Seminar @ Keio Topological Science May 18, 2016

Exotic Brane Junctions from F-theory

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This is a story of string theory.

Various objects (standard branes) in 10D string theory

F-string : (1 + 1)-dim., coupled to 2-form potential $B_{(2)}$.

Dp-brane : (1 + p)-dim., coupled to (p + 1)-form potential $C_{(p+1)}$.
F-string is ending on it.

NS5-brane : (1 + 5)-dim., magnetically coupled to $B_{(2)}$.
D1-brane (D-string) is ending on it.

They have been well investigated for 20 years.

D-brane is a useful object

becasue it captures (non)perturbative dynamics of gauge theory.

(Gauge theory with matters live on a D-brane.)

There also exist exotic branes in string theory in lower dimensions.

They are “vortex-like” (codim 2) objects.

However, their properties are less known. Are they also useful or not?

I have studied them for three years.

Exotic branes

- ✔ from standard branes via string dualities in lower dim (less than 10)
- ✔ vortex-like (codim 2)
- ✔ unclear their roles in string theory and gauge theory

NOTE

D7-brane is an object of codim 2 in 10D.

D7-brane physics has also well studied for 20 years : F-theory

F-theory

- ✔ a part of IIB string theory featured by $SL(2,\mathbb{Z})$ S-duality
- ✔ D7-brane (i.e., vortex-like object) physics
- ✔ monodromy; branch cut and junctions

In F-theory, we can consider D7-branes connected with other branes.

Their configurations provide various (new) gauge theories.

F-theory will also provide the properties and applications of the exotic branes.

Contents

- Exotic brane junctions
- Application:
  SCFTs with E_{n+1} symmetry in 5D, 4D, 3D
- Summary

Exotic brane junctions

Bergshoeff, Ortín and Riccioni: arXiv:1109.4484

de Boer and Shigemori: arXiv:1209.6056

Sakatani: arXiv:1412.8769

TK: arXiv:1410.8403, 1601.02175, 1602.08606

We will find ...

- which object is ending on exotic brane.

  (F-string is ending on D-brane, while D-string on NS5-brane.)

- which is sensitive to branch cut of exotic brane.

  (F- and D-string are sensitive to 7-brane's.)

We study exotic b_n^c-brane whose tension is

$$\frac{R_1 R_2 \cdots R_b \, (R_{b+1} \cdots R_{b+c})^2}{g_s^n \, \ell_s^{b+2c+1}}$$

Performing the following string dualities, the tension is transformed :

$$\mathrm{T}_y : \quad R_y \to \frac{\ell_s^2}{R_y}, \quad g_s \to \frac{\ell_s}{R_y} g_s$$

$$\mathrm{S} : \quad g_s \to \frac{1}{g_s}, \quad \ell_s \to g_s^{1/2} \ell_s$$

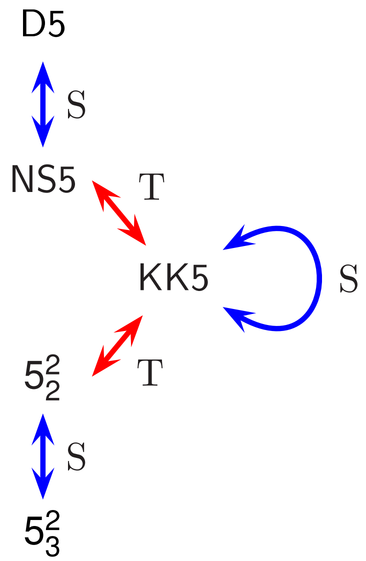
R_y : compact radius of y-direction

ℓ_s : string length

g_s : string coupling constant

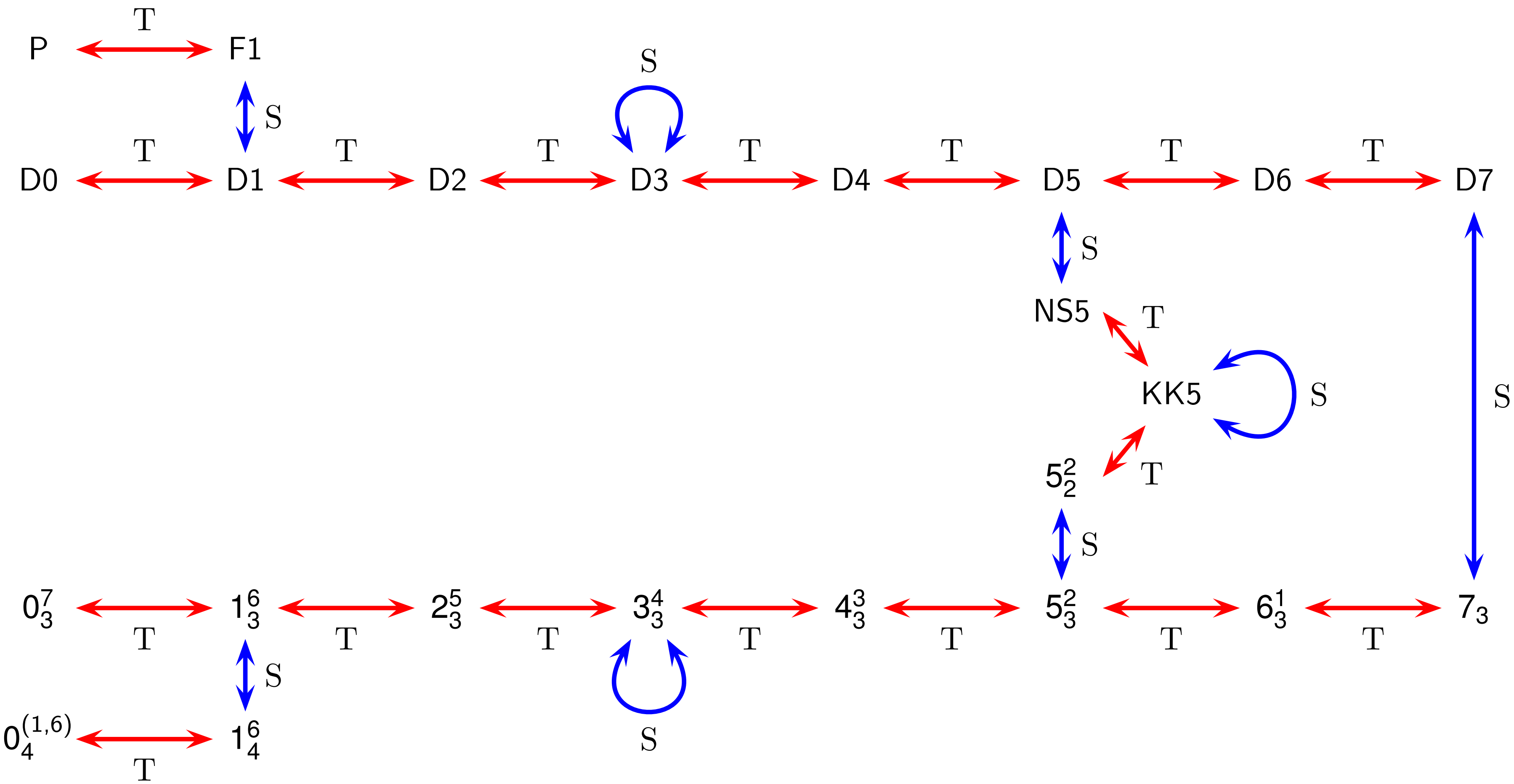
For example, we exhibit the duality chain of 5-branes :

$$\underset{5_1}{\text{D5(12345)}} \xrightarrow{\mathrm{S}} \underset{5_2}{\text{NS5(12345)}} \xrightarrow{\mathrm{T}_9} \underset{5_2^1}{\text{KK5(12345,9)}} \xrightarrow{\mathrm{T}_8} 5_2^2\text{(12345,89)} \xrightarrow{\mathrm{S}} 5_3^2\text{(12345,89)}$$

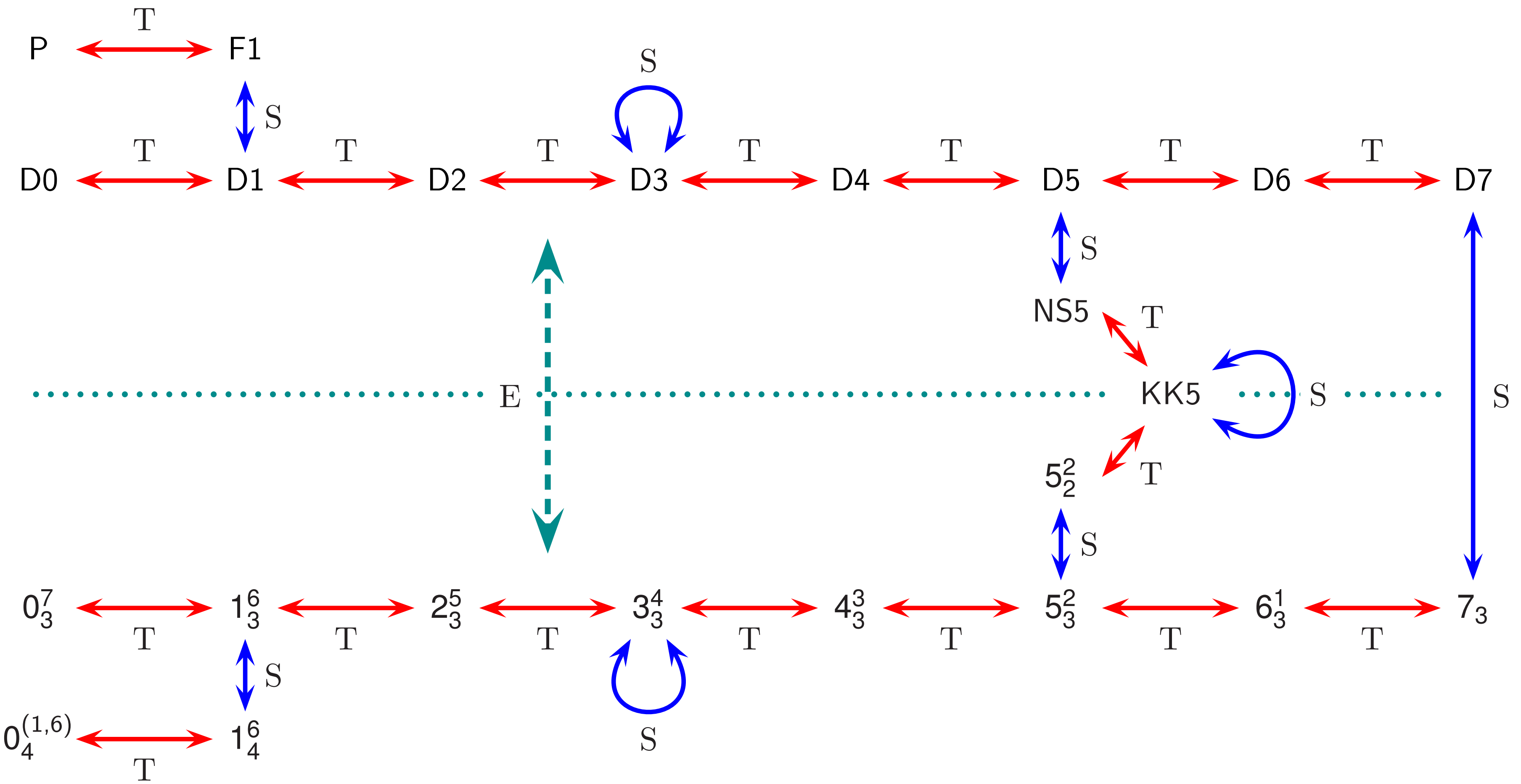


Duality chains of 5-branes in 8D.

$SL(2,Z)$ exotic dualities

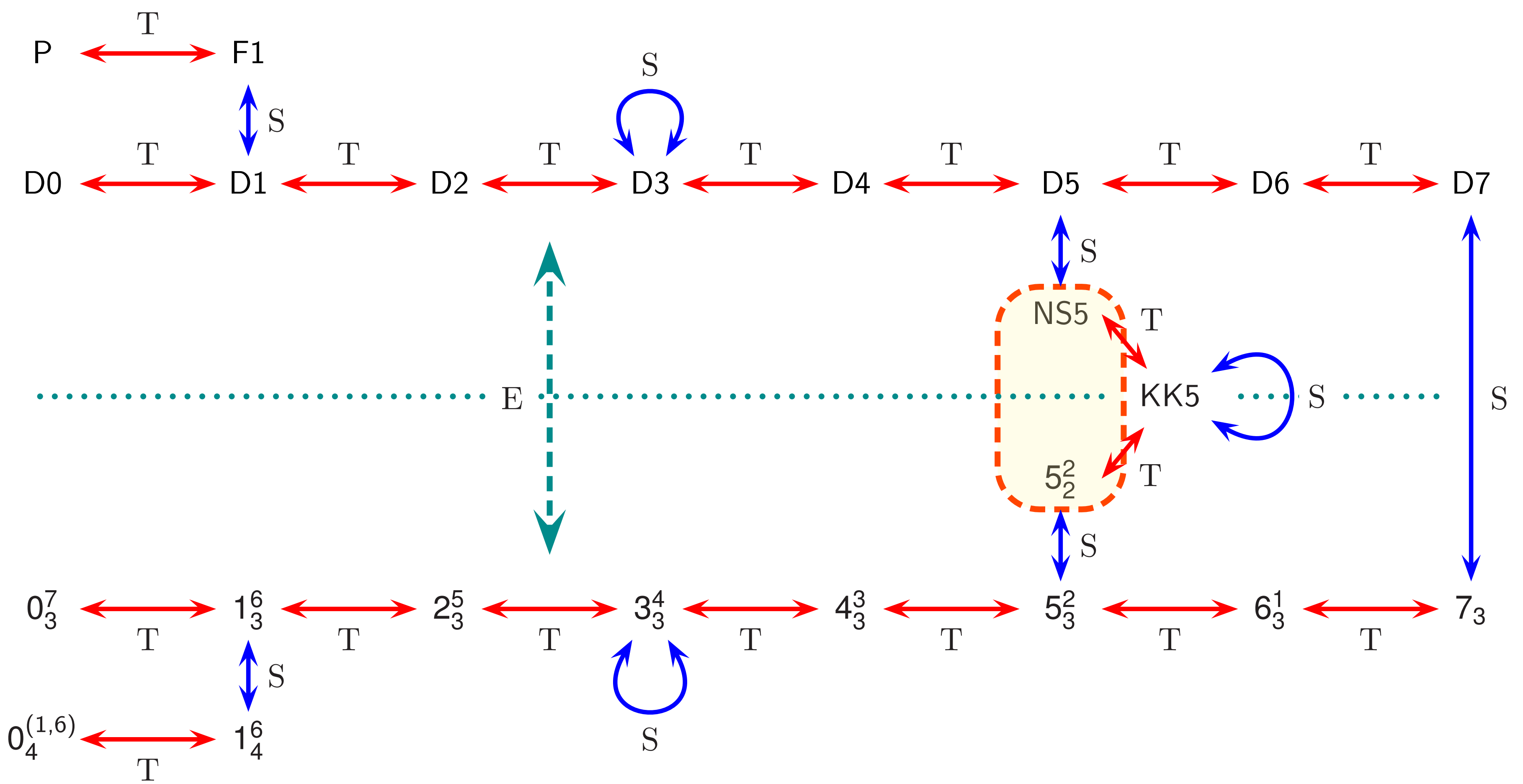


Branes of codim 2 in diverse dimensions.



Pairs (Dp, p^{7-p}_3), (NS5, 5^2_2), (F1, 1^6_4), (P, $0^{(1,6)}_4$) are $SL(2,\mathbb{Z})$ doublets.

Each $SL(2,\mathbb{Z})$ is a subgroup of U-duality.



We focus on a pair of p NS5-brane and q 5^2_2-brane in 8D.

