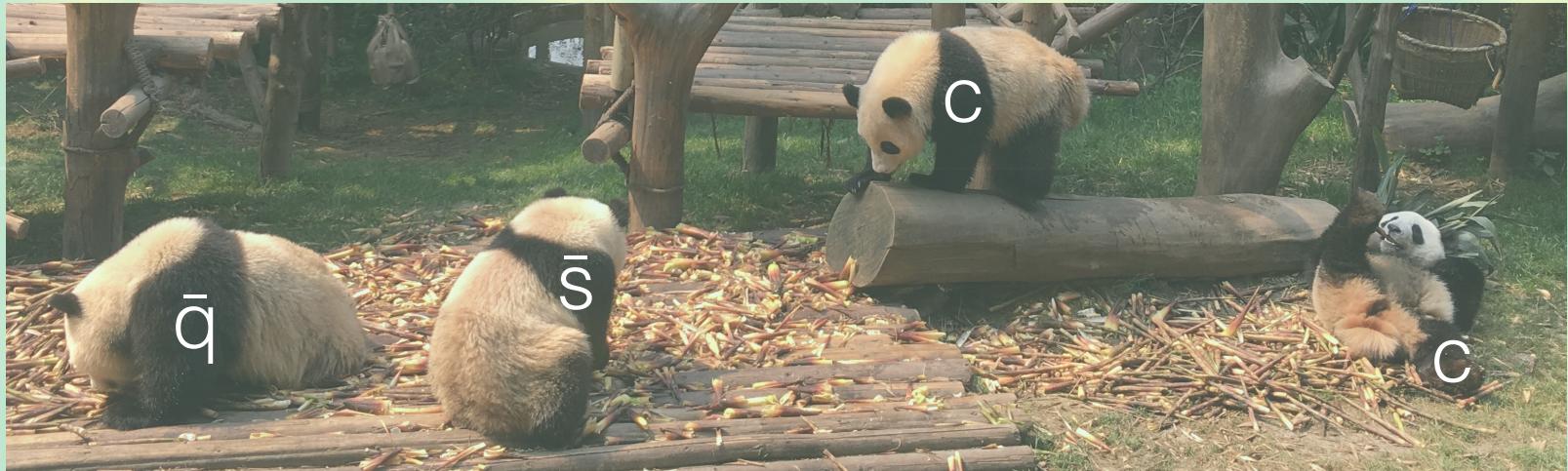


Exotic hadrons and emergent long range correlation in QCD



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2019, Jan. 29th

Classification of hadrons

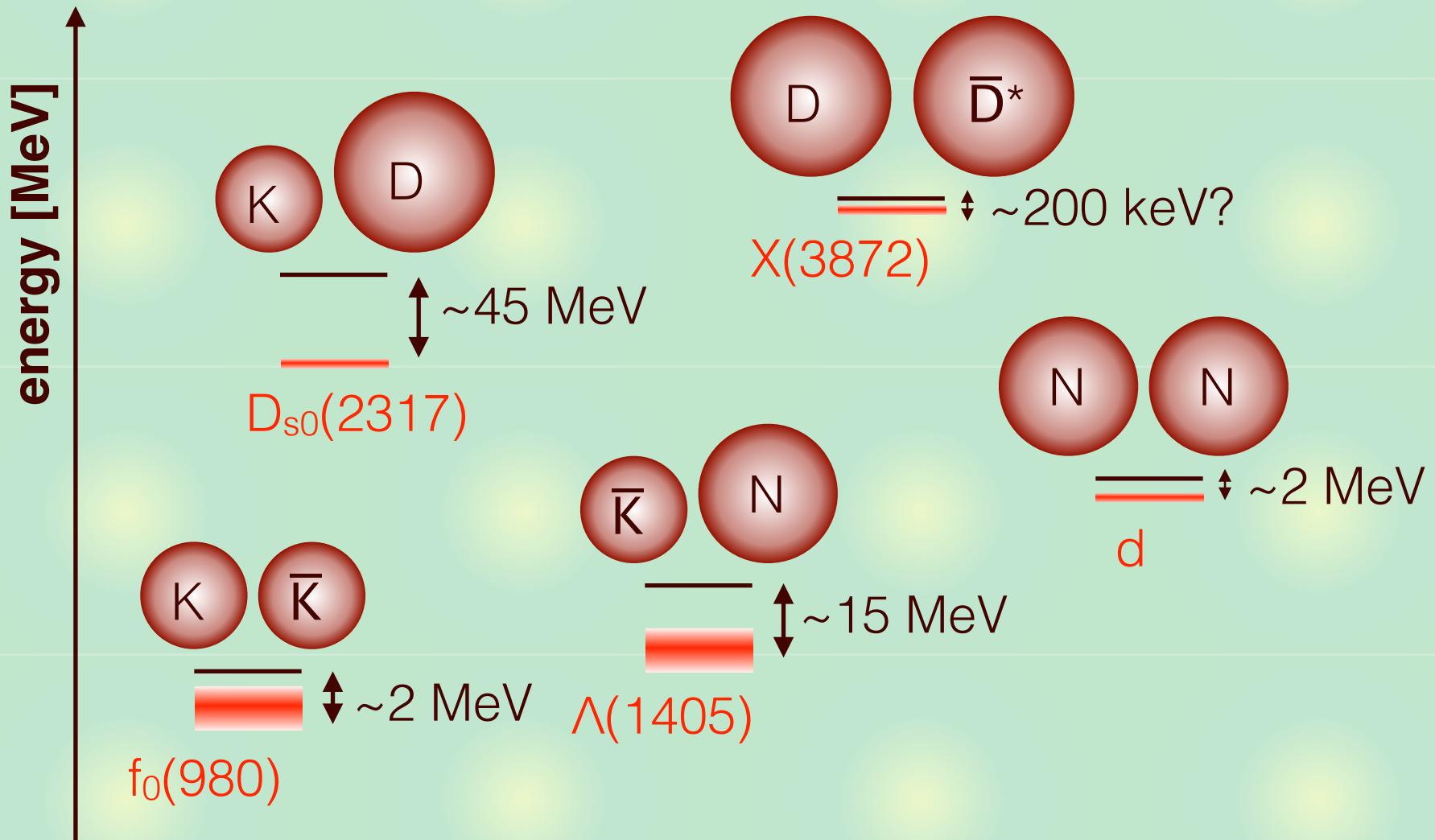
Observed hadrons

PDG2018 : <http://pdg.lbl.gov/>

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****	Λ_c^+	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	***	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1530)$	$3/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1480)$	$1/2^+$	****	$\Xi(1620)$	*		$\Lambda_c(2765)^+$	*	
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$	*		$\Xi(1690)$	***		$\Lambda_c(2880)^+$	$5/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	**	$\Sigma(1560)$	**		$\Xi(1820)$	$3/2^-$	***	$\Lambda_c(2940)^+$	***	
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$	***		$\Sigma_c(2455)$	$1/2^+$	****
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq \frac{5}{2}^?$	***	$\Sigma_c(2520)$	$3/2^+$	***
$N(1685)$	*		$\Delta(1920)$	$3/2^-$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$	*		$\Sigma_c(2800)$	***	
$N(1700)$	$3/2^-$	***	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	***	$\Xi(2250)$	**		$\Xi_c(2645)$	$3/2^+$	***
$N(1710)$	$1/2^+$	***	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$	**		$\Xi(2370)$	**		$\Xi_c(2790)$	$1/2^-$	***
$N(1720)$	$3/2^+$	****	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1730)$	$3/2^+$	*	$\Xi(2500)$	*		$\Xi_c(2815)$	$3/2^-$	***
$N(1860)$	$5/2^+$	**	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1750)$	$1/2^-$	***	$\Xi(2500)$	*		$\Xi_c(2930)$	*	
$N(1875)$	$3/2^-$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1770)$	$1/2^+$	*	Ω^-	$3/2^+$	****	$\Xi_c(2980)$	***	
$N(1880)$	$1/2^+$	**	$\Delta(2200)$	$7/2^-$	*	$\Sigma(1775)$	$5/2^-$	***	$\Omega(2250)$	***		$\Xi_c(3055)$	***	
$N(1895)$	$1/2^-$	**	$\Delta(2300)$	$9/2^-$	***	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2380)$	**		$\Xi_c(3080)$	***	
$N(1900)$	$3/2^+$	***	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2470)$	**		$\Xi_c(3123)$	*	
$N(1990)$	$7/2^-$	**	$\Delta(2390)$	$7/2^-$	*	$\Sigma(1900)$	$1/2^-$	*	$\Omega(2470)$	$1/2^+$	***	$\Xi_c(2980)$	***	
$N(2000)$	$5/2^+$	**	$\Delta(2400)$	$9/2^-$	**	$\Sigma(1915)$	$5/2^+$	****	$\Xi_c(2645)$	$3/2^+$	***	$\Xi_c(2790)$	$1/2^-$	***
$N(2040)$	$3/2^+$		$\Delta(2420)$	$11/2^-$	****	$\Sigma(1940)$	$3/2^+$	*	$\Xi_c(2815)$	$3/2^-$	***	$\Xi_c(2930)$	*	
$N(2060)$	$5/2^-$	**	$\Delta(2750)$	$13/2^-$	**	$\Sigma(1940)$	$3/2^-$	***	$\Xi_c(2980)$	*		$\Xi_c(3055)$	***	
$N(2100)$	$1/2^+$	*	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2000)$	$1/2^-$	*	$\Xi_c(3080)$	*		$\Xi_c(3123)$	*	
$N(2120)$	$3/2^-$	**	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2030)$	$7/2^+$	****	$\Xi_c(3123)$	*		$\Xi_c(2980)$	***	
$N(2190)$	$7/2^-$	****	Λ	$1/2^+$	****	$\Sigma(2030)$	$7/2^+$	****	$\Xi_c(3123)$	*		$\Xi_c(2980)$	***	
$N(2220)$	$9/2^-$	****	$\Lambda(1405)$	$1/2^-$	****	$\Sigma(2080)$	$3/2^+$	**	$\Xi_c(3123)$	*		$\Xi_c(2980)$	***	
$N(2250)$	$9/2^-$	****	$\Lambda(1520)$	$3/2^-$	****	$\Sigma(2100)$	$7/2^-$	*	$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$N(2300)$	$1/2^+$	**	$\Lambda(1600)$	$1/2^+$	***	$\Sigma(2250)$	***		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$N(2570)$	$5/2^-$	**	$\Lambda(1670)$	$1/2^-$	****	$\Sigma(2455)$	**		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$N(2600)$	$11/2^-$	***	$\Lambda(1690)$	$3/2^-$	****	$\Sigma(2620)$	**		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$N(2700)$	$13/2^+$	**	$\Lambda(1710)$	$1/2^+$	*	$\Sigma(3000)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1800)$	$1/2^-$	***	$\Lambda(1810)$	$1/2^+$	***	$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1810)$	$1/2^+$	***	$\Lambda(1820)$	$5/2^+$	****	$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1820)$	$5/2^+$	****	$\Lambda(1830)$	$5/2^-$	***	