Exotic Five-branes

arXiv:1304.4061, and so forth

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well-known objects	String Dualities → in lower dimensions	less known objects
D-branes, NS-branes		Exotic Branes

✓ co-dimension 2 (“defect” branes), or less

✓ non–single-valued metric of spacetime

✓ monodromy from string dualities

(Exotic) Branes are symbolized as

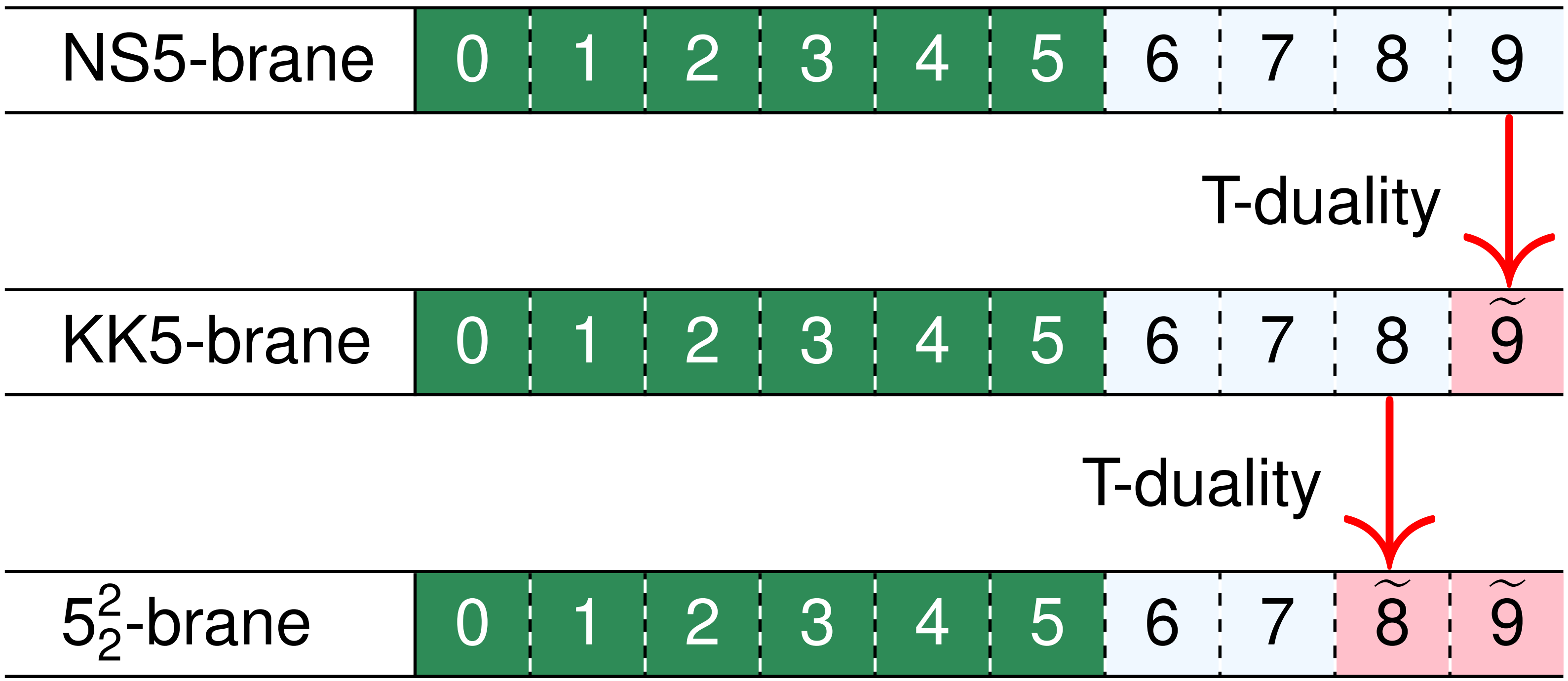
$$b^{c}_{n}$$

b : spatial dimensions

c : # of isometry directions

n : tension $\sim g_{\mathsf{s}}^{-n}$

5^2_2-brane is constructed from NS5-brane.

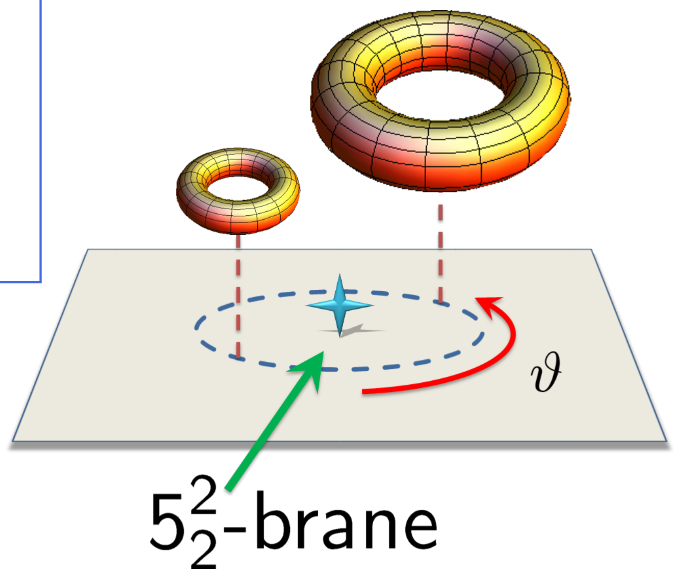


5^2_2-brane : co-dim. 2

$$
\begin{aligned}
ds^2 &= dx^2_{012345} + H\big[d\varrho^2 + \varrho^2 (d\vartheta)^2\big] + \frac{H}{K}\big[(d\widetilde{x}^8)^2 + (d\widetilde{x}^9)^2\big] \\
B_{89} &= -\frac{\vartheta}{K}, \qquad e^{2\phi} = \frac{H}{K} \\
H &= 1 + \log\frac{\Lambda}{\varrho}, \qquad K = H^2 + \vartheta^2
\end{aligned}
$$

$$\vartheta = 0 \quad : \quad G_{88} = G_{99} = \frac{1}{H}$$

$$\vartheta = 2\pi \quad : \quad G_{88} = G_{99} = \frac{H}{H^2 + (2\pi)^2}$$



We never remove the shift 2π by $\begin{cases} \text{coordinate transformations,} \\ \text{B-field gauge transformation.} \end{cases}$

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✓ ill-defined asymptotic behavior $\quad H = 1 + \log \frac{\Lambda}{\varrho}$

Should we abandon it *??*

✓ never remove the shift 2π by $\begin{cases} \text{coordinate transformations,} \\ \text{B-field gauge transformation.} \end{cases}$

✓ ill-defined asymptotic behavior $H = 1 + \log \frac{\Lambda}{\varrho}$

Should we abandon it *??*

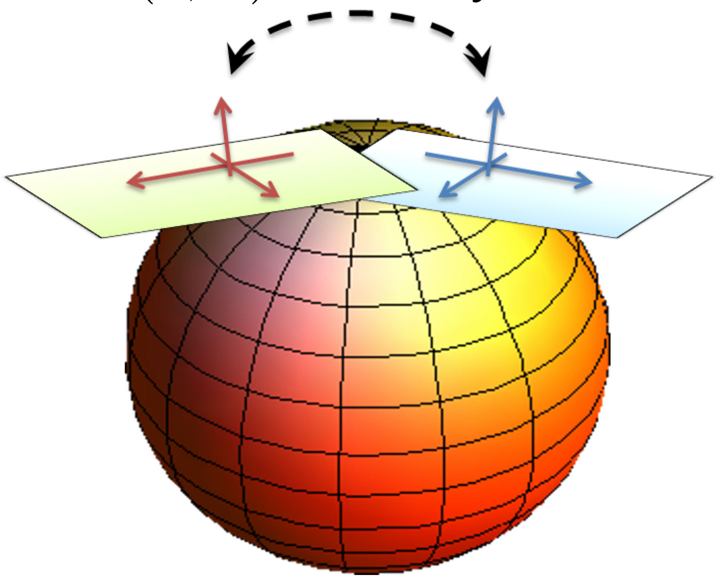
No!

"Supergravity limit" is not globally well-defined.

Non-geometric structure

structure group = diffeom. $(GL(d,\mathbb{R}))$ + String duality group

$GL(d,\mathbb{R}) \cup$ duality transf.



Generalized Geometry (N. Hitchin)
Doubled Geometry (C. Hull)

5^2_2-brane is a concrete example (T-fold).
(D7-brane is an example of S-fold.)

Exotic brane shows us a new insight of stringy spacetime

5_2^2-brane has been analyzed in SUGRA picture.

Ready to study **string worldsheet** picture!

- ☞ nonlinear sigma model (NLSM)
- ☞ conformal field theory (CFT)
- ☞ gauged linear sigma model (GLSM)

Quantum aspects of NLSM/CFT can be traced by Gauge fields in GLSM

Indeed, GLSM is the UV completion of string worldsheet theory

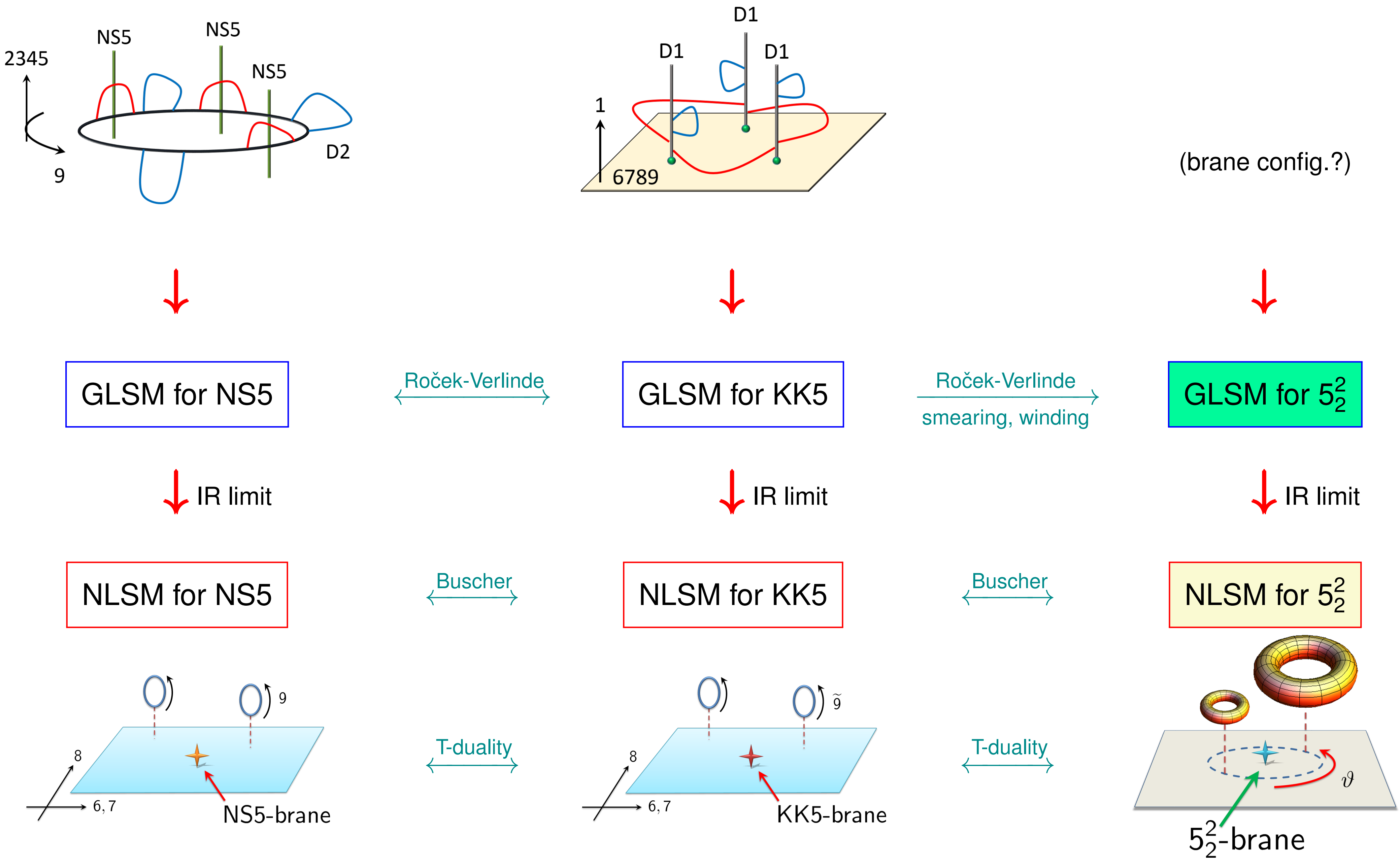
GLSM is a powerful tool to explore non-trivial configurations in string theory :

