Resonances in hadron physics

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2013, Oct. 24th

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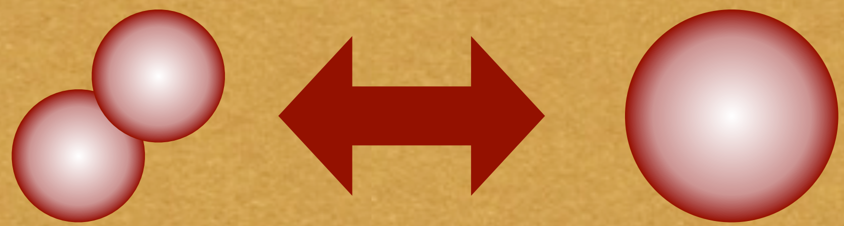
Part I : Compositeness of hadron resonances

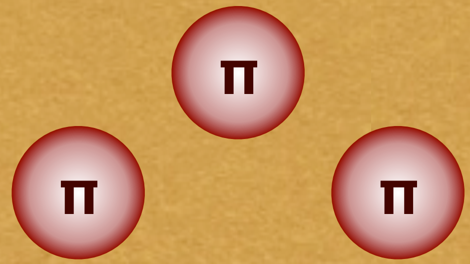
T. Hyodo, Phy. Rev. Lett. 111, 132002 (2013)

