Compositeness of hadrons from effective field theory

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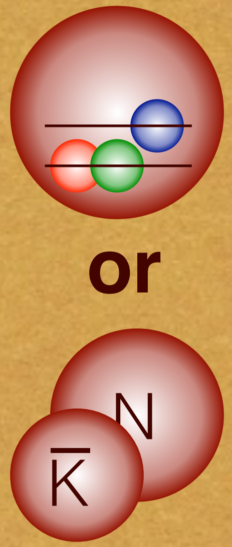


T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)
T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)
Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016) + in preparation

Classification of hadrons

Observed hadrons

PDG2015 : http://pdg.lbl.gov/

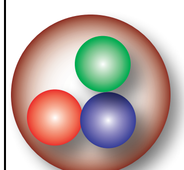
p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**
$N(1900)$	$3/2^+$	***
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$		*
$\Lambda(2020)$	$7/2^+$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1730)$	$3/2^+$	*
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1770)$	$1/2^+$	*
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1840)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	*
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(1940)$	$3/2^-$	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	***
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2980)$		***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ_{cc}^+		*
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b^*(5955)^-$	$3/2^+$	***
Ω_b^-	$1/2^+$	***



~ 150 baryons

LIGHT UNFLAVORED ($S = C = B = 0$)	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
$a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
$a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(?^{-+})$
$X(1840)$	$?^?(?^{??})$
• $\phi_3(1850)$	$0^-(3^{--})$
$\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
• $f_2(1950)$	$0^+(2^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $a_4(2040)$	$1^-(4^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$a_6(2450)$	$1^-(6^{++})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Further States

STRANGE ($S = \pm1$, $C = B = 0$)	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)	
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(0^-)$
$D(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2750)$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(?^?)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{sJ}^*(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

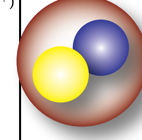
BOTTOM ($B = \pm1$)	
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
• $B(5970)^+$	$?(?^?)$
• $B(5970)^0$	$?(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)	
• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)	
• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$?^?(?^{??})$

$c\bar{c}$	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{?-})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)^\pm$	$?(1^+)$
$X(3900)^0$	$?(?^?)$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(4020)^\pm$	$?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4140)$	$0^+(?^{?+})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4230)$	$?^?(1^{--})$
$X(4240)^\pm$	$?^?(0^-)$
$X(4250)^\pm$	$?(?^?)$
• $X(4260)$	$?^?(1^{--})$
$X(4350)$	$0^+(?^{?+})$
• $X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $X(4430)^\pm$	$?(1^+)$
• $X(4660)$	$?^?(1^{--})$

$b\bar{b}$	
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$1^+(1^+)$
$X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$



~ 200 mesons

All ~ 350 hadrons emerge from single QCD Lagrangian.

All quantum numbers are described by qqq/q$\bar{\text{q}}$.

States with more than qqq/$q\bar{q}$

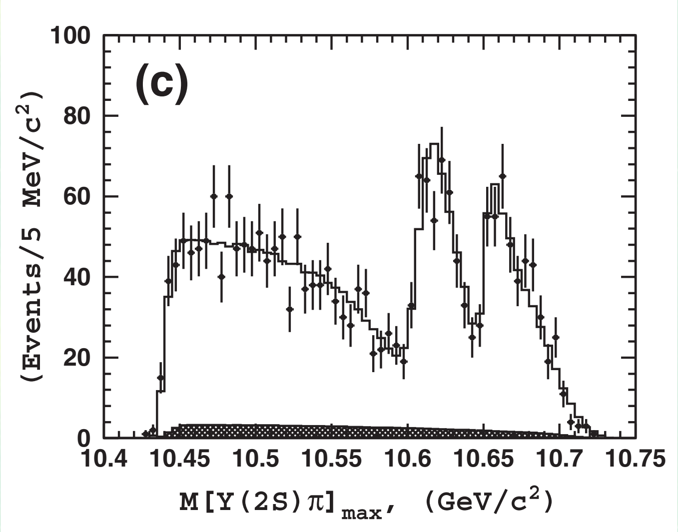
Tetraquark candidate (Belle)

