Universal physics of three-bosons with isospin

Tetsuo Hyodo

Yukawa Institute for Theoretical Physics, Kyoto

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- Introduction: universal few-body physics
- Tuning pion interaction
- Three-pion systems
- Realization and consequences


T. Hyodo, T. Hatsuda, Y. Nishida, arXiv:1311.6289 [hep-ph]

Universal physics

Universal: **different systems share the identical feature**

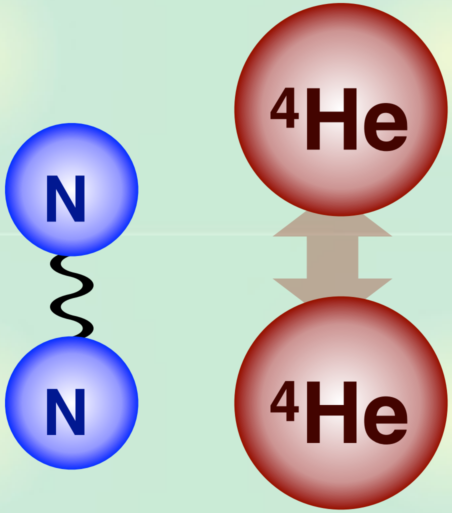
Critical phenomena around phase transition

- large correlation length ξ
- scaling, critical exponent, ...
- liquid-gas transition ~ ferromagnet

N. Goldenfeld, *"Lectures on phase transitions and the renormalization group"* (1992)

Universal physics in few-body system

- large two-body scattering length |a|
- "scaling", Efimov effect, ...
- ^{4}He atom (vdW) ~ nucleon (strong)



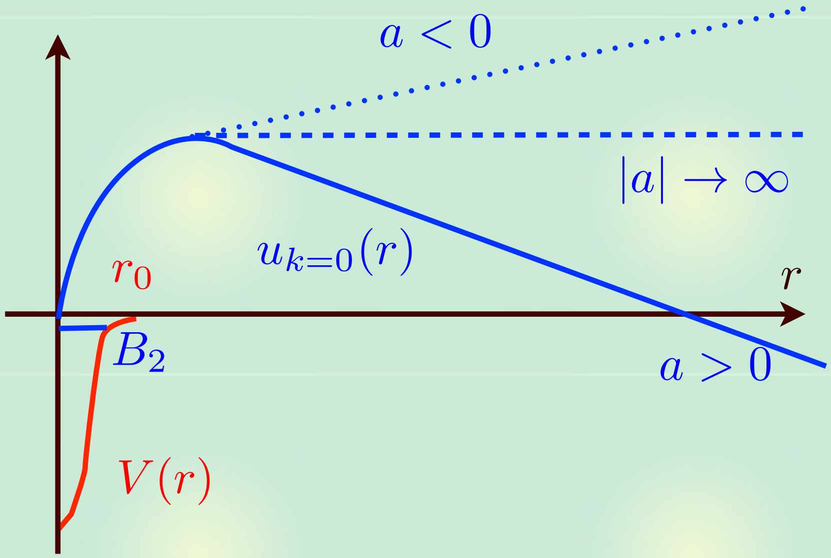
E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)

^{4}He

Two-body system

We consider the low-energy phenomena (1/p ≫ r_0) of the system with large scattering length (|a| ≫ r_0).

$$f(\theta, p) = \sum_l (2l+1) f_l(p) P_l(\cos\theta)$$
$$\to f_0(p)$$
$$= \frac{1}{p\cot\delta_0(p) - ip}$$
$$\to \frac{1}{-1/a - ip}$$



Consequence: one shallow bound state exists for a ≫ 0

$$B_2 = \frac{1}{2\mu a^2}, \quad \hbar = 1,$$

- **determined only by** a
- **scale invariance**

$$a \to \lambda a, \quad p \to \lambda^{-1} p \quad E \to \lambda^{-2} E$$

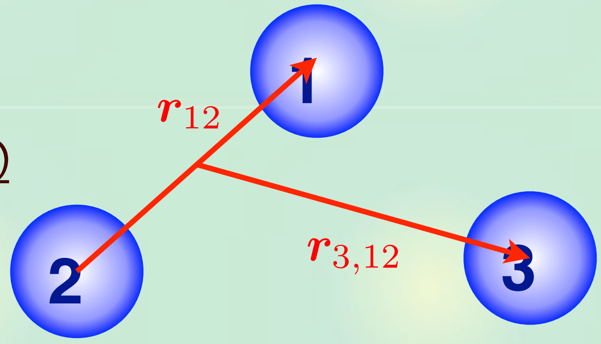
	N [MeV]	^{4}He [mK]
B_2	**2.22**	**1.31**
$1/2\mu a^2$	**1.41**	**1.12**

Three-body system: scaling and its violation

Three-body system in hyperspherical coordinates

$$(\boldsymbol{r}_{12}, \boldsymbol{r}_{3,12}) \leftrightarrow (\underline{R}, \underline{\alpha_3, \hat{\boldsymbol{r}}_{12}, \hat{\boldsymbol{r}}_{3,12}})$$

hyperradius **hyperangular variables Ω (dimensionless)**



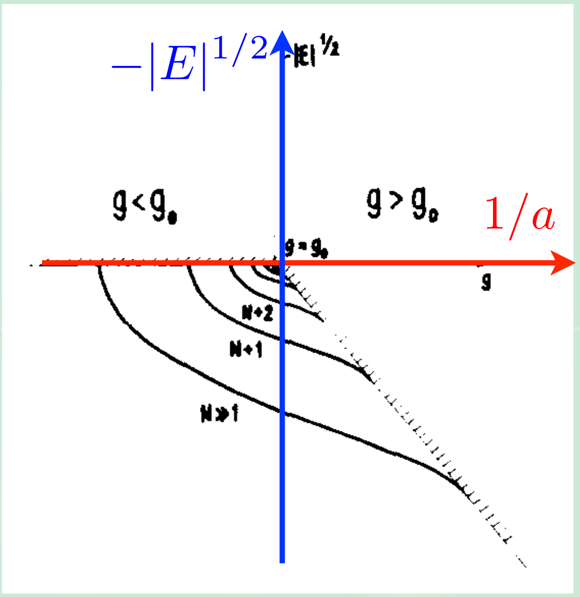
For $r_0/a \to 0$, system is scale invariant.