(as the UV completion of string worldsheet theory)

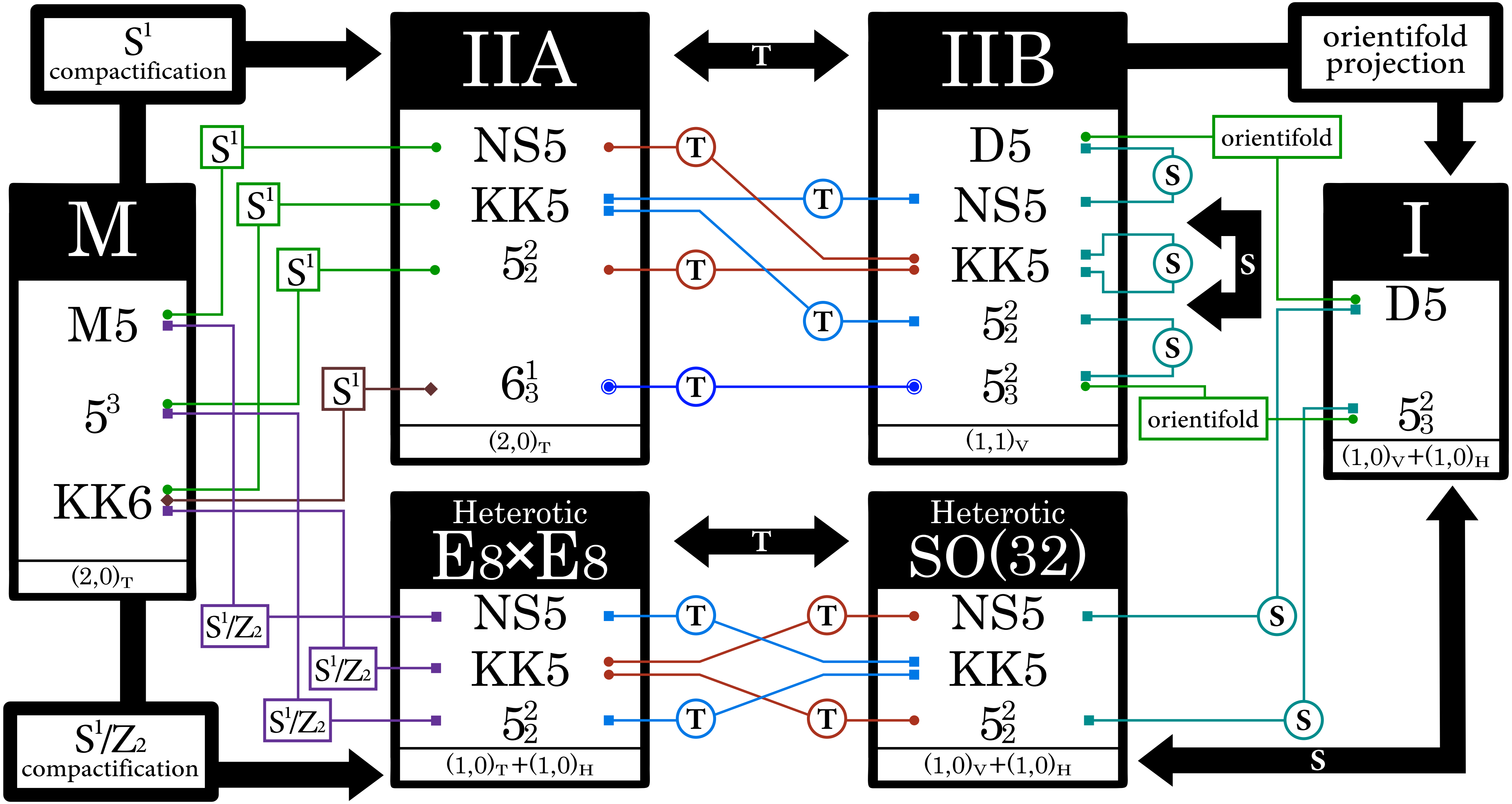
- Calabi-Yau / Landau-Ginzburg
  $\mathcal{N} = (2,2)$ SUSY

- ALE, ALF spaces (hyper-Kähler) and NS5-branes
  $\mathcal{N} = (4,4)$ SUSY

In particular, GLSM for ALE/ALF space can be constructed in brane configuration.



NS5
NS5
NS5
2345
9
D2
D1
D1
D1
1
6789
(brane config.?)
GLSM for NS5
Roček-Verlinde
GLSM for KK5
Roček-Verlinde
smearing, winding
GLSM for 5^2_2
IR limit
IR limit
IR limit
NLSM for NS5
Buscher
NLSM for KK5
Buscher
NLSM for 5^2_2
9
8
6, 7
NS5-brane
T-duality
$\tilde{9}$
8
6, 7
KK5-brane
T-duality
ϑ
5^2_2-brane



S. Sasaki, M. Yata and TK arXiv:1404.5442

Defect branes in diverse dimensions (codim.2)

	fundamental	Dirichlet	solitonic	S_D-dual of (Dirichlet)	S_D-dual of (fundamental)
D	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
IIB		C_8 [D7]		$E_8 = \mathrm{S}_{10}(C_8)$ [7_3]	
9		C_7 [D6]		$E_{8,1} = \mathrm{S}_9(C_7)$ [6^1_3]	
8		C_6 [D5]	D_6 [NS5] $D_{7,1}$ [KK5] $D_{8,2}$ [5^2_2]	$E_{8,2} = \mathrm{S}_8(C_7)$ [5^2_3]	
7		C_5 [D4]		$E_{8,3} = \mathrm{S}_7(C_5)$ [4^3_3]	
6		C_4 [D3]		$E_{8,4} = \mathrm{S}_6(C_4)$ [3^4_3]	
5		C_3 [D2]		$E_{8,5} = \mathrm{S}_5(C_3)$ [2^5_3]	
4	B_2 [F1]	C_2 [D1]		$E_{8,6} = \mathrm{S}_4(C_2)$ [1^6_3]	$F_{8,6} = \mathrm{S}_4(B_2)$ [1^6_4]
3	[P]	C_1 [D0]		$E_{8,7} = \mathrm{S}_3(C_1)$ [0^7_3]	$F_{8,7,1}$ [$0^{(6,1)}_4$]

and more...

E.A. Bergshoeff et al arXiv:1009.4657, 1102.0934, 1108.5067, etc.

「これからの弦理論」のひとつ

$$b_n^c$$

エキゾチックブレーン

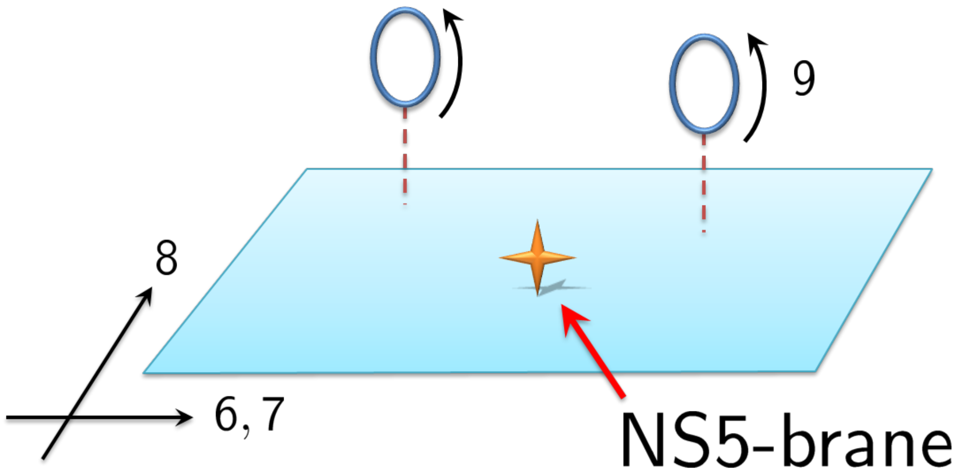
with SUGRA, NLSM, CFT, GLSM, GG, DG, DFT, EGG, etc.

Thanks

Appendix

NS5-brane (smeared along 9th-direction) G_{MN}, B_{MN}, ϕ

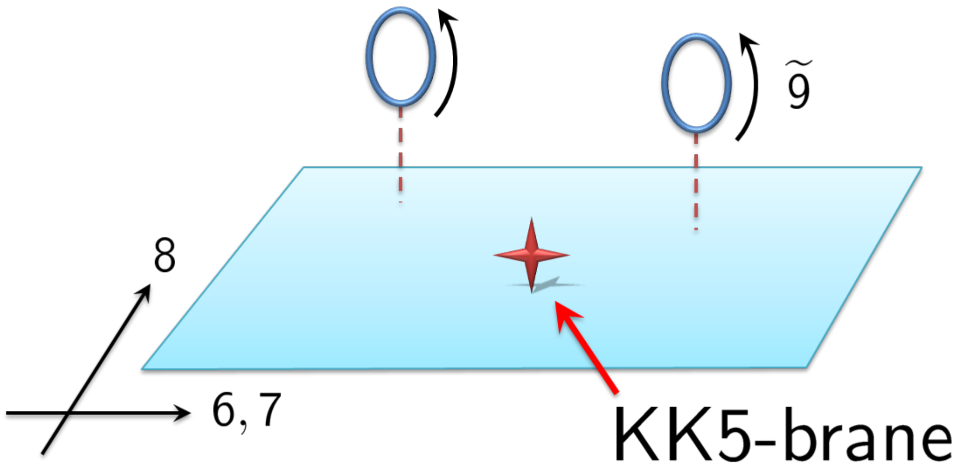
- co-dim. 3 ($\mathbb{R}^3 \times S^1, \quad \vec{x} \in \mathbb{R}^3$)
- $ds^2 = dx^2_{012345} + H\Big[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2\Big]$
- $H = 1 + \dfrac{1}{|\vec{x}|}$
- $H_{mnp} = 3\partial_{[m}B_{np]} = \varepsilon_{mnp}{}^{q}\,\partial_q \log H\,, \qquad e^{2\phi} = H$



KK5-brane (Taub-NUT space)

G_{MN}, $\cancel{B_{MN}}$, $\cancel{\phi}$

- co-dim. 3 ($\mathbb{R}^3 \times \widetilde{S}^1$, $\vec{x} \in \mathbb{R}^3$)
- $$ds^2 = dx^2_{012345} + H\Big[(dx^6)^2 + (dx^7)^2 + (dx^8)^2\Big] + \frac{1}{H}\big(d\widetilde{x}^9 - \vec{V} \cdot d\vec{x}\big)^2$$
- $H = 1 + \frac{1}{|\vec{x}|}$, $\vec{\nabla} H = \vec{\nabla} \times \vec{V}$
- $H_{mnp} = 0 = \phi$



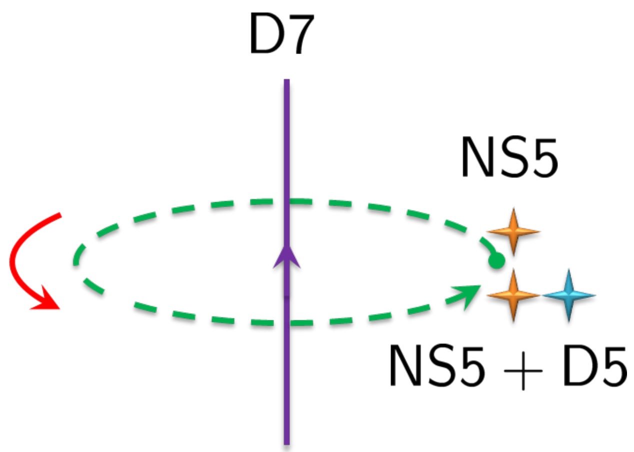
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2,\mathbb{Z})$ monodromy charge q

$$q : (C_2, B_2) \to (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

(0, NS5) $\to$ (D5, NS5)



an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

4D asymptotic geometry is locally T^k-fibration over $\mathbb{R}^{4-k}$

	k	harmonic function	
ALE	(1)	$\frac{1}{\lvert\vec{x}\rvert}$	$\mathbb{C}^2/\Gamma$ with ADE-singularities
ALF	1	$A + \frac{1}{\lvert\vec{x}\rvert}$	Taub-NUT
ALG	2	$A + B \log \lvert\varrho\rvert$	exotic objects such as D7, 5^2_2, etc.
ALH	3	$A + B\lvert r\rvert$	linear potential

