Efimov physics of hadrons

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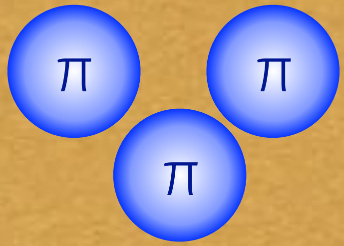
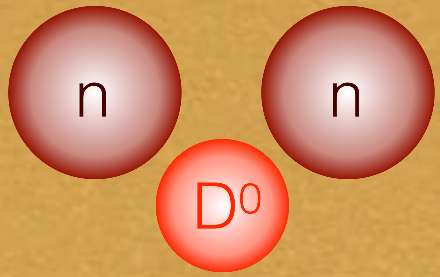


T. Hyodo, T. Hatsuda, Y. Nishida, Phys. Rev. C89, 032201(R) (2014)

- Two neutrons with a flavored meson (K^-/D^0)

U. Raha, Y. Kamiya, S.-I. Ando, T. Hyodo, arXiv:1708.03369 [nucl-th]

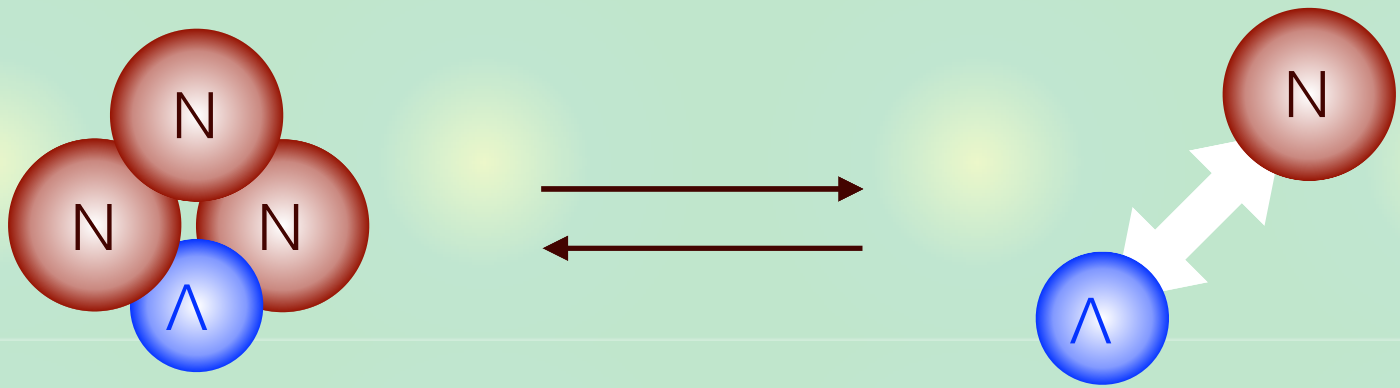


- Summary

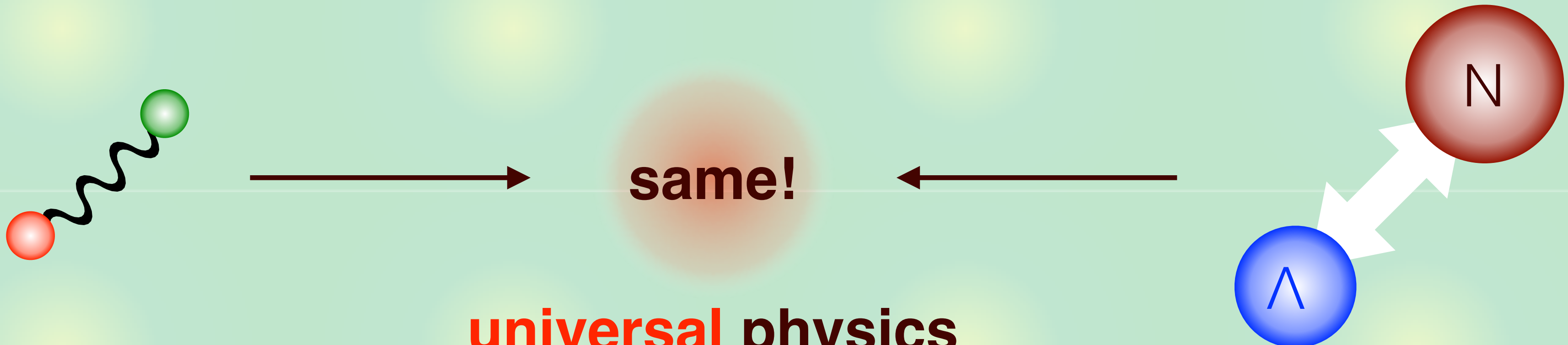
Study of few-body systems

Properties of few-body systems <—> two-body interaction

- c.f. hypernuclei



In some cases, different interactions give the same physics.



universal physics

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006);
P. Naidon, S. Endo, Rept. Prog. Phys. 80, 056001 (2017)

