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Exotic Five-branes and Generalized Geometry in String Theory

木村 哲士

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佐々木伸氏 (北里大学) と 矢田雅哉氏 (KEK) との共同研究

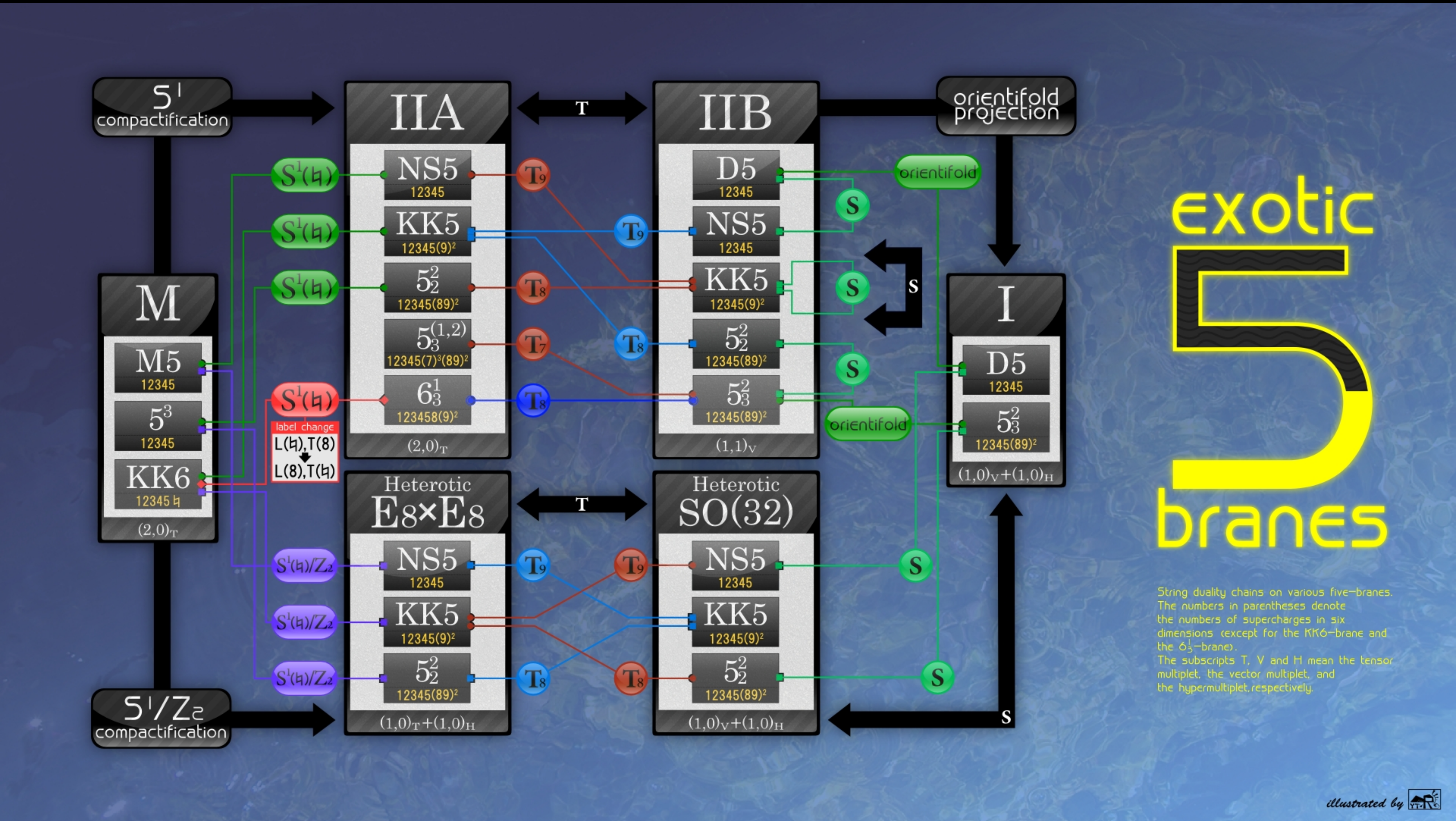
(Exotic) Branes are labelled as b_n^c-branes.

$$b_n^c$$

b : spatial dimensions

c : # of isometry directions

n : mass of brane $\sim g_s^{-n}$



S. Sasaki, M. Yata and TK arXiv:1404.5442

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- 研究動機 (比較的長いイントロダクション)
- T-duality
- GLSM for exotic five-brane
- Summary

研究動機

研究対象:超弦理論
時空のコンパクト化と新しい幾何学

- ヘテロティック弦理論におけるフラックスコンパクト化

- コンパクト空間の定義の拡大

- エキゾチックブレーン

研究対象：超弦理論
時空のコンパクト化と新しい幾何学

- ヘテロティック弦理論におけるフラックスコンパクト化

  トーションがある空間の超対称解とディラック指数定理

- コンパクト空間の定義の拡大

  弦理論の双対変換を用いた新しい幾何学

- エキゾチックブレーン

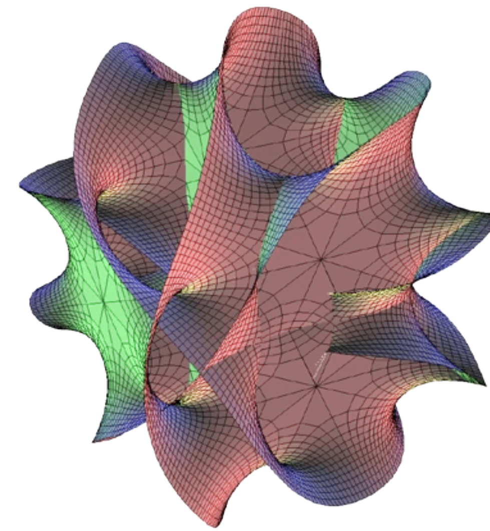
  弦の巻き付きが時空構造を深化させる

ヘテロティック弦理論において

仮定 $d\varphi = 0, \ \ H_3 = 0$ の下で

「４次元平坦時空に超対称ゲージ理論が出現すべし」とすると、

コンパクト化される6次元空間は Calabi-Yau 空間となる。



６次元 Calabi-Yau 空間

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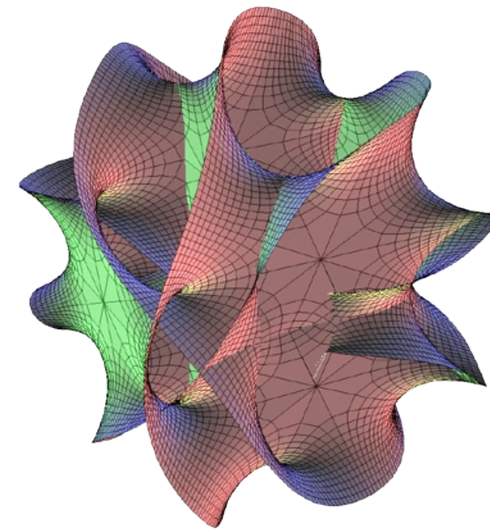


６次元 Calabi-Yau 空間

この仮定は必要か？

仮定を外しても
４次元時空に超対称ゲージ理論が出現するとき、
６次元空間はどこまで一般化できるか？

ヘテロティック弦理論において

特別な仮定をせず

「4次元平坦時空に超対称ゲージ理論が出現すべし」とする。

コンパクト化される空間は…

ヘテロティック弦理論において

特別な仮定をせず

「4次元平坦時空に超対称ゲージ理論が出現すべし」とする。

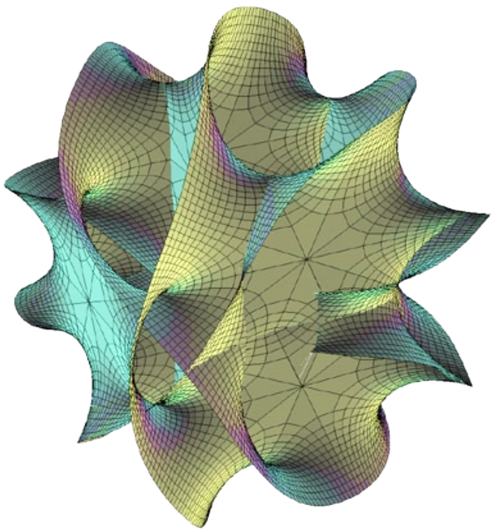
コンパクト化される空間は…

P. Yi and TK, hep-th/0605247

現在の研究の起点のひとつ

滑らかでコンパクトならば、
必ず $d\varphi = 0, H_3 = 0$ となり、Calabi-Yau 空間。

「滑らか」「コンパクト」のいずれかを緩めたら、
H_3 がトーションとなり、Calabi-Yau空間が変形。



６次元 非 Calabi-Yau 空間
(トーションあり)

テンソル場 B_{MN} が6次元空間に直接寄与する新しい幾何学

N.J. Hitchin “Generalized Geometry”

計量 G_{MN} ： 点粒子や弦の重心の運動量モード

テンソル場 B_{MN} ： 弦の巻き付きモード

テンソル場 B_{MN} が6次元空間に直接寄与する新しい幾何学

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Generalized Geometry は、弦の T-duality を自然に含む

geometry associated with G_{MN}	Conventional Geometry (manifold) $O(6)$ symmetry
geometry associated with G_{MN}, B_{MN}	Generalized Geometry $O(6,6)$ T-duality symmetry

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geometry associated with G_{MN}	Conventional Geometry (manifold) $GL(6)$ symmetry
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弦の巻き付きが
時空構造に組み込まれる

C. Albertsson, R.A. Reid-Edwards and TK, arXiv:0806.1783

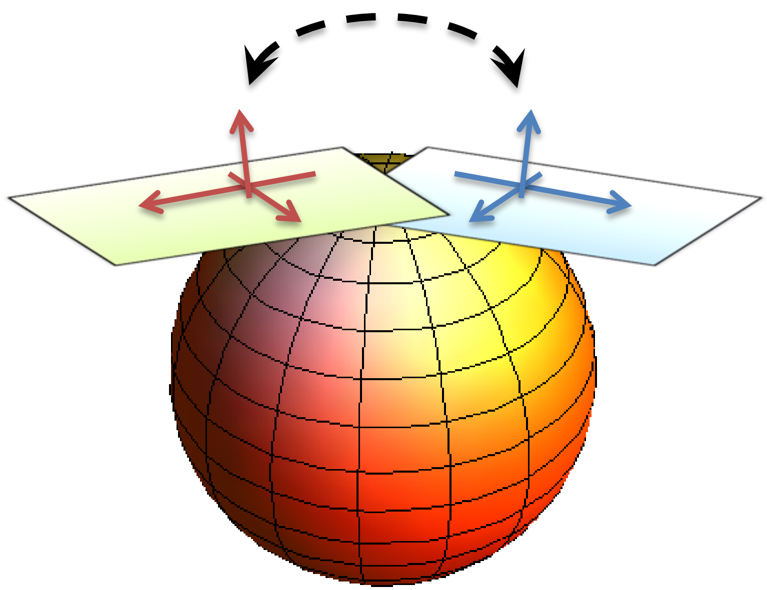
TK, arXiv:0810.0937

M. Hatsuda and TK, arXiv:1203.5499

Generalized Geometry は非常に一般的

具体性に欠ける

物理の探究に使いづらい



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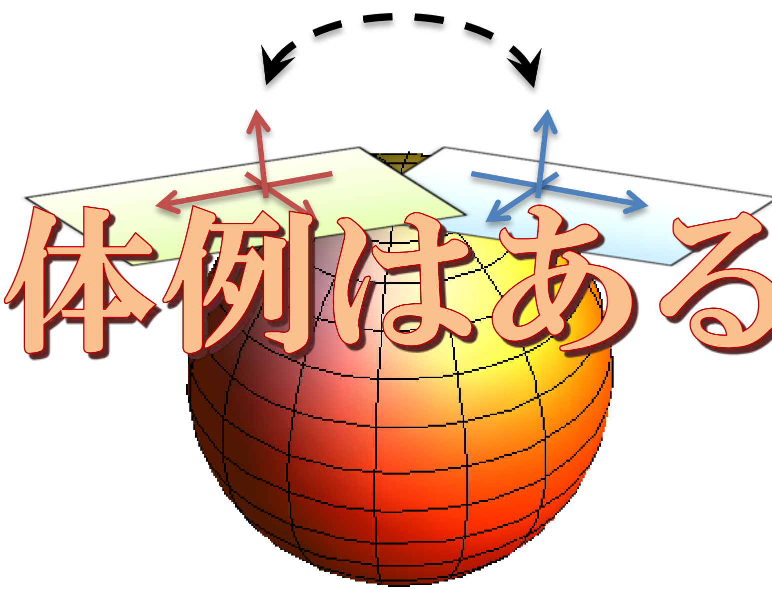