S.A. Cherkis and A. Kapustin hep-th/0006050, hep-th/0109141

Brane configuration

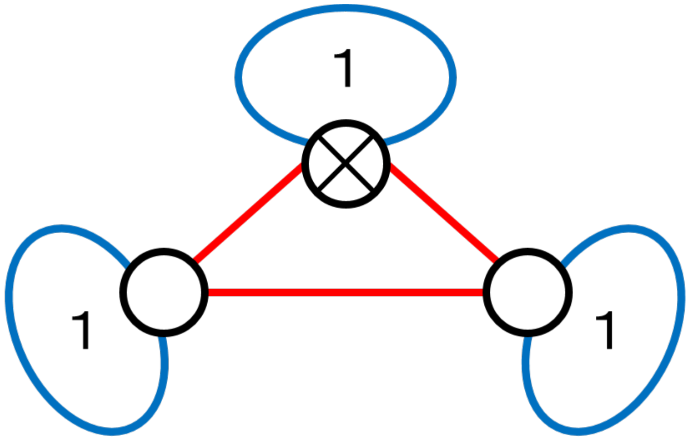
M.R. Douglas and G.W. Moore hep-th/9603167

C.V. Johnson and R.C. Myers hep-th/9610140

D. Tong hep-th/0204186

ADE-type singularities in ALE/ALF space can be illustrated as

Affine ADE Dynkin diagram :



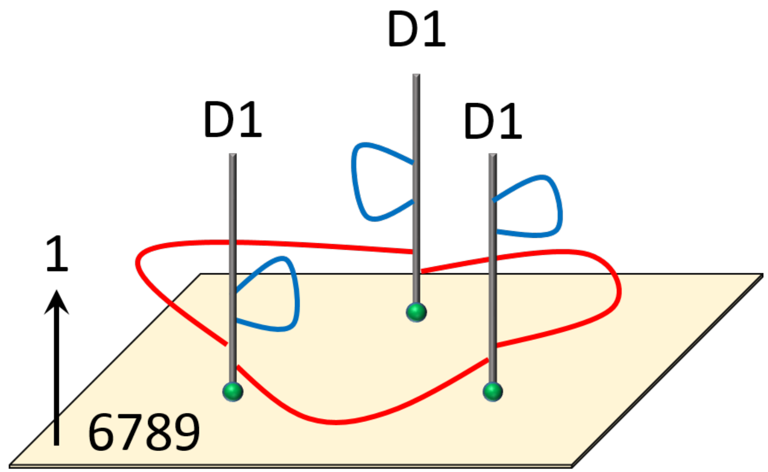
Each Affine ADE Dynkin diagram is mapped (identical) to

ADE quiver diagram.

nodes :	gauge groups
blue lines :	matters (adjoint repr.)
red lines :	matters (bi-fundamental repr.)

The corresponding picture by D-branes is

	0	1	2	3	4	5	6	7	8	(9)
3 KK5	○	○	○	○	○	○	A_2-ALF			
D1	○	○								



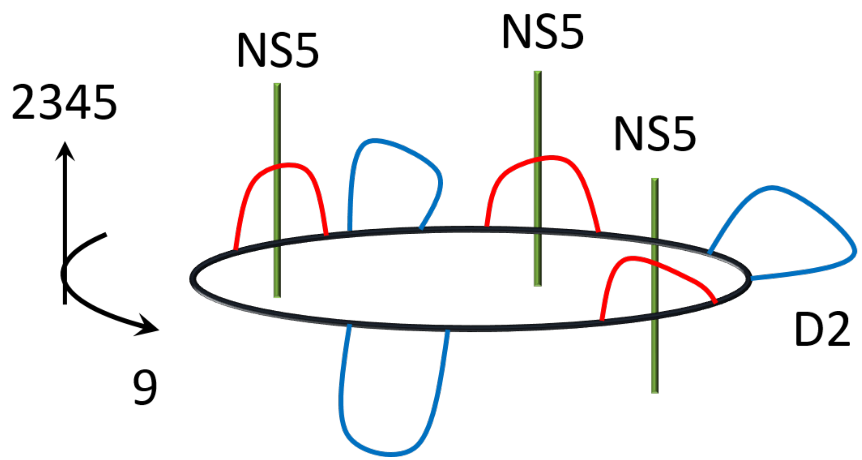
green nodes :	singularity points in ALF space
blue lines :	open strings (adjoint repr.)
red lines :	open strings (bi-fundamental repr.)
6789-plane :	ALF space

gauge theory on (compact) D1-branes $\to$ $\mathcal{N} = (4, 4)$ GLSM for A_2-ALE/ALF space

(large radius limit of 9^{th}-direction : ALF $\to$ ALE)

T-duality along 9th-direction :

	0	1	2	3	4	5	6	7	8	(9)
3 NS5	◯	◯	◯	◯	◯	◯	flat			
D2	◯	◯								◯



Singularity points in ALE/ALF are mapped to centers of NS5-branes.

gauge theory on (compact) D2-branes $\rightarrow$ $\mathcal{N} = (4,4)$ GLSM for NS5-branes

Field contents of 2D $\mathcal{N} = (4,4)$ GLSM for A_2-type ALE space :

	$U(1)_1$	$U(1)_2$	$U(1)_3$
$\omega_1 = (a_1, \bar{b}_1)$	$+1$	-1	0
$\omega_2 = (a_2, \bar{b}_2)$	0	$+1$	-1
$\omega_3 = (a_3, \bar{b}_3)$	-1	0	$+1$
(A^1_m, ϕ_1)	adj	–	–
(A^2_m, ϕ_2)	–	adj	–
(A^3_m, ϕ_3)	–	–	adj

$SU(2)$ moment map

$$\begin{aligned} \vec{\mu}_1 &= \omega_1^\dagger \vec{\tau} \omega_1 - \omega_3^\dagger \vec{\tau} \omega_3 \\ \vec{\mu}_2 &= \omega_2^\dagger \vec{\tau} \omega_2 - \omega_1^\dagger \vec{\tau} \omega_1 \\ \vec{\mu}_3 &= \omega_3^\dagger \vec{\tau} \omega_3 - \omega_2^\dagger \vec{\tau} \omega_2 \\ 0 &= \vec{\mu}_1 + \vec{\mu}_2 + \vec{\mu}_3 \end{aligned}$$

Hyper-Kähler quotient construction

cf.) reconstruction by toric data: M. Yata and TK arXiv:1402.5580

Field contents of 2D $\mathcal{N} = (4,4)$ GLSM for A_2-type ALF space :

	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_F$
$\omega_1 = (a_1, \bar{b}_1)$	$+1$	-1	0	$+1$
$\omega_2 = (a_2, \bar{b}_2)$	0	$+1$	-1	$+1$
$\omega_3 = (a_3, \bar{b}_3)$	-1	0	$+1$	$+1$
(A^1_m, ϕ_1)	adj	–	–	–
(A^2_m, ϕ_2)	–	adj	–	–
(A^3_m, ϕ_3)	–	–	adj	–
(X^6, X^7, X^8)	neutral			
(A^F_m, ϕ_F)	–	–	–	adj

$SU(2)$ moment map

$$\begin{aligned}
\vec{\mu}_1 &= \omega_1^\dagger \vec{\tau} \omega_1 - \omega_3^\dagger \vec{\tau} \omega_3 \\
\vec{\mu}_2 &= \omega_2^\dagger \vec{\tau} \omega_2 - \omega_1^\dagger \vec{\tau} \omega_1 \\
\vec{\mu}_3 &= \omega_3^\dagger \vec{\tau} \omega_3 - \omega_2^\dagger \vec{\tau} \omega_2 \\
0 &= \vec{\mu}_1 + \vec{\mu}_2 + \vec{\mu}_3 \\
\vec{\mu}_F &= \sum_{i=1}^{3} \omega_i^\dagger \vec{\tau} \omega_i - \vec{X}
\end{aligned}$$

Diagonalize $U(1)_{i,F} \rightarrow$ D. Tong hep-th/0204186

Duality transformations with F-term

in $\mathcal{N} = (2,2)$ framework

M. Yata and TK arXiv:1402.5580, 1406.0087

Duality : chiral Ψ $\longleftrightarrow$ twisted chiral Y

Target space geometry is T-dualized under the above transformation.

Duality : chiral $\Psi \longleftrightarrow$ twisted chiral Y

Target space geometry is T-dualized under the above transformation.

- dualize neutral chiral in D-term (ex. torus)

$$\int \mathrm{d}^4\theta \, |\Psi|^2 \; \leftrightarrow \; \int \mathrm{d}^4\theta \left[\frac{1}{2} R^2 - R(Y + \overline{Y})\right] \; \leftrightarrow \; - \int \mathrm{d}^4\theta \, |Y|^2$$

- dualize charged chiral in D-term (ex. projective space)

$$\int \mathrm{d}^4\theta \, |\Psi|^2 \, \mathrm{e}^{2V} \; \leftrightarrow \; \int \mathrm{d}^4\theta \left[\mathrm{e}^{2V+R} - R(Y + \overline{Y})\right] \; \leftrightarrow \; - \int \mathrm{d}^4\theta \, (Y + \overline{Y}) \Big[\log(Y + \overline{Y}) - 2V \Big]$$

These are well established.

"Global symmetry" $\Psi + \alpha$ (or $e^{i\alpha}\,\Psi$) is preserved in D-term, but broken in F-term.

— Make sense? —

► dualize neutral / charged chiral in D-term and F-term

They can be interpreted as T-duality transformations

under conversion from F-term to D-term with trick(es).

- Dualize neutral chiral in D-term and F-term

$$\mathscr{L}_1 = \int \mathrm{d}^4\theta \, |\Psi|^2 + \Big\{ \int \mathrm{d}^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\mathrm{h.c.}) \Big\}$$

1. Convert F-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int \mathrm{d}^2\theta \, \Psi \, \mathcal{W} + (\mathrm{h.c.}) = \int \mathrm{d}^4\theta \Big[(\Psi + \overline{\Psi})(C + \overline{C}) + (\Psi - \overline{\Psi})(C - \overline{C}) \Big]$$

2. Replace $\Psi \pm \overline{\Psi}$ to auxiliary fields R and $\mathrm{i}S$:

$$\mathscr{L}_2 = \int \mathrm{d}^4\theta \, \Big[\frac{1}{2} R^2 + R(C + \overline{C}) + \mathrm{i}S(C - \overline{C}) - R(Y + \overline{Y}) - \mathrm{i}S(\Upsilon - \overline{\Upsilon}) \Big]$$

3. Integrating out R and Υ, we obtain the "dual" system :

$$\mathscr{L}_3 = \int \mathrm{d}^4\theta \, \Big[-\frac{1}{2} \Big((Y + \overline{Y}) + (C + \overline{C}) \Big)^2 + (\Psi - \overline{\Psi})(C - \overline{C}) \Big]$$

Instead, integrate out Y and $\Upsilon \to \mathscr{L}_1$ appears

- Dualize charged chiral in D-term and F-term

$$\mathscr{L}_4 = \int d^4\theta \, |\Psi|^2 \, e^{2V} + \Big\{ \int d^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\text{h.c.}) \Big\}$$

1. Convert F-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int d^2\theta \, \Psi \, \mathcal{W} + (\text{h.c.}) = 2 \int d^4\theta \, \Big[\Psi C + \overline{\Psi C} \Big]$$

2. Replace $\Psi \pm \overline{\Psi}$ to auxiliary fields R and $\mathrm{i}S$:

$$\mathscr{L}_5 = \int d^4\theta \, \Big[e^{2V+R} + 2 \Big\{ e^{\frac{1}{2}(R+\mathrm{i}S)} C + e^{\frac{1}{2}(R-\mathrm{i}S)} \overline{C} \Big\} - R(Y + \overline{Y}) - \mathrm{i}S(\Upsilon - \overline{\Upsilon}) \Big]$$

3. Integrating out R and Υ, we obtain the "dual" system :

$$\mathscr{L}_6 = \int d^4\theta \, \Big[-2(Y + \overline{Y}) \log \mathcal{F} + \frac{1}{2} \mathcal{F} \mathcal{T} + 2V(Y + \overline{Y}) \Big]$$

$$\mathcal{F} = -\mathcal{T} + \sqrt{\mathcal{T}^2 + 4(Y + \overline{Y})}, \quad \mathcal{T} = e^{-V} \Big[e^{+\frac{1}{2}(\Omega - \overline{\Omega})} C + e^{-\frac{1}{2}(\Omega - \overline{\Omega})} \overline{C} \Big], \quad \Psi = e^{\Omega}$$

$\Psi - \overline{\Psi}$ still remains after the transformation.