Two-body universal physics

Universal two-body physics: unitary limit

1) s-wave short range interaction

2) scattering length : $|a| \gg r_s$ **: interaction range**

- system is scale invariant

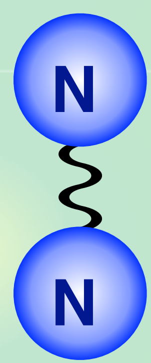
- a shallow bound state exists if $a > 0$

$$B_2 = \frac{1}{ma^2}\left[1 + \mathcal{O}\left(\frac{r_s}{a}\right)\right]$$

vdW

Examples: nucleons and ^{4}He **atoms**

strong



	N [MeV]	^{4}He [mK]
B_2	**2.22**	**1.31**
$1/ma^2$	**1.41**	**1.12**

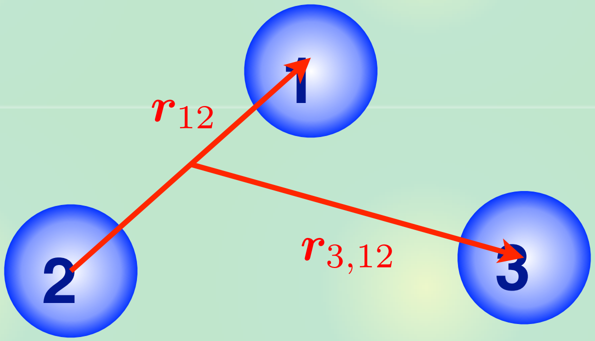
^{4}He

Three-body universal physics

Three-body system in hyperspherical coordinates

$(\boldsymbol{r}_{12}, \boldsymbol{r}_{3,12}) \leftrightarrow (R, \alpha_3, \hat{\boldsymbol{r}}_{12}, \hat{\boldsymbol{r}}_{3,12})$

hyperradius **hyperangular variables Ω (dimensionless)**



If |a| —> ∞, system is scale invariant.

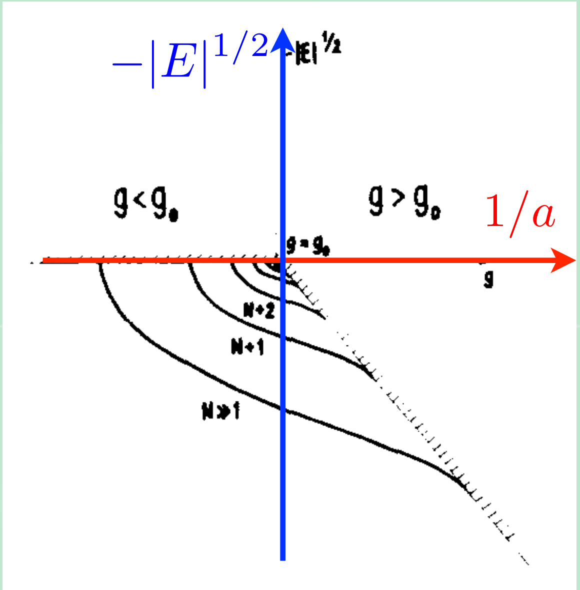
$$V(R,\Omega) \propto \frac{1}{R^2}$$

Efimov effect: attractive 1/R²

V. Efimov, Phys. Lett. B 33, 563 (1970)

$$B_3^n / B_3^{n+1} \approx 22.7^2$$

- **infinitely many bound states**
- **discrete scale invariance: RG limit cycle**

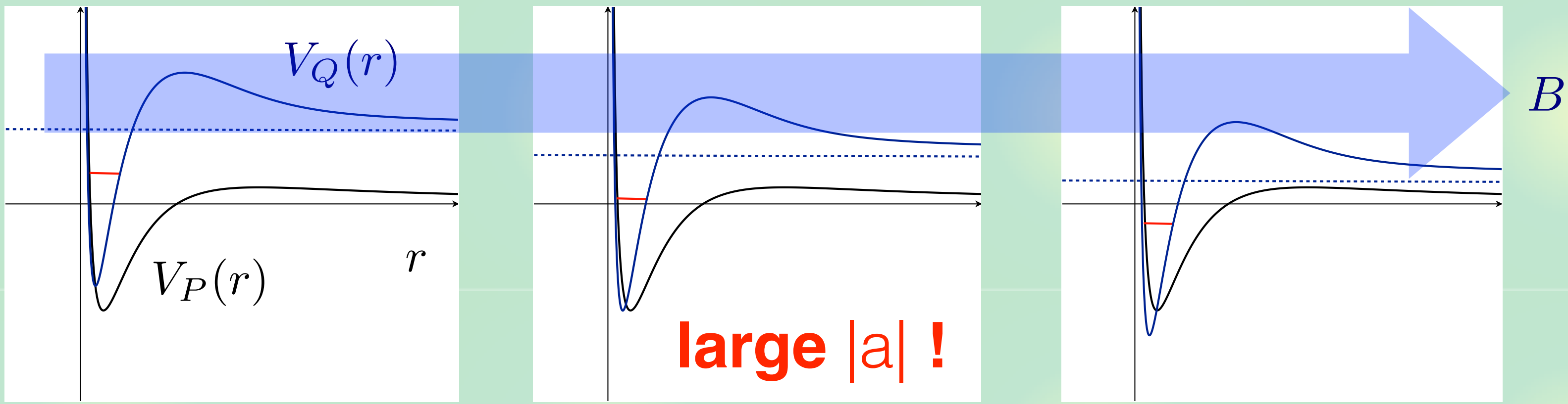


P.F. Bedaque, H.-W. Hammer, U. van Kolck, Phys. Rev. Lett. 82, 463 (1999)

Tuning two-body interactions

Large |a| is achieved by tuning two-body interaction

- Atomic physics: Feshbach resonance



In hadron physics, interactions are basically fixed.

