Semi-doubled GLSM for Five-branes of Codimension Two

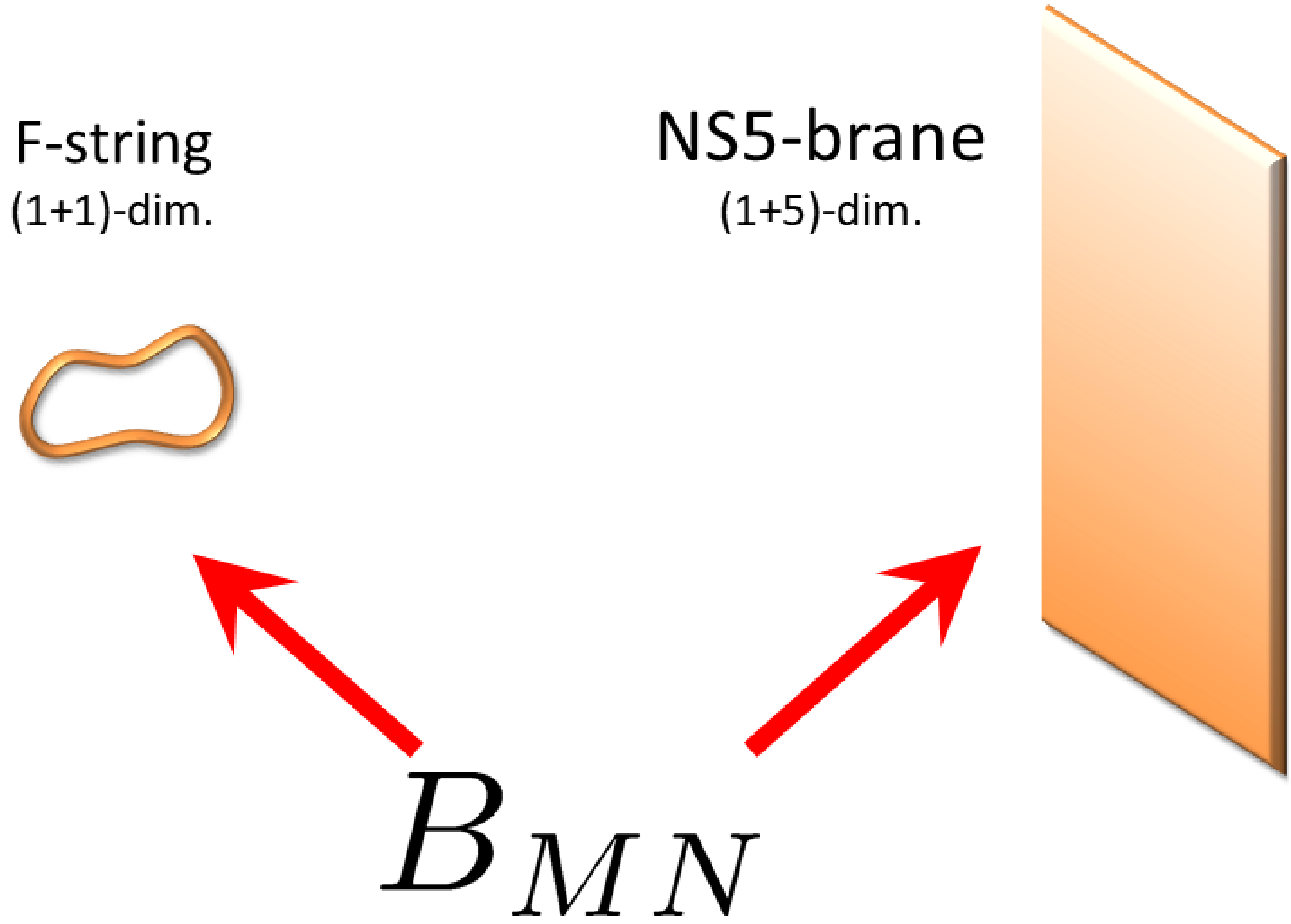
arXiv:1810.02169

木村 哲士

日本大学理工学部

佐々木伸氏、塩沢健太氏 (北里大学) との共同研究

1. 5-branes of codim-2


F-string
(1+1)-dim.
NS5-brane
(1+5)-dim.
B_{MN}

Compactify on $\boldsymbol{T}^2$

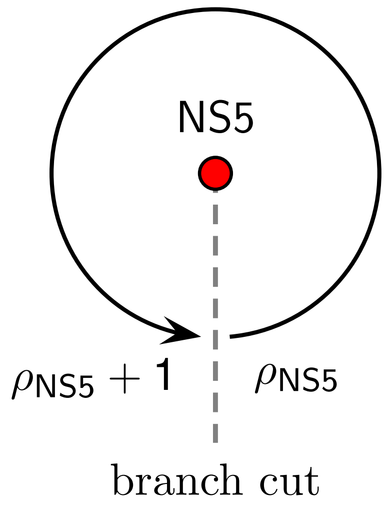
↓

NS5-brane of codim-2 in 8D string theory = defect NS5-brane

$$\mathrm{d}s^2 = \underbrace{\mathrm{d}x^2_{012345}}_{\text{NS5-brane}} + H\Big[\underbrace{(\mathrm{d}r)^2 + r^2(\mathrm{d}\theta)^2}_{\text{codim-2}} + \underbrace{(\mathrm{d}\varphi)^2 + (\mathrm{d}\vartheta)^2}_{\boldsymbol{T}^2}\Big]$$

$$B_{\varphi\vartheta} = \frac{\theta}{2\pi}, \qquad \mathrm{e}^{2\phi} = H = \frac{1}{2\pi}\log\frac{\Lambda}{r}$$

$$\rho_{\text{NS5}} = B_{\varphi\vartheta} + \mathrm{i}H = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\frac{\Lambda}{r}$$