: $Z_b(10610)$, $Z_b(10650)$

$\Upsilon(5S) \rightarrow \pi^{\pm} + Z_b$

↳ $\Upsilon(nS)(b\bar{b}) + \pi^{\mp}(u\bar{d}/d\bar{u})$

A. Bondar, *et al.*, Phys. Rev. Lett. 108, 122001 (2012)



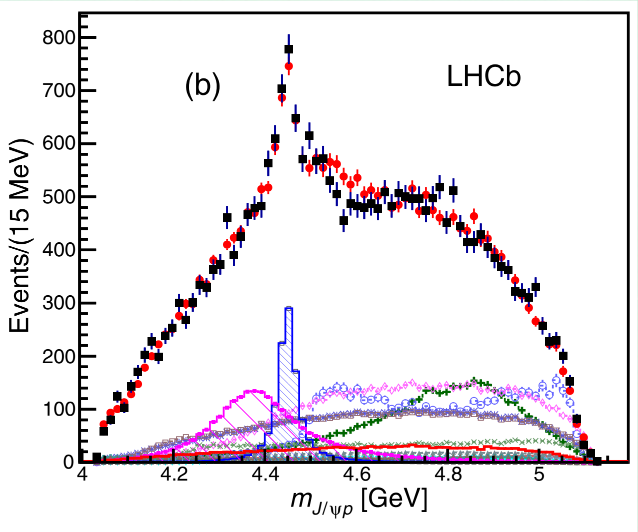
Pentaquark candidate (LHCb)

: $P_c(4450)$, $P_c(4380)$

$\Lambda_b \rightarrow K^- + P_c$

↳ $J/\psi(c\bar{c}) + p(uud)$

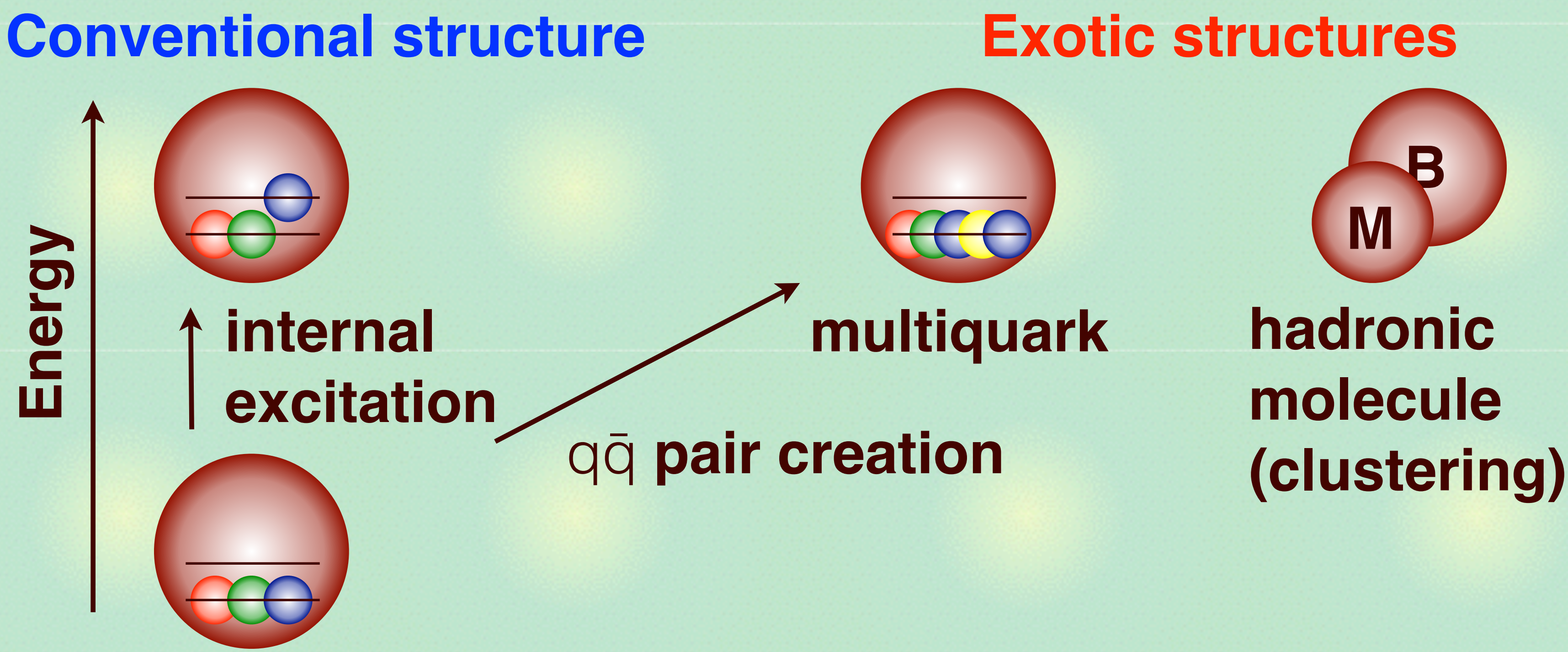
R. Aaij, *et al.*, Phys. Rev. Lett. 115, 072001 (2015)



Only a few are observed. Why only a few?

