$\Lambda(1405)$ and $\bar{K}N$ interaction

§0. Introduction

• $\bar{K}N$ interaction

Two aspects of $\bar{K}$ meson

① $\bar{K}$ is lighter than other strangeness mesons ($m_{\bar{K}} \sim 0.5\, m_{K^*}$)

$\leftarrow$ NG boson of chiral symmetry breaking (Spontaneous χSB)

② $\bar{K}$ is heavior than π ($m_{\bar{K}} \sim 3.5\, m_\pi$)

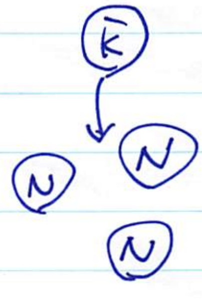
$\leftarrow$ strange quark, $m_{NG}^2 \propto m_q$ (Explicit χSB)

Interplay between ① and ② $\Rightarrow$ strong $\bar{K}N$ dynamics : §1

Consequences [HJ 2012]

- $\Lambda(1405)$ as a quasi-bound state (Feshbach resonance)
  "$\bar{K}N$ bound state embedded in resonating $\pi\Sigma$ continuum"
- $\bar{K}$-few nucleon systems [AY 2002]

	NN	$\bar{K}N$
$I=0$	d (2MeV)	$\Lambda(1405)$ (15-30 MeV)
$I=1$	attractive	attractive



Nuclei with $\bar{K}$ can form a quasi-bound state

Rigorous few-body calculation with reliable interaction up to mass number $A=6$ [Ohnishi 2017] : §2

· Current status of $\Lambda(1405)$

PDG 2017 update [PDG]

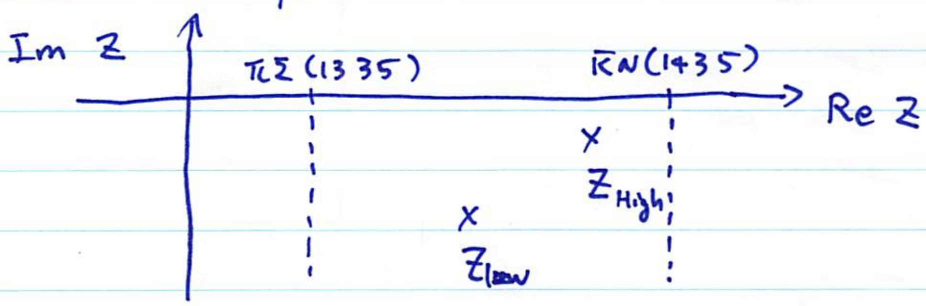
$$\begin{cases} \text{strangeness} & S = -1 \\ \text{isospin} & I = 0 \\ \text{spin-parity} & J^P = 1/2^- \quad \text{(experimentally established in 2014)} \\ \text{status} & **** \quad \left(\begin{array}{l}\text{Existence is certain, and properties} \\ \text{are at least fairly well explored}\end{array}\right) \end{cases}$$

Pole positions (prior to "mass and width")

- Four results from different analyses are shown.

e.g.) [IHW 2011]

	Re z_R [MeV]	$-2 \times$ Im z_R [MeV]
High-mass pole	1424	52
Low-mass pole	1381	162



What is pole?

- pole of scattering amplitude $\simeq$ discrete eigenstate of Hamiltonian [Note 1]
- Ideal case : Breit-Wigner amplitude

$$T(\sqrt{s}) \propto \frac{1}{\sqrt{s} - M_R + \frac{i}{2}\Gamma_R} \Rightarrow \begin{array}{l} \mathrm{Re}\, z_R = M_R : \text{Mass} \\ -2\, \mathrm{Im}\, z_R = \Gamma_R : \text{Width} \end{array}$$

Advantages of "pole position" compared with "mass and width"

- Resonances (such as $\Lambda(1405)$) are not always expressed by B-W form
- Reason: background amplitude $T(\sqrt{s}) = T_{pole}(\sqrt{s}) + \underset{\sim\sim\sim}{T_{BG}(\sqrt{s})}$
- Pole positions are less ambiguous theoretically.