T. Hyodo, arXiv:1310.1176 [hep-ph]



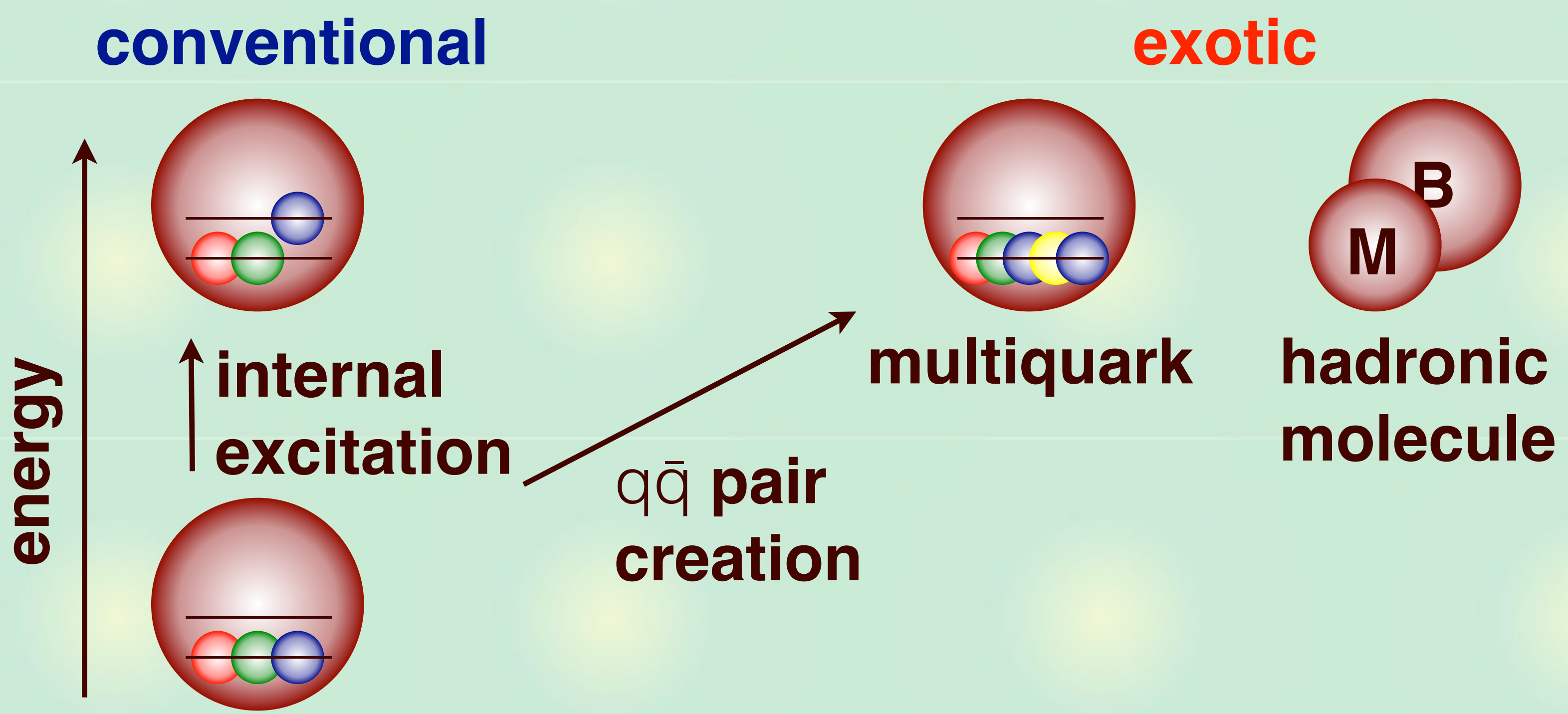
Part II : Universal thee-pion physics

T. Hyodo, T. Hatsuda, Y. Nishida, *in preparation*



Exotic structure of hadrons

Various excitations of baryons



Physical state: superposition of 3q, 5q, MB, ...

$$|\,\Lambda(1405)\,\rangle = \underline{N_{3q}}|\,uds\,\rangle + \underline{N_{5q}}|\,uds\;q\bar{q}\,\rangle + \underline{N_{\bar{K}N}}|\,\bar{K}N\,\rangle + \cdots$$

Find out the dominant component among others.

Structure of resonances?

Excited states : finite width (unstable against strong decay)

- **stable (ground) states**
- **unstable states**

Most of hadrons are unstable!

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**
$N(1900)$	$3/2^+$	***
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$		*
$\Lambda(2020)$	$7/2^+$	*
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	**
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1770)$	$1/2^+$	*
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1840)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^-$	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	***
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2980)$		***
$\Xi_c(3055)$		**
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ_{cc}^+		*
Λ_b^0	$1/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
Ω_b^-	$1/2^+$	***

PDG12

State vector of resonance?

?

$$|\,\Lambda(1405)\,\rangle = N_{3q}|\,uds\,\rangle + N_{5q}|\,uds\;q\bar{q}\,\rangle + N_{\bar{K}N}|\,\bar{K}N\,\rangle + \cdots$$

We need a classification scheme applicable to resonances.

Compositeness of bound states

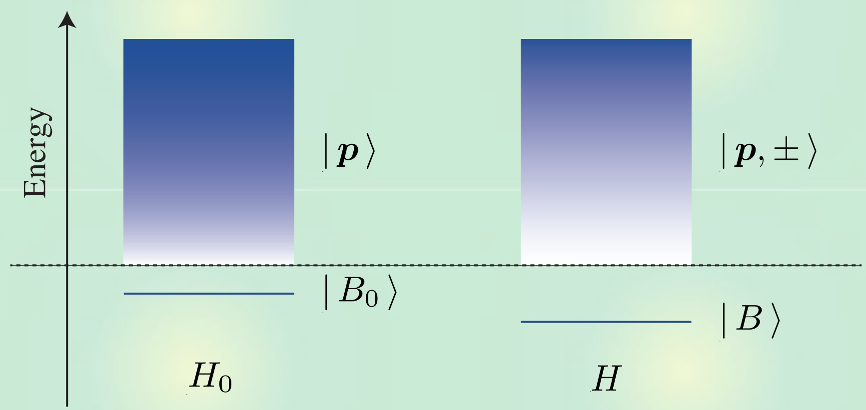
Compositeness approach: decompose Hamiltonian

S. Weinberg, Phys. Rev. 137, B672 (1965); T. Hyodo, arXiv:1310.1176 [hep-ph]

$$H = H_0 + V$$

Complete set for free Hamiltonian: bare $|B_0>$ + continuum

$$1 = |\, B_0 \,\rangle\langle\, B_0 \,| + \int d\boldsymbol{p} |\, \boldsymbol{p} \,\rangle\langle\, \boldsymbol{p} \,|$$

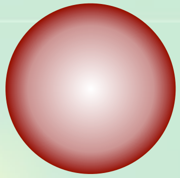


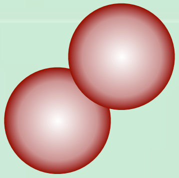
Physical bound state $|B>$

$$H|\, B \,\rangle = -B|\, B \,\rangle, \quad \langle\, B \,|\, B \,\rangle = 1$$

$$1 = \underline{\langle\, B \,|\, B_0 \,\rangle\langle\, B_0 \,|\, B \,\rangle} + \int \underline{d\boldsymbol{p}\langle\, B \,|\, \boldsymbol{p} \,\rangle\langle\, \boldsymbol{p} \,|\, B \,\rangle}$$

Z : elementariness **X : compositeness**





Z, X : real and nonnegative --> probabilistic interpretation

$$\Rightarrow 0 \leq Z \leq 1, \quad 0 \leq X \leq 1$$

Weak binding limit

In general, Z depends on the choice of the potential V.