$$V(R, \Omega) \propto \frac{1}{R^2}$$

Efimov effect : attractive $1/R^2$ for identical three bosons

V. Efimov, Phys. Lett. B 33, 563-564 (1970)

- **infinitely many bound states**
- **discrete scale invariance --> limit cycle**



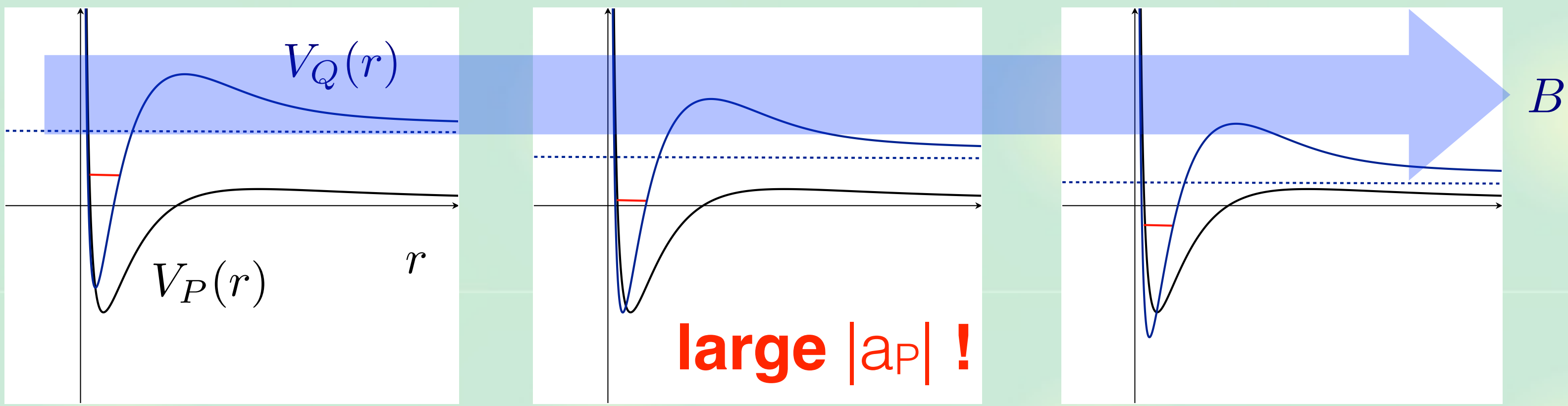
P.F. Bedaque, H.-W. Hammer, U. van Kolck, Phys. Rev. Lett. 82, 463-437 (1999)

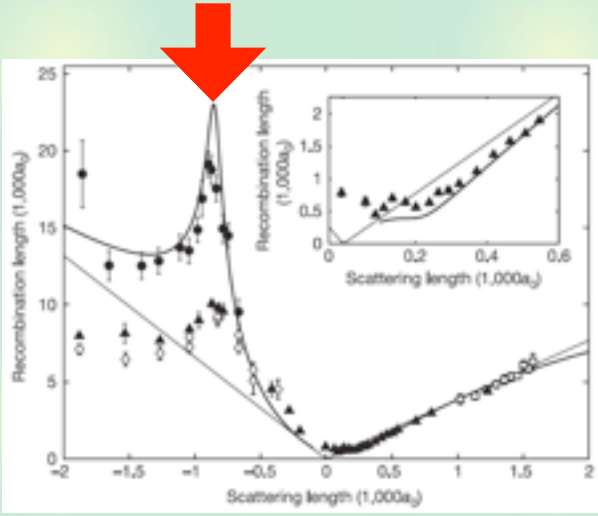
Experimental realization

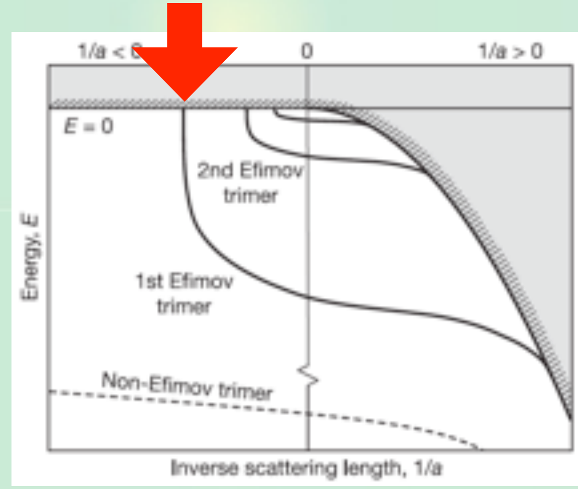
Experimental realization by ultracold cesium atoms

T. Kraemer *et al.*, Nature 440, 315 (2006)

- tuning a by magnetic field (Feshbach resonance)







Universal theory <==> data (three-body recombination rate)

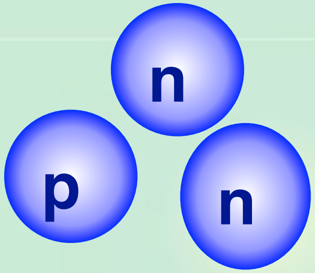
Hadrons with a large scattering length

Hadron systems ($r_0 \sim 1$ fm) with a large scattering length

- nucleon system

V. Efimov, Phys. Lett. B 33, 563-564 (1970)

E. Braaten, H.-W. Hammer, Phys. Rev. Lett. 91, 102002 (2003)



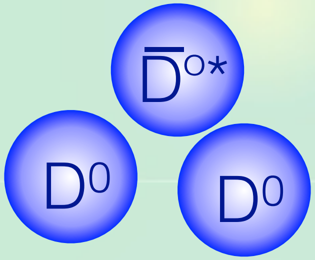
2.2 MeV ↕ —— p+n / —— d

$a_{pn} \sim -22$ fm ($^1S_0$)
~ 5 fm ($^3S_1$)

=> ^{3}H

- charmed meson system (D~c$\bar{u}$, c$\bar{d}$)

E. Braaten, M. Kusunoki, Phys. Rev. D 69, 074005 (2004)



0.1-0.5 MeV? ↕ —— $D^0+\bar{D}^{0*}$ / —— X(3872)

$a_{D^0\bar{D}^{0*}} \sim 6$-$14$ fm

=> not bound

These are the examples of accidental fine tuning.
Is there a "Feshbach resonance"?

