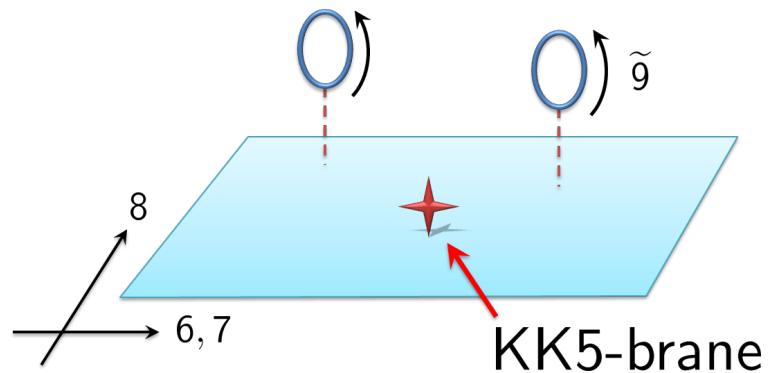


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Dualize the neutral twisted chiral $\Theta \rightarrow \Gamma$: T-duality along 9th-direction

KK5-brane = Taub-NUT space (ALF)

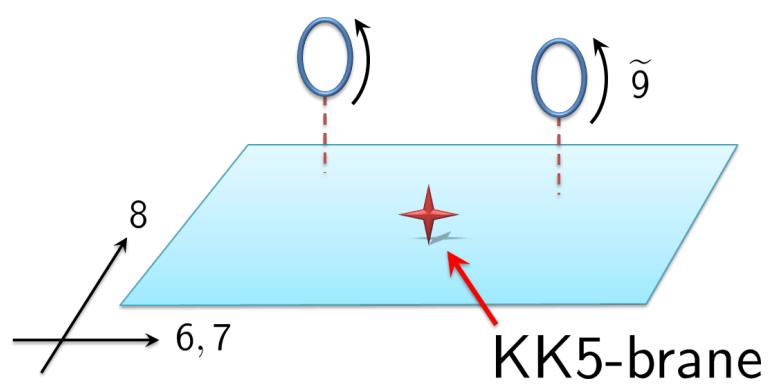


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IR limit \rightarrow gauge multiplets are integrated out \rightarrow NLSM on Taub-NUT space

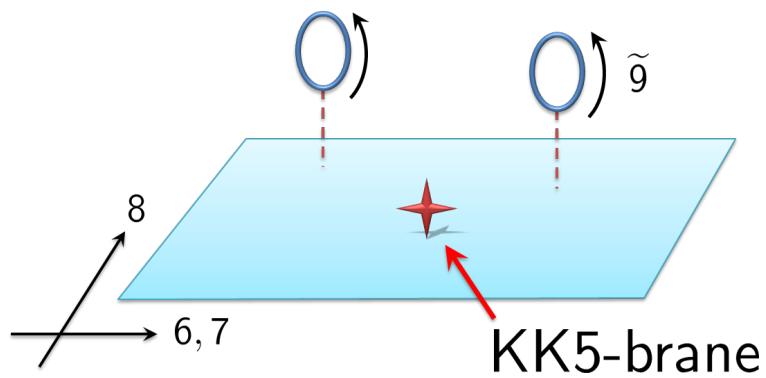
$$\begin{aligned}\mathcal{L}_{\text{KK5}}^{\text{NLSM}} &= -\frac{H}{2}(\partial_m \vec{X})^2 - \frac{1}{2H}(\partial_m \tilde{X}^9 - \Omega_i \partial_m X^i)^2 \\ &\quad + \varepsilon^{mn} \partial_m ((\textcolor{blue}{X}^9 - t^9) A_n)\end{aligned}$$

Target space geometry is

$$G_{IJ} = \text{Taub-NUT} \quad B_{i9} = 0$$

$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|} \quad \nabla_i H = (\nabla \times \Omega)_i$$

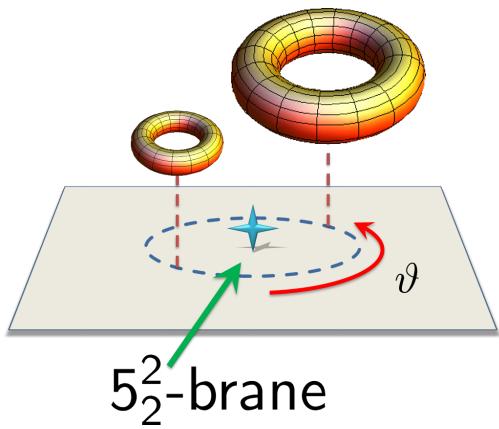
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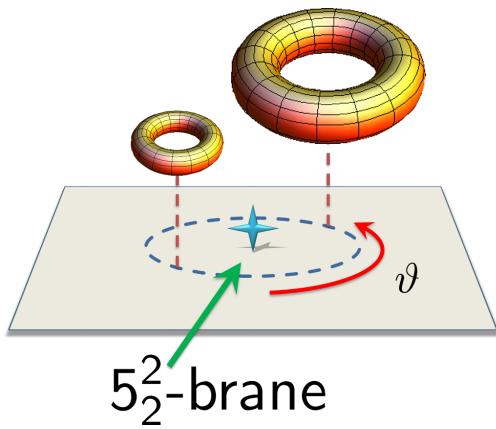
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Dualize the neutral chiral $\Psi \rightarrow \Xi$: T-duality along 8th-direction



$$\begin{aligned}
 \mathcal{L} = & \int d^4\theta \left\{ \frac{1}{e^2} (|\Phi|^2 - |\Sigma|^2) + |Q|^2 e^{+2V} + |\tilde{Q}|^2 e^{-2V} \right\} \\
 & + \int d^4\theta \left\{ -\frac{g^2}{2} (\Xi + \bar{\Xi} - (C + \bar{C}))^2 + \frac{g^2}{2} (\Gamma + \bar{\Gamma} + V)^2 \right\} \\
 & + \left\{ \int d^2\theta \left(-\tilde{Q}\Phi Q + s\Phi \right) + (\text{h.c.}) \right\} - \int d^4\theta (\Psi - \bar{\Psi})(C - \bar{C}) \\
 & + \left\{ \int d^2\tilde{\theta} (t\Sigma) + (\text{h.c.}) \right\} + \varepsilon^{mn} \partial_m (X^9 A_n)
 \end{aligned}$$

$$X^6 + i\tilde{X}^8 \in \Xi \text{ (twisted chiral)}, \quad X^7 + i\tilde{X}^9 \in \Gamma \text{ (chiral)}$$



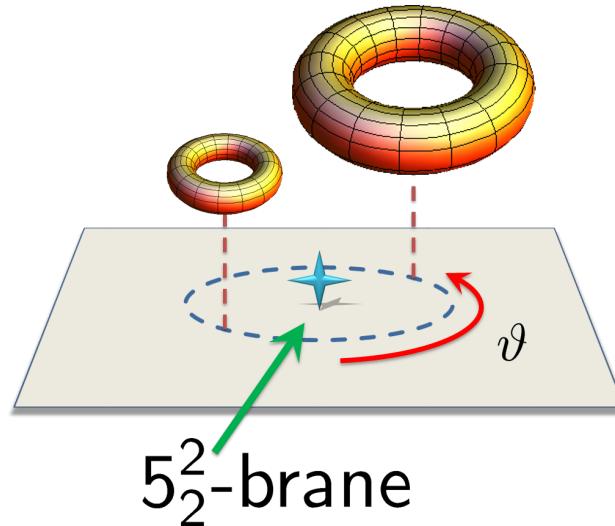
$$\begin{aligned}
 \mathcal{L} = & \int d^4\theta \left\{ \frac{1}{e^2} (|\Phi|^2 - |\Sigma|^2) + |Q|^2 e^{+2V} + |\tilde{Q}|^2 e^{-2V} \right\} \\
 & + \int d^4\theta \left\{ -\frac{g^2}{2} (\Xi + \bar{\Xi} - (C + \bar{C}))^2 + \frac{g^2}{2} (\Gamma + \bar{\Gamma} + V)^2 \right\} \\
 & + \left\{ \int d^2\theta \left(-\tilde{Q}\Phi Q + s\Phi \right) + (\text{h.c.}) \right\} - \int d^4\theta (\Psi - \bar{\Psi})(C - \bar{C}) \\
 & + \left\{ \int d^2\tilde{\theta} (t\Sigma) + (\text{h.c.}) \right\} + \varepsilon^{mn} \partial_m (X^9 A_n)
 \end{aligned}$$

$$X^6 + i\tilde{X}^8 \in \Xi \text{ (twisted chiral)}, \quad X^7 + i\tilde{X}^9 \in \Gamma \text{ (chiral)}$$

$\Psi - \bar{\Psi}$ is no longer dynamical $\rightarrow \begin{cases} \Xi + \bar{\Xi} : \text{ momentum modes} \\ \Psi - \bar{\Psi} : \text{ winding modes!} \end{cases}$

IR limit \rightarrow gauge multiplets are integrated out
 Further, integrate out $\Psi - \bar{\Psi}$: Exotic Five-brane!