$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1830)$	$5/2^-$	***	$\Lambda(1840)$	$3/2^+$	****	$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1840)$	$3/2^+$	****	$\Lambda(1850)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1850)$	*		$\Lambda(1860)$	$7/2^+$	*	$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1860)$	$7/2^+$	*	$\Lambda(1870)$	$3/2^-$	*	$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1870)$	$3/2^-$	*	$\Lambda(1880)$	$3/2^+$	*	$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1880)$	$3/2^+$	*	$\Lambda(1890)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1890)$	*		$\Lambda(1900)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1900)$	*		$\Lambda(1910)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1910)$	*		$\Lambda(1920)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1920)$	*		$\Lambda(1930)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1930)$	*		$\Lambda(1940)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1940)$	*		$\Lambda(1950)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1950)$	*		$\Lambda(1960)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1960)$	*		$\Lambda(1970)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1970)$	*		$\Lambda(1980)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1980)$	*		$\Lambda(1990)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(1990)$	*		$\Lambda(2000)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
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$\Lambda(2010)$	*		$\Lambda(2020)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2020)$	*		$\Lambda(2030)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2030)$	*		$\Lambda(2040)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2040)$	*		$\Lambda(2050)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2050)$	*		$\Lambda(2060)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
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$\Lambda(2080)$	*		$\Lambda(2090)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2090)$	*		$\Lambda(2100)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2100)$	*		$\Lambda(2110)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2110)$	*		$\Lambda(2120)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2120)$	*		$\Lambda(2130)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2130)$	*		$\Lambda(2140)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
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$\Lambda(2160)$	*		$\Lambda(2170)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2170)$	*		$\Lambda(2180)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2180)$	*		$\Lambda(2190)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2190)$	*		$\Lambda(2200)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2200)$	*		$\Lambda(2210)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2210)$	*		$\Lambda(2220)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2220)$	*		$\Lambda(2230)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2230)$	*		$\Lambda(2240)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2240)$	*		$\Lambda(2250)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2250)$	*		$\Lambda(2260)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2260)$	*		$\Lambda(2270)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
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$\Lambda(2300)$	*		$\Lambda(2310)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2310)$	*		$\Lambda(2320)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2320)$	*		$\Lambda(2330)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2330)$	*		$\Lambda(2340)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2340)$	*		$\Lambda(2350)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2350)$	*		$\Lambda(2360)$	*		$\Sigma(3170)$	*		$\Xi_c(3123)$	*		$\Xi_c(3123)$	*	
$\Lambda(2360)$	*		$\Lambda(2370)$	*		$\Sigma($								