Various hadronic excitations

Description of excited baryons



In QCD, non-qqq structures naturally arise.

- **Baryons: superposition of qqq + exotic structures**

—> How can we disentangle different components?

Unstable states via strong interaction

Hadon resonances

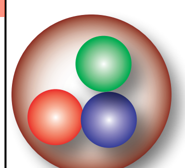
PDG2015 : http://pdg.lbl.gov/

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**
$N(1900)$	$3/2^+$	***
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$		*
$\Lambda(2020)$	$7/2^+$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1730)$	$3/2^+$	*
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1770)$	$1/2^+$	*
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1840)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	*
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(1940)$	$3/2^-$	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**



Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	***
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2980)$		***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ_{cc}^+		*
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b^*(5955)^-$	$3/2^+$	***
Ω_b^-	$1/2^+$	***

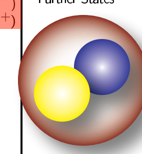
LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
$a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$
• $\pi_2(1670)$	$1^-(2^{-+})$

	$I^G(J^{PC})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
$a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(?^{-+})$
$X(1840)$	$?^?(?^{??})$
• $\phi_3(1850)$	$0^-(3^{--})$
$\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
• $f_2(1950)$	$0^+(2^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $a_4(2040)$	$1^-(4^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$a_6(2450)$	$1^-(6^{++})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Further States



STRANGE ($S = \pm 1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm 1$)

	$I(J^P)$
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(0^-)$
$D(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2750)$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm 1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(?^?)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{sJ}^*(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm 1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
• $B(5970)^+$	$?(?^?)$
• $B(5970)^0$	$?(?^?)$

BOTTOM, STRANGE ($B = \pm 1$, $S = \mp 1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm 1$)

• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$?^?(?^{??})$

$c\bar{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{?-})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)^\pm$	$?(1^+)$
$X(3900)^0$	$?(?^?)$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(4020)^\pm$	$?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4140)$	$0^+(?^{?+})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4230)$	$?^?(1^{--})$
$X(4240)^\pm$	$?^?(0^-)$
$X(4250)^\pm$	$?(?^?)$
• $X(4260)$	$?^?(1^{--})$
$X(4350)$	$0^+(?^{?+})$
• $X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $X(4430)^\pm$	$?(1^+)$
• $X(4660)$	$?^?(1^{--})$

$b\bar{b}$

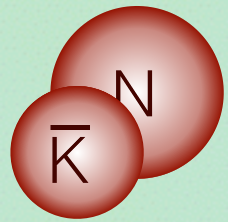
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$1^+(1^+)$
$X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

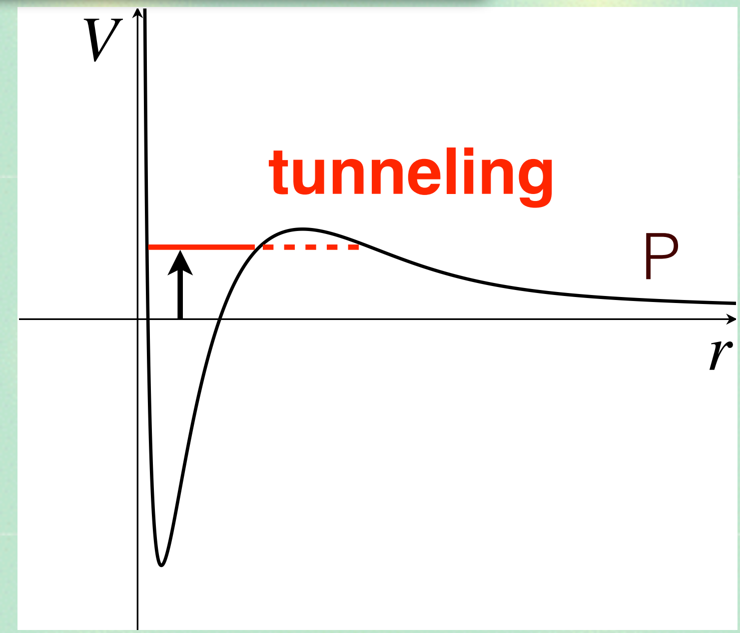
- stable/unstable via strong interaction
- Excited states are mostly unstable. —> resonances

Resonances in quantum mechanics

1) Potential (shape) resonance

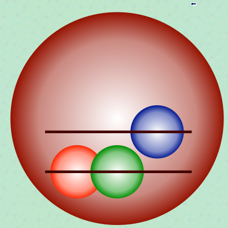
- **1 channel (P)**
- **potential barrier :** E>0
- **unstable via tunneling**
- **(composite of P-channel)**

