Structure of near-threshold hadrons

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Contents

- Introduction: structure of hadrons
- Compositeness of hadrons and near-threshold bound states

  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

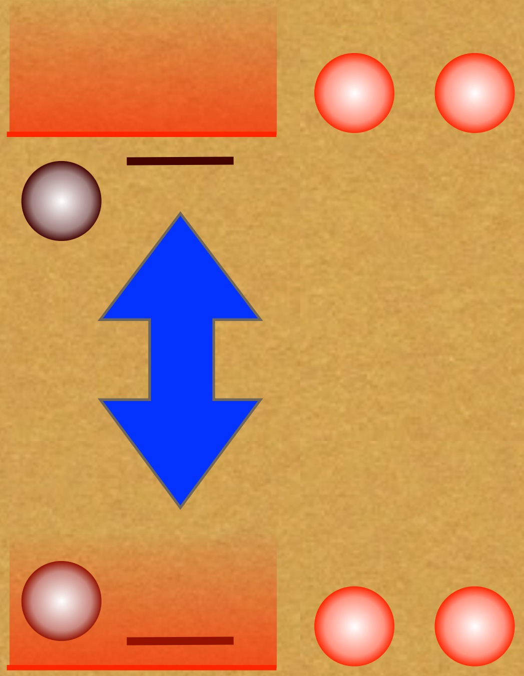
- Near-threshold resonances

  T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

- Near-threshold mass scaling

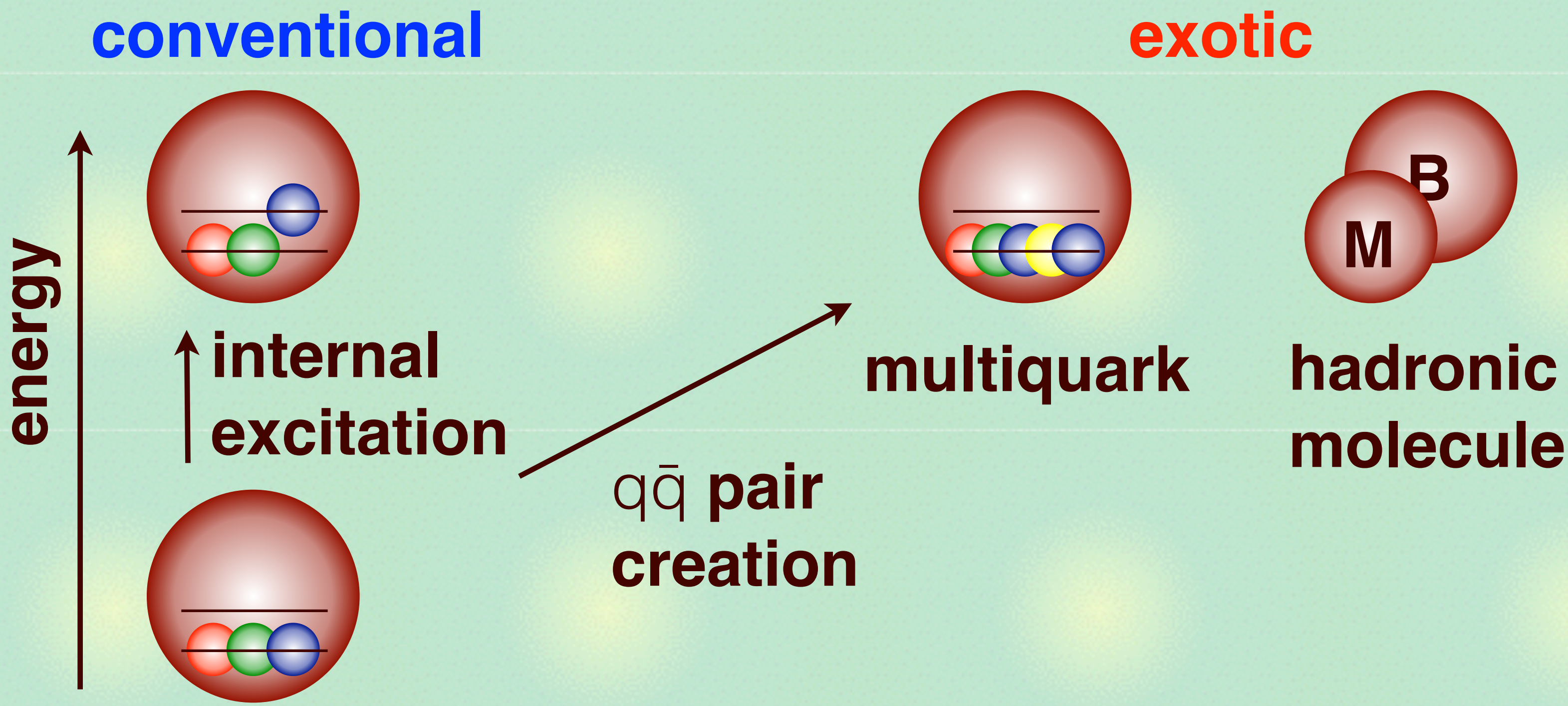
  T. Hyodo, Phys. Rev. C90, 055208 (2014)

  (Three-body case)



Exotic structure of hadrons

Various excitations of baryons



Physical state: superposition of 3q, 5q, MB, ...

$$|\,\Lambda(1405)\,\rangle = \underline{N_{3q}}|\,uds\,\rangle + \underline{N_{5q}}|\,uds\;q\bar{q}\,\rangle + \underline{N_{\bar{K}N}}|\,\bar{K}N\,\rangle + \cdots$$

Is this relevant strategy?