Hadron clusters

Hadrons near an s-wave two-body threshold



“hadronic molecules” (various flavors, baryon numbers, ...)

Two-body universal physics

Near-threshold s-wave state: universal physics

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006);

P. Naidon, S. Endo, Rept. Prog. Phys. 80, 056001 (2017)

- scattering length $|a| \gg$ interaction range r_e
- size of (quasi-)bound state $\sim |a|$: loosely bound
- relation with eigenenergy E

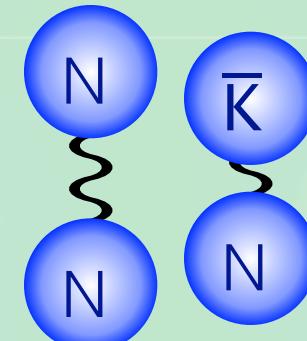
$$a(E) \sim \frac{i}{k} = \frac{i}{\sqrt{2\mu E}}$$

vdW

Examples: d, $\Lambda(1405)$, ${}^4\text{He}$ dimer

	NN [fm]	$\bar{K}N$ [fm]	${}^4\text{He}$ [a_0]
$a(E)$	4.3	1.2-0.8i	178
a_{emp}	5.1	1.4-0.9i	189
r_e	1.4	0.4	10

strong

 ${}^4\text{He}$

Classification of hadrons

Observed hadrons

PDG2018 : <http://pdg.lbl.gov/>

2	-1/2+ ***	$\Lambda(1222)$	2/3+ ****	Σ^+	-1/2+ ***	-0	1/2+ ****	Ξ^+	1/2+ ***
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LIGHT UNFLAVORED $(C=C=0,0)$	STRANGE $(C=1,C=0,0)$	CHARMED, STRANGE $(C,C=1,0)$	CHARMED $C=0,0,0$
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Only color singlet states are observed.

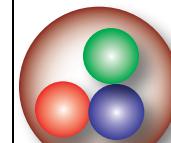
→ Color confinement problem

Flavor quantum numbers are described by $qqq/q\bar{q}$.

Why no $qq\bar{q}\bar{q}$, $qqqq\bar{q}\bar{q}$, ... states (exotic hadrons)?

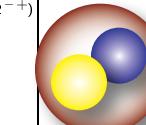
→ Exotic hadron problem, as not trivial as confinement!