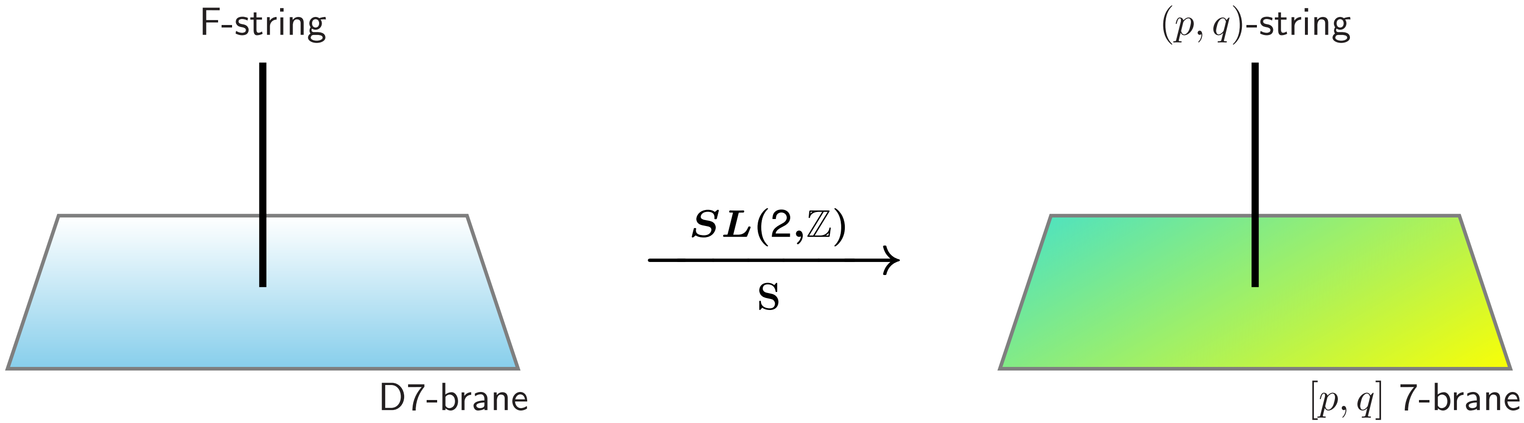
(referred to as $[p,q]^T_{s5}$-brane)

F-string : couple to $B_{(2)}$

D-string : couple to $C_{(2)}$

D7(1234567) : couple to $\rho(z) = C + \mathrm{i}e^{-\phi}$ $\quad (z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta})$

$$\rho(z) = \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{z}\right) = \theta + \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{r}\right)$$



$(1,0)$-string $=$ F1 $\qquad [1,0]$ 7-brane $=$ D7(1234567)

$(0,1)$-string $=$ D1 $\qquad [0,1]$ 7-brane $=$ 7_3(1234567)

Open D-string is ending on 7_3(1234567).

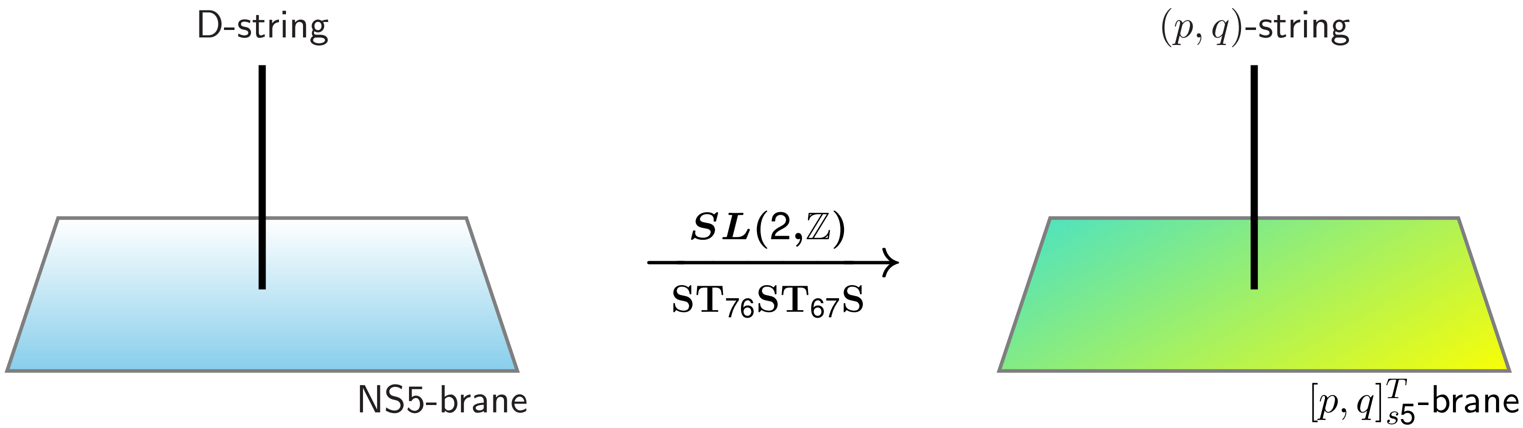
This is a setup in F-theory. We perform ST_{67}-duality and reduce 67-directions.

D-string : couple to $C_{(2)}$

D3-brane : couple to $C_{(4)}$

NS5(12345) : couple to $\rho(z) = B^{(2)}_{67} + \mathrm{i}e^{+2\phi}$ $\quad (z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta})$

$$\rho(z) = \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{z}\right) = \theta + \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{r}\right)$$



$(1,0)$-string $=$ D1 $\qquad [1,0]^T_{s5}$-brane $=$ NS5(12345)

$(0,1)$-string $=$ D3 wrapped on T^2_{67} $\qquad [0,1]^T_{s5}$-brane $=$ 5^2_2(12345,67)

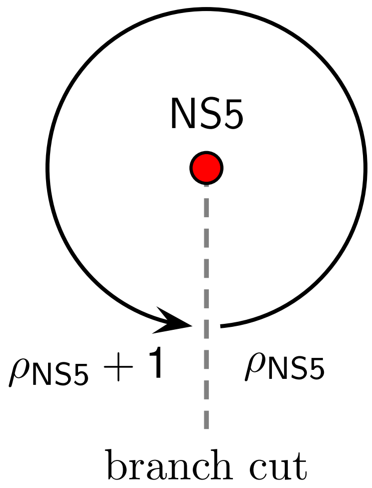
Open D3-brane wrapped on T^2_{67} is ending on exotic 5^2_2(12345,67).

NS5-brane solution in 8D :

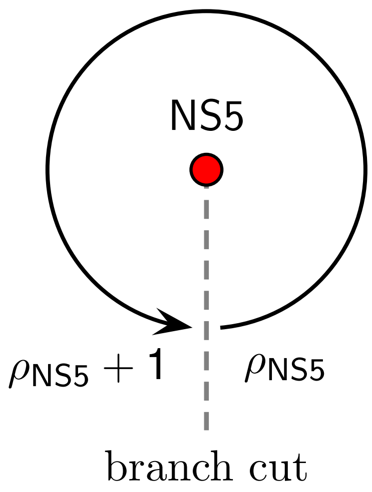
$$\rho(z) \;=\; \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{z}\right) \;=\; \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{r}\right) \qquad (z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta})$$

When ρ moves around NS5-brane counterclockwise $\theta \to \theta + 2\pi$,

it receives a magnetic "charge" of NS5-brane (monodromy) : $\rho \to \rho + 1$



There exists a branch cut in z-plane.



$$\boldsymbol{\rho} \;\to\; \boldsymbol{\rho} + 1 \;=\; \frac{\boldsymbol{a\rho} + \boldsymbol{b}}{\boldsymbol{c\rho} + \boldsymbol{d}} \;\equiv\; \boldsymbol{M}_{[1,0]} \cdot \boldsymbol{\rho}\,, \qquad \boldsymbol{M}_{[1,0]} \;=\; \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \;\in\; \boldsymbol{SL}(2,\mathbb{Z})$$

or

$$\boldsymbol{K}_{[1,0]} \cdot (\boldsymbol{\rho} + 1) \;=\; \boldsymbol{\rho}\,, \qquad \boldsymbol{K}_{[1,0]} \;=\; (\boldsymbol{M}_{[1,0]})^{-1} \;=\; \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

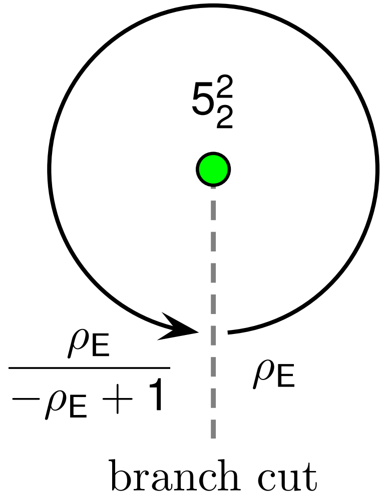
$\boldsymbol{M}_{[\boldsymbol{p},\boldsymbol{q}]}$: moving around the NS5-brane

$\boldsymbol{K}_{[\boldsymbol{p},\boldsymbol{q}]}$: going across the branch cut

By $SL(2,\mathbb{Z})$, the monodromy matrix for general $[p,q]$ 5-brane is given as

$$K_{[p,q]} = g\,K_{[1,0]}\,g^{-1} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix} \qquad g \in SL(2,\mathbb{Z})$$

ex) monodromy $K_{[0,1]}$ for exotic 5^2_2-brane :
$$K_{[0,1]} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$



$$\rho_E = -\frac{1}{\rho_{NS5}}$$

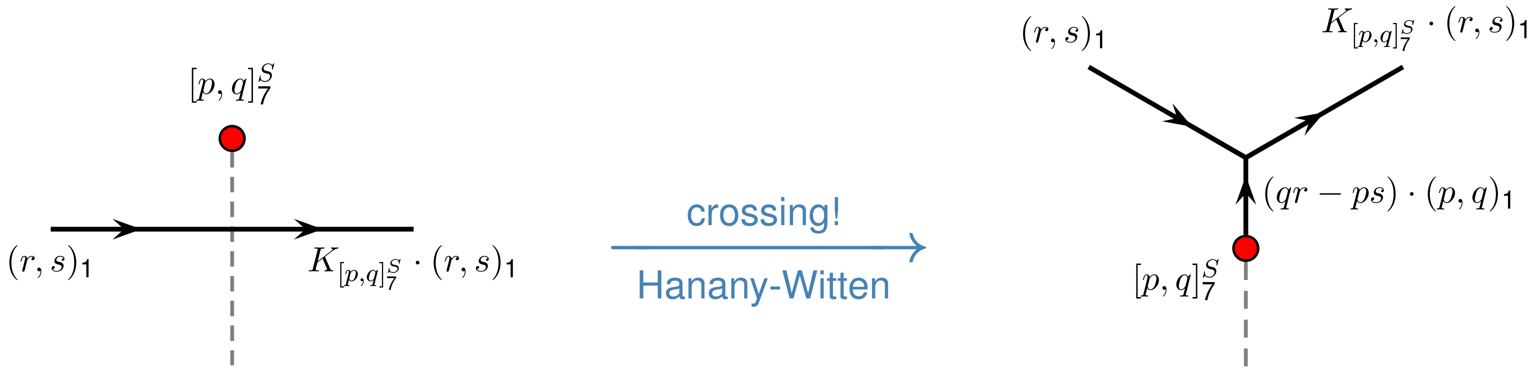
Consider an $(\boldsymbol{r}, \boldsymbol{s})$-string crossing the branch cut of $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane from the left.

The string charge is jumped by monodromy.

If the 7-brane goes across the string, the string is no longer crossing the branch cut.

Further, a new string and a junction appear (Hanany-Witten effect).

"$[\boldsymbol{p}, \boldsymbol{q}]_7^S$" means $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane



Note: 7-brane is stretched in 1234567-directions.

This is a string junction in F-theory.

Perform $\mathbf{ST}_{67}$-duality and reduce 67-directions.

We obtain a config. of an (r, s)-“string” crossing the branch cut of exotic $[p, q]$ 5-brane.

The “string” charge is jumped by monodromy.

If the 5-brane goes across the “string”, the “string” is no longer crossing the branch cut.

Further, a new “string” with a junction appears (Hanany-Witten effect).

(r, s)-“string” : r D1 $+$ s D3 wrapped on T^2_{67}

If we set $(p, q) = (0, 1)$,

we find that D3-brane wrapped on T^2_{67} is ending on the 5^2_2(12345,67).

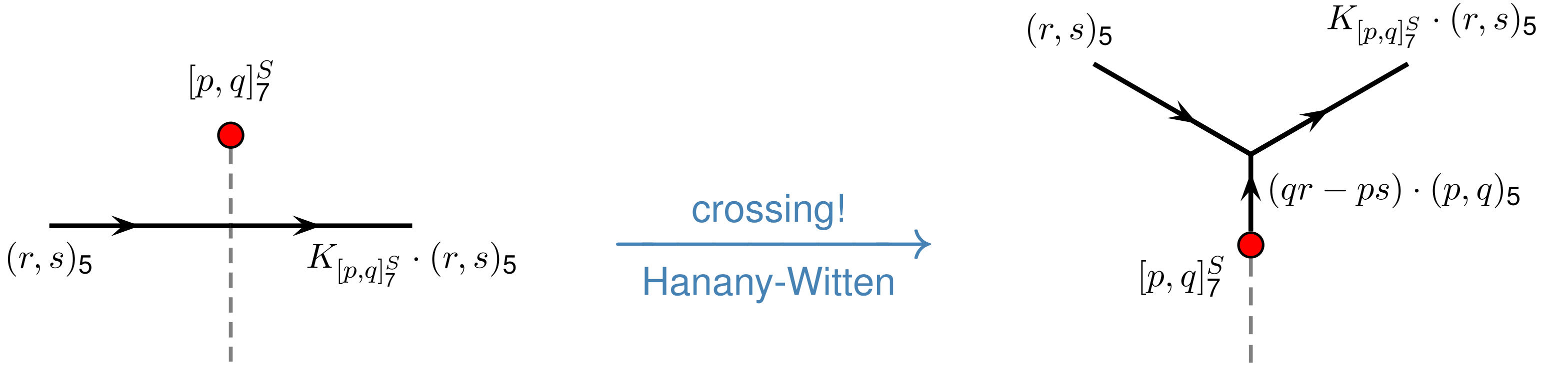
Consider an $(\boldsymbol{r}, \boldsymbol{s})$ 5-brane crossing the branch cut of $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane from the left.

The 5-brane charge is jumped by monodromy.

If the 7-brane goes across the 5-brane, the 5-brane is no longer crossing the branch cut.

Further, a new 5-brane and a junction appear (Hanany-Witten effect).

$(\boldsymbol{r}, \boldsymbol{s})$ 5-brane : $\boldsymbol{r}$ D5-branes $+$ $\boldsymbol{s}$ NS5-branes on which $(\boldsymbol{r}, \boldsymbol{s})$-string is ending



Note: 7-brane(1234567), D5(1234X), NS5(1234Y), X, Y $\in z$-plane

This is a brane junction in F-theory.

Next, we perform $\mathbf{ST}_{67}$-, $\mathbf{ST}_{47}$- or $\mathbf{ST}_{34}$-duality.

Perform the $\mathbf{ST}_{67}$-duality of $(\boldsymbol{r}, \boldsymbol{s})$ 5-branes with $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane in F-theory.

We obtain a config. that a new $(\boldsymbol{r}, \boldsymbol{s})$ "5-brane" with exotic $[\boldsymbol{p}, \boldsymbol{q}]$ 5-brane :

$(\boldsymbol{r}, \boldsymbol{s})$ "5-brane" : $\boldsymbol{r}$ 7_3(123467X) wrapped on $\boldsymbol{T}^2_{67}$ $+$ $\boldsymbol{s}$ 5^2_3(1234Y,67)

When we set $(\boldsymbol{p}, \boldsymbol{q}) = (0, 1)$,

we find that 5^2_3(1234Y,67)-brane is ending on 5^2_2(12345,67).

Perform the $\mathbf{ST}_{47}$-duality of $(\boldsymbol{r}, \boldsymbol{s})$ 5-branes with $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane in F-theory.