- quark mass term in QCD: external scalar field

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}^{(0)}_{\mathrm{QCD}} - m\bar{q}q$$

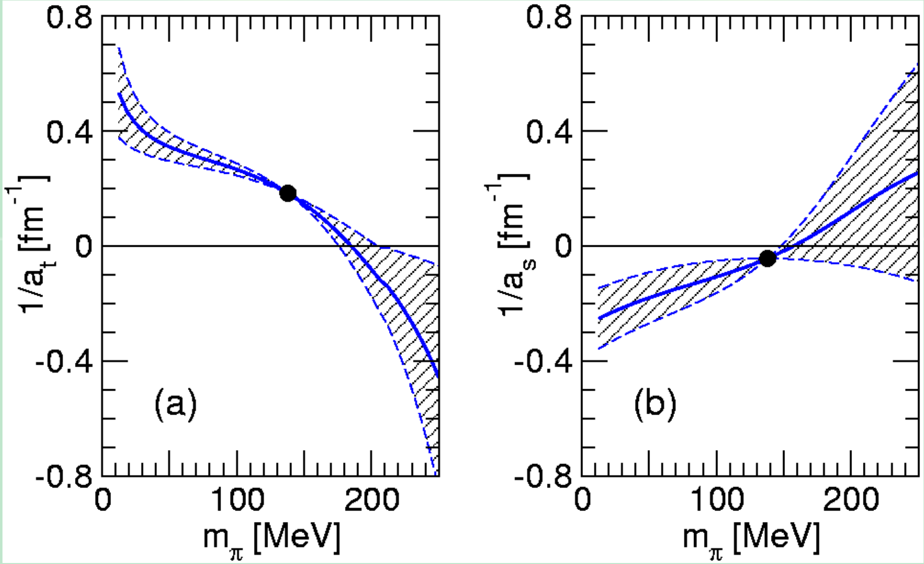
- variation of quark mass -> tuning hadron interaction?

Tuning two-hadron interactions

Nuclear force @ unphysical quark masses

- **Nuclear forces can reach unitary limit by** m_{ud} (m_π)↗

E. Braaten, H.-W. Hammer, Phys. Rev. Lett. 91, 102002 (2003)



- **predictable by chiral EFT for small** m_{ud}
- **calculable by lattice QCD for large** m_{ud}

c.f.) N. Barnea, L. Contessi, D. Gazit, F. Pederiva, U. van Kolck, Phys. Rev. Lett. 114, 052501 (2015)

Two-pion interaction

$\pi\pi$ scattering length <— chiral low energy theorem

S. Weinberg, Phys. Rev. Lett. 17, 616-621 (1966)

$$a^{I=0} \propto -\frac{7}{4}\frac{m_\pi}{f_\pi^2}, \quad a^{I=2} \propto \frac{1}{2}\frac{m_\pi}{f_\pi^2}$$

- $m_\pi \sim (m_{ud})^{1/2}$ ~ explicit breaking of chiral symmetry

Physical $\pi\pi$ (I=0) is unbound but has a resonance "σ"

(resonance: unstable eigenstate above threshold)

$f_0(500)$ or σ
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

A REVIEW GOES HERE – Check our WWW List of Reviews

$f_0(500)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\,\mathrm{Im}(\sqrt{s_{pole}})$.

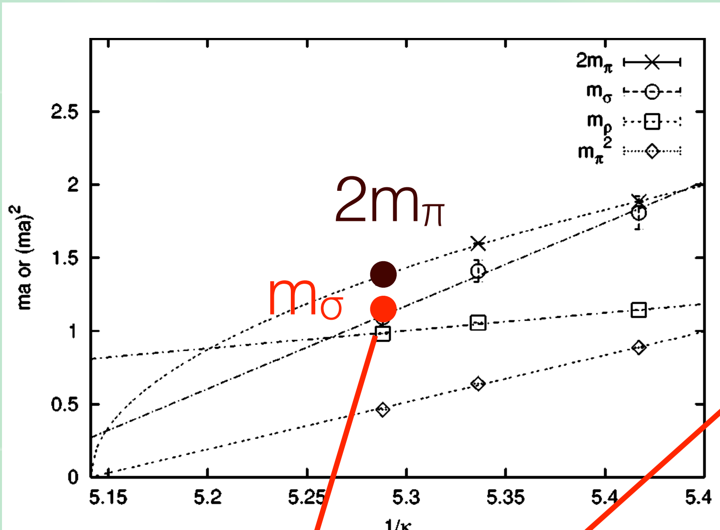
VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
(400–550)−i(200–350) OUR ESTIMATE			