$N(2700)$	13/2+ **	$\Lambda(1710)$	1/2+ *	$\Sigma(3000)$	*	Σ_b	1/2+ ***
$\Lambda(1800)$	1/2- ***			$\Sigma(3170)$	*	Ξ_b^0	3/2+ ***
$\Lambda(1810)$	1/2+ ***					Ξ_b^0, Ξ_b^-	1/2+ ***
$\Lambda(1820)$	5/2+ ****					$\Xi_b'(5935)^-$	1/2+ ***
$\Lambda(1830)$	5/2- ****					$\Xi_b(5945)^0$	3/2+ ***
$\Lambda(1890)$	3/2+ ****					$\Xi_b(5955)^-$	3/2+ ***
$\Lambda(2000)$	*					Ω_b^-	1/2+ ***
$\Lambda(2020)$	7/2+ *						
$\Lambda(2050)$	3/2- *						
$\Lambda(2100)$	7/2- ****						
$\Lambda(2110)$	5/2+ ***						
$\Lambda(2325)$	3/2- *						
$\Lambda(2350)$	9/2+ ***						
$\Lambda(2585)$	**						



~ 150 baryons

$a_1(1640)$	1- (1+-)	$a_0(2450)$	1- (6+-)	$D_0(2400)^0$	1/2(0+)	BOTTOM, CHARMED ($B = C = \pm 1$)	$\chi_{b1}(1^P)$
$f_0(1640)$	0+(2++)	$f_0(2510)$	0+(5++)	$D_0^*(2400)^{\pm}$	1/2(0+)	$\bullet h_1(1P)$	$\gamma^2(1^{+-})$
$\bullet \omega_2(1645)$	0+(2-+)			$D_1(2420)^0$	1/2(1+)	$\bullet \chi_{b2}(1P)$	$\pi^+(2^{++})$
$\bullet \omega_0(1650)$	0-(1--)			$D_1(2420)^{\pm}$	1/2(2??)	$\eta_2(2S)$	$\pi^+(0^-)$
$\bullet \omega_3(1670)$	0-(3--)			$D_2(2430)^0$	1/2(1+)	$\bullet T(2S)$	$\pi^0(1^-)$
$\bullet \pi_2(1670)$	1-(2-+)			$D_2(2460)^0$	1/2(2+)	$\bullet T(1D)$	$\pi^0(2^-)$
				$D_2^*(2460)^{\pm}$	1/2(2+)	$\bullet \chi_{b3}(2P)$	$\pi^0(0^{++})$
				$D_3(2550)^0$	1/2(0-)	$\bullet \chi_{b4}(2P)$	$\pi^+(1^{++})$
				$D_3(2600)$	1/2(2?)	$\bullet h_2(2P)$	$\gamma^2(1^{+-})$
				$D^*(2640)^{\pm}$	1/2(2?)	$\bullet \chi_{b5}(3S)$	$\pi^0(1^{--})$
				$D(2750)$	1/2(2?)	$\bullet T(4S)$	$\pi^0(1^{++})$
						$\bullet X(10610)^{\pm}$	$\pi^0(1^{+-})$
						$X(10610)^0$	$\pi^+(1^{++})$
						$X(10650)^{\pm}$	$\pi^+(1^{++})$
						$\bullet T(1060)$	$\pi^0(1^{--})$
						$\bullet T(11020)$	$\pi^0(1^{--})$



~ 210 mesons

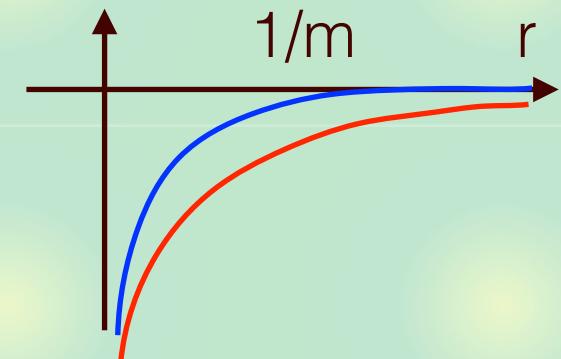
All ~ 360 hadrons emerge from single QCD Lagrangian.

Long range correlation in QCD?

Two-body potential

$$V(r) \propto \frac{1}{r} \quad : \text{long (infinite) range}$$

$$V(r) \propto \frac{e^{-mr}}{r} \quad : \text{finite } (\sim 1/m) \text{ range}$$



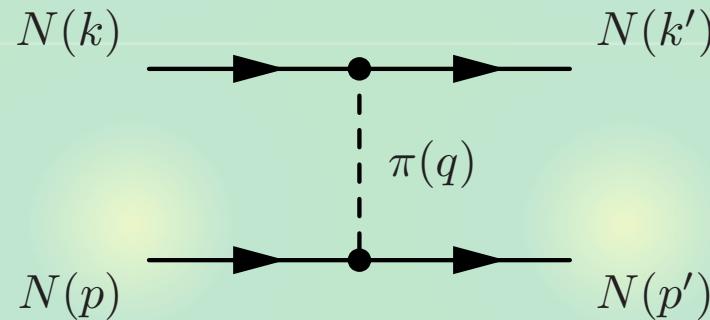
Hadron-hadron interaction is considered to be **finite range**.

- Longest interaction range
← exchange of lightest particle (π) ~ 1 fm
- Absence of the long range force is the basis for the (standard) scattering theory, Lüscher/HALQCD method, etc.

There can be (quasi) **long range force beyond 1 fm**.