We obtain a config. that a new $(\boldsymbol{r}, \boldsymbol{s})$ "4-brane" with exotic $[\boldsymbol{p}, \boldsymbol{q}]$ 5-brane :

$(\boldsymbol{r}, \boldsymbol{s})$ "4-brane" : $\boldsymbol{r}$ NS5(1237X) wrapped on S^1_7 + $\boldsymbol{s}$ KK5(1234Y,7) wrapped on S^1_4

When we set $(\boldsymbol{p}, \boldsymbol{q}) = (0, 1)$,

we see that KK5(1234Y,7)-brane wrapped on S^1_4 is ending on 5^2_2(12345,67).

Perform the $\mathbf{ST}_{34}$-duality of $(\boldsymbol{r}, \boldsymbol{s})$ 5-branes with $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane in F-theory.

We obtain a config. that a new $(\boldsymbol{r}, \boldsymbol{s})$ "3-brane" with exotic $[\boldsymbol{p}, \boldsymbol{q}]$ 5-brane :

$(\boldsymbol{r}, \boldsymbol{s})$ "3-brane" : $\boldsymbol{r}$ D3(12X) $+$ $\boldsymbol{s}$ D5(1234Y) wrapped on $\boldsymbol{T}^2_{34}$

When we set $(\boldsymbol{p}, \boldsymbol{q}) = (0, 1)$,

we find that D5(1234Y)-brane wrapped on $\boldsymbol{T}^2_{34}$ is ending on 5^2_2(12345,67).

Now, we understood

- ✔ An object ending on 5^2_2-brane is a wrapped D3-brane.

  (Its oscillations provide excitation modes on the 5^2_2-brane worldvolume.)

- ✔ Objects sensitive to 5^2_2-brane branch cut are

  wrapped D3-brane, 5^2_3-brane, wrapped KK5-brane, wrapped D5-brane.

  (They are created/annihilated by the Hanany-Witten transitions.)

Application:

SCFTs with E_{n+1} symmetry in 5D, 4D, 3D

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

Benini, Benvenuti and Tachikawa: arXiv:0906.0359

Kim, Taki and Yagi: arXiv:1504.03672

TK: arXiv:1602.08606

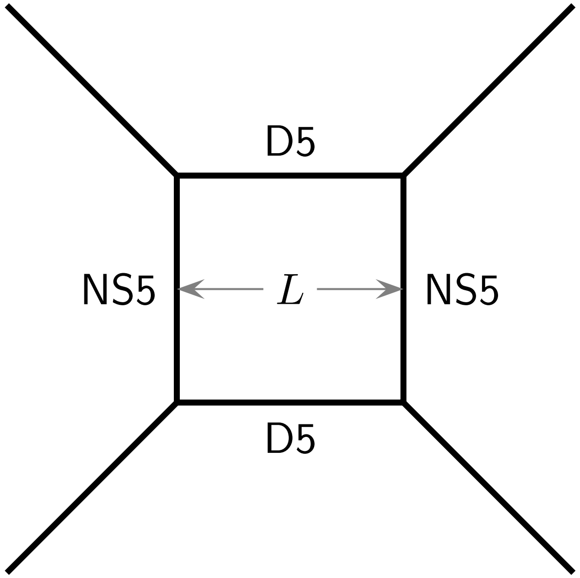
We will study ...

- how exotic branes can contribute to gauge theories.
  (# of 7-branes gives rise to # of flavors.)

- how global symmetry appears in strong coupling limit.
  (E_n symmetry appears by 7-branes' alignment.)

This topic is rather technical...

5D SUSY gauge theory can be realized by brane construction :



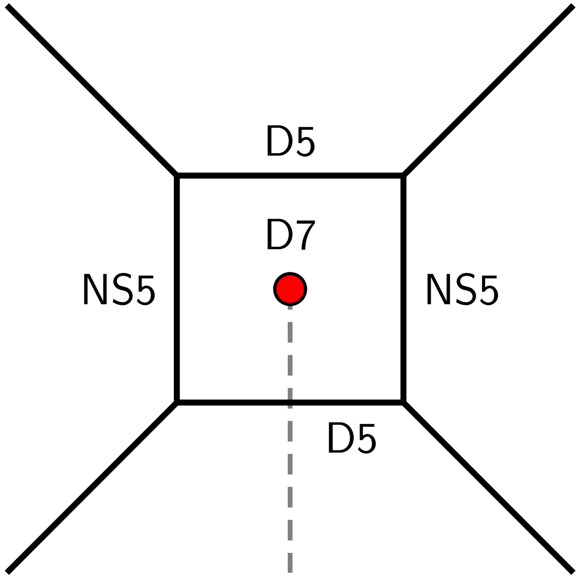
IIB		0	1	2	3	4	5	6	7	8	9
	n D7	—	—	—	—	—	—	—	—		
	D5	—	—	—	—	—				—	
	NS5	—	—	—	—	—					—
$(r,s)_5$	r D5	—	—	—	—	—				angle	
	s NS5	—	—	—	—	—				angle	

color : N_c D5 between 2 NS5 $= SU(N_c)$ gauge symmetry

flavor : N_f D5 outside 2 NS5 $= N_f$ flavors

coupling : $\dfrac{1}{g_{\text{YM}}^2} \simeq \dfrac{L}{g_s\, \ell_s^2}$ (This can be derived from Dirac-Born-Infeld action of D5-brane.)

5D SUSY gauge theory can be realized by brane construction :



IIB		0	1	2	3	4	5	6	7	8	9
	n D7	–	–	–	–	–	–	–	–		
	D5	–	–	–	–	–				–	
	NS5	–	–	–	–	–					–
$(r,s)_5$	r D5	–	–	–	–	–				angle	
	s NS5	–	–	–	–	–					

color : N_c D5 between 2 NS5 $=$ $SU(N_c)$ gauge symmetry

flavor : N_f D5 outside 2 NS5 $=$ N_f flavors

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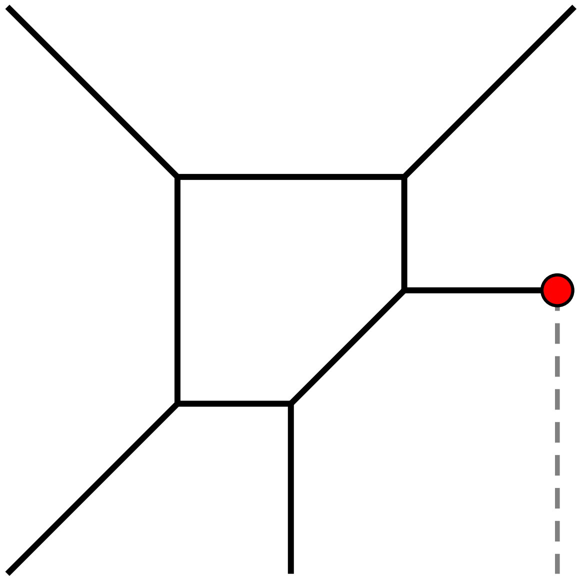

IIB		0	1	2	3	4	5	6	7	8	9
	n D7	—	—	—	—	—	—	—	—		
	D5	—	—	—	—	—				—	
	NS5	—	—	—	—	—					—
$(r,s)_5$	r D5	—	—	—	—	—				angle	
	s NS5	—	—	—	—	—					

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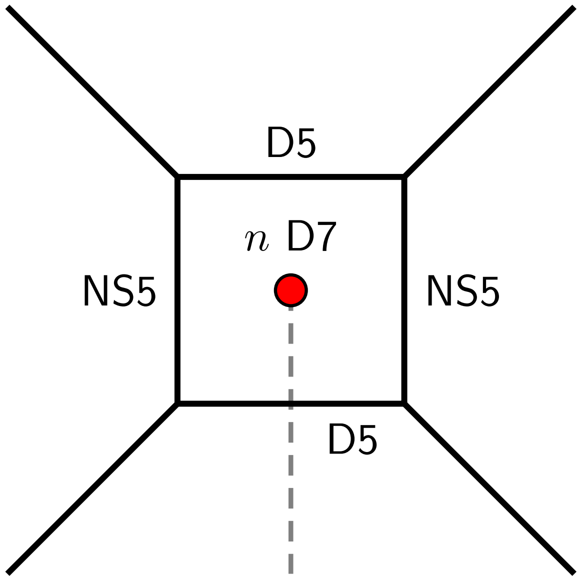


IIB		0	1	2	3	4	5	6	7	8	9
	n D7	—	—	—	—	—	—	—	—		
	D5	—	—	—	—	—				—	
	NS5	—	—	—	—	—					—
$(r,s)_5$	r D5	—	—	—	—	—				angle	
	s NS5	—	—	—	—	—					

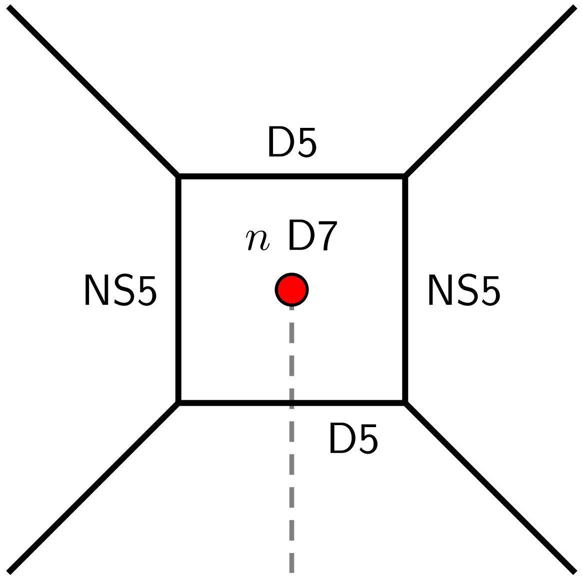
This config. indicates 5D $SU(2)$ gauge symmetry with n flavors on 01234-directions.

In the $L \propto 1/g_{\text{YM}}^2 \to 0$ limit, this gauge theory flows to SCFT with E_{n+1} symmetry.

(UV fixed point)

Let us consider a config. with $n = 5, 6, 7$ D7-branes.

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with n flavors on D5-brane can be illustrated as



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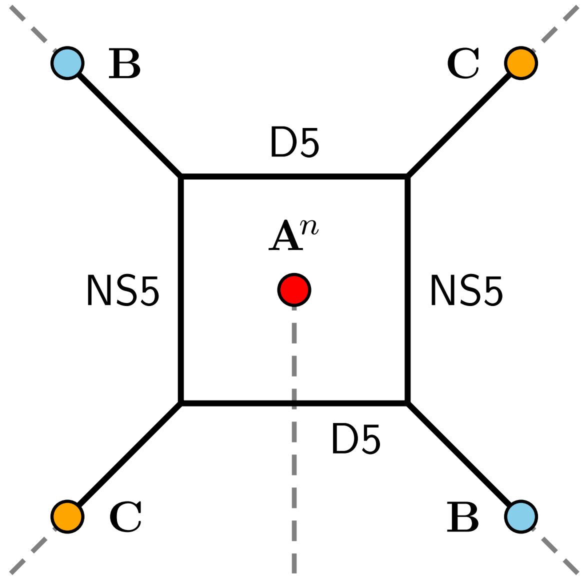


$\mathbf{A}$-brane : D7-brane

$\mathbf{B}$-brane : $[1, -1]$ 7-brane

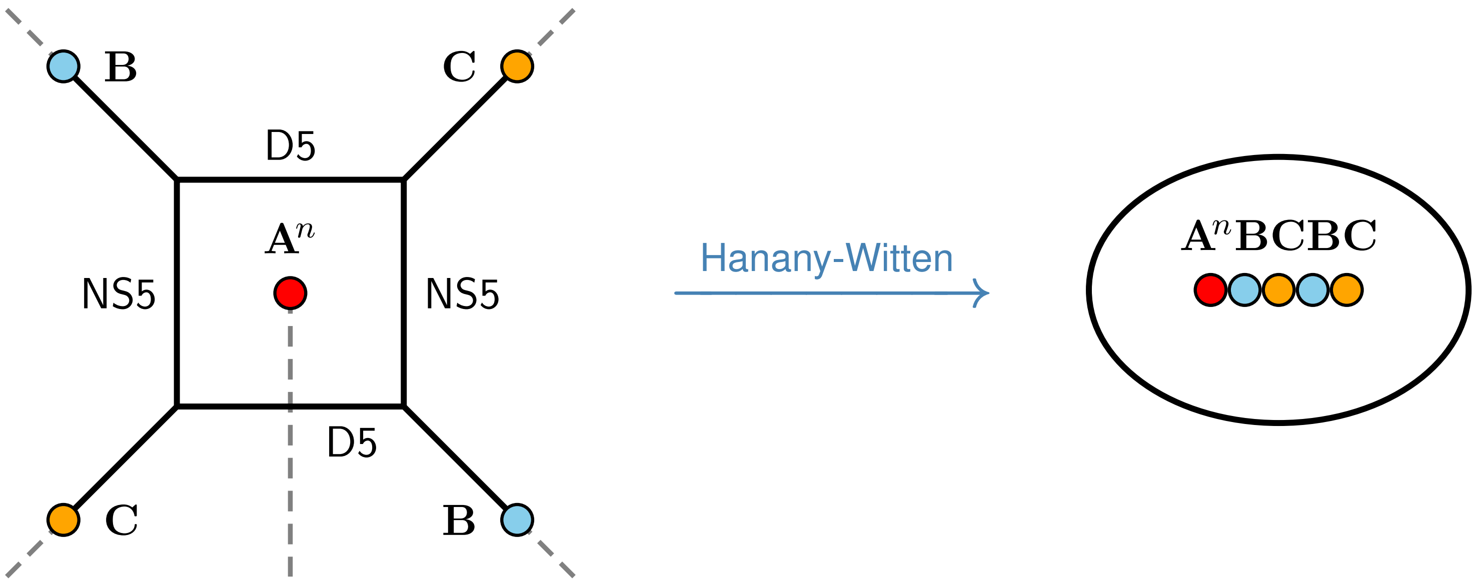
$\mathbf{C}$-brane : $[1, 1]$ 7-brane

Without changing the 5D gauge theory on D5-brane,

semi-infinite (p, q) 5-branes are terminated by $[p, q]$ 7-branes.

Let us consider a config. with $n = 5, 6, 7$ D7-branes.

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with n flavors on D5-brane can be illustrated as



Moving $\mathbf{B}$- and $\mathbf{C}$-branes along the (p, q) 5-branes and going inside the "box",

the (p, q) 5-branes are annihilated by the Hanany-Witten effect.

Further, the "box" becomes "loop" by back reaction of $\mathbf{A}^n$-, $\mathbf{B}$-, $\mathbf{C}$-, $\mathbf{B}$-, and $\mathbf{C}$-branes.

(skipped drawing the branch cuts.)

Let us consider a config. with $n = 5, 6, 7$ D7-branes.

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with n flavors on D5-brane can be illustrated as

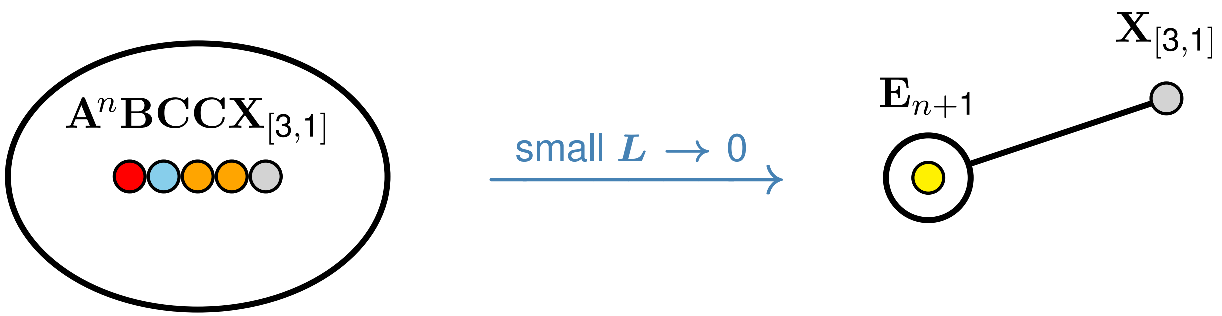
Perform the re-ordering of 7-branes with branch cuts.