The existence is rather important to complete the Duality transformations.

$\Psi - \overline{\Psi}$ appears as an auxiliary field, which must be removed finally.

Indeed, the procedure "integrating-out of $\Psi - \overline{\Psi}$" leads to

the correct involution of the dual fields in the system *!*

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Indeed, the procedure "integrating-out of $e^{\Omega-\overline{\Omega}}$" leads to

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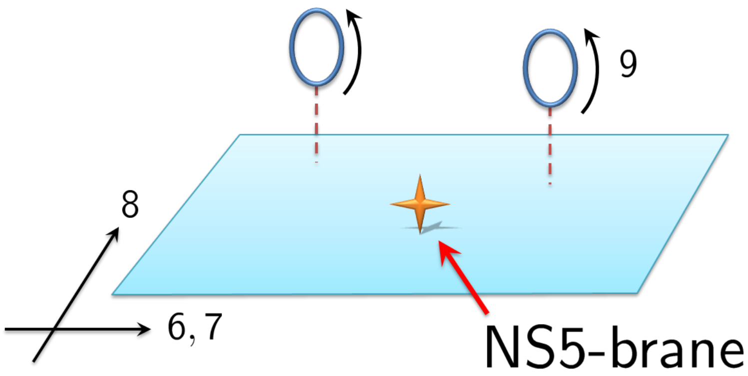
GLSM for five-branes

NS5-branes, KK5-branes and exotic 5^2_2-brane

(w/ 2D $\mathcal{N} = (2,2)$ superfields)

S. Sasaki and TK arXiv:1304.4061

NS5-brane

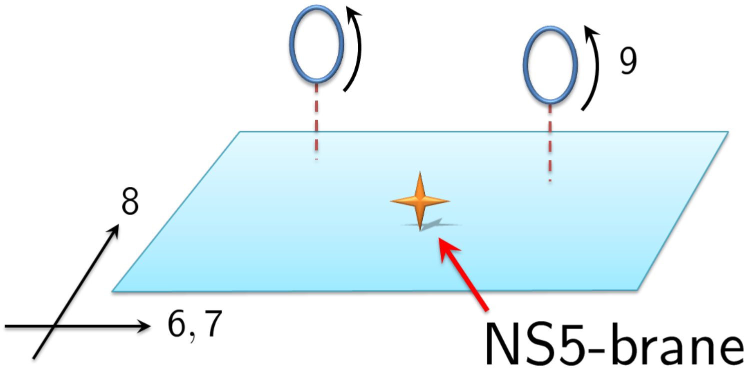


$$\begin{aligned}\mathscr{L} \;&=\; \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} (|\Phi|^2 - |\Sigma|^2) + |Q|^2 \, \mathrm{e}^{+2V} + |\widetilde{Q}|^2 \, \mathrm{e}^{-2V} \Big\} \\ &\quad + \frac{1}{g^2} \int \mathrm{d}^4\theta \, \Big\{ |\Psi|^2 - |\Theta|^2 \Big\} \\ &\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(- \widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\mathrm{h.c.}) \Big\} \\ &\quad + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, \Big((t - \Theta)\Sigma \Big) + (\mathrm{h.c.}) \Big\}\end{aligned}$$

$X^6 + \mathrm{i}X^8 \in \Psi$ (chiral), $\quad X^7 + \mathrm{i}X^9 \in \Theta$ (twisted chiral)

D. Tong hep-th/0204186

NS5-brane



$$\begin{aligned}\mathscr{L} &= \int \mathrm{d}^4\theta \Big\{ \frac{1}{e^2}(|\Phi|^2 - |\Sigma|^2) + |Q|^2 \, \mathrm{e}^{+2V} + |\widetilde{Q}|^2 \, \mathrm{e}^{-2V} \Big\} \\ &\quad + \frac{1}{g^2} \int \mathrm{d}^4\theta \Big\{ |\Psi|^2 - |\Theta|^2 \Big\} \\ &\quad + \Big\{ \int \mathrm{d}^2\theta \Big(- \widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.}) \Big\} \\ &\quad + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \Big((t - \Theta)\Sigma \Big) + (\text{h.c.}) \Big\}\end{aligned}$$

$X^6 + \mathrm{i}X^8 \in \Psi$ (chiral), $\quad X^7 + \mathrm{i}X^9 \in \Theta$ (twisted chiral)

Position is labelled by FI parameters (s, t)

IR limit $\to$ gauge multiplets are integrated out $\to$ NLSM on NS5-brane geometry

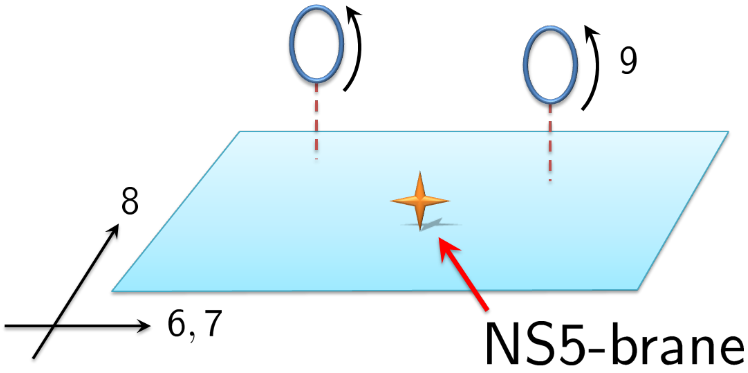
$$\mathscr{L}^{\text{NLSM}}_{\text{NS5}} = -\frac{H}{2}\Big[(\partial_m \vec{X})^2 + (\partial_m X^9)^2\Big] + \varepsilon^{mn}\,\Omega_i\,\partial_m X^i\,\partial_n X^9$$

Target space geometry is

$$G_{IJ} = H\,\delta_{IJ} \qquad B_{i9} = \Omega_i$$
$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|} \qquad \nabla_i H = (\nabla \times \Omega)_i$$

Quantum corrections can be traced by vortex corrections of 2D gauge fields.

NS5-brane



$$
\begin{aligned}
\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} (|\Phi|^2 - |\Sigma|^2) + |Q|^2 \, \mathrm{e}^{+2V} + |\widetilde{Q}|^2 \, \mathrm{e}^{-2V} \Big\} \\
& + \frac{1}{g^2} \int \mathrm{d}^4\theta \, \Big\{ |\Psi|^2 - |\Theta|^2 \Big\} \\
& + \Big\{ \int \mathrm{d}^2\theta \, \Big(- \widetilde{Q} \Phi Q + (s - \Psi) \Phi \Big) + (\mathrm{h.c.}) \Big\} \\
& + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, \Big((t - \Theta) \Sigma \Big) + (\mathrm{h.c.}) \Big\}
\end{aligned}
$$

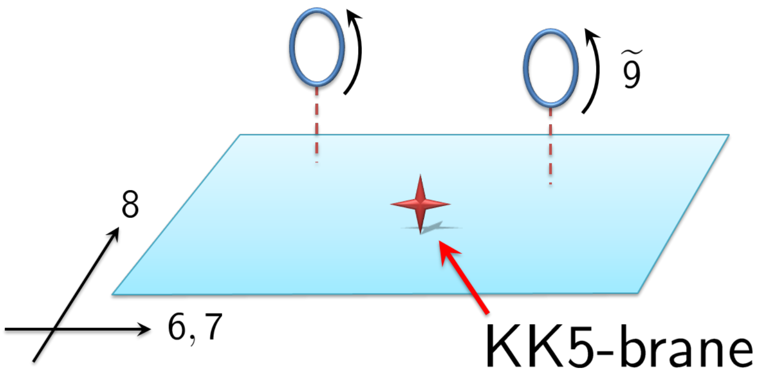
$X^6 + \mathrm{i} X^8 \in \Psi$ (chiral), $\quad X^7 + \mathrm{i} X^9 \in \Theta$ (twisted chiral)

Dualize the neutral twisted chiral $\Theta \to \Gamma$: T-duality along 9th-direction

T-duality transformations

(free) string	sign flip (parity) in right-mover	momentum $\leftrightarrow$ winding
spacetime	Buscher rule	$(G_{IJ}, B_{IJ}, \phi) \leftrightarrow (G'_{IJ}, B'_{IJ}, \phi')$
SUSY sigma model	Roček-Verlinde formula	chiral $\leftrightarrow$ twisted chiral

KK5-brane



$$\begin{aligned}\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \left\{ \frac{1}{e^2}(|\Phi|^2 - |\Sigma|^2) + |Q|^2\, \mathrm{e}^{+2V} + |\widetilde{Q}|^2\, \mathrm{e}^{-2V} \right\} \\ & + \int \mathrm{d}^4\theta \left\{ \frac{1}{g^2}|\Psi|^2 + \frac{g^2}{2}(\Gamma + \overline{\Gamma} + V)^2 \right\} \\ & + \left\{ \int \mathrm{d}^2\theta \Big(- \widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\mathrm{h.c.}) \right\} \\ & + \left\{ \int \mathrm{d}^2\widetilde{\theta}\, (t\Sigma) + (\mathrm{h.c.}) \right\} + \varepsilon^{mn}\partial_m(X^9 A_n)\end{aligned}$$

$X^6 + \mathrm{i}X^8 \in \Psi$ (chiral), $\quad X^7 + \mathrm{i}\widetilde{X}^9 \in \Gamma$ (chiral)

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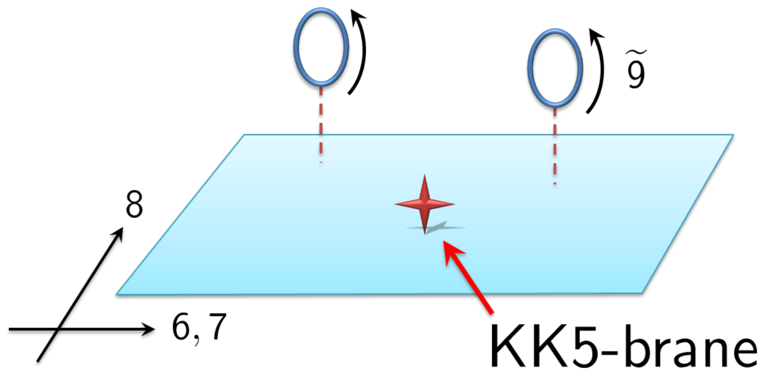
KK5-brane



$$
\begin{aligned}
\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \Big\{ \frac{1}{e^2}(|\Phi|^2 - |\Sigma|^2) + |Q|^2 \, \mathrm{e}^{+2V} + |\widetilde{Q}|^2 \, \mathrm{e}^{-2V} \Big\} \\
& + \int \mathrm{d}^4\theta \Big\{ \frac{1}{g^2}|\Psi|^2 + \frac{g^2}{2}(\Gamma + \overline{\Gamma} + V)^2 \Big\} \\
& + \Big\{ \int \mathrm{d}^2\theta \Big(- \widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.}) \Big\} \\
& + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t\Sigma) + (\text{h.c.}) \Big\} + \varepsilon^{mn}\partial_m(X^9 A_n)
\end{aligned}
$$

