4th String Theory in Greater Tokyo @ University of Tokyo, Hongo Campus April 12, 2016

Exotic Brane Junctions from F-theory

arXiv:1602.08606

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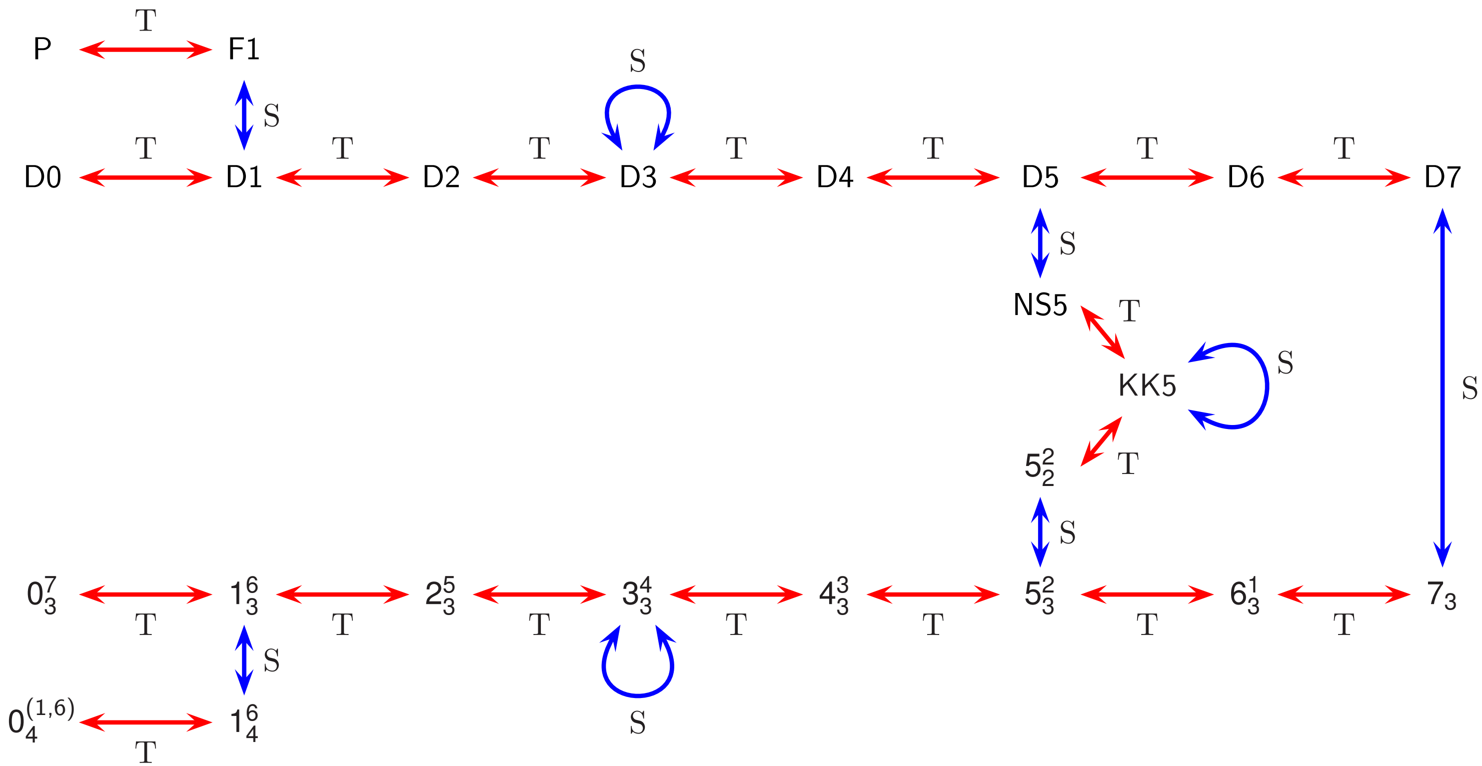
F-theory

- ✔ $SL(2,\mathbb{Z})$ S-duality monodromy
- ✔ $[p,q]$ 7-brane on which (p,q)-string is ending
- ✔ branch cut, string junctions, and non-trivial (gauge) groups

$[1,0]$ 7-brane	=	D7-brane
$[0,1]$ 7-brane	=	"NS7-brane" or exotic 7_3-brane

Exotic branes

- ✔ from standard branes via string dualities in lower dim
- ✔ codim $= \mathbf{2}$ $\rightarrow$ monodromy by string dualities



Contents

- Exotic brane junctions
- SCFTs with E_{n+1} symmetry in 5D, 4D, 3D
- Summary

Exotic brane junctions

Lozano-Tellechea and Ortín: hep-th/0012051

Bergshoeff, Ortín and Riccioni: arXiv:1109.4484

de Boer and Shigemori: arXiv:1209.6056

Sakatani: arXiv:1412.8769

TK: arXiv:1410.8403, 1601.02175, 1602.08606

Nomenclature $b_n^{(d,c)}$:

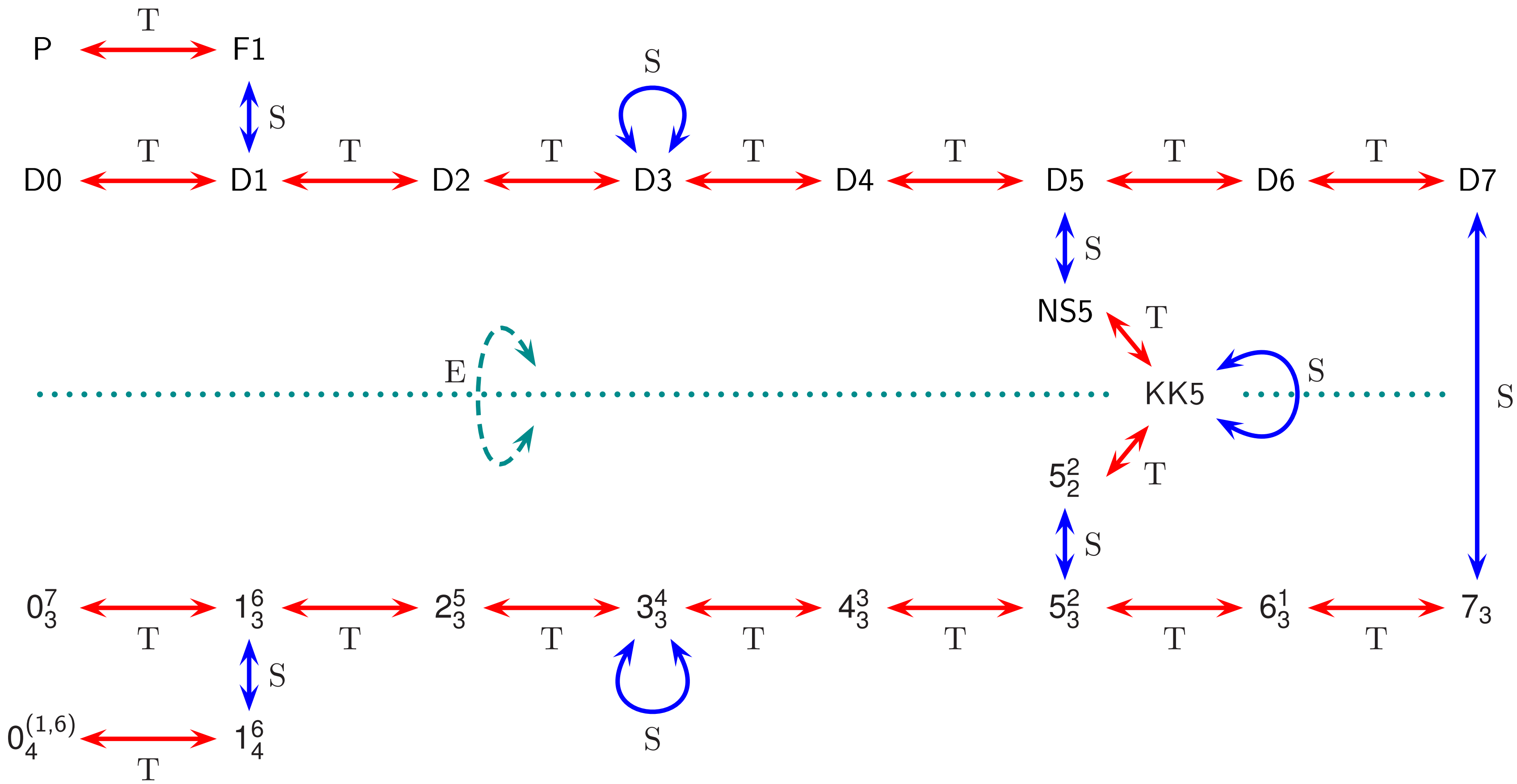
$$\text{mass} = \frac{R_1 R_2 \cdots R_b (R_{b+1} \cdots R_{b+c})^2 (R_{b+c+1} \cdots R_{b+c+d})^3}{g_s^n \, \ell_s^{b+2c+3d+1}}$$

String dualities

$$\mathrm{T}_y \; : \; R_y \to \frac{\ell_s^2}{R_y}, \quad g_s \to \frac{\ell_s}{R_y} g_s$$

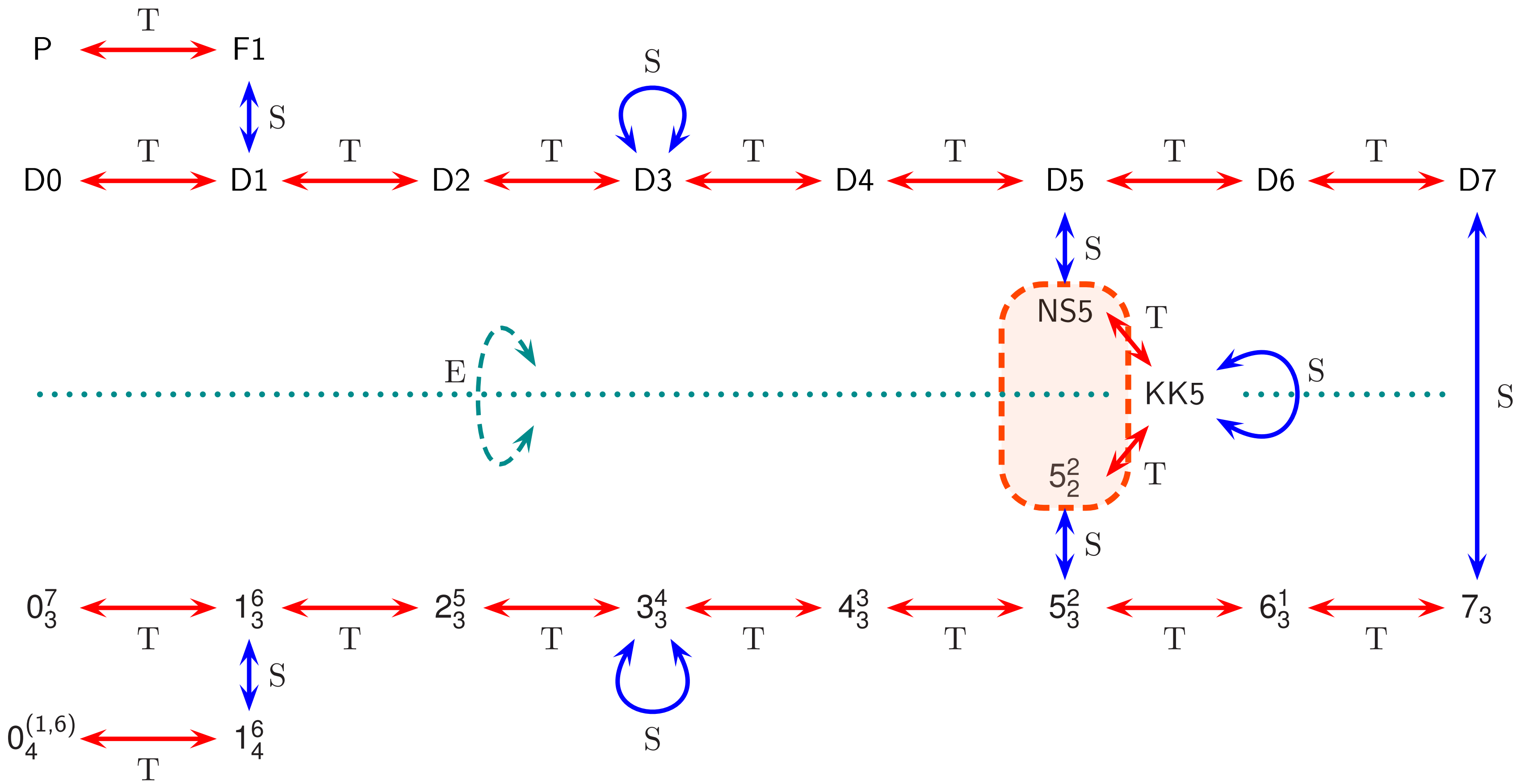
$$\mathrm{S} \; : \; g_s \to \frac{1}{g_s}, \quad \ell_s \to g_s^{1/2} \ell_s$$

$$\underset{5_1}{\text{D5(12345)}} \xrightarrow{\ S\ } \underset{5_2}{\text{NS5(12345)}} \xrightarrow{\ T_9\ } \underset{5_2^1}{\text{KK5(12345,9)}} \xrightarrow{\ T_8\ } 5_2^2\text{(12345,89)} \xrightarrow{\ S\ } 5_3^2\text{(12345,89)}$$



The $SL(2,\mathbb{Z})$ pairs (Dp, p^{7-p}_3), (NS5, 5^2_2), (F1, 1^6_4), (P, $0^{(1,6)}_4$)

yield the same physics as that of (D7, 7_3).