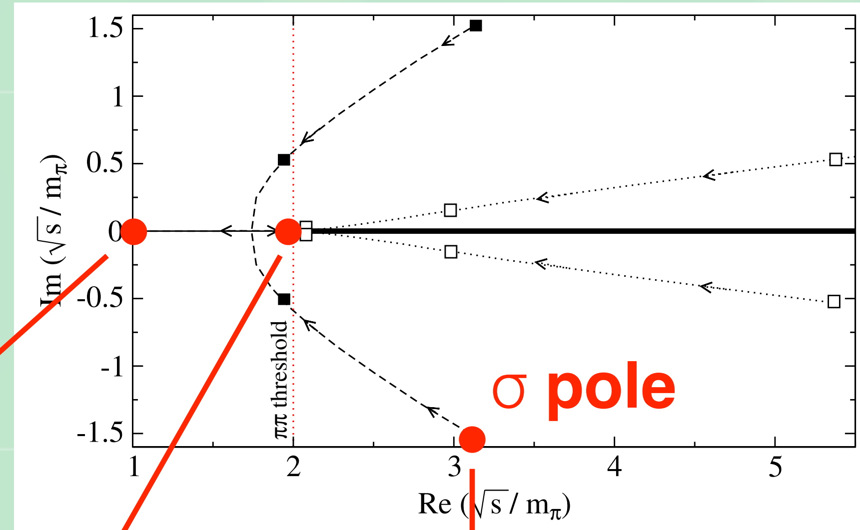
Low energy theorem is valid only for small m_{ud}

How about large m_{ud}? —> lattice QCD

Increasing ud quark mass

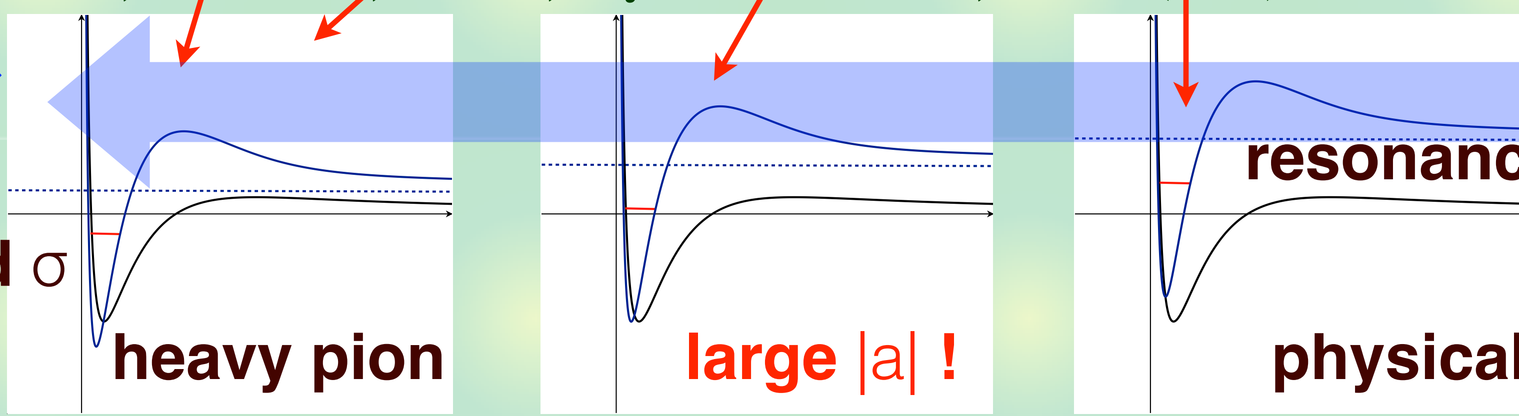
Lattice QCD/chiral EFT can tune the $\pi\pi$ interaction