$X^6 + \mathrm{i}X^8 \in \Psi$ (chiral), $\quad X^7 + \mathrm{i}\widetilde{X}^9 \in \Gamma$ (chiral)

Position is labelled by FI parameters (s, t)

IR limit $\rightarrow$ gauge multiplets are integrated out $\rightarrow$ NLSM on KK5-branes (ALF)

$$\mathscr{L}_{\text{KK5}}^{\text{NLSM}} = -\frac{H}{2}(\partial_m \vec{X})^2 - \frac{1}{2H}\big(\partial_m \widetilde{X}^9 - \Omega_i\, \partial_m X^i\big)^2$$

$$+\varepsilon^{mn}\, \partial_m((X^9 - t^9)A_n)$$

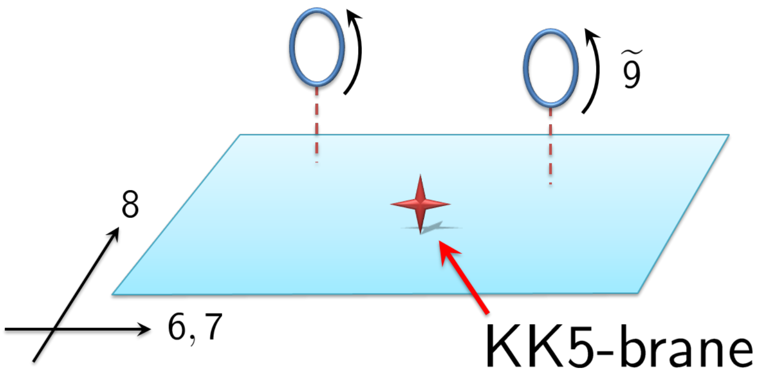
Target space geometry is

$$G_{IJ} = \text{Taub-NUT} \qquad B_{i9} = 0$$

$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|} \qquad \nabla_i H = (\nabla \times \Omega)_i$$

Quantum corrections can be traced by vortex corrections of 2D gauge fields.

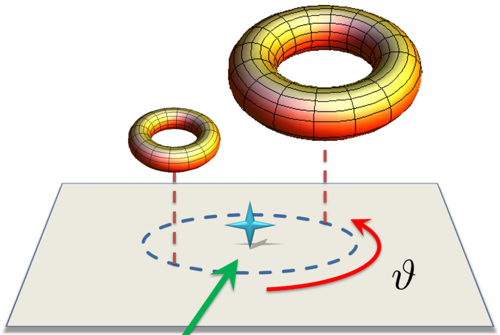
KK5-brane



$$\begin{aligned}\mathscr{L} &= \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2}(|\Phi|^2 - |\Sigma|^2) + |Q|^2 \, \mathrm{e}^{+2V} + |\widetilde{Q}|^2 \, \mathrm{e}^{-2V} \Big\} \\ &\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{g^2}|\Psi|^2 + \frac{g^2}{2}(\Gamma + \overline{\Gamma} + V)^2 \Big\} \\ &\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(- \widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + (\text{h.c.}) \Big\} \\ &\quad + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t\Sigma) + (\text{h.c.}) \Big\} + \varepsilon^{mn}\partial_m(X^9 A_n)\end{aligned}$$

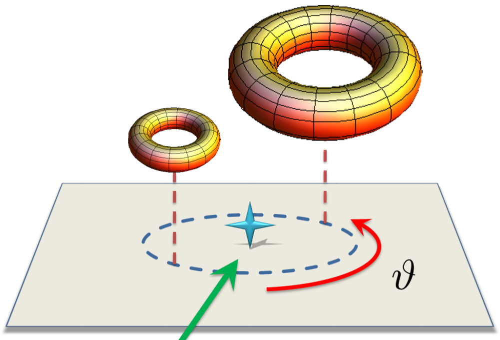
$X^6 + \mathrm{i}X^8 \in \Psi$ (chiral), $\quad X^7 + \mathrm{i}\widetilde{X}^9 \in \Gamma$ (chiral)

Dualize the neutral chiral $\Psi \to \Xi$: T-duality along 8$^{\text{th}}$-direction



$$\begin{aligned}\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e^2} (|\Phi|^2 - |\Sigma|^2) + |Q|^2 \, \mathrm{e}^{+2V} + |\widetilde{Q}|^2 \, \mathrm{e}^{-2V} \Big\} \\ & + \int \mathrm{d}^4\theta \, \Big\{ - \frac{g^2}{2} (\Xi + \overline{\Xi} - (C + \overline{C}))^2 + \frac{g^2}{2} (\Gamma + \overline{\Gamma} + V)^2 \Big\} \\ & + \Big\{ \int \mathrm{d}^2\theta \, \Big(- \widetilde{Q} \Phi Q + s \Phi \Big) + (\mathrm{h.c.}) \Big\} - \int \mathrm{d}^4\theta \, (\Psi - \overline{\Psi})(C - \overline{C}) \\ & + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t \Sigma) + (\mathrm{h.c.}) \Big\} + \varepsilon^{mn} \partial_m (X^9 A_n) \end{aligned}$$

$X^6 + \mathrm{i}\widetilde{X}^8 \in \Xi$ (twisted chiral), $\quad X^7 + \mathrm{i}\widetilde{X}^9 \in \Gamma$ (chiral)



$$\begin{aligned}\mathscr{L} \;=\; & \int d^4\theta \Big\{\frac{1}{e^2}(|\Phi|^2 - |\Sigma|^2) + |Q|^2 e^{+2V} + |\widetilde{Q}|^2 e^{-2V}\Big\} \\ & + \int d^4\theta \Big\{ -\frac{g^2}{2}(\Xi + \overline{\Xi} - (C + \overline{C}))^2 + \frac{g^2}{2}(\Gamma + \overline{\Gamma} + V)^2 \Big\} \\ & + \Big\{ \int d^2\theta \Big(-\widetilde{Q}\Phi Q + s\Phi \Big) + (\text{h.c.}) \Big\} - \int d^4\theta \, (\Psi - \overline{\Psi})(C - \overline{C}) \\ & + \Big\{ \int d^2\widetilde{\theta} \, (t\Sigma) + (\text{h.c.}) \Big\} + \varepsilon^{mn}\partial_m (X^9 A_n)\end{aligned}$$

$X^6 + i\widetilde{X}^8 \in \Xi$ (twisted chiral), $\quad X^7 + i\widetilde{X}^9 \in \Gamma$ (chiral)

$\Psi - \overline{\Psi}$ is no longer dynamical $\rightarrow \begin{cases} \Xi + \overline{\Xi}: & \text{momentum modes} \\ \Psi - \overline{\Psi}: & \text{winding modes}\,! \end{cases}$

IR limit $\rightarrow$ gauge multiplets are integrated out

Further, integrate out $\Psi - \overline{\Psi}$: Exotic Five-brane*!*

From GLSM to NLSM

LESSON 1 : GLSM for NS5-brane

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K. Okuyama hep-th/0508097

Bosonic Lagrangian after integrating-out auxiliary fields :

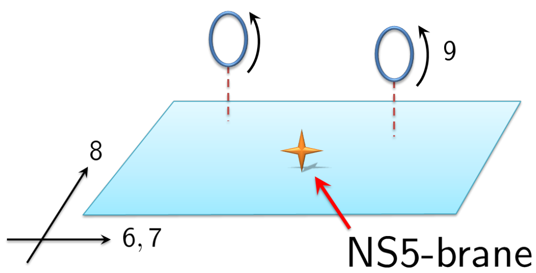
$$\begin{aligned}
\mathscr{L}^{\text{kin}}_{\text{NS5}} &= \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big] \\
&\quad -\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2\Big] + (X^9 - t^9)\, F_{01}
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}^{\text{pot}}_{\text{NS5}} &= -2\big(|\sigma|^2 + |\phi|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big) \\
&\quad -\frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} + (X^6 - s^6) + \mathrm{i}(X^8 - s^8)\Big|^2
\end{aligned}$$

Steps to NLSM for NS5-brane

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, X^9)$

1. SUSY vacua

$$\sigma = 0 = \phi \,, \qquad |q|^2 - |\widetilde{q}|^2 = X^7 - t^7 \,, \qquad -q\widetilde{q} = (X^6 - s^6) + \mathrm{i}(X^8 - s^8)$$

2. solve constraints on $(q, \widetilde{q})$

$$q = \mathrm{i}\, \mathrm{e}^{+\mathrm{i}\alpha} \sqrt{|\vec{X}| + (X^7 - t^7)} \,, \qquad \widetilde{q} = \mathrm{i}\, \mathrm{e}^{-\mathrm{i}\alpha_a} \frac{(X^6 - s^6) + \mathrm{i}(X^8 - s^8)}{\sqrt{|\vec{X}| + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \widetilde{q}|^2 = \frac{1}{2|\vec{X}|} \Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \Big] + |\vec{X}| \Big(\partial_m \alpha - A_m + \Omega_i \, \partial_m X^i \Big)^2$$

$$|\vec{X}| = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2 + (X^8 - s^8)^2}$$

$$\Omega_i \, \partial_m X^i = \frac{-(X^6 - s^6) \partial_m X^8 + (X^8 - s^8) \partial_m X^6}{|\vec{X}| (|\vec{X}| + (X^7 - t^7))}$$

3. IR limit $e \to \infty$, and integrate out A_m

$$A_m = \Omega_i \, \partial_m X^i + \frac{1}{|\vec{X}|} \, \varepsilon_{mn} \, \partial^n X^9$$

$$\Longrightarrow \quad \mathscr{L}^{\text{NLSM}}_{\text{NS5}} = -\frac{1}{2} \Big(\frac{1}{g^2} + \frac{1}{|\vec{X}|} \Big) \Big[(\partial_m \vec{X})^2 + (\partial_m X^9)^2 \Big] + \varepsilon^{mn} \, \Omega_i \, \partial_m X^i \, \partial_n X^9$$

LESSON 2 : GLSM for KK5-brane

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$\Theta \to \Gamma$:

$$\begin{aligned} \mathscr{L}_{\text{NS5}} \ni \mathscr{L}_{\Theta} &= \int \mathrm{d}^4\theta \Big(- \frac{1}{g^2} \overline{\Theta} \Theta \Big) + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (-\Theta) \Sigma + \text{(h.c.)} \Big\} \\ &= \int \mathrm{d}^4\theta \Big\{ - \frac{1}{2g^2} (\Theta + \overline{\Theta})^2 - (\Theta + \overline{\Theta}) V \Big\} + \varepsilon^{mn} \, \partial_m (X^9 A_n) \end{aligned}$$

$$\downarrow$$

$$\mathscr{L}_{B\Gamma} = \int \mathrm{d}^4\theta \, \Big\{ - \frac{1}{2g^2} B^2 - BV - B(\Gamma + \overline{\Gamma}) \Big\} + \varepsilon^{mn} \, \partial_m (X^9 A_n)$$

real $\overline{B} = B$; chiral $\overline{D}_{\pm}\Gamma = 0$

$\Theta \to \Gamma$:

$$\mathscr{L}_{B\Gamma} = \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2} B^2 - BV - (\Gamma + \overline{\Gamma}) B \Big\} + \varepsilon^{mn} \, \partial_m (X^9 A_n)$$

Integrating out Γ, $\overline{\Gamma}$: $\to$ GLSM for NS5-brane

$$B = \Theta + \overline{\Theta}$$

or, Integrating out B : $\to$ GLSM for KK5-brane

$$\frac{1}{g^2} B = -(\Gamma + \overline{\Gamma}) - V$$

duality relation :

$$\Theta = X^7 + \mathrm{i} X^9 + \ldots, \quad \Gamma = \widetilde{X}^7 + \mathrm{i} \widetilde{X}^9 + \ldots$$

$$\Theta + \overline{\Theta} = -g^2 (\Gamma + \overline{\Gamma}) - g^2 V \quad \to \quad \boxed{\begin{aligned} X^7 &= -g^2 \, \widetilde{X}^7 \\ \pm(\partial_0 \pm \partial_1) X^9 &= -g^2 (D_0 \pm D_1) \widetilde{X}^9 \\ D_m \widetilde{X}^9 &= \partial_m \widetilde{X}^9 - A_m \end{aligned}}$$

$$\mathscr{L}_{\text{KK5}} = \int d^4\theta \left\{ \frac{1}{e^2}\Big(-\overline{\Sigma}\Sigma + \overline{\Phi}\Phi \Big) + \overline{Q}\, e^{+2V} Q + \overline{\widetilde{Q}}\, e^{-2V} \widetilde{Q} \right\} + \int d^4\theta \left\{ \frac{g^2}{2}\Big(\Gamma + \overline{\Gamma} + V \Big)^2 + \frac{1}{g^2}\overline{\Psi}\Psi \right\}$$