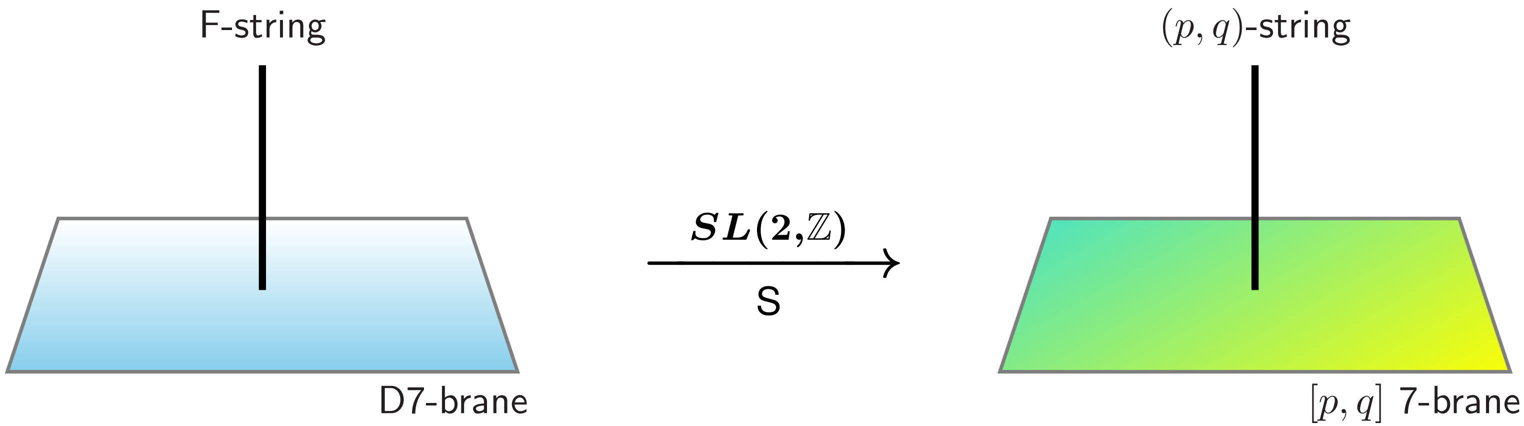
We focus on $[p, q]^T_{s5}$-brane $=$ (p NS5, q 5^2_2).

F-string : couple to $B_{(2)}$

D-string : couple to $C_{(2)}$

D7(1234567) : couple to $\rho(z) = C + \mathrm{i}e^{-\phi}$ $(z = x^8 + \mathrm{i}x^9)$

F-theory



$(1, 0)$-string = F1 $[1, 0]$ 7-brane = D7(1234567)

$(0, 1)$-string = D1 $[0, 1]$ 7-brane = 7_3(1234567)

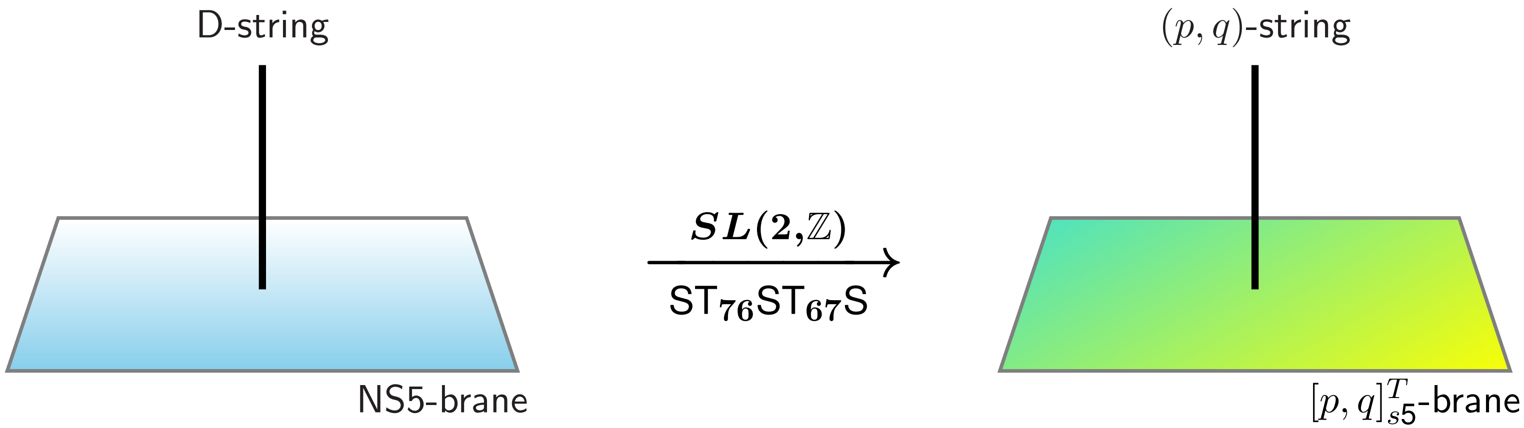
Open D-string is ending on 7_3(1234567).

D-string : couple to $C_{(2)}$

D3-brane : couple to $C_{(4)}$

NS5(12345) : couple to $\rho(z) = B^{(2)}_{67} + \mathrm{i}e^{+2\phi}$ $(z = x^8 + \mathrm{i}x^9)$

ST$_{67}$-dualizied



$(1,0)$-string = D1

$(0,1)$-string = D3 wrapped on T^2_{67}

$[1,0]^T_{s5}$-brane = NS5(12345)

$[0,1]^T_{s5}$-brane = 5^2_2(12345,67)

Open D3-brane wrapped on T^2_{67} is ending on 5^2_2(12345,67).

String junctions

$(\boldsymbol{r}, \boldsymbol{s})$-string crossing the branch cut of $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane :

w/ charge conservation law : $\boldsymbol{qr - ps = \pm 1}$

monodromy matrix : $\boldsymbol{K_{[p,q]^S_7} = \begin{pmatrix} 1 + pq & -p^2 \\ q^2 & 1 - pq \end{pmatrix}}$

"$[\boldsymbol{p}, \boldsymbol{q}]^S_7$" means $[\boldsymbol{p}, \boldsymbol{q}]$ 7-brane

(r, s)-string crossing the branch cut of $[p, q]^{T}_{s5}$-brane :

D3-brane wrapped on T^2_{67} is ending on 5^2_2(12345,67).

monodromy matrix : $K_{[p,q]^T_5} = K_{[p,q]^S_7}$

$(p, q)_1$: p D1 + q "D3 wrapped on T^2_{67}"

SCFTs with E_{n+1} symmetry in 5D, 4D, 3D

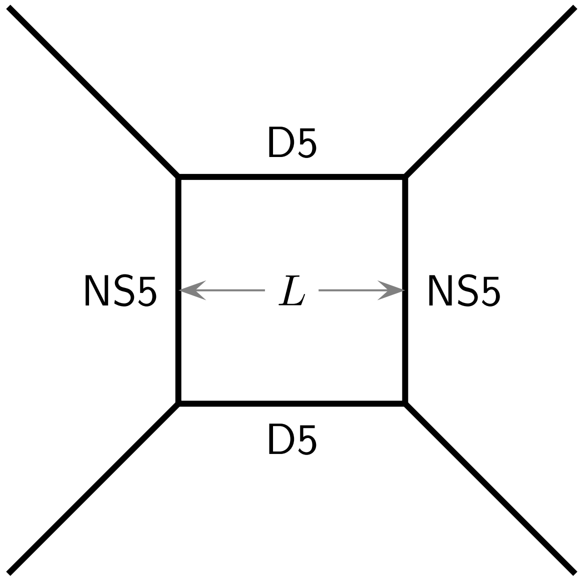
DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

Benini, Benvenuti and Tachikawa: arXiv:0906.0359

Kim, Taki and Yagi: arXiv:1504.03672

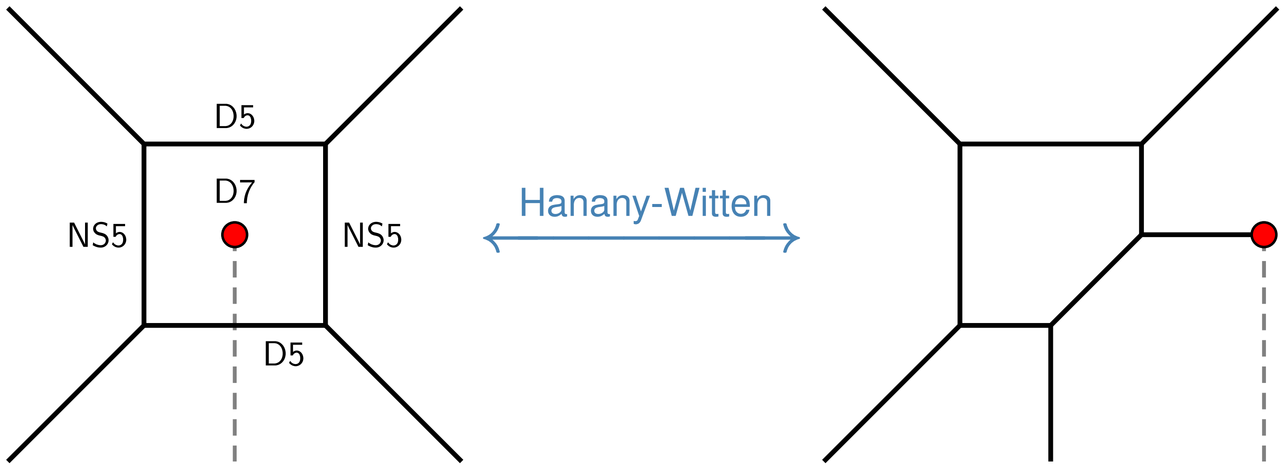
TK: arXiv:1602.08606

5D SUSY gauge theory by brane construction :



- ✔ N_c D5 between 2 NS5 $=$ $SU(N_c)$ gauge symmetry
- ✔ N_f D5 outside 2 NS5 $=$ N_f flavors
- ✔ $\dfrac{1}{g_{\text{YM}}^2} \simeq \dfrac{L}{g_s \, \ell_s^2}$

5D SUSY gauge theory by brane construction :

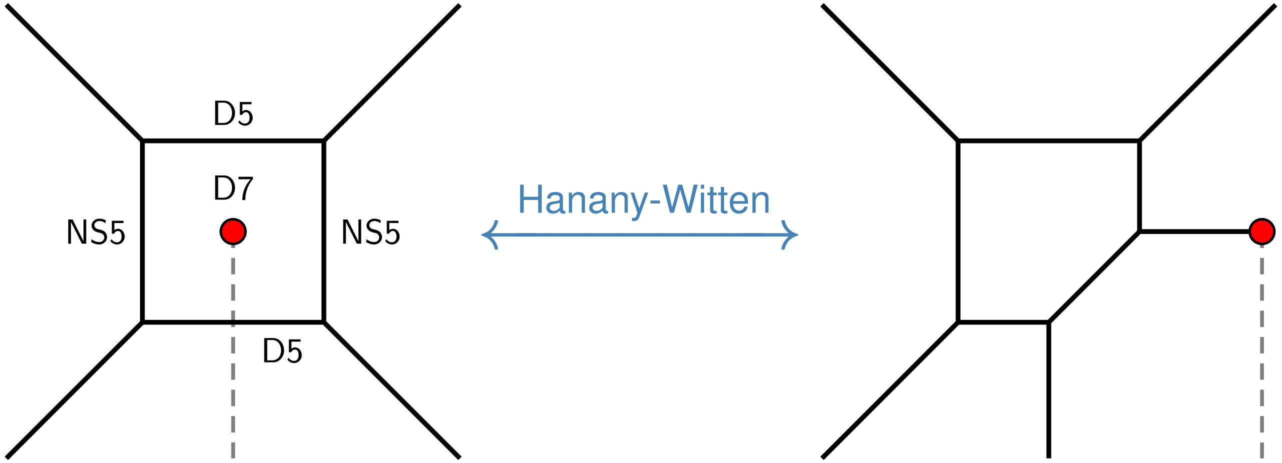


IIB		0	1	2	3	4	5	6	7	8	9
	n D7	—	—	—	—	—	—	—	—		
	D5	—	—	—	—	—				—	
	NS5	—	—	—	—	—					—
$(r, s)_5$	r D5	—	—	—	—	—				angle	
	s NS5	—	—	—	—	—					