Introduction to pion

Yukawa: pion mediates the nuclear force

H. Yukawa, Proc. Phys. Math. Soc. Jap. 17, 48-57 (1935)

- pseudoscalar particle
- isospin I=1
- lightest hadron (~ 140 MeV)

On the Interaction of Elementary Particles. I.

By Hideki YUKAWA.

(Read Nov. 17, 1934)

§1. Introduction

At the present stage of the quantum theory litt
the nature of interaction of elementary particles. He
the interaction of "Platzwechsel" between the neut
to be of importance to the nuclear structure.(1)

Nambu: spontaneous breaking of chiral symmetry

Y. Nambu, G. Jona-Lasinio, Phys. Rev. 124, 246-254 (1961)

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†

The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.
The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

Pion interaction

Interaction <- chiral low energy theorem

- S-wave ππ scattering length

S. Weinberg, Phys. Rev. Lett. 17, 616-621 (1966)

$$a^{I=0} \propto -\frac{7}{4}\frac{m_\pi}{f_\pi^2}, \quad a^{I=2} \propto \frac{1}{2}\frac{m_\pi}{f_\pi^2}$$

attractive **repulsive**

The scattering lengths are proportional to

- $1/f_\pi^2$ **~ spontaneous breaking of chiral symmetry**
- m_π **~ explicit breaking of chiral symmetry**

In nature, the scattering lengths are small:

- $a^{I=0}$ ~ -0.31 fm, $a^{I=2}$ ~ 0.06 fm **/ QCD scale ~** 1 fm

<-- explicit symmetry breaking is small.

Tuning pion interaction

If we can adjust m_π or f_π, $|a|$ increases by m_π ↗ or f_π ↘

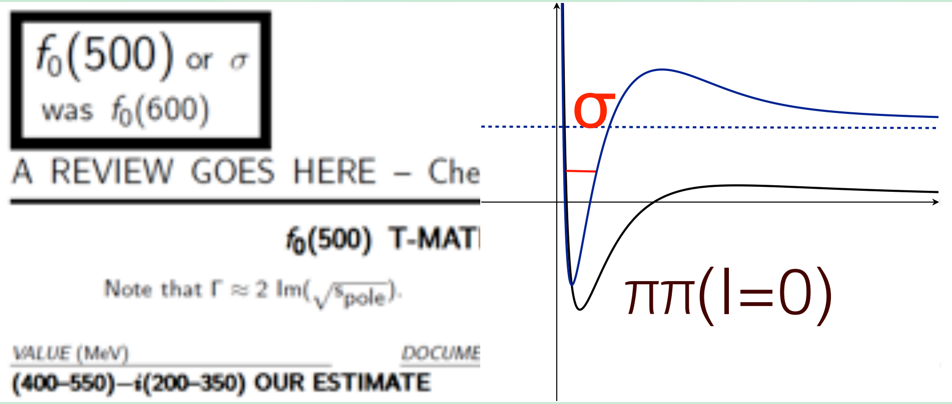
$$a^{I=0} \propto -\frac{7}{4}\frac{m_\pi}{f_\pi^2}, \quad a^{I=2} \propto \frac{1}{2}\frac{m_\pi}{f_\pi^2}$$

Can $|a|$ be extremely large?

- **low energy theorem ~ Born approximation**
- **sufficient attraction --> bound state in I=0 --> diverging $|a|$**

σ meson: resonance in ππ scattering

- **scalar particle**
- **isospin I=0**
- **experimentally established**
- **chiral partner of π**

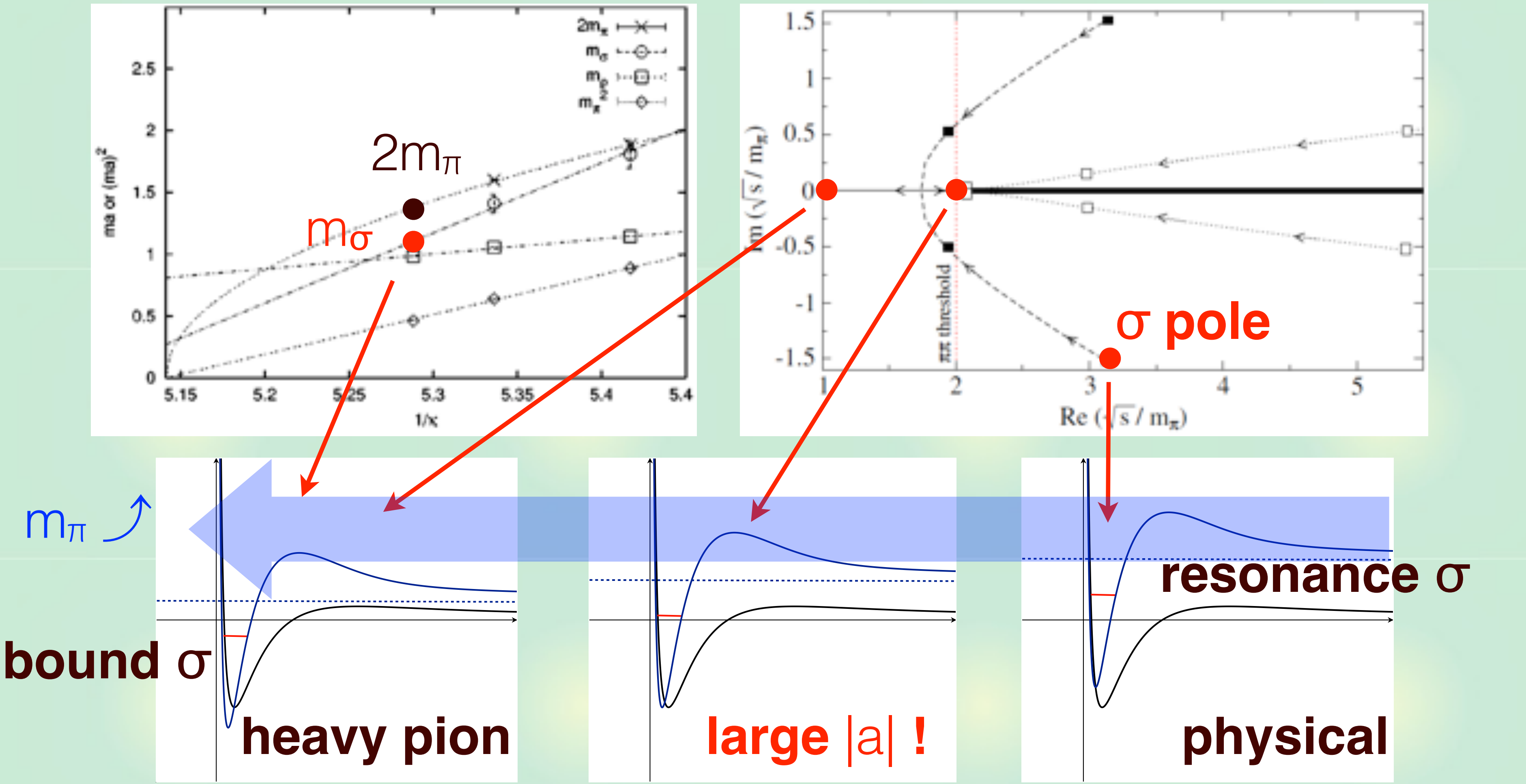


Increase pion mass

Lattice QCD and chiral effective field theory (EFT)