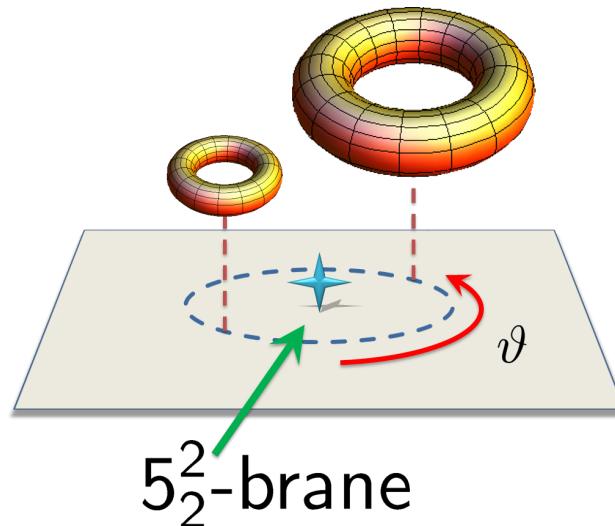
$$\begin{aligned}\mathcal{L}_{\text{exotic}}^{\text{NLSM}} = & -\frac{H}{2} \left[(\partial_m \varrho)^2 + \varrho^2 (\partial_m \vartheta)^2 \right] - \frac{H}{2K} \left[(\partial_m \tilde{X}^8)^2 + (\partial_m \tilde{X}^9)^2 \right] \\ & - \frac{\sigma \vartheta}{K} \varepsilon^{mn} (\partial_m \tilde{X}^8) (\partial_n \tilde{X}^9) + \varepsilon^{mn} \partial_m ((\textcolor{blue}{X}^9 - t^9) A_n)\end{aligned}$$



We obtained NLSM for **Exotic** 5_2^2 -brane.

Exotic feature comes from the **string winding modes**.

$$\begin{aligned}\mathcal{L}_{\text{exotic}}^{\text{NLSM}} = & -\frac{H}{2} \left[(\partial_m \varrho)^2 + \varrho^2 (\partial_m \vartheta)^2 \right] - \frac{H}{2K} \left[(\partial_m \tilde{X}^8)^2 + (\partial_m \tilde{X}^9)^2 \right] \\ & - \frac{\sigma \vartheta}{K} \varepsilon^{mn} (\partial_m \tilde{X}^8) (\partial_n \tilde{X}^9) + \varepsilon^{mn} \partial_m ((\textcolor{blue}{X}^9 - t^9) A_n)\end{aligned}$$



Spacetime picture involving string winding modes

beyond the Einstein's view of gravity !

Quantum corrections to five-branes

S. Sasaki and TK arXiv:1305.4439

String Worldsheet Instanton Corrections



Deform target space geometry by momentum and/or winding effects

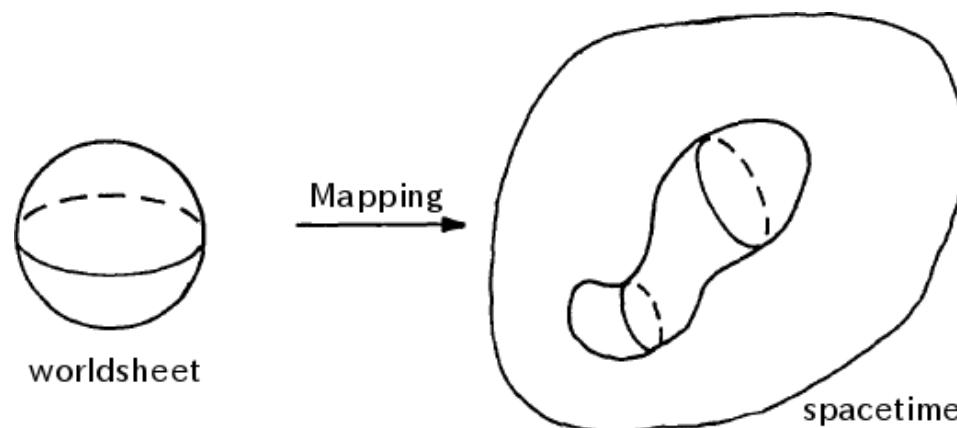


Figure from E. Witten CMP 92 (1984) 455

Again, GLSM is a powerful tool :

Worldsheet instantons in NLSM can be captured by
vortex solution in gauge theory

► NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|} : \text{radius of } X^9$$

$g \rightarrow 0$ case	general $ \vec{X} $	$g \rightarrow \infty$ case	$ \vec{X} \rightarrow 0$	$ \vec{X} \rightarrow \infty$
radius of X^9	∞	radius of X^9	∞	0
KK-modes	light	KK-modes	light	heavy
winding modes	heavy	winding modes	heavy	light

► NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|} : \text{radius of } X^9$$

$g \rightarrow 0$ case	general $ \vec{X} $	$g \rightarrow \infty$ case	$ \vec{X} \rightarrow 0$	$ \vec{X} \rightarrow \infty$
radius of X^9	∞	radius of X^9	∞	0
KK-modes	light	KK-modes	light	heavy
winding modes	heavy	winding modes	heavy	light

GLSM for NS5-brane has $X^9 F_{01}$

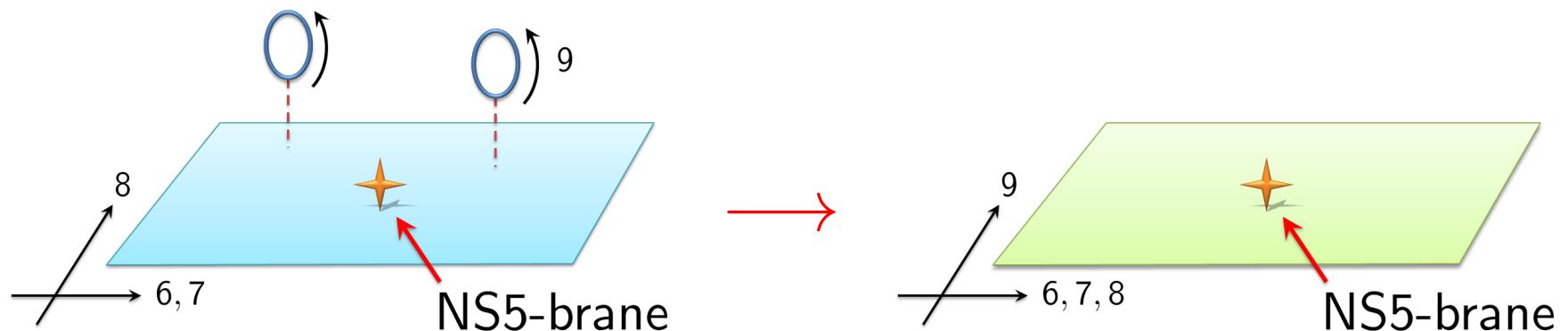


KK-mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to NS5-brane :

$$X^9 F_{01} \text{ in GLSM}$$

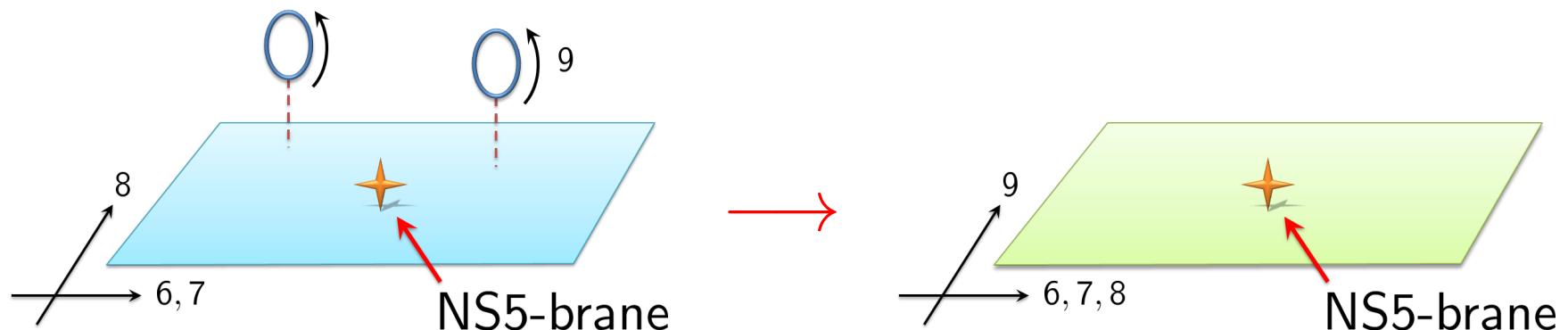
→ unfolding effect on compactified circle X^9



Worldsheet instanton corrections to NS5-brane :

$$X^9 F_{01} \text{ in GLSM}$$

→ unfolding effect on compactified circle X^9



$$\begin{aligned} H &= \frac{1}{g^2} + \frac{1}{|\vec{X}|} \rightarrow \frac{1}{g^2} + \frac{1}{|\vec{X}|} \sum_{n=1}^{\infty} e^{-n|\vec{X}|} [e^{+i n X^9} + e^{-i n X^9}] \\ &= \frac{1}{g^2} + \frac{1}{|\vec{X}|} \frac{\sinh(|\vec{X}|)}{\cosh(|\vec{X}|) - \cos(X^9)} \end{aligned}$$

D. Tong hep-th/0204186

► KK5-brane

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{|\vec{X}|} \right)^{-1} : \text{radius of } \tilde{X}^9$$

$g \rightarrow 0$ case	general $ \vec{X} $	$g \rightarrow \infty$ case	$ \vec{X} \rightarrow 0$	$ \vec{X} \rightarrow \infty$
radius of \tilde{X}^9	0	radius of \tilde{X}^9	0	∞
KK-modes	heavy	KK-modes	heavy	light
winding modes	light	winding modes	light	heavy

► KK5-brane

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{|\vec{X}|} \right)^{-1} : \text{radius of } \tilde{X}^9$$

$g \rightarrow 0$ case	general $ \vec{X} $	$g \rightarrow \infty$ case	$ \vec{X} \rightarrow 0$	$ \vec{X} \rightarrow \infty$
radius of \tilde{X}^9	0	radius of \tilde{X}^9	0	∞
KK-modes	heavy	KK-modes	heavy	light
winding modes	light	winding modes	light	heavy