- Z : model-(scheme-)dependent quantity

$$1 - Z = \int d\boldsymbol{p} \frac{|\langle \boldsymbol{p} |V| B \rangle|^2}{(E_p + B)^2}$$

In the weak binding limit, Z is related to observables

S. Weinberg, Phys. Rev. 137, B672 (1965); T. Hyodo, arXiv:1310.1176 [hep-ph]

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(R_{\rm typ}), \quad r_e = \frac{-Z}{1-Z} R + \mathcal{O}(R_{\rm typ}),$$

a **: scattering length,** r_e **: effective range**

$R = (2\mu B)^{-1/2}$ **: radius (binding energy)**

$R_{\rm typ}$ **: typical length scale of the interaction**

Criterion for the structure:

$$\begin{cases} a \sim R_{\rm typ} \ll -r_e & \text{(elementary dominance)}, \quad Z \sim 1 \\ a \sim R \gg r_e \sim R_{\rm typ} & \text{(composite dominance)}. \quad Z \sim 0 \end{cases}$$

Interpretation of negative effective range

For $Z>0$, effective range is always negative.

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(R_{\rm typ}), \quad r_e = \frac{-Z}{1-Z} R + \mathcal{O}(R_{\rm typ}),$$

$$\begin{cases} a \sim R_{\rm typ} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\rm typ} & \text{(composite dominance)}. \end{cases}$$

Simple attractive potential: $r_e > 0$

--> only "composite dominance" is possible.

$r_e < 0$ **: energy- (momentum-)dependence of the potential**

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998)
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

<-- pole term/Feshbach projection of coupled-channel effect

Negative r_e **--> Something other than** $|p\rangle$ **: CDD pole**

Application to resonances

Compositeness approach at the weak binding:

- Model-independent (no potential, wavefunction, ...)
- Related to experimental observables
- Only for bound states with small binding

Application to general resonances

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)
F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012)

- Z and X are in general complex. Interpretation?

$$\langle R | R \rangle \to \infty, \quad \langle \tilde{R} | R \rangle = 1$$

$$1 = \langle \tilde{R} | B_0 \rangle \langle B_0 | R \rangle + \int d\boldsymbol{p} \langle \tilde{R} | \boldsymbol{p} \rangle \langle \boldsymbol{p} | R \rangle$$

What about near-threshold resonances (~ small binding) ?