Questions

- How are the poles in PDG determined? § 2
- Why are there two poles for one resonance? § 1

c.f. "old story" on $\Lambda(1405)$

$\Lambda(1405)$ in a constitute quark model has

- too light mass for $\ell = 1$ $\leftrightarrow$ $N(1535)$
- too small L·S splitting with $3/2^-$ $\Lambda(1520)$

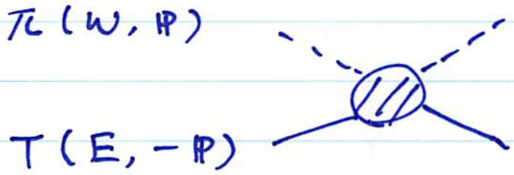
$\leftrightarrow$ $N(1535)$, $N(1520)$

From a modern viewpoint, such static picture is too naive.
Coupling to meson-baryon states drastically changes the situation
(bare state of $N(1440)$, XYZ states above $D\bar{D}$ threshold, ---)

§ 1 Chiral dynamics for meson-baryon scattering

1.1 Weinberg - Tomozawa theorem

low-energy s-wave interaction of the NG boson (π) and a target hadron (T)



$$\Big) l=0 \Big|_{|\mathbf{p}| \to \text{small}} \sim V^{WT}$$

Chiral symmetry gives constraints on V^{WT}

- Original derivation [Weinberg 1966, Tomozawa 1966, Cheng-Li, Coleman]
- Chiral perturbation theory [Scherer - Schindler]

• <u>Important properties</u>

① V^{WT} is proportional to $\omega = \sqrt{m_{NG}^2 + \mathbf{p}^2}$

- Interaction is energy dependent
- Chiral limit ($m_{NG} = 0$) $\Rightarrow V^{WT} = 0$ at threshold ($\mathbf{p} = 0$)
- Finite quark mass ($m_{NG} \neq 0$) $\Rightarrow V^{WT} \propto m_{NG}$ at threshold

② Sign (attractive / repulsive) and strength are determined by group theoretical factor of $SU(N_F)_V$

- No adjustable parameter. Model-independent prediction.

  c.f. p-wave interaction $\propto g_A^2$, reflecting the structure of T

• $\bar{K}N - \pi\Sigma$ ($I=0$) interaction

SU(3) meson-baryon Lagrangian for the WT interaction

$$\mathcal{L}^{WT} = \mathrm{Tr}\left(\bar{B}\gamma^\mu \frac{i}{4f^2}[\Phi \overleftrightarrow{\partial_\mu} \Phi, B]\right)$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \quad \text{baryon octet}$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \quad \begin{array}{l}\text{meson octet} \\ \text{(NG bosons)}\end{array}$$

f : meson decay constant ~ 93 MeV

$\leftarrow$ covariant derivative of $\mathrm{Tr}(\bar{B}\, i\gamma^\mu \partial_\mu B)$: property ②

WT interaction (see [Note 3] for derivation)

$$V^{WT}_{ij} = -\frac{C_{ij}}{4f^2}(\omega_i + \omega_j)$$

$\leftarrow$ energy of meson in channel i

$$C_{ij} = \begin{pmatrix} 3 & -\sqrt{3/2} \\ & 4 \end{pmatrix} \quad \begin{array}{l} \bar{K}N\ (I=0) \\ \pi\Sigma\ (I=0) \end{array}$$

- $\omega_i \leftarrow$ time derivative in $\mathcal{L}^{WT}$: property ①
- $C > 0 \Rightarrow V < 0 \Rightarrow$ attractive interaction.
- No dependence on momentum transfer $\rightarrow$ δ-fn interaction in coordinate space

1.2 3d δ function potential

• Schrödinger equation

$$\left(-\frac{\nabla^2}{2\mu} + \lambda\,\delta(\mathbb{x})\right)\psi(\mathbb{x}) = E\,\psi(\mathbb{x})$$

- $d=1$: Attractive interaction ($\lambda < 0$) always gives one bound state.

- $d \geq 2$: Pathology (ultraviolet divergence)

  $\leftarrow$ short range behavior is too singular

option 1) Renormalization

Regularize UV by cutoff Λ, let the coupling λ depends on Λ, so that the observables are independent of Λ. c.f. [Note 2]

$\boxed{\text{option 2)}}$ Physical cutoff Λ

Our description is applicable up to the UV scale Λ

Potential is defined for a given Λ.