GLSM for KK5-brane has $\varepsilon^{mn} \partial_m (\tilde{X}^9 A_n)$

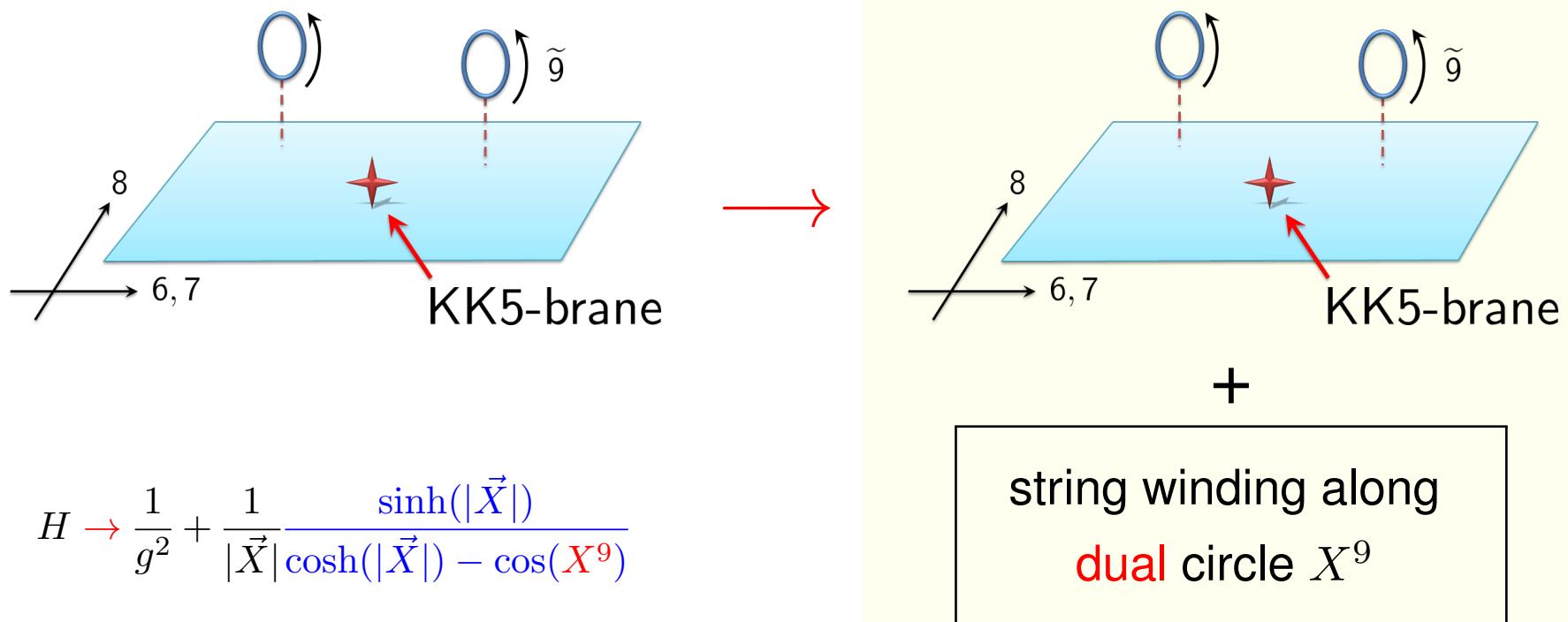


Winding mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to KK5-brane :

$$\varepsilon^{mn} \partial_m (\textcolor{blue}{X}^9 A_n) \text{ in GLSM}$$

→ string **winding modes** along X^9



D. Tong hep-th/0204186; J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097

► 5_2^2 -brane

$$\frac{H}{K} : \text{radius of } \tilde{X}^9 \quad H = \frac{1}{g^2} + \sigma \log \frac{\Lambda}{\varrho}, K = H^2 + (\sigma\vartheta)^2$$

$g \rightarrow 0$ case	general ϱ	$g \rightarrow \infty$ case	$\varrho \rightarrow 0$	$\varrho \rightarrow \Lambda$
radius of \tilde{X}^9	0	radius of \tilde{X}^9	0	0
KK-modes	heavy	KK-modes	heavy	heavy
winding modes	light	winding modes	light	light at $\vartheta = 0$

► 5_2^2 -brane

$$\frac{H}{K} : \text{radius of } \tilde{X}^9 \quad H = \frac{1}{g^2} + \sigma \log \frac{\Lambda}{\varrho}, K = H^2 + (\sigma \vartheta)^2$$

$g \rightarrow 0$ case	general ϱ	$g \rightarrow \infty$ case	$\varrho \rightarrow 0$	$\varrho \rightarrow \Lambda$
radius of \tilde{X}^9	0	radius of \tilde{X}^9	0	∞ at $\vartheta = 0$
KK-modes	heavy	KK-modes	heavy	heavy at $\vartheta = 0$
winding modes	light	winding modes	light	heavy at $\vartheta = 0$

GLSM for 5_2^2 with one gauged isometry has $\varepsilon^{mn} \partial_m (\tilde{X}^9 A_n)$

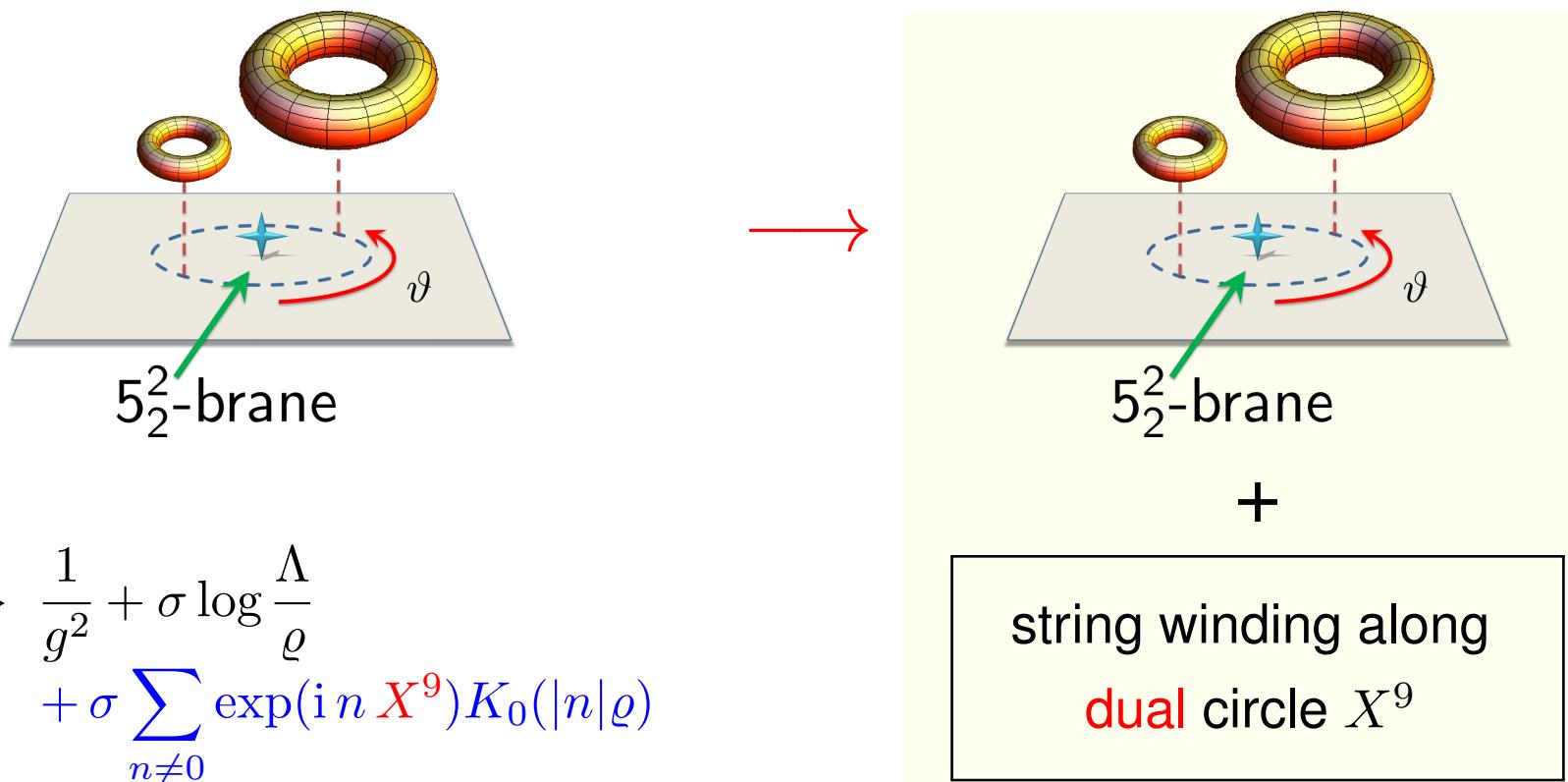


Winding mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to 5_2^2 -brane with **one** gauged isometry :

$$\varepsilon^{mn} \partial_m (X^9 A_n) \text{ in GLSM}$$

→ string **winding modes** along X^9



S. Sasaki and TK arXiv:1305.4439

Result

Quantum corrections can be traced by Vortices in GLSM:

$$\mathcal{L}_{\text{topological}}^{\text{GLSM}} = \textcolor{blue}{X}^9 F_{01} + \dots$$

$$H = 1 + \sigma \log \left(\frac{\Lambda}{\varrho} \right) \rightarrow 1 + \sigma \log \left(\frac{\Lambda}{\varrho} \right) + \sum_{n \neq 0} \exp(in\textcolor{blue}{X}^9) K_0(|n| \varrho)$$

$K_0(x)$: modified Bessel function of the second kind

	physical coordinate in NS5
$\textcolor{blue}{X}^9$ is ...	dual coordinate in KK5
	dual coordinate in 5_2^2

S. Sasaki and TK arXiv:1305.4439

Summary and Discussions

We enjoy studying exotic branes, right now!