良い具体例はあるのか？

エキゾチック
ブレーン
特に 5^2_2-brane

木村哲士

5^2_2-brane は

NS5-brane を T-duality 変換することで得られる

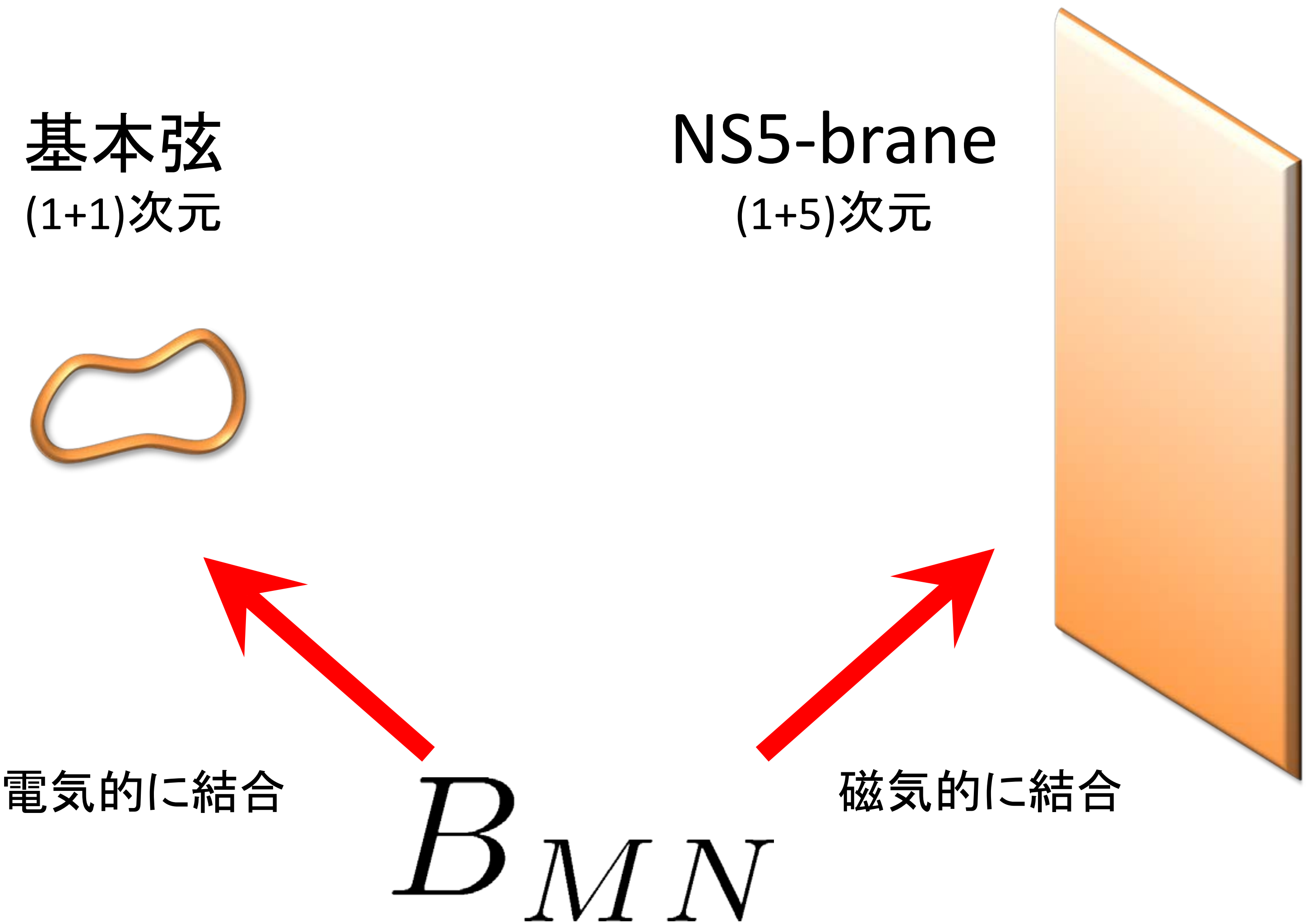
奇妙な物体である

5^2_2-brane 周囲の時空計量が

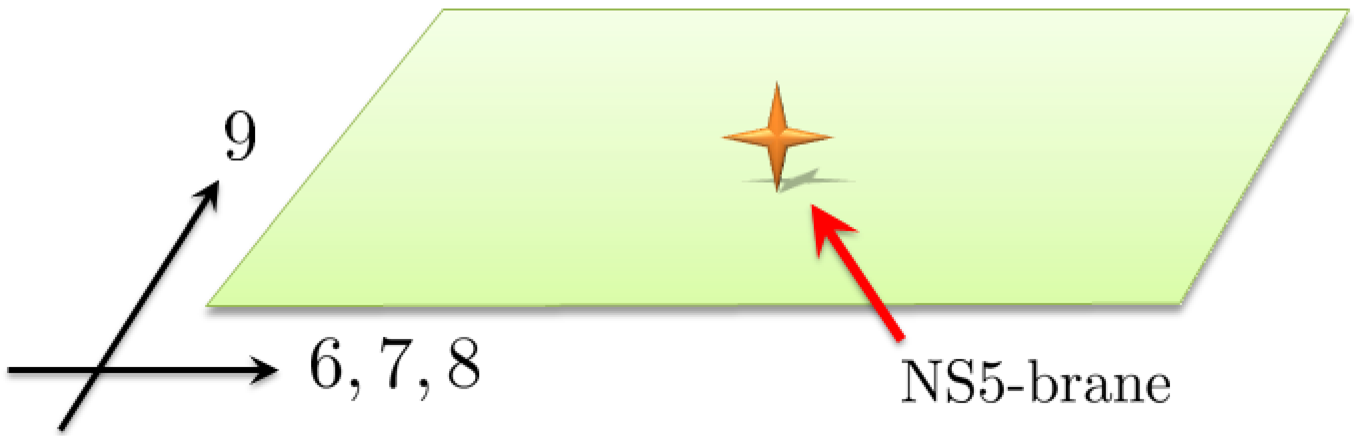
一価関数で表現できない

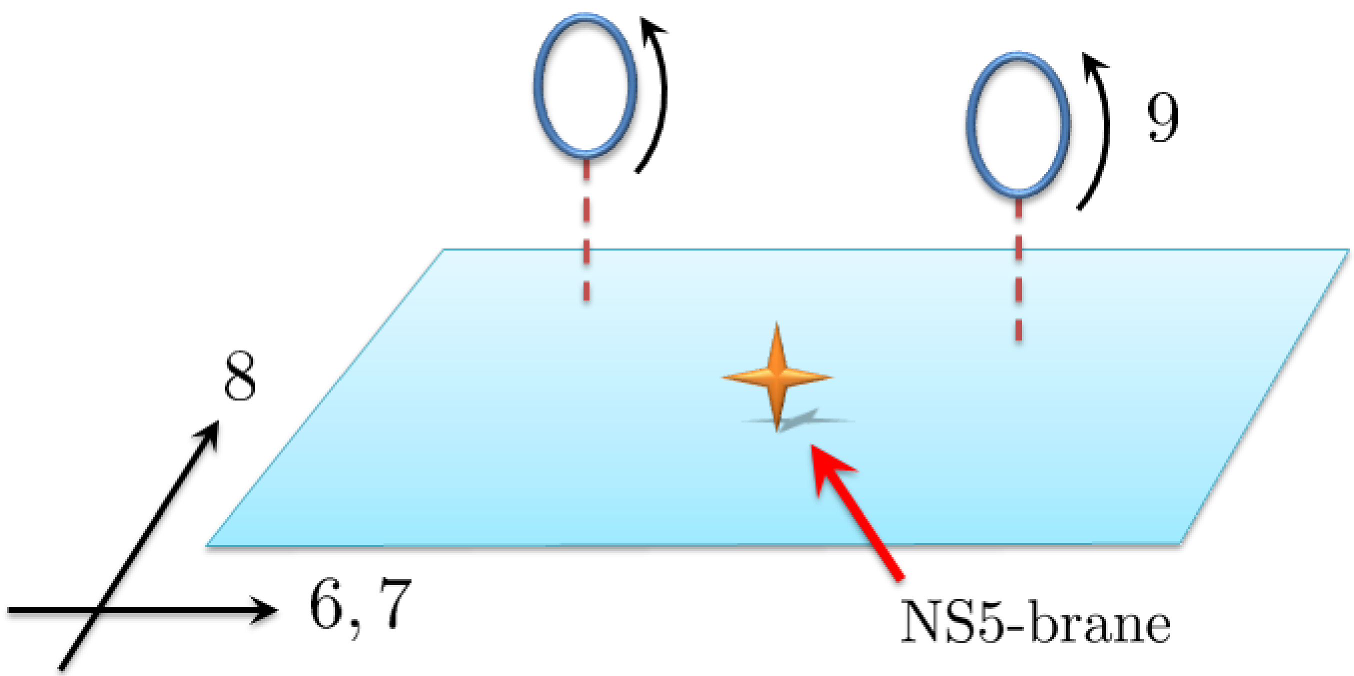
弦の巻き付き自由度が時空構造を深化させている

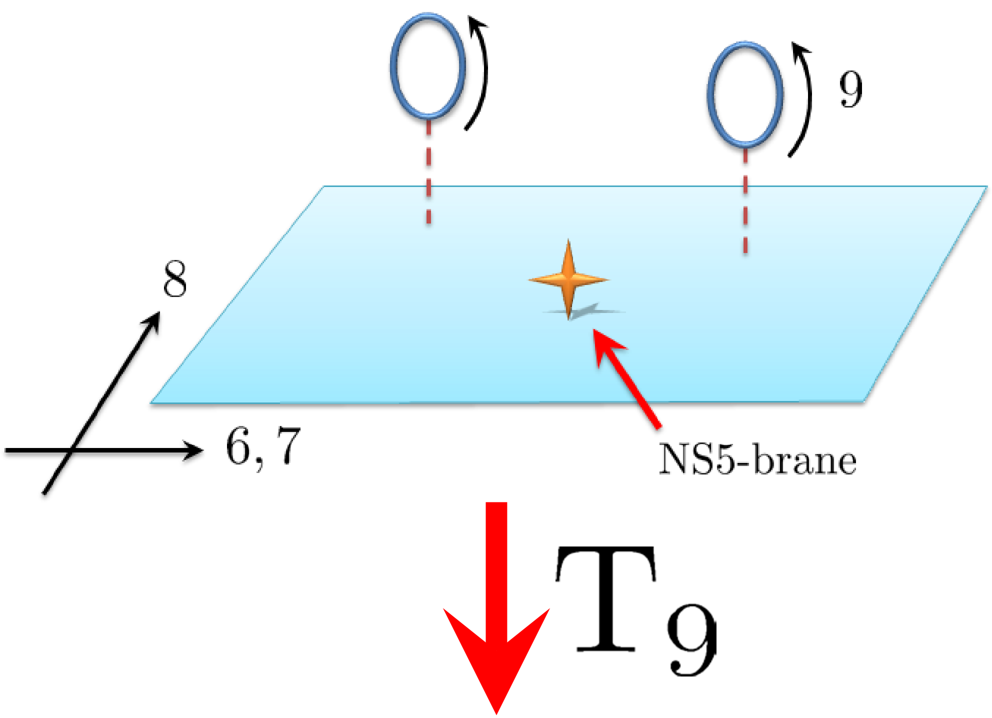
Generalized Geometry の具体例！

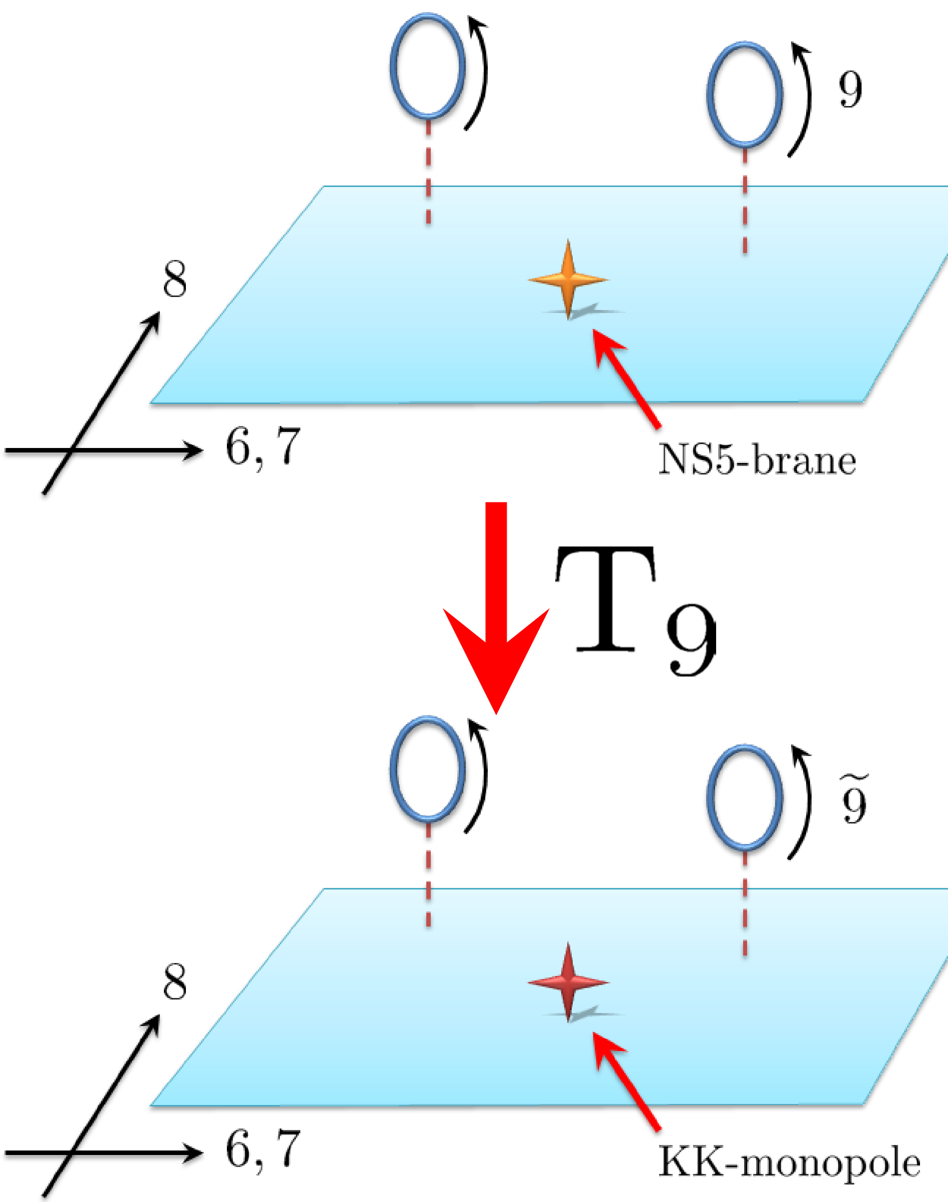

基本弦
(1+1)次元
NS5-brane
(1+5)次元
電気的に結合
磁気的に結合
B_{MN}

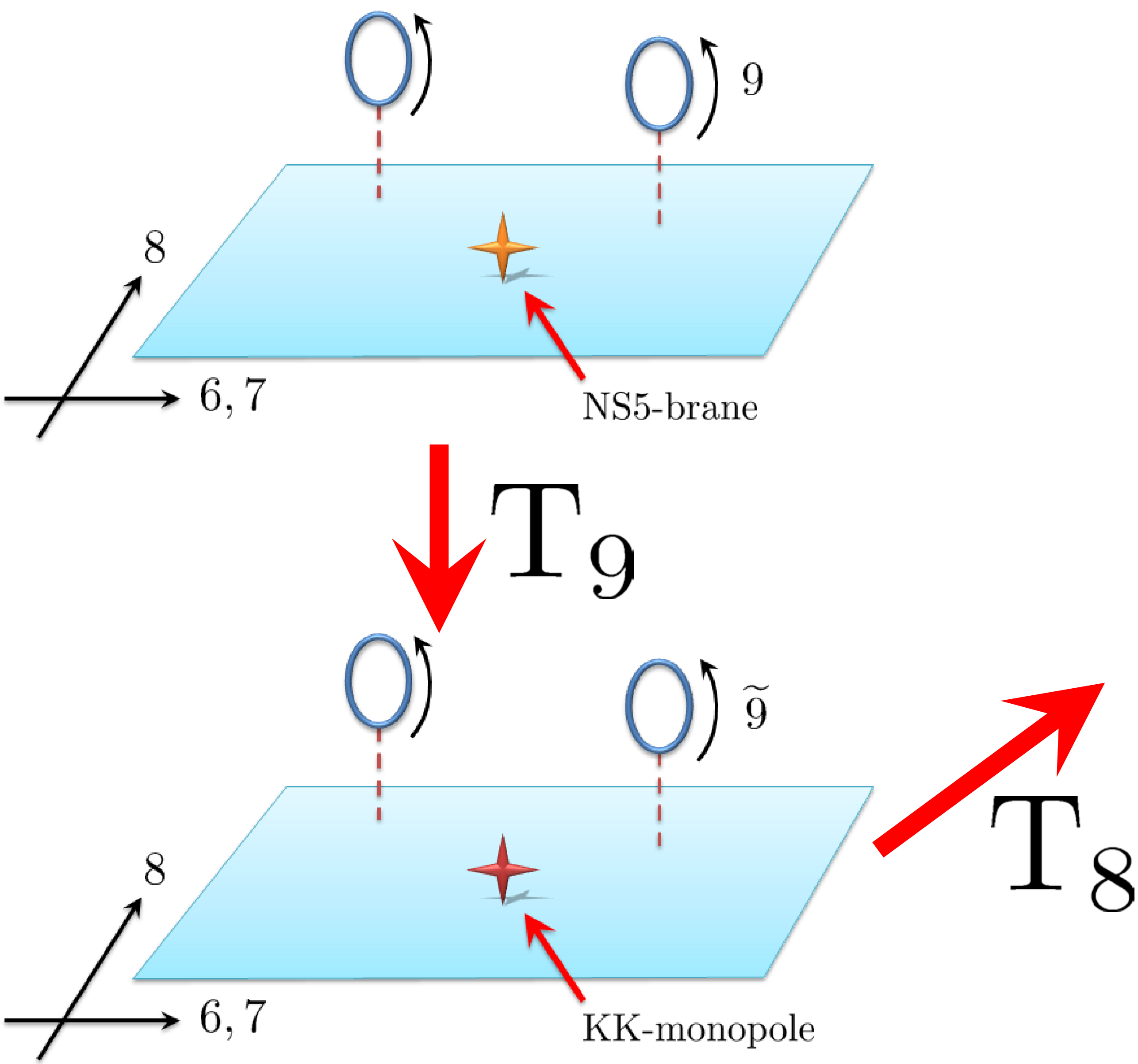
NS5-brane に垂直な4方向に着目する

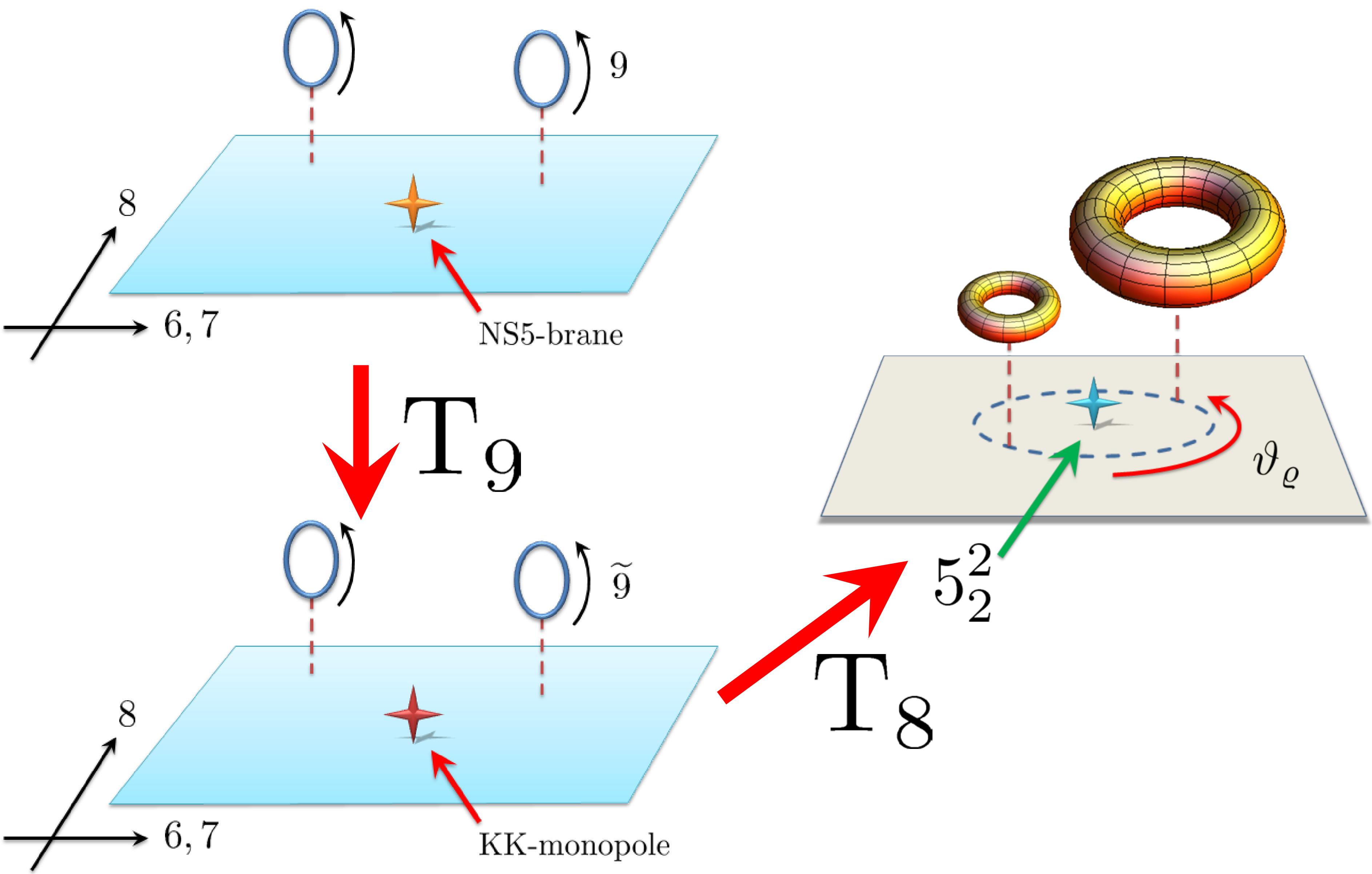



9
8
6, 7
NS5-brane


9
8
6, 7
NS5-brane
T9


9
8
6, 7
NS5-brane
T9
9̃
8
6, 7
KK-monopole


9
8
6, 7
NS5-brane
T9
$\tilde{9}$
8
6, 7
KK-monopole
T8



木村哲士

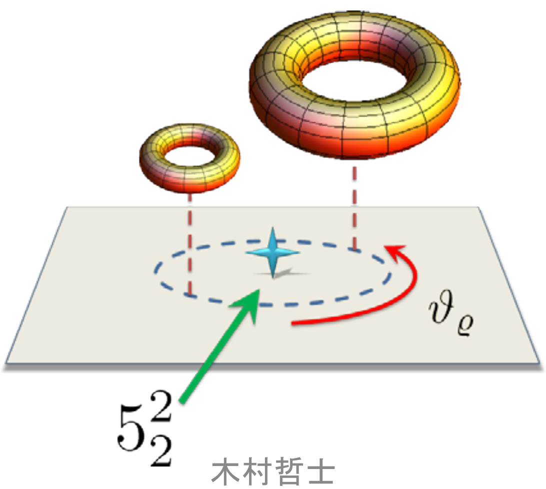
$$ds^2 = dx^2_{012345} + H\left[d\varrho^2 + \varrho^2(d\vartheta_\varrho)^2\right] + \frac{H}{K}\left[(dy^8)^2 + (dy^9)^2\right]$$

$$B_{89} = -\frac{\sigma\vartheta_\varrho}{K}\,, \qquad e^{2\Phi} = \frac{H}{K}$$

$$H = \frac{1}{g^2} + \sigma\log\frac{\Lambda}{\varrho}\,, \qquad K = H^2 + (\sigma\vartheta_\varrho)^2$$

5^2_2-brane の周りを一周しても元の位置に戻って来ない！

T-duality 変換 (計量 G_{MN} と反対称テンソル場 B_{MN} の混合) に起因



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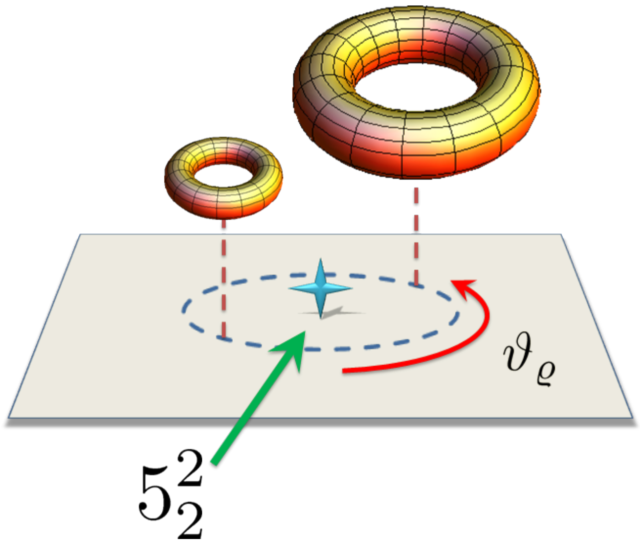
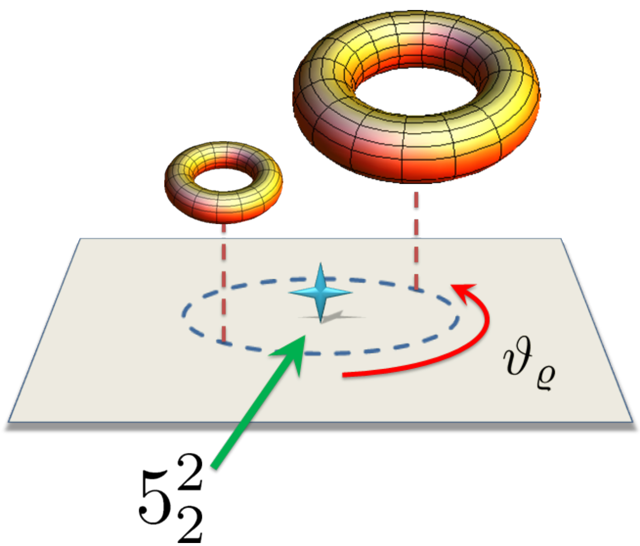


これまでは 5^2_2-brane を
超重力理論の枠内でしか考察されなかった

しかし「計量の多価性」は弦の巻き付きに起因

超重力理論は超弦理論の点粒子極限なので
弦の巻き付きが自然には扱えていない



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しかし「計量の多価性」は弦の巻き付きに起因