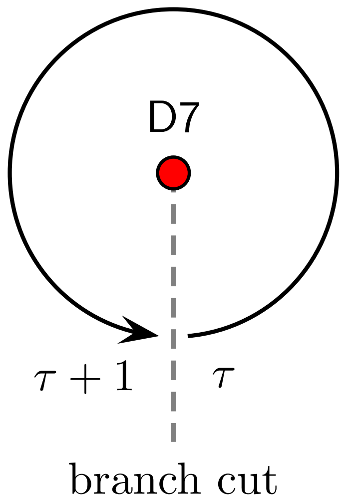


similar to D7-brane in type IIB string

D7-brane in 10D type IIB string is coupled to

$$\tau(z) \equiv C + \mathrm{i}e^{-\phi} = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi} \log \frac{\Lambda}{\rho}$$

magnetic “charge” (monodromy) : $\tau \to \tau + 1$



feature of τ $\to$ F-theory

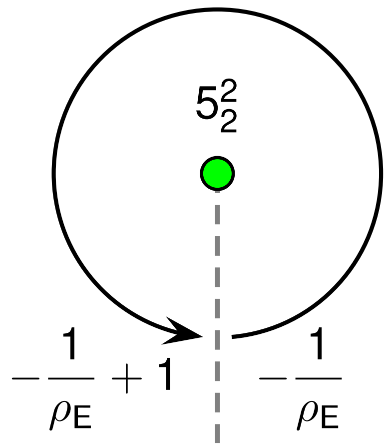
Vafa (1996)

defect NS5-brane $\xrightarrow{\text{T-dual along } T^2}$ exotic 5^2_2-brane

$$ds^2 = \underbrace{dx^2_{012345}}_{5^2_2\text{-brane}} + H\Big[\underbrace{(dr)^2 + r^2(d\theta)^2}_{\text{codim-2}}\Big] + \frac{H}{K}\Big[\underbrace{(d\widetilde{\varphi})^2 + (d\widetilde{\vartheta})^2}_{\text{dual } T^2}\Big]$$

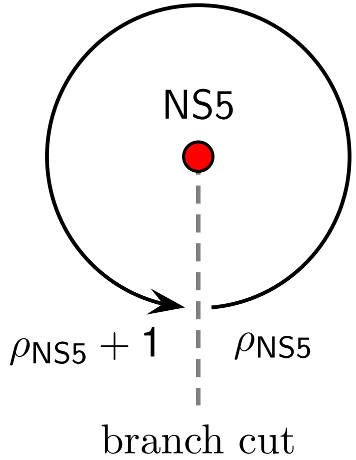
$$B_{\varphi\vartheta} = -\frac{\theta}{2\pi K}, \quad e^{2\phi} = \frac{H}{K}, \quad K = H^2 + \Big(\frac{\theta}{2\pi}\Big)^2$$

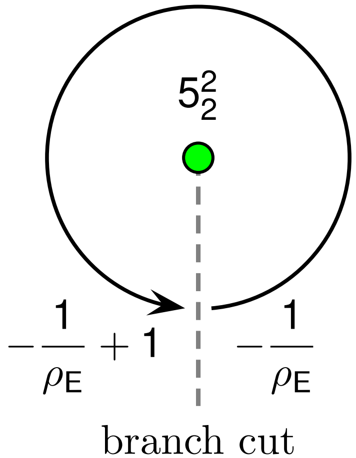
$$\rho_{\mathsf{E}} = -\Big[\frac{\theta}{2\pi} + \frac{i}{2\pi}\log\frac{\Lambda}{r}\Big]^{-1} = -\frac{1}{\rho_{\mathsf{NS5}}}$$



codim-2 branes	D7	defect NS5
D	10B	8 (+ T^2)
coupled to	$\tau = C + \mathrm{i}e^{-\phi}$	$\rho_{\mathrm{NS5}} =$ Kähler str. of T^2
monodromy	$SL(2,\mathbb{Z})_S$	$SL(2,\mathbb{Z})_K$
pair	NS7 ($[0,1]$7 or 7_3)	5^2_2

$$SL(2,\mathbb{Z})_K \subset SO(2,2;\mathbb{Z}) : \text{ T-duality along } \boldsymbol{T}^2$$