T. Kunihiro *et al.* (SCALAR Collaboration), Rev. Rev. D70, 034504 (2004)

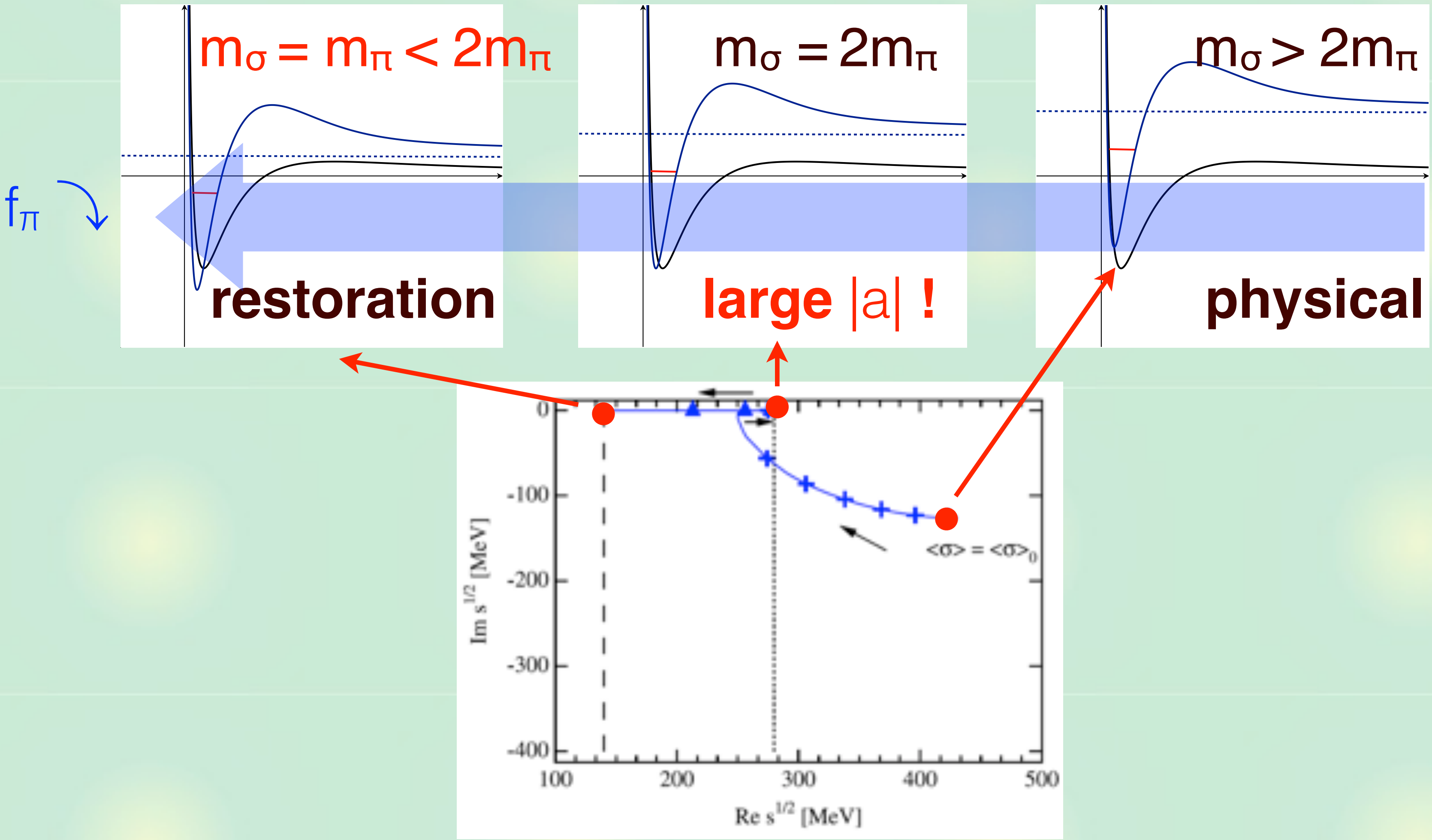
C. Hanhart, J.R. Pelaez, G. Rios, Phys. Rev. Lett. 100, 152001 (2008)



==> Numerical experiment (lattice QCD)!

Decrease pion decay constant

Chiral symmetry restoration ~ reduction of f_π



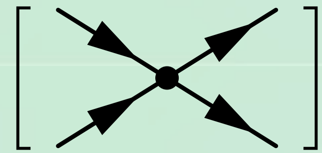
T. Hyodo, D. Jido, T. Kunihiro, Nucl. Phys. A848, 341-365 (2010)

==> Real experiment (in-medium symmetry restoration) !

