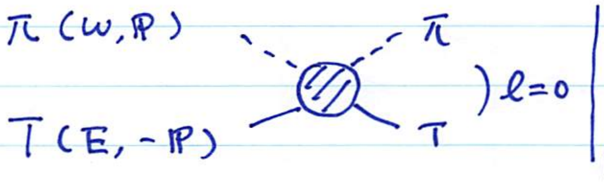
§ 3 Chiral dynamics for meson-baryon scattering

3.1 Weinberg - Tomozawa theorem

Chiral symmetry $SU(N_F)_R \otimes SU(N_F)_L \to SU(N_F)_V$ constrains the low-energy behavior of the s-wave interaction of the NG boson (π) and a target hadron (T).

$V^{WT} \sim$



Weinberg 1966, Tomozawa 1966

- Important properties

① V^{WT} is proportional to $\omega = \sqrt{m_{NG}^2 + \mathbb{P}^2}$

- Interaction is energy dependent
- Chiral limit ($m_{NG} = 0$) $\Rightarrow$ $V^{WT} = 0$ at threshold ($\mathbb{P} = 0$)
- Finite quark mass ($m_{NG} \neq 0$) $\Rightarrow$ $V^{WT} \propto m_{NG}$ at threshold

② Sign (attractive/repulsive) and strength are determined by group theoretical factor of $SU(N_F)_V$.

- No adjustable parameter. Model-independent prediction.

  c.f. p-wave interaction $\propto g_A^2$, reflecting the structure of T

- $N_F = 2$ case

$$V^{WT} \propto \underset{\uparrow \text{isospin of } \pi T \text{ system}}{I_\alpha (I_\alpha + 1)} - 2 - \overset{\swarrow \text{isospin of } T}{I_T (I_T + 1)} = \begin{cases} -2 & : \pi N \,(I_\alpha = 1/2) \\ +1 & : \pi N \,(I_\alpha = 3/2) \end{cases}$$

• Current algebra (original derivation)

Consider hadron matrix elements of vector (V) and axial vector (A) currents

$[V,V] \propto V$, $[V,A] \propto A$, $[A,A] \propto V$

- one π process → Goldberger-Treiman relation $g_{\pi NN} = \frac{2M_N g_A}{f_\pi}$
- two π process → Weinberg-Tomozawa theorem

$$\left\{ \begin{array}{l} \text{property ① ← Goldstone's theorem} \quad \underset{\text{broken current}}{\overset{\uparrow}{A^\mu}} \propto \partial^\mu \underset{\text{NG field}}{\overset{\uparrow}{\pi}} \\ \text{property ② ← conservation of vector current} \end{array} \right.$$

See textbooks by Cheng & Li or Coleman for detail.

• Chiral perturbation theory

- low-energy effective field theory of QCD
- nonlinear realization of chiral symmetry
  (SSB is incorporated from the beginning)
- systematic expansion in powers of NG boson momentum
  $\mathcal{O}(p)$, $\mathcal{O}(p^2)$, ---
- Current algebra results ← lowest order Lagrangian

See textbook by Scherer & Schindler

• $\bar{K}N - \pi\Sigma$ interaction

SU(3) meson-baryon Lagrangian for the WT interaction

$$\mathcal{L}^{WT} = \mathrm{Tr}\left(\bar{B}\gamma^\mu \frac{i}{4f^2}[\Phi \overleftrightarrow{\partial_\mu}\Phi , B]\right)$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \quad \text{baryon octet}$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \quad \begin{array}{c}\text{meson octet} \\ \text{(NG bosons)}\end{array}$$

f : meson decay constant

$\frac{i}{4f^2}\Phi \overleftrightarrow{\partial_\mu}\Phi \sim V_\mu$: vector current

- $\mathcal{L}^{WT}$ ← covariant derivative of kinetic term $\mathrm{Tr}(\bar{B} i\gamma^\mu \partial_\mu B)$