M. Sanchez Sanchez, L.S. Geng, J. Lu, T. Hyodo, M.P. Valderrama, PRD98, 054001 (2018)

NN potential

Low energy NN interaction : π exchange

- **Static approx.** $p^\mu = (M_N, \mathbf{p})$, $p'^\mu = (M_N, \mathbf{p}')$, $q^\mu = p'^\mu - p^\mu = (0, \mathbf{q})$

- **Coupling** $g \bar{N} i \gamma_5 \pi N \sim g \chi^\dagger \sigma \cdot q \chi$ **(isospin ignored)**

Potential

$$V(r) \sim \text{F.T.} \left\{ g^2 (\sigma_1 \cdot q) (\sigma_2 \cdot q) \frac{-1}{q^2 + m_\pi^2} \right\}$$

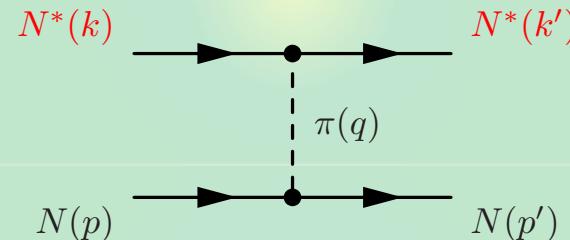
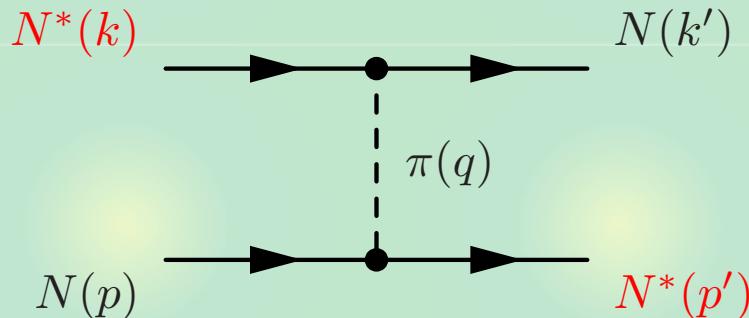
$\frac{1}{(q^0)^2 - \mathbf{q}^2 - m_\pi^2}$

Tensor op. **Yukawa** $\frac{e^{-m_\pi r}}{r}$

Emergence of long range correlation

NN* potential (exchange)

NN*(J^P=1/2-) interaction



**Mass difference
= energy transfer**

$$\Delta = M_{N^*} - M_N$$

- **Static approx.** $p^\mu = (M_N, \mathbf{p})$, $p'^\mu = (M_{N^*}, \mathbf{p}')$, $q^\mu = (\Delta, \mathbf{q})$

- **Coupling** $\tilde{g} \bar{N}^* \pi N + \text{h.c.} \sim \tilde{g} \chi^\dagger \mathbf{1} \chi$

Potential (P_σ: spin exchange factor)

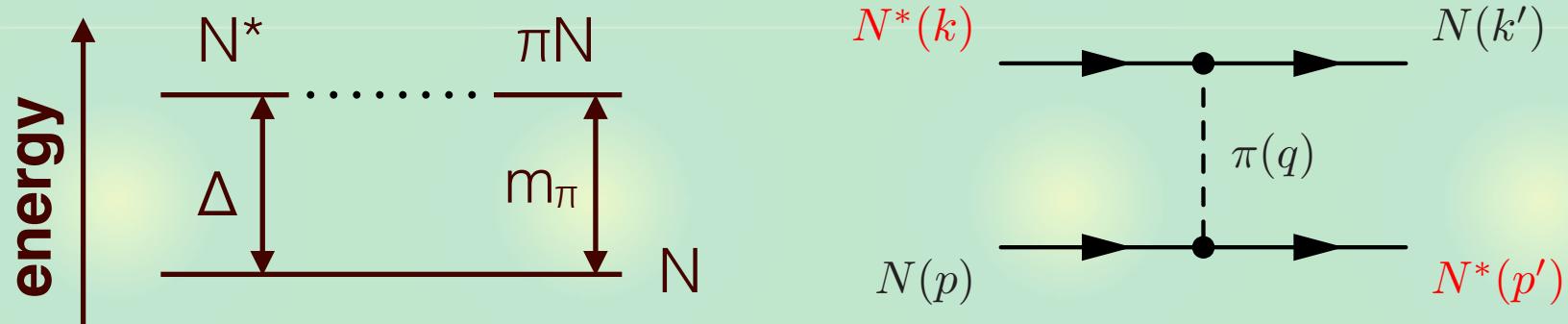
$$\mu = \sqrt{m_\pi^2 - \Delta^2}$$

$$V(r) \sim \text{F.T.} \left\{ \tilde{g}^2 \frac{1}{\Delta^2 - q^2 - m_\pi^2} \right\} P_\sigma = \text{F.T.} \left\{ \tilde{g}^2 \frac{-1}{q^2 + \mu^2} \right\} P_\sigma \sim \tilde{g}^2 P_\sigma \frac{e^{-\mu r}}{r}$$