Isospin symmetric three pions

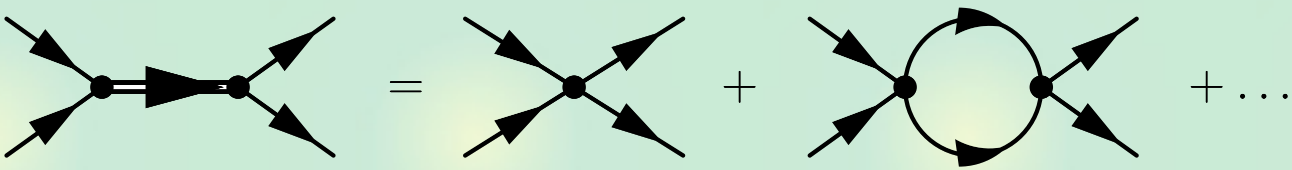
Large scattering length: zero range theory (I=0 interaction)

$$\mathcal{L} = \sum_{i=1,2,3} \phi_i^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \phi_i + v \left| \sum_{i=1,2,3} \phi_i \phi_i \right|^2$$

$I = 0$  $I = 0$

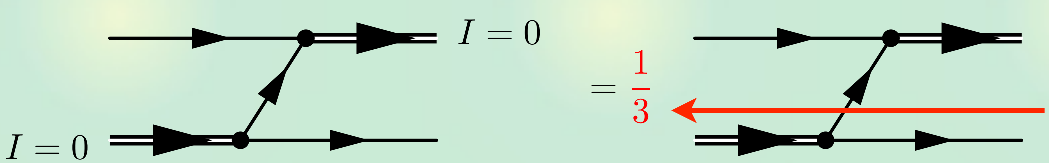
- two-body amplitude: I=0 and I=2

$$it_0(p) = \frac{8\pi}{m} \frac{i}{\frac{1}{a} - \sqrt{\frac{\boldsymbol{p}^2}{4} - mp_0 - i0^+}}, \quad it_2(p) = 0$$



S-wave three-pion system in total I=1

$$\begin{pmatrix} |\, \pi \otimes [\pi \otimes \pi]_{I=0} \,\rangle_{I=1} \\ |\, \pi \otimes [\pi \otimes \pi]_{I=2} \,\rangle_{I=1} \end{pmatrix} = \begin{pmatrix} 1/3 & \sqrt{5}/3 \\ \sqrt{5}/3 & 1/6 \end{pmatrix} \begin{pmatrix} |\, [\pi \otimes \pi]_{I=0} \otimes \pi \,\rangle_{I=1} \\ |\, [\pi \otimes \pi]_{I=2} \otimes \pi \,\rangle_{I=1} \end{pmatrix}$$

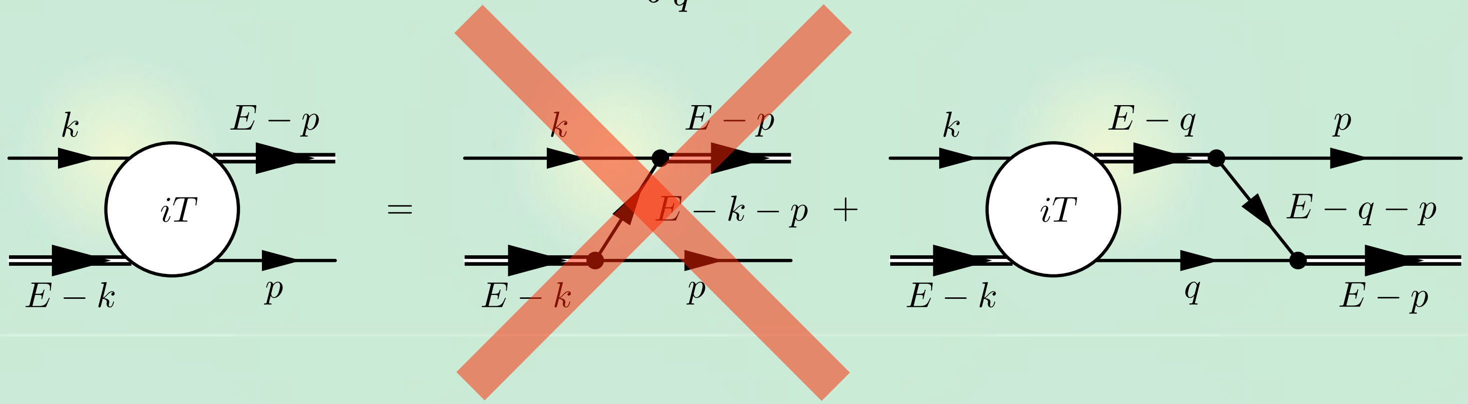


coupled-channel effect

Isospin symmetric three pions

Three-body scattering equation

$$iT(E;k,p) = iG(P-k-p) - \int_q T(E;k,q)t_K(P-q)G(q)G(P-q-p)$$



Eigenstate: homogeneous equation with pole condition

$$T^{\rm on}(E;|\boldsymbol{k}|,|\boldsymbol{p}|) \to \frac{z^*(|\boldsymbol{k}|)z(|\boldsymbol{p}|)}{E+B_3}$$

Eigenvalue equation (eigenvalue B_3 for eigenfunction $z(|\mathbf{p}|)$)

$$z(|\boldsymbol{p}|) = \frac{2}{3\pi}\int_0^\infty d|\boldsymbol{q}|\frac{|\boldsymbol{q}|}{|\boldsymbol{p}|}\ln\left(\frac{\boldsymbol{q}^2+\boldsymbol{p}^2+|\boldsymbol{q}||\boldsymbol{p}|+mB_3}{\boldsymbol{q}^2+\boldsymbol{p}^2-|\boldsymbol{q}||\boldsymbol{p}|+mB_3}\right)\frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2+mB_3}-\frac{1}{a}}$$

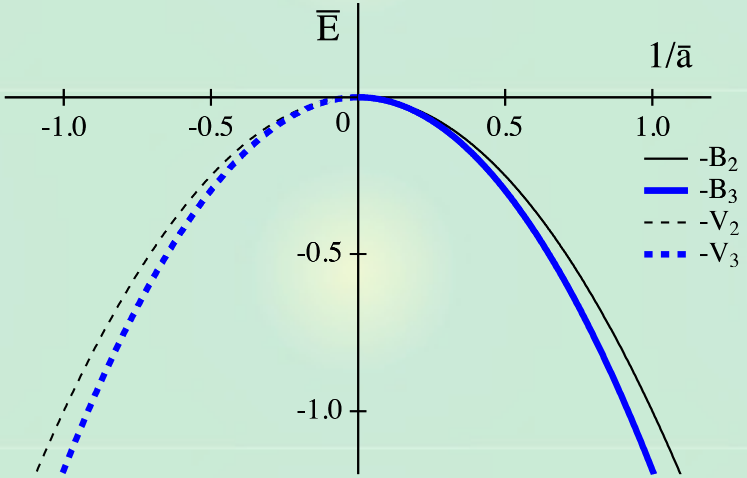
Factor 1/3 difference from the identical boson case

Spectrum in the isospin symmetric limit

Result: one universal three-pion bound state

$$B_3 = \frac{1.04391}{ma^2} \quad \text{for } 1/a > 0$$

$$\text{c.f.} \quad B_2 = \frac{1}{ma^2}$$



Resonances?