When a 7-brane goes across another's branch cut, its monodromy is modified.

$$\mathbf{A}^n\mathbf{BCBC} \ \to \ \mathbf{A}^n\mathbf{BCCX}_{[3,1]} \qquad \text{with } \mathbf{X}_{[3,1]}\text{-brane} \equiv [3,1]\text{-brane}$$

Let us consider a config. with $n = 5, 6, 7$ D7-branes.

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with n flavors on D5-brane can be illustrated as



Perform the small loop limit $L \to 0$. This implies the strong coupling limit $g_{\text{YM}} \to \infty$.

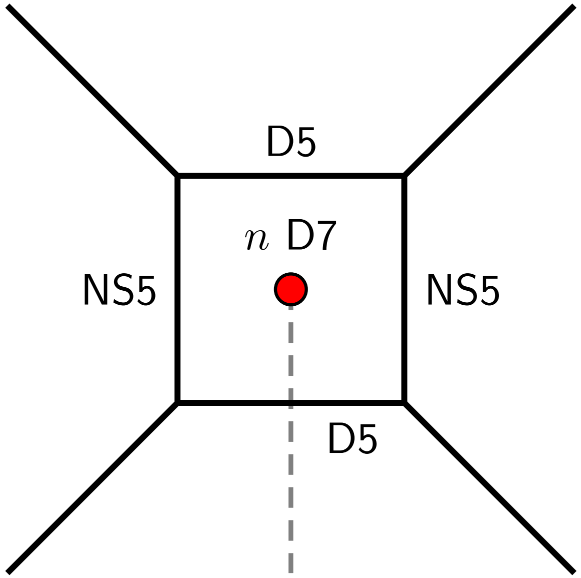
There exists a non-trivial UV fixed point of 5D gauge theory $\to$ CFT.

$\mathbf{A}^n$-, $\mathbf{B}$-, and $\mathbf{C}^2$-branes are collapsed to $\mathbf{E}_{n+1}$-brane.

$\mathbf{X}_{[3,1]}$-brane is gone far away from $\mathbf{E}_{n+1}$-brane.

Open string ending on 5-brane loop and $\mathbf{E}_{n+1}$-brane provides $\mathbf{E}_{n+1}$ symmetry.

We argued 5D SUSY gauge theory and its strong coupling limit by brane construction.



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	n D7	—	—	—	—	—	—	—	—		
	D5	—	—	—	—	—				—	
	NS5	—	—	—	—	—					—
$(r, s)_5$	r D5	—	—	—	—	—				angle	
	s NS5	—	—	—	—	—					

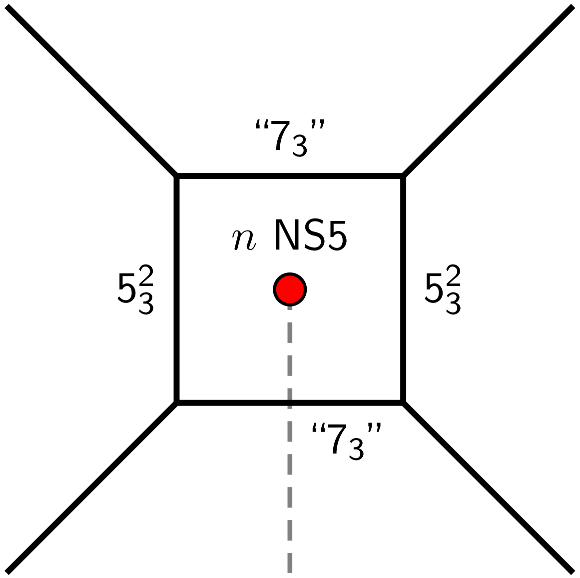
Perform string dualities :

ST_{67}-dual $\rightarrow$ 5D

ST_{47}-dual $\rightarrow$ 4D

ST_{34}-dual $\rightarrow$ 3D

$\mathbf{ST}_{67}$-dualized system :

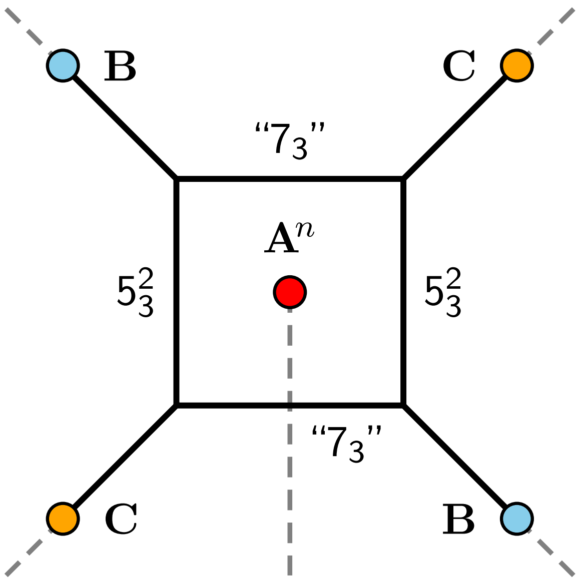


IIB		0	1	2	3	4	5	⑥	⑦	8	9
$\mathbf{A}^n$	n NS5	—	—	—	—	—	—				
	"7_3"	—	—	—	—	—		—	—	—	
	5^2_3	—	—	—	—	—		$\bullet^2$	$\bullet^2$		—
$(r, s)_5$	r "7_3"	—	—	—	—	—		—	—	angle	
	s 5^2_3	—	—	—	—	—		$\bullet^2$	$\bullet^2$		

There exits 5D $SU(2)$ gauge symmetry with n flavors on 01234-directions.

This gauge theory has non-trivial UV fixed point by RG flow.

$\mathbf{ST}_{67}$-dualized system :



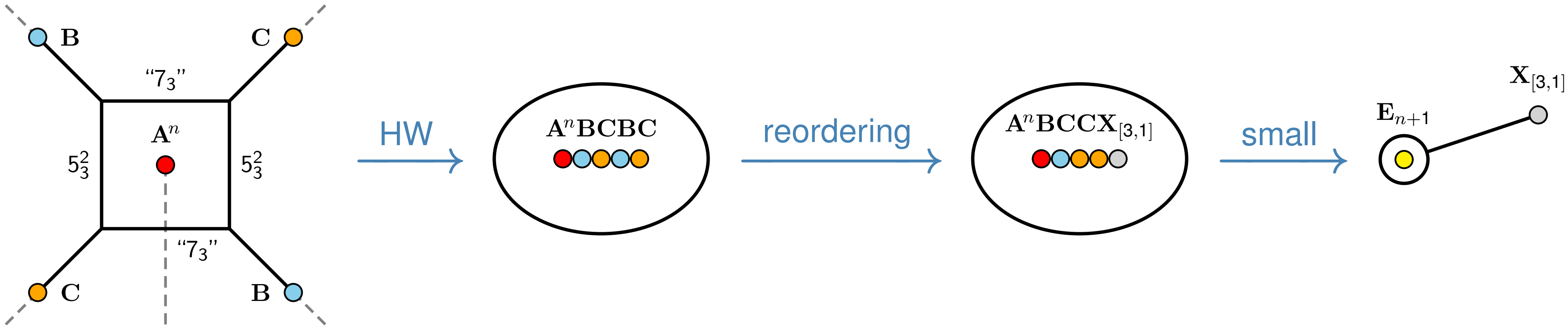
IIB		0	1	2	3	4	5	⑥	⑦	8	9
$\mathbf{A}^n$	n NS5	—	—	—	—	—	—				
	"7_3"	—	—	—	—	—		—	—	—	
	5^2_3	—	—	—	—	—		$\bullet^2$	$\bullet^2$		—
$(r, s)_5$	r "7_3"	—	—	—	—	—		—	—	angle	
	s 5^2_3	—	—	—	—	—		$\bullet^2$	$\bullet^2$		

$\mathbf{A}$-brane : $[1, 0]^{T}_{s5}$-brane $=$ NS5

$\mathbf{B}$-brane : $[1, -1]^{T}_{s5}$-brane

$\mathbf{C}$-brane : $[1, 1]^{T}_{s5}$-brane

5D $\mathcal{N} = 1$ $SU(2)$ gauge with n flavors $\rightarrow$ SCFT with E_{n+1} symmetry



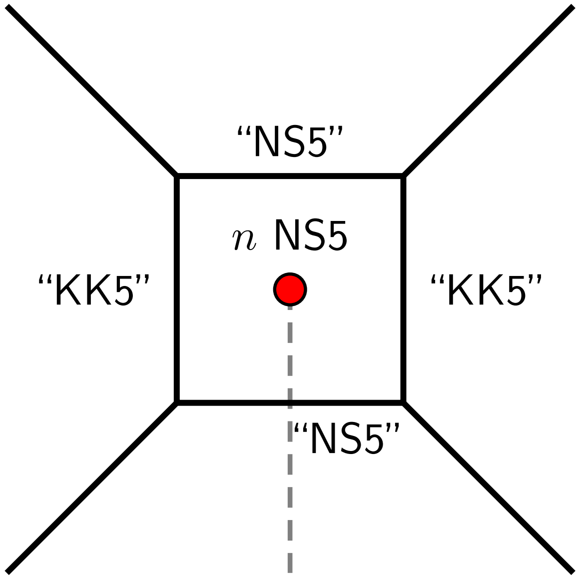
5D gauge coupling is given by Dirac-Born-Infeld action of 7_3-brane :

$$\frac{1}{g^2_{\rm YM}} \simeq \frac{V(T^2_{67})L}{g_s \, \ell_s^4} = \frac{L}{g_s} \cdot \frac{2\pi \widetilde{R}_6}{\ell_s^2} \cdot \frac{2\pi \widetilde{R}_7}{\ell_s^2}$$

Then, the strong coupling limit can be controlled as

$$\ell_s \rightarrow 0 \,, \qquad \frac{\widetilde{R}_i}{\ell_s^2} \rightarrow 0 \,, \qquad L \rightarrow 0$$

$\mathbf{ST}_{47}$-dualized system :

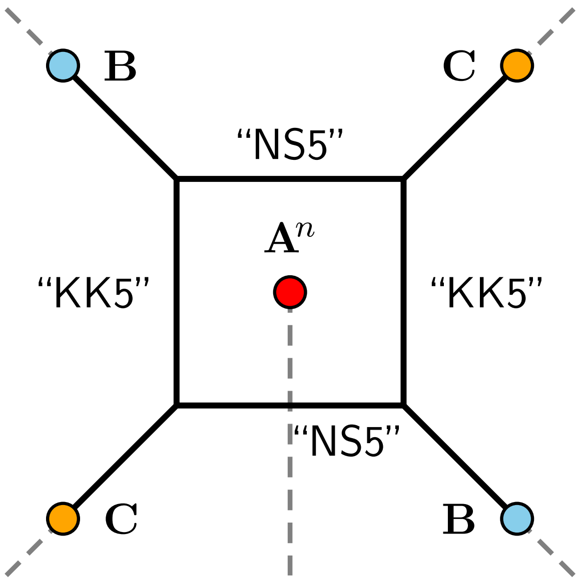


IIB		0	1	2	3	④	5	6	⑦	8	9
$\mathbf{A}^n$	n NS5	—	—	—	—		—	—			
	"NS5"	—	—	—	—				—	—	
	"KK5"	—	—	—	—	—			$\bullet^2$		—
$(r,s)_4$	r "NS5"	—	—	—	—				—	angle	
	s "KK5"	—	—	—	—	—			$\bullet^2$		

We can read off 4D $SU(2)$ gauge symmetry with n flavors on 0123-directions.

$$\beta(g_{\rm YM}) \ = \ -\frac{4-n}{16\pi^2}\, g_{\rm YM}^3 \qquad \to \qquad \begin{cases} 0 \leq n \leq 3 : & \text{IR fixed point} \\ n = 4 : & \text{conformal} \\ 5 \leq n \leq 8 : & \text{UV fixed point} \end{cases}$$

$\mathbf{ST}_{47}$-dualized system :



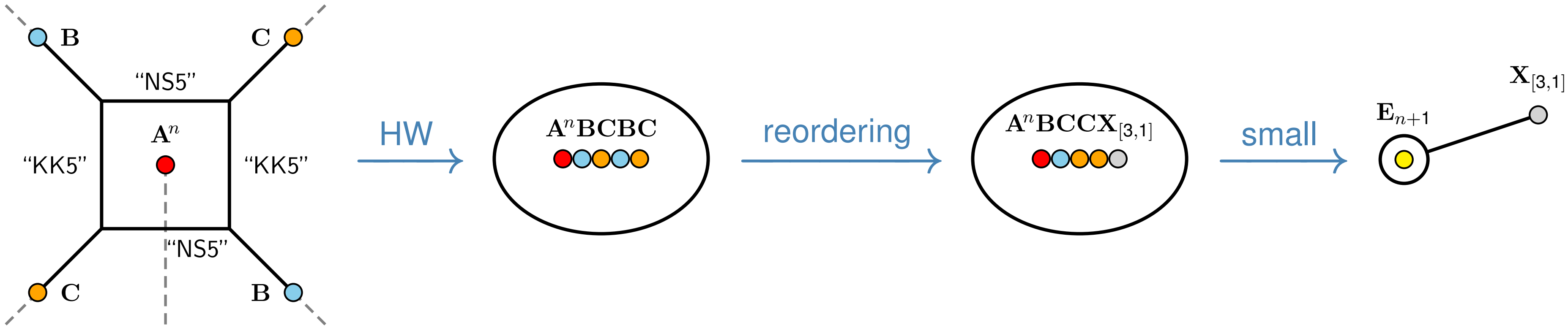
IIB		0	1	2	3	④	5	6	⑦	8	9
$\mathbf{A}^n$	n NS5	–	–	–	–		–	–			
	"NS5"	–	–	–	–				–	–	
	"KK5"	–	–	–	–	–			$\bullet^2$		–
$(r, s)_4$	r "NS5"	–	–	–	–				–	angle	
	s "KK5"	–	–	–	–	–			$\bullet^2$		

$\mathbf{A}$-brane : $[1, 0]^{T}_{s5}$-brane $=$ NS5

$\mathbf{B}$-brane : $[1, -1]^{T}_{s5}$-brane

$\mathbf{C}$-brane : $[1, 1]^{T}_{s5}$-brane

4D $\mathcal{N} = 2$ $SU(2)$ gauge with n flavors $\rightarrow$ SCFT with E_{n+1} symmetry



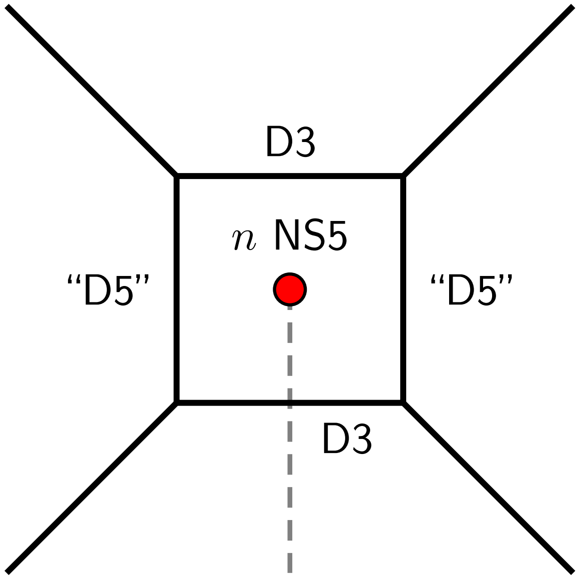
4D gauge coupling is derived from Dirac-Born-Infeld action of NS5-brane :

$$\frac{1}{g_{\text{YM}}^2} \simeq \frac{(2\pi \widetilde{R}_7)L}{\ell_s^2}$$

Then, the strong coupling limit is given by

$$\ell_s \to 0\,, \qquad \frac{\widetilde{R}_7}{\ell_s^2} \to 0\,, \qquad L \to 0$$

$\mathbf{ST}_{34}$-dualized system :

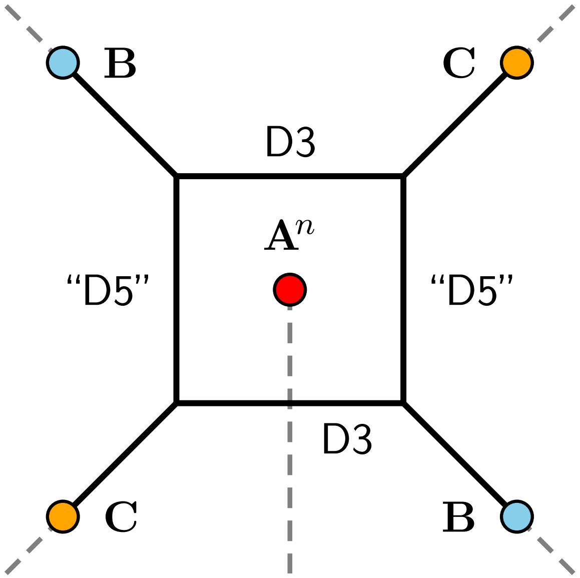


IIB		0	1	2	③	④	5	6	7	8	9
$\mathbf{A}^n$	n NS5	—	—	—			—	—	—		
	D3	—	—	—						—	
	"D5"	—	—	—	—	—					—
$(r,s)_3$	r D3	—	—	—						angle	
	s "D5"	—	—	—	—	—					

We can see 3D $SU(2)$ gauge symmetry with n flavors on 012-directions.