Ambiguity of definition of hadron structure

Decomposition of hadron "wave function"

$$|\,\Lambda(1405)\,\rangle = N_{3q}|\,uds\,\rangle + N_{5q}|\,uds\ q\bar{q}\,\rangle + N_{\bar{K}N}|\,\bar{K}N\,\rangle + \cdots$$

- **$N_X \neq$ probability?**
- **5q v.s. MB: double counting (orthogonality)?**

$$\langle\,udsq\bar{q}\,|\,\bar{K}N\,\rangle \neq 0$$

- **3q v.s. 5q: not clearly separated in QCD**

$$\langle\,uds\,|\,udsq\bar{q}\,\rangle \neq 0$$

- **hadron resonances: unstable, finite decay width**

$$|\,\Lambda(1405)\,\rangle = \,?$$

What is the suitable basis to classify the hadron structure?

Strategy

Elementary/composite nature of bound states near the lowest energy two-body threshold

elementary

- 6q **for deuteron**
- $c\bar{c}$ **for** X(3872)

composite

- NN **for deuteron**
- $\bar{D}D^*$ **for** X(3872)

- **orthogonality <— eigenstates of bare Hamiltonian**
- **normalization <— eigenstate of full Hamiltonian**
- **model dependence <— low energy universality**

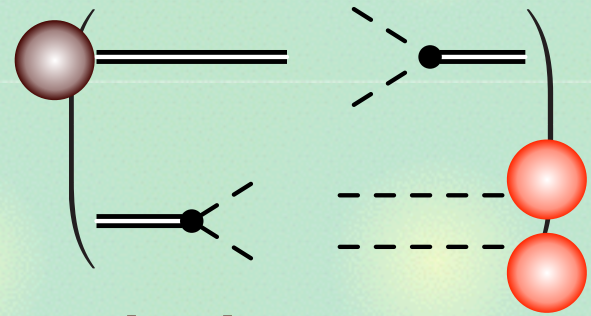
*** Basis must be asymptotic states (in QCD, hadrons).**

*** "Elementary" stands for any states other than two-body composite (missing channels, CDD pole, ...).**

Formulation

Coupled-channel Hamiltonian (bare state + continuum)

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu}(+\hat{V}_{\rm sc}) \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$



S. Weinberg, Phys. Rev. 131, 330 (1963)

Elementariness by field renormalization constant

- Bound state normalization + completeness relation

$$\langle\,\Psi\,|\,\Psi\,\rangle = 1 \qquad 1 = |\,\psi_0\,\rangle\langle\,\psi_0\,| + \int d^3q|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|$$

$$1 = \left|\langle\,\Psi\,|\begin{pmatrix} |\,\psi_0\,\rangle \\ 0 \end{pmatrix}\right|^2 + \int d^3q\left|\langle\,\Psi\,|\begin{pmatrix} 0 \\ |\,\boldsymbol{q}\,\rangle \end{pmatrix}\right|^2 \equiv Z + X$$

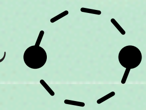
bare state contribution — **continuum contribution**

Z, X **: real and nonnegative —> probabilistic interpretation**

Weak binding limit

In general, Z is determined by the potential V.

$$Z(B) = \frac{1}{1 - \frac{d}{dE}\int \frac{|\langle \psi_0 | \hat{V} | \boldsymbol{q} \rangle|^2}{E - q^2/(2\mu) + i0^+} d^3q \Big|_{E=-B}} \equiv \frac{1}{1 - \Sigma'(-B)}$$

$\Sigma(E) \sim$ 

- Z is a model-(scheme-)dependent (c.f. potential)

Z of weakly-bound (R ≫ R_{typ}) s-wave state <— observables.

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(R_{\rm typ}), \quad r_e = \frac{-Z}{1-Z} R + \mathcal{O}(R_{\rm typ}),$$

a **: scattering length,** r_e **: effective range**
$R = (2\mu B)^{-1/2}$ **: radius <— binding energy**
R_{typ} **: typical length scale of the interaction**

- can be explicitly derived by the expansion of the amplitude

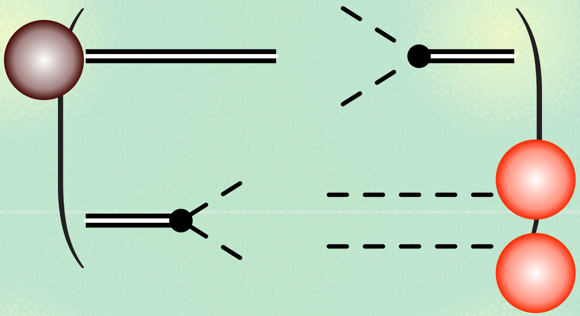
T. Sekiara, T. Hyodo, D. Jido arXiv: 1411.2308 [hep-ph]

Model independence in weak binding limit

Model independence <— low energy universality

- Weak binding: bound state size ≫ interaction range

—> two-channel model with a contact interaction



$\langle \boldsymbol{q} | \hat{V} | \psi_0 \rangle = g_0$ constant

: resonance model without four-point interaction

E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

—> full (exact) amplitude: only two observable

$$f(p) = \left(-\frac{1}{a} + \frac{r_e}{2} p^2 - ip \right)^{-1}$$

—> system can be completely specified by a and r_e

Interpretation of negative effective range

For $Z > 0$, effective range is always negative.