- **phase rotation of binding energy = phase rotation of** a

$$B_3 \to B_3 e^{i\theta} \quad \Leftrightarrow \quad \frac{1}{a} \to \frac{1}{a} e^{-i\theta/2}$$

Negative a: virtual state

J.R. Taylor, *"Scattering theory: the quantum theory on nonrelativistic collisions"* (1972)

<-- rotation of B_3 **by** 2π **= sign flip of** a

No resonance for all a

<-- interchange of Riemann sheet = sign flip of a

With isospin breaking

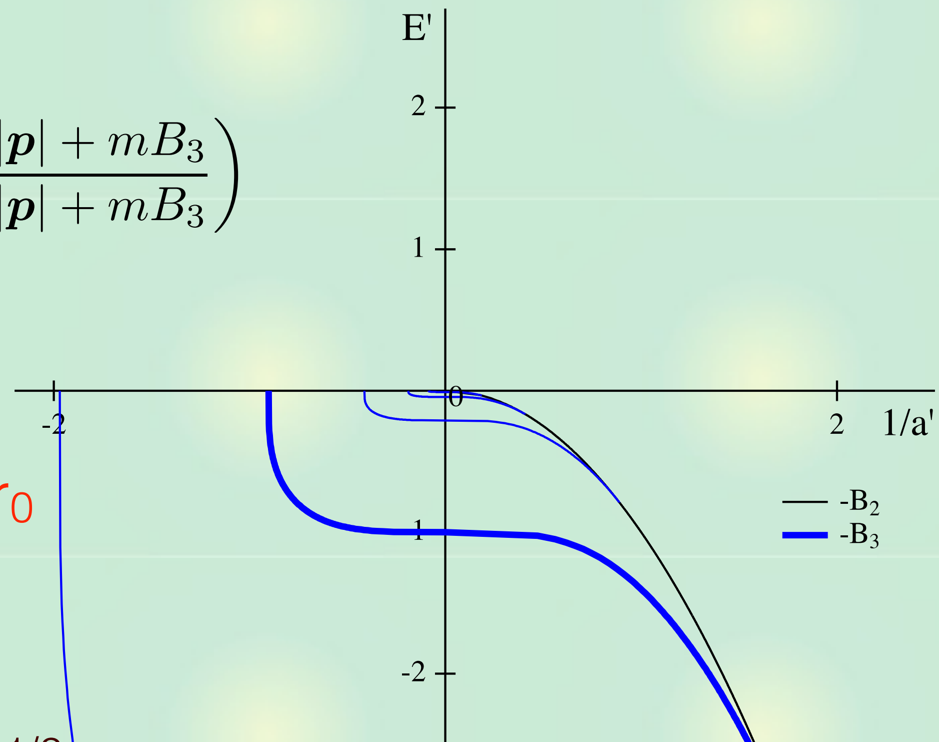
In nature, $m_{\pi^{\pm}} = m_{\pi^0} + \Delta$ **with** $\Delta > 0$

- In the energy region $E \ll \Delta$**, heavy** $\pi^{\pm}$ **can be neglected.**

Identical three-boson system with a large scattering length
--> Efimov effect

$$z(|\boldsymbol{p}|) = \frac{2}{\pi}\int_0^\infty d|\boldsymbol{q}|\frac{|\boldsymbol{q}|}{|\boldsymbol{p}|}\ln\left(\frac{\boldsymbol{q}^2+\boldsymbol{p}^2+|\boldsymbol{q}||\boldsymbol{p}|+mB_3}{\boldsymbol{q}^2+\boldsymbol{p}^2-|\boldsymbol{q}||\boldsymbol{p}|+mB_3}\right) \times \frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2+mB_3}-\frac{1}{a}} f_\Lambda(|\boldsymbol{q}|)$$