5D $SU(2)$ gauge symmetry with n flavors on 01234-directions

$L \to 0$ limit : SCFT with E_{n+1} symmetry

5D SUSY gauge theory by brane construction :



IIB		0	1	2	3	4	5	6	7	8	9
	n D7	–	–	–	–	–	–	–	–		
	D5	–	–	–	–	–				–	
	NS5	–	–	–	–	–					–
$(r,s)_5$	r D5	–	–	–	–	–				angle	
	s NS5	–	–	–	–	–					

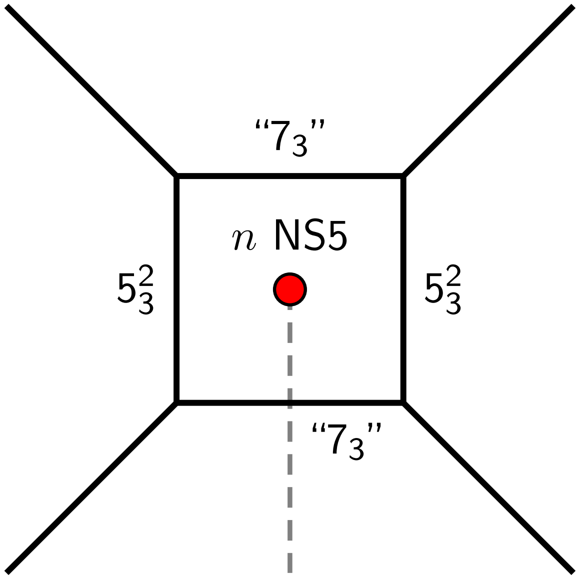
Perform string dualities :

ST_{67}-dual $\to$ 5D

ST_{47}-dual $\to$ 4D

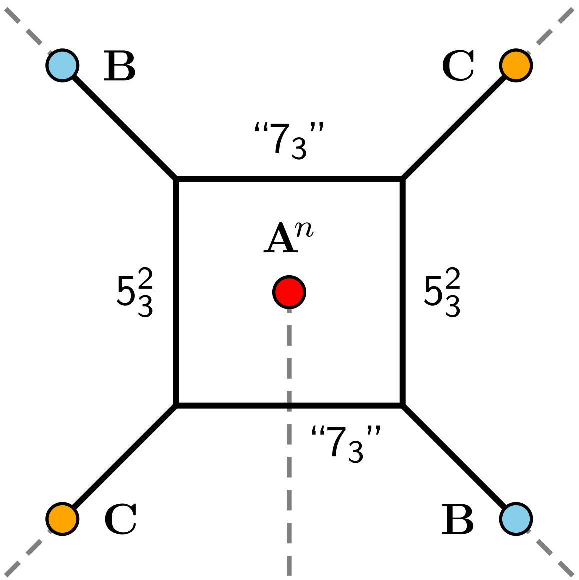
ST_{34}-dual $\to$ 3D

ST_{67}-dualized system :



IIB		0	1	2	3	4	5	⑥	⑦	8	9
$\mathbf{A}^n$	n NS5	—	—	—	—	—	—				
	“7_3”	—	—	—	—	—		—	—	—	
	5^2_3	—	—	—	—	—		$\bullet^2$	$\bullet^2$		—
$(r, s)_5$	r “7_3”	—	—	—	—	—		—	—	angle	
	s 5^2_3	—	—	—	—	—		$\bullet^2$	$\bullet^2$		

ST$_{67}$-dualized system :



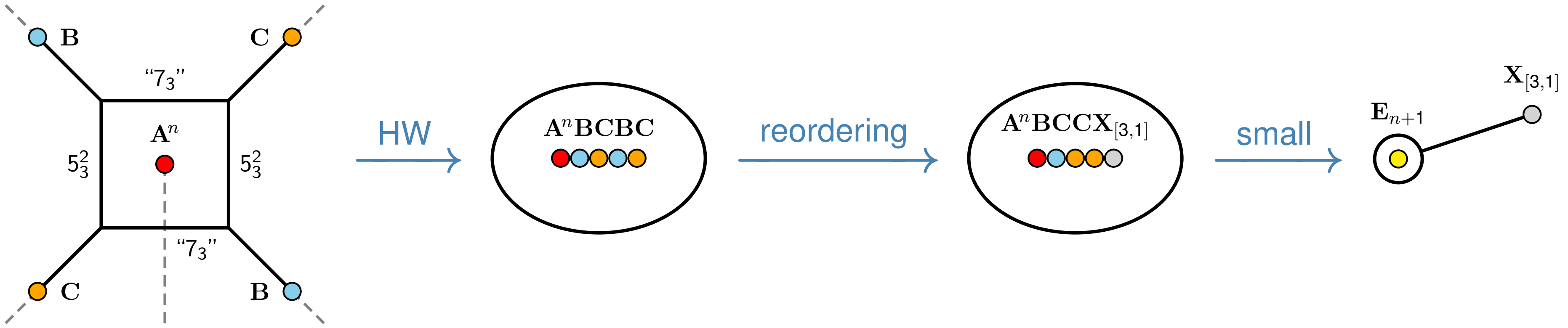
IIB		0	1	2	3	4	5	⑥	⑦	8	9
$\mathbf{A}^n$	n NS5	—	—	—	—	—	—				
	“7_3”	—	—	—	—	—		—	—	—	
	5^2_3	—	—	—	—	—		$\bullet^2$	$\bullet^2$		—
$(r,s)_5$	r “7_3”	—	—	—	—	—		—	—	angle	
	s 5^2_3	—	—	—	—	—		$\bullet^2$	$\bullet^2$		

$\mathbf{A}$-brane : $[1,0]^T_{s5}$-brane $=$ NS5

$\mathbf{B}$-brane : $[1,-1]^T_{s5}$-brane

$\mathbf{C}$-brane : $[1,1]^T_{s5}$-brane

5D $\mathcal{N} = 1$ $SU(2)$ gauge w/ n flavors $\rightarrow$ SCFT w/ E_{n+1} symmetry



$n \geq 9$: no fixed point

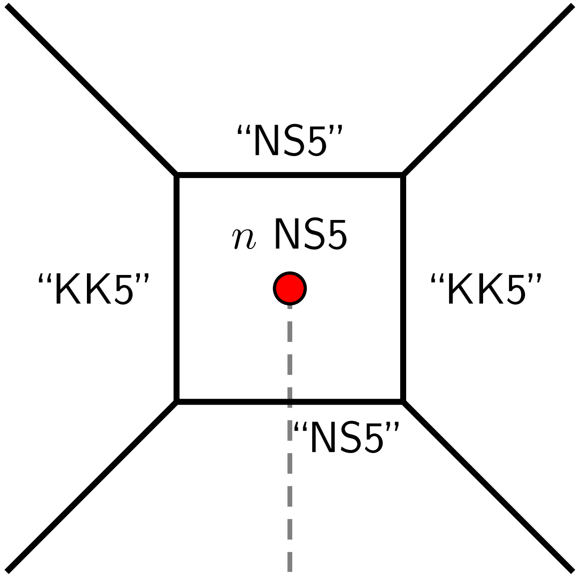
gauge coupling on wrapped 7_3 :

$$\frac{1}{g^2_{\text{YM}}} \simeq \frac{V(T^2_{67})L}{g_s\,\ell_s^4} = \frac{L}{g_s} \cdot \frac{2\pi\widetilde{R}_6}{\ell_s^2} \cdot \frac{2\pi\widetilde{R}_7}{\ell_s^2}$$

strong coupling limit :

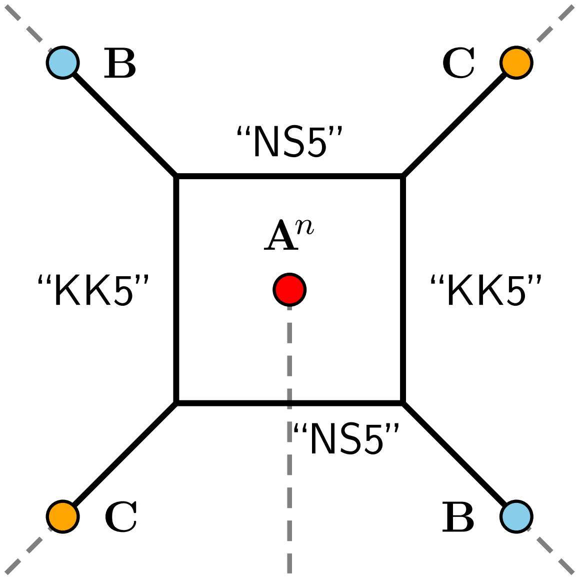
$$\ell_s \to 0\,, \qquad \frac{\widetilde{R}_i}{\ell_s^2}\,, \qquad L \to 0$$

ST$_{47}$-dualized system :



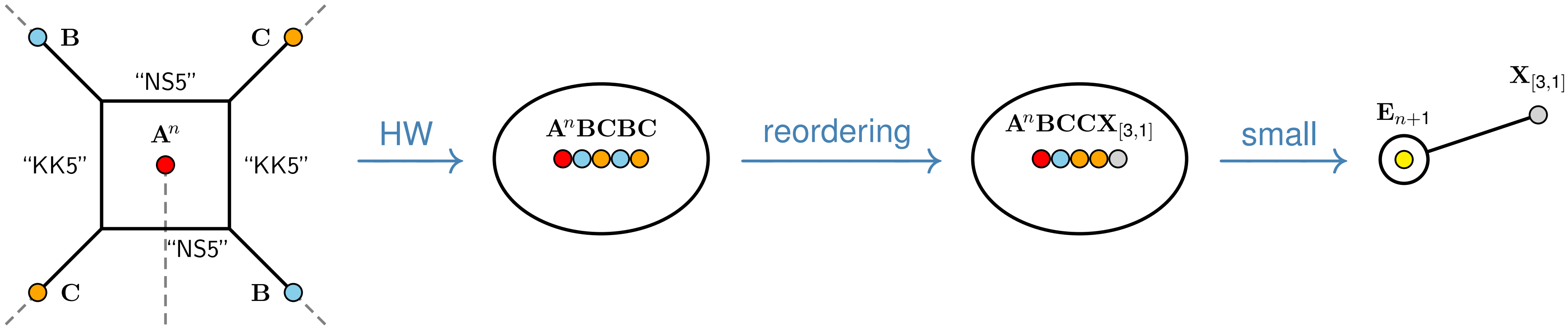
IIB		0	1	2	3	④	5	6	⑦	8	9
$\mathbf{A}^n$	n NS5	—	—	—	—		—	—			
	"NS5"	—	—	—	—				—	—	
	"KK5"	—	—	—	—	—			$\bullet^2$		—
$(r, s)_4$	r "NS5"	—	—	—	—				—	angle	
	s "KK5"	—	—	—	—	—			$\bullet^2$		

ST_{47}-dualized system :



IIB		0	1	2	3	④	5	6	⑦	8	9
$\mathbf{A}^n$	n NS5	—	—	—	—		—	—			
	"NS5"	—	—	—	—				—	—	
	"KK5"	—	—	—	—	—			$\bullet^2$		—
$(r,s)_4$	r "NS5"	—	—	—	—				—	angle	
	s "KK5"	—	—	—	—	—			$\bullet^2$		

4D $\mathcal{N} = 2$ $SU(2)$ gauge w/ n flavors $\to$ SCFT w/ E_{n+1} symmetry



$n \geq 9$: no fixed point

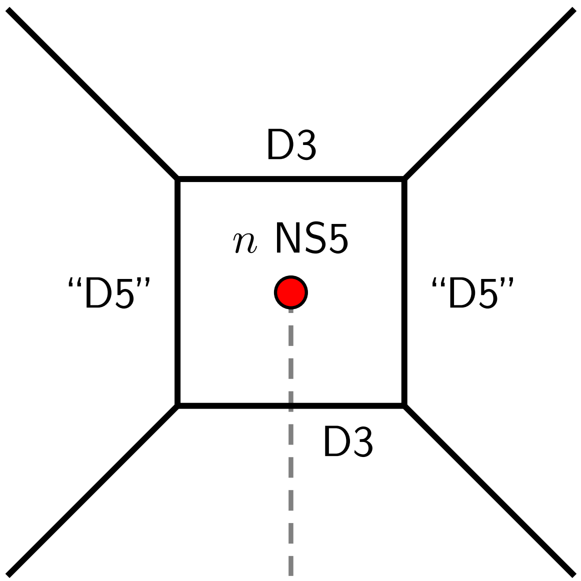
gauge coupling on wrapped NS5 :

$$\frac{1}{g_{\text{YM}}^2} \simeq \frac{(2\pi \widetilde{R}_7) L}{\ell_s^2}$$

strong coupling limit :