Poles of the amplitude

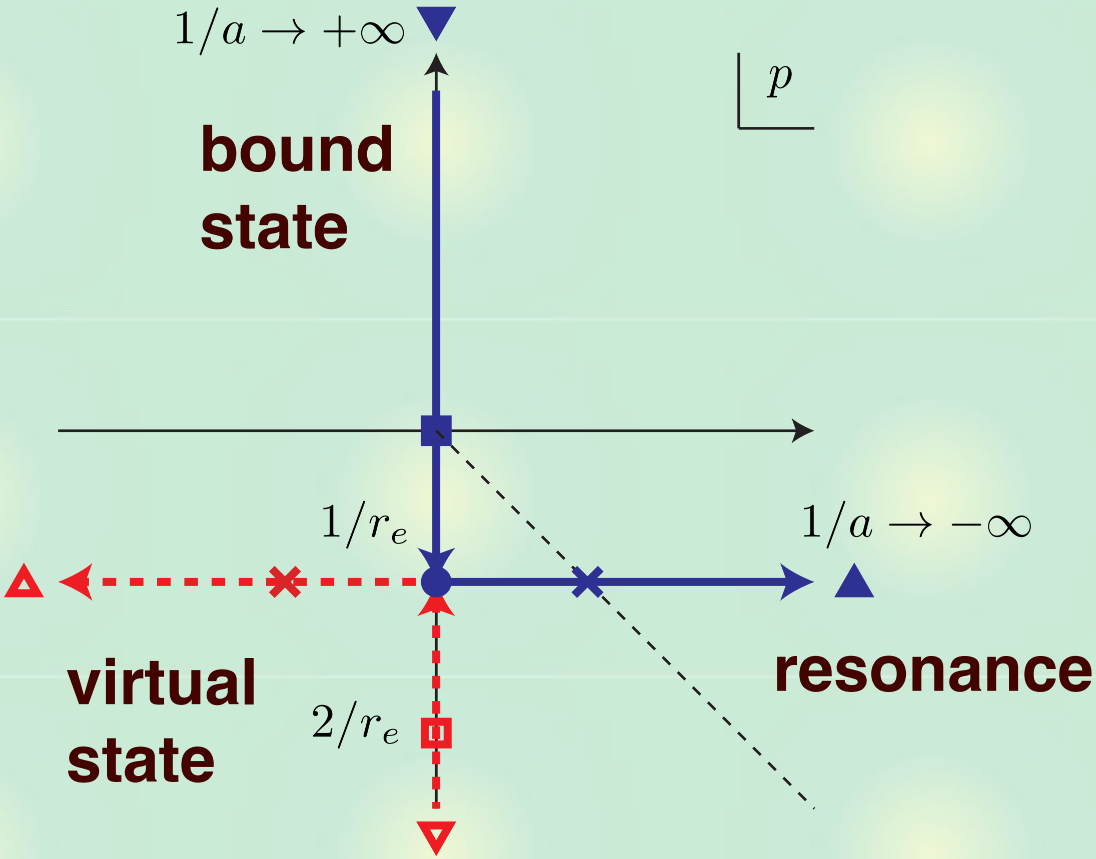
Near-threshold phenomena: effective range expansion

T. Hyodo, Phy. Rev. Lett. 111, 132002 (2013) with opposite sign of scattering length

$$f(p) = \left(-\frac{1}{a} - pi + \frac{r_e}{2} p^2 \right)^{-1}$$

$$p^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e} \sqrt{\frac{2r_e}{a} - 1}$$

Pole trajectories with a fixed $r_e < 0$



Resonance pole position <--> (a, r_e)

Example of resonance: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering

- central values in PDG

$$E = 0.67 \text{ MeV}, \quad \Gamma = 2.59 \text{ MeV}$$

- deduced threshold parameters

$$a = -\frac{p^+ + p^-}{ip^+p^-} = -10.5 \text{ fm}, \quad r_e = \frac{2i}{p^+ + p^-} = -19.5 \text{ fm}$$

- field renormalization constant: complex

$$Z = 1 - 0.608i$$

Large negative effective range

<-- substantial elementary contribution other than $\pi\Sigma_c$ (three-quark, other meson-baryon channel, or ...)

$\Lambda_c(2595)$ **is not likely a $\pi\Sigma_c$ molecule**

Part I : Summary

Composite/elementary nature of resonances

- Renormalization constant Z measures elementariness of a stable bound state.
- In general, Z of a resonance is complex.
- Negative effective range r_e : CDD pole
- Near-threshold resonance: pole position is related to r_e --> elementariness

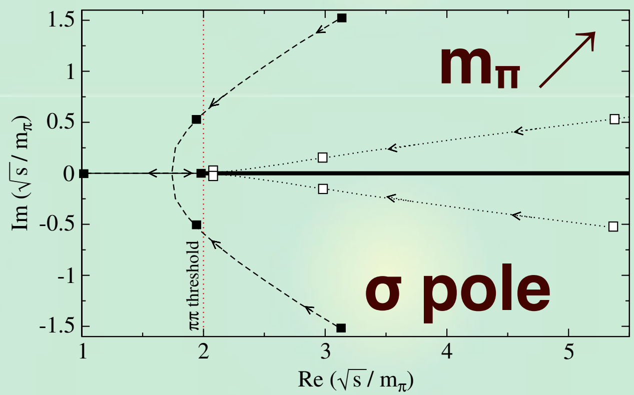
T. Hyodo, Phy. Rev. Lett. 111, 132002 (2013)
T. Hyodo, arXiv:1310.1176 [hep-ph]