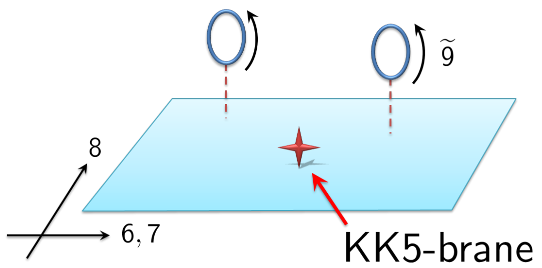
$$+ \int d^2\theta \,\Big(-\widetilde{Q}\Phi Q + (s - \Psi)\Phi \Big) + \text{(h.c.)}$$

$$+ \Big\{ \int d^2\widetilde{\theta}\, t\, \Sigma + \text{(h.c.)} \Big\} + \varepsilon^{mn}\, \partial_m (X^9 A_n)$$

Steps to NLSM for KK5-brane

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q, \widetilde{q})$
3. IR limit $e \to \infty$, and integrate out A_m

$\Longrightarrow$



$(X^6, X^7; X^8, \widetilde{X}^9)$

1. SUSY vacua

$$\sigma = 0 = \phi \,, \qquad |q|^2 - |\widetilde{q}|^2 = X^7 - t^7 \,, \qquad -q\widetilde{q} = (X^6 - s^6) + \mathrm{i}(X^8 - s^8)$$

2. solve constraints on $(q, \widetilde{q})$

$$q = \mathrm{i}\,\mathrm{e}^{+\mathrm{i}\alpha}\sqrt{|\vec{X}| + (X^7 - t^7)} \,, \qquad \widetilde{q} = \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\alpha}\frac{(X^6 - s^6) + \mathrm{i}(X^8 - s^8)}{\sqrt{|\vec{X}| + (X^7 - t^7)}}$$

$$|D_m q|^2 + |D_m \widetilde{q}|^2 = \frac{1}{2|\vec{X}|}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] + |\vec{X}|\Big(\partial_m \alpha - A_m + \Omega_i\,\partial_m X^i\Big)^2$$

3. IR limit $e \to \infty$, and integrate out A_m

$$A_m = \frac{1}{|\vec{X}|H}\big(\partial_m \widetilde{X}^9 - \Omega_i\,\partial_m X^i\big) + \Omega_i\,\partial_m X^i \,, \qquad H = \frac{1}{g^2} + \frac{1}{|\vec{X}|}$$

$$\mathscr{L}^{\text{NLSM}}_{\text{KK5}} \;=\; -\frac{H}{2}(\partial_m \vec{X})^2 - \frac{1}{2H}(\partial_m \widetilde{X}^9 - \Omega_i\,\partial_m X^i)^2 + \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)$$

LESSON 3 : GLSM for 5^2_2-brane

S. Sasaki and TK arXiv:1304.4061

M. Yata and TK arXiv:1406.0087

$\Psi \to \Xi$:

$$
\begin{aligned}
\mathscr{L}_{\text{KK5}} \ni \mathscr{L}_{\Psi} &= \int d^4\theta \, \frac{1}{g^2} \overline{\Psi} \Psi + \Big\{ \int d^2\theta \, (-\Psi) \Phi + \text{(h.c.)} \Big\} \\
&= \int d^4\theta \, \Big\{ \frac{a}{g^2} (\Psi + \overline{\Psi})^2 - (\Psi + \overline{\Psi})(C + \overline{C}) \Big\} \\
&\quad + \int d^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\Psi - \overline{\Psi})^2 - (\Psi - \overline{\Psi})(C - \overline{C}) \Big\}
\end{aligned}
$$

$$\downarrow$$

$$
\begin{aligned}
\mathscr{L}_{RSX\Xi} &= \int d^4\theta \, \Big\{ \frac{a}{g^2} R^2 - R(C + \overline{C}) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} \\
&\quad + \int d^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - (\mathrm{i}S)(C - \overline{C}) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}
\end{aligned}
$$

$\overline{R} = R, \ \overline{S} = S, \ \overline{D}_+ \Xi_{1,2} = 0 = D_- \Xi_{1,2}, \ \overline{D}_\pm X = 0, \ \Phi_a = \overline{D}_+ \overline{D}_- C_a$

$\Psi \to \Xi$:

$$\widetilde{\mathscr{L}} = \int d^4\theta \Big\{ \frac{a}{g^2} R^2 - R(C + \overline{C}) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} + \int d^4\theta \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - (\mathrm{i}S)(C - \overline{C}) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}$$

Integrating out Ξ_1, Ξ_2, X : $\to$ GLSM for KK5-brane

or, Integrating out R, Ξ_2 : $\to$ GLSM for 5^2_2-brane

$$\frac{2a}{g^2} R = -(\Xi_1 + \overline{\Xi}_1) + (C + \overline{C})$$

duality relation at $a = \frac{1}{2}$:

$$\Psi = X^6 + \mathrm{i}\, X^8 + \ldots$$

$$\Psi + \overline{\Psi} = -g^2(\Xi_1 + \overline{\Xi}_1) + g^2(C + \overline{C})$$

$X^6 \sim$ real part of Ξ
$\partial X^8 \sim \partial$(imaginary part of Ξ) + "gauge" fields in C_a

$$
\begin{aligned}
\mathscr{L}^{\text{kin}} &= \frac{1}{e^2}\Big[\frac{1}{2}(F_{01})^2 - |\partial_m \sigma|^2 - |\partial_m \phi|^2\Big] - \Big[|D_m q|^2 + |D_m \widetilde{q}|^2\Big] \\
&\quad - \frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2\Big] - \frac{g^2}{2}\Big[(\partial_m \widetilde{X}^8)^2 + (D_m \widetilde{X}^9)^2\Big] + (X^9 - t^9)\, F_{01}
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}^{\text{pot}} &= -2\big(|\sigma|^2 + 4|M_c|^2\big)\big(|q|^2 + |\widetilde{q}|^2 + g^2\big) \\
&\quad - \frac{e^2}{2}\Big(|q|^2 - |\widetilde{q}|^2 - (X^7 - t^7)\Big)^2 - e^2\Big|q\widetilde{q} + (X^6 - s^6) + \mathrm{i}(X^8 - s^8)\Big|^2 \\
&\quad + \frac{g^2}{2}(A_{c=} + \overline{A}_{c=})(B_{c\#} + \overline{B}_{c\#})
\end{aligned}
$$

$$
\begin{aligned}
(\partial_0 + \partial_1) X^8 &= -g^2 (\partial_0 + \partial_1)\widetilde{X}^8 - g^2 (B_{c\#} + \overline{B}_{c\#}) \\
(\partial_0 - \partial_1) X^8 &= +g^2 (\partial_0 - \partial_1)\widetilde{X}^8 - g^2 (A_{c=} + \overline{A}_{c=}) \\
+\frac{g^2}{2}(A_{c=} + \overline{A}_{c=})(B_{c\#} + \overline{B}_{c\#}) &= -\frac{1}{2g^2}(\partial_m X^8)^2 + \frac{g^2}{2}(\partial_m \widetilde{X}^8)^2 + \varepsilon^{mn}\,(\partial_m X^8)(\partial_n \widetilde{X}^8)
\end{aligned}
$$

Steps to NLSM for 5^2_2-brane

1. search SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints of charged HM $(Q, \widetilde{Q})$
3. integrate out VM (V, Φ) in IR

★4. integrate s^8, and solve EOM for X^8 in $\Psi - \overline{\Psi}$

Why is $\Psi - \overline{\Psi}$ integrated out at the final step?

→ X^8 in $\Psi - \overline{\Psi}$ still appears as a winding coordinate in string worldsheet sigma model.

We should not integrate it out before integrating-out of VM.

Step 1.,2.,3. :

$$\begin{aligned}
\mathscr{L} &= -\frac{1}{2}H\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] - \frac{1}{2H}(\partial_m \widetilde{X}^9)^2 \\
&\quad -\frac{(\Omega_8)^2}{2H}(\partial_m X^8)^2 + \frac{\Omega_8}{H}(\partial_m X^8)(\partial^m \widetilde{X}^9) \\
&\quad -\frac{(\Omega_6)^2}{2H}(\partial_m X^6)^2 - \frac{\Omega_6\Omega_8}{H}(\partial_m X^6)(\partial^m X^8) + \frac{\Omega_6}{H}(\partial_m X^6)(\partial^m \widetilde{X}^9) \\
&\quad +\varepsilon^{mn}\partial_m((X^9 - t^9)A_n) + \varepsilon^{mn}(\partial_m X^8)(\partial_n \widetilde{X}^8)
\end{aligned}$$

$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|}, \quad \Omega_6 = \frac{X^8 - s^8}{|\vec{X}|(|\vec{X}| + (X^7 - t^7))}, \quad \Omega_8 = -\frac{X^6 - s^6}{|\vec{X}|(|\vec{X}| + (X^7 - t^7))}$$

$$A_m = \frac{1}{|\vec{X}|H}\big(\partial_m \widetilde{X}^9 - \Omega_i\,\partial_m X^i\big) + \Omega_i\,\partial_m X^i$$

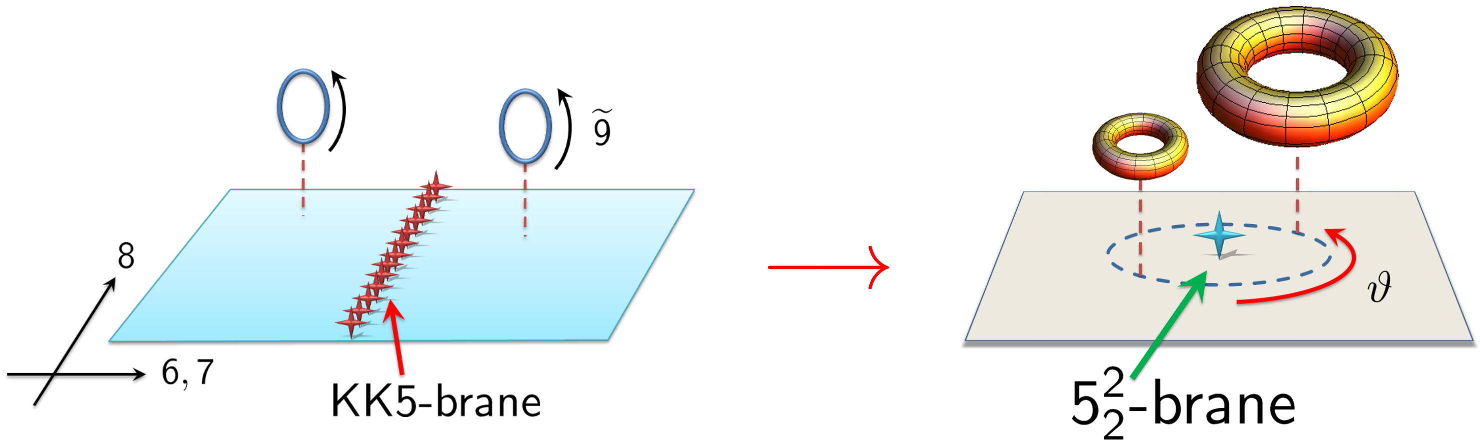
Step 4. : $s^8 = (2\pi R_8)\, s \xrightarrow{\text{integral over } s}$ emerge isometry

$$\begin{cases} H \to 1/g^2 + \log(\Lambda/\varrho) & \text{: co-dim. 2} \\ \Omega_6 \to 0 & \text{: isometry along } X^8 \\ \Omega_8 \to \arctan\left(\dfrac{X^7 - t^7}{X^6 - s^6}\right) \equiv \vartheta & \text{: “non–single-valued” metric} \end{cases}$$