T. Kunihiro *et al.* (SCALAR Collaboration), Rev. Rev. D70, 034504 (2004)

C. Hanhart, J.R. Pelaez, G. Rios, Phys. Rev. Lett. 100, 152001 (2008)



$\pi\pi$ scattering can reach unitary limit by increasing m_{ud}

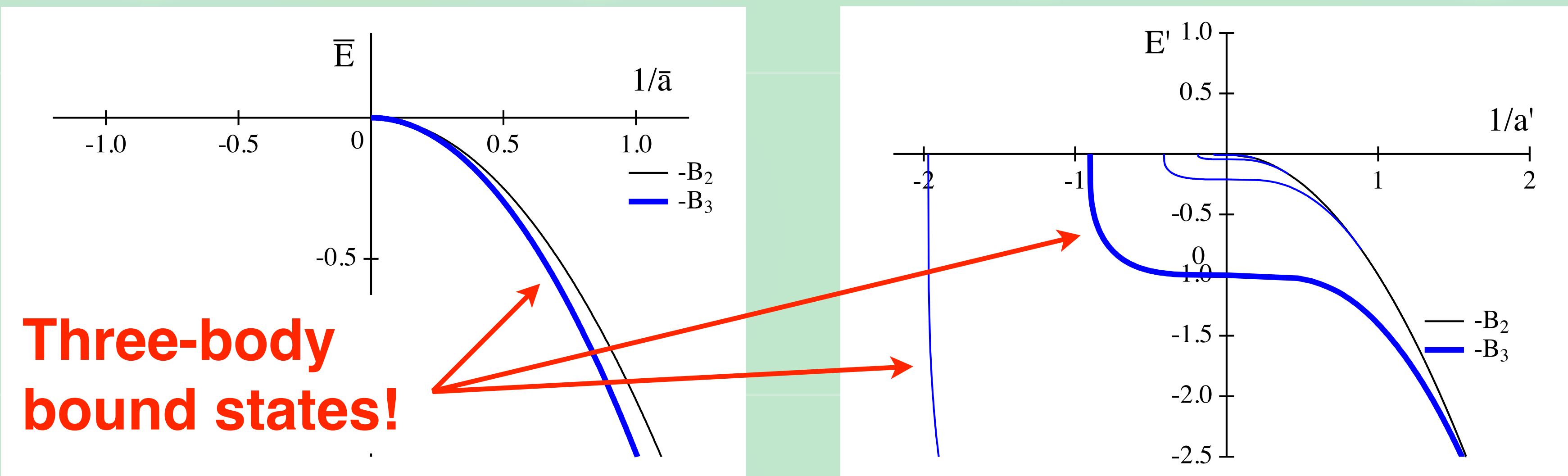
Universal physics of three pions

Universal physics of $\pi\pi\pi$ with large $\pi\pi$(I=0) scattering length

T. Hyodo, T. Hatsuda, Y. Nishida, Phys. Rev. C89, 032201(R) (2014)

- Let I=0 $\pi\pi$ scattering length large by **changing** m_q

isospin sym. ($\pi^0\pi^0\pi^0$-$\pi^0\pi^-\pi^+$) **isospin breaking** ($\pi^0\pi^0\pi^0$)



- Universal physics of pions @ unphysical m_{ud}
- Coupled-channel effect reduces the attraction.

Implication for real world

Universality -> a $\pi\pi\pi$ bound state @ heavy m_{ud}

- **Heavy m_{ud} is continuously connected to physical point.**

- **Existence of a pole (eigenstate) is stable against the continuous change of parameters (such as m_{ud}).**

[Y. Kamiya, T. Hyodo, arXiv:1711.04558 [hep-ph]]

$$\frac{1}{2\pi}\oint_C dz \frac{d\arg\mathcal{F}(z)}{dz} = (\#\text{ of zeros}) - (\#\text{ of poles})$$

—> $\pi\pi\pi$ state may exist as a resonance at physical point.

Possible candidates ~ I=1, J=0 state : excited state of π?

$\pi(1800)$ $I^G(J^P$

See also minireview under non-$q\bar{q}$ candid
Physics **G33** 1 (2006).

π(1800) MASS

VALUE (MeV) *EVTS* *DOCUMENT ID* *TECN* *CHG* *COMMENT*

$\pi(1300)$ $I^G(J^{PC}) = 1^-(0^{-+})$

π(1300) MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1300±100 OUR ESTIMATE				

Two neutrons and one flavored meson

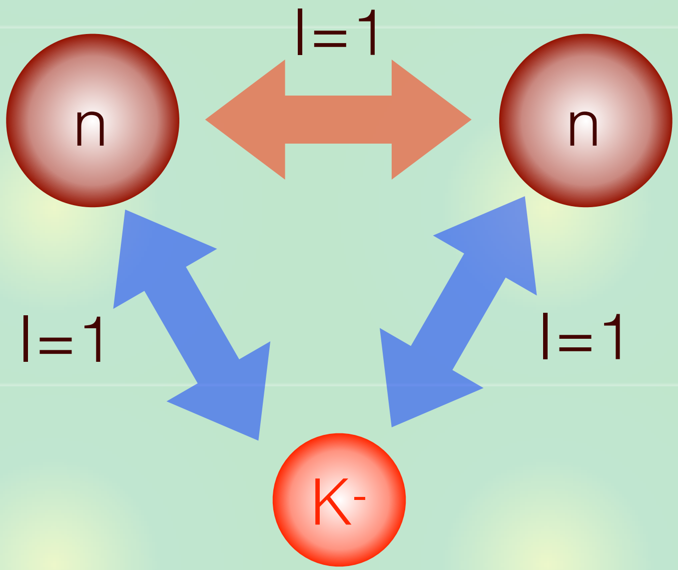
Flavored meson: K^- **(**$s\bar{u}$**, strangeness),** D^0 **(**$c\bar{u}$**, charm)**

K^-nn/D^0nn **system with** $J=0$, $I=3/2$, $I_3=-3/2$

- **different from** $J=0$, $I=1/2$ **(so-called** $K^-pp-\bar{K}^0np$**)**
- **all interactions: isospin** $I=1$ **(no** $\Lambda(1405)$**)**

Desirable features for Efimov effect

- **no coupled channels**
- **no Coulomb interaction**
- $|a_{nn}| \sim 20$ fm $\gg r_s \sim O(1)$ fm