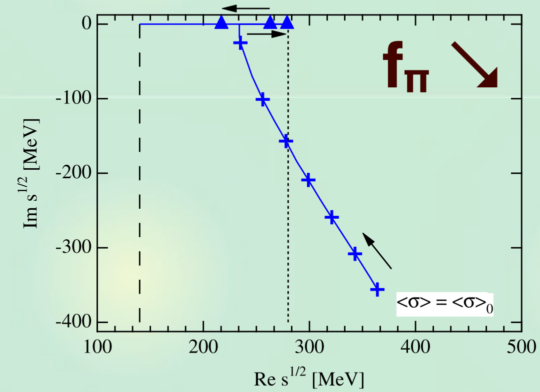
Universal phenomena in hadron physics

Universal few-body physics <-- large scattering length

S-wave ππ scattering length

- $a_{I=0}$ ~ -0.31 fm, $a_{I=2}$ ~ 0.06 fm / QCD scale ~ 1 fm
- I=0 component **can be increased** by m_π ↗ or f_π ↘





C. Hanhart, J.R. Pelaez, G. Rios, Phys. Rev. Lett. 100, 152001 (2008)
T. Hyodo, D. Jido, T. Kunihiro, Nucl. Phys. A848, 341-365 (2010)

- Realizable by lattice QCD / nuclear medium

==> Three-pion system with a large scattering length

Isospin symmetric three pions

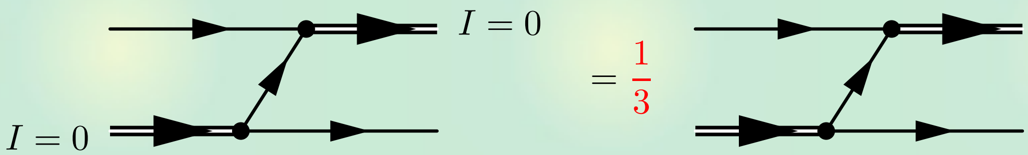
Pion has an internal degree of freedom : isospin I=1

- **s-wave two-body amplitude:** I=0 **and** I=2

$$it_0(p) = \frac{8\pi}{m} \frac{i}{\frac{1}{a} - \sqrt{\frac{\boldsymbol{p}^2}{4} - mp_0 - i0^+}}, \quad it_2(p) = 0$$

S-wave three-pion system in total I=1

$$\begin{pmatrix} |\,\pi \otimes [\pi \otimes \pi]_{I=0}\,\rangle_{I=1} \\ |\,\pi \otimes [\pi \otimes \pi]_{I=2}\,\rangle_{I=1} \end{pmatrix} = \begin{pmatrix} 1/3 & \sqrt{5}/3 \\ \sqrt{5}/3 & 1/6 \end{pmatrix} \begin{pmatrix} |\,[\pi \otimes \pi]_{I=0} \otimes \pi\,\rangle_{I=1} \\ |\,[\pi \otimes \pi]_{I=2} \otimes \pi\,\rangle_{I=1} \end{pmatrix}$$



Eigenvalue equation (eigenvalue B$_3$ for eigenfunction z(|p|))

$$z(|\boldsymbol{p}|) = \frac{2}{3\pi} \int_0^\infty d|\boldsymbol{q}| \frac{|\boldsymbol{q}|}{|\boldsymbol{p}|} \ln\left(\frac{\boldsymbol{q}^2 + \boldsymbol{p}^2 + |\boldsymbol{q}||\boldsymbol{p}| + mB_3}{\boldsymbol{q}^2 + \boldsymbol{p}^2 - |\boldsymbol{q}||\boldsymbol{p}| + mB_3} \right) \frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2 + mB_3} - \frac{1}{a}}$$

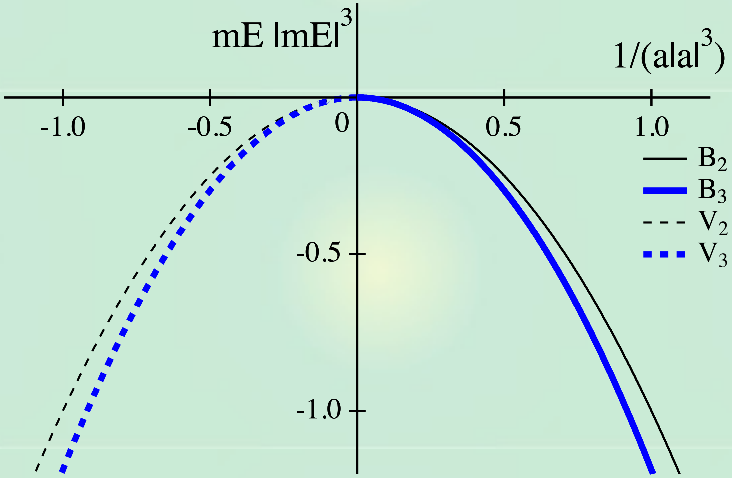
Factor 1/3 difference from the identical boson case