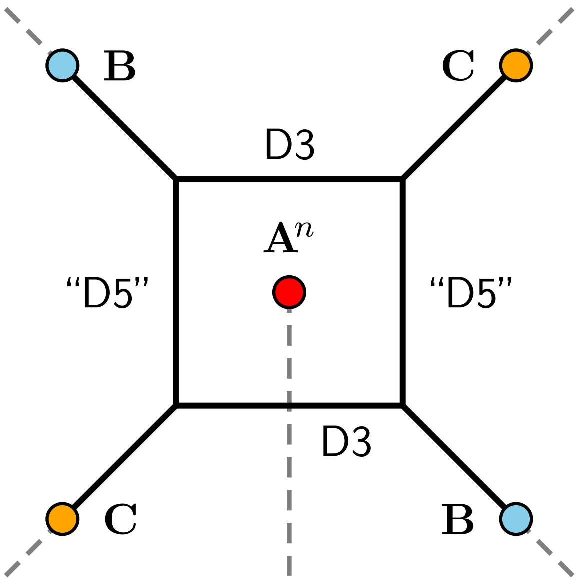
$$\ell_s \to 0 \,, \qquad \frac{\widetilde{R}_7}{\ell_s^2} \,, \qquad L \to 0$$

ST_{34}-dualized system :



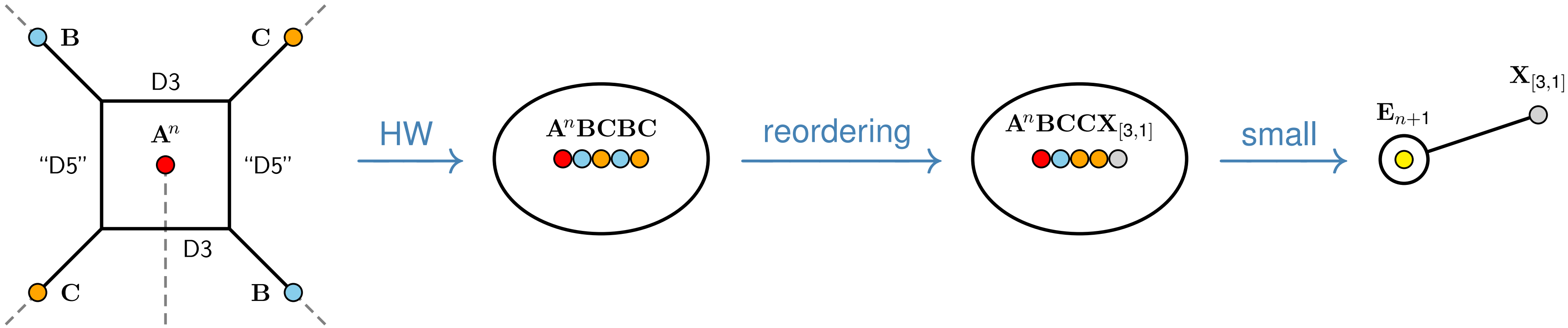
IIB		0	1	2	③	④	5	6	7	8	9
$\mathbf{A}^n$	n NS5	—	—	—			—	—	—		
	D3	—	—	—						—	
	"D5"	—	—	—	—	—					—
$(r, s)_3$	r D3	—	—	—						angle	
	s "D5"	—	—	—	—	—					

ST$_{34}$-dualized system :



IIB		0	1	2	③	④	5	6	7	8	9
$\mathbf{A}^n$	n NS5	—	—	—			—	—	—		
	D3	—	—	—						—	
	"D5"	—	—	—	—	—					—
$(r, s)_3$	r D3	—	—	—						angle	
	s "D5"	—	—	—	—	—					

3D $\mathcal{N} = 4$ $SU(2)$ gauge w/ n flavors $\to$ SCFT w/ E_{n+1} symmetry



$n \geq 9$: no fixed point

gauge coupling on NS5 :

$$\frac{1}{g_{\text{YM}}^2} \simeq \frac{L}{g_s}$$

strong coupling limit :

$$L \to 0$$

Summary

Eventually, we understood

- ✔ objects ending on 5^2_2-brane

  $=$ wrapped D3-brane

  (Its oscillations provide excitation modes on the 5^2_2-brane worldvolume.)

- ✔ objects sensitive to 5^2_2-brane branch cut

  $=$ wrapped D3 (and 5^2_3, wrapped KK5, wrapped D5)

  (They are created/annihilated by the Hanany-Witten transitions.)

We also found the brane constructions which give rise to

✔ SCFTs with E_{n+1} symmetry in 5D, 4D, 3D (qualitatively)

We can further construct various brane configurations involving exotic branes.

Exotic F-theories

If you adapt the techniques of the 7-branes to the exotic branes,
you can construct F-theories in any dimensions.

Thanks

Appendix

- Solitonic five-branes in IIA theory :

$$\begin{aligned} \text{NS5(12345)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \\ \text{KK5(12345,9)} : \quad \pm\epsilon &= \Gamma^{012345}\Gamma\epsilon \\ 5^2_2\text{(12345,89)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \\ 5^3_2\text{(12345,789)} : \quad \pm\epsilon &= \Gamma^{012345}\Gamma\epsilon \\ 5^4_2\text{(12345,6789)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \end{aligned}$$

Lozano-Tellechea and Ortín: hep-th/0012051

TK: arXiv:1601.02175

- Solitonic five-branes in IIB theory :

$$\begin{aligned}
\text{NS5(12345)} : \quad \pm\epsilon &= \Gamma^{012345}(\sigma_3)\epsilon \\
\text{KK5(12345,9)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \\
5^2_2\text{(12345,89)} : \quad \pm\epsilon &= \Gamma^{012345}(\sigma_3)\epsilon \\
5^3_2\text{(12345,789)} : \quad \pm\epsilon &= \Gamma^{012345}\epsilon \\
5^4_2\text{(12345,6789)} : \quad \pm\epsilon &= \Gamma^{012345}(\sigma_3)\epsilon
\end{aligned}$$

Lozano-Tellechea and Ortín: hep-th/0012051

TK: arXiv:1601.02175

- Defect branes in IIA theory :

$$6_3^1(123456,7): \ \pm\epsilon = \mathbf{\Gamma}^{0123456}(\boldsymbol{\sigma}_1)\epsilon$$

$$4_3^3(1234,567): \ \pm\epsilon = \mathbf{\Gamma}^{01234}\mathbf{\Gamma}(\boldsymbol{\sigma}_1)\epsilon$$

$$2_3^5(12,34567): \ \pm\epsilon = \mathbf{\Gamma}^{012}(\boldsymbol{\sigma}_1)\epsilon$$

$$0_3^7(,1234567): \ \pm\epsilon = \mathbf{\Gamma}^{0}\mathbf{\Gamma}(\boldsymbol{\sigma}_1)\epsilon$$

$$1_4^6(1,234567): \ \pm\epsilon = \mathbf{\Gamma}^{01}\mathbf{\Gamma}\epsilon$$

$$0_4^{(1,6)}(,234567,1): \ \pm\epsilon = \mathbf{\Gamma}^{01}\epsilon$$

Lozano-Tellechea and Ortín: hep-th/0012051

TK: arXiv:1601.02175

- Defect branes in IIB theory :

$$
\begin{aligned}
7_3(1234567):\quad \pm\epsilon &= \Gamma^{01234567}(\mathrm{i}\sigma_2)\epsilon \\
5^2_3(12345,67):\quad \pm\epsilon &= \Gamma^{012345}(\sigma_1)\epsilon \\
3^4_3(123,4567):\quad \pm\epsilon &= \Gamma^{0123}(\mathrm{i}\sigma_2)\epsilon \\
1^6_3(1,234567):\quad \pm\epsilon &= \Gamma^{01}(\sigma_1)\epsilon \\
1^6_4(1,234567):\quad \pm\epsilon &= \Gamma^{01}(\sigma_3)\epsilon \\
0^{(1,6)}_4(,234567,1):\quad \pm\epsilon &= \Gamma^{01}\epsilon
\end{aligned}
$$

Lozano-Tellechea and Ortín: hep-th/0012051

TK: arXiv:1601.02175

- NS5(12345) :

$$\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}x^2_{012345} + \rho_2\, \mathrm{d}x^2_{67} + \rho_2 |f|^2\, \mathrm{d}z \mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \rho_2 \\
B_{(2)} &= \rho_1\, \mathrm{d}x^6 \wedge \mathrm{d}x^7\,, \qquad B_{(6)} \;=\; \frac{1}{\rho_2}\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^5 \\
\rho &= \rho_1 + \mathrm{i}\rho_2 \;=\; B^{(2)}_{67} + \mathrm{i}\mathrm{e}^{2\phi} \;=\; B^{(2)}_{67} + \mathrm{i}\sqrt{\det G_{mn}} \\
\tau &= \text{(complex structure of } T^2_{67}\text{)} \;=\; \mathrm{i} \\
f &= 1\,, \qquad m,n = 6,7
\end{aligned}$$

Sakatani: arXiv:1412.8769

- 5^2_2(12345,67) :

$$
\begin{aligned}
\mathrm{d}s'^2 &= \mathrm{d}x^2_{012345} + \rho'_2\, \mathrm{d}x^2_{67} + \rho'_2 |f'|^2\, \mathrm{d}z \mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= \rho'_2 \\
B'_{(2)} &= \rho'_1\, \mathrm{d}x^6 \wedge \mathrm{d}x^7 \,, \qquad B'_{(6)} \;=\; \frac{1}{\rho'_2} \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^5 \\
\rho' &= B'^{(2)}_{67} + \mathrm{i}\mathrm{e}^{2\phi'} \;=\; B'^{(2)}_{67} + \mathrm{i}\sqrt{\det G'_{mn}} \;=\; -\frac{1}{\rho_{\mathrm{NS5}}} \\
\rho'_1 &= -\frac{\rho_1}{|\rho|^2} \,, \qquad \rho'_2 \;=\; \frac{\rho_2}{|\rho|^2} \\
\tau' &= (\text{complex structure of } \widetilde{T}^2_{67}) \;=\; \mathrm{i} \;=\; -\frac{1}{\tau_{\mathrm{NS5}}} \\
\rho'_2 |f'|^2 &= \rho_2 |f|^2 \,, \qquad m, n = 6, 7
\end{aligned}
$$

Sakatani: arXiv:1412.8769

- KK5(12345,7) smeared along 6-th direction :

$$
\begin{aligned}
ds^2 &= dx^2_{012345} + \tau_2\, dx^2_6 + \frac{1}{\tau_2}\big(dx^7 - \tau_1\, dx^6\big)^2 + \tau_2 |f|^2\, dz d\bar{z} \\
e^{2\phi} &= 1\,, \qquad B_{(2)} \;=\; 0\,, \qquad B_{(6)} \;=\; 0 \\
\rho &= B^{(2)}_{67} + i\sqrt{\det G_{mn}} \;=\; i \\
\tau &= (\text{complex structure of } T^2_{67}) \;=\; \tau_1 + i\tau_2 \\
f &= 1\,, \qquad m, n = 6, 7
\end{aligned}
$$

Sakatani: arXiv:1412.8769

- KK5(12345,6) smeared along 7-th direction :

$$
\begin{aligned}
ds'^2 &= dx^2_{012345} + \tau'_2\, dx^2_6 + \frac{1}{\tau'_2}\big(dx^7 - \tau'_1\, dx^6\big)^2 + \tau'_2|f'|^2\, dz d\bar{z} \\
e^{2\phi'} &= 1\,, \qquad B'_{(2)} = 0\,, \qquad B'_{(6)} = 0 \\
\rho' &= B'^{(2)}_{67} + i\sqrt{\det G'_{mn}} = i = -\frac{1}{\rho_{\text{KK5}}} \\
\tau' &= \text{(complex structure of } \widetilde{T}^2_{67}) = \tau'_1 + i\tau'_2 = -\frac{1}{\tau_{\text{KK5}}} \\
\tau'_2|f'|^2 &= \tau_2|f|^2\,, \qquad m, n = 6, 7
\end{aligned}
$$

Sakatani: arXiv:1412.8769

- D$p(12\cdots p)$:

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{(\rho_2)^{1/2}}\,\mathrm{d}x^2_{012\cdots p} + (\rho_2)^{1/2}\,\mathrm{d}x^2_{a_1\cdots a_{7-p}} + (\rho_2)^{1/2}|f|^2\,\mathrm{d}z\mathrm{d}\overline{z} \\
\mathrm{e}^{2\phi} &= (\rho_2)^{\frac{3-p}{2}} \\
C_{(7-p)} &= \rho_1\,\mathrm{d}x^{a_1}\wedge\cdots\wedge\mathrm{d}x^{a_{7-p}} \\
C_{(p+1)} &= -\frac{1}{\rho_2}\,\mathrm{d}x^0\wedge\mathrm{d}x^1\wedge\cdots\wedge\mathrm{d}x^p \\
\rho &= C^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{\frac{4}{3-p}\phi} \;=\; C^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{-\phi}(\det G_{mn})^{1/2} \\
f &= 1\,, \qquad m,n = a_1,\ldots,a_{7-p}
\end{aligned}
$$

Sakatani: arXiv:1412.8769

- $p_3^{7-p}(12\cdots p, a_1\cdots a_{7-p})$:

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{(\rho_2')^{1/2}}\,\mathrm{d}x^2_{012\cdots p} + (\rho_2')^{1/2}\,\mathrm{d}x^2_{a_1\cdots a_{7-p}} + (\rho_2')^{1/2}|f'|^2\,\mathrm{d}z\mathrm{d}\overline{z} \\
\mathrm{e}^{2\phi'} &= (\rho_2')^{\frac{3-p}{2}} \\
C'_{(7-p)} &= \rho_1'\,\mathrm{d}x^{a_1}\wedge\cdots\wedge\mathrm{d}x^{a_{7-p}}\,, \qquad C'_{(p+1)} \;=\; -\frac{1}{\rho_2'}\,\mathrm{d}x^0\wedge\mathrm{d}x^1\wedge\cdots\wedge\mathrm{d}x^p \\
\rho' &= C'^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{\frac{4}{3-p}\phi'} \;=\; -\frac{1}{\rho_{\mathrm{D}p}} \\
\rho_1' &= -\frac{\rho_1}{|\rho|^2}\,, \qquad \rho_2' \;=\; \frac{\rho_2}{|\rho|^2} \\
\rho_2'|f'|^2 &= \rho_2|f|^2\,, \qquad |f'| \;=\; |\rho||f|\,, \qquad m,n = a_1,\ldots,a_{7-p}
\end{aligned}
$$

Sakatani: arXiv:1412.8769

- F1(1) :

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{\rho_2}\,\mathrm{d}x_{01}^2 + \mathrm{d}x_{234567}^2 + |f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \frac{1}{\rho_2} \\
B_{(6)} &= \rho_1\,\mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \cdots \wedge \mathrm{d}x^7\,, \qquad B_{(2)} \;=\; -\frac{1}{\rho_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \\
\rho &= B^{(6)}_{234567} + \mathrm{i}\mathrm{e}^{-2\phi}\,, \qquad f \;=\; 1
\end{aligned}$$

Sakatani: arXiv:1412.8769

- 1^6_4(1,234567) :

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{\rho_2'}\,\mathrm{d}x_{01}^2 + \mathrm{d}x_{234567}^2 + |f'|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= \frac{1}{\rho_2'} \\
B'_{(6)} &= \rho_1'\,\mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \cdots \wedge \mathrm{d}x^7\,, \qquad B'_{(2)} \;=\; -\frac{1}{\rho_2'}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \\
\rho' &= B'^{(6)}_{234567} + \mathrm{i}\mathrm{e}^{-2\phi'} \;=\; -\frac{1}{\rho_{\mathsf{F1}}} \\
\rho_2'|f'|^2 &= \rho_2|f|^2\,, \qquad |f'| \;=\; |\rho||f|
\end{aligned}
$$

Sakatani: arXiv:1412.8769

- P(1) :

$$
\begin{aligned}
\mathrm{d}s^2 &= -2\mathrm{d}x^0\mathrm{d}x^1 + \rho_2\,\mathrm{d}x_1^2 + \mathrm{d}x_{234567}^2 + |f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \frac{1}{\rho_2}\,, \qquad B_{(2)} = 0\,, \qquad B_{(6)} = 0 \\
\rho &= \mathrm{i}\mathrm{e}^{-2\phi}\,, \qquad f = 1
\end{aligned}
$$

- $0_4^{(1,6)}(,234567,1)$:

$$
\begin{aligned}
\mathrm{d}s^2 &= -2\mathrm{d}x^0\mathrm{d}x^1 + \rho_2'\,\mathrm{d}x_1^2 + \mathrm{d}x_{234567}^2 + |f'|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi'} &= \frac{1}{\rho_2'} = \frac{|\rho|^2}{\rho_2}\,, \qquad B'_{(2)} = 0\,, \qquad B'_{(6)} = 0 \\
\rho' &= \mathrm{i}\mathrm{e}^{-2\phi'} = -\frac{1}{\rho_{\mathrm{P}}}\,, \qquad \rho_2'|f'|^2 = \rho_2|f|^2\,, \qquad |f'| = |\rho||f|
\end{aligned}
$$

Sakatani: arXiv:1412.8769

F-theory

Vafa: hep-th/9602022

Zwiebach et al: hep-th/9709013, 9801205, 9804210, 9812028, 9812209, etc.

Zwiebach: “Seven-brane, string junctions and Lie algebra,” 1998年国際シンポジウム「ゲージ理論の力学と弦双対性」

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

IIB action in Einstein frame :

$$S_{\text{IIB}} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G_{\text{E}}}\,\Big\{R_{\text{E}} - \frac{\partial_M\overline{\rho}\,\partial^M\rho}{2(\rho_2)^2} - \frac{1}{2}F^i_{(3)}\cdot\mathcal{M}_{ij}F^j_{(3)} - \frac{1}{4}|\widetilde{F}_{(5)}|^2\Big\} - \frac{\epsilon_{ij}}{8\kappa^2}\int C_{(4)}\wedge F^i_{(3)}\wedge F^j_{(3)}$$

$$\rho \equiv C + \mathrm{i}e^{-\phi} \equiv \rho_1 + \mathrm{i}\rho_2\,, \qquad \mathcal{M}_{ij} \equiv \frac{1}{\rho_2}\begin{pmatrix} 1 & -\rho_1 \\ -\rho_1 & |\rho|^2 \end{pmatrix}$$

$$F^i_{(3)} \equiv \begin{pmatrix} dC_{(2)} \\ dB_{(2)} \end{pmatrix}, \qquad \widetilde{F}_{(5)} \equiv dC_{(4)} - \frac{1}{2}C_{(2)}\wedge H_{(3)} + \frac{1}{2}B_{(2)}\wedge F_{(3)}$$

$SL(2,\mathbb{Z})$ S-duality

$$\rho \to \frac{a\rho+b}{c\rho+d}, \qquad \Lambda^i{}_j = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

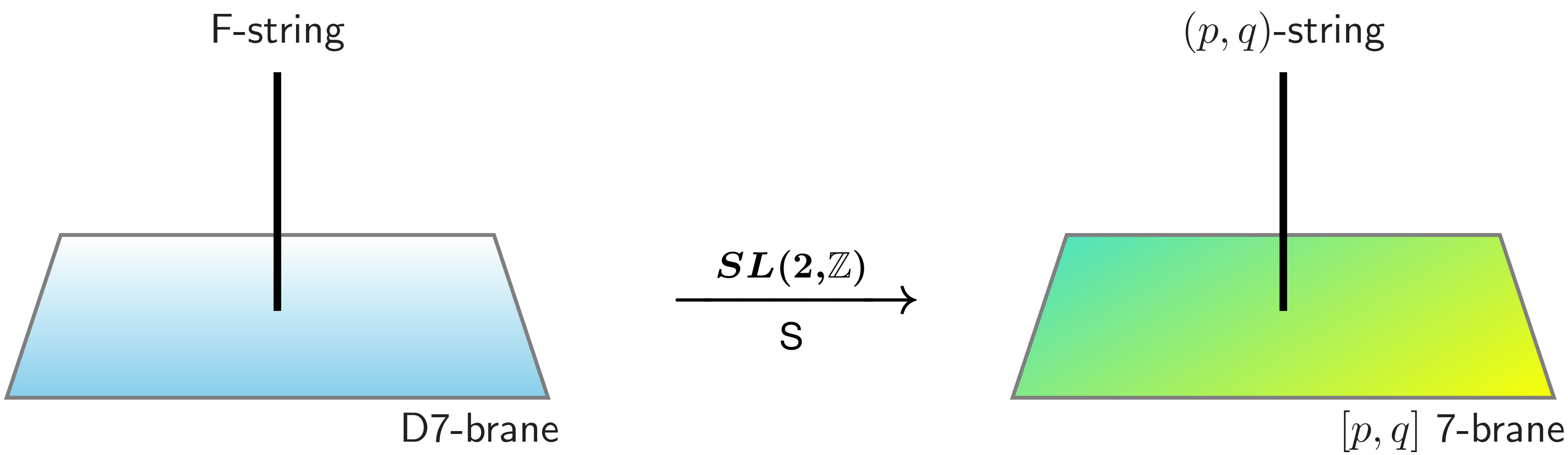
$$F^i_{(3)} \to \Lambda^i{}_j\,F^j_{(3)}\,, \qquad \widetilde{F}_{(5)} \to \widetilde{F}_{(5)}\,, \qquad G^{\text{E}}_{MN} \to G^{\text{E}}_{MN}$$

$$\mathcal{M} \to \Lambda^{-\mathrm{T}}\mathcal{M}\Lambda^{-1}$$

F-string : couple to $B_{(2)}$

D-string : couple to $C_{(2)}$

D7-brane : (magnetically) couple to $C \in \rho(z)$

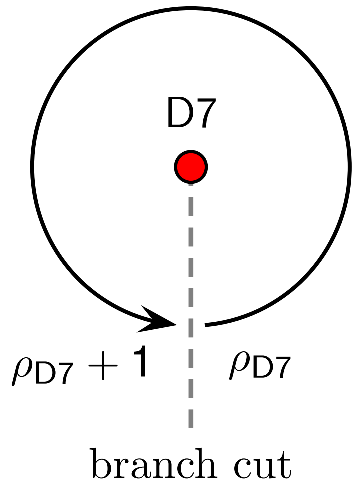


$(1,0)$-string = F-string $[1,0]$ 7-brane = D7-brane

$(0,1)$-string = D-string $[0,1]$ 7-brane = 7_3-brane (or NS7-brane)

When ρ moves around a D7-brane counterclockwise,

it receives a magnetic "charge" of D7-brane (monodromy!) $\rightarrow \;\; \rho + 1$



$$\rho \;\rightarrow\; \rho + 1 \;=\; \frac{a\rho + b}{c\rho + d} \;\equiv\; M_{[1,0]} \cdot \rho \,, \qquad M_{[1,0]} \;=\; \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \;\in\; SL(2,\mathbb{Z})$$

or equivalently

$$K_{[1,0]} \cdot (\rho + 1) \;=\; \rho \,, \qquad K_{[1,0]} \;=\; (M_{[1,0]})^{-1} \;=\; \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

Monodromy matrix $\boldsymbol{K}_{[1,0]}$ is $\boldsymbol{SL(2,\mathbb{Z})}$ transformed to $\boldsymbol{K}_{[p,q]}$ for $[\boldsymbol{p},\boldsymbol{q}]$ 7-brane :

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to g \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}, \quad g = \begin{pmatrix} p & r \\ q & s \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$K_{[p,q]} = g\, K_{[1,0]}\, g^{-1} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix}$$

ex) Monodromy $\boldsymbol{K}_{[0,1]}$ for 7_3-brane : $\quad K_{[0,1]} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

(r, s) 5-brane crossing the branch cut of $[p, q]$ 7-brane :

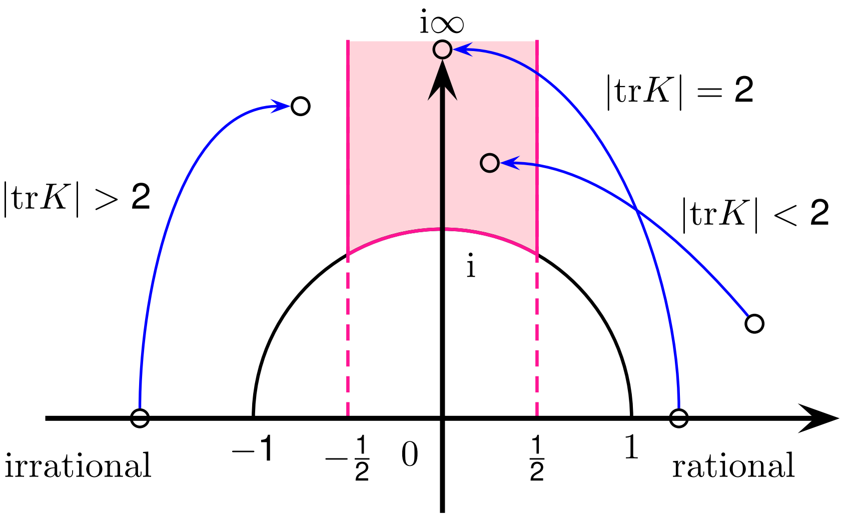
w/ charge conservation law : $qr - ps = \pm 1$

$(p, q)_5$-brane : p D5-branes $+$ q NS5-branes on which (p, q)-string is ending

$|\mathrm{tr}K|$ is a good character to classify 7-branes :

$$K \cdot \rho_* = \frac{a\rho_* + b}{c\rho_* + d} = \rho_* \, , \qquad K = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$\therefore \quad \rho_* = \frac{1}{2c}\Big\{(a-d) \pm \sqrt{(\mathrm{tr}K)^2 - 4}\Big\}$$



$|\mathrm{tr}K| = 2$: parabolic (collapsible)

$|\mathrm{tr}K| < 2$: elliptic (collapsible)

$|\mathrm{tr}K| > 2$: hyperbolic (non-collapsible)