弦理論特有の性質は
弦理論の言葉で追究すべき！

超重力理論は超弦理論の点粒子極限なので
弦の巻き付きが自然には扱えていない

2D Gauged Linear Sigma Model (GLSM) as String Worldsheet Theory

S. Sasaki and TK

arXiv:1304.4061, 1305.4439, 1310.6361

非線形で多価な時空計量の性質を

２次元の線形な超対称ゲージ理論で記述する

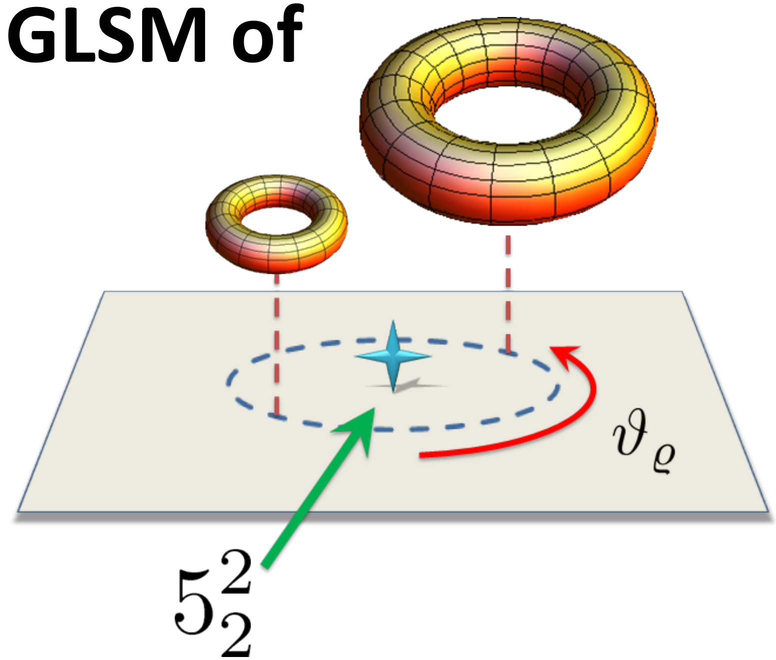
理解できたこと

✓ 計量 G_{MN} とテンソル場 B_{MN} の振る舞いを
2Dゲージ場と荷電カイラル超場で記述

⇨ 2D場の理論として自然な振舞いで記述できた

✓ 5^2_2-brane に対する弦理論のインスタントン補正が
2Dゲージ場のボーテックス配位で記述できた

GLSM of



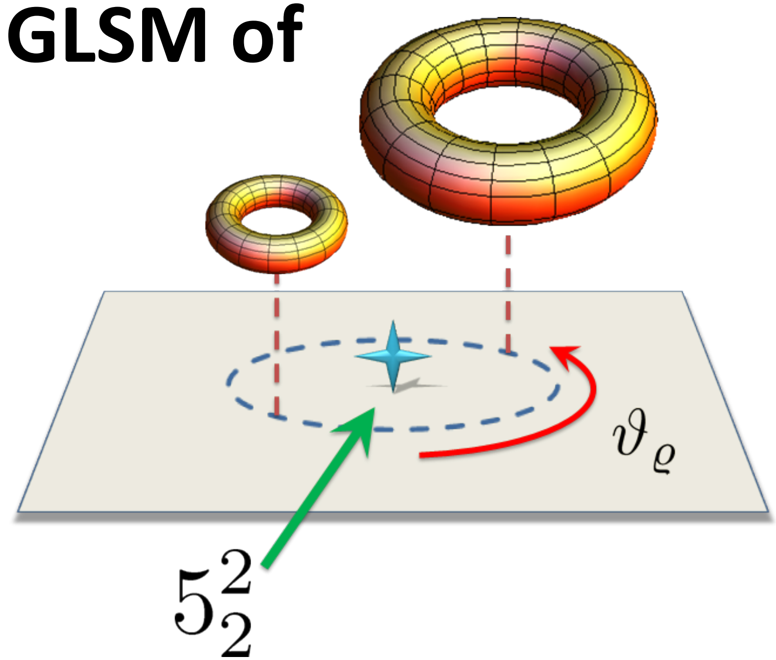
理解できたこと

✓ 計量 G_{MN} とテンソル場 B_{MN} の振る舞いを 2Dゲージ場と荷電カイラル超場で記述

⇨ 2D場の理論として自然な振舞いで記述できた

✓ 5^2_2-brane に対する弦理論のインスタントン補正が 2Dゲージ場のボーテックス配位で記述できた

GLSM of



エキゾチックブレーンの理解が大きく前進!!

GLSM description

as an example of nongeometric backgrounds in string theory

5^2_2-brane has been analyzed in spacetime (SUGRA) picture.

de Boer and Shigemori (2010, 2012), etc.

Ready to study string worldsheet picture!

string worldsheet picture

- ☞ nonlinear sigma model (NLSM)
- ☞ conformal field theory (CFT)
- ☞ gauged linear sigma model (GLSM)

GLSM is a powerful tool to investigate non-trivial configurations in string theory :

GLSM = Gauged Linear Sigma Model (2D gauge theory)

Calabi-Yau sigma model and its corresponding CFT

Witten (1993)

D-branes and gauge theories

ex.) review by Giveon and Kutasov (1998)

Analysis of NS5-branes

Tong (2002), Harvey and Jensen (2005), Okuyama (2005)

In particular, it is important to study T-duality of their configurations.