Spectrum in the isospin symmetric limit

Result: one universal three-pion bound state

$$B_3 = \frac{1.04391}{ma^2} \quad \text{for } 1/a > 0$$

$$\text{c.f.} \quad B_2 = \frac{1}{ma^2}$$



Resonances?

- phase rotation of binding energy = phase rotation of a

$$B_3 \to B_3 e^{i\theta} \quad \Leftrightarrow \quad \frac{1}{a} \to \frac{1}{a} e^{-i\theta/2}$$

Negative a: virtual state

<-- rotation of B_3 by 2π = sign flip of a

No resonance for all a

<-- interchange of Riemann sheet = sign flip of a

With isospin breaking

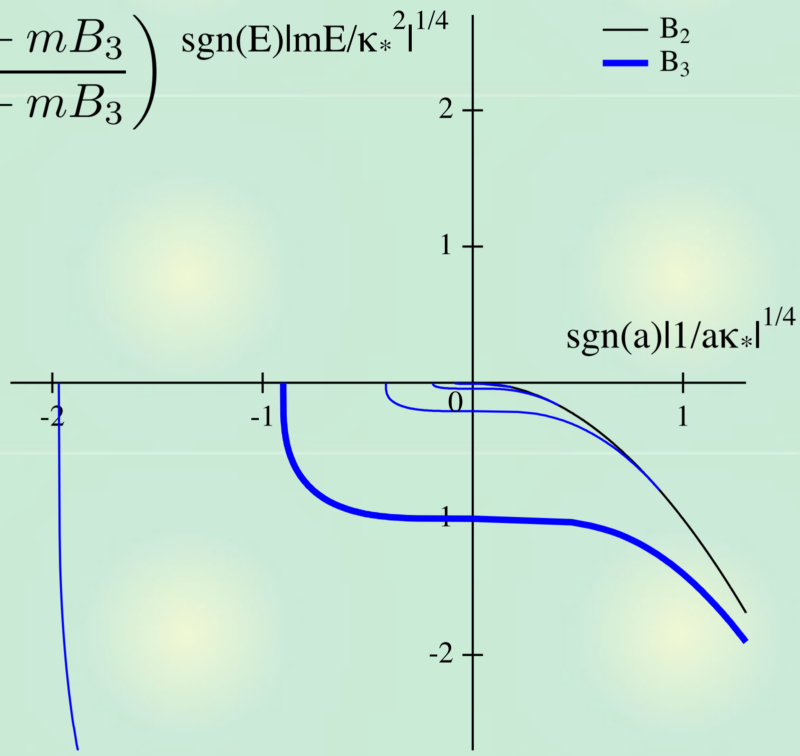
In nature, $m_{\pi^\pm} = m_{\pi^0} + \Delta$ **with** $\Delta > 0$

- In the energy region $E \ll \Delta$**, heavy** $\pi^\pm$ **can be neglected.**

Identical three-boson system with a large scattering length
--> Efimov effect

$$z(|\boldsymbol{p}|) = \frac{2}{\pi}\int_0^\infty d|\boldsymbol{q}|\frac{|\boldsymbol{q}|}{|\boldsymbol{p}|}\ln\left(\frac{\boldsymbol{q}^2+\boldsymbol{p}^2+|\boldsymbol{q}||\boldsymbol{p}|+mB_3}{\boldsymbol{q}^2+\boldsymbol{p}^2-|\boldsymbol{q}||\boldsymbol{p}|+mB_3}\right) \times \frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2+mB_3}-\frac{1}{a}} f_\Lambda(|\boldsymbol{q}|)$$