There exits IR fixed point.

$\mathbf{ST}_{34}$-dualized system :



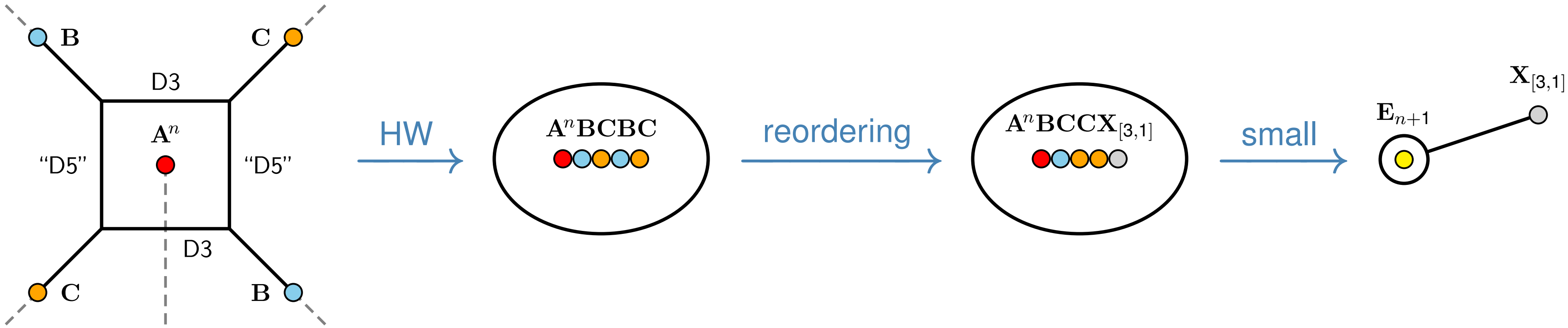
IIB		0	1	2	③	④	5	6	7	8	9
$\mathbf{A}^n$	n NS5	—	—	—			—	—	—		
	D3	—	—	—						—	
	"D5"	—	—	—	—	—					—
$(r, s)_3$	r D3	—	—	—						angle	
	s "D5"	—	—	—	—	—					

$\mathbf{A}$-brane : $[1, 0]^T_{s5}$-brane $=$ NS5

$\mathbf{B}$-brane : $[1, -1]^T_{s5}$-brane

$\mathbf{C}$-brane : $[1, 1]^T_{s5}$-brane

3D $\mathcal{N} = 4$ $SU(2)$ gauge with n flavors $\rightarrow$ SCFT with E_{n+1} symmetry



3D gauge coupling is given by Dirac-Born-Infeld action of D3-brane :

$$\frac{1}{g_{\mathrm{YM}}^2} \simeq \frac{L}{g_s}$$

Then the strong coupling limit is given by

$$L \rightarrow 0$$

Here we understood

- ✔ # of exotic 5-branes gives # of flavors in gauge theory.
- ✔ Alignment of exotic 5-branes in the loop gives global symmetry in the strong coupling limit.

We can construct SCFTs in various dimensions by exotic branes.

This is beyond F-theory result.

Summary

Eventually, we understood

✔ objects ending on 5^2_2-brane

= wrapped D3-brane

(Its oscillations provide excitation modes on the 5^2_2-brane worldvolume.)

✔ objects sensitive to 5^2_2-brane branch cut

= wrapped D3 (and 5^2_3, wrapped KK5, wrapped D5)

(They are created/annihilated by the Hanany-Witten transitions.)

We also found the brane constructions which give rise to

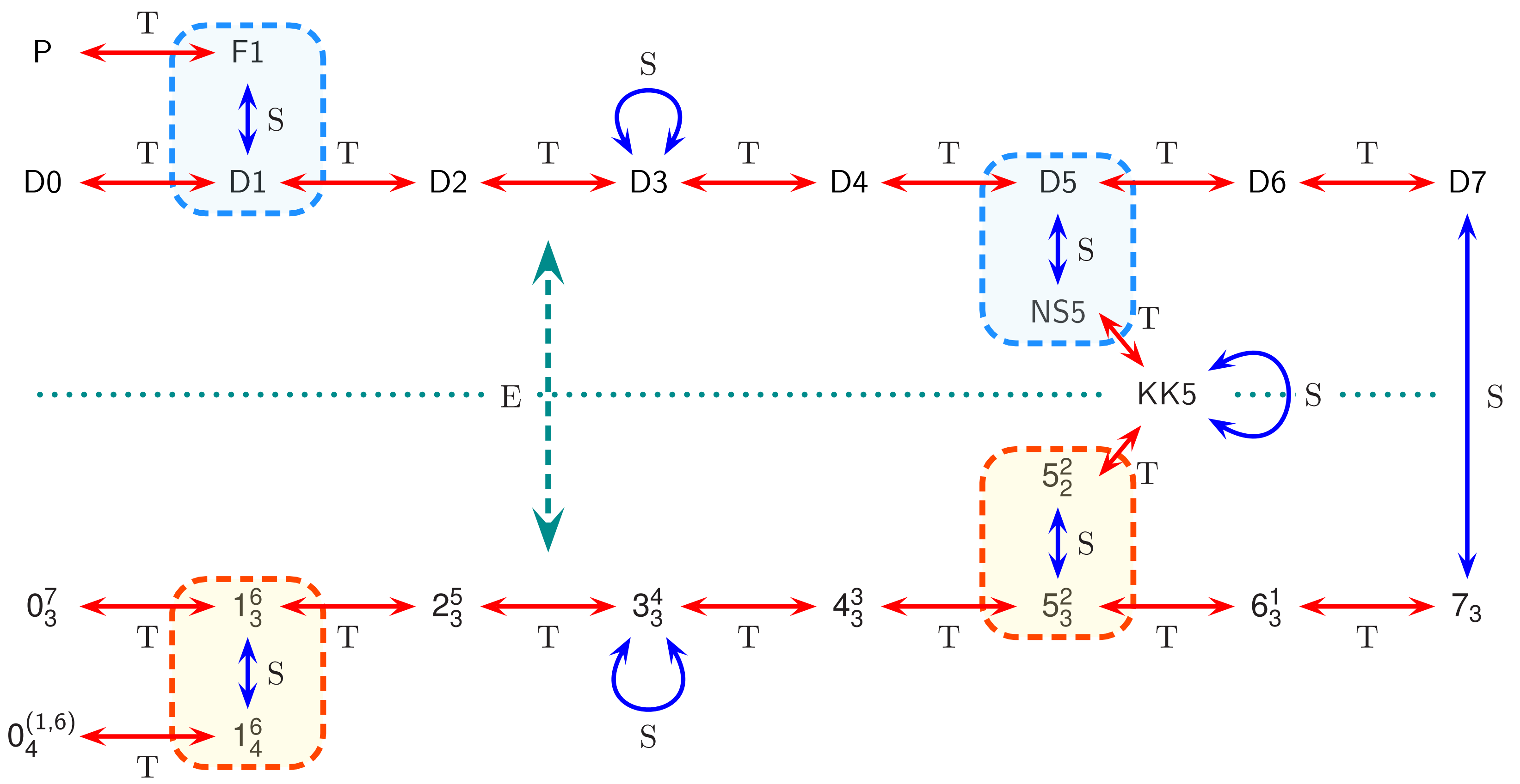
- ✔ SCFTs with E_{n+1} symmetry in 5D, 4D, 3D (qualitatively)

We can further construct various brane configurations involving exotic branes.

If you adapt the techniques of the 7-branes to the exotic branes,

you can construct F-theories in any dimensions.

Nongeometric $SL(2,\mathbb{Z})$ doublets : $(1^6_4, 1^6_3)$-string and $(5^2_3, 5^2_2)$-brane



These pairs do not couple to $[p, q]$ 7-brane in the conventional type IIB supergravity

Go beyond it $\rightarrow$ Exceptional Field Theories (EFTs)

Thanks

Appendix

Properties of exotic branes

Lozano-Tellechea and Ortín: hep-th/0012051

de Boer and Shigemori: arXiv:1209.6056

Sakatani: arXiv:1412.8769

TK: arXiv:1601.02175

- Solitonic five-branes in IIA theory :

$$\begin{aligned} \text{NS5(12345)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \\ \text{KK5(12345,9)} : \quad \pm\epsilon &= \Gamma^{012345}\Gamma\epsilon \\ 5^2_2\text{(12345,89)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \\ 5^3_2\text{(12345,789)} : \quad \pm\epsilon &= \Gamma^{012345}\Gamma\epsilon \\ 5^4_2\text{(12345,6789)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \end{aligned}$$

- Solitonic five-branes in IIB theory :

$$
\begin{aligned}
\text{NS5(12345)} : \quad \pm\epsilon &= \Gamma^{012345}(\sigma_3)\epsilon \\
\text{KK5(12345,9)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \\
5^2_2\text{(12345,89)} : \quad \pm\epsilon &= \Gamma^{012345}(\sigma_3)\epsilon \\
5^3_2\text{(12345,789)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \\
5^4_2\text{(12345,6789)} : \quad \pm\epsilon &= \Gamma^{012345}(\sigma_3)\epsilon
\end{aligned}
$$

- Defect branes in IIA theory :

$$
\begin{aligned}
6^1_3(123456,7): \quad \pm\epsilon &= \boldsymbol{\Gamma}^{0123456}(\boldsymbol{\sigma}_1)\epsilon \\
4^3_3(1234,567): \quad \pm\epsilon &= \boldsymbol{\Gamma}^{01234}\boldsymbol{\Gamma}(\boldsymbol{\sigma}_1)\epsilon \\
2^5_3(12,34567): \quad \pm\epsilon &= \boldsymbol{\Gamma}^{012}(\boldsymbol{\sigma}_1)\epsilon \\
0^7_3(,1234567): \quad \pm\epsilon &= \boldsymbol{\Gamma}^{0}\boldsymbol{\Gamma}(\boldsymbol{\sigma}_1)\epsilon \\
1^6_4(1,234567): \quad \pm\epsilon &= \boldsymbol{\Gamma}^{01}\boldsymbol{\Gamma}\epsilon \\
0^{(1,6)}_4(,234567,1): \quad \pm\epsilon &= \boldsymbol{\Gamma}^{01}\epsilon
\end{aligned}
$$

- Defect branes in IIB theory :

$$\begin{aligned}
7_3(1234567):\quad \pm\epsilon &= \Gamma^{01234567}(\mathbf{i}\boldsymbol{\sigma}_2)\epsilon \\
5^2_3(12345,67):\quad \pm\epsilon &= \Gamma^{012345}(\boldsymbol{\sigma}_1)\epsilon \\
3^4_3(123,4567):\quad \pm\epsilon &= \Gamma^{0123}(\mathbf{i}\boldsymbol{\sigma}_2)\epsilon \\
1^6_3(1,234567):\quad \pm\epsilon &= \Gamma^{01}(\boldsymbol{\sigma}_1)\epsilon \\
1^6_4(1,234567):\quad \pm\epsilon &= \Gamma^{01}(\boldsymbol{\sigma}_3)\epsilon \\
0^{(1,6)}_4(,234567,1):\quad \pm\epsilon &= \Gamma^{01}\epsilon
\end{aligned}$$

- NS5(12345) :

$$\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}x^2_{012345} + \rho_2\,\mathrm{d}x^2_{67} + \rho_2|f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \rho_2 \\
B_{(2)} &= \rho_1\,\mathrm{d}x^6 \wedge \mathrm{d}x^7\,, \qquad B_{(6)} = \frac{1}{\rho_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^5 \\
\rho &= \rho_1 + \mathrm{i}\rho_2 = B^{(2)}_{67} + \mathrm{i}\mathrm{e}^{2\phi} = B^{(2)}_{67} + \mathrm{i}\sqrt{\det G_{mn}} \\
\tau &= (\text{complex structure of } T^2_{67}) = \mathrm{i} \\
f &= 1\,, \qquad m,n = 6,7
\end{aligned}$$

- 5^2_2(12345,67) :

$$
\begin{aligned}
\mathrm{d}s'^2 &= \mathrm{d}x^2_{012345} + \rho'_2\, \mathrm{d}x^2_{67} + \rho'_2 |f'|^2\, \mathrm{d}z \mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= \rho'_2 \\
B'_{(2)} &= \rho'_1\, \mathrm{d}x^6 \wedge \mathrm{d}x^7 \,, \qquad B'_{(6)} = \frac{1}{\rho'_2} \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^5 \\
\rho' &= B'^{(2)}_{67} + \mathrm{i}\mathrm{e}^{2\phi'} = B'^{(2)}_{67} + \mathrm{i}\sqrt{\det G'_{mn}} = -\frac{1}{\rho_{\text{NS5}}} \\
\rho'_1 &= -\frac{\rho_1}{|\rho|^2} \,, \qquad \rho'_2 = \frac{\rho_2}{|\rho|^2} \\
\tau' &= (\text{complex structure of } \widetilde{T}^2_{67}) = \mathrm{i} = -\frac{1}{\tau_{\text{NS5}}} \\
\rho'_2 |f'|^2 &= \rho_2 |f|^2 \,, \qquad m,n = 6,7
\end{aligned}
$$

- KK5(12345,7) smeared along 6-th direction :

$$
\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}x^2_{012345} + \tau_2\, \mathrm{d}x_6^2 + \frac{1}{\tau_2}\big(\mathrm{d}x^7 - \tau_1\, \mathrm{d}x^6\big)^2 + \tau_2 |f|^2\, \mathrm{d}z \mathrm{d}\overline{z} \\
\mathrm{e}^{2\phi} &= 1 \,, \qquad B_{(2)} \;=\; 0 \,, \qquad B_{(6)} \;=\; 0 \\
\rho &= B_{67}^{(2)} + \mathrm{i}\sqrt{\det G_{mn}} \;=\; \mathrm{i} \\
\tau &= (\text{complex structure of } T^2_{67}) \;=\; \tau_1 + \mathrm{i}\tau_2 \\
f &= 1 \,, \qquad m, n = 6, 7
\end{aligned}
$$