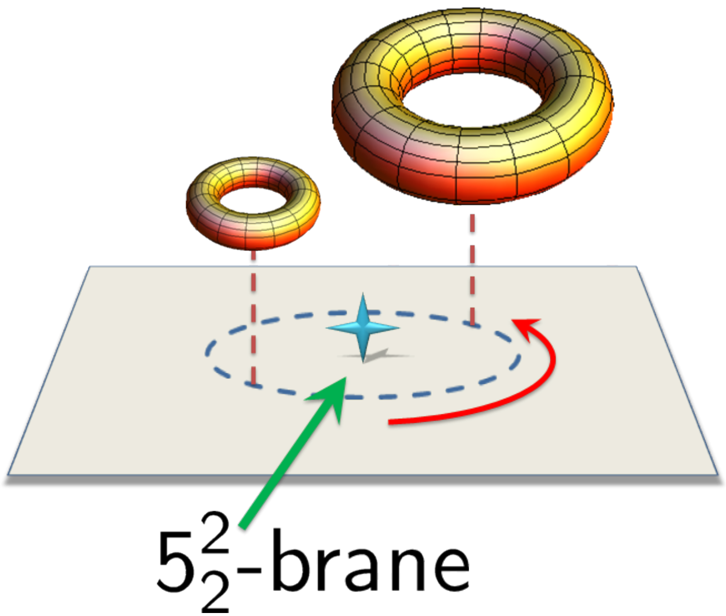


We investigated 5-branes of codim-2 in string worldsheet sigma model,

and understood the classical background geometry.

We also investigated the worldsheet instanton corrections along $S^1 \subset T^2$

in terms of ANO vortex corrections in the UV gauge theory (GLSM).



How can we obtained the full corrections along T^2 ?

Worldsheet instantons on

defect NS5 : point-like instantons, i.e., small instantons arrayed along T^2

$\to$ deforms background geometry

$\to$ KK-modes along T^2

Witten (1996); Tong (2002)

5^2_2-brane : disk instantons

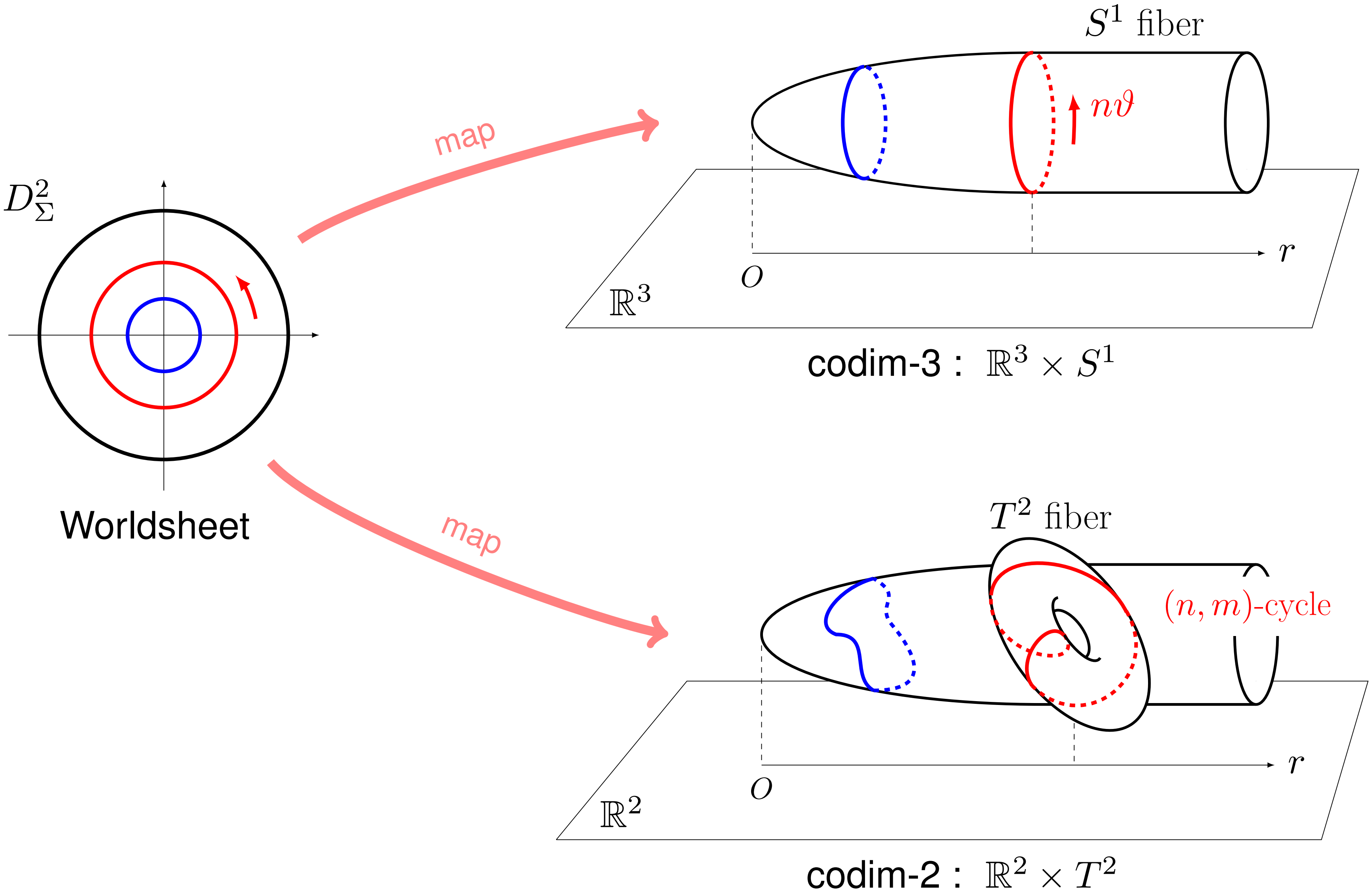
$\to$ does not appear as deformation of background geometry