  ⇒ no low energy constant in the Lagrangian, property ②

- $K^-p \to K^-p$

$$V^{WT}_{K^-pK^-p} = -\langle K^-(k_f)\, p \,|\, \mathcal{L}_{WT} \,|\, K^-(k_i)\, p\rangle$$

$$= -\frac{1}{4f^2}\langle K^-p \,|\, \cdots -2(\bar{p}(i\gamma K^+)K^- p - \bar{p}K^+(i\gamma K^-)p)\cdots \,|\, K^-p\rangle$$

$$= -\frac{1}{4f^2}(-2)\left[\bar{u}_p\, i\, i \not{k}_f\, u_p - \bar{u}_p\, i(-i\not{k}_i)\, u_p\right]$$

$$= -\frac{2}{4f^2}\left[\bar{u}_p(\not{k}_f + \not{k}_i)\, u_p\right]$$

Dirac representation of spinor

$$u \propto \begin{pmatrix} \text{large} \\ \text{small} \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

→ γ^0 term is the leading order in nonrelativistic expansion

$$V^{WT}_{\bar{K}p\,\bar{K}p} = -\frac{2}{4f^2}\underbrace{(\omega_i + \omega_f)}_{\text{property ①}}\chi^\dagger\chi \quad (\mathbb{P} \to \text{small})$$

Calculating $\bar{K}^0 n$, $\pi^0\Sigma^0$, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, we obtain

$$V^{WT}_{ij} = -\frac{C_{ij}}{4f^2}\bar{u}(K_i + K_j)u$$

$$C_{ij} = \begin{array}{c} \\ \end{array}\left(\begin{array}{ccccc} 2 & 1 & \frac{1}{2} & 0 & 1 \\ & 2 & \frac{1}{2} & 1 & 0 \\ & & 0 & 2 & 2 \\ & & & 2 & 0 \\ & & & & 2 \end{array}\right)\begin{array}{l} K^-p \\ \bar{K}^0 n \\ \pi^0\Sigma^0 \\ \pi^+\Sigma^- \\ \pi^-\Sigma^+ \end{array}$$

Ex. 10) Derive this C_{ij} in particle basis.

In the isospin basis, this matrix is given by

$$C_{ij} = \left(\begin{array}{cc} 3 & -\sqrt{3/2} \\ & 4 \end{array}\right)\begin{array}{l} \bar{K}N\,(I=0) \\ \pi\Sigma\,(I=0) \end{array}$$

$C > 0 \Rightarrow V < 0 \Rightarrow$ attractive.

Both $\bar{K}N$ and $\pi\Sigma$ diagonal interactions are attractive!

Ex. 11) Derive C_{ij} in isospin basis.

(N.B. To use "standard" C·G coefficients, one should assign

$$|\pi^+\rangle = -|\overset{I}{\downarrow}1, \overset{I_3}{\downarrow}1\rangle$$
$$|\Sigma^+\rangle = -|1, 1\rangle \;)$$
$$|K^-\rangle = -|1/2, -1/2\rangle$$

- Comparison of the interaction strength

	$\pi N(I=\frac{1}{2})$	$\bar{K}N(I=0)$	$\pi\Sigma(I=0)$
$C\, m_{NG}$	$2m_\pi \sim 280$ MeV	$3m_K \sim 1485$ MeV	$4m_\pi \sim 560$ MeV

V^{WT} at threshold $\propto C m_{NG}$

$$\Rightarrow \quad V^{WT}_{\bar{K}N} \gg V^{WT}_{\pi\Sigma} \gg V^{WT}_{\pi N}$$

WT interaction in the $\Lambda(1405)$ sector is very strong!

- $m_{NG} = m_K \leftarrow$ strange quark mass
- $C_{\pi\Sigma} = 4 \leftarrow$ SU(3) group structure.

i.e. unique feature in three-flavor/strangeness sector

- When V^{WT} is not strong $\Rightarrow$ perturbative calculation should work.

c.f.) $\pi\pi(I=0)$ sector

$$\text{ChPT}: \; a^{I=0}_{\pi\pi} = -(\underset{\mathcal{O}(p^2)}{0.156} + \underset{\mathcal{O}(p^4)}{0.044} + \underset{\mathcal{O}(p^6)}{0.017})\, m_\pi^{-1} \simeq -0.217\, m_\pi^{-1}$$

$$\text{Exp}: \; a^{I=0}_{\pi\pi} \simeq -0.220\, m_\pi^{-1}$$

(N.B. $a_{\pi\pi}$ is usually defined with opposite sign from §2)

- When V^{WT} is strong $\Rightarrow$ perturbative calculation may not work.