cutoff ~ $1/r_0$



Universal physics at $E \ll (2m\Lambda)^{1/2}$

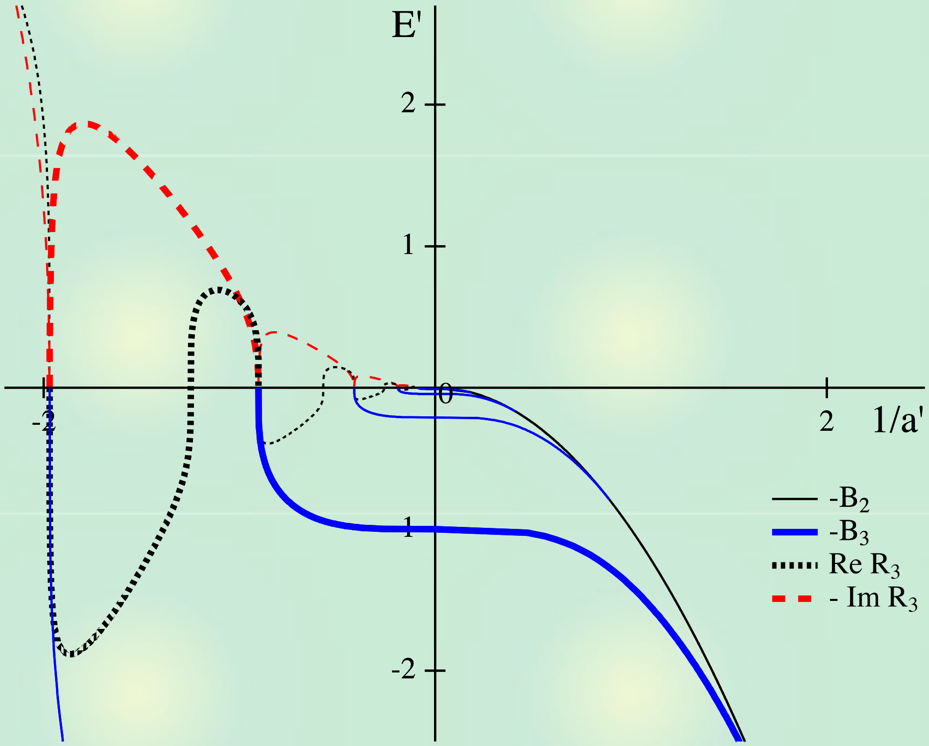
<-- Efimov parameter κ_*

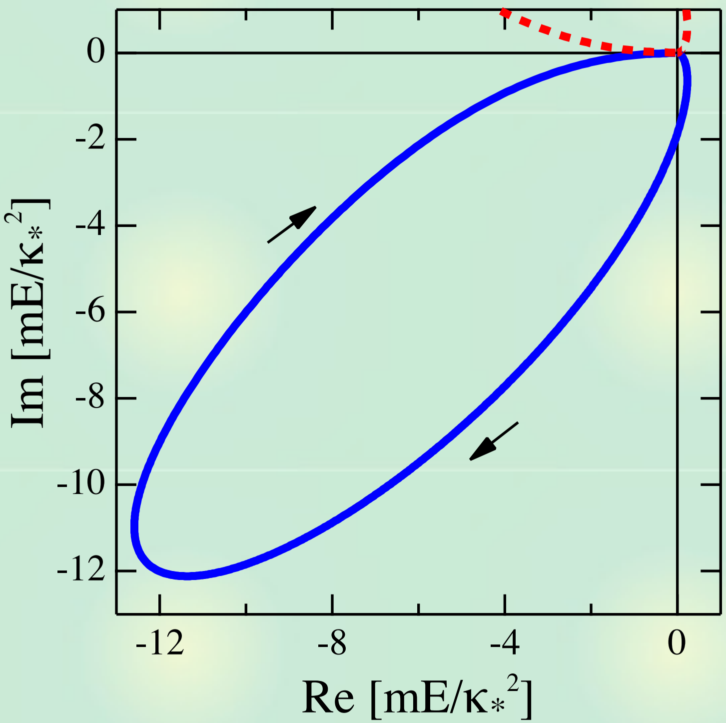
Efimov resonances

Resonance solution is now possible.

- phase rotation of binding energy = phase rotation of a **and Λ + proper treatment of singularity in** $f_\Lambda(|\mathbf{q}|)$

$$B_3 \to B_3 e^{i\theta} \quad \Leftrightarrow \quad \frac{1}{a} \to \frac{1}{a} e^{-i\theta/2} \quad \text{and} \quad \Lambda \to \Lambda e^{-i\theta/2}$$





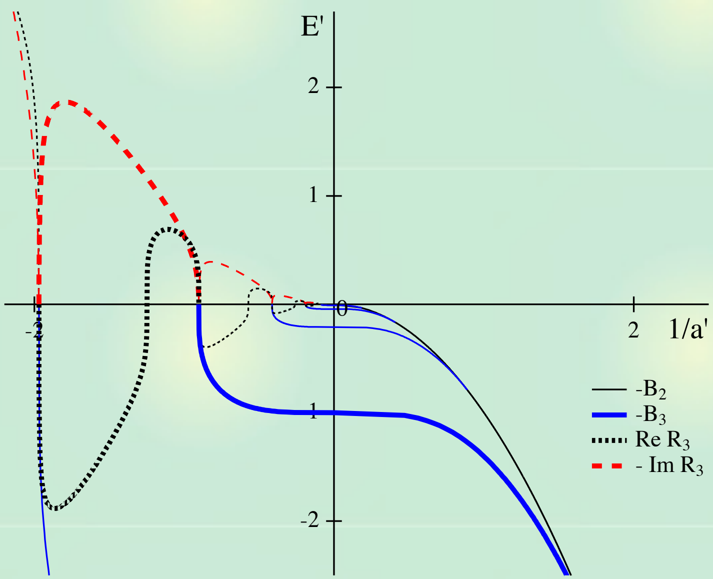
Efimov bound state --> resonance

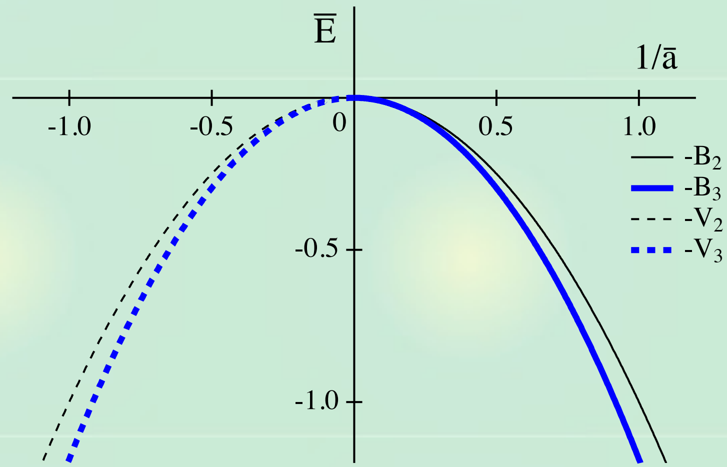
Coupled-channel effect

Two universal phenomena : existence of the coupled channel

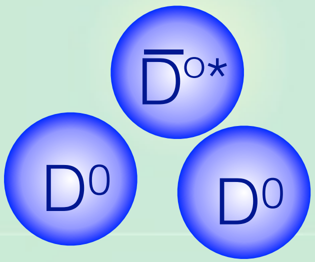
$$z(|\boldsymbol{p}|) = \frac{2}{\lambda\pi}\int_0^\infty d|\boldsymbol{q}|\frac{|\boldsymbol{q}|}{|\boldsymbol{p}|}\ln\left(\frac{\boldsymbol{q}^2+\boldsymbol{p}^2+|\boldsymbol{q}||\boldsymbol{p}|+mB_3}{\boldsymbol{q}^2+\boldsymbol{p}^2-|\boldsymbol{q}||\boldsymbol{p}|+mB_3}\right)\frac{z(|\boldsymbol{q}|)}{\sqrt{\frac{3}{4}\boldsymbol{q}^2+mB_3}-\frac{1}{a}}$$