$\to$ Winding modes along T^2

Gregory, Harvey, Moore (1997); Tong (2002); Harvey, Jensen (2005); Okuyama (2005); TK, Sasaki (2013)

analyzed in our DFT computation (TK, Sasaki, Shiozawa arXiv:1803.11087)

Worldsheet instantons as disk instantons



2. Formulation

2D $\mathcal{N} = (2,2)$ superfield formalism

$$e^{\int \vartheta F}$$

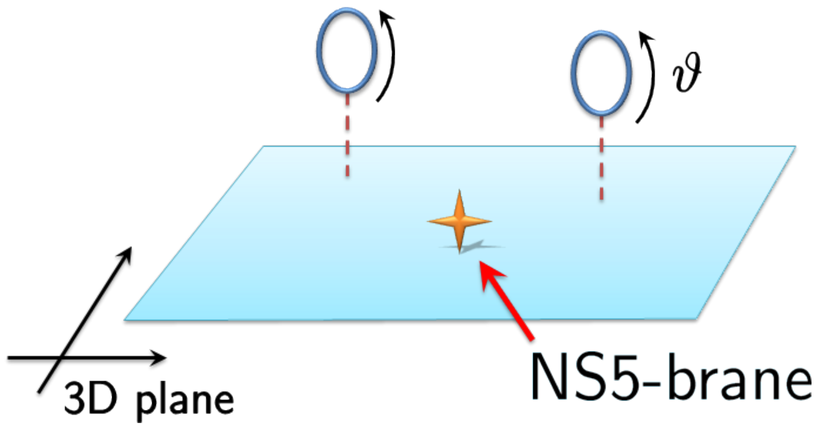
theta-angle in 2D path integral

$$e^{\int \vartheta F}$$ is provided by twisted superpotential $$e^{\int \Theta \Sigma}$$

$\Theta,\, \Sigma = \overline{D}_+ D_- V$:   twisted chiral

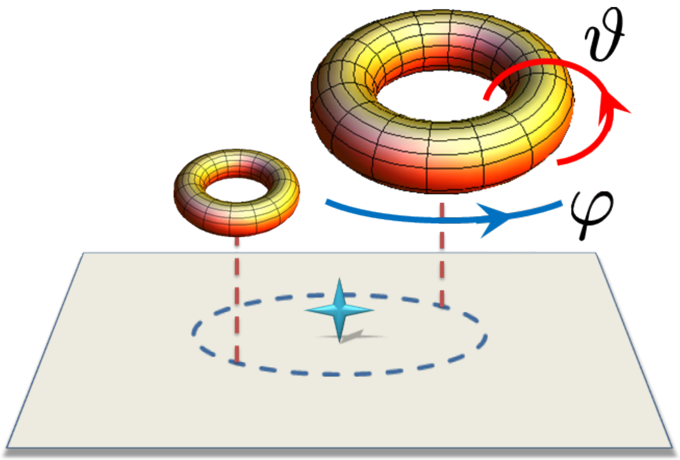
ϑ, as imaginary part of Θ, receives corrections by Chern class.

This is applied to worldsheet instanton corrections of 5-branes of codim-3.



Tong (2002); Harvey, Jensen (2005); Okuyama (2005)

Apply this to gauge theory for 5-brane of codim-2 with $\boldsymbol{S^1_{\vartheta} \times S^1_{\varphi} = T^2}$.



$$\rightarrow \quad \mathrm{e}^{\int \vartheta F + \int \varphi \widehat{F}}$$

$$\rightarrow \quad \mathrm{e}^{\int \Theta \Sigma + \mathrm{i} \int \Theta \widehat{\Sigma}}$$

Worldsheet instantons can be mapped to corrections to T^2.

Available duality transformation w/ irreducible superfields ?

Roček, Verlinde (1991); Hori, Vafa (2000)

$$-\frac{1}{g^2}\int \mathrm{d}^4\theta\,|\Theta|^2 - \int \mathrm{d}^2\widetilde{\theta}\,\Theta\big(\Sigma + \mathrm{i}\widehat{\Sigma}\big) + \mathrm{h.c.}$$

dualize imaginary part $\downarrow$ twisted chiral $\Theta \to$ chiral Γ

$$+\frac{g^2}{2}\int \mathrm{d}^4\theta\,(\Gamma + \overline{\Gamma} + 2(V + \mathrm{i}\widehat{V}))^2$$

dualize real part $\downarrow$ chiral $\Gamma \to$ twisted chiral

??

use reducible superfields

Grisaru, Massar, Sevrin, Troost (1998); TK (2015)