$\mathrm{tr}K$	monodromy	branes	collapsible?	symmetry	
$+2$	$T^{-n} = K_A^n$	A^n	yes	$A_{n-1} = SU(n)$	$n \geq 1$
	$\mathbb{1} = T^0 = K_C K_B K_C K_B K_A^8$	$\widehat{\mathrm{E}}_9 \equiv \mathrm{A}^8\mathrm{BCBC}$	yes	$\widehat{E}_9$	$n = 0$
	$T^{\lvert n\rvert} = K_C K_B K_C K_B K_A^{8-\lvert n\rvert}$	$\mathrm{A}^{8-\lvert n\rvert}\mathrm{BCBC}$	no	$\widehat{E}_{9-\lvert n\rvert}$	$n \leq -1$
$+1$	$ST \sim K_C K_A$	$\mathrm{H}_0 \equiv \mathrm{AC}$	yes	H_0	
	$(ST)^{-1} \sim K_C^2 K_B K_A^7$	$\mathrm{E}_8 \equiv \mathrm{A}^7\mathrm{BC}^2$	yes	E_8	
0	$S \sim K_C K_A^2$	$\mathrm{H}_1 \equiv \mathrm{A}^2\mathrm{C}$	yes	$H_1 = SU(2)$	"$\sim$" up to
	$-S \sim K_C^2 K_B K_A^6$	$\mathrm{E}_7 \equiv \mathrm{A}^6\mathrm{BC}^2$	yes	E_7	G tf.
-1	$-(ST)^{-1} \sim K_C K_A^3$	$\mathrm{H}_2 \equiv \mathrm{A}^3\mathrm{C}$	yes	$H_2 = SU(3)$	
	$-ST \sim K_C^2 K_B K_A^5$	$\mathrm{E}_6 \equiv \mathrm{A}^5\mathrm{BC}^2$	yes	E_6	
-2	$-T^{-n} = K_C K_B K_A^{n+4}$	$\mathrm{D}_{n+4} \equiv \mathrm{A}^{n+4}\mathrm{BC}$	yes	$D_{n+4} = SO(2n+8)$	$n \geq 1$
	$-\mathbb{1} = -T^0 = K_C K_B K_A^4$	$\mathrm{D}_4 \equiv \mathrm{A}^4\mathrm{BC}$	yes	$D_4 = SO(8)$	$n = 0$
	$-T = K_C K_B K_A^3$	$\mathrm{A}^3\mathrm{BC}$	no	$D_3 = SO(6) \simeq SU(4)$	$n = -1$
	$-T^2 = K_C K_B K_A^2$	$\mathrm{A}^2\mathrm{BC}$	no	$D_2 = SO(4) \simeq SU(2) \times SU(2)$	$n = -2$
	$-T^3 = K_C K_B K_A$	ABC	no	$D_1 = SO(2) \simeq U(1)$	$n = -3$
	$-T^4 = K_C K_B$	BC	no	$-$	$n = -4$

$[p,q]$-brane is expressed by $\mathrm{z} = (p,q)^{\mathrm{T}}$ vector and $\mathbf{X}_{\mathrm{z}}$.

The monodromy matrix K_{z} is also given as

$$K_{\mathrm{z}} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix} = \mathbb{1} + \mathrm{z}\mathrm{z}^{\mathrm{T}} S \,, \qquad S \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Re-ordering the branes $\mathbf{X}_{\mathrm{z}_1}\mathbf{X}_{\mathrm{z}_2}$:

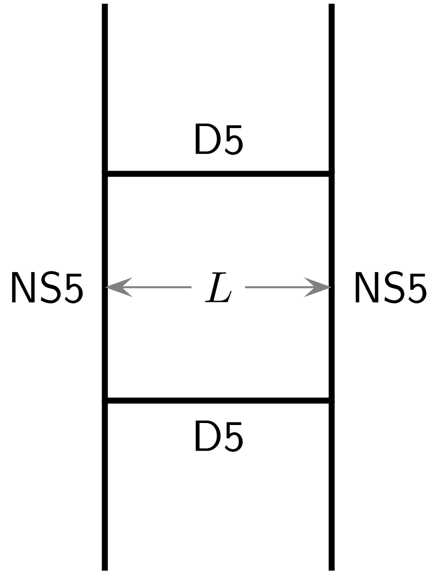
$$\mathbf{X}_{\mathrm{z}_1}\mathbf{X}_{\mathrm{z}_2} = \mathbf{X}_{(\mathrm{z}_2+(\mathrm{z}_1\times\mathrm{z}_2)\mathrm{z}_1)}\mathbf{X}_{\mathrm{z}_1} = \mathbf{X}_{\mathrm{z}_2}\mathbf{X}_{(\mathrm{z}_1+(\mathrm{z}_1\times\mathrm{z}_2)\mathrm{z}_2)}$$

$$\mathrm{z}_1 \times \mathrm{z}_2 \equiv -\mathrm{z}_1^{\mathrm{T}} S\, \mathrm{z}_2 = \mathrm{z}_2\, S\, \mathrm{z}_1 = \det \begin{pmatrix} p_1 & p_2 \\ q_1 & q_2 \end{pmatrix}$$

$$K_{\mathrm{z}_2}K_{\mathrm{z}_1} = K_{(\mathrm{z}_1+(\mathrm{z}_1\times\mathrm{z}_2)\mathrm{z}_2)}K_{\mathrm{z}_2} = K_{\mathrm{z}_1}K_{(\mathrm{z}_2+(\mathrm{z}_1\times\mathrm{z}_2)\mathrm{z}_1)}$$

Application

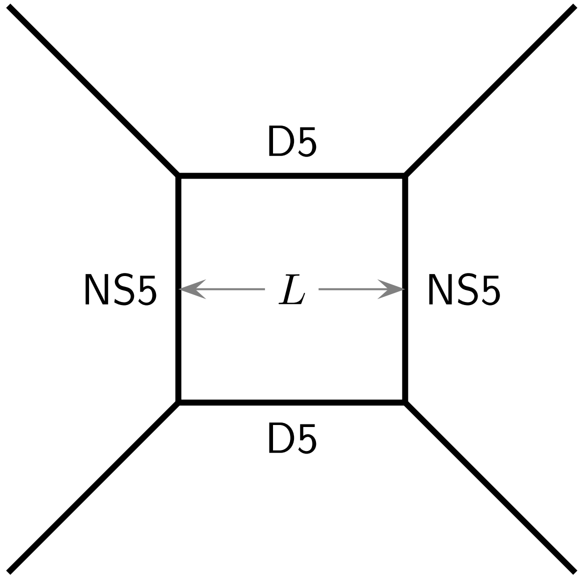
SUSY gauge theory by brane construction :



- ✔ N_c D5 between 2 NS5 $=$ $SU(N_c)$ gauge symmetry
- ✔ N_f D5 outside 2 NS5 $=$ N_f flavors

Application

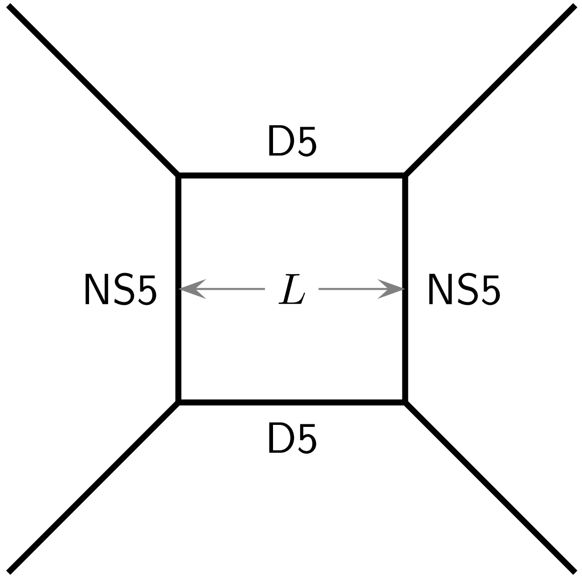
SUSY gauge theory by brane construction :



- ✔ N_c D5 between 2 NS5 $= SU(N_c)$ gauge symmetry
- ✔ N_f D5 outside 2 NS5 $= N_f$ flavors

no D7-branes :

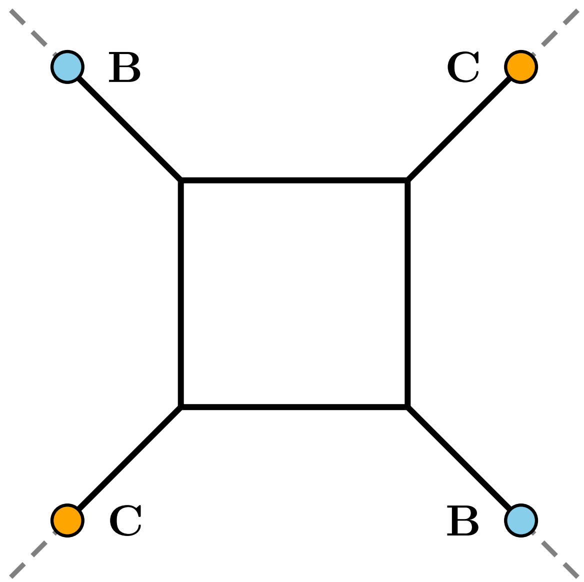
5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane



$$\frac{1}{g_{\mathrm{YM}}^2} \simeq \frac{L}{g_s \, \ell_s^2}$$

no D7-branes :

5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane

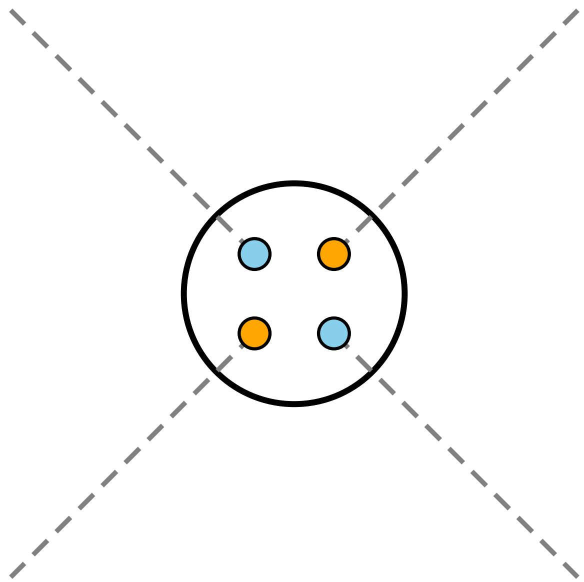


$\mathbf{B}$-brane : $[1, -1]$ 7-brane

$\mathbf{C}$-brane : $[1, 1]$ 7-brane

no D7-branes :

5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane

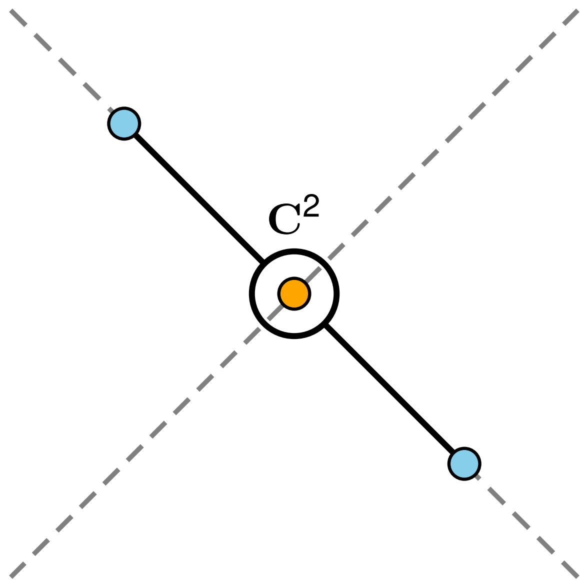


Hanany-Witten transitions

"Box" becomes "loop" by back reactions from $\mathbf{B}$- and $\mathbf{C}$-branes

no D7-branes :

5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane

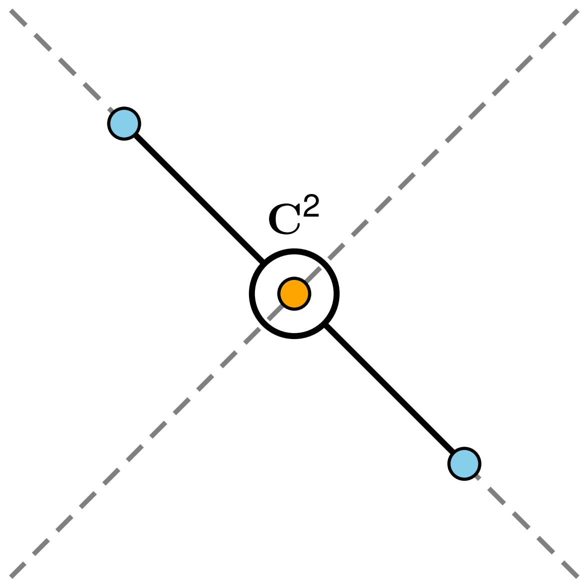


strong gauge coupling limit on 5-branes $=$ small loop limit

$$\ell_s \to 0 \,, \quad L \to 0 \,, \quad \frac{L}{\ell_s^2} \to 0$$

no D7-branes :