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\text{typ}}),$$

$$\begin{cases} a \sim R_{\text{typ}} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\text{typ}} & \text{(composite dominance)}. \end{cases}$$

Simple (e.g. square-well) attractive potential: $r_e > 0$

- only "composite dominance" is possible.

$r_e < 0$ **: energy- (momentum-)dependence of the potential**

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998);
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

- pole term/Feshbach projection of coupled-channel effect

Negative r_e **—> something other than** $|p\rangle$ **: CDD pole**

Compositeness theorem

Exact B -> 0 **limit:**

If the s-wave scattering amplitude has a pole exactly at the threshold with a finite range interaction, then the field renormalization constant vanishes.

T. Hyodo, Phys. Rev. C90, 055208 (2014)

For bare state-continuum model (c**: nonzero constant)**

$$Z(B) = \frac{1}{1-\Sigma'(-B)} \approx \frac{1}{1-c\frac{g_0^2}{\sqrt{B}}}$$

$$\mathrm{Im}\ \Sigma(p^2/2\mu) \propto p^{2l+1}$$

Z(0) **vanishes for** $g_0 \neq 0$**. If** $g_0=0$**, no pole in the amplitude.**

For general potentials: poles in the effective range expansion

$$p_1 = i\sqrt{2\mu B}, \quad p_2 = -i\sqrt{2\mu B}\frac{2-Z(B)}{Z(B)}$$

If $Z(0) \neq 0$**, then both** p_1 **and** p_2 **go to zero for** B -> 0

: contradict with simple pole at p=0 **—>** Z(0)=0

R.G. Newton, J. Math. Phys. 1, 319 (1960)

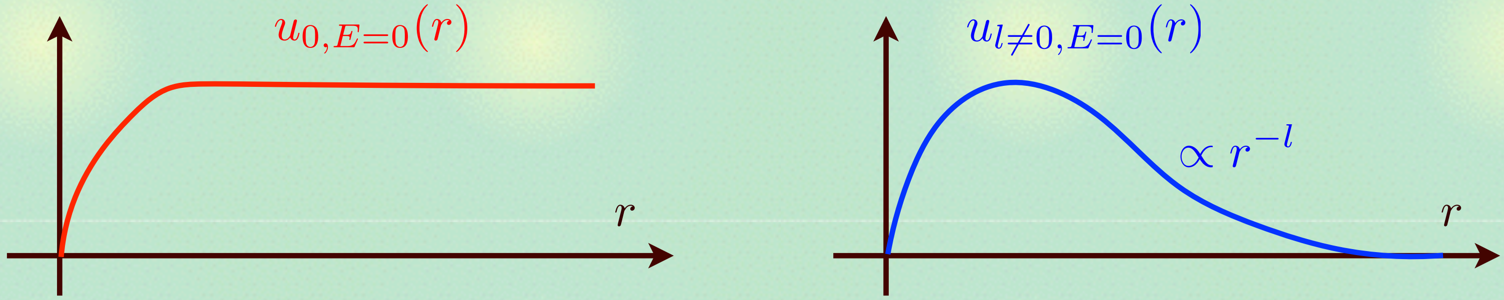
Interpretation of the compositeness theorem

Z(B): **overlap of the bound state with bare state**

$$\left| \langle \Psi | \begin{pmatrix} |\psi_0\rangle \\ 0 \end{pmatrix} \right|^2 + \int \left| \langle \Psi | \begin{pmatrix} 0 \\ |\boldsymbol{q}\rangle \end{pmatrix} \right|^2 d^3q = 1$$

- Z(B≠0)=0 **—> Bound state is completely composite.**

Two-body wave function at E=0**:** $u_{l,E=0}(r) \xrightarrow{r\to\infty} r^{-l}$



Z(0)=0**: ~~Bound state is completely composite.~~**

Composite component is infinitely large so that the fraction of any finite admixture of bare state is zero.

Generalization to resonances

Compositeness of bound states

$$Z(B) = \frac{1}{1 - \Sigma'(-B)}$$

Naive generalization to resonances:

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

$$Z(E_R) = \frac{1}{1 - \Sigma'(-E_R)}$$

complex ↑ complex

- Problem of interpretation (probability?)

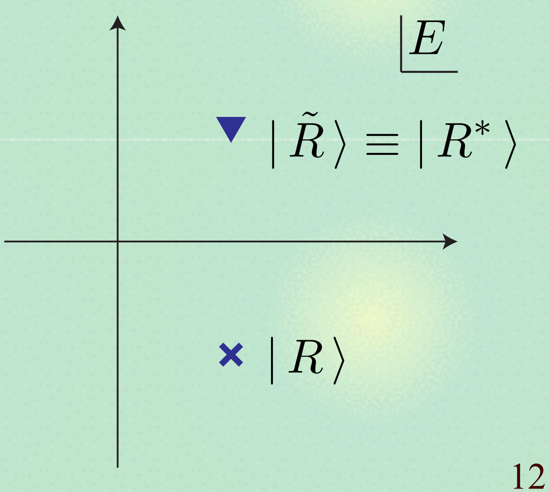
<— Normalization of resonances

$$\langle R | R \rangle \to \infty, \quad \langle \tilde{R} | R \rangle = 1$$

$$1 = \langle \tilde{R} | B_0 \rangle \langle B_0 | R \rangle + \int d\boldsymbol{p} \langle \tilde{R} | \boldsymbol{p} \rangle \langle \boldsymbol{p} | R \rangle$$

complex

$$\langle \tilde{R} | B_0 \rangle = \langle B_0 | R \rangle \neq \langle B_0 | R \rangle^*$$