Duality transformation by reducible superfield :

$$\mathscr{L} = \int \mathrm{d}^4\theta \Big(-\frac{1}{g^2}|R|^2 - 2(R+\overline{R})V - 2\mathrm{i}(R-\overline{R})\widehat{V} - R\,L - \overline{R}\,\overline{L} \Big)$$

- integrate out L : $\mathscr{L} = -\frac{1}{g^2}\int \mathrm{d}^4\theta\,|\Theta|^2 - \int \mathrm{d}^2\widetilde{\theta}\,\Theta\big(\Sigma + \mathrm{i}\widehat{\Sigma}\big) + \mathrm{h.c.}$

$\updownarrow$ dual $(1/g^2 \leftrightarrow g^2)$

- integrate out R : $\mathscr{L} = +g^2 \int \mathrm{d}^4\theta\,\big|L + 2(V + \mathrm{i}\widehat{V})\big|^2$

L : twisted linear supefield $0 = \overline{D}_+ D_- L$

$$L = X + \overline{W} + Y$$

X, W : chiral, Y : twisted chiral

Grisaru, Massar, Sevrin, Troost (1998); TK (2015)

3. Worldsheet instantons

Intrinsic part :

$$S = \int d^2\sigma \left\{ \frac{1}{2e^2}F_{12}^2 + |D_m q|^2 + \frac{e^2}{2}(|q|^2 - t)^2 + i\vartheta F_{12} + \frac{1}{2\hat{e}^2}\hat{F}_{12}^2 + |D_m p|^2 + \frac{\hat{e}^2}{2}(|p|^2 - \hat{t})^2 + i\varphi \hat{F}_{12} \right\} + \dots$$

$$\geq |\vec{t}|^2 \sqrt{|\vec{Q}|^2} - i\vec{\vartheta}\cdot\vec{Q}$$

$$\vec{\vartheta} = (\vartheta, \varphi)\,, \quad \vec{t} = (t, \hat{t})\,, \quad \vec{Q} = \left(-\int d^2\sigma\, F_{12}\,, -\int d^2\sigma\, \hat{F}_{12} \right)$$

Chern numbers

BPS eqs are given as Abrikosov-Nielsen-Olesen (ANO) vortex eq.

$$F_{12} \mp e^2(|q|^2 - t) = 0\,, \quad (D_1 \pm iD_2)q = 0$$

- Semi-doubled NLSM (classical)

$$\begin{aligned}\mathscr{L} \;&=\; -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2 + (\partial_m \varphi)^2 + (\partial_m \vartheta)^2\big\} + \Omega\,\epsilon^{mn}(\partial_m \varphi)(\partial_n \vartheta) \\ &\quad +\epsilon^{mn}(\partial_m \varphi)(\partial_n \widetilde{\gamma}) + \epsilon^{mn}(\partial_m \vartheta)(\partial_m \gamma)\end{aligned}$$

provides string sigma model for defect NS5 and 5^2_2 (skip)

with $H = \log \frac{\Lambda}{r}\,, \quad \Omega = \arctan\left(\frac{y}{x}\right), \quad r = \sqrt{x^2 + y^2}$

- Worldsheet instantons originate from ANO vortices

$$H \;\to\; H + \sum_{(n,m)\neq(0,0)} \mathrm{e}^{\mathrm{i}n\vartheta}\mathrm{e}^{\mathrm{i}m\varphi}\mathrm{e}^{-r\sqrt{n^2+m^2}}$$

Witten (1993); Morrison, Plesser (1994); Schroers (1996)

Worldsheet instantons on

defect NS5 : point-like instantons, i.e., small instantons arrayed along T^2

$\rightarrow$ deforms background geometry

$\rightarrow$ KK-modes along T^2

Witten (1996); Tong (2002)