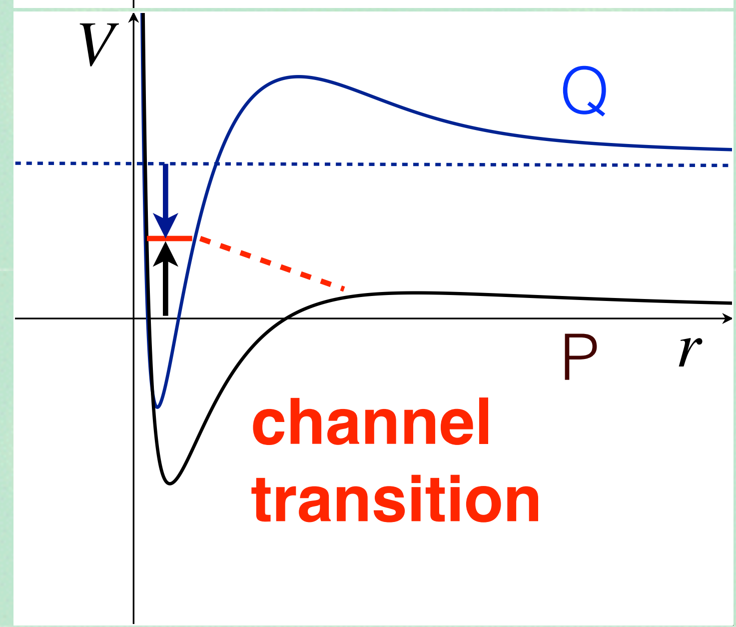




2) Feshbach resonance

- **coupled-channel (P+Q)**
- **bound state of Q:** $E_Q<0$, $E_P>0$
- **unstable via transition**
- **("elementary": other than P)**





Difficulty of resonances

Resonance as an "eigenstate" of Hamiltonian

- complex energy

G. Gamow, Z. Phys. 51, 204 (1928)

Zur Quantentheorie des Atomkernes.

Von G. Gamow, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und E komplex setzen:

$$E = E_0 + \boxed{i\,\frac{h\,\lambda}{4\,\pi}},$$

wo E_0 die gewöhnliche Energie ist und λ das Dämpfungsdekrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),

- Wave function cannot be normalized.

$$\langle R | R \rangle = \int d\boldsymbol{r} |\psi_R(\boldsymbol{r})|^2 \sim \int_0^\infty dr e^{-2\mathrm{Im}[k]r} \to \infty$$

Normalization by bi-orthogonal basis (Gamow vector)

N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)

T. Berggren, Nucl. Phys. A 109, 265 (1968)

$$| \tilde{R} \rangle = | R^* \rangle, \quad |\langle \tilde{R} | R \rangle| = \left| \int d\boldsymbol{r} [\psi_R(\boldsymbol{r})]^2 \right| < \infty$$

- Complex expectation value (e.g. <r^2>) —> interpretation?

Compositeness of hadrons

- Internal structure of excited hadrons
- Weak binding relation for stable bound states

S. Weinberg, Phys. Rev. 137, B672 (1965)

Compositeness X threshold channel	or	"Elementariness" Z other contributions

↑
observables

- Study compositeness of hadron resonances by generalizing weak binding relation to unstable states with effective field theory.

Weak binding relation for stable states

Compositeness of s-wave weakly bound state ($R \gg R_{\mathrm{typ}}$**)**

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)\right\}, \quad r_e = R\left\{\frac{X-1}{X} + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)\right\}$$

a_0**: scattering length,** r_e**: effective range**
$R = (2\mu B)^{-1/2}$**: radius of wave function**
R_{typ}**: length scale of interaction**

- deuteron is NN **composite (**$a_0 \sim R \gg r_e$**) —>** $X \sim 1$

- internal structure from observable

- no nuclear force potential / wavefunction of deuteron

Problem: applicable only for stable states.