T. Berggren, Nucl. Phys. A 109, 265 (1968)

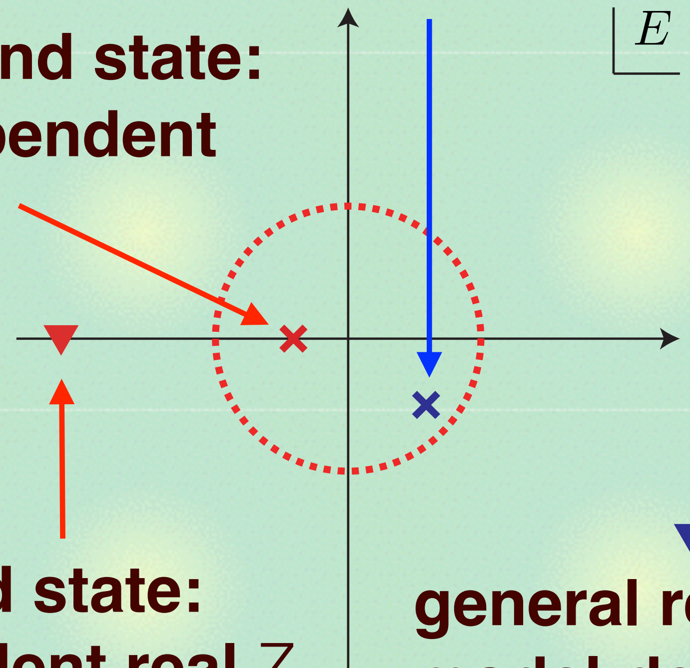
Near-threshold resonances

Weak binding limit for bound states

- Model-independent (no potential, wavefunction, ...)
- Related to experimental observables

What about near-threshold resonances (~ small binding)?

shallow bound state: model-independent structure



general bound state: model-dependent real Z

general resonance: model-dependent complex Z

Poles in the effective range expansion

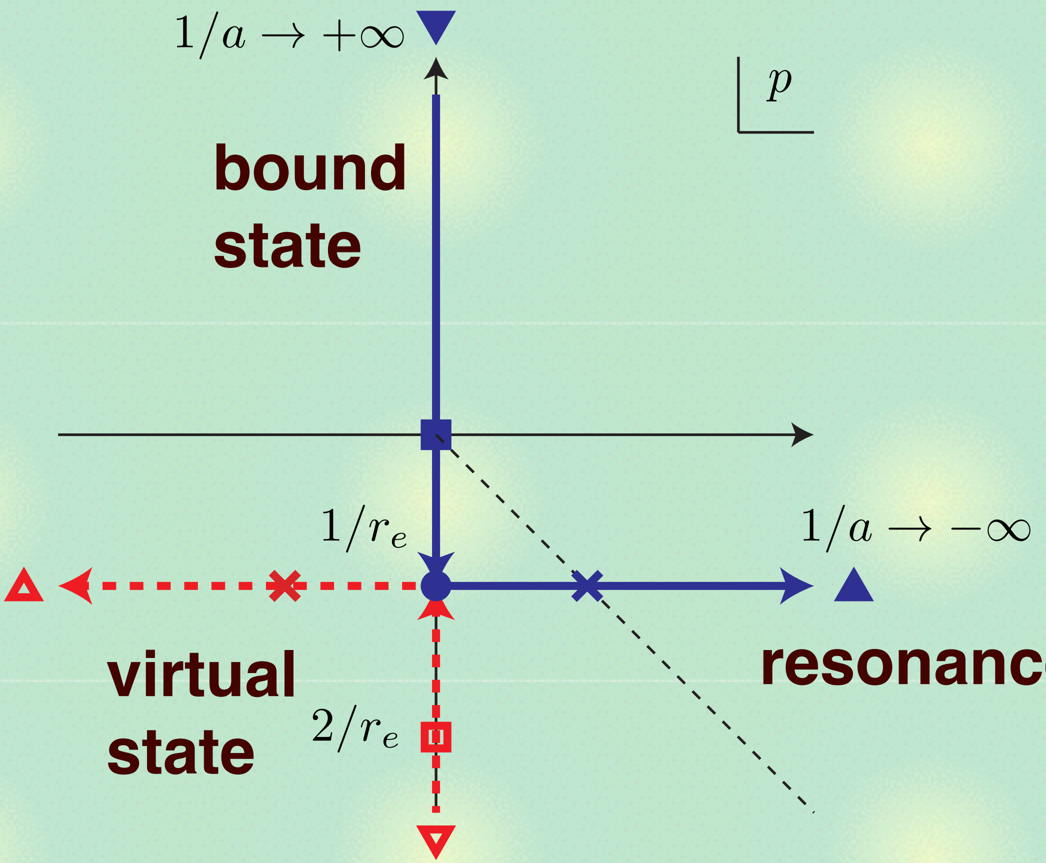
Near-threshold pole: effective range expansion

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013) with opposite sign of scattering length

$$f(p) = \left(-\frac{1}{a} + \frac{r_e}{2} p^2 - ip \right)^{-1}$$

$$p^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e} \sqrt{\frac{2r_e}{a} - 1}$$

- pole trajectories with a fixed $r_e < 0$



Resonance **pole position <--> (a, r_e)**

Application: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering

- central values in PDG

$$E = 0.67 \text{ MeV}, \quad \Gamma = 2.59 \text{ MeV} \qquad p^{\pm} = \sqrt{2\mu(E \mp i\Gamma/2)}$$

- deduced threshold parameters of $\pi\Sigma_c$ scattering

$$a = -\frac{p^+ + p^-}{ip^+p^-} = -10.5 \text{ fm}, \quad r_e = \frac{2i}{p^+ + p^-} = -19.5 \text{ fm}$$

- field renormalization constant: complex

$$Z = 1 - 0.608i$$

Large negative effective range

<— substantial elementary contribution other than $\pi\Sigma_c$ (three-quark, other meson-baryon channel, or ...)