5^2_2-brane : disk instantons

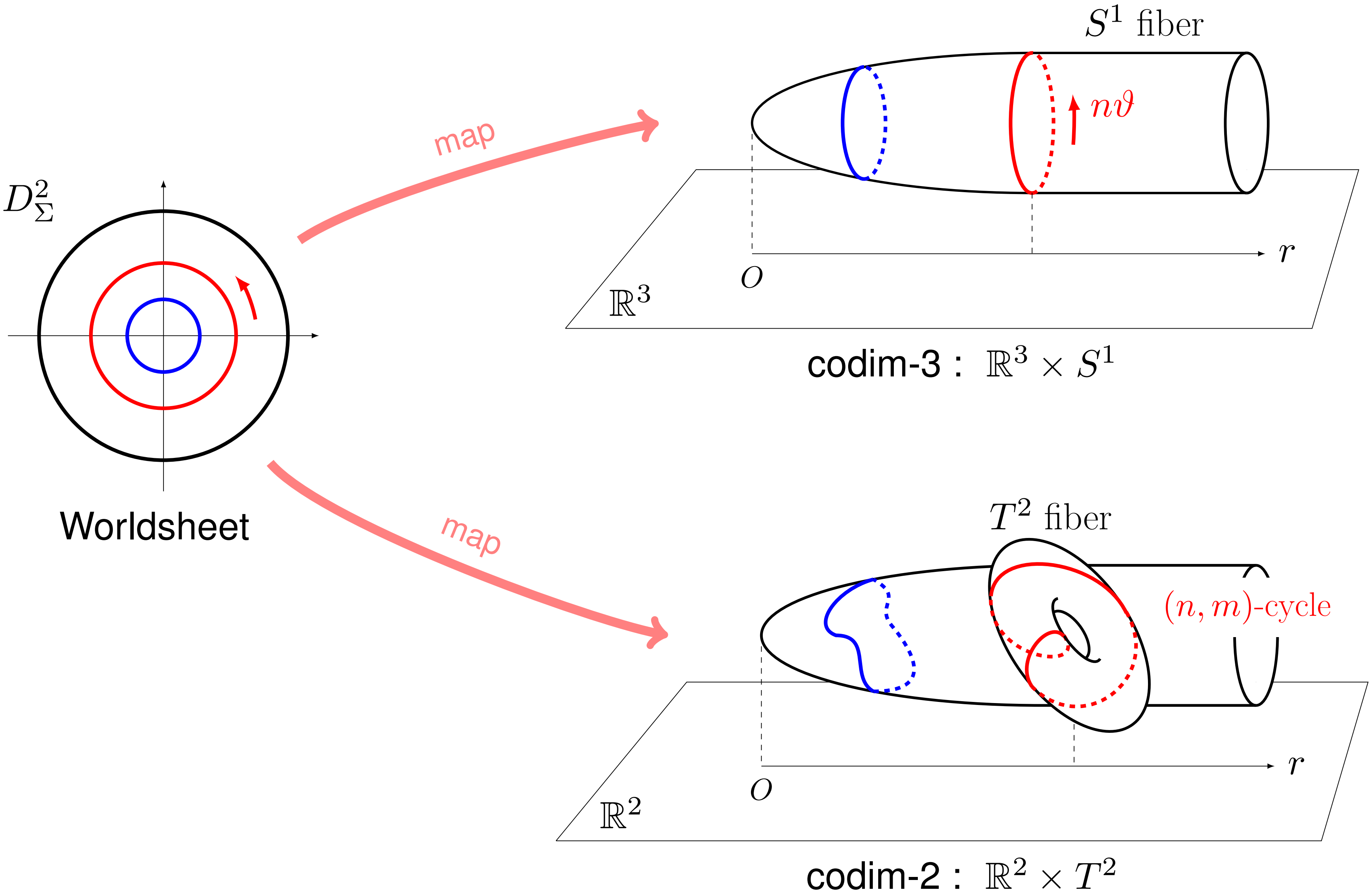
$\rightarrow$ does not appear as deformation of background geometry

$\rightarrow$ Winding modes along T^2

Gregory, Harvey, Moore (1997); Tong (2002); Harvey, Jensen (2005); Okuyama (2005); TK, Sasaki (2013)

confirmed our DFT computation (TK, Sasaki, Shiozawa arXiv:1803.11087)

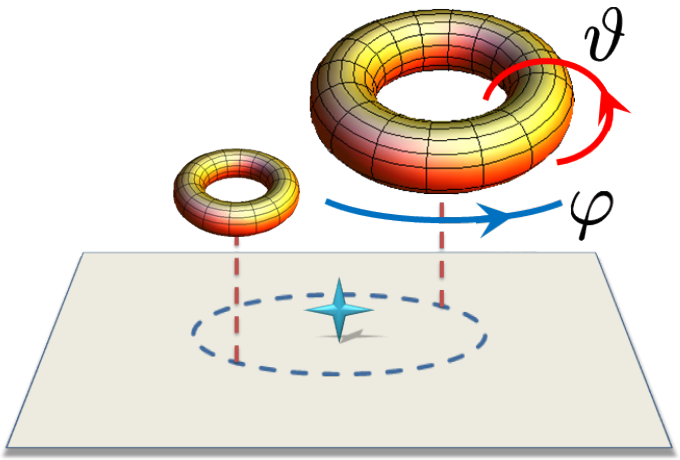
Worldsheet instantons as disk instantons



4. Summary

- We captured string worldsheet instanton corrections via ANO vortices in GLSM.
- These instanton corrections are understood as corrections by KK-modes and/or winding modes along $\boldsymbol{T^2}$.

To realize above, we developed duality transformations by reducible superfield.



Thanks

Appendix

A. Duality transformations

Duality transformation :

$$\mathscr{L} \; = \; -\frac{1}{2g^2}(\partial_m \vartheta)^2 + \epsilon^{mn}(\partial_m \vartheta)(\partial_n \gamma)$$

- integrate out γ : $\mathscr{L} \; = \; -\frac{1}{2g^2}(\partial_m \vartheta)^2$
- integrate out ϑ : $\mathscr{L} \; = \; -\frac{g^2}{2}(\partial_m \gamma)^2$

It is well known if only ϑ in Θ is dualized.