Effective field theory

Low-energy scattering with near-threshold bound state

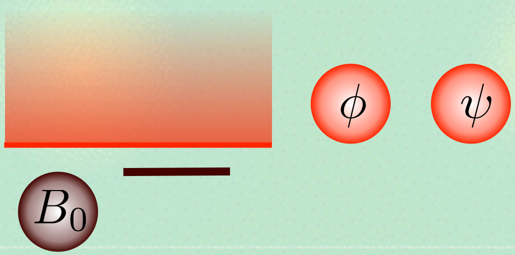
- Nonrelativistic EFT with contact interaction

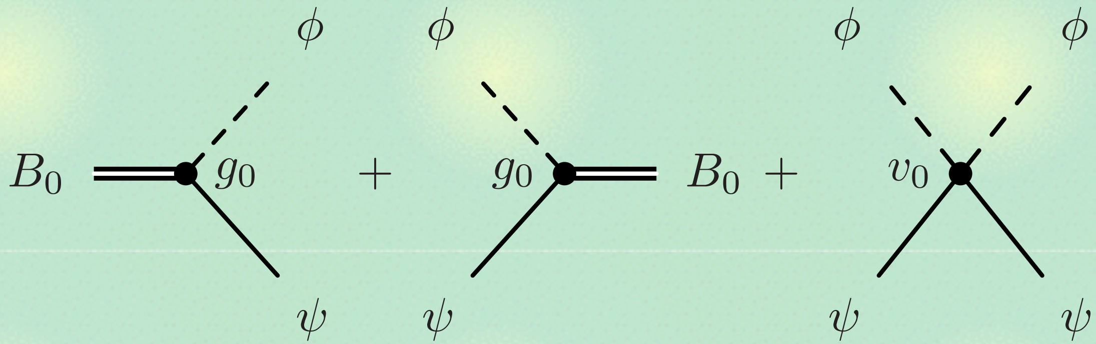
D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

$$H_{\text{free}} = \int d\boldsymbol{r}\left[\frac{1}{2M}\nabla\psi^{\dagger}\cdot\nabla\psi + \frac{1}{2m}\nabla\phi^{\dagger}\cdot\nabla\phi + \frac{1}{2M_0}\nabla B_0^{\dagger}\cdot\nabla B_0 + \nu_0 B_0^{\dagger}B_0\right],$$

$$H_{\text{int}} = \int d\boldsymbol{r}\left[g_0\left(B_0^{\dagger}\phi\psi + \psi^{\dagger}\phi^{\dagger}B_0\right) + v_0\psi^{\dagger}\phi^{\dagger}\phi\psi\right]$$





- cutoff : $\Lambda \sim 1/R_{\text{typ}}$ **(interaction range of microscopic theory)**

- At low energy $p \ll \Lambda$**, interaction ~ contact**

Compositeness and “elementariness”

Eigenstates

$$H_{\text{free}}|\,B_0\,\rangle = \nu_0|\,B_0\,\rangle, \quad H_{\text{free}}|\,\boldsymbol{p}\,\rangle = \frac{p^2}{2\mu}|\,\boldsymbol{p}\,\rangle$$ **free (discrete + continuum)**

$$(H_{\text{free}} + H_{\text{int}})|\,B\,\rangle = -B|\,B\,\rangle$$ **full (bound state)**

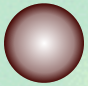
- normalization of |B> + completeness relation

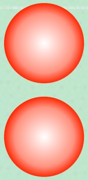
$$\langle\,B\,|\,B\,\rangle = 1, \quad 1 = |\,B_0\,\rangle\langle\,B_0\,| + \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\,\boldsymbol{p}\,\rangle\langle\,\boldsymbol{p}\,|$$

- projections onto bare states

$$1 = Z + X, \quad \underline{Z \equiv |\langle\,B_0\,|\,B\,\rangle|^2}, \quad \underline{X \equiv \int \frac{d\boldsymbol{p}}{(2\pi)^3}|\langle\,\boldsymbol{p}\,|\,B\,\rangle|^2}$$

“elementariness” **compositeness**



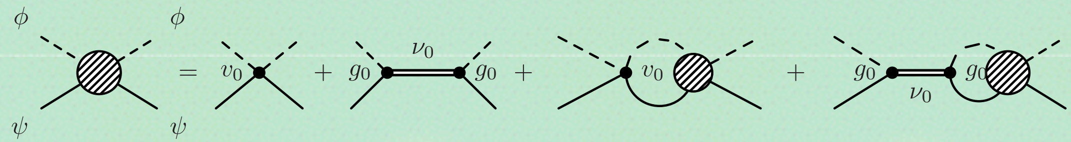


Z, X**: real and nonnegative —> interpreted as probability**

Weak binding relation

ΨΦ scattering amplitude (exact result)

$$f(E)=-\frac{\mu}{2\pi}\frac{1}{[v(E)]^{-1}-G(E)}$$



$$v(E)=v_0+\frac{g_0^2}{E-\nu_0},\quad G(E)=\frac{1}{2\pi^2}\int_0^\Lambda dp\frac{p^2}{E-p^2/(2\mu)+i0^+}$$