• Bound state solution ($d=3$)

$$E = -B \qquad (B>0)$$

Fourier transformation of w.f. $\psi(\mathbb{x}) = \int \frac{d^3\mathbb{p}}{(2\pi)^3} e^{i\mathbb{p}\cdot\mathbb{x}} \phi(\mathbb{p})$

Schrödinger eq. $\Rightarrow$ $\phi(\mathbb{p}) = -\frac{\lambda}{B + \mathbb{p}^2/2\mu} \underbrace{\int \frac{d^3\mathbb{k}}{(2\pi)^3}\phi(\mathbb{k})}_{=\,\psi(\mathbb{x}=0)}$

Integrate over $\mathbb{P}$: consistency condition for a discrete eigenstate

$$-\lambda \int \frac{d^3\mathbb{P}}{(2\pi)^3} \frac{1}{B + \mathbb{P}^2/2\mu} = 1$$

$$-\lambda \frac{4\pi}{(2\pi)^3} \int_0^{\cancel{\infty} \to \Lambda} dp \frac{p^2}{B + p^2/2\mu} = 1 \quad \leftarrow \text{UV divergence!}$$

$$-\lambda \frac{\mu}{\pi^2} \left(\Lambda - \sqrt{2\mu B} \arctan \frac{\Lambda}{\sqrt{2\mu B}} \right) = 1$$

Binding energy $B \leftarrow$ coupling λ for a given cutoff Λ

- Critical coupling

Consider $B \to 0$ limit $\left(\lim_{x \to 0} x \arctan \frac{1}{x} = 0 \right)$

$$-\lambda \frac{\mu}{\pi^2} \Lambda = 1 \quad (B \to 0) \quad \Rightarrow \quad \lambda_c \equiv -\frac{\pi^2}{\mu \Lambda}$$

critical coupling

- $\lambda < 0$ and $|\lambda| \geq |\lambda_c|$ $\Rightarrow$ one bound state
- else $\Rightarrow$ no bound state

For $d = 3$, attractive δ-fn potential does not always give a bound state

N.B.) $\lambda_c \propto 1/\mu$

$\Rightarrow$ if μ is large, more chance to have a bound state

Can we have a resonance for $0 > \lambda > \lambda_c$?

$\rightarrow$ It is shown that the resonance solution is not allowed.

[HJH 2007] Appendix B

1.3 $\Lambda(1405)$ and meson-baryon interaction

V^{WT} at threshold $\propto C m_{NG}$

$C^{\bar{K}N} = 3$, $C^{\pi\Sigma} = 4$, $C^{\pi N (I=1/2)} = 2$ (all attractive)

	$\bar{K}N (I=0)$	$\pi\Sigma (I=0)$	$\pi N (I=1/2)$
$C m_{NG}$	$3 m_K \sim 1485$ MeV	$4 m_\pi \sim 560$ MeV	$2 m_\pi \sim 280$ MeV

$$\Rightarrow \quad V^{WT}_{\bar{K}N} \gg V^{WT}_{\pi\Sigma} \gg V^{WT}_{\pi N}$$

WT interaction in the $\Lambda(1405)$ sector is very strong.

- $m_{NG} = m_K \leftarrow$ strange quark mass
- $C_{\pi\Sigma} = 4 \leftarrow$ SU(3) group structure

unique feature in three-flavor/strangeness sector!

- Critical coupling

Similar discussion with § 1.2 for relativistic dispersion relation [HJH 2006, 2007]

$$C_{crit} = \frac{2f^2}{m_{NG}[-G(M_T + m_{NG})]}$$

some function

$\bar{K}N$: $C_{crit} \sim 1.8 < C^{\bar{K}N} = 3$ → bound

$\pi\Sigma$: $C_{crit} \sim 10.8 > C^{\pi\Sigma} = 4$

πN : $C_{crit} \sim 11.9 > C^{\pi N} = 2$

→ unbound (π is too light to bind)

$\bar{K}N$ system should support a bound state. → $\Lambda(1405)$

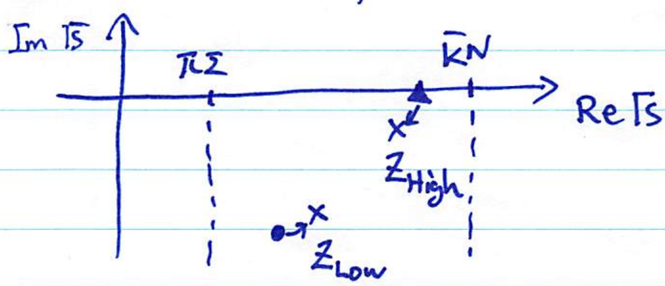
- **Two-pole structure**

In contrast to § 1.2, V^{WT} can generate resonances through E-dependence

$\rightarrow$ $\pi\Sigma$ channel has a resonance above threshold

- Origin of two-pole structure of $\Lambda(1405)$ in isospin basis [HW 2008]

$$\begin{cases} V^{WT}_{\bar{K}N} \rightarrow \text{bound state } \blacktriangle \\ V^{WT}_{\pi\Sigma} \rightarrow \text{resonance } \bullet \end{cases}$$



two eigenstates between $\bar{K}N$ and $\pi\Sigma$ thresholds

- Origin in SU(3) basis (flavor singlet + octet) [Jido 2003]