Roček, Verlinde (1991); Hori, Vafa (2000)

However, how to dualize both ϑ and φ in Θ ?

$$e^{\int \Theta \Sigma + i \int \Theta \widehat{\Sigma}}$$

technically disturbs the standard T-duality (Hori-Vafa) transformation :

$$\begin{aligned} \Theta + \overline{\Theta} \quad &\Leftrightarrow \quad -(\Gamma + \overline{\Gamma}) - 2V \\ \text{and/or} \quad \Theta - \overline{\Theta} \quad &\Leftrightarrow \quad -(\Gamma - \overline{\Gamma}) + 2i\widehat{V} \end{aligned}$$

by parity transform along x^--direction

Duality transformation by irreducible superfields : Roček, Verlinde (1991); Hori, Vafa (2000)

$$\mathscr{L} \; = \; \int \mathrm{d}^4\theta \Big(- \frac{1}{2g^2} B^2 - 2BV - 2(\Gamma + \overline{\Gamma})B \Big)$$

- integrate out Γ : $\mathscr{L} \; = \; -\dfrac{1}{g^2} \displaystyle\int \mathrm{d}^4\theta \, |\Theta|^2 - \int \mathrm{d}^2\widetilde{\theta} \, \Theta\Sigma + \mathrm{h.c.}$
- integrate out B : $\mathscr{L} \; = \; +\dfrac{g^2}{2} \displaystyle\int \mathrm{d}^4\theta \, (\Gamma + \overline{\Gamma} + 2V)^2$

How to generalize this method to $\Theta(\Sigma + \mathrm{i}\widehat{\Sigma})$?

Role of X, W, Y :

$$\mathrm{Im}\,(X + \overline{W}) = \mathrm{Im}\,\Theta \quad \xleftrightarrow{\text{dual}} \quad \mathrm{Im}\,(X - \overline{W})$$

$$\mathrm{Re}\,(X + \overline{W}) = \mathrm{Re}\,\Theta \quad \xleftrightarrow{\text{dual}} \quad \mathrm{Re}\,(X - \overline{W})$$

Y : link LHS with RHS

Different from the duality transformation by irreducible superfields,

we can dualize real/imaginary part of Θ without any disturbance !

$L = X + \overline{W} + Y$ carries both original and dual fields $\to$ “doubled” GLSM

B. **Another GLSM for 5-branes of codim-2**

“Semi-doubled” GLSM for 5-branes of codim-2 $\mathscr{L} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{CHM}} + \mathscr{L}_{\text{NHM}}$:

$$\mathscr{L}_{\text{gauge}} = \int d^4\theta \left\{ \frac{1}{e^2}(-|\Sigma|^2 + |\Phi|^2) + \frac{1}{\widehat{e}^2}(-|\widehat{\Sigma}|^2 + |\widehat{\Phi}|^2) \right\}$$

$$\begin{aligned}\mathscr{L}_{\text{CHM}} &= \int d^4\theta \left\{ |Q|^2 e^{+2V} + |\widetilde{Q}|^2 e^{-2V} \right\} - \int d^2\theta \, \widetilde{Q}\Phi Q + \text{h.c.} \\ &\quad + \int d^4\theta \left\{ |P|^2 e^{+2\widehat{V}} + |\widetilde{P}|^2 e^{-2\widehat{V}} \right\} - \int d^2\theta \, \widetilde{P}\widehat{\Phi} P + \text{h.c.}\end{aligned}$$

$$\begin{aligned}\mathscr{L}_{\text{NHM}} &= \int d^4\theta \left\{ \frac{1}{g^2}|\Psi|^2 + g^2 \big| L + 2(V + i\widehat{V}) \big|^2 \right\} \\ &\quad + \left\{ \int d^2\theta \, (s - \Psi)\Phi + \int d^2\widetilde{\theta} \, t\Sigma + \text{h.c.} \right\} + \epsilon^{mn} \partial_m (\vartheta A_n) \\ &\quad + \left\{ \int d^2\theta \, (\widehat{s} - \Psi)\widehat{\Phi} + \int d^2\widetilde{\theta} \, \widehat{t\Sigma} + \text{h.c.} \right\} + \epsilon^{mn} \partial_m (\varphi \widehat{A}_n)\end{aligned}$$

skip the detail...

Semi-doubled NLSM

$$\begin{aligned}\mathscr{L} &= -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2 + (\partial_m \varphi)^2 + (\partial_m \vartheta)^2\big\} + \Omega\, \epsilon^{mn}(\partial_m \varphi)(\partial_n \vartheta) \\ &\quad + \epsilon^{mn}(\partial_m \varphi)(\partial_n \widetilde{\gamma}) + \epsilon^{mn}(\partial_m \vartheta)(\partial_m \gamma)\end{aligned}$$

provides string sigma model for defect NS5, KK-vortex, and 5^2_2.

dNS5 : $$\mathscr{L} = -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2 + (\partial_m \varphi)^2 + (\partial_m \vartheta)^2\big\} + \Omega\, \epsilon^{mn}(\partial_m \varphi)(\partial_n \vartheta)$$