T-duality transformations

(free) string	sign flip (parity) in right-mover	momentum $\leftrightarrow$ winding
spacetime	Buscher rule	$(G_{IJ}, B_{IJ}) \to (G'_{IJ}, B'_{IJ})$
SUSY sigma model	Roček-Verlinde formula	chiral $\leftrightarrow$ twisted chiral

$$\mathscr{L}_{\text{GLSM}} = \int \mathrm{d}^4\theta \, \Big\{ - \frac{1}{e^2} |\Sigma|^2 + |Q|^2 \, \mathrm{e}^{2V} + |\Psi|^2 - 2rV \Big\}$$

$$\mathscr{L}_{\text{GLSM}} = \int d^4\theta \Big\{ -\frac{1}{e^2}|\Sigma|^2 + |Q|^2\, e^{2V} + |\Psi|^2 - 2rV \Big\}$$

T-duality

might be violated if an F-term exists.

We would like to find a consistent formula to perform T-duality

even in the presence of F-terms.

Duality transformations with F-term

in $\mathcal{N} = (2, 2)$ framework

Duality : chiral $\longleftrightarrow$ twisted chiral

Target space geometry is T-dualized under the above transformation.

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Target space geometry is T-dualized under the above transformation.

- dualize neutral chiral in D-term (ex. torus)

$$\int d^4\theta \, |\Psi|^2 \leftrightarrow \int d^4\theta \, \Big[\frac{1}{2}R^2 - R(Y + \overline{Y})\Big] \leftrightarrow -\int d^4\theta \, |Y|^2$$

- dualize charged chiral in D-term (ex. projective space)

$$\int d^4\theta \, |Q|^2 \, e^{2V} \leftrightarrow \int d^4\theta \, \Big[e^{2V+R} - R(Y+\overline{Y})\Big] \leftrightarrow -\int d^4\theta \, (Y+\overline{Y})\Big[\log(Y+\overline{Y}) - 2V\Big]$$

These are well established.

"Global symmetry" $\Psi \to \Psi + \alpha$ is preserved in D-term, but broken in F-term.

— Make sense? —

► dualize chiral in D-term and F-term

They can be interpreted as T-duality transformations

under conversion from F-term to D-term with trick(es).

- Dualize neutral chiral in D-term and F-term

$$\mathscr{L}_1 = \int d^4\theta \, |\Psi|^2 + \Big\{ \int d^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\text{h.c.}) \Big\}$$

► Dualize neutral chiral in D-term and F-term

$$\mathscr{L}_1 = \int d^4\theta \, |\Psi|^2 + \Big\{ \int d^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\text{h.c.}) \Big\}$$

1. Convert F-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int d^2\theta \, \Psi \, \mathcal{W} + (\text{h.c.}) = \int d^4\theta \Big[(\Psi + \overline{\Psi})(C + \overline{C}) + (\Psi - \overline{\Psi})(C - \overline{C}) \Big]$$

- Dualize neutral chiral in D-term and F-term

$$\mathscr{L}_1 = \int \mathrm{d}^4\theta \, |\Psi|^2 + \Big\{ \int \mathrm{d}^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\mathrm{h.c.}) \Big\}$$

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2. Replace $\Psi \pm \overline{\Psi}$ to auxiliary fields R and $\mathrm{i}S$:

$$\mathscr{L}_2 = \int \mathrm{d}^4\theta \, \Big[\frac{1}{2} R^2 + R(C + \overline{C}) + \mathrm{i}S(C - \overline{C}) - R(Y + \overline{Y}) - \mathrm{i}S(\Upsilon - \overline{\Upsilon}) \Big]$$

► Dualize neutral chiral in D-term and F-term

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3. Integrating out R and Υ, we obtain the "dual" system :

$$\mathscr{L}_3 = \int d^4\theta \, \Big[-\frac{1}{2} \Big((Y + \overline{Y}) + (C + \overline{C}) \Big)^2 + (\Psi - \overline{\Psi})(C - \overline{C}) \Big]$$

Instead, integrate out Y and $\Upsilon \to \mathscr{L}_1$ appears

$\Psi - \overline{\Psi}$ still remains after the transformation.

The existence is rather important to complete the Duality transformations.

$\Psi - \overline{\Psi}$ appears as an auxiliary field, which must be removed finally.

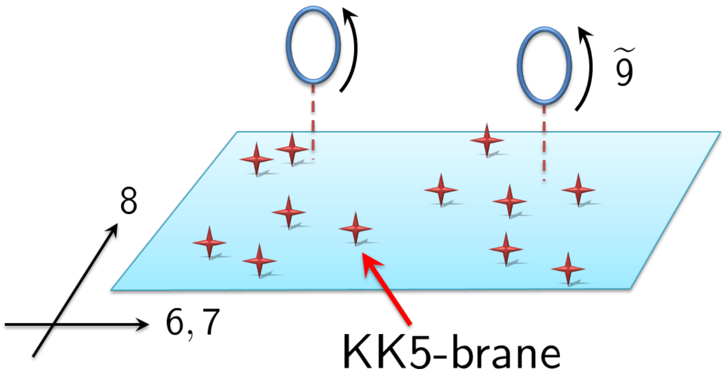
Indeed, the procedure "integrating-out of $\Psi - \overline{\Psi}$" leads to

the correct involution of the dual fields in the system !

GLSM for 5^2_2-brane

as $\mathcal{N} = (4, 4)$ GLSMs with F-term

Start by GLSM for ALF space (KK5-branes)

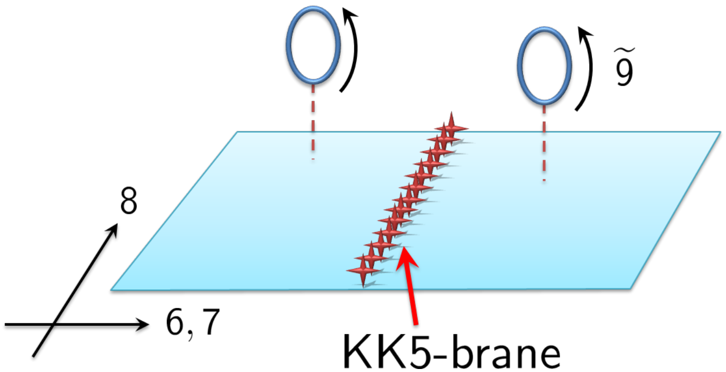


$$
\begin{aligned}
\mathscr{L} \ = \ & \sum_a \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} (|\Phi_a|^2 - |\Sigma_a|^2) + |Q_a|^2 \, \mathrm{e}^{-2V_a} + |\widetilde{Q}_a|^2 \, \mathrm{e}^{2V_a} \Big\} \\
& + \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{g^2} |\Psi|^2 + \frac{g^2}{2} (\Gamma + \overline{\Gamma} + 2 \sum_a V_a)^2 \Big\} \\
& + \sum_a \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\text{h.c.}) \Big\} \\
& + \sum_a \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t_a \Sigma_a) + (\text{h.c.}) \Big\} - \sum_a \varepsilon^{mn} \partial_m (X^9 A_n^a)
\end{aligned}
$$

$(6,8)$-,$(7,\widetilde{9})$-directions are described by chirals Ψ, Γ

D. Tong (2002)

Start by GLSM for ALF space (KK5-branes)



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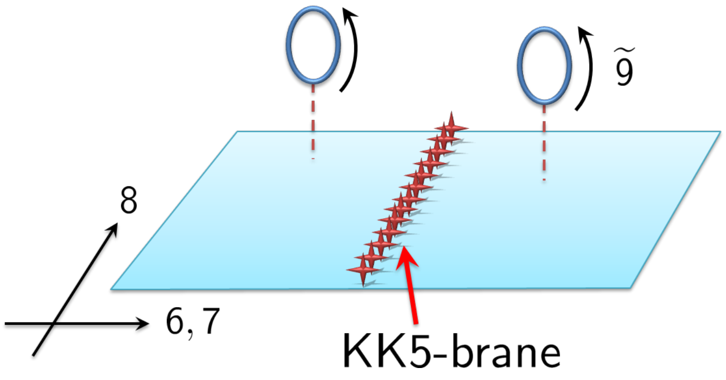
\end{aligned}
$$

$(6,8)$-,$(7,\widetilde{9})$-directions are described by chirals Ψ, Γ

Each position is labelled by FI parameters (s_a, t_a)

D. Tong (2002)

Start by GLSM for ALF space (KK5-branes)



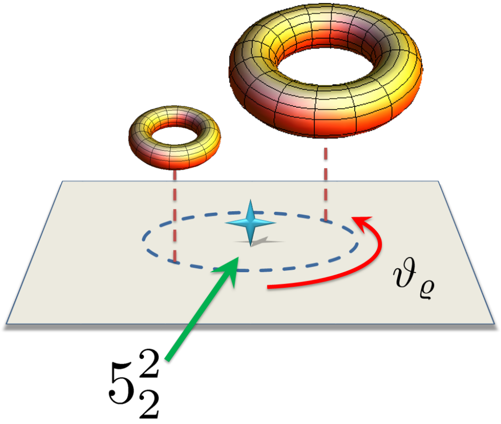
$$\begin{aligned}\mathscr{L} \ &= \sum_a \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2}(|\Phi_a|^2 - |\Sigma_a|^2) + |Q_a|^2 \, \mathrm{e}^{-2V_a} + |\widetilde{Q}_a|^2 \, \mathrm{e}^{2V_a} \Big\} \\ &+ \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{g^2}|\Psi|^2 + \frac{g^2}{2}(\Gamma + \overline{\Gamma} + 2 \sum_a V_a)^2 \Big\} \\ &+ \sum_a \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi)\Phi_a \Big) + (\text{h.c.}) \Big\} \\ &+ \sum_a \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t_a \Sigma_a) + (\text{h.c.}) \Big\} - \sum_a \varepsilon^{mn} \partial_m (X^9 A_n^a)\end{aligned}$$

$(6,8)$-,$(7,\widetilde{9})$-directions are described by chirals Ψ, Γ

Dualize the neutral chiral $\Psi \to \Xi$: T-duality along 8th-direction

D. Tong (2002)

T-duality!



$$\begin{aligned}
\mathscr{L}' \;=\; & \sum_a \int d^4\theta \Big\{ \frac{1}{e_a^2}(|\Phi_a|^2 - |\Sigma_a|^2) + |Q_a|^2 e^{-2V_a} + |\widetilde{Q}_a|^2 e^{2V_a} \Big\} \\
& + \int d^4\theta \Big\{ -\frac{g^2}{2}(\Xi + \overline{\Xi} - \sum_a (C_a + \overline{C}_a))^2 + \frac{g^2}{2}(\Gamma + \overline{\Gamma} + 2\sum_a V_a)^2 \Big\} \\
& + \sum_a \Big\{ \int d^2\theta \Big(\widetilde{Q}_a \Phi_a Q_a + s_a \Phi_a \Big) + (\text{h.c.}) \Big\} \\
& + \sum_a \Big\{ \int d^2\widetilde{\theta}\, (t_a \Sigma_a) + (\text{h.c.}) \Big\} - \sum_a \varepsilon^{mn} \partial_m (X^9 A^a_n) \\
& - \int d^4\theta\, (\Psi - \overline{\Psi}) \sum_a (C_a - \overline{C}_a)
\end{aligned}$$

$(6, \widetilde{8})$-,$(7, \widetilde{9})$-directions are described by (twisted) chirals Ξ, Γ

Integrate out $\Psi - \overline{\Psi}$: Exotic Five-brane!

S. Sasaki and TK arXiv:1304.4061

Remark

Quantum corrections to the geometry can be traced by Vortices in GLSM:

$$\mathscr{L}^{\text{GLSM}}_{\text{topological}} = \sum_a X^9 F^a_{01} + \ldots$$

$$H = h + \sigma \log\left(\frac{\mu}{\varrho}\right) \quad \to \quad h + \sigma \log\left(\frac{\mu}{\varrho}\right) + \sum_{n \neq 0} K_0(|n|\,\varrho)\,\exp(\mathrm{i} n X^9)$$

X^9 is ...

X^9 is ...	physical coordinate in NS5	Tong
	dual coordinate in KKM	Tong, Harvey and Jensen, Okuyama
	dual coordinate in 5^2_2	Sasaki and TK

S. Sasaki and TK arXiv:1305.4439

Summary

✓ Dualized chiral multiplets in D-term and F-term

✓ Applied to $\mathcal{N} = (4,4)$ GLSM with F-term

✓ Calabi-Yau and its T-duality, revisited

Conifold $\leftrightarrow$ Intersecting NS5-branes

✓ GLSM for exotic 5^2_2-brane with D-branes

Brane construction with string dualities

We want to understand nongeometric backgrounds in string theory

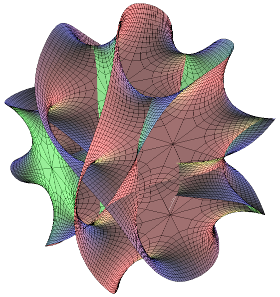
We want to see a deeper insight of exotic five-branes

from various viewpoints *!*

Thanks

Appendix

Calabi-Yau 3-fold $\mathcal{M}_{\text{CY}}$



Ricci-flat, torsionless, (compact) Kähler manifold with $SU(3)$ holonomy group

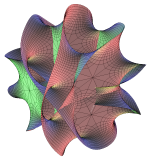
$$ds^2_{\text{10D}} = \eta_{\mu\nu}(x)\, dx^\mu\, dx^\nu + g_{mn}(x,y)\, dy^m\, dy^n$$

4D   CY

Invariant two-form J and three-form Ω on CY w.r.t. Levi-Civita connection:

$$dJ = \nabla_{[m} J_{np]} = 0 \qquad d\Omega = \nabla_{[m} \Omega_{npq]} = 0$$

Calabi-Yau 3-fold $\mathcal{M}_{\text{CY}}$



Ricci-flat, torsionless, (compact) Kähler manifold with $SU(3)$ holonomy group

Invariant two-form J and three-form Ω on CY w.r.t. Levi-Civita connection:

$$\mathrm{d}J = \nabla_{[m} J_{np]} = 0, \quad \mathrm{d}\Omega = \nabla_{[m} \Omega_{npq]} = 0$$

This is suitable for 1/4-SUSY condition with vanishing background fields

$$\delta_{\text{SUSY}} \psi_{m\pm} = \nabla_m \, \varepsilon_{\pm}^{\text{(10D)}} = 0$$

$$\varepsilon_{+}^{\text{(10D)}} = \varepsilon_{1+}^{\text{(4D)}} \otimes \eta_{+}^{1} + \text{(c.c.)}, \quad \varepsilon_{-}^{\text{(10D)}} = \varepsilon_{2+}^{\text{(4D)}} \otimes \eta_{-}^{2} + \text{(c.c.)}$$

$$(\varepsilon_{1,2+}^{\text{(4D)}})^c = \varepsilon_{1,2-}^{\text{(4D)}}, \quad (\eta_{-}^{1,2})^* = \eta_{+}^{1,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ * \end{pmatrix} \ : \ SU(3)\text{-invariant} \subset SU(4) \sim SO(6)$$

$$\eta_{\pm}^{1} = \eta_{\pm}^{2} \equiv \eta_{\pm}, \qquad J_{mn} \sim \mp \mathrm{i}\, \eta_{\pm}^{\dagger} \gamma_{mn} \eta_{\pm}, \qquad \Omega \sim -\mathrm{i}\, \eta_{-}^{\dagger} \gamma_{mnp} \eta_{+}$$