- Exotic branes are strange objects from supergravity regime.
- We revealed that the exotic feature originates from the string winding modes by virtue of GLSM (the UV completion of string worldsheet theory)!
- We also evaluated string worldsheet instanton corrections to exotic branes.
- Worldvolume description for various five-branes arXiv:1404.5442
- Monodromy structures, composite states, & global description
 - defect (p, q) five-branes similar/different from D7-branes
arXiv:1410.8403, 1411.3457 (local)
cf.) S. Hellerman, J. McGreevy and B. Williams hep-th/0208174 (global)

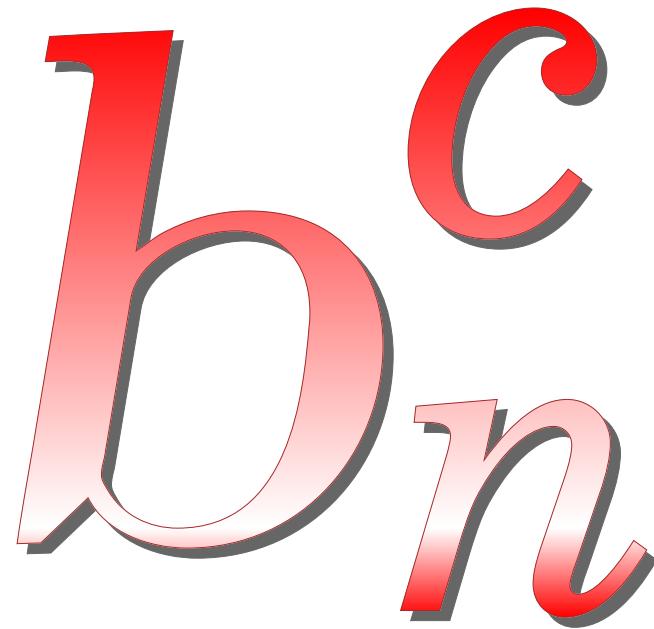
Defect/exotic branes in diverse dimensions

	fundamental	Dirichlet	solitonic	S_D -dual of (Dirichlet)	S_D -dual of (fundamental)
D	$\alpha = 0$	$\alpha = -1$	$\alpha = -2$	$\alpha = -3$	$\alpha = -4$
IIB		C_8 [D7]		$E_8 = S_{10}(C_8)$ [7 ₃]	
9		C_7 [D6]		$E_{8,1} = S_9(C_7)$ [6 ₃ ¹]	
8		C_6 [D5]	D_6 [NS5] $D_{7,1}$ [KK5] $D_{8,2}$ [5 ₂ ²]	$E_{8,2} = S_8(C_7)$ [5 ₃ ²]	
7		C_5 [D4]		$E_{8,3} = S_7(C_5)$ [4 ₃ ³]	
6		C_4 [D3]		$E_{8,4} = S_6(C_4)$ [3 ₃ ⁴]	
5		C_3 [D2]		$E_{8,5} = S_5(C_3)$ [2 ₃ ⁵]	
4	B_2 [F1]	C_2 [D1]		$E_{8,6} = S_4(C_2)$ [1 ₃ ⁶]	$F_{8,6} = S_4(B_2)$ [1 ₄ ⁶]
3	[P]	C_1 [D0]		$E_{8,7} = S_3(C_1)$ [0 ₃ ⁷]	$F_{8,7,1} [0_4^{(6,1)}]$

and more...

E.A. Bergshoeff et al arXiv:1009.4657, 1102.0934, 1108.5067

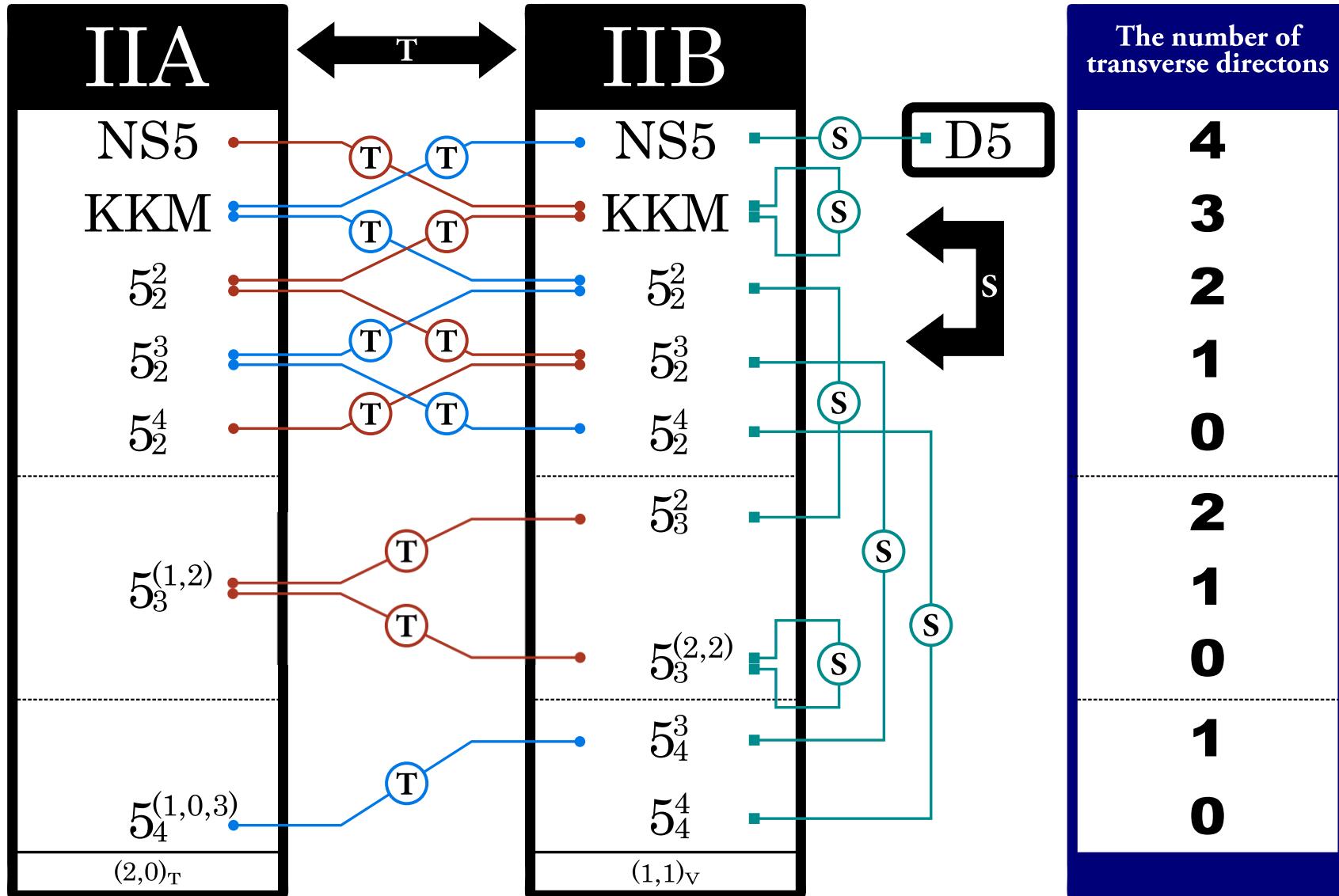
see also NU String Seminar on Jan 25, 2013



We would like to see a deeper insight of exotic branes
from various viewpoints,
in particular, from the GLSM framework !

Thanks

Appendix



S. Sasaki, M. Yata and TK arXiv:1404.5442

4D asymptotic geometry is locally T^k -fibration over \mathbb{R}^{4-k}

k	harmonic function	
ALE	0	$\frac{1}{ \vec{x} }$ \mathbb{C}^2/Γ with ADE-singularities
ALF	1	$A + \frac{1}{ \vec{x} }$ Taub-NUT
ALG	2	$A + B \log \varrho $ exotic objects such as D7, 5_2^2 , etc.
ALH	3	$A + B x $ linear potential

S.A. Cherkis and A. Kapustin hep-th/0006050, hep-th/0109141

LESSON 0 : GLSM for ALE and ALF spaces

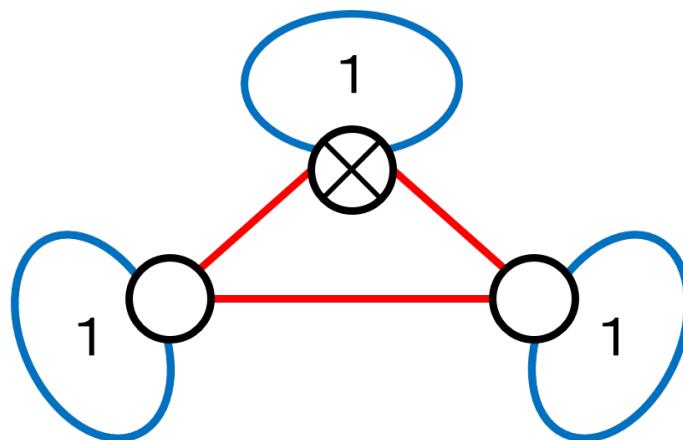
M.R. Douglas and G.W. Moore hep-th/9603167