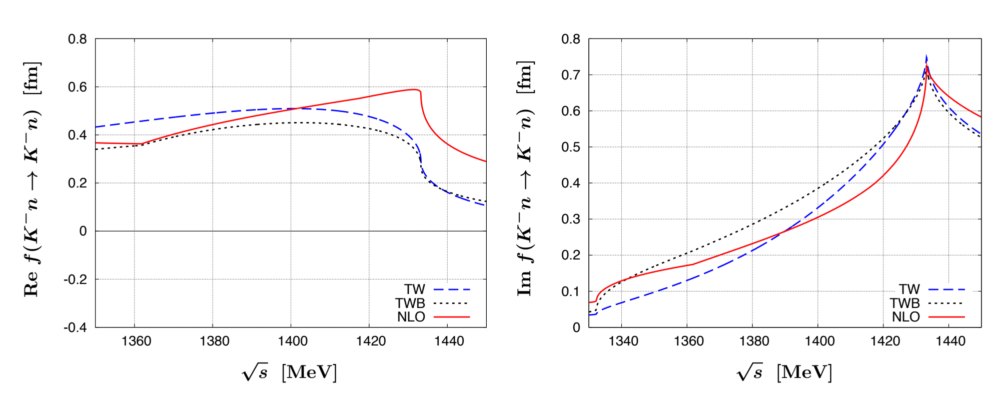
Two-body meson-neutron scattering length?

Meson-neutron interaction

K-n **system:** K̄N **scattering data**

Y. Ikeda, T. Hyodo, W. Weise, Phys. Lett. B706, 63 (2011); Nucl. Phys. A881, 98 (2012)

- **data fitted as χ^2/d.o.f. ~ 1**



$$a_{0,K^-n} = -0.57^{+0.21}_{-0.04} - i0.72^{+0.41}_{-0.26}\ \text{fm}$$

decay

—> Strangeness sector is unbound

D^{0}n **system: identify** Σ_c(2800) **as a** J^P=1/2- **state**

$\Sigma_c(2800)$ $\quad I(J^P) = 1(?^?)$ Status: $***$

Seen in the $\Lambda_c^+\pi^+$, $\Lambda_c^+\pi^0$, and $\Lambda_c^+\pi^-$ mass spectra.

Σ_c(2800) MASSES

- D^{0}n **threshold ~ 2804 MeV**

—> Charm sector has a shallow quasi-bound state

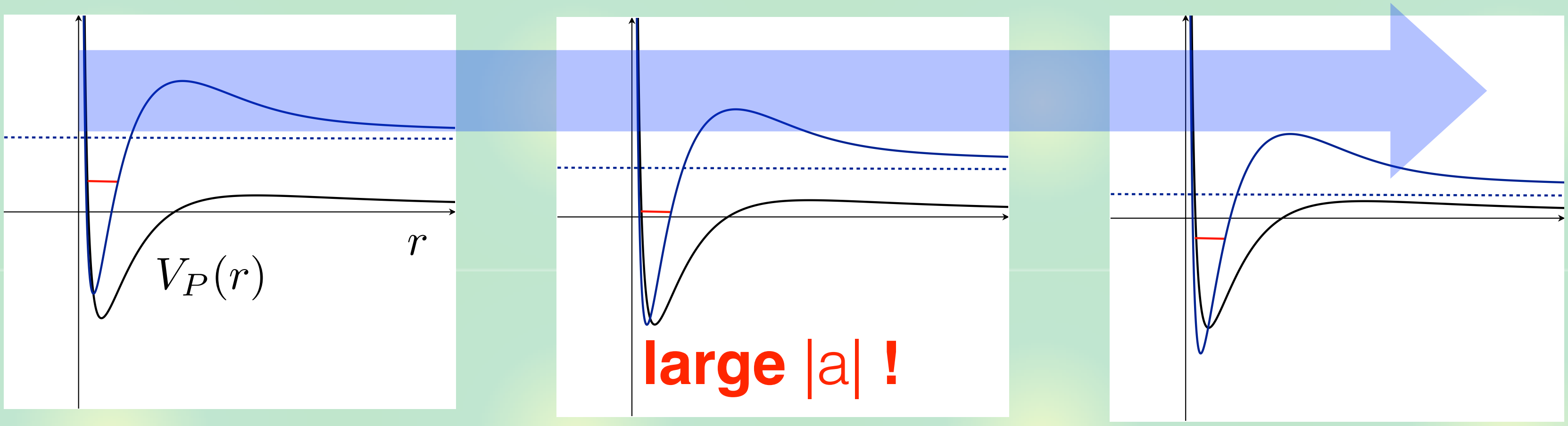
Idealization

Zero coupling limit (ZCL)

- coupling to decay channels are switched off
- $a_{K^-n} < 0$ (attractive), $a_{D^0n} > 0$ (repulsive, with bound state)