$\Lambda_c(2595)$ **is not likely a $\pi\Sigma_c$ molecule**

Hadron mass scaling and threshold effect

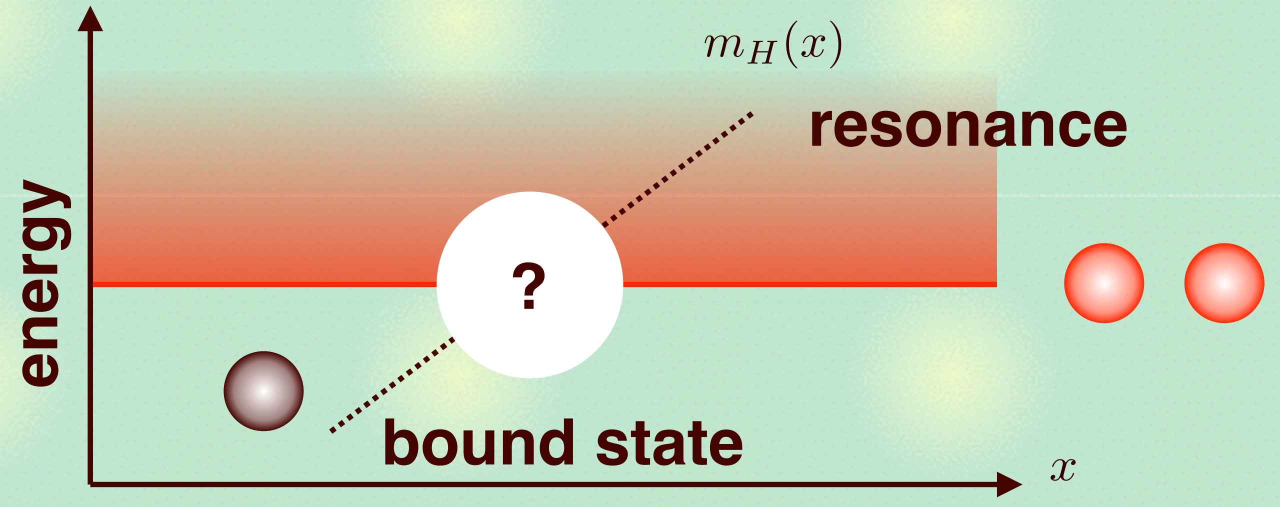
Systematic expansion of hadron masses

- **ChPT: light quark mass** m_q
- **HQET: heavy quark mass** m_Q
- **large Nc: number of colors** N_c

Hadron mass scaling

$$m_H(x); \quad x = \frac{m_q}{\Lambda}, \frac{\Lambda}{m_Q}, \frac{1}{N_c}$$

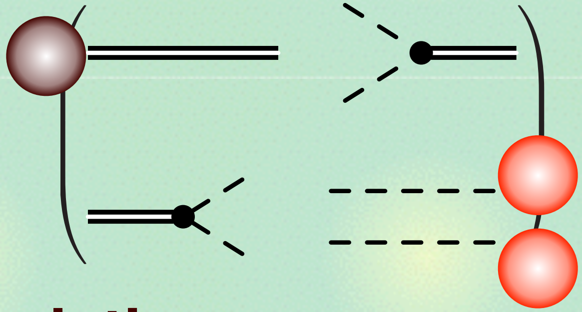
What happens at two-body threshold?



Formulation

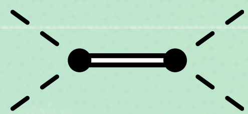
Coupled-channel Hamiltonian (bare state + continuum)

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu}(+\hat{V}_{\mathrm{sc}}) \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$

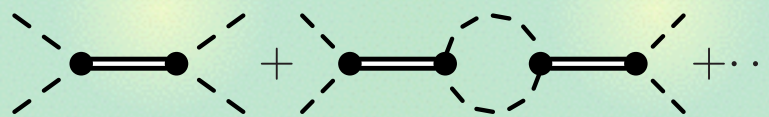


Equivalent single-channel scattering formulation

$$\hat{V}_{\mathrm{eff}}(E) = \frac{\hat{V}|\,\psi_0\,\rangle\langle\,\psi_0\,|\hat{V}}{E - M_0} \sim$$



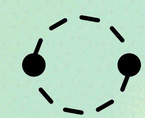
$$f(\boldsymbol{p}, \boldsymbol{p}', E) = -\frac{4\pi^2\mu\langle\,\boldsymbol{p}\,|\hat{V}|\,\psi_0\,\rangle\langle\,\psi_0\,|\hat{V}|\,\boldsymbol{p}'\,\rangle}{E - M_0 - \Sigma(E)} \sim$$