c.f.) $\bar{K}N(I=0)$ sector

$$\text{ChPT}: \; a_{K^-p} \simeq -0.79 \text{ fm} \quad (\text{WT term})$$

$$\text{Exp}: \; a_{K^-p} \simeq +0.65 - 0.81i \text{ fm} \quad (\text{SIDDHARTA})$$

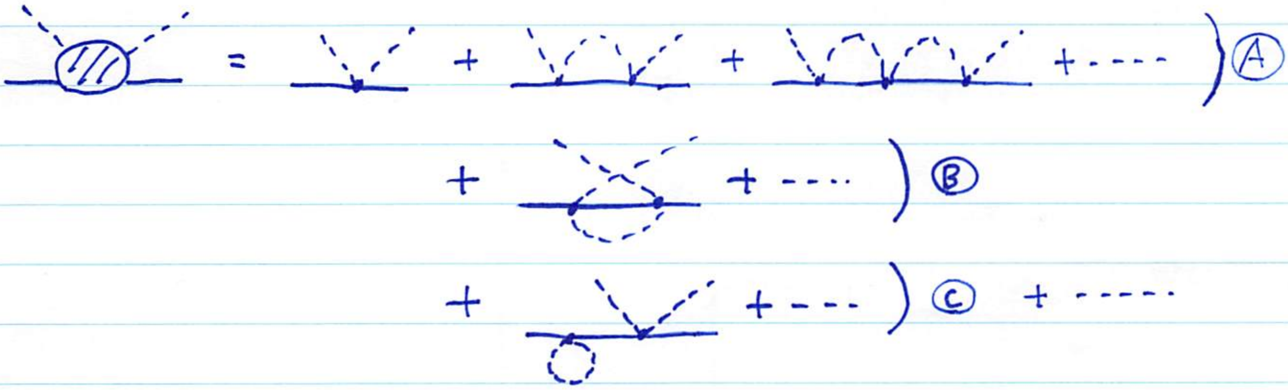
- sign of the real part is wrong. $\leftarrow$ $\Lambda(1405)$ below threshold
- imaginary part is absent $\leftarrow$ decay into $\pi\Sigma, \pi\Lambda$ channels

3.2 Chiral dynamics

• Nonperturbative resummation

Feynman diagram of V^{WT} :  4-point contact interaction

Diagrams for full 4-point function with V^{WT}



In contrast to § 2.2, diagrams Ⓑ and Ⓒ are allowed in relativistic theory
← propagator contains negative energy part

Ⓑ : crossed diagrams Ⓒ : mass renormalization

Summing up diagrams in Ⓐ, we schematically obtain

$$T = V + VGT$$
$$= V + VGV + VGVGV + \cdots$$

$G \doteq$ 

Good
- T satisfies unitarity (derivation in N/D method)
- Resonances can by dynamically generated (c.f. bound state in § 2)

Bad
- T violates crossing symmetry (← lack of Ⓑ)
- T violates power counting scheme in ChPT

In Λ(1405) sector, "Good" >> "Bad" ⇒ resummation is mandatory

• Numerical results with V^{WT}

Ikeda-Hyodo-Weise 2011, 2012 "TW" approach

- $V = V^{WT}$
- $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Lambda$, $\eta\Sigma$, $K\Xi$ channels
- 6 subtraction constants ($\approx$ UV cutoff) : free parameters
- Systematic χ^2 fit to experimental data

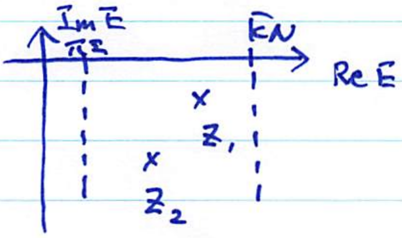
$\Rightarrow$ $\chi^2/\text{d.o.f.} = 1.12$