$\lambda < 2.41480$ | $2.41480 < \lambda < 3.66811$ | $3.66811 < \lambda$





no universal bound state



$\lambda = 4$

discrete scale invariance

scale invariance

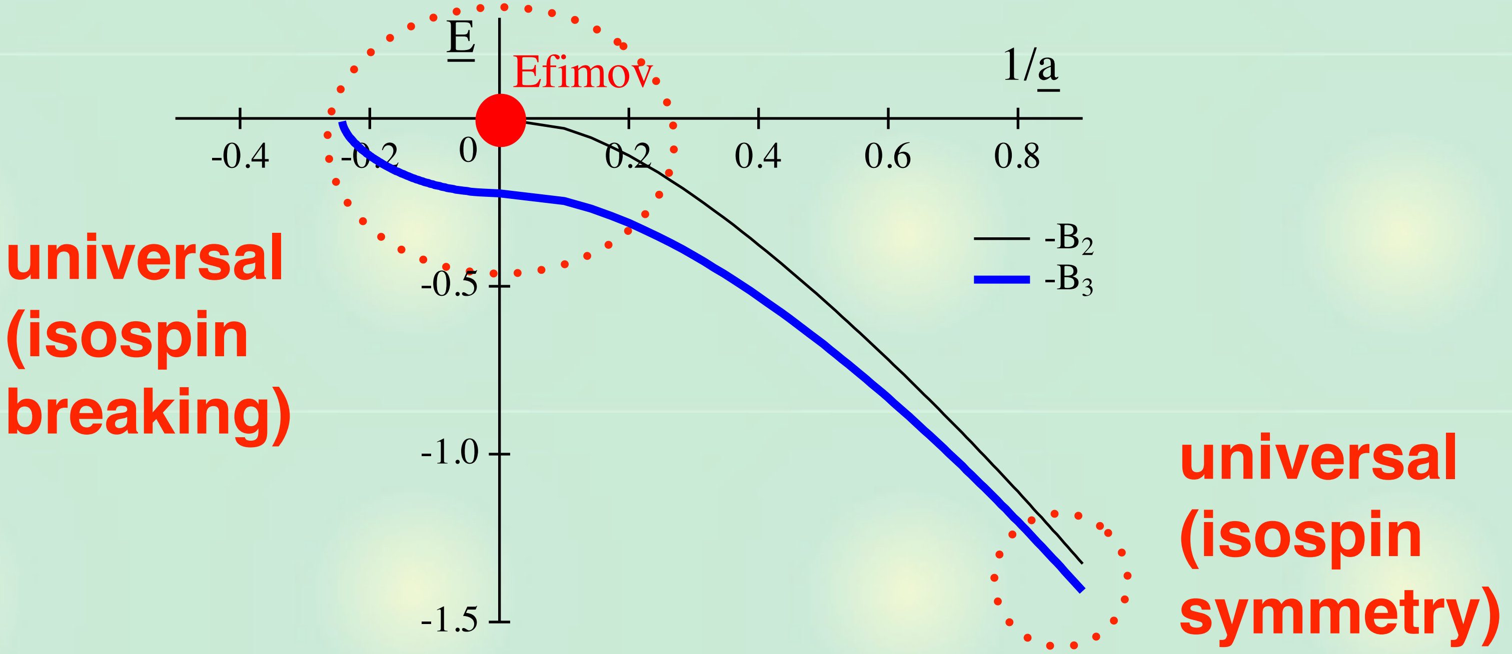
Both can be realized in three-pion systems.

Interpolation by model

A model with finite mass difference Δ = $m_\pm$ - m_0

$$\mathcal{L} = \sum_{i=0,\pm} \pi_i^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_i} - m_i \right) \pi_i + \frac{g}{4} \frac{\pi_0^\dagger \pi_0^\dagger - 2\pi_+^\dagger \pi_-^\dagger}{\sqrt{3}} \frac{\pi_0 \pi_0 - 2\pi_- \pi_+}{\sqrt{3}}$$

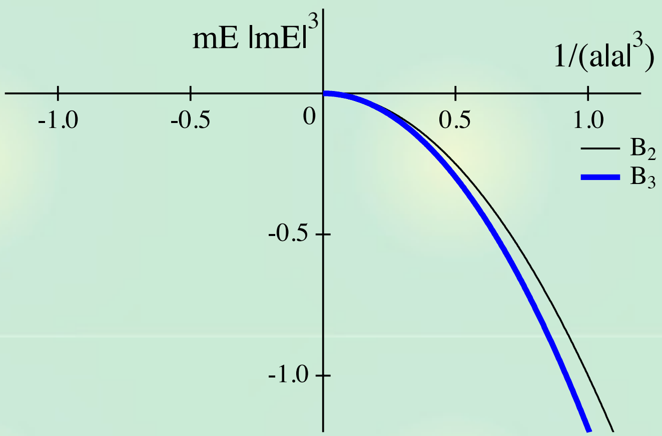
- E ≪ Δ : **Efimov states,** (Λ ≫) E ≫ Δ : **single bound state**
- **cutoff for the Efimov effect is introduced by Δ.**

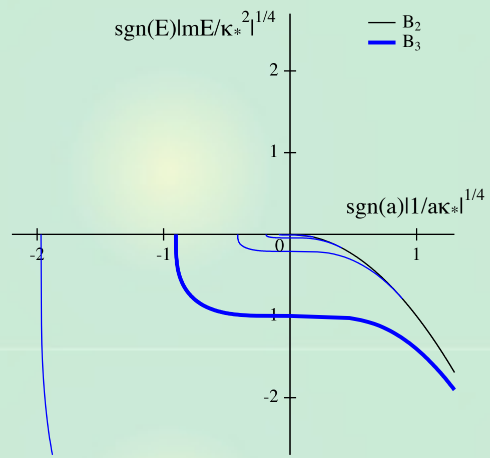


Lowest Efimov level --> universal bound state

Implication in hadron physics

Two-body ππ bound state (σ) --> at least one bound state in three-body channel with I=1 **and** J=0 **channel:** π^*





Remnant of universal bound state : π^*(1300)

M = 1300 ± 100 MeV, Γ = 200-600 MeV,

$\Gamma(\pi(\pi\pi)_{\text{s-wave}})/\Gamma(\pi\rho) \sim 2.2$

When the σ softens, π^* **also softens simultaneously.**

- caveats for the σ softening in practice: final state interaction, mixing with quark number fluctuation, ...

Summary

Universal physics of three pions

- **Large $\pi\pi$ scattering length (I=0) can be obtained by $m_\pi \nearrow$ or $f_\pi \searrow$.**

- **Universal phenomena with large a:**
  - **single bound state (isospin symmetry)**
  - **Efimov states (isospin breaking)**

- **Consequence in hadron physics:**
  - **realization in lattice QCD**
  - **simultaneous softening of σ and π^***

T. Hyodo, T. Hatsuda, Y. Nishida, arXiv:1311.6289 [hep-ph]