NS-NS fields in 10D are expanded around CY :

$$\begin{aligned}
\phi(x,y) &= \varphi(x) \\
g_{\mathfrak{m}\bar{\mathfrak{n}}}(x,y) &= \mathrm{i}v^a(x)\,(\omega_a)_{\mathfrak{m}\bar{\mathfrak{n}}}(y), \quad g_{\mathfrak{m}\mathfrak{n}}(x,y) = \mathrm{i}\,\overline{z}^{\bar{\jmath}}(x)\left(\frac{(\overline{\chi}_{\bar{\jmath}})_{\mathfrak{m}\bar{\mathfrak{p}}\bar{\mathfrak{q}}}\Omega^{\bar{\mathfrak{p}}\bar{\mathfrak{q}}}{}_{\mathfrak{n}}}{||\Omega||^2}\right)(y) \\
\widehat{B}_2(x,y) &= B_2(x) + b^a(x)\omega_a(y) \\
\mathfrak{t}^a(x) &\equiv b^a(x) + \mathrm{i}v^a(x)
\end{aligned}$$

R-R fields :

$$\begin{aligned}
\widehat{C}_1(x,y) &= A_1^0(x) \\
\widehat{C}_3(x,y) &= A_1^a(x)\wedge\omega_a(y) + \xi^I(x)\alpha_I(y) - \widetilde{\xi}_I(x)\beta^I(y)
\end{aligned}$$

cohomology class on CY	basis	degrees
$H^{(1,1)}$	ω_a	$a = 1,\ldots,h^{(1,1)}$
$H^{(0)}\oplus H^{(1,1)}$	$\omega_\Lambda = (1,\omega_a)$	$\Lambda = 0,1,\ldots,h^{(1,1)}$
$H^{(2,2)}\oplus H^{(6)}$	$\widetilde{\omega}^\Lambda = (\widetilde{\omega}^a, \frac{\mathrm{vol.}}{\lvert\mathrm{vol.}\rvert})$	
$H^{(2,1)}$	χ_i	$i = 1,\ldots,h^{(2,1)}$
$H^{(3)}$	(α_I,β^I)	$I = 0,1,\ldots,h^{(2,1)}$

$$\mathrm{d}\omega_\Lambda = 0 = \mathrm{d}\widetilde{\omega}^\Lambda$$

$$\mathrm{d}\alpha_I = 0 = \mathrm{d}\beta^I$$

non-CY 3-fold $\mathcal{M}_6$

vanishing Ricci 2-form, torsionful, (compact) non-Kähler manifold

with (a pair of) $SU(3)$-structure

$\mathrm{d}J \neq 0$ and/or $\mathrm{d}\Omega \neq 0$

- $\eta^1_\pm = \eta^2_\pm$ at any points on $\mathcal{M}_6$:

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}(\overline{\mathcal{W}}_1 \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

(It is possible to consider a geometry with $\eta^1_\pm \neq \eta^2_\pm$ on a certain point of $\mathcal{M}_6$.)

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}(\overline{\mathcal{W}}_1\Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \qquad \mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

complex ($1/4$-SUSY Minkowski$_{1,3}$)	hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
	balanced	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$
	special hermitian	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	conformally CY	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$
almost complex ($1/4$-SUSY AdS$_4$)	symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
	nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	quasi Kähler	$\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
	semi Kähler	$\mathcal{W}_4 = \mathcal{W}_5 = 0$
	half-flat	$\mathrm{Im}\mathcal{W}_1 = \mathrm{Im}\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$

Non-vanishing $\mathrm{d}J$ and $\mathrm{d}\Omega$ are expanded by "non"-closed basis forms:

NS-NS

$$\mathrm{d}\begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix} \sim \begin{pmatrix} e_\Lambda{}^I & m^{\Lambda I} \\ e_{\Lambda I} & m^\Lambda{}_I \end{pmatrix} \begin{pmatrix} \widetilde{\omega}^\Lambda \\ \omega_\Lambda \end{pmatrix}$$

Σ_- $\quad Q^{\mathrm{T}}$ $\quad \Sigma_+$

$e_0{}^I, e_{0I}$: H-flux charges ($H^{\mathrm{fl}} = -e_0{}^I \alpha_I + e_{0I}\beta^I$)
$e_a{}^I, e_{aI}$: geometric flux charges (torsion)
$m^{\Lambda I}, m^\Lambda{}_I$: nongeometric flux charges (magnetic dual of $e_\Lambda{}^I, e_{\Lambda I}$)

R-R

$$\widehat{\mathbf{F}} \equiv \widehat{F}_0 + \widehat{F}_2 + \ldots + \widehat{F}_{10} \equiv \mathrm{e}^{\widehat{B}} \widehat{\mathbf{G}}$$

with self-dual cond. $\widehat{\mathbf{F}} = \lambda(*\widehat{\mathbf{F}})\,, \quad \lambda(\widehat{F}_k) \equiv (-)^{[\frac{k+1}{2}]}\widehat{F}_k$

$$\begin{aligned} \tfrac{1}{\sqrt{2}}\widehat{\mathbf{G}} &= (G_0^\Lambda + G_2^\Lambda + G_4^\Lambda)\,\omega_\Lambda - (\widetilde{G}_{0\Lambda} + \widetilde{G}_{2\Lambda} + \widetilde{G}_{4\Lambda})\,\widetilde{\omega}^\Lambda \\ &\quad + (G_1^I + G_3^I)\,\alpha_I - (\widetilde{G}_{1I} + \widetilde{G}_{3I})\,\beta^I \end{aligned}$$

$$G_0^\Lambda \equiv p^\Lambda\,, \qquad \widetilde{G}_{0\Lambda} \equiv q_\Lambda - \xi^I e_{\Lambda I} + \widetilde{\xi}_I e_\Lambda{}^I$$

$c \equiv (p^\Lambda, q_\Lambda)^{\mathrm{T}}$: R-R flux charges

(p^0: Romans' mass)

10D string theory = D-dim spacetime $\otimes$ compact space $\mathcal{M}_d$

$\mathcal{M}_d$	geometry associated with G_{mn}	Conventional geometry (manifold) $O(d)$ global symmetry [Calabi-Yau, etc]	ordinary compactifications
	geometry associated with G_{mn}, B_{mn}	Generalized geometry $O(d,d;\mathbb{Z})$ T-duality symmetry [T-fold]	flux compactifications
	geometry associated with G_{mn}, $\tau = C_{(0)} + \mathrm{i}\,\mathrm{e}^{-\Phi}$	Generalized geometry $SL(2,\mathbb{Z})$ S-duality symmetry [S-fold]	F-theory
	geometry associated with G_{mn}, B_{mn}, Φ, $C_{(p)}$	Generalized geometry $E_{d+1(d+1)}(\mathbb{Z})$ U-duality symmetry [U-fold]	compactifications with non-abelian gauge

almost complex structure $J_m{}^n$ on $T\mathcal{M}_6$ s.t.

$$J_m{}^n : T\mathcal{M}_6 \longrightarrow T\mathcal{M}_6$$

$$J^2 = -\mathbb{1}_6$$

$^\exists$ $O(6)$ invariant metric η, s.t. $J^{\rm T} \eta J = \eta$

Structure group on $T\mathcal{M}_6$:

η_{mn}	$GL(6)$	$\dashrightarrow$	$O(6)$
$\eta_{mn}\,,\ \varepsilon_{m_1 \cdots m_6}$	$O(6)$	$\dashrightarrow$	$SO(6)$
$\eta_{mn}\,,\ \varepsilon_{m_1 \cdots m_6}\,,\ J_{mn}$	$SO(6)$	$\dashrightarrow$	$U(3)$
$\eta_{mn}\,,\ \varepsilon_{m_1 \cdots m_6}\,,\ J_{mn}\,,\ \Omega_{mnp}$	$U(3)$	$\dashrightarrow$	$SU(3)$

Connection between geometry and physics :

$$J_{mn} = \mp 2{\rm i}\, \eta_\pm^\dagger \gamma_{mn} \eta_\pm \,, \qquad \Omega_{mnp} = -2{\rm i}\, \eta_-^\dagger \gamma_{mnp} \eta_+$$

a generalized almost complex structure ${\mathcal{J}_\Pi}^\Sigma$ on $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$ s.t.

$$ {\mathcal{J}_\Pi}^\Sigma : \ T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 \ \longrightarrow \ T\mathcal{M}_6 \oplus T^*\mathcal{M}_6 $$

$$ \mathcal{J}^2 \ = \ -\mathbb{1}_{12} $$

$^\exists$ $O(6,6)$ invariant metric L, s.t. $\mathcal{J}^{\mathrm{T}} L \mathcal{J} \ = \ L$

Structure group on $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$:

L	$GL(12)$	$\dashrightarrow$	$O(6,6)$
$\mathcal{J}^2 = -\mathbb{1}_{12}$	$O(6,6)$	$\dashrightarrow$	$U(3,3)$
$\mathcal{J}_1$, $\mathcal{J}_2$	$U_1(3,3) \cap U_2(3,3)$	$\dashrightarrow$	$U(3) \times U(3)$
integrable $\mathcal{J}_{1,2}$	$U(3) \times U(3)$	$\dashrightarrow$	$SU(3) \times SU(3)$

$$ \mathcal{J}_{\pm \Pi\Sigma} \ = \ \big\langle \mathrm{Re}\Phi_\pm , \Gamma_{\Pi\Sigma} \mathrm{Re}\Phi_\pm \big\rangle $$

$$ \Phi_+ \ = \ \frac{1}{4} \sum_{k=0}^{6} \frac{1}{k!} \eta_+^\dagger \gamma_{m_k \cdots m_1} \eta_+ \gamma^{m_1 \cdots m_k} \ \sim \ \mathrm{e}^{-\mathrm{i}J} , \qquad \Phi_- \ = \ \frac{1}{4} \sum_{k=0}^{6} \frac{1}{k!} \eta_-^\dagger \gamma_{m_k \cdots m_1} \eta_+ \gamma^{m_1 \cdots m_k} \ \sim \ -\mathrm{i}\Omega $$

Further extend $SU(3) \times SU(3)$ to $E_{7(7)}$:

$T\mathcal{M}_6$	$\oplus$	$T^*\mathcal{M}_6$
momentum		winding
(GCT)		(B_2 gauge)
$\mathbf{6}$		$\mathbf{6}$

$\downarrow$

$T\mathcal{M}_6$	$\oplus$	$T^*\mathcal{M}_6$	$\oplus$	$\Lambda^5 T^*\mathcal{M}_6$	$\oplus$	$\Lambda^5 T\mathcal{M}_6$	$\oplus$	$\Lambda^{\text{even}} T^*\mathcal{M}_6$
momentum		winding		NS5		KK5		D0, D2, D4, D6
(GCT)		(B_2 gauge)		(B_6 gauge)		(GCT of dual vielbein)		($C_{(p)}$, $C_{(8-p)}$ gauges)
$\mathbf{6}$		$\mathbf{6}$		$\mathbf{6}$		$\mathbf{6}$		$\mathbf{1} + \mathbf{15} + \mathbf{15} + \mathbf{1}$

$\mathrm{d}B_6 = *_{10}\mathrm{d}B_2$; $\mathrm{d}C_{(8-p)} = *_{10}\mathrm{d}C_{(p)}$ $(p = 1, 3)$ in type IIA

$\mathbf{6} + \mathbf{6} = \mathbf{12}$: fund. repr. of $O(6,6)$

$\mathbf{6} + \mathbf{6} + \mathbf{6} + \mathbf{6} + (\mathbf{1} + \mathbf{15} + \mathbf{15} + \mathbf{1}) = \mathbf{56}$: fund. repr. of $E_{7(7)}$

hep-th/0701203, arXiv:0904.2333, arXiv:1007.5509, arXiv:1202.0770, etc.

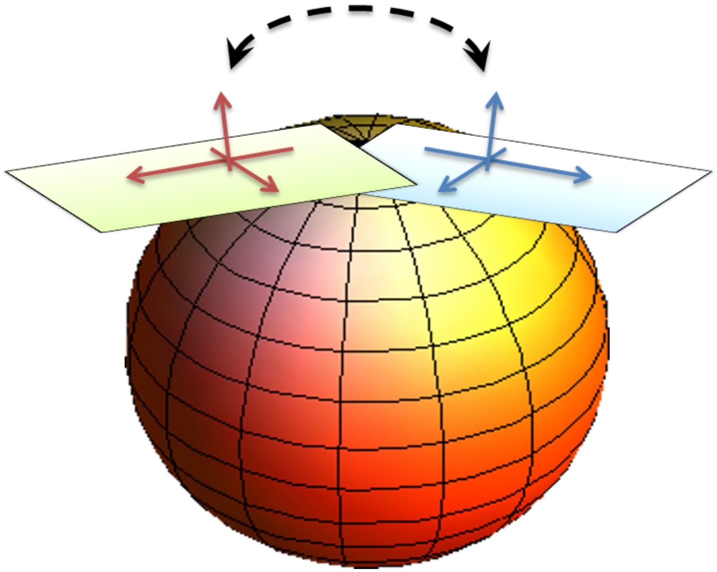
Non-geometric structure

structure groups = diffeom. $(GL(d,\mathbb{R}))$ $\cup$ String duality groups

↑

T-duality, U-duality, etc.

$GL(d,\mathbb{R}) \cup$ duality transf.