C.V. Johnson and R.C. Myers hep-th/9610140

D. Tong hep-th/0204186

ADE-type singularities in ALE space (\mathbb{C}^2/Γ) can be illustrated as

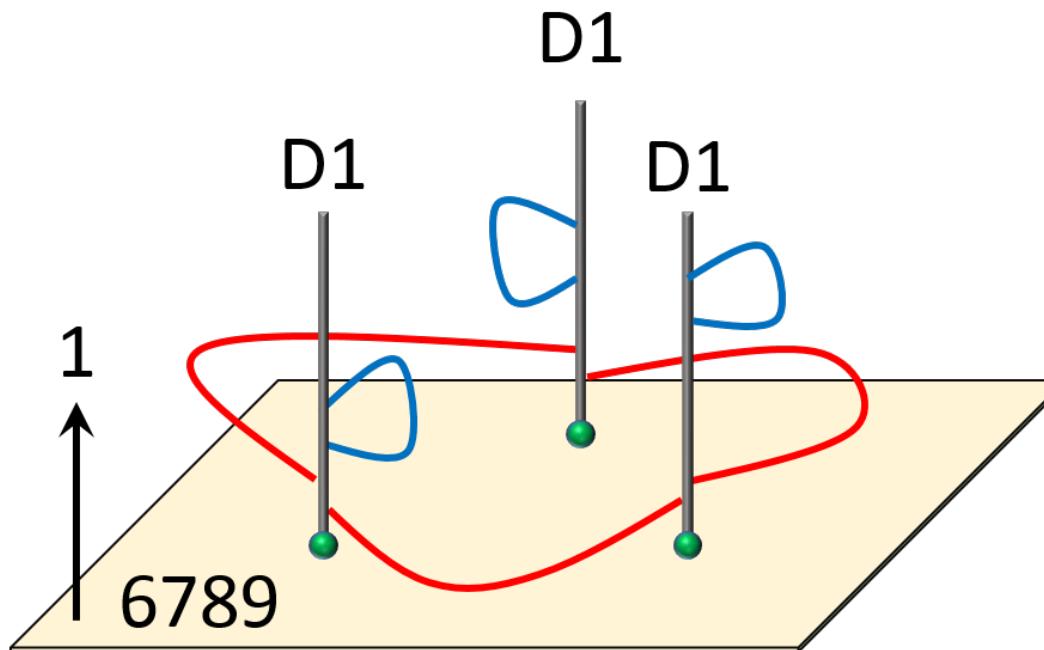
Affine ADE Dynkin diagram :



Each Affine ADE Dynkin diagram is mapped (identical) to
ADE quiver diagram.

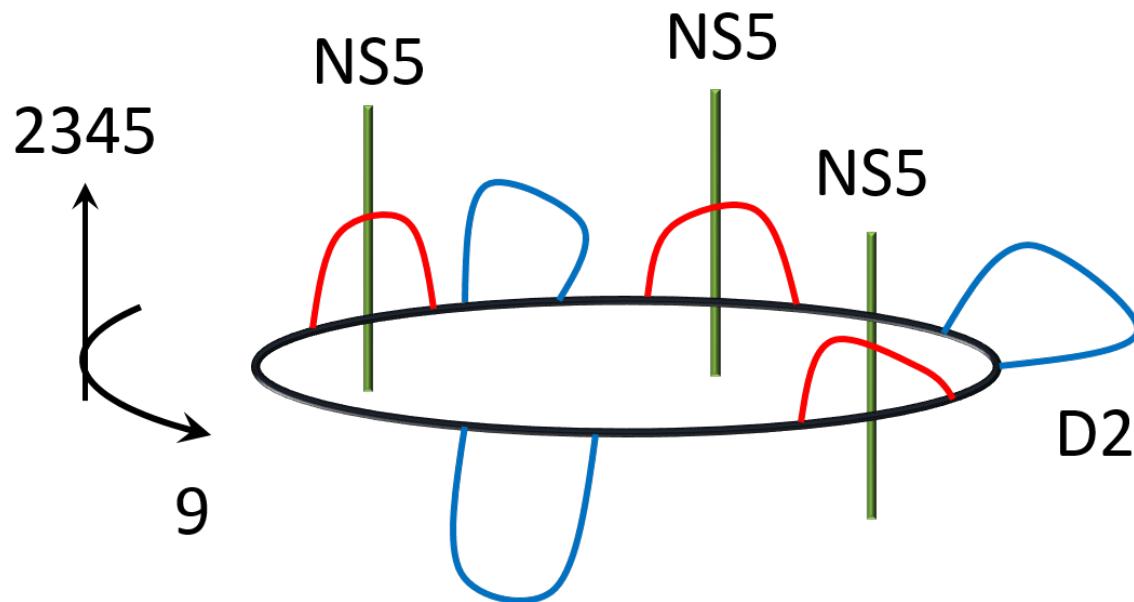
- nodes : gauge groups
- blue lines : matters (adjoint repr.)
- red lines : matters (bi-fundamental repr.)

The corresponding picture by D-branes is



- green nodes : singularity points in ALE space
- blue lines : open strings (adjoint repr.)
- red lines : open strings (bi-fundamental repr.)
- 6789-plane : ALE space (\mathbb{C}^2/Γ)

T-duality along 9th-direction :



Singularity points in ALE are mapped to
centers of NS5-branes.

(singularity points in ALE = centers of KK5-branes)

Field contents of $\mathcal{N} = (4, 4)$ GLSM for A_2 -type ALE space :

	$U(1)_1$	$U(1)_2$	$U(1)_3$	
$\omega_1 = (a_1, \bar{b}_1)$	+1	-1	0	$SU(2)$ moment map
$\omega_2 = (a_2, \bar{b}_2)$	0	+1	-1	$\vec{\mu}_1 = \omega_1^\dagger \vec{\tau} \omega_1 - \omega_3^\dagger \vec{\tau} \omega_3$
$\omega_3 = (a_3, \bar{b}_3)$	-1	0	+1	$\vec{\mu}_2 = \omega_2^\dagger \vec{\tau} \omega_2 - \omega_1^\dagger \vec{\tau} \omega_1$
(A_m^1, ϕ_1)	adj	-	-	$\vec{\mu}_3 = \omega_3^\dagger \vec{\tau} \omega_3 - \omega_2^\dagger \vec{\tau} \omega_2$
(A_m^2, ϕ_2)	-	adj	-	$0 = \vec{\mu}_1 + \vec{\mu}_2 + \vec{\mu}_3$
(A_m^3, ϕ_3)	-	-	adj	

Hyper-Kähler quotient construction

cf.) reconstruction by toric data: M. Yata and TK arXiv:1402.5580

Field contents of $\mathcal{N} = (4, 4)$ GLSM for A_2 -type **ALF** space :

	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_F$
$\omega_1 = (a_1, \bar{b}_1)$	+1	-1	0	+1
$\omega_2 = (a_2, \bar{b}_2)$	0	+1	-1	+1
$\omega_3 = (a_3, \bar{b}_3)$	-1	0	+1	+1
(A_m^1, ϕ_1)	adj	-	-	-
(A_m^2, ϕ_2)	-	adj	-	-
(A_m^3, ϕ_3)	-	-	adj	-
(X^6, X^7, X^8)	neutral			
(A_m^F, ϕ_F)	-	-	-	adj

$SU(2)$ moment map

$$\vec{\mu}_1 = \omega_1^\dagger \vec{\tau} \omega_1 - \omega_3^\dagger \vec{\tau} \omega_3$$

$$\vec{\mu}_2 = \omega_2^\dagger \vec{\tau} \omega_2 - \omega_1^\dagger \vec{\tau} \omega_1$$

$$\vec{\mu}_3 = \omega_3^\dagger \vec{\tau} \omega_3 - \omega_2^\dagger \vec{\tau} \omega_2$$

$$0 = \vec{\mu}_1 + \vec{\mu}_2 + \vec{\mu}_3$$

$$\vec{\mu}_F = \sum_{i=1}^3 \omega_i^\dagger \vec{\tau} \omega_i - \vec{X}$$

Diagonalize $U(1)_{i,F} \rightarrow$ [D. Tong hep-th/0204186](https://arxiv.org/abs/hep-th/0204186)

LESSON 1 : GLSM for NS5-brane

D. Tong hep-th/0204186

J.A. Harvey and S. Jensen hep-th/0507204

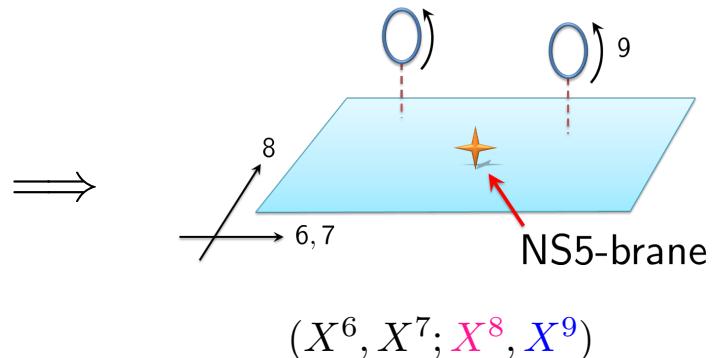
K. Okuyama hep-th/0508097

Bosonic Lagrangian after integrating-out auxiliary fields :

$$\begin{aligned}\mathcal{L}_{\text{NS5}}^{\text{kin}} &= \frac{1}{e^2} \left[\frac{1}{2} (F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2 \right] - \left[|D_m q|^2 + |D_m \tilde{q}|^2 \right] \\ &\quad - \frac{1}{2g^2} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2 \right] + (X^9 - t^9) F_{01} \\ \mathcal{L}_{\text{NS5}}^{\text{pot}} &= -2(|\sigma|^2 + |\phi|^2)(|q|^2 + |\tilde{q}|^2 + g^2) \\ &\quad - \frac{e^2}{2} \left(|q|^2 - |\tilde{q}|^2 - (X^7 - t^7) \right)^2 - e^2 \left| q\tilde{q} + (X^6 - s^6) + i(X^8 - s^8) \right|^2\end{aligned}$$