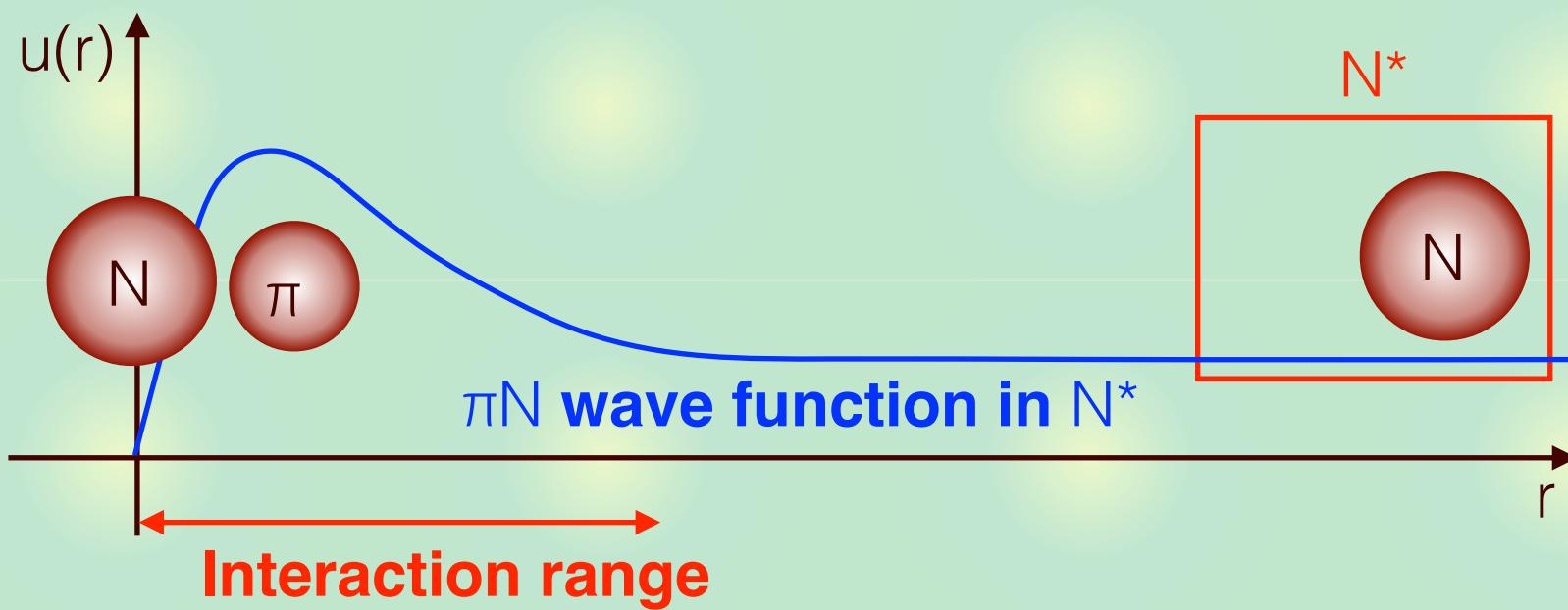
- Sign of $V(r)$ is fixed and attractive (c.f. σ exchange in NN)
- Effective mass $\mu=0 \rightarrow$ long range force (Coulomb like)

Unitary limit and zero-energy resonance

What does $\mu = (m_\pi^2 - \Delta^2)^{1/2} = 0 \Leftrightarrow \Delta = m_\pi$ mean?

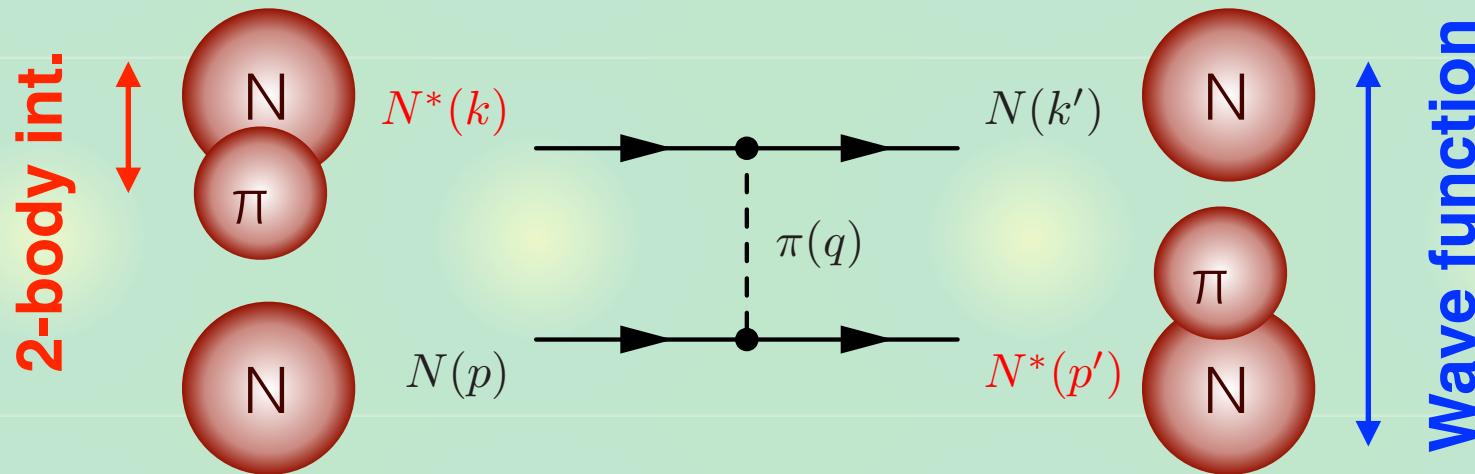


- $\Delta = m_\pi$: N^* lies on top of the πN threshold $\rightarrow a_{\pi N} = \infty$



Remarks and toward physical realization

$N^*N \sim \pi NN$: effective description of three-body system



Similarity with the Efimov effect

- spatially large three-body system via unitary two-body int.
- $1/r$ attraction (not $1/r^2$)?