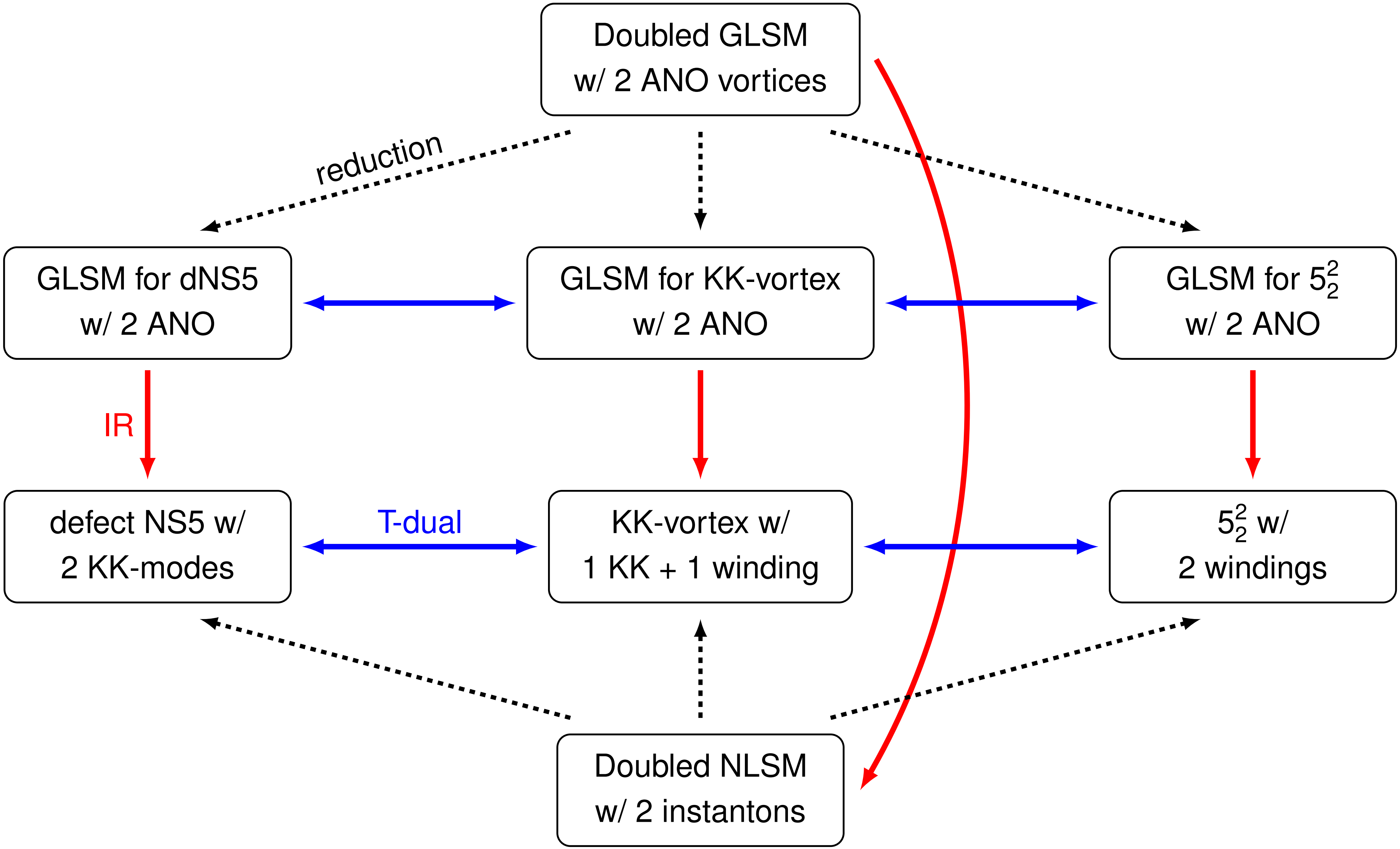
KK-v : $$\mathscr{L} = -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2 + (\partial_m \varphi)^2\big\} - \frac{1}{2H}(\partial_m \gamma - \Omega\, \partial_m \varphi)^2$$

5^2_2 : $$\mathscr{L} = -\frac{H}{2}\big\{(\partial_m x)^2 + (\partial_m y)^2\big\} - \frac{H}{K}\big\{(\partial_m \widetilde{\gamma})^2 + (\partial_m \gamma)^2\big\} - \frac{\Omega}{K}\epsilon^{mn}(\partial_m \widetilde{\gamma})(\partial_n \gamma)$$

$$H = \log\frac{\Lambda}{r}\,, \quad \Omega = \arctan\Big(\frac{y}{x}\Big)\,, \quad r = \sqrt{x^2+y^2}\,, \quad K = H^2 + \Omega^2$$

instanton corrections : $$H \ \to \ H + \sum_{(n,m)\neq(0,0)} \mathrm{e}^{\mathrm{i}n\vartheta}\mathrm{e}^{\mathrm{i}m\varphi} \ K_0\big(r\sqrt{n^2+m^2}\big)$$

modified Bessel of 2nd kind


Doubled GLSM
w/ 2 ANO vortices
reduction
GLSM for dNS5
w/ 2 ANO
GLSM for KK-vortex
w/ 2 ANO
GLSM for 5^2_2
w/ 2 ANO
IR
defect NS5 w/
2 KK-modes
T-dual
KK-vortex w/
1 KK + 1 winding
5^2_2 w/
2 windings
Doubled NLSM
w/ 2 instantons