- KK5(12345,6) smeared along 7-th direction :

$$
\begin{aligned}
ds'^2 &= dx^2_{012345} + \tau'_2\, dx^2_6 + \frac{1}{\tau'_2}\big(dx^7 - \tau'_1\, dx^6\big)^2 + \tau'_2 |f'|^2\, dz d\bar{z} \\
e^{2\phi'} &= 1\,, \qquad B'_{(2)} = 0\,, \qquad B'_{(6)} = 0 \\
\rho' &= B'^{(2)}_{67} + i\sqrt{\det G'_{mn}} = i = -\frac{1}{\rho_{\text{KK5}}} \\
\tau' &= (\text{complex structure of } \widetilde{T}^2_{67}) = \tau'_1 + i\tau'_2 = -\frac{1}{\tau_{\text{KK5}}} \\
\tau'_2 |f'|^2 &= \tau_2 |f|^2\,, \qquad m,n = 6,7
\end{aligned}
$$

- D$p(12 \cdots p)$:

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{(\rho_2)^{1/2}}\,\mathrm{d}x^2_{012\cdots p} + (\rho_2)^{1/2}\,\mathrm{d}x^2_{a_1\cdots a_{7-p}} + (\rho_2)^{1/2}|f|^2\,\mathrm{d}z\mathrm{d}\overline{z} \\
\mathrm{e}^{2\phi} &= (\rho_2)^{\frac{3-p}{2}} \\
C_{(7-p)} &= \rho_1\,\mathrm{d}x^{a_1}\wedge\cdots\wedge\mathrm{d}x^{a_{7-p}} \\
C_{(p+1)} &= -\frac{1}{\rho_2}\,\mathrm{d}x^0\wedge\mathrm{d}x^1\wedge\cdots\wedge\mathrm{d}x^p \\
\rho &= C^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{\frac{4}{3-p}\phi} \;=\; C^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{-\phi}(\det G_{mn})^{1/2} \\
f &= 1\,, \qquad m,n = a_1,\ldots,a_{7-p}
\end{aligned}$$

- $p_3^{7-p}(12 \cdots p, a_1 \cdots a_{7-p})$:

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{(\rho_2')^{1/2}}\, \mathrm{d}x^2_{012\cdots p} + (\rho_2')^{1/2}\, \mathrm{d}x^2_{a_1 \cdots a_{7-p}} + (\rho_2')^{1/2} |f'|^2\, \mathrm{d}z \mathrm{d}\overline{z} \\
\mathrm{e}^{2\phi'} &= (\rho_2')^{\frac{3-p}{2}} \\
C'_{(7-p)} &= \rho_1'\, \mathrm{d}x^{a_1} \wedge \cdots \wedge \mathrm{d}x^{a_{7-p}}\,, \qquad C'_{(p+1)} \;=\; -\frac{1}{\rho_2'}\, \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^p \\
\rho' &= C'^{(7-p)}_{a_1 \cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{\frac{4}{3-p}\phi'} \;=\; -\frac{1}{\rho_{\mathrm{D}p}} \\
\rho_1' &= -\frac{\rho_1}{|\rho|^2}\,, \qquad \rho_2' \;=\; \frac{\rho_2}{|\rho|^2} \\
\rho_2' |f'|^2 &= \rho_2 |f|^2\,, \qquad |f'| \;=\; |\rho||f|\,, \qquad m, n = a_1, \ldots, a_{7-p}
\end{aligned}
$$

- F1(1) :

$$\begin{aligned}
\mathbf{d}s^2 &= \frac{1}{\rho_2}\,\mathbf{d}x_{01}^2 + \mathbf{d}x_{234567}^2 + |f|^2\,\mathbf{d}z\mathbf{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \frac{1}{\rho_2} \\
B_{(6)} &= \rho_1\,\mathbf{d}x^2 \wedge \mathbf{d}x^3 \wedge \cdots \wedge \mathbf{d}x^7\,, \qquad B_{(2)} \;=\; -\frac{1}{\rho_2}\,\mathbf{d}x^0 \wedge \mathbf{d}x^1 \\
\rho &= B^{(6)}_{234567} + \mathrm{i}\mathrm{e}^{-2\phi}\,, \qquad f \;=\; 1
\end{aligned}$$

- $1^6_4(1,234567)$:

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{\rho'_2}\,\mathrm{d}x^2_{01} + \mathrm{d}x^2_{234567} + |f'|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= \frac{1}{\rho'_2} \\
B'_{(6)} &= \rho'_1\,\mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \cdots \wedge \mathrm{d}x^7\,, \qquad B'_{(2)} \;=\; -\frac{1}{\rho'_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \\
\rho' &= B'^{(6)}_{234567} + \mathrm{i}\mathrm{e}^{-2\phi'} \;=\; -\frac{1}{\rho_{\mathrm{F1}}} \\
\rho'_2|f'|^2 &= \rho_2|f|^2\,, \qquad |f'| \;=\; |\rho||f|
\end{aligned}
$$

- P(1) :

$$\begin{aligned} \mathrm{d}s^2 &= -2\mathrm{d}x^0\mathrm{d}x^1 + \rho_2\,\mathrm{d}x_1^2 + \mathrm{d}x_{234567}^2 + |f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\ \mathrm{e}^{2\phi} &= \frac{1}{\rho_2}\,, \quad B_{(2)} \;=\; 0\,, \quad B_{(6)} \;=\; 0 \\ \rho &= \mathrm{i}\mathrm{e}^{-2\phi}\,, \quad f \;=\; 1 \end{aligned}$$

- $0_4^{(1,6)}$(,234567,1) :

$$\begin{aligned} \mathrm{d}s^2 &= -2\mathrm{d}x^0\mathrm{d}x^1 + \rho'_2\,\mathrm{d}x_1^2 + \mathrm{d}x_{234567}^2 + |f'|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\ \mathrm{e}^{2\phi'} &= \frac{1}{\rho'_2} = \frac{|\rho|^2}{\rho_2}\,, \quad B'_{(2)} \;=\; 0\,, \quad B'_{(6)} \;=\; 0 \\ \rho' &= \mathrm{i}\mathrm{e}^{-2\phi'} = -\frac{1}{\rho_{\mathrm{P}}}\,, \quad \rho'_2|f'|^2 \;=\; \rho_2|f|^2\,, \quad |f'| \;=\; |\rho||f| \end{aligned}$$

F-theory

Vafa, “Evidence for F-theory” : hep-th/9602022

Zwiebach et al: hep-th/9709013, 9801205, 9804210, 9812028, 9812209, etc.

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

IIB action in Einstein frame :

$$S_{\rm IIB} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G_{\rm E}}\,\Big\{R_{\rm E} - \frac{\partial_M\bar{\rho}\,\partial^M\rho}{2(\rho_2)^2} - \frac{1}{2}F^i_{(3)}\cdot\mathcal{M}_{ij}\,F^j_{(3)} - \frac{1}{4}|\widetilde{F}_{(5)}|^2\Big\}$$

$$-\frac{\epsilon_{ij}}{8\kappa^2}\int C_{(4)}\wedge F^i_{(3)}\wedge F^j_{(3)}$$

$$\rho \equiv C + i e^{-\phi} \equiv \rho_1 + i\rho_2\,, \qquad \mathcal{M}_{ij} \equiv \frac{1}{\rho_2}\begin{pmatrix} 1 & -\rho_1 \\ -\rho_1 & |\rho|^2 \end{pmatrix}$$

$$F^i_{(3)} \equiv \begin{pmatrix} dC_{(2)} \\ dB_{(2)} \end{pmatrix}, \qquad \widetilde{F}_{(5)} \equiv dC_{(4)} - \frac{1}{2}C_{(2)}\wedge H_{(3)} + \frac{1}{2}B_{(2)}\wedge F_{(3)}$$

$SL(2,\mathbb{Z})$ S-duality

$$\rho \to \frac{a\rho + b}{c\rho + d}, \qquad \Lambda^i{}_j = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

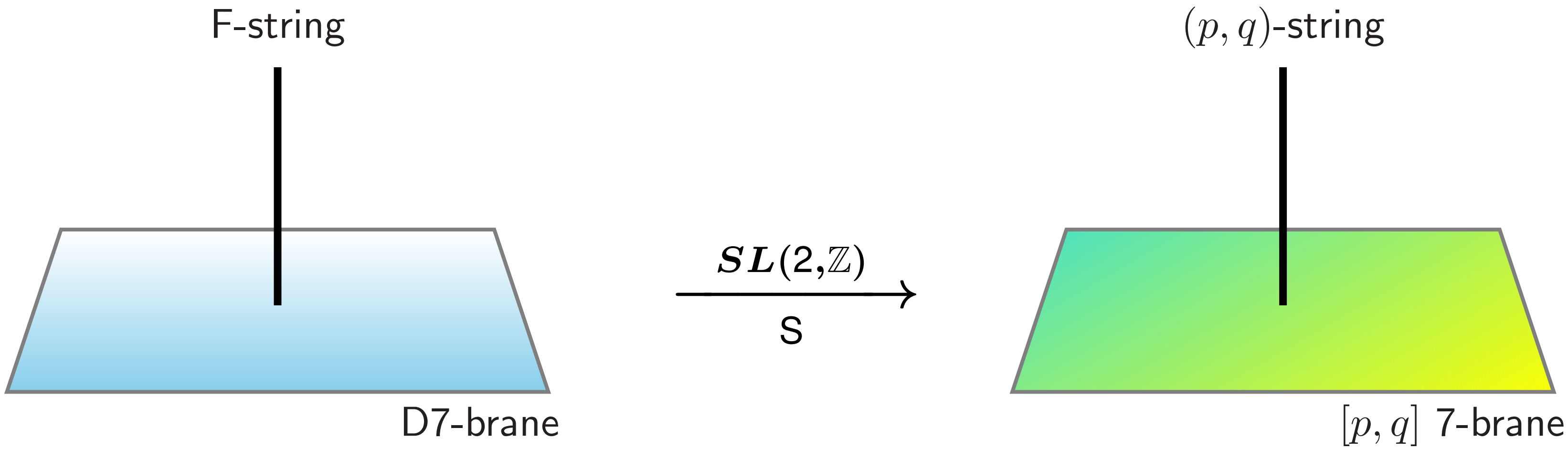
$$F^i_{(3)} \to \Lambda^i{}_j\,F^j_{(3)}\,, \qquad \widetilde{F}_{(5)} \to \widetilde{F}_{(5)}\,, \qquad G^{\rm E}_{MN} \to G^{\rm E}_{MN}$$

$$\mathcal{M} \to \Lambda^{-{\rm T}}\mathcal{M}\Lambda^{-1}$$

F-string : couple to $B_{(2)}$

D-string : couple to $C_{(2)}$

D7-brane : (magnetically) couple to $\rho = C + \mathrm{i}e^{-\phi}$



$(1, 0)$-string $=$ F-string $[1, 0]$ 7-brane $=$ D7-brane

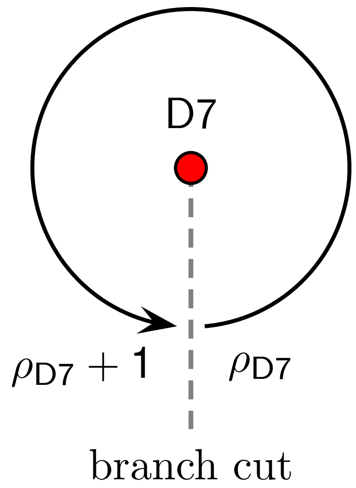
$(0, 1)$-string $=$ D-string $[0, 1]$ 7-brane $=$ NS7-brane

D7-brane solution :

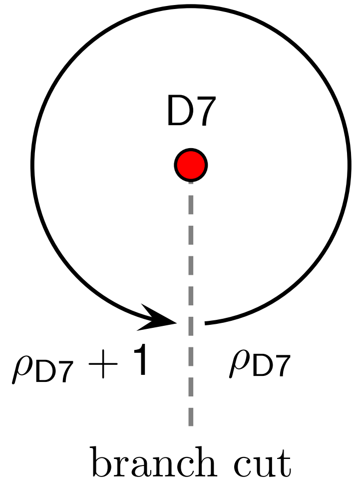
$$\rho(z) \;=\; \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{z}\right) \;=\; \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\left(\frac{\Lambda}{r}\right) \qquad (z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta})$$

When ρ moves around D7-brane counterclockwise $\theta \to \theta + 2\pi$,

it receives a magnetic "charge" of D7-brane (monodromy) : $\rho \to \rho + 1$



There exists a branch cut in z-plane.



$$\rho \;\to\; \rho + 1 \;=\; \frac{a\rho + b}{c\rho + d} \;\equiv\; M_{[1,0]} \cdot \rho \,, \qquad M_{[1,0]} \;=\; \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \;\in\; SL(2, \mathbb{Z})$$

or

$$K_{[1,0]} \cdot (\rho + 1) \;=\; \rho \,, \qquad K_{[1,0]} \;=\; (M_{[1,0]})^{-1} \;=\; \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

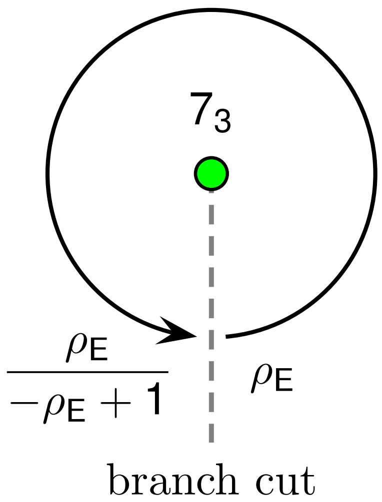
$M_{[p,q]}$: moving around the 7-brane

$K_{[p,q]}$: going across the branch cut

By $SL(2,\mathbb{Z})$, the monodromy matrix for general $[p,q]$ 7-brane is given as

$$K_{[p,q]} = g\,K_{[1,0]}\,g^{-1} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix} \qquad g \in SL(2,\mathbb{Z})$$

ex) monodromy $K_{[0,1]}$ for 7_3-brane : $\quad K_{[0,1]} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

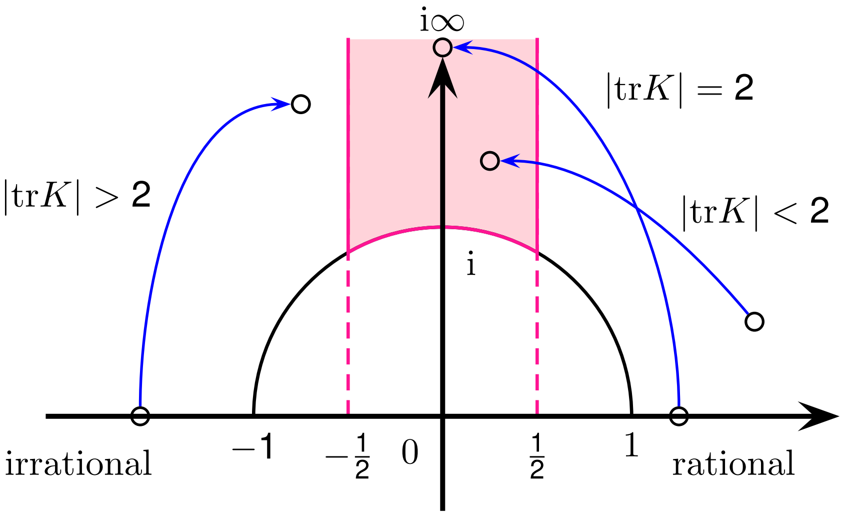


$$\rho_E = -\frac{1}{\rho_{D7}}$$

$|\mathrm{tr}\boldsymbol{K}|$ is a good character to classify 7-branes :

$$\boldsymbol{K} \cdot \boldsymbol{\rho}_* = \frac{a\rho_* + b}{c\rho_* + d} = \rho_* \, , \qquad \boldsymbol{K} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \boldsymbol{SL}(2, \mathbb{Z})$$

$$\therefore \quad \rho_* = \frac{1}{2c}\Big\{(a-d) \pm \sqrt{(\mathrm{tr}\boldsymbol{K})^2 - 4}\Big\}$$



$|\mathrm{tr}\boldsymbol{K}| = 2$: parabolic (collapsible)