Realization in physical hadron systems

- No system with exact $\mu=0$ (N^* : $\Delta \sim 595$ MeV / $m_\pi \sim 140$ MeV)
- Is there any system with small μ ?

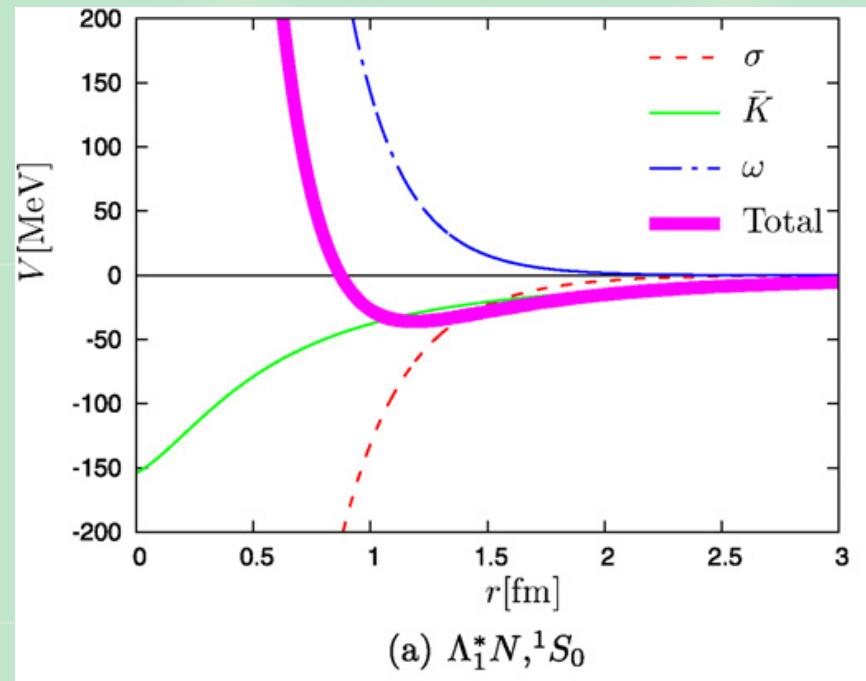
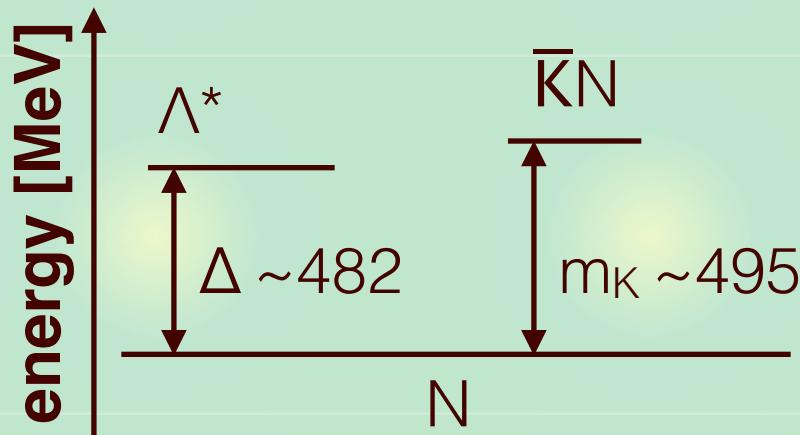
Strange dibaryon

$\Lambda(1405)=\Lambda^*$: $\bar{K}N$ quasibound state near the threshold

T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

- \bar{K} exchange between Λ^* and N

Λ^* (at 1420 MeV), $\bar{K}N$ threshold



- $\mu \sim 91$ MeV: \bar{K} exchange has longer tail than expected
- attractive in spin singlet channel $\rightarrow \bar{K}NN$ as $\Lambda^* N$ system

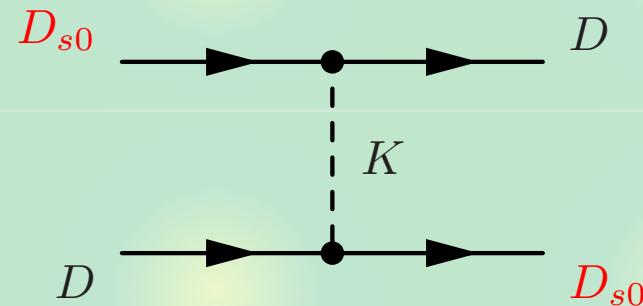
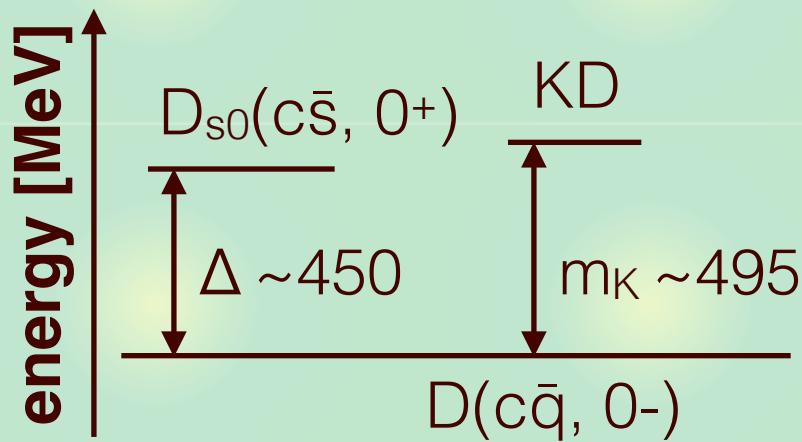
T. Uchino, T. Hyodo, M. Oka, Nucl. Phys. A, 868-869, 53 (2011)