Generalized Geometry (N. Hitchin)
Doubled Geometry (C. Hull)

5^2_2-brane is a concrete example (T-fold)

Exotic brane shows us a new insight of stringy spacetime

M-theory on $S^1(R_s)$	mass/tension ($l_s \equiv 1$)	type IIA
longitudinal M2	1	F1
transverse M2	$\frac{1}{g_s}$	D2
longitudinal M5	$\frac{1}{g_s}$	D4
transverse M5	$\frac{1}{g_s^2}$	NS5
longitudinal KK6	$\frac{R_{TN}^2}{g_s^2}$	KK5
KK6 with $R_{TN} = R_s$	$\frac{1}{g_s}$	D6
transverse KK6	$\frac{R_{TN}^2}{g_s^3}$	6_3^1

0	1	2	3	4	5	6	7	8	9	M
✓	✓	✓	✓	✓	✓	✓	S^1	$\mathbb{R}^3$		
KK6 → 6_3^1							Taub-NUT			

$$b_n^c : M = \frac{(R_1 \cdots R_c)^2}{g_s^n}$$

for review: N. Obers and B. Pioline hep-th/9809039

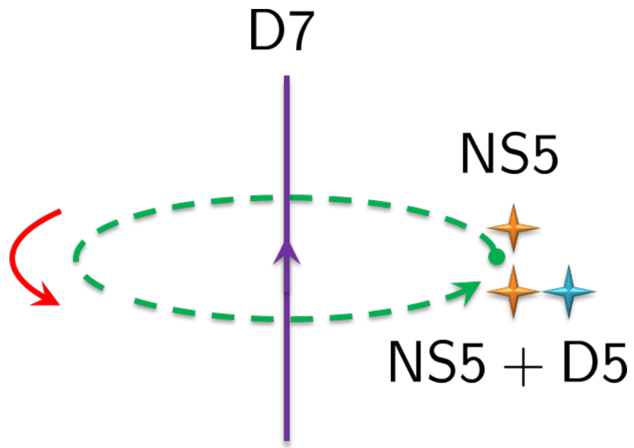
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2,\mathbb{Z})$ monodromy charge q

$$q : (C_2, B_2) \to (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

$$(0, \text{NS5}) \to (\text{D5}, \text{NS5})$$



an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

Five-branes and GLSM

ALE / ALF spaces

4D asymptotic geometry is locally T^k-fibration over $\mathbb{R}^{4-k}$

	k	harmonic function	
ALE	0	$\frac{1}{\lvert\vec{x}\rvert}$	$\mathbb{C}^2/\Gamma$, ADE-singularities
ALF	1	$A+\frac{1}{\lvert\vec{x}\rvert}$	Taub-NUT
ALG	2	$A+B\log\lvert\rho\rvert$	exotic objects as D7, 5^2_2, etc.
ALH	3	$A+B\lvert x\rvert$	linear potential

$$\mathscr{L}_{\text{NS5}} = + \int d^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta} \Theta + \overline{\Psi} \Psi \Big)$$

GLSM for NS5-brane

D. Tong hep-th/0204186

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$	role
neutral HM	chiral $\Psi = X^6 + \mathrm{i}\, X^8 + \ldots$ twisted chiral $\Theta = X^7 + \mathrm{i}\, X^9 + \ldots$	spacetime coord.

$$\mathscr{L}_{\text{NS5}} = \int d^4\theta \Big\{ \frac{1}{e^2}\Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) \Big\} + \int d^4\theta \, \frac{1}{g^2}\Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \Big(+ (s - \Psi)\Phi \Big) + \text{(h.c.)}$$

$$+ \int d^2\widetilde{\theta} \, (t - \Theta)\Sigma + \text{(h.c.)}$$

GLSM for NS5-brane

D. Tong hep-th/0204186

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		role
neutral HM	chiral $\Psi = X^6 + \mathrm{i}\, X^8 + \ldots$	twisted chiral $\Theta = X^7 + \mathrm{i}\, X^9 + \ldots$	spacetime coord.
VMs	twisted chiral $\Sigma = \overline{D}_+ D_- V$	chiral Φ	gauging isometry
FI parameters	$s = s^6 + \mathrm{i}\, s^8$	$t = t^7 + \mathrm{i}\, t^9$	position of five-branes

$$\mathscr{L}_{\text{NS5}} = \int d^4\theta \Big\{ \frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \Big\} + \int d^4\theta \frac{1}{g^2}\Big(-\overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + \text{(h.c.)}$$

$$+ \int d^2\widetilde{\theta}\, (t - \Theta)\Sigma + \text{(h.c.)}$$

GLSM for NS5-brane

D. Tong hep-th/0204186

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		role
neutral HM	chiral $\Psi = X^6 + i\, X^8 + \ldots$	twisted chiral $\Theta = X^7 + i\, X^9 + \ldots$	spacetime coord.
VMs	twisted chiral $\Sigma = \overline{D}_+ D_- V$	chiral Φ	gauging isometry
charged HM	chiral $Q\ (+)$	chiral $\widetilde{Q}\ (-)$	curving geometry
FI parameters	$s = s^6 + i\, s^8$	$t = t^7 + i\, t^9$	position of five-branes

Bosonic Lagrangian after integrating-out auxiliary fields :

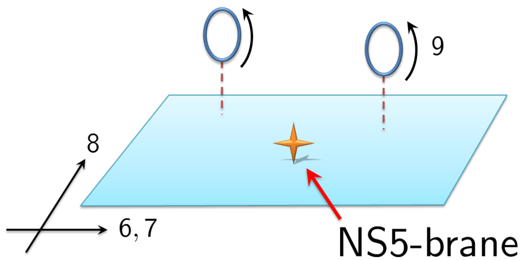
$$\begin{aligned}\mathscr{L}^{\text{kin}}_{\text{NS5}} &= \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big] \\ &\quad -\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2\Big] - (X^9 - t^9)\, F_{01}\end{aligned}$$

$$\begin{aligned}\mathscr{L}^{\text{pot}}_{\text{NS5}} &= -2\big(|\sigma|^2 + |\phi|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big) \\ &\quad -\frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2\end{aligned}$$

Steps to NLSM of NS5-brane

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, X^9)$

$$\mathscr{L}^{\text{NLSM}}_{\text{NS5}} = -\frac{1}{2}\Big(\frac{1}{g^2} + \frac{1}{r}\Big)\Big[(\partial_m \vec{X})^2 + (\partial_m X^9)^2\Big] + \varepsilon^{mn}\,\Omega_i\,\partial_m X^i\,\partial_n X^9$$

$$= -\frac{1}{2} G_{IJ}\,\partial_m X^I\,\partial^m X^J + \frac{1}{2} B_{IJ}\,\varepsilon^{mn}\partial_m X^I\,\partial_n X^J$$

Target space geometry is...

$$G_{IJ} = H\,\delta_{IJ}\,, \qquad B_{i9} = \Omega_i$$

$$H = \frac{1}{g^2} + \frac{1}{r}\,, \qquad \nabla_i H = (\nabla \times \Omega)_i$$

$$\mathscr{L}_{\text{NS5}} = \int \mathrm{d}^4\theta \left\{ \frac{1}{e^2}\Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\, \mathrm{e}^{-2V} \widetilde{Q} \right\} + \int \mathrm{d}^4\theta\, \frac{1}{g^2}\Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int \mathrm{d}^2\theta \left(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \right) + \text{(h.c.)}$$

$$+ \int \mathrm{d}^2\widetilde{\theta}\, (t - \Theta)\Sigma + \text{(h.c.)}$$

T-duality transformation to KK-monopole

$$\mathscr{L}_{\text{NS5}} = \int d^4\theta \Big\{ \frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \Big\} + \int d^4\theta\, \frac{1}{g^2}\Big(-\overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$

$$+ \int d^2\theta \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + \text{(h.c.)}$$

$$+ \int d^2\widetilde{\theta}\, (t - \Theta)\Sigma + \text{(h.c.)}$$

T-duality transformation to KK-monopole

Duality (Roček-Verlinde) transformation

$$-\frac{1}{g^2}(\Theta + \overline{\Theta}) = (\Gamma + \overline{\Gamma}) + V$$

$$\pm(\partial_0 \pm \partial_1) X^9 = -g^2 (D_0 \pm D_1) Y^9$$

$$D_m Y^9 = \partial_m Y^9 + A_m$$

$$\mathscr{L}_{\text{KK}} = \int \mathrm{d}^4\theta \left\{ \frac{1}{e^2}\Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\,\mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\,\mathrm{e}^{-2V} \widetilde{Q} \right\} + \int \mathrm{d}^4\theta \left\{ \frac{g^2}{2}\Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2} \overline{\Psi}\Psi \right\}$$

$$+ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.})$$

$$+ \left\{ \int \mathrm{d}^2\widetilde{\theta}\, t\, \Sigma + (\text{h.c.}) \right\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

GLSM for KK-monopole

Bosonic Lagrangian after integrating-out auxiliary fields :

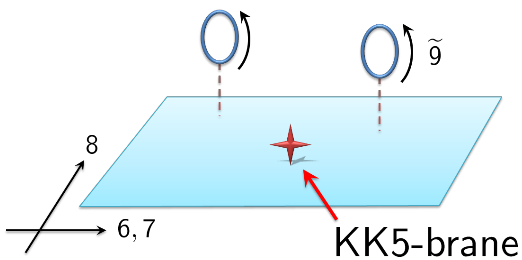
$$\begin{aligned}\mathscr{L}^{\text{kin}}_{\text{KK}} &= \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m\sigma|^2 - |\partial_m\phi|^2\Big] - \Big[|D_m q|^2 + |D_m\widetilde{q}|^2\Big] \\ &\quad -\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] - \frac{g^2}{2}(D_m Y^9)^2 - \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)\end{aligned}$$

$$\begin{aligned}\mathscr{L}^{\text{pot}}_{\text{KK}} &= -2\big(|\sigma|^2 + |\phi|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big) \\ &\quad -\frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2\end{aligned}$$

Steps to NLSM of KK-monopole

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, Y^9)$

$$\mathscr{L}_{\text{KK}}^{\text{NLSM}} = -\frac{1}{2}H\Big[(\partial_m \vec{X})^2\Big] - \frac{1}{2}H^{-1}(\partial_m Y^9 - \Omega_i\,\partial_m X^i)^2$$

$$= -\frac{1}{2}G_{IJ}\,\partial_m X^I\,\partial^m X^J + \frac{1}{2}B_{IJ}\,\varepsilon^{mn}\partial_m X^I\,\partial_n X^J$$

Target space geometry is...

$$G_{ij} = H\,\delta_{ij}\,, \qquad G_{99} = H^{-1}\,, \qquad B_{IJ} = 0$$

$$H = \frac{1}{g^2} + \frac{1}{r}\,, \qquad \nabla_i H = (\nabla \times \Omega)_i$$

$$\mathscr{L}_{\text{KK}} = \int d^4\theta \left\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \right\} + \int d^4\theta \left\{ \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2} \overline{\Psi}\Psi \right\}$$

$$+ \int d^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + \text{(h.c.)}$$

$$+ \Big\{ \int d^2\widetilde{\theta}\, t\, \Sigma + \text{(h.c.)} \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

T-duality transformation to 5^2_2-brane!

$$\mathscr{L}_{\text{KK}} = \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}}\, \mathrm{e}^{-2V} \widetilde{Q} \Big\} + \int \mathrm{d}^4\theta \, \Big\{ \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2} \overline{\Psi}\Psi \Big\}$$

$$+ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + \text{(h.c.)}$$

$$+ \Big\{ \int \mathrm{d}^2\widetilde{\theta}\, t\, \Sigma + \text{(h.c.)} \Big\} - \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

T-duality transformation to 5^2_2-brane*!*

Duality (Roček-Verlinde) transformation

$$-\frac{1}{g^2}(\Psi + \overline{\Psi}) = (\Xi + \overline{\Xi}) - (C + \overline{C})$$

$$\Phi = \overline{D}_+ \overline{D}_- C \qquad \text{cf. } \Sigma = \overline{D}_+ D_- V$$

$$
\begin{aligned}
\mathscr{L}_{\mathsf{E}} \;=\; & \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} \Big(- \overline{\Sigma} \Sigma + \overline{\Phi} \Phi \Big) + \overline{Q} \, \mathrm{e}^{2V} Q + \overline{\widetilde{Q}} \, \mathrm{e}^{-2V} \widetilde{Q} \Big\} \\
& + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big\{ \Big(\Gamma + \overline{\Gamma} + V \Big)^2 - \Big(\Xi + \overline{\Xi} - (C + \overline{C}) \Big)^2 \Big\} \\
& + \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q} \Phi Q + s \Phi \Big) + \text{(h.c.)} \Big\} - \int \mathrm{d}^4\theta \, (\Psi - \overline{\Psi})(C - \overline{C}) \\
& + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, t \, \Sigma + \text{(h.c.)} \Big\} - \varepsilon^{mn} \, \partial_m (X^9 A_n)
\end{aligned}
$$

GLSM for exotic 5^2_2-brane

S. Sasaki and TK arXiv:1304.4061

Steps to NLSM of 5^2_2-brane

1. search SUSY vacua $\mathscr{L}^{\text{potential}} = 0$
2. solve constraints of charged HM $(Q, \widetilde{Q})$
3. integrate out VM (V, Φ) in IR

★4. integrate s^8, and solve EOM for T-dual field X^8

$$\mathscr{L}_{\mathsf{E}}^{\mathsf{kin}} = \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big]$$
$$-\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2\Big] - \frac{g^2}{2}\Big[(\partial_m Y^8)^2 + (D_m Y^9)^2\Big] - (X^9 - t^9)\, F_{01}$$

$$\mathscr{L}_{\mathsf{E}}^{\mathsf{pot}} = -2\big(|\sigma|^2 + 4|M_c|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big)$$
$$-\frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} - (X^6 - s^6) - \mathrm{i}(X^8 - s^8)\Big|^2$$
$$+ \frac{g^2}{2}(A_{c=} + \overline{A}_{c=})(B_{c+\!\!+} + \overline{B}_{c+\!\!+})$$

$$(\partial_0 + \partial_1)X^8 = -g^2(\partial_0 + \partial_1)Y^8 + g^2(B_{c+\!\!+} + \overline{B}_{c+\!\!+})$$
$$(\partial_0 - \partial_1)X^8 = +g^2(\partial_0 - \partial_1)Y^8 + g^2(A_{c=} + \overline{A}_{c=})$$
$$+\frac{g^2}{2}(A_{c=} + \overline{A}_{c=})(B_{c+\!\!+} + \overline{B}_{c+\!\!+}) = -\frac{1}{2g^2}(\partial_m X^8)^2 + \frac{g^2}{2}(\partial_m Y^8)^2 + \varepsilon^{mn}\,(\partial_m X^8)(\partial_n Y^8)$$

Step 1.,2.,3. :

$$\begin{aligned}
\mathscr{L} \;=\; & -\frac{1}{2}H\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] - \frac{1}{2H}(\partial_m Y^9)^2 \\
& -\frac{(\Omega_8)^2}{2H}(\partial_m X^8)^2 + \frac{\Omega_8}{H}(\partial_m X^8)(\partial^m Y^9) \\
& -\frac{(\Omega_6)^2}{2H}(\partial_m X^6)^2 - \frac{\Omega_6 \Omega_8}{H}(\partial_m X^6)(\partial^m X^8) + \frac{\Omega_6}{H}(\partial_m X^6)(\partial^m Y^9) \\
& -\varepsilon^{mn}\partial_m((X^9 - t^9)A_n) + \varepsilon^{mn}(\partial_m X^8)(\partial_n Y^8)
\end{aligned}$$