- different coupling strengths : $z_{High} \rightarrow \bar{K}N$, $z_{Low} \rightarrow \pi\Sigma$

  $\Rightarrow$ $\pi\Sigma$ lineshape can be reaction dependent

1.4 Summary

Weinberg-Tomozawa theorem (chiral symmetry constraint)

$$V_{\bar{K}N} \propto 3 m_{\bar{K}} \quad , \quad V_{\pi\Sigma} \propto 4 m_{\pi}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

bound state $\qquad$ resonance

$\searrow \qquad \swarrow$

two-pole structure of $\Lambda(1405)$

Strangeness / SU(3) is essential.

§ 2 Recent developments

2.1 Determination of pole positions in PDG

- **Experimental database**

① K^-p total cross sections to K^-p, $\bar{K}^0 n$, $\pi^-\Sigma^+$, $\pi^0\Sigma^0$, $\pi^+\Sigma^-$, $\pi^0\Lambda$

- Old bubble chamber data with sizable error bars
- Constraints above $\bar{K}N$ threshold

② Threshold branching ratios γ, R_n, R_c

- Stopped K^- in hydrogen, relatively accurate
- Constraints at $\bar{K}N$ threshold

③ Kaonic hydrogen shift ΔE and width Γ

- Transition X-ray from 2P to 1S
- Related to K^-p scattering length $\rightarrow$ constraint at $\bar{K}N$ threshold
- "Inconsistency" with ①, ② was resolved [SIDDHARTA 2011]

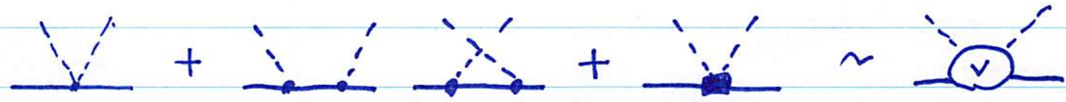
④ $\pi\Sigma$ spectra

- Invariant mass distribution in production experiments
  $\gamma p \rightarrow K^+ \underline{\underline{\pi\Sigma}}$ (LEPS, CLAS), $pp \rightarrow K^+ p\, \underline{\underline{\pi\Sigma}}$ (HADES), etc.
- Constraints below $\bar{K}N$ threshold
- Reaction model is needed to relate with T_{MB} $\qquad \frac{d\sigma}{dM_I} \sim |\underline{\underline{C_{reac}}} \otimes T_{MB}|^2$

  $\rightarrow$ ④ is NOT a direct constraint.

• NLO chiral SU(3) dynamics

Systematic improvement of V based on ChPT

$V =$ 

WT term	Born terms	NLO terms
$\mathcal{O}(p)$	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$

- S-wave component of Born terms is in higher order than WT term in nonrelativistic expansion.
- Born terms contain axial couplings D, F ← hyperon decays
- NLO (next-to-leading order) terms have 7 low-energy constants
  ↳ free parameters

Scattering amplitude : nonperturbative resummation

$$T(\sqrt{s}) = V(\sqrt{s}) + V(\sqrt{s})\,G(\sqrt{s})\,T(\sqrt{s})$$

$$= V + VGV + VGVGV + \cdots \sim$$ 

- Loop function $G(\sqrt{s})$ has 6 subtraction constants ($\simeq$ UV cutoff)
  ↳ free parameters

χ^2 fitting to data ①, ②, ③ [IHW 2011]

- $\chi^2/\mathrm{d.o.f.} = 0.96$ (~ 120 data)
- Pole positions ← $T(\sqrt{s})$: $z_{High} = 1424 - 26i$ MeV, $z_{Low} = 1381 - 81i$ MeV → PDG

Analyses by other groups (fitting strategy, choice of data, ---)

- Position of z_{High} converges in a small region
- " z_{Low} shows a sizable scatter [MH 2015]

2.2 Realistic $\bar{K}N$ potential

$\Lambda(1405) \sim \bar{K}N$ quasi-bound state

→ Quasi-bound states of $\bar{K}$ + few nucleons?

- Rigorous few-body techniques : (many-body) Schrödinger equation
- How can we apply the results of chiral dynamics?