$a_{K^-p} = +0.93 - 0.82i$ fm (in convention of §1, 2)

comparable with SIDDHARTA

Pole positions $z_1 = 1422 - 16i$ MeV

$z_2 = 1384 - 90i$ MeV



• Two-pole structure

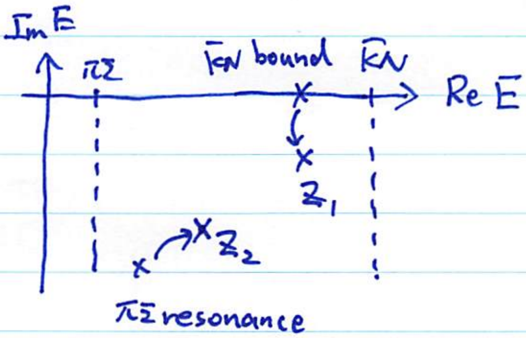
- Jido et al. 2003
  - Origin in SU(3) basis (flavor singlet + octet)
  - different coupling strengths : $z_1 \to \bar{K}N$, $z_2 \to \pi\Sigma$

  $\Rightarrow$ $\pi\Sigma$ lineshape can be reaction dependent.

- Hyodo-Weise 2008
  - Origin in isospin basis

    $V_{\bar{K}N} \propto 3m_K \to$ bound state

    $V_{\pi\Sigma} \propto 4m_\pi \to$ resonance



3.3 Determination of pole positions in PDG

- Experimental database

① K^-p total cross sections to $K^-p, \bar{K}^0n, \pi^-\Sigma^+, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^0\Lambda$

- Old bubble chamber data with sizable error bars
- Constraints above $\bar{K}N$ threshold

② Threshold branching ratios γ, R_n, R_c

- Stopped K^- in hydrogen, relatively accurate
- Constraints at $\bar{K}N$ threshold

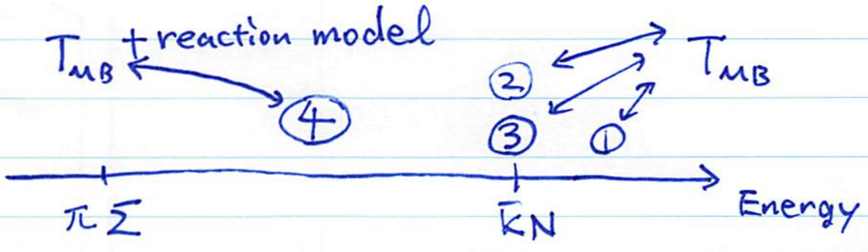
③ Kaonic hydrogen shift ΔE and width Γ

- Transition X-ray from 2P to 1S
- Related to K^-p scattering length → constraint at $\bar{K}N$ threshold
- "Inconsistency" with ①, ② was resolved by SIDDHARTA 2011

④ $\pi\Sigma$ spectra

- Invariant mass distribution in production experiments
  $\gamma p \to K^+ \underline{\underline{\pi\Sigma}}$ (LEPS, CLAS) $pp \to K^+p\, \underline{\underline{\pi\Sigma}}$ (HADES), etc.
- Constraints below $\bar{K}N$ threshold
- Reaction model is needed to relate with T_{MB} (indirect constraints)

Schematically...



• NLO chiral dynamics

Systematic improvement of V based on ChPT

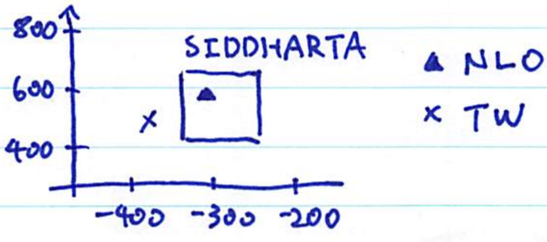
$V =$

WT term — $\mathcal{O}(p)$
Born terms — $\mathcal{O}(p)$
NLO terms — $\mathcal{O}(p^2)$

- S-wave component of Born terms is in higher order than WT term in nonrelativistic expansion
- Born terms contain axial coupling D, F ← hyperon decays.
- NLO (next-to-leading order) terms have 7 low-energy constants
  ↳ free parameters

χ^2 fitting with NLO terms to data ① ② ③

- $\chi^2/\text{d.o.f.} = 0.96$
- Pole positions $z_1 = 1424 - 26i$ MeV, $z_2 = 1381 - 81i$ MeV $\rightarrow$ PDG
- $\Delta E, \Gamma$



TW gives a reasonable result, while best-fit requires NLO

Analyses by other groups (difference in fitting strategy, choice of data, etc.)

- Position of z_1 converges in a small region
- 〃 z_2 shows a sizable scatter

See "Pole structure of the $\Lambda(1405)$ region" in PDG for more detail.