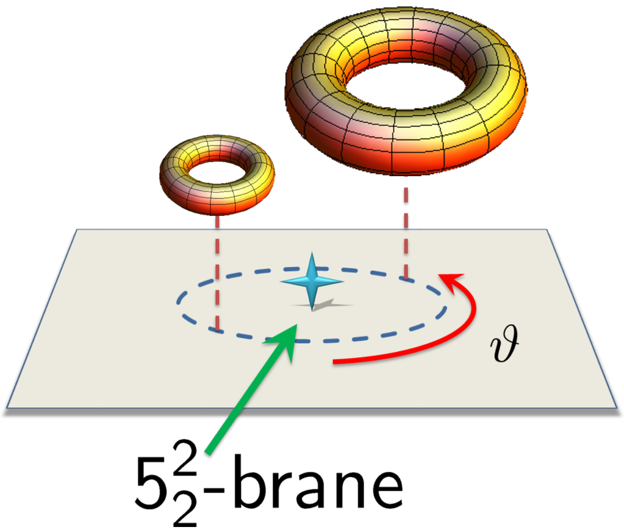
$\varrho = \sqrt{(X^6 - s^6)^2 + (X^7 - t^7)^2}$

EOM for X^8 : $\partial_m X^8 = \dfrac{H}{K}\Big[\dfrac{\vartheta}{H}(\partial_m \widetilde{X}^9) + \varepsilon_{mn}(\partial^n \widetilde{X}^8)\Big]$

$K = H^2 + \vartheta^2$



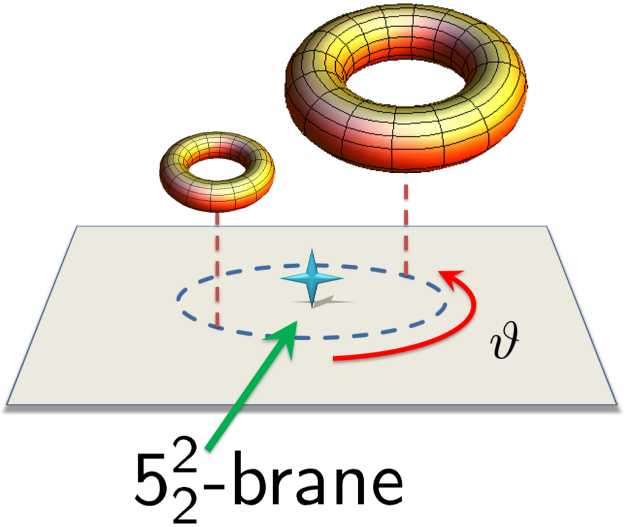
$$\mathscr{L}^{\text{NLSM}}_{\text{exotic}} = -\frac{H}{2}\Big[(\partial_m \varrho)^2 + \varrho^2 (\partial_m \vartheta)^2\Big] - \frac{H}{2K}\Big[(\partial_m \widetilde{X}^8)^2 + (\partial_m \widetilde{X}^9)^2\Big]$$
$$-\frac{\vartheta}{K}\,\varepsilon^{mn}(\partial_m \widetilde{X}^8)(\partial_n \widetilde{X}^9) + \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)$$



We obtained NLSM for Exotic 5^2_2-brane.

Exotic feature comes from the string winding modes.

$$\mathscr{L}^{\text{NLSM}}_{\text{exotic}} = -\frac{H}{2}\Big[(\partial_m \varrho)^2 + \varrho^2(\partial_m \vartheta)^2\Big] - \frac{H}{2K}\Big[(\partial_m \widetilde{X}^8)^2 + (\partial_m \widetilde{X}^9)^2\Big]$$
$$-\frac{\vartheta}{K}\,\varepsilon^{mn}(\partial_m \widetilde{X}^8)(\partial_n \widetilde{X}^9) + \varepsilon^{mn}\,\partial_m((X^9 - t^9)A_n)$$



Spacetime picture involving string winding modes

beyond the Einstein's view of gravity !

Quantum corrections to five-branes

S. Sasaki and TK arXiv:1305.4439

String Worldsheet Instanton Corrections

↓

Deform target space geometry by momentum and/or winding effects

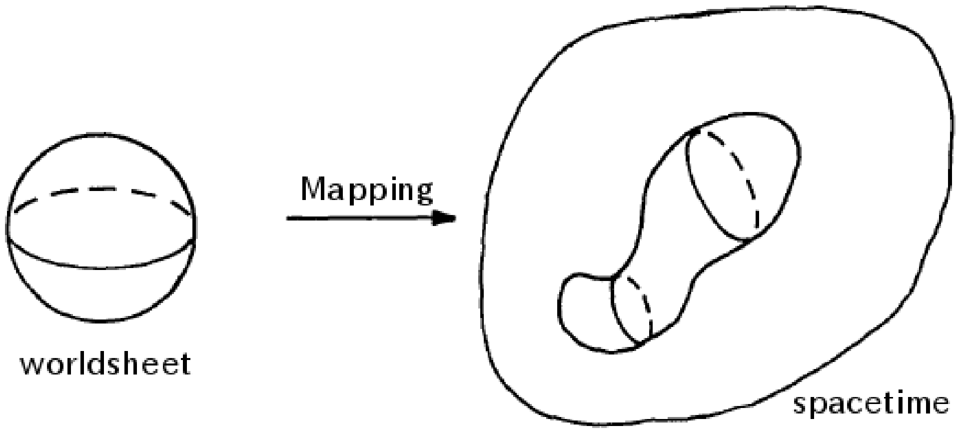


Figure from E. Witten CMP 92 (1984) 455

Again, GLSM is a powerful tool :

Worldsheet instantons in NLSM can be captured by vortex solution in gauge theory

- NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|} \text{ : radius of } X^9$$

$g \to 0$ case	general $\lvert\vec{X}\rvert$
radius of X^9	∞
KK-modes	light
winding modes	heavy

$g \to \infty$ case	$\lvert\vec{X}\rvert \to 0$	$\lvert\vec{X}\rvert \to \infty$
radius of X^9	∞	0
KK-modes	light	heavy
winding modes	heavy	light

- NS5-brane

$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|} \text{ : radius of } X^9$$

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radius of X^9	∞	0
KK-modes	light	heavy
winding modes	heavy	light

GLSM for NS5-brane has $X^9 F_{01}$

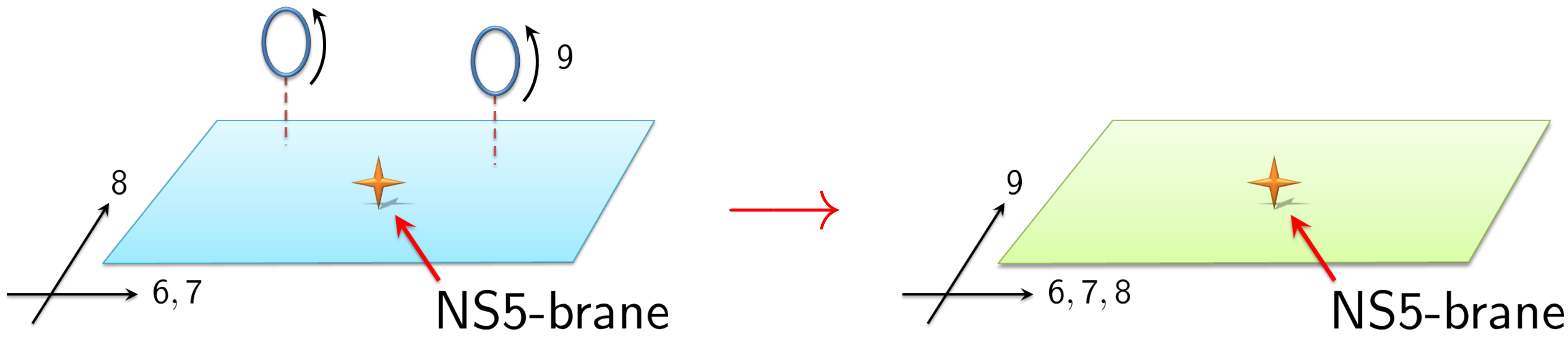
$\downarrow$

KK-mode corrections can be captured by vortex solution in gauge theory

Worldsheet instanton corrections to NS5-brane :

$X^9 F_{01}$ in GLSM

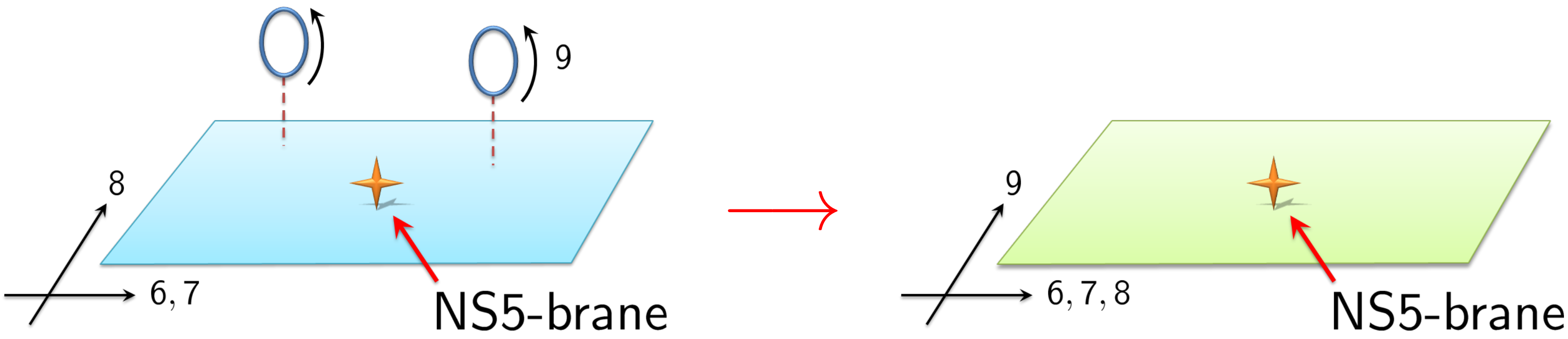
$\rightarrow$ unfolding effect on compactified circle X^9



Worldsheet instanton corrections to NS5-brane :

$X^9 F_{01}$ in GLSM

$\rightarrow$ unfolding effect on compactified circle X^9



$$H = \frac{1}{g^2} + \frac{1}{|\vec{X}|} \rightarrow \frac{1}{g^2} + \frac{1}{|\vec{X}|} \sum_{n=1}^{\infty} \mathrm{e}^{-n|\vec{X}|} \Big[\mathrm{e}^{+\mathrm{i}\, n X^9} + \mathrm{e}^{-\mathrm{i}\, n X^9} \Big]$$

$$= \frac{1}{g^2} + \frac{1}{|\vec{X}|} \frac{\sinh(|\vec{X}|)}{\cosh(|\vec{X}|) - \cos(X^9)}$$

D. Tong hep-th/0204186

- KK5-brane

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{|\vec{X}|}\right)^{-1} \text{ : radius of } \widetilde{X}^9$$

$g \to 0$ case	general $\lvert\vec{X}\rvert$
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\lvert\vec{X}\rvert \to 0$	$\lvert\vec{X}\rvert \to \infty$
radius of $\widetilde{X}^9$	0	∞
KK-modes	heavy	light
winding modes	light	heavy

- KK5-brane

$$H^{-1} = \left(\frac{1}{g^2} + \frac{1}{|\vec{X}|}\right)^{-1} \text{ : radius of } \widetilde{X}^9$$

$g \to 0$ case	general $\lvert\vec{X}\rvert$	$g \to \infty$ case	$\lvert\vec{X}\rvert \to 0$	$\lvert\vec{X}\rvert \to \infty$
radius of $\widetilde{X}^9$	0	radius of $\widetilde{X}^9$	0	∞
KK-modes	heavy	KK-modes	heavy	light
winding modes	light	winding modes	light	heavy