• Construction of $\bar{K}N$ potential

Equivalent single-channel local potential [HW 2008]

$$V_{ij}^{\text{chiral}}(E) \xrightarrow{①} V^{\bar{K}N}(E) \xrightarrow{②} U^{\bar{K}N}(E;r)$$

channel	coupled-channel	single-channel	single-channel
E-dep.	chiral	chiral + Feshbach	chiral + Feshbach
strength	real	complex	complex
scattering eq.	relativistic	relativistic	Schrödinger

process ① : exact transformation

- Feshbach projection (channel elimination) causes E-dependence
- Elimination of lower energy $\pi\Sigma$ channel induces $\mathrm{Im}\, V^{\bar{K}N}$

process ② : approximate parameterization

- Spatial distribution is assumed to be gaussian
- Strength is tuned to reproduce original amplitude by $V_{ij}^{\text{chiral}}(E)$

• Kyoto $\bar{K}N$ potential [MH 2016]

- V_{ij}^{chiral} ← NLO chiral dynamics with SIDDHARTA constraint
- Precision in complex energy plane improved (pole positions $\lesssim 1$ MeV)
- Reproduces whole experimental data with $\chi^2/\text{d.o.f.} \sim 1$

⇒ realistic $\bar{K}N$ potential (c.f. nuclear force)

• Spatial distribution of $\Lambda(1405)$

Potential → wavefunction of $\Lambda(1405)$ → spatial structure

Note: Unstable states have a complex eigenenergy

→ complex expectation value $\langle \psi_R | \hat{O} | \psi_R \rangle \in \mathbb{C}$

Root mean squared radius of $\Lambda(1405)$

$$\langle \Lambda(1405) | \hat{r}^2 | \Lambda(1405) \rangle^{1/2} = 1.04 - 0.61i \text{ fm}$$

(Kyoto $\bar{K}N$ potential, High-mass pole)

Interpretation?

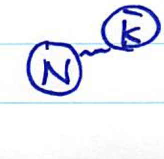
complex eigenmomentum $k = k_R + i k_I$

$$\psi(r) \xrightarrow{r \to \infty} \frac{e^{ikr}}{r} = \frac{1}{r} \underbrace{e^{ik_R r}}_{\text{oscillation}} \underbrace{e^{-k_I r}}_{\text{dumping (quasi-bound state, } k_I > 0)}$$

extruction of dumping factor → $\sqrt{\langle r^2 \rangle} \sim 1.44$ fm

potential range ~ 0.4 fm, hadron size ~ 0.7 fm

⇒ $\Lambda(1405)$ as loosely bound $\bar{K}N$ molecule



2.3 Application to few-nucleon systems

- **$\bar{K}$-nuclei**
  - $\bar{K}N^A$ systems with $A \leq 6$ [Ohnishi 2017]
  - $\hat{V} = \hat{V}^{\bar{K}N} + \hat{V}^{NN}$
  - Stochastic variational method + correlated gaussians [Suzuki 1998]

	$\bar{K}NN$	$\bar{K}NNN$	$\bar{K}NNNN$	$\bar{K}NNNNNN$
B [MeV]	25-28	45-50	68-76	70-81
Γ [MeV]	31-59	26-70	28-74	24-76

  - uncertainty $\leftarrow$ treatment of E-dependence of $\hat{V}^{\bar{K}N}$
  - quasi-bound state below the lowest threshold
  - Γ (without multi-N absorption) $\sim$ B
  - interplay between $\hat{V}^{\bar{K}N}$ and $\hat{V}^{NN}$ (see [Ohnishi 2017])

- **Kaonic deuterium**
  - $K^-pn - \bar{K}^0nn$ three-body system with spin 1 [Hoshino 2017]
  - $\hat{V} = \hat{V}^{\bar{K}N} + \hat{V}^{NN} + \hat{V}^{\text{Coulomb}}$
  - eV precision $\leftarrow$ basis function up to hundreds of fm

$$\Delta E - \frac{i}{2}\Gamma = 670 - i508 \text{ eV}$$

  - Result is sensitive to $I=1$ $\bar{K}N$ potential
  - Planned experiments (J-PARC E57, SIDDHARTA-2) will give stringend constraint on $I=1$ component.

2.4 Summary

NLO chiral SU(3) dynamics
PDG value of $\Lambda(1405)$ pole position

$\xrightarrow{\chi^2/\mathrm{d.o.f.} \sim 1}$ $\bar{K}N$ scattering, Kaonic hydrogen

$\updownarrow$ equivalent scattering amplitude

Realistic $\bar{K}N$ potential $\xrightarrow{\mathrm{w.f.}}$ spatial structure of $\Lambda(1405)$

$\downarrow$ few-body technique

$\bar{K}$-nuclei
Kaonic deuterium

: future J-PARC experiments