Pole condition:

$$E_h - M_0 = \Sigma(E_h)$$

$$\Sigma(E) = \int \frac{\langle\,\psi_0\,|\hat{V}|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|\hat{V}|\,\psi_0\,\rangle}{E - q^2/(2\mu) + i0^+} d^3q \sim$$

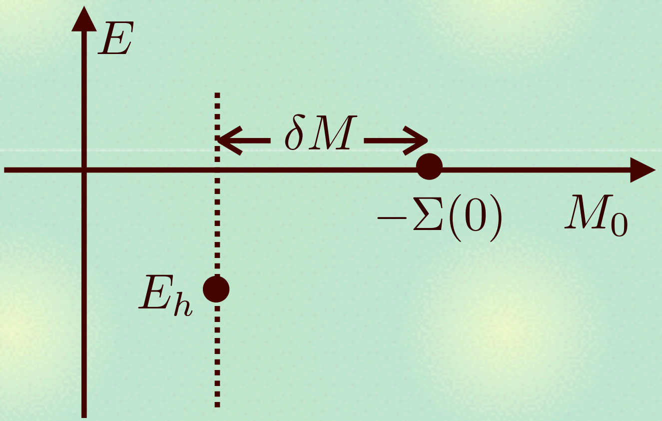


Question: How E_h behaves against M_0 around E_h=0?

Near-threshold bound state

Bound state condition around $E_h=0$

$$E_h + \Sigma(0) - \delta M = \Sigma(E_h)$$



Leading contribution of the expansion:

$$E_h = \frac{1}{1-\Sigma'(0)}\delta M = Z(0)\delta M, \quad \Sigma'(E) \equiv \frac{d\Sigma(E)}{dE}$$

Field renormalization constant

$$\left|\langle \Psi | \begin{pmatrix} |\psi_0\rangle \\ 0 \end{pmatrix}\right|^2 + \int \left|\langle \Psi | \begin{pmatrix} 0 \\ |\boldsymbol{q}\rangle \end{pmatrix}\right|^2 d^3q = 1$$

Z(0) vanishes for l=0: compositeness theorem

$$E_h \propto \begin{cases} \mathcal{O}(\delta M^2) & l=0 \\ \delta M & l \neq 0 \end{cases}$$

Near-threshold bound state (general)

General argument by Jost function (Fredholm determinant)

J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972)

$$f_l(p) = \frac{\not{f}_l(-p) - \not{f}_l(p)}{2ip\not{f}_l(p)}$$

pole (eigenstate) = Jost function zero

Expansion of the Jost function:

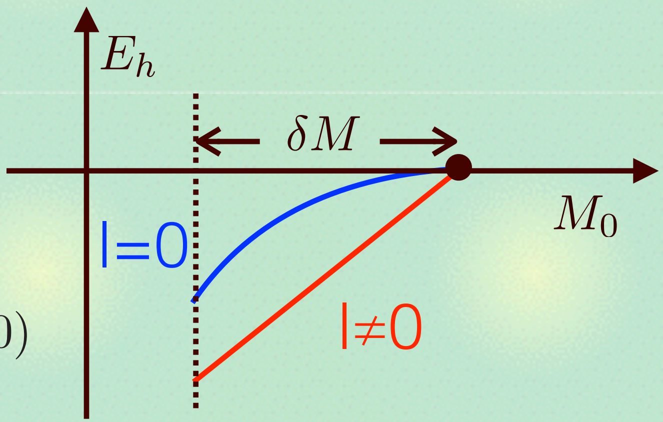
$$\not{f}_l(p) = \begin{cases} 1 + \alpha_0 + i\gamma_0 p + \mathcal{O}(p^2) & l = 0 \\ 1 + \alpha_l + \beta_l p^2 + \mathcal{O}(p^3) & l \neq 0 \end{cases}$$

- **γ_0 and β_l are nonzero for a general potential**
- **zero at $p=0$ ($1+\alpha_l=0$) must be simple (double) for $l=0$ ($l\neq0$)**

R.G. Newton, J. Math. Phys. 1, 319 (1960)
H.-W. Hammer, D. Lee, Annals Phys. 325, 2212 (2010)

Near-threshold scaling:

$$1 + \alpha_l \sim \delta M \quad \Rightarrow \quad E_h \propto \begin{cases} -\delta M^2 & l = 0 \\ \delta M & l \neq 0 \end{cases} \quad (\delta M < 0)$$



General threshold behavior

Near threshold scaling:

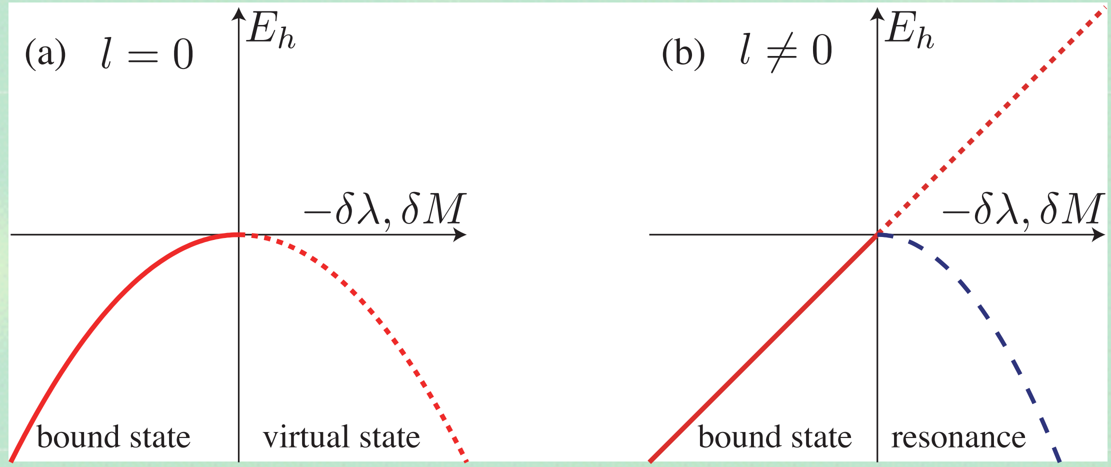
- $\delta M < 0$

$$E_h \propto \begin{cases} -\delta M^2 & l = 0 \\ \delta M & l \neq 0 \end{cases}$$

- $\delta M > 0$

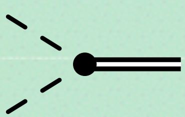
$$E_h \propto -\delta M^2 \quad l = 0$$

$$\begin{cases} \mathrm{Re}\ E_h \propto \delta M \\ \mathrm{Im}\ E_h \propto -(\delta M)^{l+1/2} \end{cases} \quad l \neq 0$$



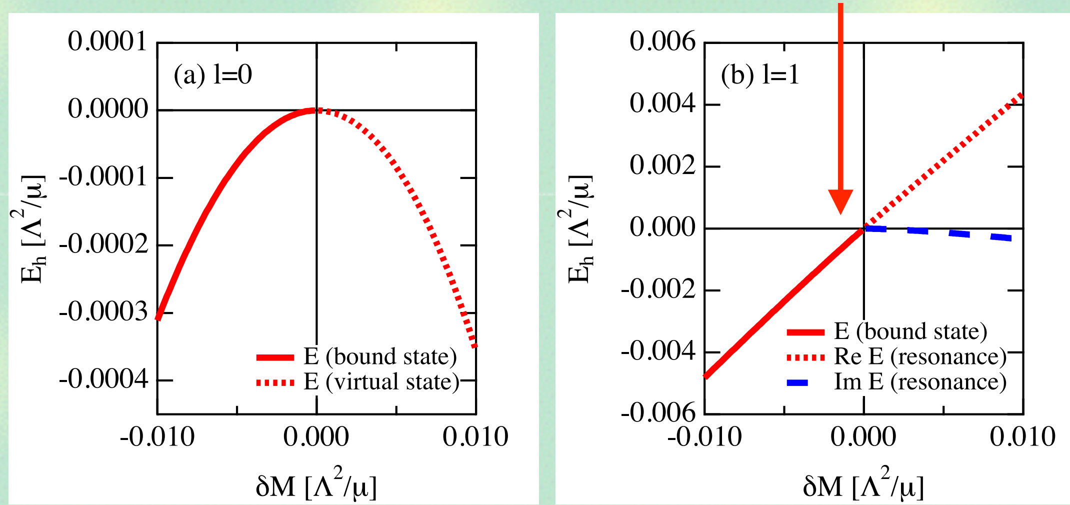
c.f. NN $^1S_0$

Numerical calculation



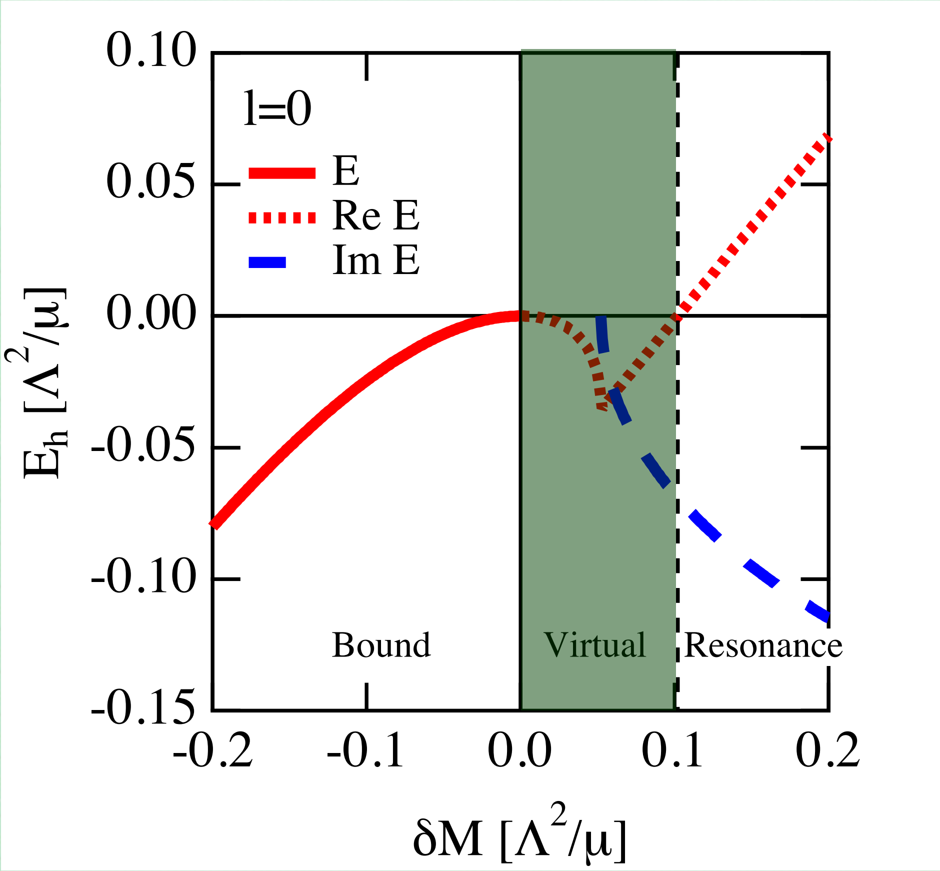
$$\langle \boldsymbol{q} | \hat{V} | \psi_0 \rangle = g_l |\boldsymbol{q}|^l \Theta(\Lambda - |\boldsymbol{q}|)$$

slope: Z(0)