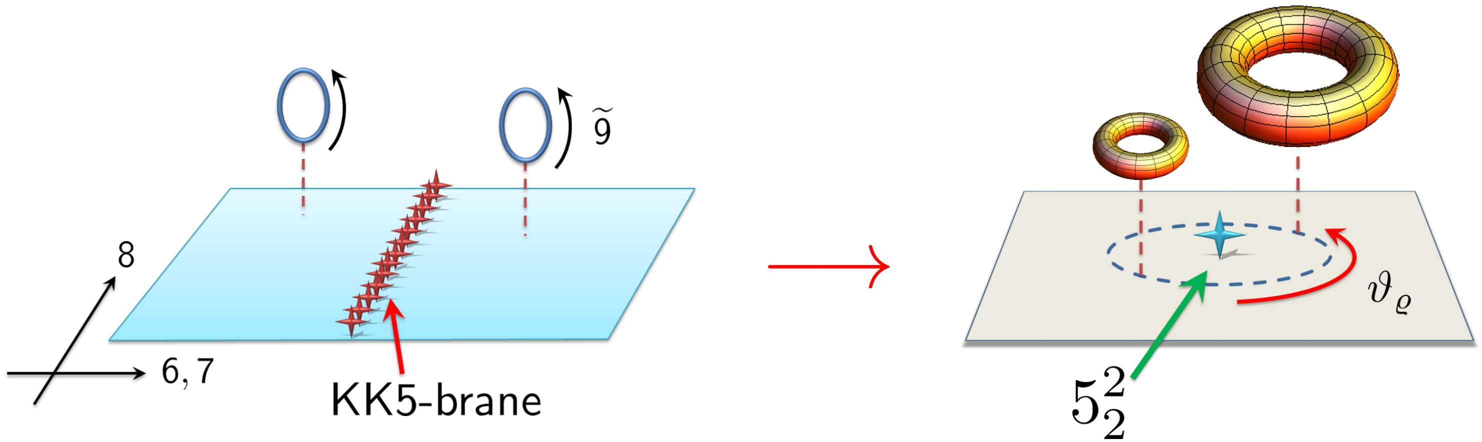
$$H = \frac{1}{g^2} + \frac{1}{r}, \qquad \Omega_6 = \frac{X^8 - s^8}{r(r + (X^7 - t^7))}, \quad \Omega_8 = -\frac{X^6 - s^6}{r(r + (X^7 - t^7))}$$

$$A_m = -\frac{1}{rH}(\partial_m Y^9 - \Omega_i \, \partial_m X^i) - \Omega_i \, \partial_m X^i$$

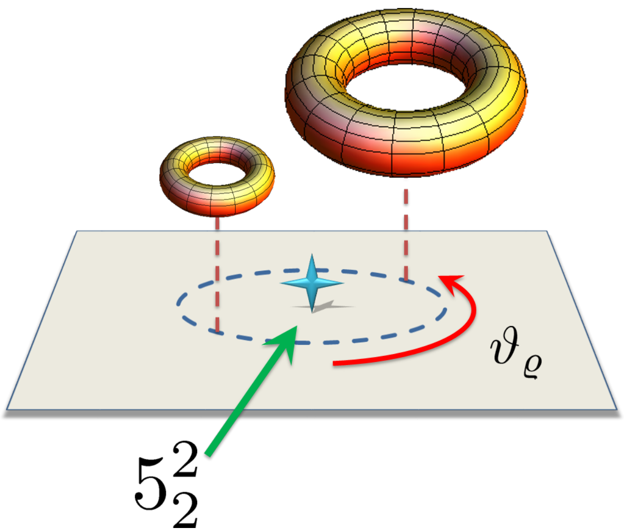
Step 4. : $s^8 = 2\pi \mathcal{R}_8 \, s \xrightarrow{\text{integral of } s}$ emerge isometry

$$
\begin{cases}
H \to h_0 + \sigma \log(\mu/\varrho) & \text{: co-dim. 2} \quad \varrho = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2} \\
\Omega_6 \to 0 & \text{: isometry along } X^8 \\
\Omega_8 \to \sigma \arctan\left(\dfrac{X^7 - t^7}{X^6 - s^6}\right) \equiv \sigma \, \vartheta_\varrho & \text{: “non–single-valued” metric}
\end{cases}
$$

EOM for X^8 : $\partial_m X^8 = \dfrac{H}{K}\Big[\dfrac{\sigma \vartheta_\varrho}{H}(\partial_m Y^9) + \varepsilon_{mn}(\partial^n Y^8)\Big]$ $\qquad K = H^2 + (\sigma \vartheta_\varrho)^2$



$$\mathscr{L}_{\text{exotic}} = -\frac{H}{2}\Big[(\partial_m \varrho)^2 + \varrho^2(\partial_m \vartheta_\varrho)^2\Big] - \frac{H}{2K}\Big[(\partial_m Y^8)^2 + (\partial_m Y^9)^2\Big]$$
$$-\frac{\sigma\vartheta_\varrho}{K}\,\varepsilon^{mn}(\partial_m Y^8)(\partial_n Y^9) - \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)$$



We obtained GLSM/NLSM of Exotic Five-brane!

Quantum corrections on five-branes

S. Sasaki and TK arXiv:1305.4439

String Worldsheet Instanton Corrections

↓

Deform target space geometry by momentum and/or winding effects

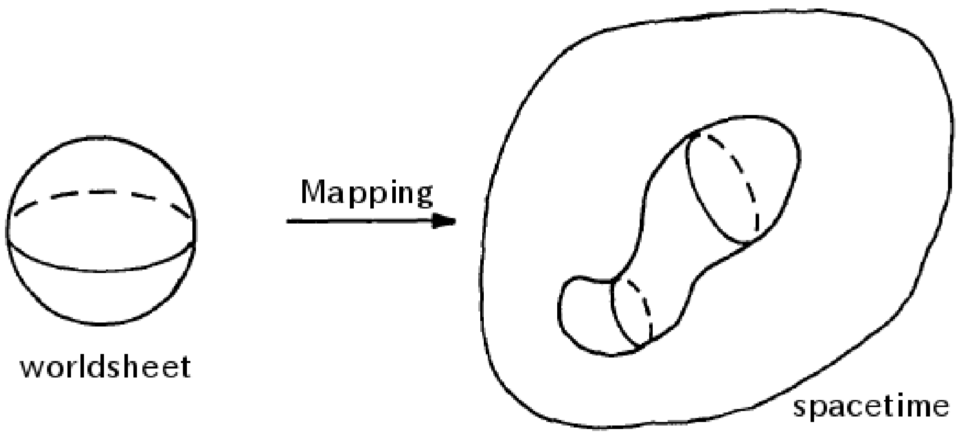


Figure from E. Witten CMP 92 (1984) 455

GLSM is a powerful tool in this stage :

Worldsheet instantons in NLSM can be captured by vortex solution in gauge theory

- NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{r} \text{ : radius of } X^9$$

$g \to 0$ case	general r
radius of X^9	∞
KK-modes	light
winding modes	heavy

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of X^9	∞	0
KK-modes	light	heavy
winding modes	heavy	light

- NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{r} \text{ : radius of } X^9$$

$g \to 0$ case	general r
radius of X^9	∞
KK-modes	light
winding modes	heavy

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of X^9	∞	0
KK-modes	light	heavy
winding modes	heavy	light

GLSM for NS5-brane has $X^9 F_{01}$

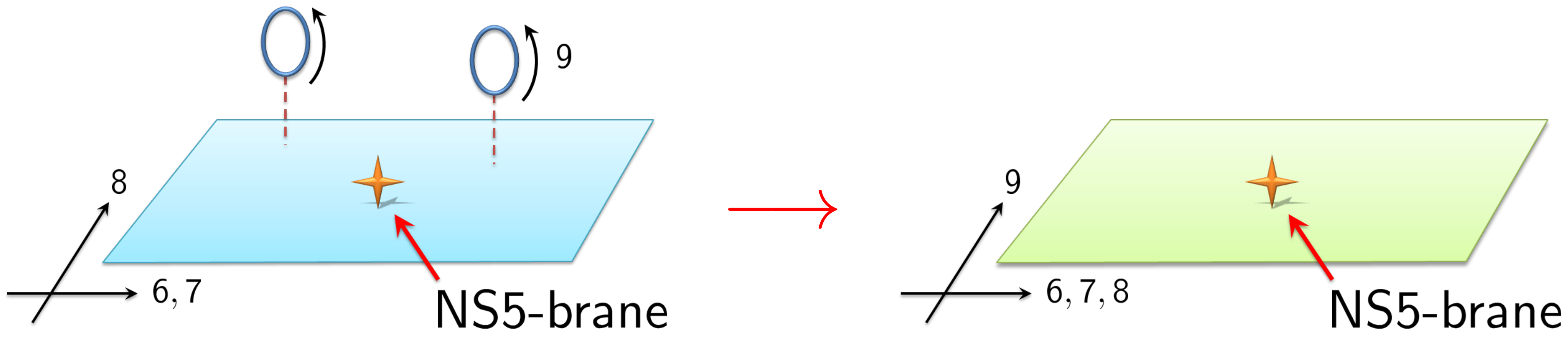
$\downarrow$

KK-mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to NS5-brane :

$X^9 F_{01}$ in GLSM

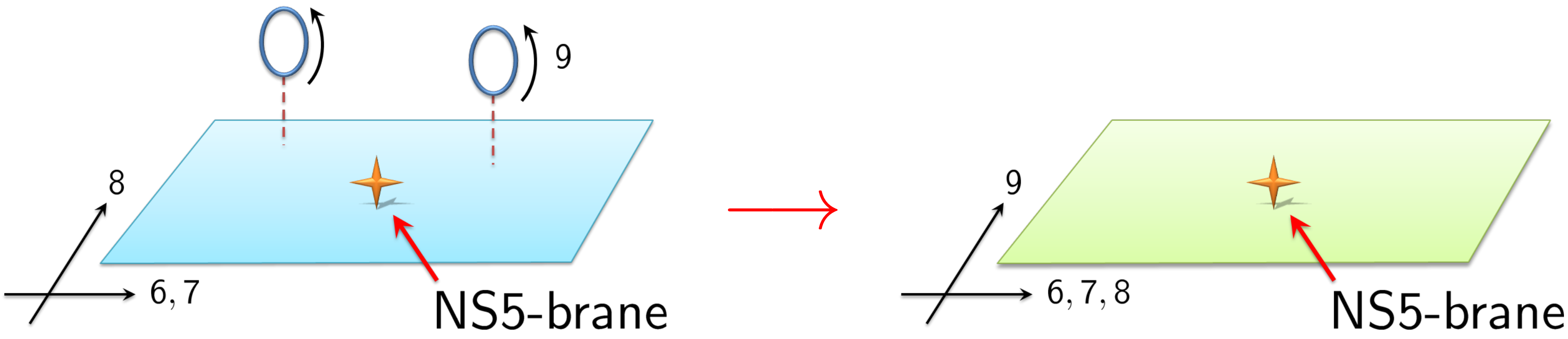
$\rightarrow$ unfolding effect on compactified circle X^9



Worldsheet instanton corrections to NS5-brane :

$X^9 F_{01}$ in GLSM

$\rightarrow$ unfolding effect on compactified circle X^9



$$H = \frac{1}{g^2} + \frac{1}{r} \rightarrow \frac{1}{g^2} + \frac{1}{r}\sum_{n=1}^{\infty} e^{-nr}\Big[e^{+i\,nX^9} + e^{-i\,nX^9}\Big]$$
$$= \frac{1}{g^2} + \frac{1}{r}\frac{\sinh(r)}{\cosh(r) - \cos(X^9)}$$

D. Tong hep-th/0204186

▶ KK-monopole

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{r}\right)^{-1} \text{ : radius of } \widetilde{X}^9$$

$g \to 0$ case	general r
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of $\widetilde{X}^9$	0	∞
KK-modes	heavy	light
winding modes	light	heavy

▶ KK-monopole

$$H^{-1} = \Big(\frac{1}{g^2} + \frac{1}{r} \Big)^{-1} \text{ : radius of } \widetilde{X}^9$$

$g \to 0$ case	general r	$g \to \infty$ case	$r \to 0$	$r \to \infty$
radius of $\widetilde{X}^9$	0	radius of $\widetilde{X}^9$	0	∞
KK-modes	heavy	KK-modes	heavy	light
winding modes	light	winding modes	light	heavy

GLSM for KK-monopole has $\varepsilon^{mn} \partial_m (X^9 A_n)$

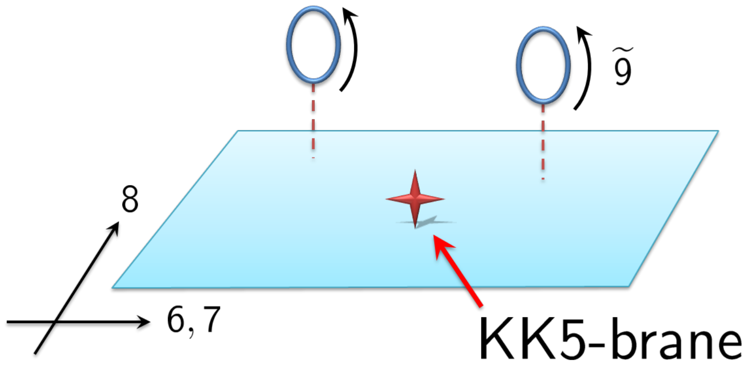
$\downarrow$

Winding mode corrections can be captured by vortex solution in gauge theory

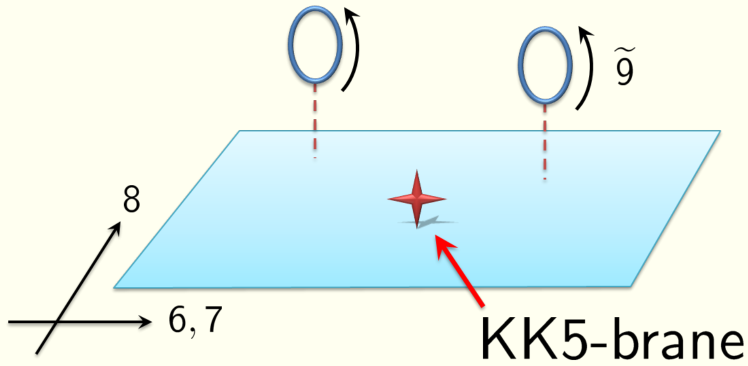
Worldsheet instanton corrections to KK-monopole :

$\varepsilon^{mn}\,\partial_m(X^9 A_n)$ in GLSM

$\rightarrow$ string winding modes along X^9



$\longrightarrow$



+

string winding along
dual circle X^9

$$H \rightarrow \frac{1}{g^2} + \frac{1}{r}\frac{\sinh(r)}{\cosh(r) - \cos(X^9)}$$

J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097

- 5^2_2-brane

$\dfrac{H}{K}$: radius of $\widetilde{X}^9$ $\qquad H = \dfrac{1}{g^2} + \sigma \log\left(\dfrac{\Lambda}{\varrho}\right), \; K = H^2 + (\sigma \vartheta_\varrho)^2$

$g \to 0$ case	general ϱ
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\varrho \to 0$	$\varrho \to \Lambda$	
radius of $\widetilde{X}^9$	0	0	∞ at $\vartheta_\varrho = 0$
KK-modes	heavy	heavy	light at $\vartheta_\varrho = 0$
winding modes	light	light	heavy at $\vartheta_\varrho = 0$