Steps to NLSM for NS5-brane

1. SUSY vacua $\mathcal{L}^{\text{pot}} = 0$
2. solve constraints on (q, \tilde{q})
3. IR limit $e \rightarrow \infty$, and integrate out A_m



1. SUSY vacua

$$\sigma = 0 = \phi , \quad |q|^2 - |\tilde{q}|^2 = X^7 - t^7 , \quad -q\tilde{q} = (X^6 - s^6) + i(X^8 - s^8)$$

2. solve constraints on (q, \tilde{q})

$$q = i e^{+i\alpha} \sqrt{|\vec{X}| + (X^7 - t^7)} , \quad \tilde{q} = i e^{-i\alpha_a} \frac{(X^6 - s^6) + i(X^8 - s^8)}{\sqrt{|\vec{X}| + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \tilde{q}|^2 = \frac{1}{2|\vec{X}|} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] + |\vec{X}| \left(\partial_m \alpha - A_m + \Omega_i \partial_m X^i \right)^2$$

$$|\vec{X}| = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2 + (X^8 - s^8)^2}$$

$$\Omega_i \partial_m X^i = \frac{-(X^6 - s^6) \partial_m X^8 + (X^8 - s^8) \partial_m X^6}{|\vec{X}|(|\vec{X}| + (X^7 - t^7))}$$

3. IR limit $e \rightarrow \infty$, and integrate out A_m

$$A_m = \Omega_i \partial_m X^i + \frac{1}{|\vec{X}|} \varepsilon_{mn} \partial^n X^9$$

$$\implies \mathcal{L}_{\text{NS5}}^{\text{NLSM}} = -\frac{1}{2} \left(\frac{1}{g^2} + \frac{1}{|\vec{X}|} \right) \left[(\partial_m \vec{X})^2 + (\partial_m X^9)^2 \right] + \varepsilon^{mn} \Omega_i \partial_m X^i \partial_n X^9$$

LESSON 2 : GLSM for KK5-brane

D. Tong hep-th/0204186

J.A. Harvey and S. Jensen hep-th/0507204

K. Okuyama hep-th/0508097

$\Theta \rightarrow \Gamma :$

$$\begin{aligned}\mathcal{L}_{\text{NS5}} \ni \mathcal{L}_\Theta &= \int d^4\theta \left(-\frac{1}{g^2} \overline{\Theta} \Theta \right) + \left\{ \int d^2\tilde{\theta} (-\Theta) \Sigma + (\text{h.c.}) \right\} \\ &= \int d^4\theta \left\{ -\frac{1}{2g^2} (\Theta + \overline{\Theta})^2 - (\Theta + \overline{\Theta}) V \right\} + \varepsilon^{mn} \partial_m (X^9 A_n)\end{aligned}$$

↓

$$\mathcal{L}_{B\Gamma} = \int d^4\theta \left\{ -\frac{1}{2g^2} \overline{B^2} - \overline{BV} - \overline{B}(\Gamma + \overline{\Gamma}) \right\} + \varepsilon^{mn} \partial_m (X^9 A_n)$$

real $\overline{B} = B$; chiral $\overline{D}_\pm \Gamma = 0$

$\Theta \rightarrow \Gamma :$

$$\mathcal{L}_{B\Gamma} = \int d^4\theta \left\{ -\frac{1}{2g^2}B^2 - BV - (\Gamma + \bar{\Gamma})B \right\} + \varepsilon^{mn} \partial_m (X^9 A_n)$$

Integrating out $\Gamma, \bar{\Gamma}$: \rightarrow GLSM for NS5-brane

$$B = \Theta + \bar{\Theta}$$

or, Integrating out B : \rightarrow GLSM for KK5-brane

$$\frac{1}{g^2}B = -(\Gamma + \bar{\Gamma}) - V$$

duality relation :

$$\Theta = X^7 + iX^9 + \dots, \quad \Gamma = \tilde{X}^7 + i\tilde{X}^9 + \dots$$

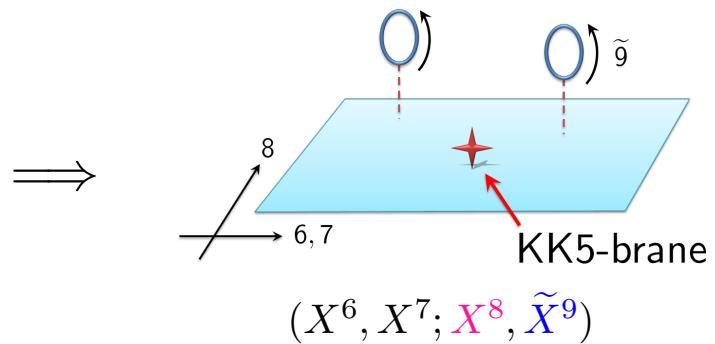
$$\Theta + \bar{\Theta} = -g^2(\Gamma + \bar{\Gamma}) - g^2V \rightarrow$$

$X^7 = -g^2 \tilde{X}^7$
$\pm(\partial_0 \pm \partial_1)X^9 = -g^2(D_0 \pm D_1)\tilde{X}^9$
$D_m \tilde{X}^9 = \partial_m \tilde{X}^9 - A_m$

$$\begin{aligned}
\mathcal{L}_{\text{KK5}} = & \int d^4\theta \left\{ \frac{1}{e^2} \left(-\bar{\Sigma}\Sigma + \bar{\Phi}\Phi \right) + \bar{Q} e^{+2V} Q + \bar{\tilde{Q}} e^{-2V} \tilde{Q} \right\} + \int d^4\theta \left\{ \frac{g^2}{2} \left(\Gamma + \bar{\Gamma} + V \right)^2 + \frac{1}{g^2} \bar{\Psi}\Psi \right\} \\
& + \int d^2\theta \left(-\tilde{Q}\Phi Q + (s - \Psi)\Phi \right) + (\text{h.c.}) \\
& + \left\{ \int d^2\tilde{\theta} t\Sigma + (\text{h.c.}) \right\} + \varepsilon^{mn} \partial_m (\textcolor{blue}{X}^9 A_n)
\end{aligned}$$

Steps to NLSM for KK5-brane

1. SUSY vacua $\mathcal{L}^{\text{pot}} = 0$
2. solve constraints on (q, \tilde{q})
3. IR limit $e \rightarrow \infty$, and integrate out A_m



1. SUSY vacua

$$\sigma = 0 = \phi , \quad |q|^2 - |\tilde{q}|^2 = X^7 - t^7 , \quad -q\tilde{q} = (X^6 - s^6) + i(X^8 - s^8)$$

2. solve constraints on (q, \tilde{q})

$$q = i e^{+i\alpha} \sqrt{|\vec{X}| + (X^7 - t^7)} , \quad \tilde{q} = i e^{-i\alpha} \frac{(X^6 - s^6) + i(X^8 - s^8)}{\sqrt{|\vec{X}| + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \tilde{q}|^2 = \frac{1}{2|\vec{X}|} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] + |\vec{X}| \left(\partial_m \alpha - A_m + \Omega_i \partial_m X^i \right)^2$$

3. IR limit $e \rightarrow \infty$, and integrate out A_m

$$A_m = \frac{1}{|\vec{X}|H} \left(\partial_m \tilde{X}^9 - \Omega_i \partial_m X^i \right) + \Omega_i \partial_m X^i , \quad H = \frac{1}{g^2} + \frac{1}{|\vec{X}|}$$

$$\mathcal{L}_{\text{KK5}}^{\text{NLSM}} = -\frac{H}{2} (\partial_m \vec{X})^2 - \frac{1}{2H} (\partial_m \tilde{X}^9 - \Omega_i \partial_m X^i)^2 + \varepsilon^{mn} \partial_m ((X^9 - t^9) A_n)$$

LESSON 3 : GLSM for 5_2^2 -brane

S. Sasaki and TK arXiv:1304.4061

M. Yata and TK arXiv:1406.0087

$\Psi \rightarrow \Xi :$

$$\begin{aligned}\mathcal{L}_{KK5} \ni \mathcal{L}_\Psi &= \int d^4\theta \frac{1}{g^2} \bar{\Psi} \Psi + \left\{ \int d^2\theta (-\Psi) \Phi + (\text{h.c.}) \right\} \\ &= \int d^4\theta \left\{ \frac{a}{g^2} (\Psi + \bar{\Psi})^2 - (\Psi + \bar{\Psi})(C + \bar{C}) \right\} \\ &\quad + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (\Psi - \bar{\Psi})^2 - (\Psi - \bar{\Psi})(C - \bar{C}) \right\}\end{aligned}$$

↓

$$\begin{aligned}\mathcal{L}_{RSX\Xi} &= \int d^4\theta \left\{ \frac{a}{g^2} \mathcal{R}^2 - \mathcal{R}(C + \bar{C}) + \mathcal{R}(\Xi_1 + \bar{\Xi}_1) + \mathcal{R}(X + \bar{X}) \right\} \\ &\quad + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (\mathbf{i}S)^2 - (\mathbf{i}S)(C - \bar{C}) + \mathbf{i}S(\Xi_2 - \bar{\Xi}_2) + \mathbf{i}S(X - \bar{X}) \right\}\end{aligned}$$

$$\bar{R} = R, \bar{S} = S, \bar{D}_+ \Xi_{1,2} = 0 = D_- \Xi_{1,2}, \bar{D}_\pm X = 0, \Phi_a = \bar{D}_+ \bar{D}_- C_a$$