Compositeness X **<—** v(E), G(E) **: renormalization dependent**

T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)

T. Hyodo, arXiv:1511.00870 [hep-ph]

$$X=\{1+G^2(-B)v'(-B)\,[G'(-B)]^{-1}\}^{-1}$$

$1/R=(2\mu B)^{1/2}$ **expansion: leading term <—** X

$$a_0=-f(E=0)=R\left\{\underline{\frac{2X}{1+X}}+\overline{\mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)}\right\}$$

renormalization dependent

renormalization independent

If $R \gg R_{\rm typ}$**, correction terms neglected:** X **<— (**B, a_0**)**

When a CDD pole exist

CDD pole: pole of the inverse amplitude $f(E_c)=0,\ 1/f(E_c)\to\infty$

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)

- weak-binding relation breaks down

V. Baru, *et al.*, Eur. Phys. J. A44, 93 (2010)

Z.-H. Guo, J.A. Oller, Phys. Rev. D93, 054014 (2016)

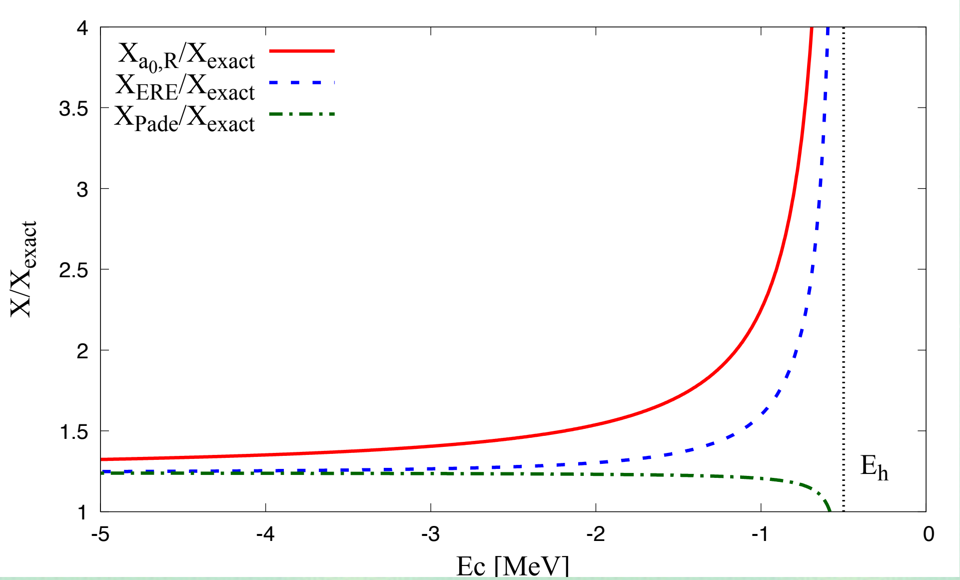
The effect of CDD pole <— Pade approximant for $1/f(E)$

Y. Kamiya, T. Hyodo, in preparation

$$X_{a_0,R} = \frac{a_0}{2R-a_0}$$

$$X_{\mathrm{Pade}} = \left[1-\frac{4R(a_0-R)}{a_0^2 r_e}\right]^{-1}$$

- model calculation with known exact X



New formula approximates the exact result even if $E_c \sim B$

Introduction of decay channel

Introduce decay channel

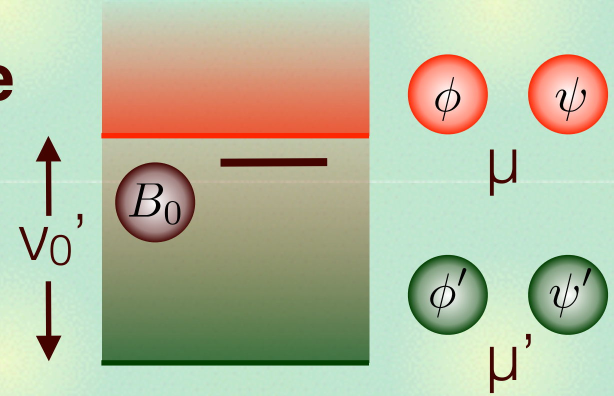
$$H'_{\text{free}} = \int d\boldsymbol{r}\left[\frac{1}{2M'}\nabla\psi'^{\dagger}\cdot\nabla\psi' - \nu_{\psi}\psi'^{\dagger}\psi' + \frac{1}{2m'}\nabla\phi'^{\dagger}\cdot\nabla\phi' - \nu_{\phi}\phi'^{\dagger}\phi'\right],$$

$$H'_{\text{int}} = \int d\boldsymbol{r}\left[g'_0\left(B_0^{\dagger}\phi'\psi' + \psi'^{\dagger}\phi'^{\dagger}B_0\right) + v'_0\psi'^{\dagger}\phi'^{\dagger}\phi'\psi' + v^t_0(\psi^{\dagger}\phi^{\dagger}\phi'\psi' + \psi'^{\dagger}\phi'^{\dagger}\phi\psi)\right],$$