GLSM for KK5-brane has $\varepsilon^{mn}\partial_m(X^9 A_n)$

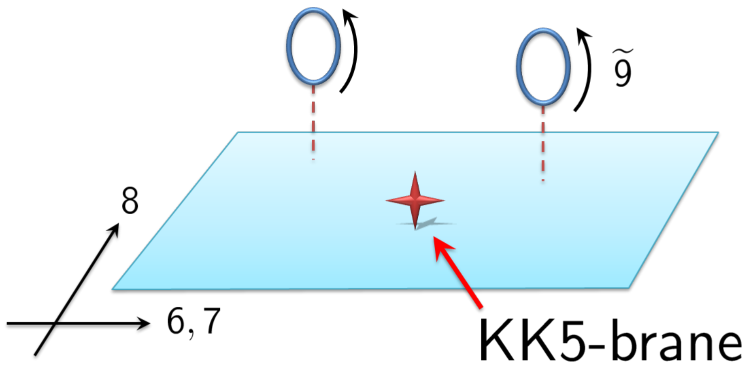
$\downarrow$

Winding mode corrections can be captured by vortex solution in gauge theory

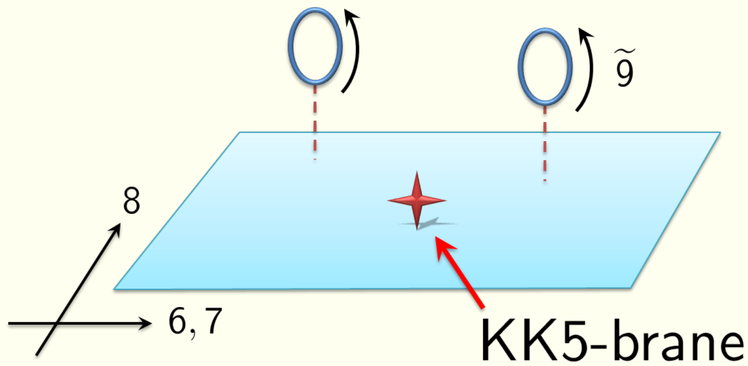
Worldsheet instanton corrections to KK5-brane :

$\varepsilon^{mn}\,\partial_m(X^9 A_n)$ in GLSM

$\to$ string winding modes along X^9



$\longrightarrow$



+

string winding along dual circle X^9

$$H \to \frac{1}{g^2} + \frac{1}{|\vec{X}|}\frac{\sinh(|\vec{X}|)}{\cosh(|\vec{X}|) - \cos(X^9)}$$

D. Tong hep-th/0204186; J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097

- 5^2_2-brane

$\frac{H}{K}$: radius of $\widetilde{X}^9$ $\qquad H = \frac{1}{g^2} + \log \frac{\Lambda}{\varrho}, \; K = H^2 + \vartheta^2$

$g \to 0$ case	general ϱ
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\varrho \to 0$	$\varrho \to \Lambda$	
radius of $\widetilde{X}^9$	0	0	∞ at $\vartheta = 0$
KK-modes	heavy	heavy	light at $\vartheta = 0$
winding modes	light	light	heavy at $\vartheta = 0$

- 5^2_2-brane

$\frac{H}{K}$: radius of $\widetilde{X}^9$ $\quad H = \frac{1}{g^2} + \log \frac{\Lambda}{\varrho}, \; K = H^2 + \vartheta^2$

$g \to 0$ case	general ϱ
radius of $\widetilde{X}^9$	0
KK-modes	heavy
winding modes	light

$g \to \infty$ case	$\varrho \to 0$	$\varrho \to \Lambda$	
radius of $\widetilde{X}^9$	0	0	∞ at $\vartheta = 0$
KK-modes	heavy	heavy	light at $\vartheta = 0$
winding modes	light	light	heavy at $\vartheta = 0$

GLSM for 5^2_2 with one gauged isometry has $\varepsilon^{mn}\partial_m(X^9 A_n)$

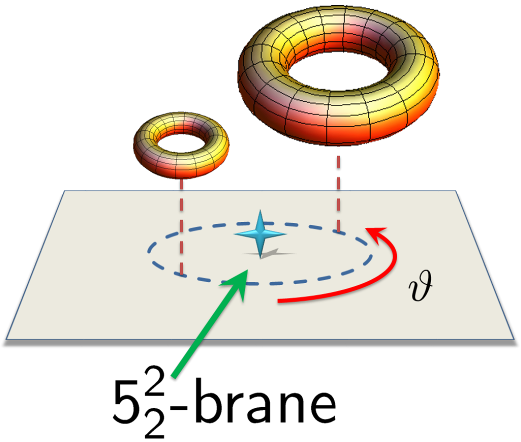
$\downarrow$

Winding mode corrections can be captured by vortex solution in gauge theory

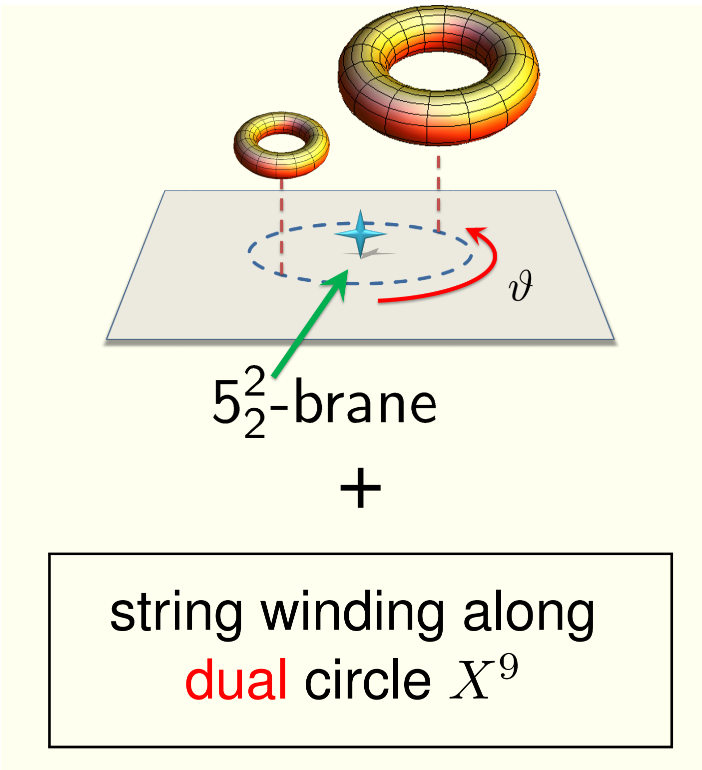
Worldsheet instanton corrections to 5^2_2-brane with one gauged isometry :

$$\varepsilon^{mn}\,\partial_m(X^9 A_n) \text{ in GLSM}$$

$\to$ string winding modes along X^9



$\longrightarrow$



$$H \;\to\; \frac{1}{g^2} + \log\frac{\Lambda}{\varrho} + \sum_{n\neq 0} \exp(\mathrm{i}\,n\,X^9) K_0(|n|\varrho)$$

S. Sasaki and TK arXiv:1305.4439

Result

Quantum corrections can be traced by Vortices in GLSM:

$$\mathscr{L}^{\text{GLSM}}_{\text{topological}} = X^9 F_{01} + \dots$$

$$H = 1 + \log\left(\frac{\Lambda}{\varrho}\right) \quad \to \quad 1 + \log\left(\frac{\Lambda}{\varrho}\right) + \sum_{n \neq 0} \exp(\mathrm{i} n X^9)\, K_0(|n|\, \varrho)$$

$K_0(x)$: modified Bessel function of the second kind

X^9 is ...
- physical coordinate in NS5
- dual coordinate in KK5
- dual coordinate in 5^2_2

S. Sasaki and TK arXiv:1305.4439

Worldvolume action of exotic 5^2_2-brane

S. Sasaki, M. Yata and TK arXiv:1404.5442

Worldvolume action of 5^2_2-brane could be obtained from

D5-brane's action via S- and T-dualities
KK6-brane action via reduction and T-duality
M5-brane action via reduction and T-dualities

Different point: Two isometries along transverse directions

How to describe its worldvolume theory with isometries?

Worldvolume action of 5^2_2-brane could be obtained from

D5-brane's action via S- and T-dualities
KK6-brane action via reduction and T-duality
M5-brane action via reduction and T-dualities

Different point: Two isometries along transverse directions

How to describe its worldvolume theory with isometries?

Already Known!

Bergshoeff et al hep-th/9706117

Worldvolume action with WZ-term is gauged with respect to isometries ex.) gravity sector of KK6-brane action with gauged isometry in M-theory

$$\begin{aligned}
\mathscr{L}^{\text{M}}_{\text{KK6}} &= -\frac{1}{2} T^{\text{M}}_{\text{KK6}} \sqrt{-\gamma} \Big(k^{\frac{4}{7}} \gamma^{ab} D_a X^{\mu} D_b X^{\nu} g_{\mu\nu} - 5 \Big) \\
&= -T^{\text{M}}_{\text{KK6}} k^2 \sqrt{-\det(D_a X^{\mu} D_b X^{\nu} g_{\mu\nu})} \\
&= -T^{\text{M}}_{\text{KK6}} k^2 \sqrt{-\det(\partial_a X^{\mu} \partial_b X^{\nu} \Pi_{\mu\nu})}
\end{aligned}$$

$$D_a X^{\mu} = \partial_a X^{\mu} + C_a k^{\mu} \qquad k^{\mu} \text{ : Killing vector}$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - \frac{k^{\mu} k^{\nu}}{k^2}$$

✓ Construct the worldvolume action with WZ-term in a covariant way:

$$\text{KK6} \xrightarrow{\text{reduction}} \text{KK5 in IIA} \xrightarrow{\text{T-duality}} 5^2_2 \text{ in IIB}$$

$$\text{M5} \xrightarrow{\text{reduction}} \text{NS5 in IIA} \xrightarrow{\text{T-dualities}} 5^2_2 \text{ in IIA}$$

✓ 5^2_2-branes in heterotic theories can be obtained by truncation:

$$5^2_2 \text{ in IIB} \xrightarrow[\mathcal{N}=(1,1)\to\mathcal{N}=(1,0)]{\text{RR}=0} 5^2_2 \text{ in HSO}$$

$$5^2_2 \text{ in IIA} \xrightarrow[\mathcal{N}=(2,0)\to\mathcal{N}=(1,0)]{\text{RR}=0} 5^2_2 \text{ in HE8}$$

$$
\begin{aligned}
S^{\text{IIB}}_{5^2_2} \;=\; & -T_{5^2_2} \int \mathrm{d}^6\xi\, \mathrm{e}^{-2\phi} (\det h_{IJ}) \sqrt{1 + \mathrm{e}^{+2\phi} (\det h_{IJ})^{-1} \big(i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B) \big)^2} \\
& \times \sqrt{- \det \Big[\Pi_{\mu\nu}(k_2)\, \partial_a X^{\mu} \partial_b X^{\nu} + \frac{K^{(1)}_a K^{(1)}_b}{(k_2)^2} - \frac{(k_2)^2 (K^{(2)}_a K^{(2)}_b - K^{(3)}_a K^{(3)}_b)}{(\det h_{IJ})} + \lambda \mathfrak{F}_{ab} \Big]} \\
& - \mu_5 \int_{\mathcal{M}_6} \Big[P[i_{k_1} i_{k_2} B^{(8,2)}] - \frac{1}{2} P[\widetilde{B} \wedge \widetilde{C}^{(2)} \wedge \widetilde{C}^{(2)}] + \lambda P[\widetilde{C}^{(4)} + \widetilde{C}^{(2)} \wedge \widetilde{B}] \wedge \widetilde{F} \\
& \quad - \frac{\lambda^2}{2!} P[\widetilde{B}] \wedge \widetilde{F} \wedge \widetilde{F} + \frac{\lambda^3}{3!} \frac{i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B)}{(i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B))^2 + \mathrm{e}^{-2\phi} (\det h_{IJ})} \widetilde{F} \wedge \widetilde{F} \wedge \widetilde{F} \Big]
\end{aligned}
$$

$$
h_{IJ} \;=\; k^{\mu}_I k^{\nu}_J (g_{\mu\nu} + B_{\mu\nu}), \quad \text{etc..}
$$

S. Sasaki, M. Yata and TK arXiv:1404.5442