Varying m_s —> m_c

strange, unbound　　　　charm, bound



Unitary limit: tuning $m_{s/c}$ in ZCL

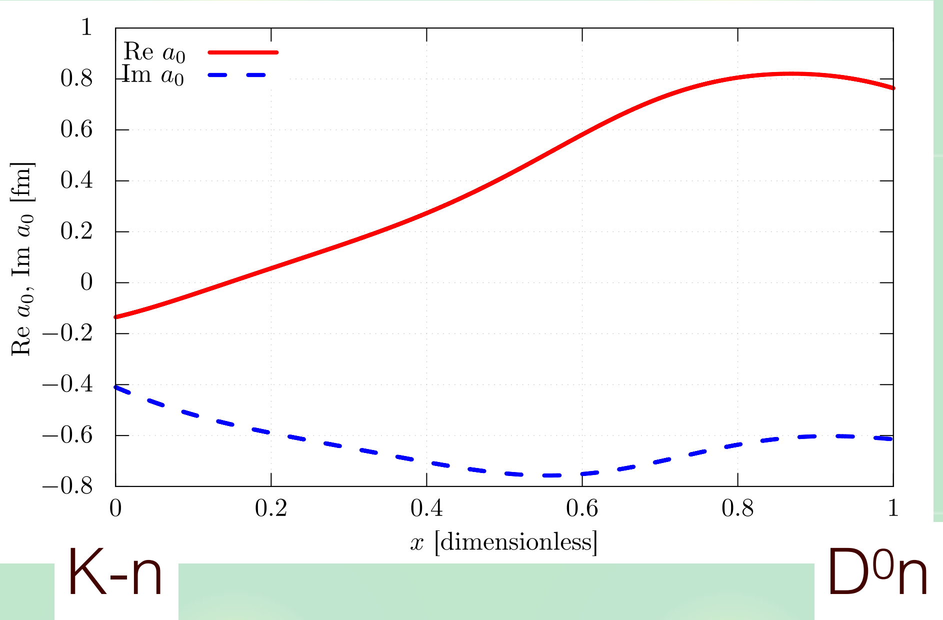
Model extrapolation

Contact interaction model with extrapolation parameter x

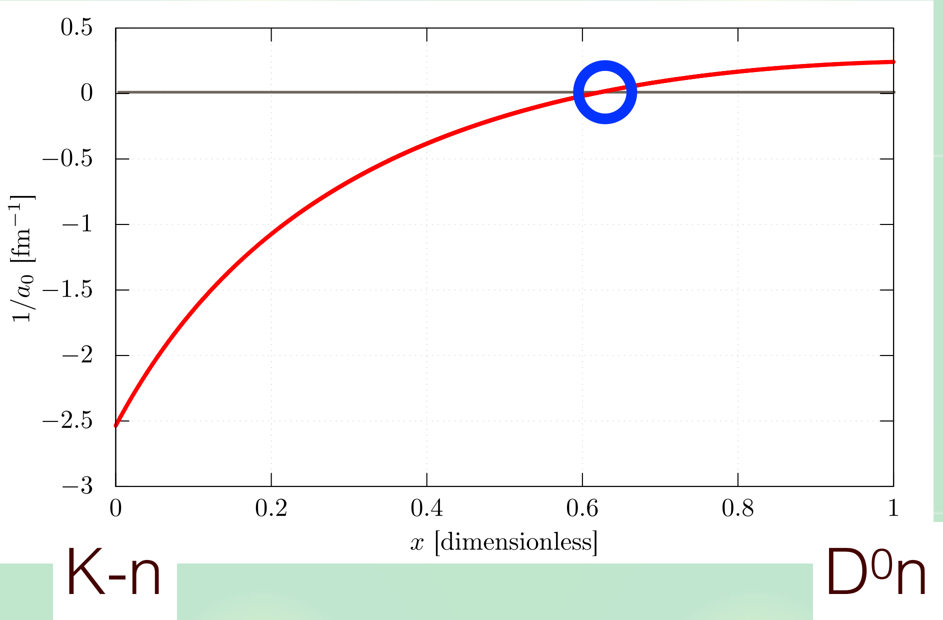
U. Raha, Y. Kamiya, S.-I. Ando, T. Hyodo, arXiv:1708.03369 [nucl-th]

- x=0 : K-n, x=1 : D^0n

with decay channels



ZCL (without decay)



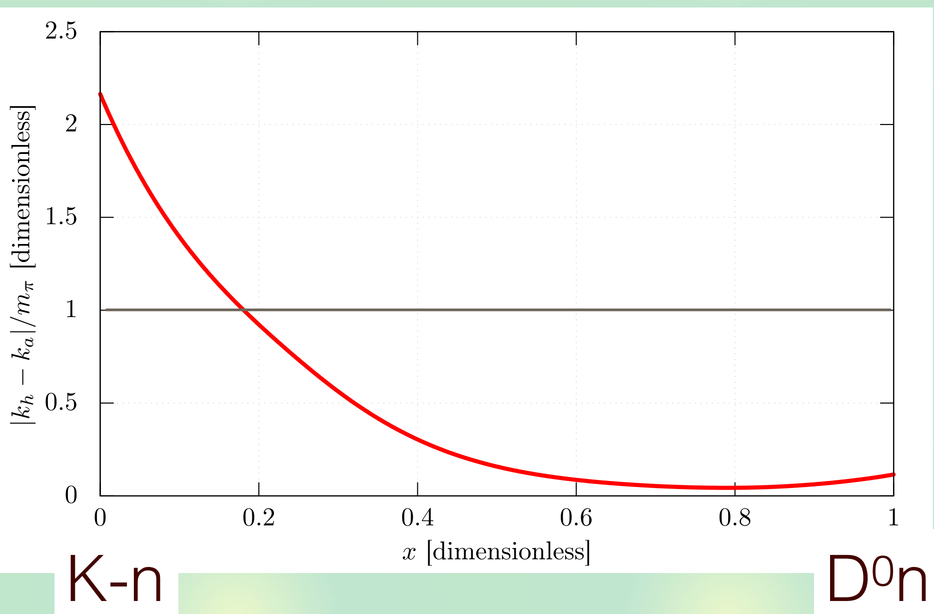
- **In ZCL, unitary limit at** m_K = 1337 **MeV** (x~0.6)
- **With decay channel, remnant is not very clear.**