$\Psi \rightarrow \Xi :$

$$\begin{aligned}\widetilde{\mathcal{L}} &= \int d^4\theta \left\{ \frac{a}{g^2} R^2 - R(C + \bar{C}) + R(\Xi_1 + \bar{\Xi}_1) + R(X + \bar{X}) \right\} \\ &\quad + \int d^4\theta \left\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - (\mathrm{i}S)(C - \bar{C}) + \mathrm{i}S(\Xi_2 - \bar{\Xi}_2) + \mathrm{i}S(X - \bar{X}) \right\}\end{aligned}$$

Integrating out $\Xi_1, \Xi_2, X : \rightarrow$ GLSM for KK5-brane

or, Integrating out $R, \Xi_2 : \rightarrow$ GLSM for 5_2^2 -brane

$$\frac{2a}{g^2} R = -(\Xi_1 + \bar{\Xi}_1) + (C + \bar{C})$$

duality relation at $a = \frac{1}{2} :$

$$\Psi = X^6 + \mathrm{i}X^8 + \dots$$

$$\Psi + \bar{\Psi} = -g^2(\Xi_1 + \bar{\Xi}_1) + g^2(C + \bar{C})$$

$X^6 \sim \text{real part of } \Xi$
 $\partial X^8 \sim \partial(\text{imaginary part of } \Xi) + \text{"gauge" fields in } C_a$

$$\begin{aligned}
\mathcal{L}_E &= \int d^4\theta \left\{ \frac{1}{e^2} \left(-\bar{\Sigma}\Sigma + \bar{\Phi}\Phi \right) + \bar{Q} e^{+2V} Q + \bar{\tilde{Q}} e^{-2V} \tilde{Q} \right\} \\
&\quad + \int d^4\theta \frac{g^2}{2} \left\{ \left(\Gamma + \bar{\Gamma} + V \right)^2 - \left(\Xi + \bar{\Xi} - (C + \bar{C}) \right)^2 \right\} \\
&\quad + \left\{ \int d^2\theta \left(-\tilde{Q}\Phi Q + s\Phi \right) + (\text{h.c.}) \right\} - \int d^4\theta (\Psi - \bar{\Psi})(C - \bar{C}) \\
&\quad + \left\{ \int d^2\tilde{\theta} t\Sigma + (\text{h.c.}) \right\} + \varepsilon^{mn} \partial_m (\textcolor{blue}{X}^9 A_n)
\end{aligned}$$

$\mathcal{N} = (4, 4)$	$\mathcal{N} = (2, 2)$		role
neutral HM	twisted chiral $\Xi = X^6 + i\tilde{X}^8 + \dots$	chiral $\Gamma = X^7 + i\tilde{X}^9 + \dots$	spacetime coord.
VMs	twisted chiral $\Sigma = \bar{D}_+ D_- V$	chiral $\Phi = \bar{D}_+ \bar{D}_- C$	gauging isometry
charged HM	chiral $Q (+)$	chiral $\tilde{Q} (-)$	curving geometry
FI parameters	$s = s^6 + i s^8$	$t = t^7 + i t^9$	position of 5_2^2 -branes

$$\begin{aligned}\mathcal{L}_{\text{E}}^{\text{kin}} &= \frac{1}{e^2} \left[\frac{1}{2} (F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2 \right] - \left[|D_m q|^2 + |D_m \tilde{q}|^2 \right] \\ &\quad - \frac{1}{2g^2} \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 \right] - \frac{g^2}{2} \left[(\partial_m \tilde{X}^8)^2 + (D_m \tilde{X}^9)^2 \right] + (X^9 - t^9) F_{01} \\ \mathcal{L}_{\text{E}}^{\text{pot}} &= -2(|\sigma|^2 + 4|M_c|^2)(|q|^2 + |\tilde{q}|^2 + g^2) \\ &\quad - \frac{e^2}{2} \left(|q|^2 - |\tilde{q}|^2 - (X^7 - t^7) \right)^2 - e^2 \left| q\tilde{q} + (X^6 - s^6) + i(\tilde{X}^8 - s^8) \right|^2 \\ &\quad + \frac{g^2}{2} (A_{c=} + \bar{A}_{c=})(B_{c\pm} + \bar{B}_{c\pm})\end{aligned}$$

$$\begin{aligned}(\partial_0 + \partial_1) X^8 &= -g^2 (\partial_0 + \partial_1) \tilde{X}^8 - g^2 (B_{c\pm} + \bar{B}_{c\pm}) \\ (\partial_0 - \partial_1) X^8 &= +g^2 (\partial_0 - \partial_1) \tilde{X}^8 - g^2 (A_{c=} + \bar{A}_{c=}) \\ + \frac{g^2}{2} (A_{c=} + \bar{A}_{c=})(B_{c\pm} + \bar{B}_{c\pm}) &= -\frac{1}{2g^2} (\partial_m X^8)^2 + \frac{g^2}{2} (\partial_m \tilde{X}^8)^2 + \varepsilon^{mn} (\partial_m X^8)(\partial_n \tilde{X}^8)\end{aligned}$$

Steps to NLSM for 5_2^2 -brane

1. search SUSY vacua $\mathcal{L}^{\text{pot}} = 0$
2. solve constraints of charged HM (Q, \tilde{Q})
3. integrate out VM (V, Φ) in IR
- ★4. integrate s^8 , and solve EOM for X^8 in $\Psi - \bar{\Psi}$

Why is $\Psi - \bar{\Psi}$ integrated out at the final step?

→ X^8 in $\Psi - \bar{\Psi}$ still appears as a winding coordinate in string worldsheet sigma model.

We should **not** integrate it out **before** integrating-out of VM.

Step 1.,2.,3. :

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{2}H \left[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \right] - \frac{1}{2H}(\partial_m \tilde{X}^9)^2 \\
 & - \frac{(\Omega_8)^2}{2H}(\partial_m \textcolor{red}{X}^8)^2 + \frac{\Omega_8}{H}(\partial_m \textcolor{red}{X}^8)(\partial^m \tilde{X}^9) \\
 & - \frac{(\Omega_6)^2}{2H}(\partial_m X^6)^2 - \frac{\Omega_6 \Omega_8}{H}(\partial_m X^6)(\partial^m \textcolor{red}{X}^8) + \frac{\Omega_6}{H}(\partial_m X^6)(\partial^m \tilde{X}^9) \\
 & + \varepsilon^{mn} \partial_m ((X^9 - t^9) A_n) + \varepsilon^{mn} (\partial_m X^8)(\partial_n \tilde{X}^8)
 \end{aligned}$$

$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|}, \quad \Omega_6 = \frac{X^8 - s^8}{|\vec{X}|(|\vec{X}| + (X^7 - t^7))}, \quad \Omega_8 = -\frac{X^6 - s^6}{|\vec{X}|(|\vec{X}| + (X^7 - t^7))}$$

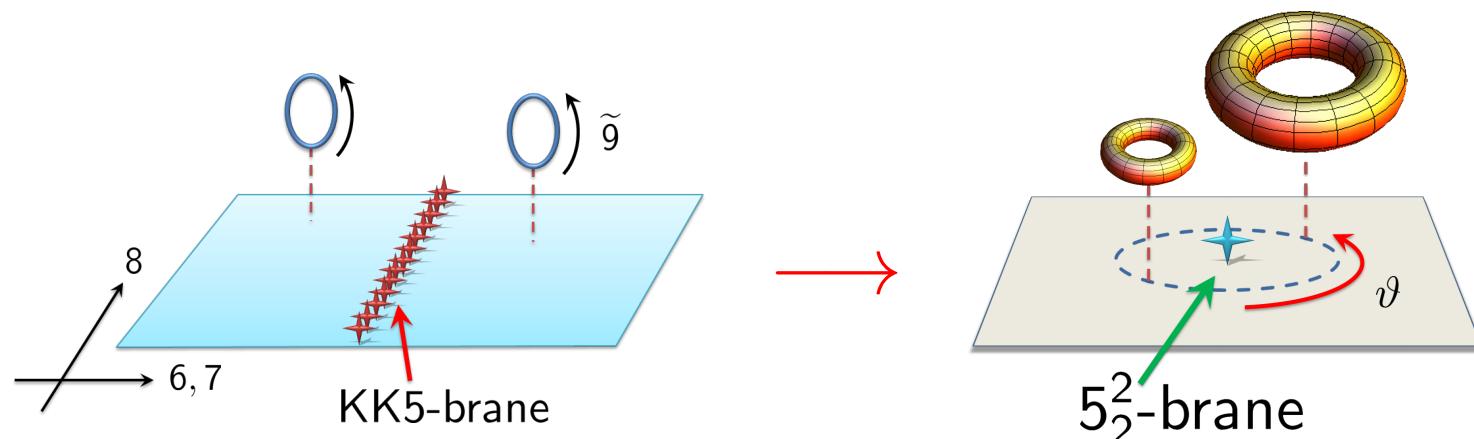
$$A_m = \frac{1}{|\vec{X}|H} (\partial_m \tilde{X}^9 - \Omega_i \partial_m X^i) + \Omega_i \partial_m X^i$$

Step 4. : $s^8 = (2\pi R_8) s \xrightarrow{\text{integral over } s} \text{emerge isometry}$

$$\left\{ \begin{array}{ll} H \rightarrow 1/g^2 + \sigma \log(\Lambda/\varrho) & : \text{co-dim. 2} \quad \varrho = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2} \\ \Omega_6 \rightarrow 0 & : \text{isometry along } X^8 \\ \Omega_8 \rightarrow \sigma \arctan \left(\frac{X^7 - t^7}{X^6 - s^6} \right) \equiv \sigma \vartheta & : \text{"non-single-valued" metric} \end{array} \right.$$

EOM for X^8 : $\partial_m X^8 = \frac{H}{K} \left[\frac{\sigma \vartheta}{H} (\partial_m \tilde{X}^9) + \varepsilon_{mn} (\partial^n \tilde{X}^8) \right]$

$$K = H^2 + (\sigma \vartheta)^2$$



Duality transformations with **F**-term

charged chiral case

Duality : chiral Ψ \longleftrightarrow twisted chiral Y

Target space geometry is T-dualized under the above transformation.