$|\mathrm{tr}\boldsymbol{K}| < 2$: elliptic (collapsible)

$|\mathrm{tr}\boldsymbol{K}| > 2$: hyperbolic (non-collapsible)

$\mathrm{tr}K$	monodromy	branes	collapsible?	symmetry	
$+2$	$T^{-n} = K_A^n$	A^n	yes	$A_{n-1} = SU(n)$	$n \geq 1$
	$\mathbb{1} = T^0 = K_C K_B K_C K_B K_A^8$	$\widehat{\mathrm{E}}_9 \equiv \mathrm{A}^8\mathrm{BCBC}$	yes	$\widehat{E}_9$	$n = 0$
	$T^{\lvert n \rvert} = K_C K_B K_C K_B K_A^{8-\lvert n \rvert}$	$\mathrm{A}^{8-\lvert n \rvert}\mathrm{BCBC}$	no	$\widehat{E}_{9-\lvert n \rvert}$	$n \leq -1$
$+1$	$ST \sim K_C K_A$	$\mathrm{H}_0 \equiv \mathrm{AC}$	yes	H_0	
	$(ST)^{-1} \sim K_C^2 K_B K_A^7$	$\mathrm{E}_8 \equiv \mathrm{A}^7\mathrm{BC}^2$	yes	E_8	
0	$S \sim K_C K_A^2$	$\mathrm{H}_1 \equiv \mathrm{A}^2\mathrm{C}$	yes	$H_1 = SU(2)$	“$\sim$” up to
	$-S \sim K_C^2 K_B K_A^6$	$\mathrm{E}_7 \equiv \mathrm{A}^6\mathrm{BC}^2$	yes	E_7	G tf.
-1	$-(ST)^{-1} \sim K_C K_A^3$	$\mathrm{H}_2 \equiv \mathrm{A}^3\mathrm{C}$	yes	$H_2 = SU(3)$	
	$-ST \sim K_C^2 K_B K_A^5$	$\mathrm{E}_6 \equiv \mathrm{A}^5\mathrm{BC}^2$	yes	E_6	
-2	$-T^{-n} = K_C K_B K_A^{n+4}$	$\mathrm{D}_{n+4} \equiv \mathrm{A}^{n+4}\mathrm{BC}$	yes	$D_{n+4} = SO(2n+8)$	$n \geq 1$
	$-\mathbb{1} = -T^0 = K_C K_B K_A^4$	$\mathrm{D}_4 \equiv \mathrm{A}^4\mathrm{BC}$	yes	$D_4 = SO(8)$	$n = 0$
	$-T = K_C K_B K_A^3$	$\mathrm{A}^3\mathrm{BC}$	no	$D_3 = SO(6) \simeq SU(4)$	$n = -1$
	$-T^2 = K_C K_B K_A^2$	$\mathrm{A}^2\mathrm{BC}$	no	$D_2 = SO(4) \simeq SU(2) \times SU(2)$	$n = -2$
	$-T^3 = K_C K_B K_A$	ABC	no	$D_1 = SO(2) \simeq U(1)$	$n = -3$
	$-T^4 = K_C K_B$	BC	no	$-$	$n = -4$

$[p, q]$-brane is expressed by $\mathrm{z} = (p, q)^{\mathrm{T}}$ vector and $\mathbf{X}_{\mathrm{z}}$.

The monodromy matrix K_{z} is also given as

$$K_{\mathrm{z}} = \begin{pmatrix} 1 + pq & -p^2 \\ q^2 & 1 - pq \end{pmatrix} = \mathbb{1} + \mathrm{z}\mathrm{z}^{\mathrm{T}} S \,, \qquad S \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Re-ordering the branes $\mathbf{X}_{\mathrm{z}_1}\mathbf{X}_{\mathrm{z}_2}$:

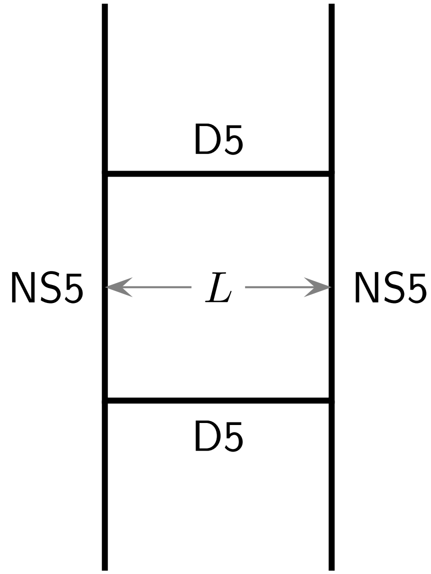
$$\mathbf{X}_{\mathrm{z}_1}\mathbf{X}_{\mathrm{z}_2} = \mathbf{X}_{(\mathrm{z}_2 + (\mathrm{z}_1 \times \mathrm{z}_2)\mathrm{z}_1)}\mathbf{X}_{\mathrm{z}_1} = \mathbf{X}_{\mathrm{z}_2}\mathbf{X}_{(\mathrm{z}_1 + (\mathrm{z}_1 \times \mathrm{z}_2)\mathrm{z}_2)}$$

$$\mathrm{z}_1 \times \mathrm{z}_2 \equiv -\mathrm{z}_1^{\mathrm{T}} S \, \mathrm{z}_2 = \mathrm{z}_2 \, S \, \mathrm{z}_1 = \det \begin{pmatrix} p_1 & p_2 \\ q_1 & q_2 \end{pmatrix}$$

$$K_{\mathrm{z}_2}K_{\mathrm{z}_1} = K_{(\mathrm{z}_1 + (\mathrm{z}_1 \times \mathrm{z}_2)\mathrm{z}_2)}K_{\mathrm{z}_2} = K_{\mathrm{z}_1}K_{(\mathrm{z}_2 + (\mathrm{z}_1 \times \mathrm{z}_2)\mathrm{z}_1)}$$

Application

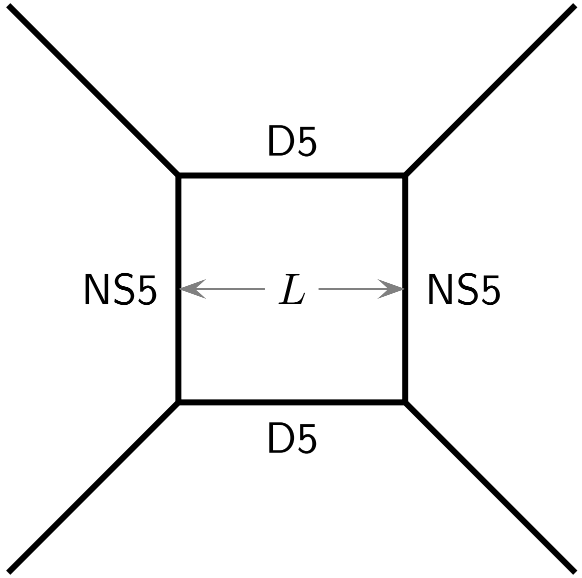
SUSY gauge theory by brane construction :



- N_c D5 between 2 NS5 $= SU(N_c)$ gauge symmetry
- N_f D5 outside 2 NS5 $= N_f$ flavors

Application

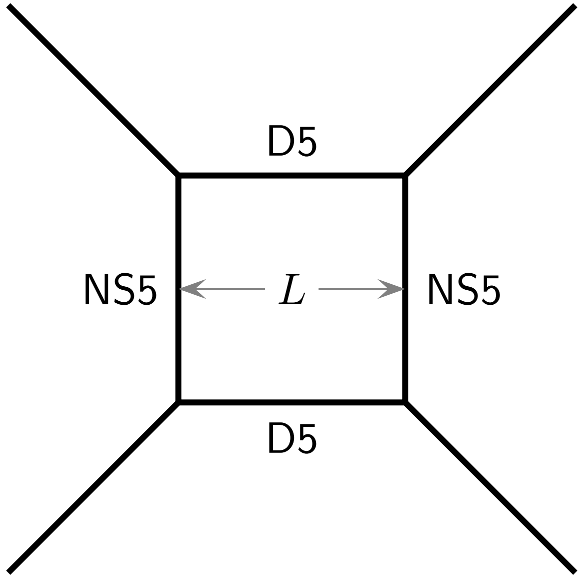
SUSY gauge theory by brane construction :



- ✔ N_c D5 between 2 NS5 $=$ $SU(N_c)$ gauge symmetry
- ✔ N_f D5 outside 2 NS5 $=$ N_f flavors

no D7-branes :

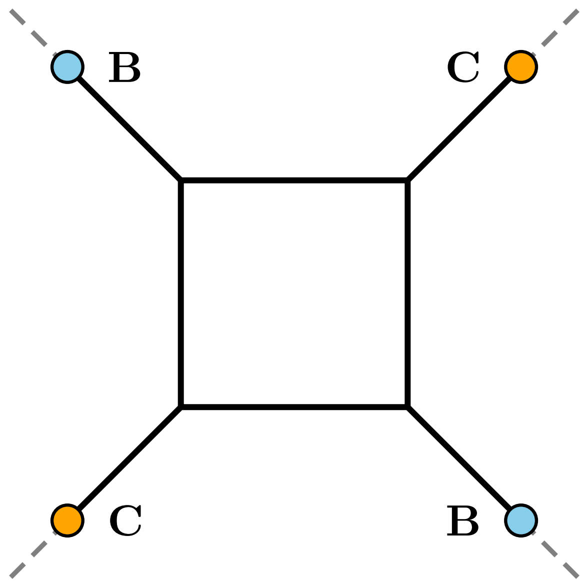
5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane



$$\frac{1}{g_{\text{YM}}^2} \simeq \frac{L}{g_s \, \ell_s^2}$$

no D7-branes :

5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane

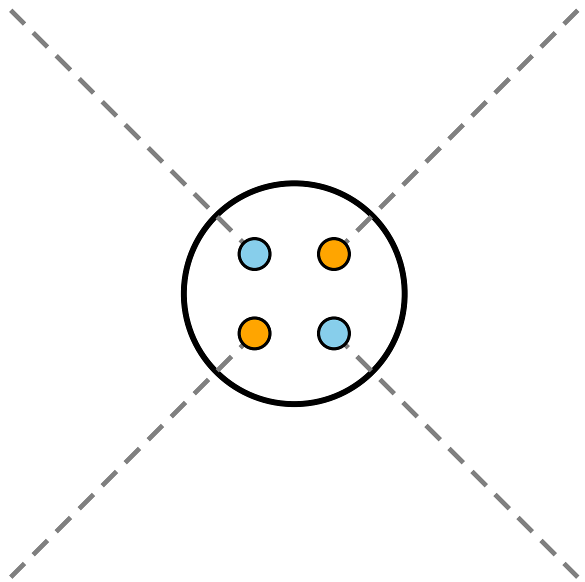


$\mathbf{B}$-brane : $[1, -1]$ 7-brane

$\mathbf{C}$-brane : $[1, 1]$ 7-brane

no D7-branes :

5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane

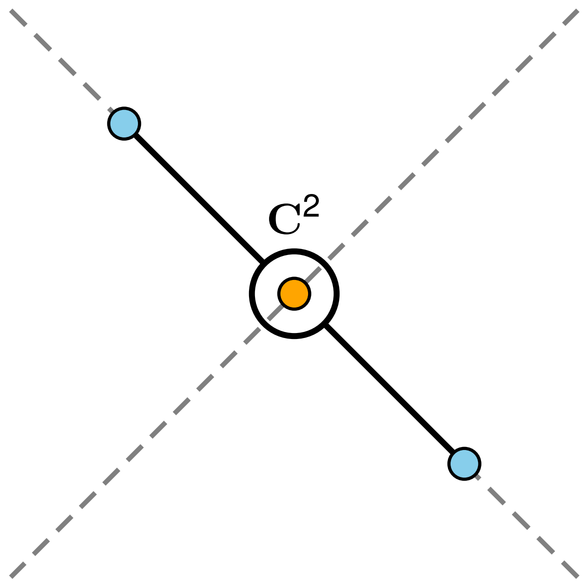


Hanany-Witten transitions

"Box" becomes "loop" by back reactions from $\mathbf{B}$- and $\mathbf{C}$-branes

no D7-branes :

5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane

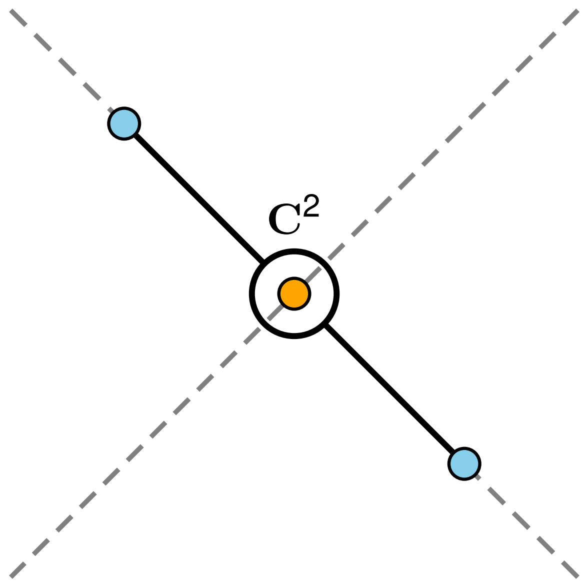


strong gauge coupling limit on 5-branes $=$ small loop limit

$$\ell_s \to 0\,, \quad L \to 0\,, \quad \frac{L}{\ell_s^2} \to 0$$

no D7-branes :

5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane

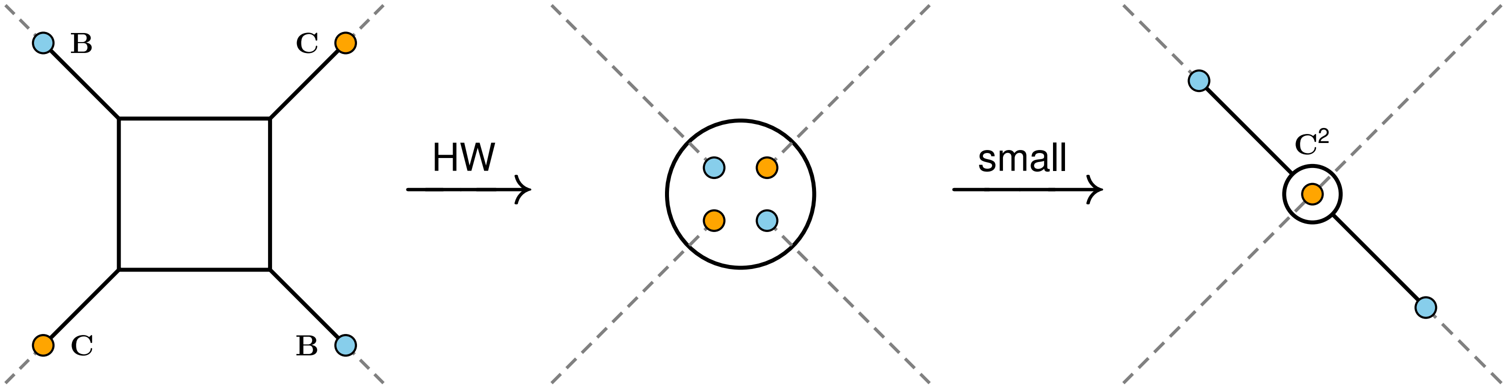


open string ending on 2 $\mathbf{C}$-branes $\rightarrow$ $SU(2)$ symmetry

open string ending on $\mathbf{C}^2$-branes and "loop" 5-branes $\rightarrow$ flavor symmetry

no D7-branes :

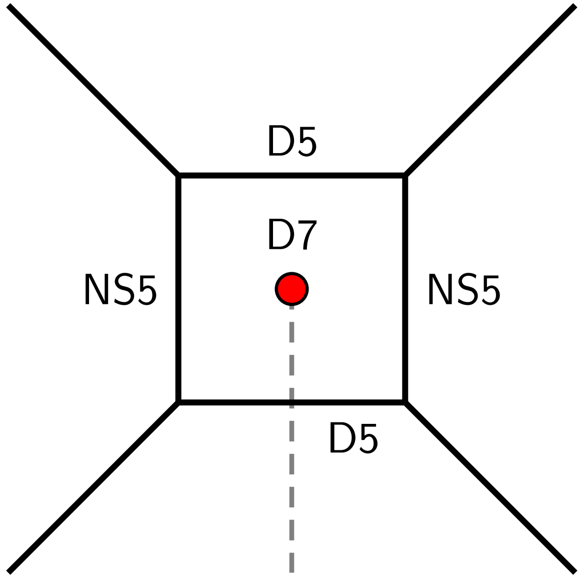
5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane



$\rightarrow$ SCFT with $E_1 \simeq SU(2)$ global symmetry emerges!