Chiral extrapolation across s-wave threshold

s-wave: bound state —> virtual state —> resonance



Near-threshold scaling: nonperturbative phenomenon

—> Naive ChPT does not work; resummation required.

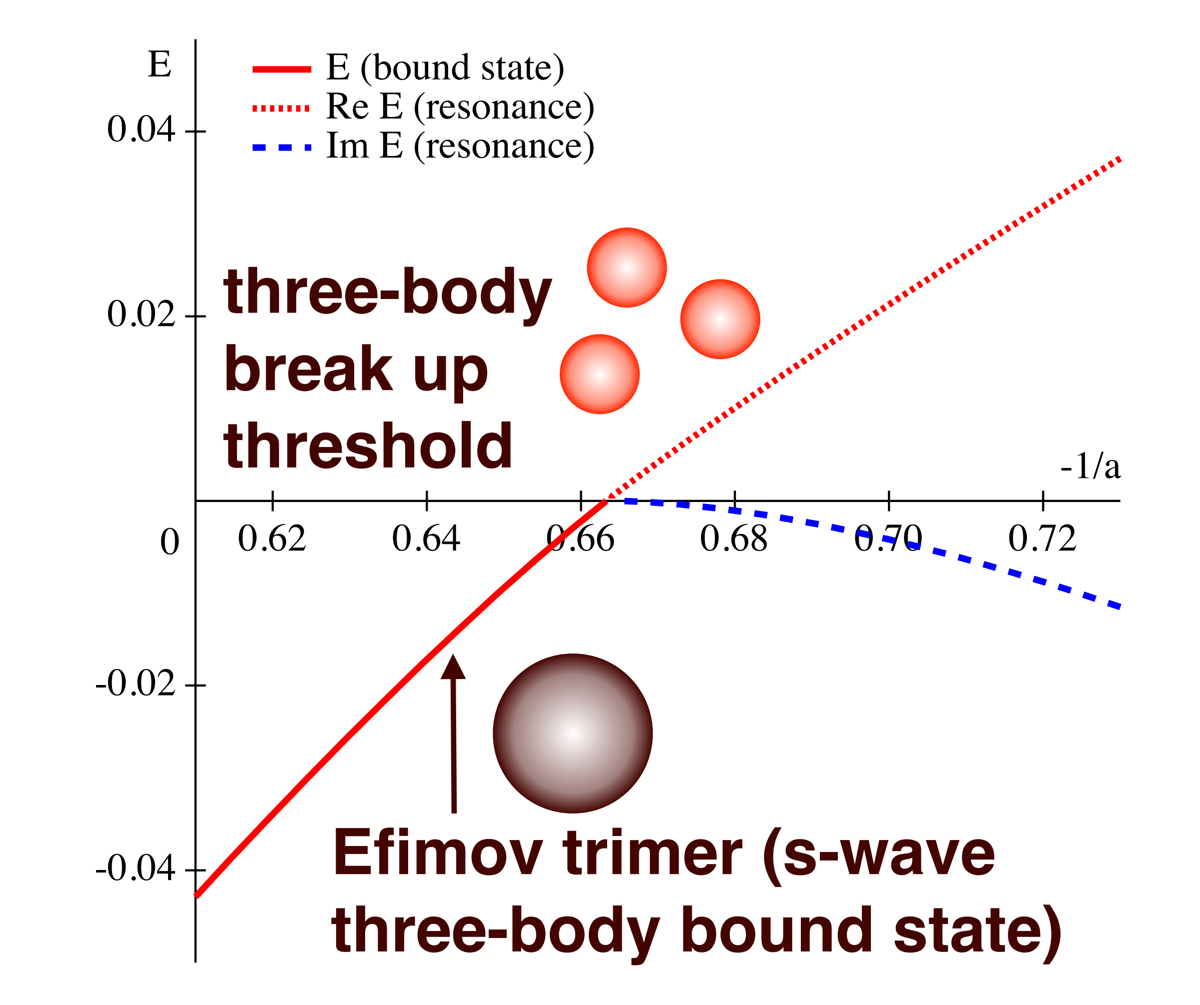
c.f.) NN **sector,** $\bar{K}$N **sector, ...**

Scaling of three-body bound state

Near-threshold scaling is universal for two-body system.

- What about three-body case?

T. Hyodo, T. Hatsuda, Y. Nishida, Phys. Rev. C89, 032201 (2014)



Summary

Compositeness of hadrons near threshold

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013);
T. Hyodo, Phys. Rev. C90, 055208 (2014)

- **Compositeness / elementariness**
  - suitable classification for hadron structure
  - model independent in the weak binding limit
- **Near-threshold resonance:**
  - elementariness from effective range
- **Near-threshold mass scaling:**
  - s-wave case is different from the others