5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane

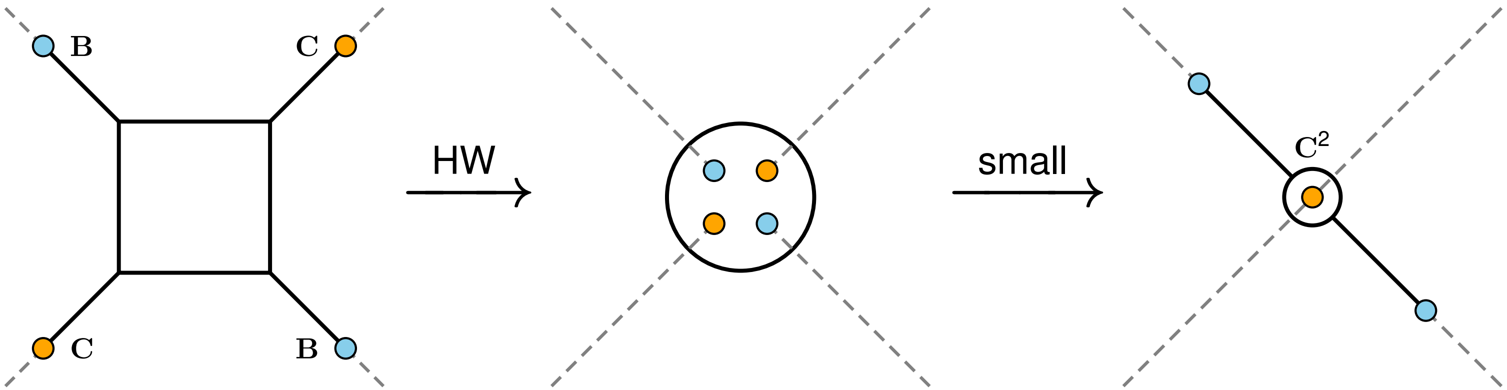


open string ending on 2 $\mathbf{C}$-branes $\rightarrow$ $SU(2)$ symmetry

open string ending on $\mathbf{C}^2$-branes and “loop” 5-branes $\rightarrow$ flavor symmetry

no D7-branes :

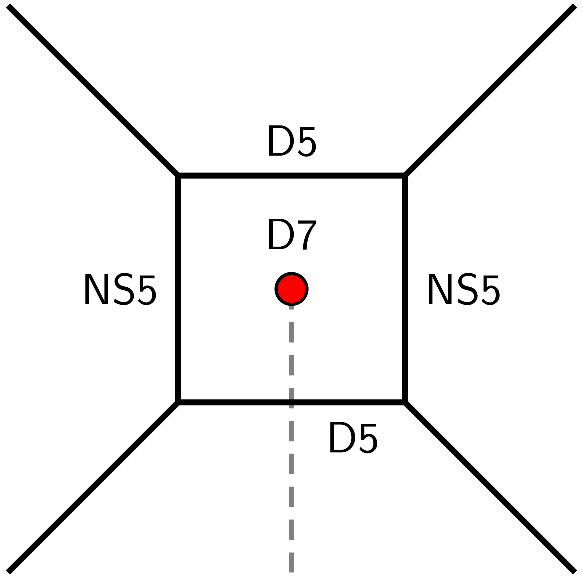
5D $\mathcal{N} = 1$ $SU(2)$ Yang-Mills theory on D5-brane



$\to$ SCFT w/ $E_1 \simeq SU(2)$ global symmetry emerges!

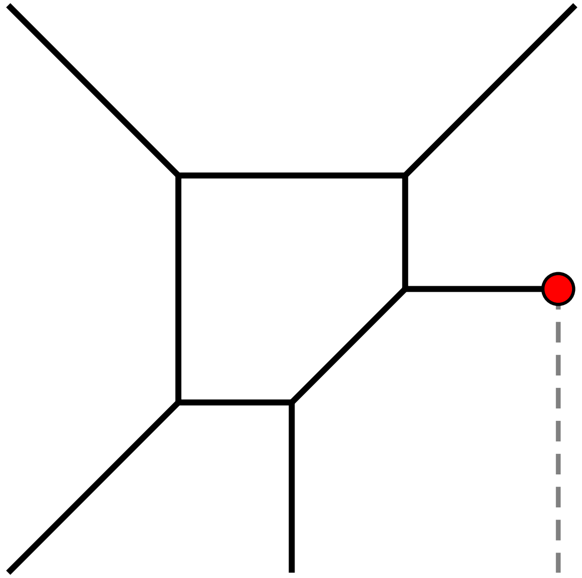
1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory w/ 1 flavor on D5-brane



1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory w/ 1 flavor on D5-brane

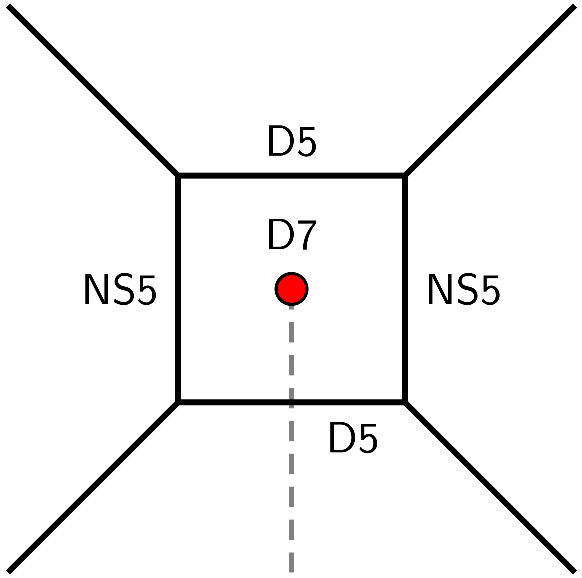


Hanany-Witten transition

new horizontal semi-infinite D5-brane appears $=$ 1 flavor

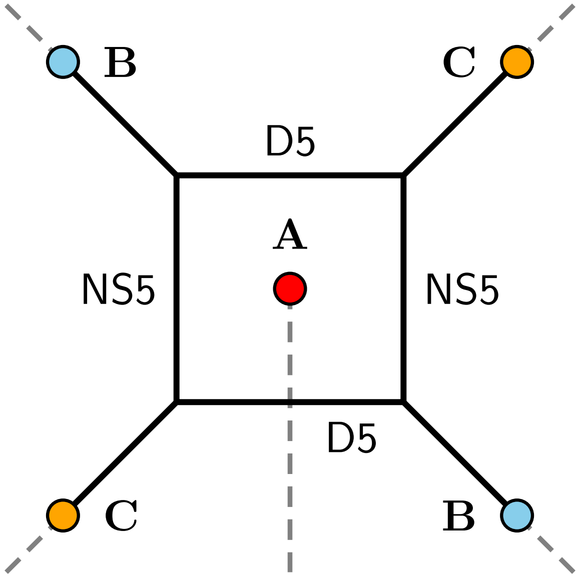
1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory w/ 1 flavor on D5-brane



1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory w/ 1 flavor on D5-brane



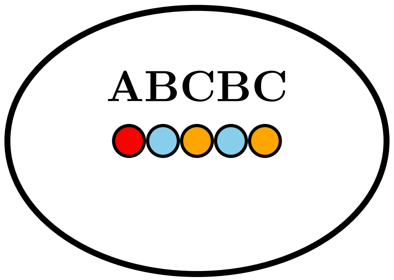
$\mathbf{A}$-brane : $[1, 0]$ 7-brane = D7-brane

$\mathbf{B}$-brane : $[1, -1]$ 7-brane

$\mathbf{C}$-brane : $[1, 1]$ 7-brane

1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory w/ 1 flavor on D5-brane

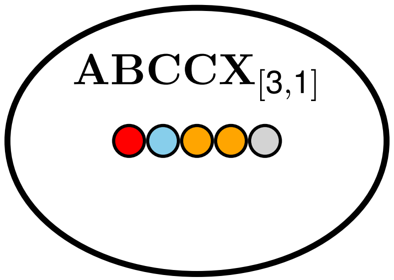


Hanany-Witten transtions

“Box” becomes “loop” by back reaction of $\mathbf{ABCBC}$-branes

1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory w/ 1 flavor on D5-brane

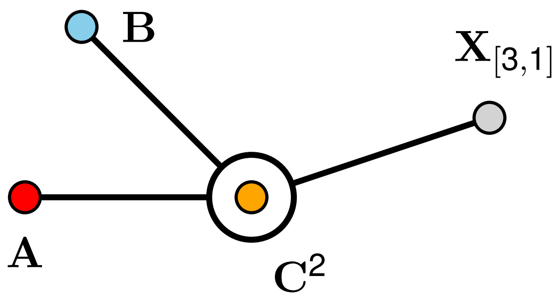


re-ordering of 7-branes : $\mathbf{ABCBC} = \mathbf{ABCCX}_{[3,1]}$

$\mathbf{X}_{[3,1]}$-brane $\equiv$ $[3,1]$-brane

1 D7-brane :

5D $\mathcal{N} = 1$ $SU(2)$ gauge theory w/ 1 flavor on D5-brane



small loop limit

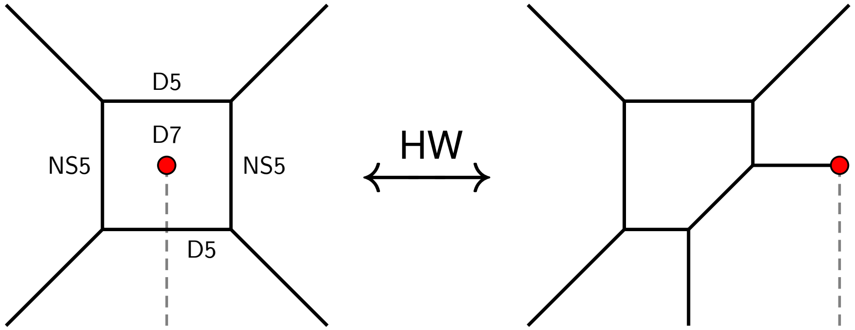
open string ending on 2 $\mathbf{C}$-branes $\rightarrow$ $SU(2)$ symmetry

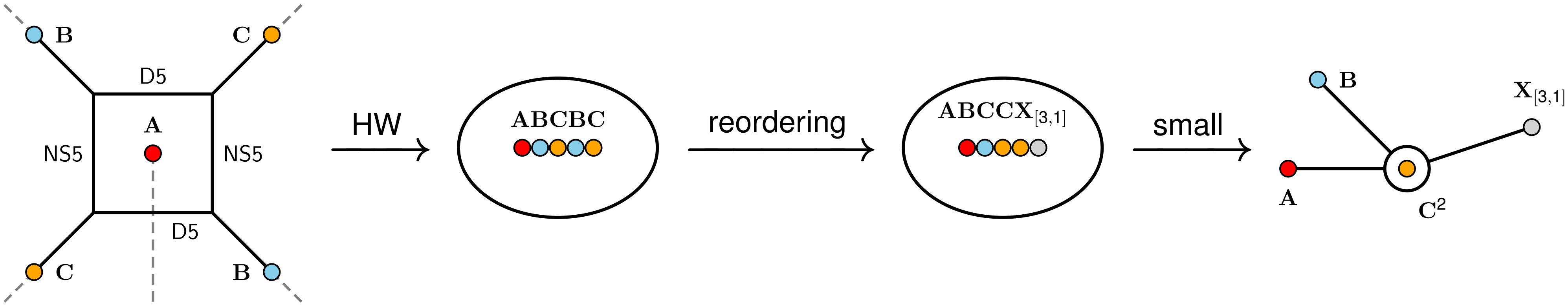
open string ending on $\mathbf{C}^2$-branes and “loop” 5-branes $\rightarrow$ flavor symmetry

$$E_2 = SU(2) \times U(1)$$

1 D7-brane :

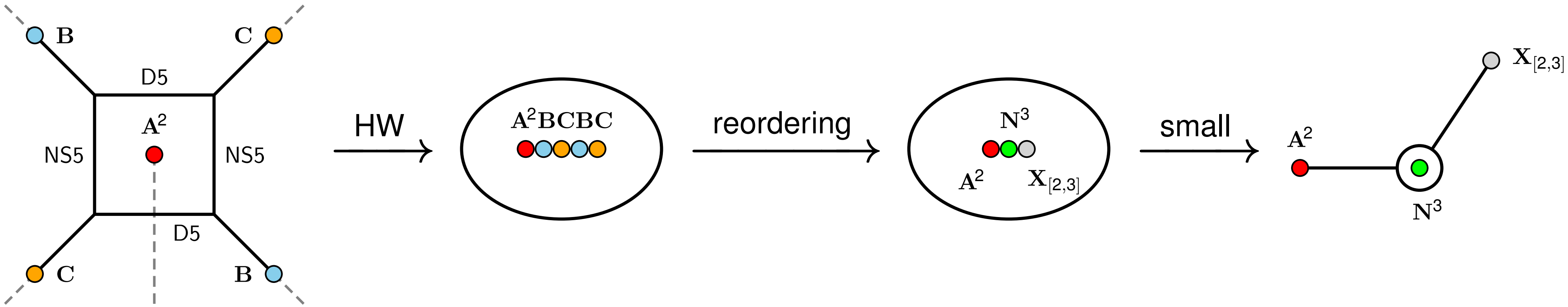
$SU(2)$ gauge w/ 1 flavor $\rightarrow$ SCFT w/ $E_2 \supset SU(2)$ symmetry