Two-body universality

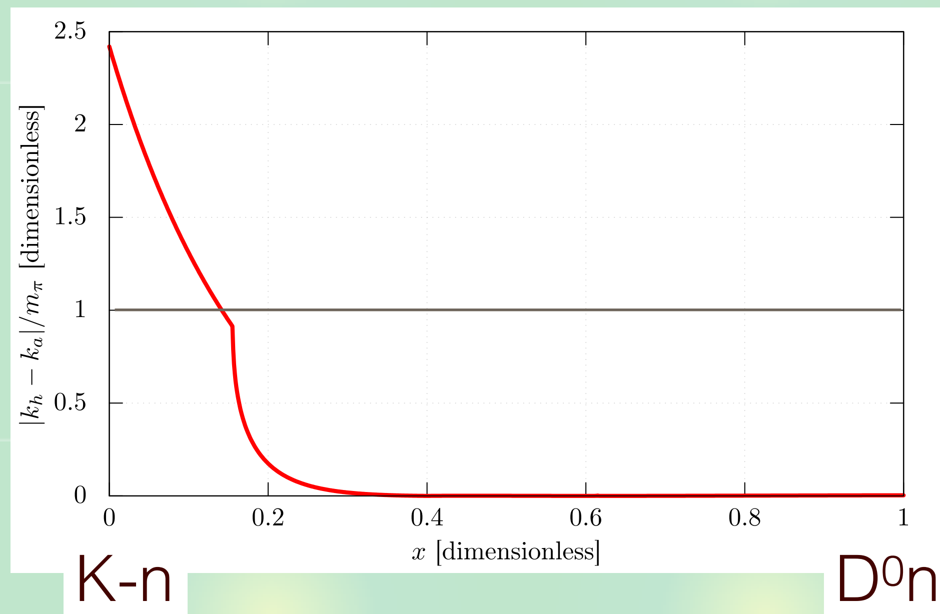
Check of universality: $|k_h - k_a|/m_\pi$

- **full eigenmomentum:** $k_h = \sqrt{2\mu E_h}$
- **universality prediction:** $k_a = \frac{i}{a}\left[1 + \mathcal{O}\left(\frac{r_s}{a}\right)\right], \quad r_s \sim 1/m_\pi$

with decay channels



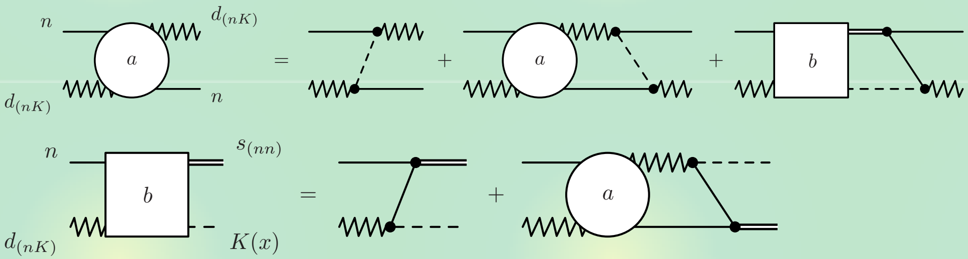
ZCL (without decay)



Universality governs the system near D^0n even with decay

Three-body system

Three-body equations for K-nn/D⁰nn system



Asymptotic behavior of K-nn/D⁰nn system at unitary limit:

- $a_{s(nn)}$ —> ∞ **and** $a_{d(nK)}$ —> ∞

$$1 = C_1 \frac{2\pi}{s} \frac{\sin[s \arcsin(a/2)]}{\cos(\pi s/2)} + C_2 \frac{4\pi^2}{s^2} \frac{\sin^2[s\, \mathrm{arccot}(\sqrt{4b-1})]}{\cos^2(\pi s/2)} \quad \Rightarrow \quad s_0 = 1.01156$$

- $a_{s(nn)}$ **fixed and** $a_{d(nK)}$ —> ∞

$$1 = C_1 \frac{2\pi}{s} \frac{\sin[s \arcsin(a/2)]}{\cos(\pi s/2)} \quad \Rightarrow \quad s_0 = 0.327675$$

- **RG limit cycle in the asymptotic expressions: Efimov effect**

Implication for real world:

—> D⁰nn **state may exist as a resonance at physical point.**

Summary

- **Hadron-hadron interactions can be tuned by changing quark masses.**
- **Increase** m_{ud} **: unitary** $\pi\pi$ **(I=0) scattering —> Efimov effect in** $\pi^0\pi^0\pi^0$

  T. Hyodo, T. Hatsuda, Y. Nishida, Phys. Rev. C89, 032201(R) (2014)
- **Decrease** m_c **(+ZCL) : unitary** D^0n **scattering —> Efimov effect in** D^0nn

  U. Raha, Y. Kamiya, S.-I. Ando, T. Hyodo, arXiv:1708.03369 [nucl-th]
- **Remnant of Efimov physics may be observed as a three-body resonance in nature.**