cutoff



Universal physics at $E \ll (2m\Lambda)^{1/2}$

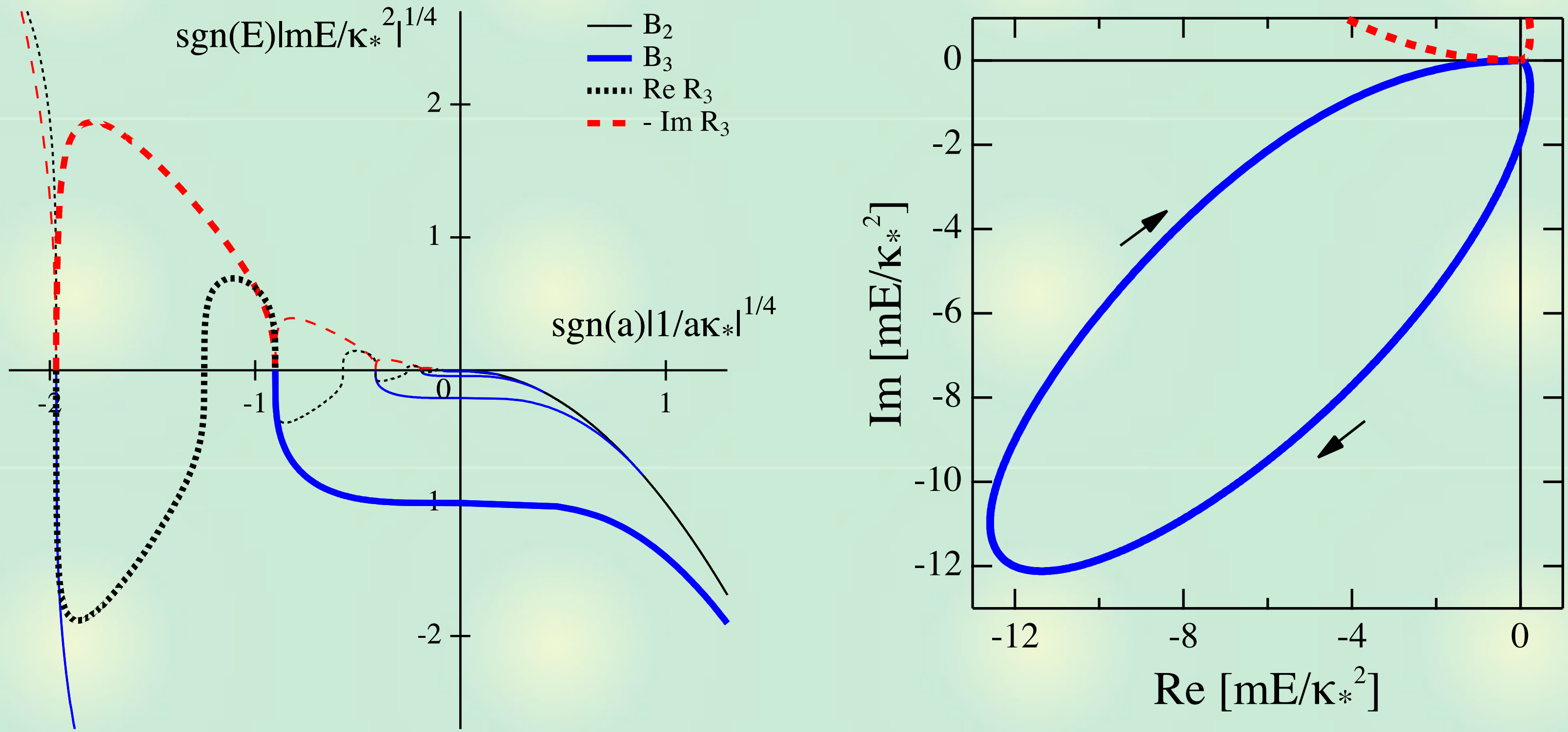
<-- Efimov parameter κ_*

Efimov resonances

Resonance solution is now possible.