► 5^2_2-brane

$\frac{H}{K}$: radius of $\widetilde{X}^9$ $\qquad H = \frac{1}{g^2} + \sigma \log\Big(\frac{\Lambda}{\varrho}\Big), \ K = H^2 + (\sigma\vartheta_{\varrho})^2$

$g \to 0$ case	general ϱ
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\varrho \to 0$	$\varrho \to \Lambda$	
radius of $\widetilde{X}^9$	0	0	∞ at $\vartheta_{\varrho} = 0$
KK-modes	heavy	heavy	light at $\vartheta_{\varrho} = 0$
winding modes	light	light	heavy at $\vartheta_{\varrho} = 0$

GLSM for 5^2_2 with one gauged isometry has $\varepsilon^{mn}\partial_m(X^9 A_n)$

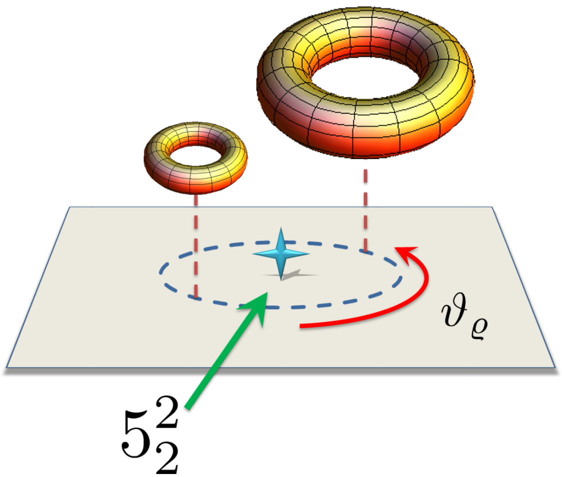
$\downarrow$

Winding mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to 5^2_2-brane with one gauged isometry :

$\varepsilon^{mn}\,\partial_m(X^9 A_n)$ in GLSM

$\to$ string winding modes along X^9



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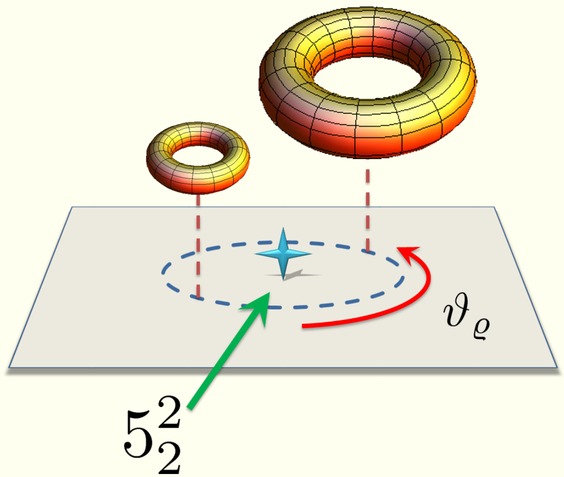

+

string winding along dual circle X^9

$$H \;\to\; h_0 + \sigma \log(\mu/\varrho) + \sigma \sum_{n\neq 0} \exp(\mathrm{i}\, n\, X^9) K_0(|n|\varrho)$$

S. Sasaki and TK arXiv:1305.4439

Duality transformation of charged chiral in F-term

M. Yata and TK arXiv:1406.0087

- Dualize charged chiral in D-term and F-term

$$\mathscr{L}_4 = \int \mathrm{d}^4\theta \, |\Psi|^2 \, \mathrm{e}^{2V} + \Big\{ \int \mathrm{d}^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\text{h.c.}) \Big\}$$

- Dualize charged chiral in D-term and F-term

$$\mathscr{L}_4 = \int \mathrm{d}^4\theta \, |\Psi|^2 \, \mathrm{e}^{2V} + \Big\{ \int \mathrm{d}^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\mathrm{h.c.}) \Big\}$$

1. Convert F-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int \mathrm{d}^2\theta \, \Psi \, \mathcal{W} + (\mathrm{h.c.}) = 2 \int \mathrm{d}^4\theta \, \Big[\Psi C + \overline{\Psi C} \Big]$$

2. Replace $\Psi \pm \overline{\Psi}$ to auxiliary fields R and $\mathrm{i}S$:

$$\mathscr{L}_5 = \int \mathrm{d}^4\theta \, \Big[\mathrm{e}^{2V+R} + 2\Big\{ \mathrm{e}^{\frac{1}{2}(R+\mathrm{i}S)} C + \mathrm{e}^{\frac{1}{2}(R-\mathrm{i}S)} \overline{C} \Big\} - R(Y + \overline{Y}) - \mathrm{i}S(\Upsilon - \overline{\Upsilon}) \Big]$$

3. Integrating out R and Υ, we obtain the "dual" system :

$$\mathscr{L}_6 = \int \mathrm{d}^4\theta \, \Big[-2(Y + \overline{Y}) \log \mathcal{F} + \frac{1}{2} \mathcal{F} \mathcal{T} + 2V(Y + \overline{Y}) \Big]$$

$$\mathcal{F} = -\mathcal{T} + \sqrt{\mathcal{T}^2 + 4(Y + \overline{Y})}, \quad \mathcal{T} = \mathrm{e}^{-V} \Big[\mathrm{e}^{+\frac{1}{2}(\Omega - \overline{\Omega})} C + \mathrm{e}^{-\frac{1}{2}(\Omega - \overline{\Omega})} \overline{C} \Big], \quad \Psi = \mathrm{e}^{\Omega}$$

$$\begin{aligned}\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \, \Big[\frac{1}{e^2}\Big(- |\Sigma|^2 + |\Phi|^2\Big) + \frac{1}{\widetilde{e}^2}\Big(- |\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2\Big)\Big] \\ & + \int \mathrm{d}^4\theta \, \Big[|A_1|^2 \mathrm{e}^{2V} + |A_2|^2 \mathrm{e}^{-4V-2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V}\Big] + \int \mathrm{d}^4\theta \, \Big[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V+2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V}\Big] \\ & + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi(-A_1 B_1 + 2 A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2\Big) + (\text{h.c.})\Big\} \\ & - \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t\Sigma) + (\text{h.c.})\Big\}\end{aligned}$$

toric data	(A_1, B_1)	(A_2, B_2)	(A_3, B_3)
$U(1)(V, \Phi)$	$(+1, -1)$	$(-2, +2)$	$(+1, -1)$
$U(1)(\widetilde{V}, \widetilde{\Phi})$	$(0, 0)$	$(-1, +1)$	$(0, 0)$

M. Yata and TK arXiv:1406.0087

$$
\begin{aligned}
\mathscr{L} \ &= \ \int \mathrm{d}^4\theta \, \Big[\frac{1}{e^2} \Big(- |\Sigma|^2 + |\Phi|^2 \Big) + \frac{1}{\widetilde{e}^2} \Big(- |\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2 \Big) \Big] \\
&\quad + \int \mathrm{d}^4\theta \, \Big[|A_1|^2 \mathrm{e}^{2V} + |A_2|^2 \mathrm{e}^{-4V - 2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V} \Big] + \int \mathrm{d}^4\theta \, \Big[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V + 2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V} \Big] \\
&\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi(-A_1 B_1 + 2 A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2 \Big) + (\text{h.c.}) \Big\} \\
&\quad - \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t \Sigma) + (\text{h.c.}) \Big\}
\end{aligned}
$$

$$\downarrow$$

$$
\begin{aligned}
\mathscr{L}_{\text{NLSM}} \ &= \ -\frac{\mathcal{A}^{-1}}{2} (\partial_m \rho)^2 - \frac{\rho^2}{8} \Big\{ (\partial_m \vartheta)^2 + (\partial_m \varphi)^2 \sin^2 \vartheta \Big\} - \frac{\rho^2 \mathcal{A}}{8} \Big\{ \partial_m \psi + (\partial_m \varphi) \cos \vartheta \Big\}^2 \\
\mathcal{A} \ &= \ 1 - \frac{\mathfrak{a}^4}{\rho^4}
\end{aligned}
$$

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$$
\begin{aligned}
\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \left[\frac{1}{e^2}\Big(-|\Sigma|^2 + |\Phi|^2\Big) + \frac{1}{\widetilde{e}^2}\Big(-|\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2\Big)\right] \\
& + \int \mathrm{d}^4\theta \left[|A_1|^2 \mathrm{e}^{2V} + |A_2|^2 \mathrm{e}^{-4V-2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V}\right] + \int \mathrm{d}^4\theta \left[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V+2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V}\right] \\
& + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi(-A_1 B_1 + 2A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2\Big) + (\text{h.c.})\Big\} \\
& - \Big\{ \int \mathrm{d}^2\widetilde{\theta}\, (t\Sigma) + (\text{h.c.})\Big\}
\end{aligned}
$$

Dualize charged chiral $A_1 \to Y_1$

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$$\begin{aligned}
\mathscr{L}' \;&=\; \int \mathrm{d}^4\theta \, \Big[\frac{1}{e^2}\Big(- |\Sigma|^2 + |\Phi|^2\Big) + \frac{1}{\widetilde{e}^2}\Big(- |\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2\Big)\Big] \\
&\quad + \int \mathrm{d}^4\theta \, \Big[|A_2|^2 \mathrm{e}^{-4V-2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V}\Big] + \int \mathrm{d}^4\theta \, \Big[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V+2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V}\Big] \\
&\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi(2A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2 \Big) + (\text{h.c.}) \Big\} \\
&\quad + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (-2Y_1 - t)\Sigma + (\text{h.c.}) \Big\} + \int \mathrm{d}^4\theta \, \Big[4(Y_1 + \overline{Y}_1) \log \mathcal{F}_1 + \frac{1}{2} \mathcal{F}_1 \mathcal{T}_1 \Big]
\end{aligned}$$

Integrate out $\mathrm{e}^{\Omega_1 - \overline{\Omega}_1}$ in $\mathcal{T}_1$ with $A_1 \;=\; \mathrm{e}^{\Omega_1}$: T-dualized configuration

$$\begin{aligned}
\mathrm{d}s^2 \;&=\; -\frac{\mathcal{A}^{-1}}{2}(\partial_m \rho)^2 - \frac{\rho^2}{8}(\partial_m \vartheta)^2 - \frac{\rho^2}{4} \frac{\mathcal{A} \sin^2 \vartheta}{\mathcal{A} \cos^2 \vartheta + \sin^2 \vartheta} (\partial_m \psi)^2 - \frac{2}{\rho^2} \frac{1}{\mathcal{A} \cos^2 \vartheta + \sin^2 \vartheta} (\partial_m \widetilde{\varphi})^2 \\
&\quad + \frac{\mathcal{A} \cos \vartheta}{\mathcal{A} \cos^2 \vartheta + \sin^2 \vartheta} \, \varepsilon^{mn} \, (\partial_m \widetilde{\varphi})(\partial_n \psi)
\end{aligned}$$

$\leftarrow$ B-field for 2 units of NS5-branes

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Worldvolume actions

- ✓ covariant form of gauged isometries and Wess-Zumino terms
- ✓ type IIB, IIA, and Heterotics

S. Sasaki, M. Yata and TK arXiv:1404.5442

Worldvolume action of 5^2_2-brane could be obtained from

D5-brane's action via S- and T-dualities	(IIB)
KK6-brane action via reduction and T-duality	(IIB)
M5-brane action via reduction and T-dualities	(IIA)

Different point: Two isometries along transverse directions

How to describe its worldvolume theory with isometries?

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Different point: Two isometries along transverse directions

How to describe its worldvolume theory with isometries?

Already Known!

Bergshoeff et al hep-th/9706117

Worldvolume action with WZ-term is gauged with respect to isometries ex) gravity sector of KK6-brane action with gauged isometry in M-theory:

$$\mathscr{L}^{\mathsf{M}}_{\mathsf{KK6}} = -\frac{1}{2} T^{\mathsf{M}}_{\mathsf{KK6}} \sqrt{-\gamma} \left(k^{\frac{4}{7}} \gamma^{ab} D_a X^\mu D_b X^\nu g_{\mu\nu} - 5 \right)$$

$$= -T^{\mathsf{M}}_{\mathsf{KK6}} k^2 \sqrt{-\det(D_a X^\mu D_b X^\nu g_{\mu\nu})}$$

$$= -T^{\mathsf{M}}_{\mathsf{KK6}} k^2 \sqrt{-\det(\partial_a X^\mu \partial_b X^\nu \Pi_{\mu\nu})}$$

$$D_a X^\mu = \partial_a X^\mu + C_a k^\mu$$ k^μ : Killing vector

$$\Pi_{\mu\nu} = g_{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

✓ Construct the worldvolume action with WZ-term in a covariant way:

$$\text{KK6} \xrightarrow{\text{reduction}} \text{KK5 in IIA} \xrightarrow{\text{T-duality}} 5^2_2 \text{ in IIB}$$

$$\text{M5} \xrightarrow{\text{reduction}} \text{NS5 in IIA} \xrightarrow{\text{T-dualities}} 5^2_2 \text{ in IIA}$$

✓ 5^2_2-branes in heterotic theories can be obtained by truncations:

$$5^2_2 \text{ in IIB} \xrightarrow[\mathcal{N}=(1,1)\to\mathcal{N}=(1,0)]{\text{RR = 0}} 5^2_2 \text{ in HSO}$$

$$5^2_2 \text{ in IIA} \xrightarrow[\mathcal{N}=(2,0)\to\mathcal{N}=(1,0)]{\text{RR = 0}} 5^2_2 \text{ in HE8}$$

$$S^{\text{IIB}}_{5^2_2} = -T_{5^2_2} \int \mathrm{d}^6\xi \, \mathrm{e}^{-2\phi} (\det h_{IJ}) \sqrt{1 + \mathrm{e}^{+2\phi} (\det h_{IJ})^{-1} \big(i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B) \big)^2}$$

$$\times \sqrt{- \det \Big[\Pi_{\mu\nu}(k_2) \, \partial_a X^\mu \partial_b X^\nu + \frac{K^{(1)}_a K^{(1)}_b}{(k_2)^2} - \frac{(k_2)^2 (K^{(2)}_a K^{(2)}_b - K^{(3)}_a K^{(3)}_b)}{(\det h_{IJ})} + \lambda \mathcal{F}_{ab} \Big]}$$

$$-\mu_5 \int_{\mathcal{M}_6} \Big[P[i_{k_1} i_{k_2} B^{(8,2)}] - \frac{1}{2} P[\widetilde{B} \wedge \widetilde{C}^{(2)} \wedge \widetilde{C}^{(2)}] + \lambda P[\widetilde{C}^{(4)} + \widetilde{C}^{(2)} \wedge \widetilde{B}] \wedge \widetilde{F}$$

$$- \frac{\lambda^2}{2!} P[\widetilde{B}] \wedge \widetilde{F} \wedge \widetilde{F} + \frac{\lambda^3}{3!} \frac{i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B)}{(i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B))^2 + \mathrm{e}^{-2\phi} (\det h_{IJ})} \widetilde{F} \wedge \widetilde{F} \wedge \widetilde{F} \Big]$$

$$h_{IJ} = k^\mu_I k^\nu_J (g_{\mu\nu} + B_{\mu\nu}), \quad \text{etc..}$$

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