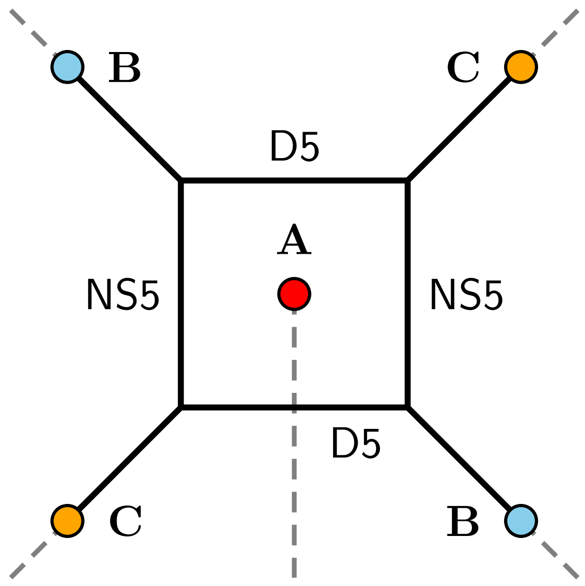
1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with 1 flavor on D5-brane



1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with 1 flavor on D5-brane



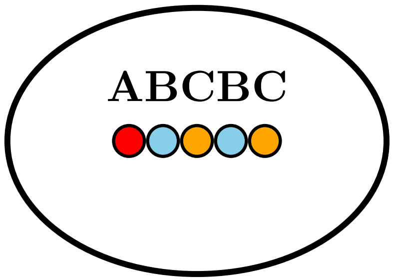
$\mathbf{A}$-brane : $[1, 0]$ 7-brane = D7-brane

$\mathbf{B}$-brane : $[1, -1]$ 7-brane

$\mathbf{C}$-brane : $[1, 1]$ 7-brane

1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with 1 flavor on D5-brane

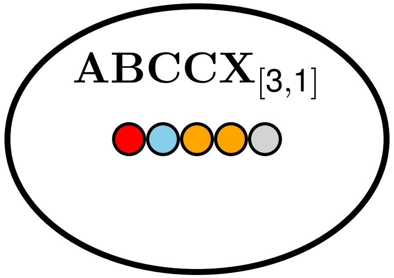


Hanany-Witten transitions

"Box" becomes "loop" by back reaction of $\mathbf{ABCBC}$-branes

1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with 1 flavor on D5-brane

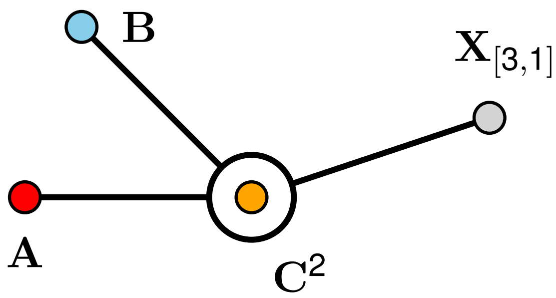


re-ordering of 7-branes : $\mathbf{ABCBC} = \mathbf{ABCCX}_{[3,1]}$

$\mathbf{X}_{[3,1]}$-brane $\equiv$ $[3, 1]$-brane

1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory with 1 flavor on D5-brane



small loop limit

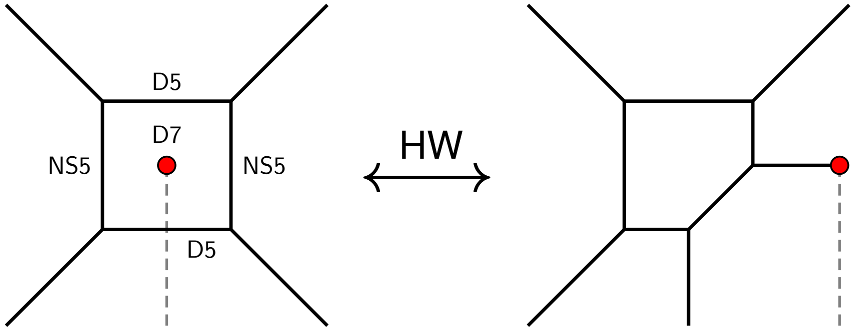
open string ending on 2 $\mathbf{C}$-branes $\to$ $SU(2)$ symmetry

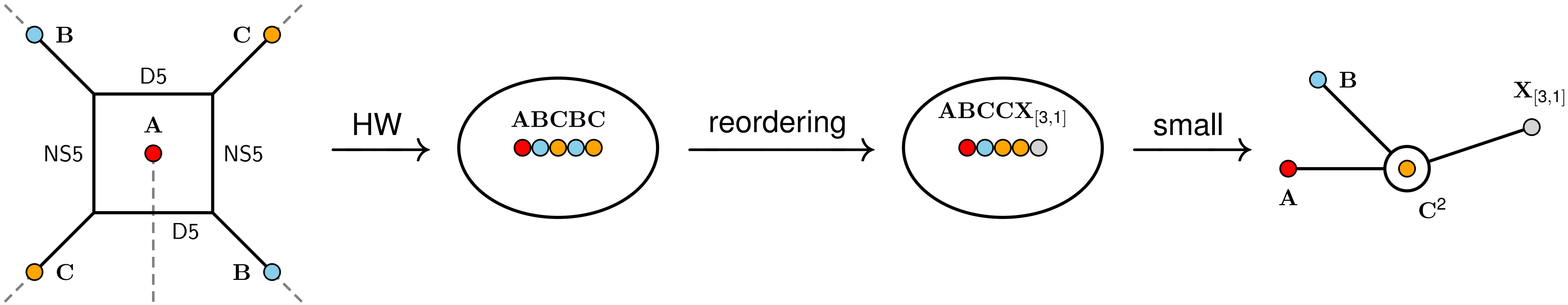
open string ending on $\mathbf{C}^2$-branes and "loop" 5-branes $\to$ flavor symmetry

$$E_2 = SU(2) \times U(1)$$

1 D7-brane :

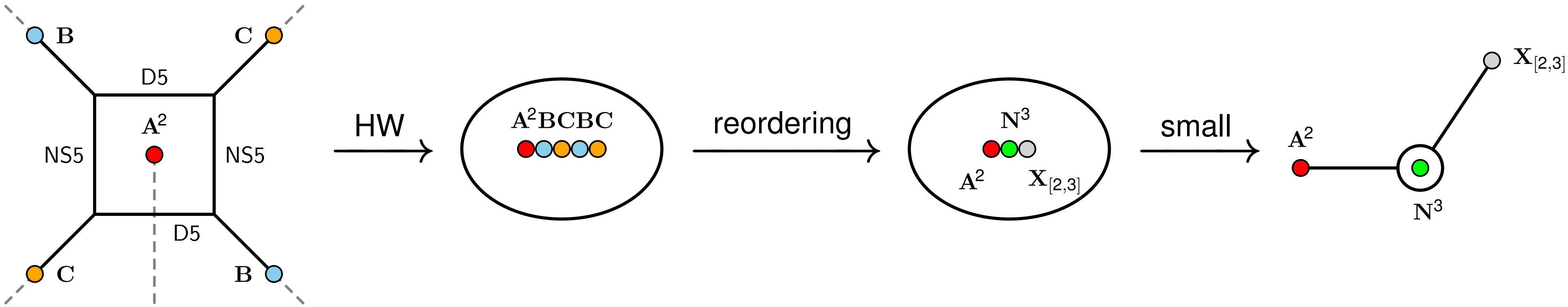
$SU(2)$ gauge with 1 flavor $\rightarrow$ SCFT with $E_2 \supset SU(2)$ symmetry





2 D7-branes :

$SU(2)$ gauge with 2 flavors $\to$ SCFT with $E_3 = SU(3) \times SU(2)$ symmetry



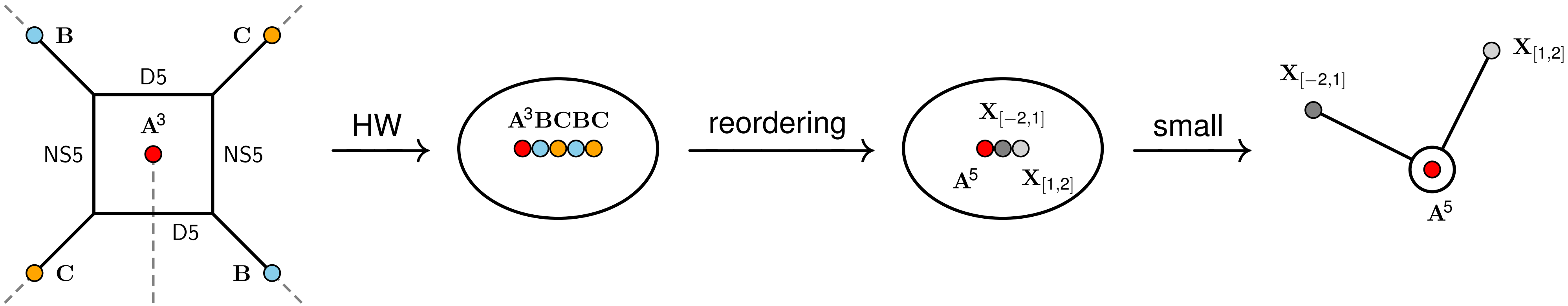
N-brane $\equiv$ [0, 1]-brane

$$\mathbf{A}^2\mathbf{BCBC} \sim \mathbf{A}^2\mathbf{N}^3\mathbf{X}_{[2,3]} \quad \to \quad \begin{cases} \mathbf{A}^2\text{-brane}: & SU(2) \\ \mathbf{N}^3\text{-brane}: & SU(3) \end{cases}$$

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

3 D7-branes :

$SU(2)$ gauge with 3 flavors $\rightarrow$ SCFT with $E_4 = SU(5)$ symmetry

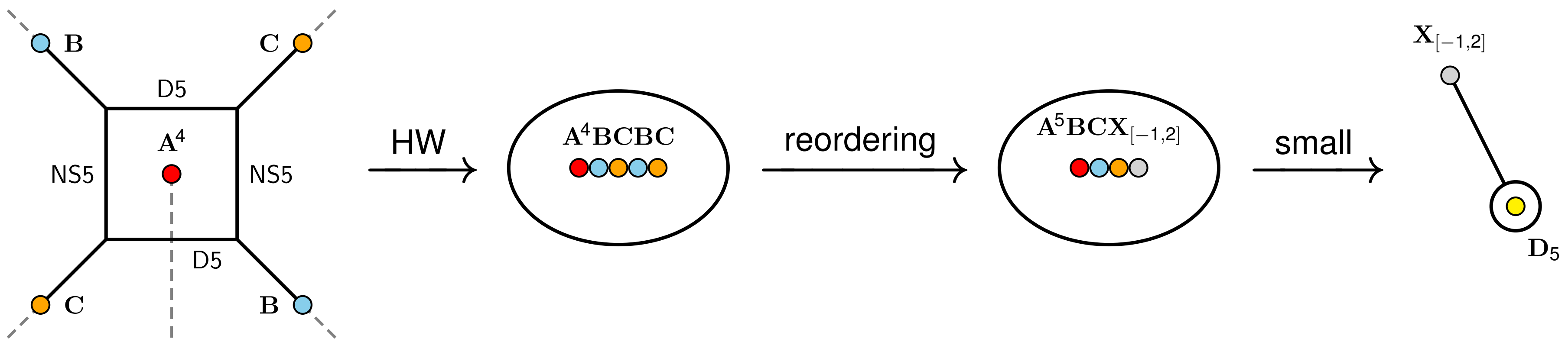


$$\mathbf{A}^3\mathbf{BCBC} \sim \mathbf{A}^5\mathbf{X}_{[-2,1]}\mathbf{X}_{[1,2]} \quad \rightarrow \quad \mathbf{A}^5\text{-brane :} \quad SU(5)$$

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

4 D7-branes :

$SU(2)$ gauge with 4 flavors $\rightarrow$ SCFT with $E_5 = SO(10)$ symmetry

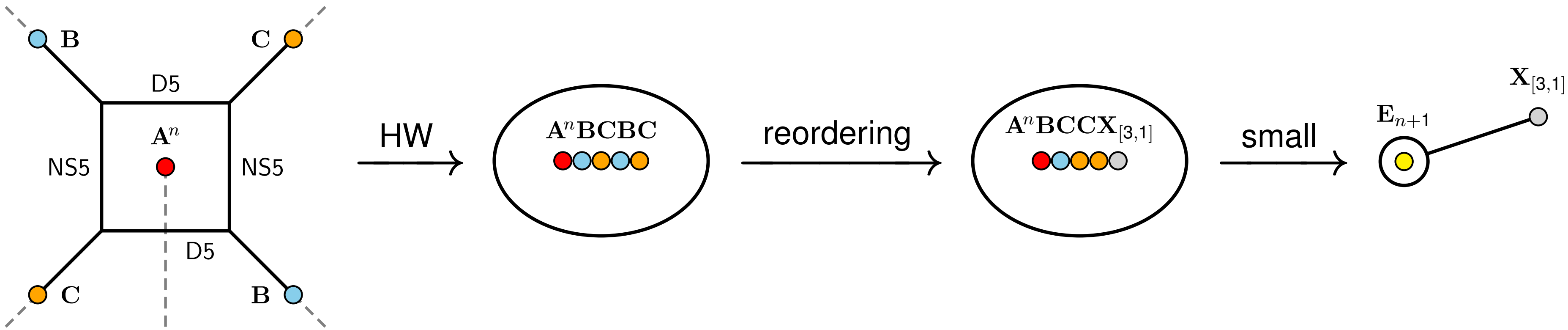


$$\mathbf{A}^4\mathbf{BCBC} \sim \mathbf{A}^5\mathbf{BCX}_{[-1,2]} \quad \rightarrow \quad \mathbf{A}^5\mathbf{BC} = \mathbf{D}_5 : \quad SO(10)$$

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

$n = 5, 6, 7$ D7-branes :

$SU(2)$ gauge with $n = 5, 6, 7$ flavors $\rightarrow$ SCFT with E_{n+1} symmetry



After re-ordering $\mathbf{A}^n\mathbf{BCBC} \ = \ \mathbf{A}^n\mathbf{BCCX}_{[3,1]}$,

$\mathbf{A}^n\mathbf{BCC}$ are collapsible at one point! $\rightarrow$ $\mathbf{E}_{n+1}$-brane

EFTs in diverse dimensions :

D	U-duality group	arXiv
9	$E_{2(2)}(\mathbb{Z}) = SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	1512.06115
8	$E_{3(3)}(\mathbb{Z}) = SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$	1501.01600
7	$E_{4(4)}(\mathbb{Z}) = SL(5,\mathbb{Z})$	1302.1652 1412.0635 1512.02163
6	$E_{5(5)}(\mathbb{Z}) = SO(5,5;\mathbb{Z})$	1504.01523

D	U-duality group	arXiv
5	$E_{6(6)}(\mathbb{Z})$	1312.0614 1412.7286
4	$E_{7(7)}(\mathbb{Z})$	1312.4542
3	$E_{8(8)}(\mathbb{Z})$	1406.3348

$SL(n+1;\mathbb{Z}) \subseteq E_{n(n)}(\mathbb{Z})$ in $(11-n)$-dim : 1402.5027