Quasi-bound state: complex eigenvalue

$$H = H_{\text{free}} + H'_{\text{free}} + H_{\text{int}} + H'_{\text{int}}$$

$$H|QB\rangle = E_{QB}|QB\rangle, \quad E_{QB}\in\mathbb{C}$$



Generalized relation: correction term <— threshold difference

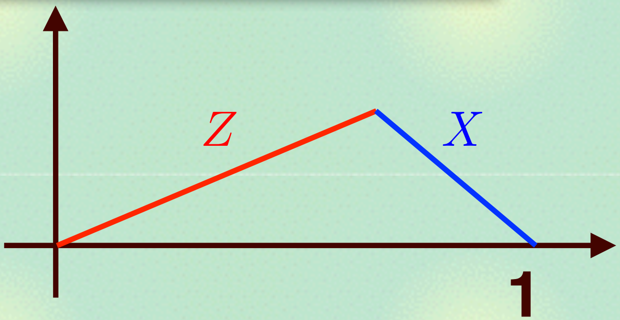
$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \underline{\sqrt{\frac{\mu'^3}{\mu^3}}\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu'_0}}$$

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

c.f. V. Baru, *et al.*, Phys. Lett. B586, 53 (2004),…

If $|R| \gg (R_{\text{typ}}, l)$ correction terms neglected: X <— (E_{QB}, a_0)

Complex compositeness and interpretation



Unstable states —> complex Z and X

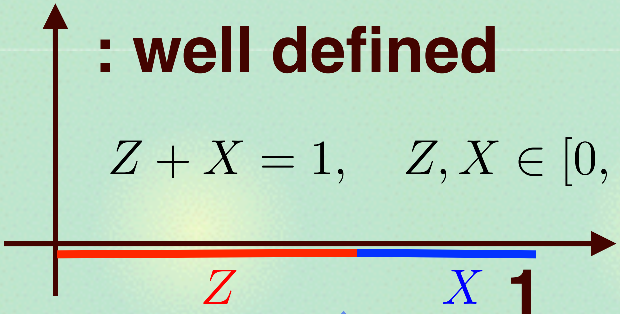
$$Z + X = 1, \quad Z, X \in \mathbb{C}$$

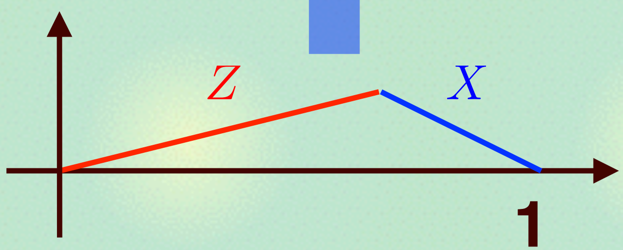
Similarity with bound state

c.f. T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)