- dualize **neutral** chiral in D-term (ex. torus)

$$\int d^4\theta |\Psi|^2 \leftrightarrow \int d^4\theta \left[\frac{1}{2}R^2 - R(Y + \bar{Y}) \right] \leftrightarrow - \int d^4\theta |Y|^2$$

- dualize **charged** chiral in D-term (ex. projective space)

$$\int d^4\theta |\Psi|^2 e^{2V} \leftrightarrow \int d^4\theta \left[e^{2V+R} - R(Y + \bar{Y}) \right] \leftrightarrow - \int d^4\theta (Y + \bar{Y}) \left[\log(Y + \bar{Y}) - 2V \right]$$

These are well established.

“Global symmetry” $e^{i\alpha} \Psi$ is preserved in D-term, but broken in F-term.

— Make sense? —

- ▶ dualize **charged chiral** in D-term and **F-term**

They can be interpreted as T-duality transformations
under conversion from **F-term** to D-term with trick(es).

- Dualize **charged** chiral in D-term and **F**-term

$$\mathcal{L}_4 = \int d^4\theta |\Psi|^2 e^{2V} + \left\{ \int d^2\theta \Psi \mathcal{W}(\Phi) + (\text{h.c.}) \right\}$$

- ▶ Dualize **charged** chiral in D-term and **F**-term

$$\mathcal{L}_4 = \int d^4\theta |\Psi|^2 e^{2V} + \left\{ \int d^2\theta \Psi \mathcal{W}(\Phi) + (\text{h.c.}) \right\}$$

1. Convert **F**-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int d^2\theta \Psi \mathcal{W} + (\text{h.c.}) = 2 \int d^4\theta [\Psi C + \overline{\Psi} \overline{C}]$$

► Dualize **charged** chiral in D-term and **F**-term

$$\mathcal{L}_4 = \int d^4\theta |\Psi|^2 e^{2V} + \left\{ \int d^2\theta \Psi \mathcal{W}(\Phi) + (\text{h.c.}) \right\}$$

1. Convert **F**-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int d^2\theta \Psi \mathcal{W} + (\text{h.c.}) = 2 \int d^4\theta [\Psi C + \overline{\Psi} \overline{C}]$$

2. Replace $\Psi \pm \overline{\Psi}$ to auxiliary fields R and iS :

$$\mathcal{L}_5 = \int d^4\theta \left[e^{2V+R} + 2 \left\{ e^{\frac{1}{2}(R+iS)} C + e^{\frac{1}{2}(R-iS)} \overline{C} \right\} - R(Y + \overline{Y}) - iS(\Upsilon - \overline{\Upsilon}) \right]$$

► Dualize **charged** chiral in D-term and **F**-term

$$\mathcal{L}_4 = \int d^4\theta |\Psi|^2 e^{2V} + \left\{ \int d^2\theta \Psi \mathcal{W}(\Phi) + (\text{h.c.}) \right\}$$

1. Convert **F**-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

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$$\mathcal{L}_5 = \int d^4\theta \left[e^{2V+R} + 2 \left\{ e^{\frac{1}{2}(R+iS)} C + e^{\frac{1}{2}(R-iS)} \overline{C} \right\} - R(Y + \overline{Y}) - iS(\Upsilon - \overline{\Upsilon}) \right]$$

3. Integrating out R and Υ , we obtain the “dual” system :

$$\begin{aligned} \mathcal{L}_6 &= \int d^4\theta \left[-2(Y + \overline{Y}) \log \mathcal{F} + \frac{1}{2} \mathcal{F} \mathcal{T} + 2V(Y + \overline{Y}) \right] \\ \mathcal{F} &= -\mathcal{T} + \sqrt{\mathcal{T}^2 + 4(Y + \overline{Y})}, \quad \mathcal{T} = e^{-V} \left[e^{+\frac{1}{2}(\Omega-\overline{\Omega})} C + e^{-\frac{1}{2}(\Omega-\overline{\Omega})} \overline{C} \right], \quad \Psi = e^\Omega \end{aligned}$$

$e^{\Omega - \bar{\Omega}}$ still remains **after** the transformation.

The existence is rather important to complete the Duality transformations.

$e^{\Omega - \bar{\Omega}}$ appears as an auxiliary field, which **must be removed** finally.

Indeed, the procedure “**integrating-out of $e^{\Omega - \bar{\Omega}}$** ” leads to
the correct involution of the dual fields in the system !

Worldvolume action of exotic 5_2^2 -brane

S. Sasaki, M. Yata and TK arXiv:1404.5442

Worldvolume action of 5_2^2 -brane could be obtained from

- D5-brane's action via S- and T-dualities
- KK6-brane action via reduction and T-duality
- M5-brane action via reduction and T-dualities

Different point: **Two** isometries along transverse directions

How to describe its worldvolume theory with isometries?

Worldvolume action of 5_2^2 -brane could be obtained from

- D5-brane's action via S- and T-dualities
- KK6-brane action via reduction and T-duality
- M5-brane action via reduction and T-dualities

Different point: **Two** isometries along transverse directions

How to describe its worldvolume theory with isometries?

Already Known!

Bergshoeff et al hep-th/9706117

Worldvolume action with WZ-term is **gauged** with respect to isometries

ex.) gravity sector of KK6-brane action with gauged isometry in M-theory

$$\begin{aligned}
 \mathcal{L}_{\text{KK6}}^{\text{M}} &= -\frac{1}{2} T_{\text{KK6}}^{\text{M}} \sqrt{-\gamma} \left(k^{\frac{4}{7}} \gamma^{ab} D_a X^\mu D_b X^\nu g_{\mu\nu} - 5 \right) \\
 &= -T_{\text{KK6}}^{\text{M}} k^2 \sqrt{-\det(D_a X^\mu D_b X^\nu g_{\mu\nu})} \\
 &= -T_{\text{KK6}}^{\text{M}} k^2 \sqrt{-\det(\partial_a X^\mu \partial_b X^\nu \Pi_{\mu\nu})}
 \end{aligned}$$

$$D_a X^\mu = \partial_a X^\mu + C_a k^\mu \quad k^\mu : \text{Killing vector}$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

✓ Construct the worldvolume action with WZ-term in a covariant way:

$$\text{KK6} \xrightarrow{\text{reduction}} \text{KK5 in IIA} \xrightarrow{\text{T-duality}} 5_2^2 \text{ in IIB}$$

$$\text{M5} \xrightarrow{\text{reduction}} \text{NS5 in IIA} \xrightarrow{\text{T-dualities}} 5_2^2 \text{ in IIA}$$

✓ 5_2^2 -branes in heterotic theories can be obtained by truncation:

$$5_2^2 \text{ in IIB} \xrightarrow[\mathcal{N}=(1,1) \rightarrow \mathcal{N}=(1,0)]{\text{RR} = 0} 5_2^2 \text{ in HSO}$$

$$5_2^2 \text{ in IIA} \xrightarrow[\mathcal{N}=(2,0) \rightarrow \mathcal{N}=(1,0)]{\text{RR} = 0} 5_2^2 \text{ in HE8}$$

$$\begin{aligned}
S_{5_2^2}^{\text{IIB}} = & -T_{5_2^2} \int d^6\xi e^{-2\phi} (\det h_{IJ}) \sqrt{1 + e^{+2\phi} (\det h_{IJ})^{-1} (i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B))^2} \\
& \times \sqrt{-\det \left[\Pi_{\mu\nu}(k_2) \partial_a X^\mu \partial_b X^\nu + \frac{K_a^{(1)} K_b^{(1)}}{(k_2)^2} - \frac{(k_2)^2 (K_a^{(2)} K_b^{(2)} - K_a^{(3)} K_b^{(3)})}{(\det h_{IJ})} + \lambda \mathcal{F}_{ab} \right]} \\
& - \mu_5 \int_{\mathcal{M}_6} \left[P[i_{k_1} i_{k_2} B^{(8,2)}] - \frac{1}{2} P[\tilde{B} \wedge \tilde{C}^{(2)} \wedge \tilde{C}^{(2)}] + \lambda P[\tilde{C}^{(4)} + \tilde{C}^{(2)} \wedge \tilde{B}] \wedge \tilde{F} \right. \\
& \left. - \frac{\lambda^2}{2!} P[\tilde{B}] \wedge \tilde{F} \wedge \tilde{F} + \frac{\lambda^3}{3!} \frac{i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B)}{(i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B))^2 + e^{-2\phi} (\det h_{IJ})} \tilde{F} \wedge \tilde{F} \wedge \tilde{F} \right]
\end{aligned}$$

$$h_{IJ} = k_I^\mu k_J^\nu (g_{\mu\nu} + B_{\mu\nu}), \quad \text{etc..}$$