2 A-branes :

$SU(2)$ gauge w/ 2 flavors $\to$ SCFT w/ $E_3 = SU(3) \times SU(2)$ symmetry



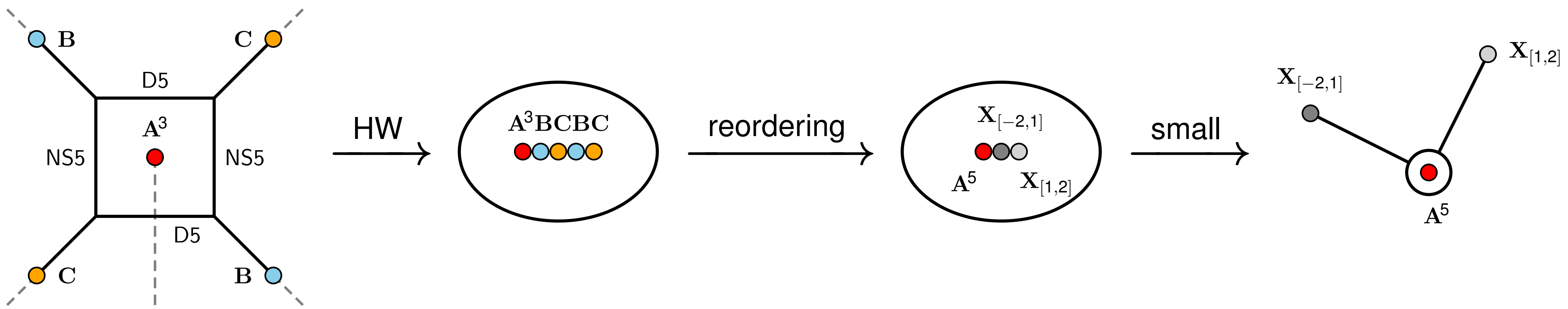
$$\text{N-brane} \equiv [0,1]\text{-brane}$$

$$\mathrm{A}^2\mathrm{BCBC} \sim \mathrm{A}^2\mathrm{N}^3\mathrm{X}_{[2,3]} \quad \to \quad \begin{cases} \mathrm{A}^2\text{-brane}: & SU(2) \\ \mathrm{N}^3\text{-brane}: & SU(3) \end{cases}$$

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

$\mathbf{3}$ $\mathbf{A}$-branes :

$SU(2)$ gauge w/ 3 flavors $\rightarrow$ SCFT w/ $E_4 = SU(5)$ symmetry

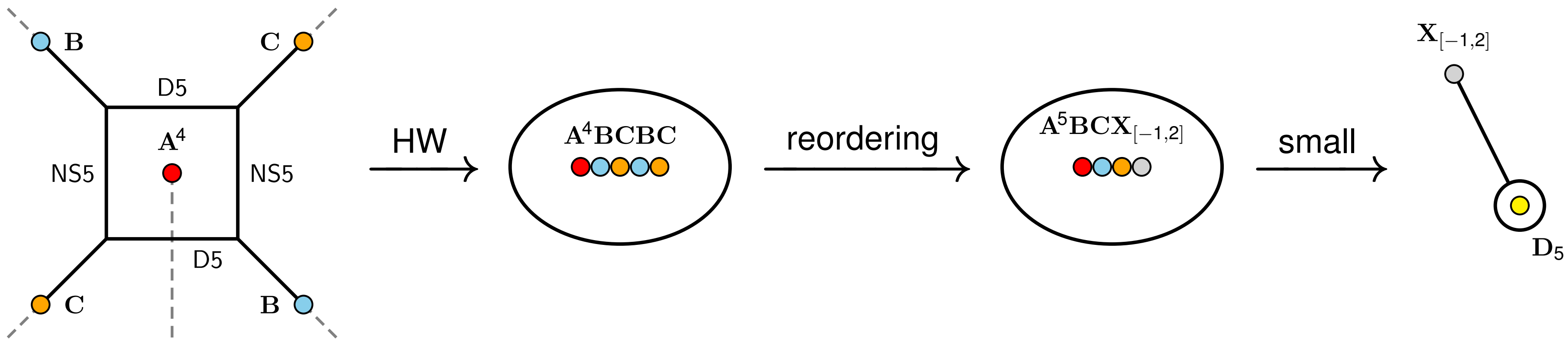


$$\mathbf{A}^3\mathbf{BCBC} \sim \mathbf{A}^5\mathbf{X}_{[-2,1]}\mathbf{X}_{[1,2]} \quad \rightarrow \quad \mathbf{A}^5\text{-brane}: \quad SU(5)$$

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

4 **A**-branes :

$SU(2)$ gauge w/ 4 flavors $\rightarrow$ SCFT w/ $E_5 = SO(10)$ symmetry

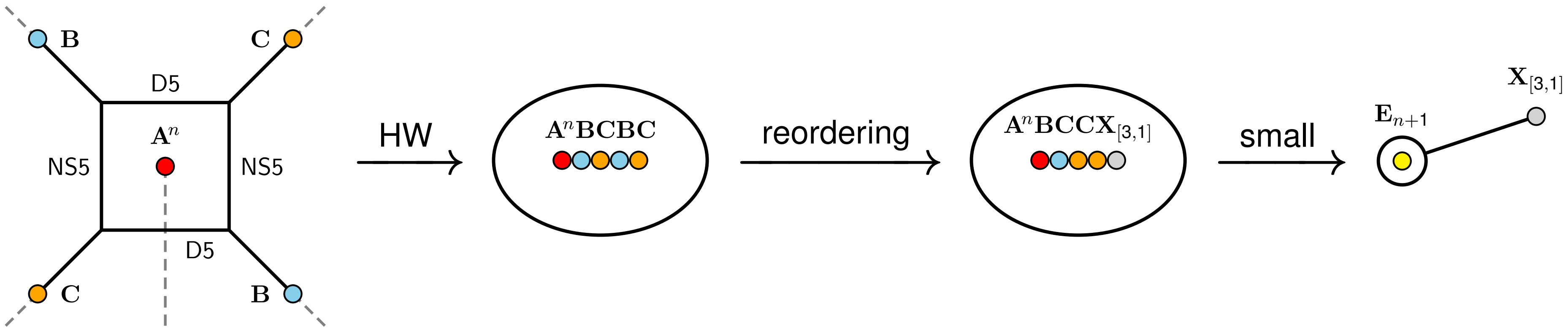


$$\mathbf{A}^4\mathbf{BCBC} \sim \mathbf{A}^5\mathbf{BCX}_{[-1,2]} \quad \rightarrow \quad \mathbf{A}^5\mathbf{BC} = \mathbf{D}_5 : \quad SO(10)$$

DeWolfe, Hanany, Iqbal and Katz: hep-th/9902179

$n = 5, 6, 7$ D7-branes :

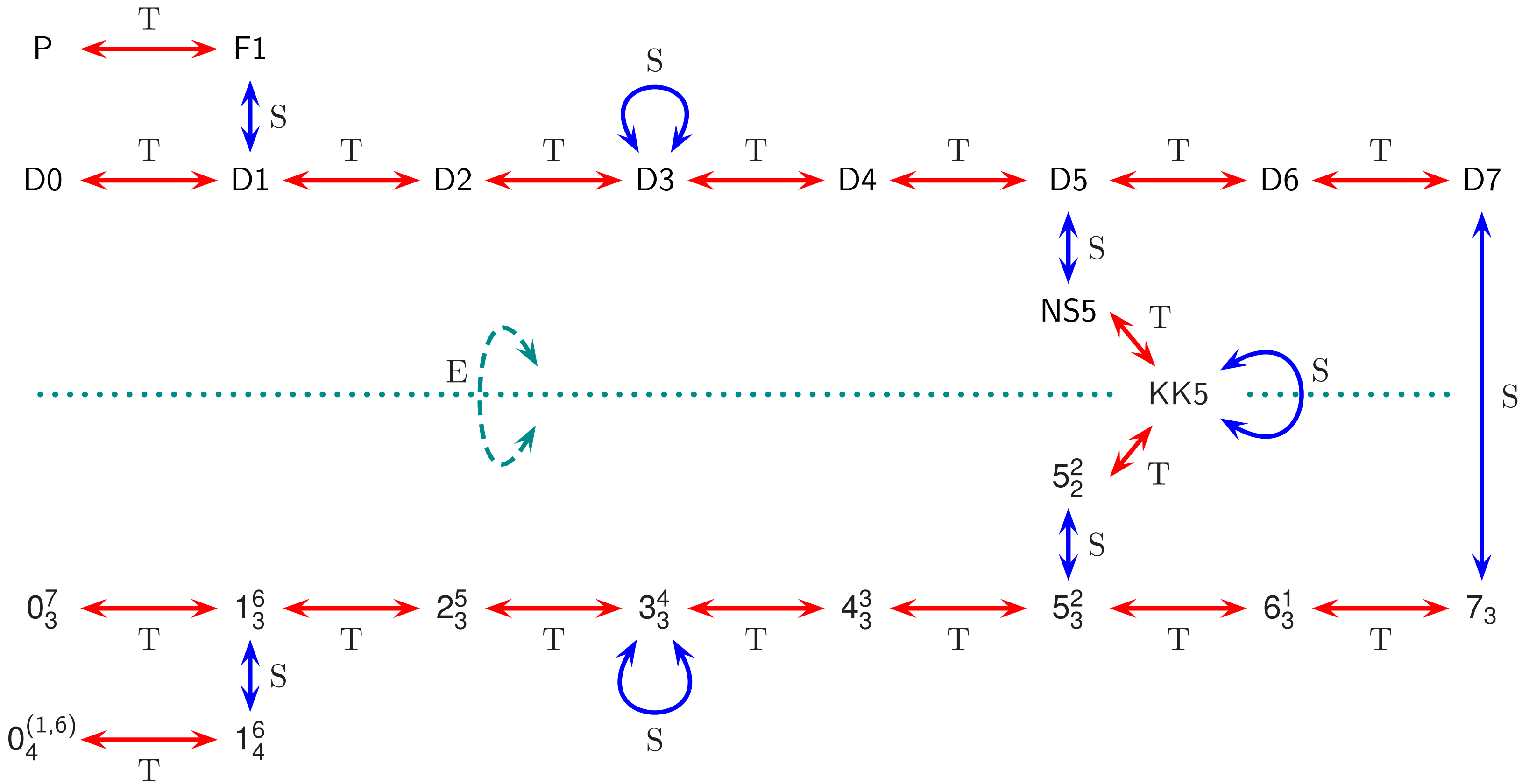
$SU(2)$ gauge w/ $n = 5, 6, 7$ flavors $\rightarrow$ SCFT w/ E_{n+1} symmetry



After re-ordering $\mathbf{A}^n\mathbf{BCBC} \;=\; \mathbf{A}^n\mathbf{BCCX}_{[3,1]}$,

$\mathbf{A}^n\mathbf{BCC}$ are collapsible at one point! $\rightarrow$ $\mathbf{E}_{n+1}$-brane

Nongeometric $\boldsymbol{SL(2,\mathbb{Z})}$ doublets : $(1^6_4, 1^6_3)$-string and $(5^2_3, 5^2_2)$-brane



These pairs never couple to $[\boldsymbol{p},\boldsymbol{q}]$ 7-brane in the conventional type IIB supergravity

Go beyond it $\rightarrow$ Exceptional Field Theories (EFTs)

EFTs in diverse dimensions :

D	U-duality group	arXiv	D	U-duality group	arXiv
9	$E_{2(2)}(\mathbb{Z}) = SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	1512.06115	5	$E_{6(6)}(\mathbb{Z})$	1312.0614 1412.7286
8	$E_{3(3)}(\mathbb{Z}) = SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$	1501.01600	4	$E_{7(7)}(\mathbb{Z})$	1312.4542
7	$E_{4(4)}(\mathbb{Z}) = SL(5,\mathbb{Z})$	1302.1652 1412.0635 1512.02163	3	$E_{8(8)}(\mathbb{Z})$	1406.3348
6	$E_{5(5)}(\mathbb{Z}) = SO(5,5;\mathbb{Z})$	1504.01523			

$SL(n+1;\mathbb{Z}) \subseteq E_{n(n)}(\mathbb{Z})$ in $(11-n)$-dim : 1402.5027