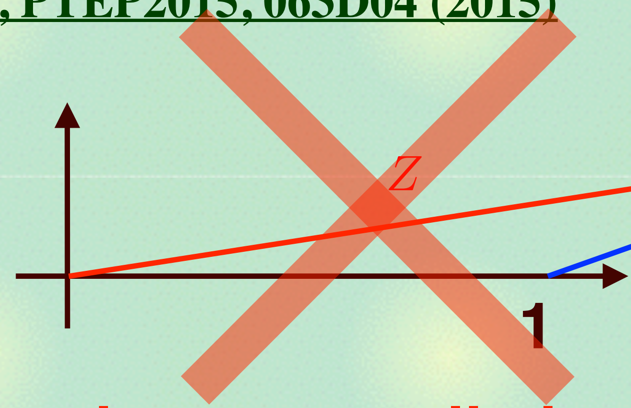
bound state
: well defined

$$Z + X = 1, \quad Z, X \in [0, 1]$$

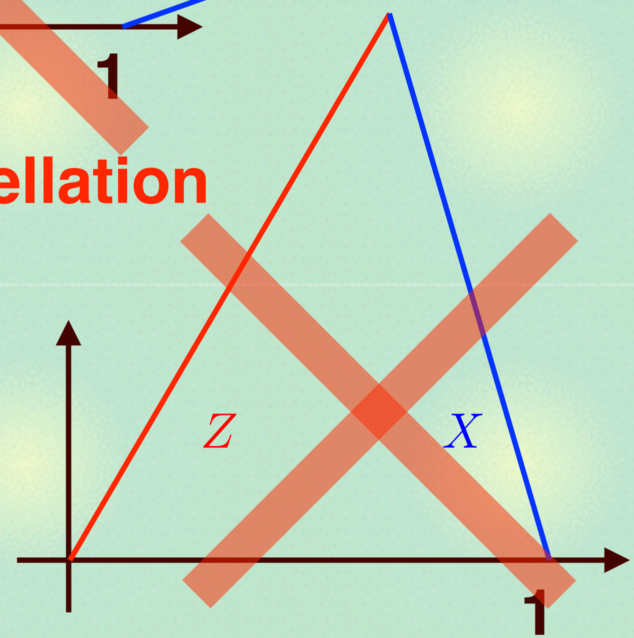




small cancellation



large cancellation



New definitions

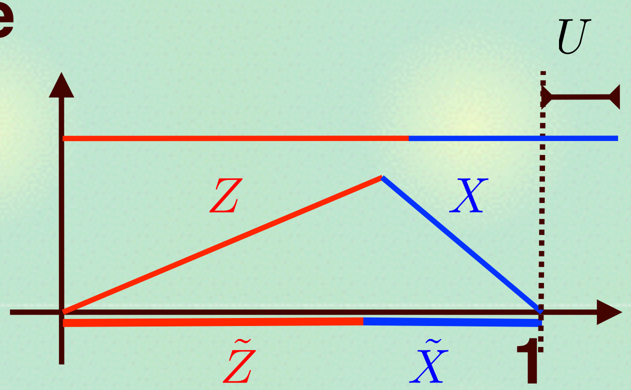
Step 1: quantify the deviation from bound state

- 0 for bound state
- becomes large when deviation is large

$$U = |Z| + |X| - 1$$

—> U: uncertainty of interpretation

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)



Step 2: define new compositeness/elementariness

- interpreted as probabilities $\tilde{Z} + \tilde{X} = 1, \quad \tilde{Z}, \tilde{X} \in [0, 1]$
- coincide with Z, X for bound state if U —> 0

$$\tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} = \frac{1 - |Z| + |X|}{2}$$

compositeness when U is small

Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016)

Application

Generalized weak binding relation X <— (E_{QB}, a_0)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \sqrt{\frac{\mu'^3}{\mu^3}}\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu'_0}}$$

- Λ(1405) **pole position and** $\bar{\rm K}$N **scattering length**

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012), …

- E_{QB} = -10 -26i **MeV —>** |R| ~ 2 fm **—> small correction term**

$$\left|\frac{R_{\rm typ}}{R}\right| \lesssim 0.12, \quad \left|\frac{l}{R}\right|^3 \lesssim 0.16$$

energy difference from πΣ

vector meson exchange

Ref.	E_{QB} (MeV)	a_0 (fm)	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	U	$\lvert r_e/a_0\rvert$
[43]	$-10-i26$	$1.39-i0.85$	$1.2+i0.1$	1.0	0.5	0.2
[44]	$-4-i8$	$1.81-i0.92$	$0.6+i0.1$	0.6	0.0	0.7
[45]	$-13-i20$	$1.30-i0.85$	$0.9-i0.2$	0.9	0.1	0.2
[46]	$2-i10$	$1.21-i1.47$	$0.6+i0.0$	0.6	0.0	0.7
[46]	$-3-i12$	$1.52-i1.85$	$1.0+i0.5$	0.8	0.6	0.4

systematic error

Λ(1405) **is** $\bar{\rm K}$N **composite <— observables**

Summary

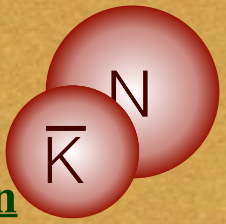
- **Compositeness of near-threshold bound state can be determined only by observables.**

  S. Weinberg, Phys. Rev. 137, B672 (1965)

- **Weak binding relation can be generalized to unstable states with effective field theory.**

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right) + \sqrt{\frac{\mu'^3}{\mu^3}}\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}, \quad R = \frac{1}{\sqrt{-2\mu E_{QB}}}, \quad l \equiv \frac{1}{\sqrt{2\mu\nu_0'}}$$

- **Precise determination of the pole position and scattering length shows that Λ(1405) is dominated by $\bar{\text{K}}$N composite component.**



Y. Kamiya, T. Hyodo, Phys. Rev. C93, 035203 (2016) + in preparation