- phase rotation of binding energy = phase rotation of a and Λ + proper treatment of singularity in $f_\Lambda(|\mathbf{q}|)$

$$B_3 \to B_3 e^{i\theta} \quad \Leftrightarrow \quad \frac{1}{a} \to \frac{1}{a} e^{-i\theta/2} \quad \text{and} \quad \Lambda \to \Lambda e^{-i\theta/2}$$



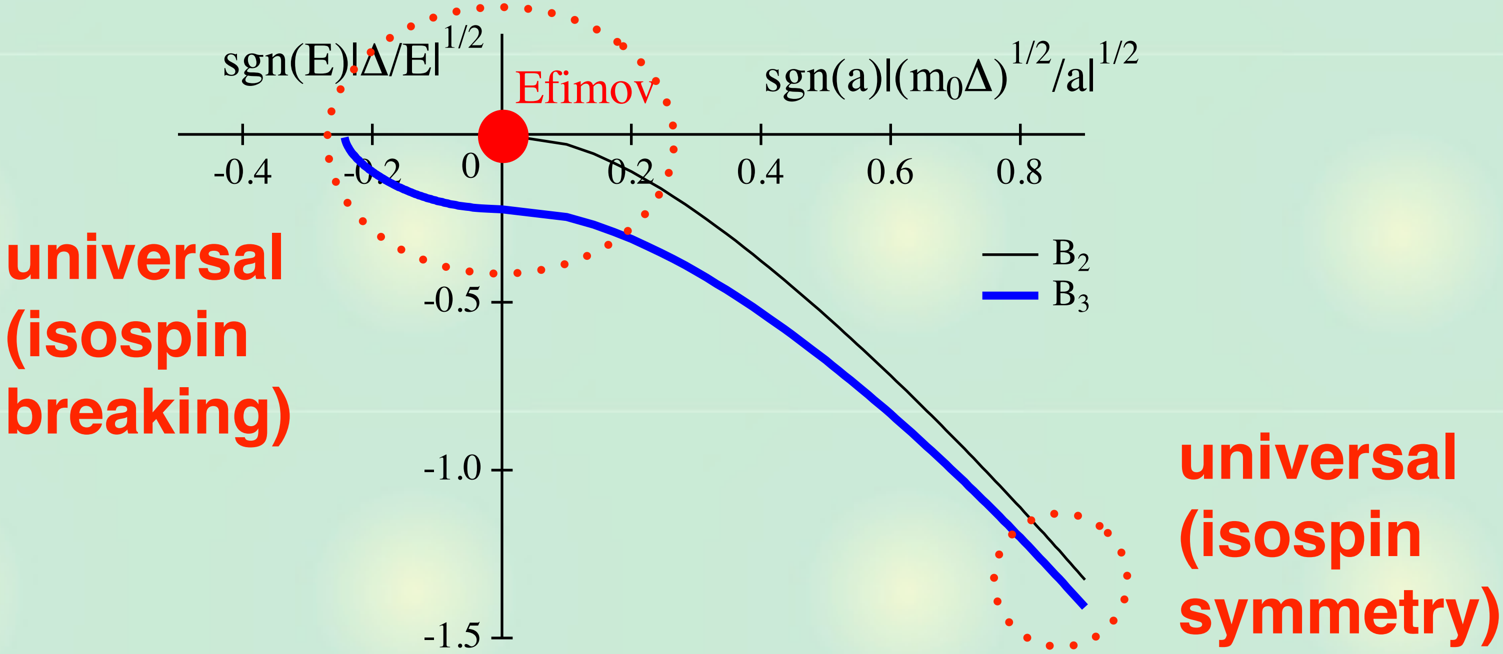
Efimov bound state --> resonance

Interpolation by model

A model with finite mass difference Δ = $m_{\pi^\pm} - m_{\pi^0}$

$$\mathcal{L} = \sum_{i=0,\pm} \pi_i^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_i} - m_i \right) \pi_i + \frac{g}{4} \frac{\pi_0^\dagger \pi_0^\dagger - 2\pi_+^\dagger \pi_-^\dagger}{\sqrt{3}} \frac{\pi_0 \pi_0 - 2\pi_- \pi_+}{\sqrt{3}}$$

- E ≪ Δ **: Efimov states,** (Λ ≫) E ≫ Δ **: single bound state**
- **cutoff for the Efimov effect is introduced by Δ.**



Lowest Efimov level --> universal bound state

Part II : Summary

Universal physics of three pions

- **Large $\pi\pi$ scattering length (I=0) can be realized by $m_\pi \nearrow$ or $f_\pi \searrow$.**
- **With isospin symmetry: single three-body bound state for I=1, J=0.**

  --> turns into virtual state
- **With isospin breaking: Efimov states for three neutral pions.**

  --> turn into resonances

T. Hyodo, T. Hatsuda, Y. Nishida, *in preparation*