Doubly charmed exotic meson

We consider $D_{s0}(c\bar{s}, 0^+)D(c\bar{q}, 0^-)$ system via K exchange

- Charm C=2: manifestly **exotic** ($cc\bar{q}\bar{s}$)

$D_{s0}(2317)$, KD threshold



- K exchange gives **quasi-long range** ($\mu \sim 200$ MeV) attraction

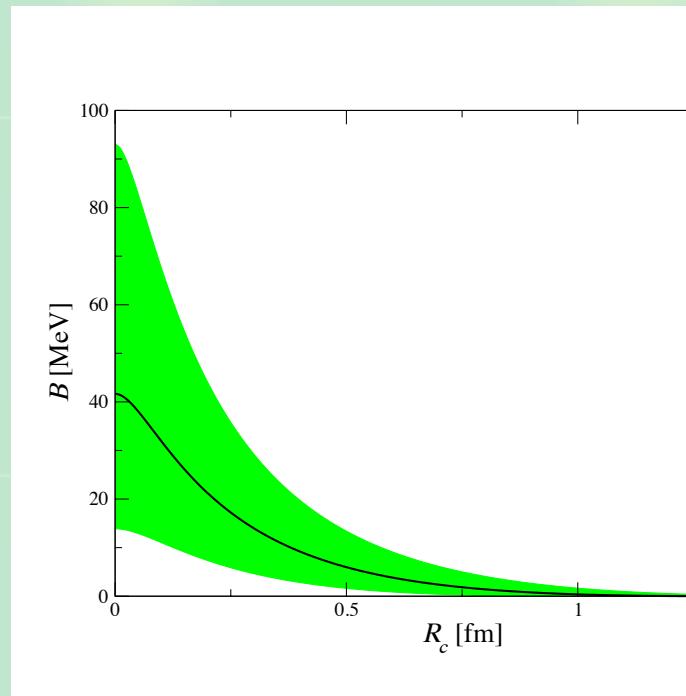
Can the attraction generate a bound state?

Prediction of binding energy

Effective Lagrangian for $D_{s0}DK$ (and HQ partners) coupling

$$\mathcal{L} = \frac{h}{2} \text{Tr}[\bar{H}_a S_b A_{ab} \gamma_5] + \text{C.C.}$$

- coupling constant h : $D_0 \rightarrow D\pi$ decay + SU(3) symmetry
- Short range cutoff $R_c \leftarrow$ hadron size

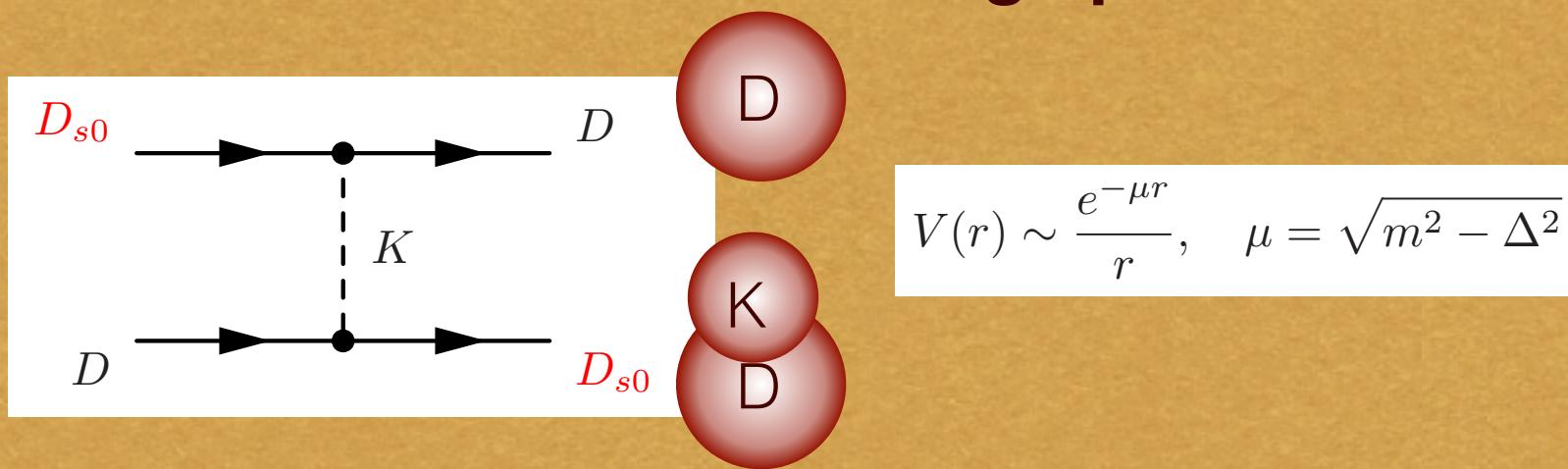


- $R_c \sim 0.5$ fm $\rightarrow \sim 6$ MeV binding

Summary



Long range correlation among hadrons emerges when the mass difference Δ matches with the mass of the exchange particle m .



**K exchange in $D_{s0}(0^+)$ $D(0^-)$ system: $\mu \sim 200$ MeV
—> prediction of exotic charmed tetraquark**

M. Sanchez Sanchez, L.S. Geng, J. Lu, T. Hyodo, M.P. Valderrama,
Phys. Rev. D98, 054001 (2018)