Scattering theory, effective field theory, and chiral dynamics

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Abstract

Solutions to the excersises in the lecture are presented.

1 Scattering theory

Ex.1)

Denote the dimention of mass, length, and time as M, L, and T, respectively. The dimensions of the momentum p and the energy E are given by

$$[p] = MLT^{-1}, \quad [E] = ML^2T^{-2}. \tag{1}$$

The dimension of the Planck constant $\hbar$ is

$$[\hbar] = ML^2T^{-1}. \tag{2}$$

By setting $\hbar = 1$, we can elimimnate T, and thee dimensions of the quantities are determined as in the third column in Table 1. If we further set $m = 1$ to measure the scale of mass, we obtain the fourth column. Thus, the momentumn and energy are now given by

$$[p] = L^{-1}, \quad [E] = L^{-2} \quad (\hbar = 1, m = 1) \tag{3}$$

For reference, we also show the natural unit where $\hbar = 1$ and $c = 1$, where the dimension of the speed of light is $[c] = LT^{-1}$. In the natural unit, we see the relation $[p] = [E]$, in contrast to the present unit $[p] \neq [E]$.

Table 1: Dimensions of physical quantities in various units.

unit	-	$\hbar = 1$	$\hbar = 1$, $m = 1$	$\hbar = 1$, $c = 1$
Mass	M	M	1	L^{-1}
Length	L	L	L	L
Time	T	ML^2	L^2	L
Momentum	MLT^{-1}	L^{-1}	L^{-1}	L^{-1}
Energy	ML^2T^{-2}	$M^{-1}L^{-2}$	L^{-2}	L^{-1}

Ex.2)

The continuity of the wavefunction $\psi(x)$ at $x = b$ gives

$$\sin(kb) = A(p)e^{-ipb} + B(p)e^{ipb} \tag{4}$$

and the continuity of the derivative of the wavefunction gives

$$k\cos(kb) = -ipA(p)e^{-ipb} + ipB(p)e^{ipb} \tag{5}$$

From Eq. (4), we obtain

$$B(p)e^{ipb} = -A(p)e^{-ipb} + \sin(kb) \tag{6}$$

$$B(p) = -A(p)e^{-2ipb} + \sin(kb)e^{-ipb} \tag{7}$$

Substituting this into Eq. (5), we obtain

$$k\cos(kb) = -ipA(p)e^{-ipb} + ip[-A(p)e^{-2ipb} + \sin(kb)e^{-ipb}]e^{ipb} \tag{8}$$

$$k\cos(kb) = -ipA(p)e^{-ipb} - ipA(p)e^{-ipb} + ip\sin(kb) \tag{9}$$

$$2ipA(p)e^{-ipb} = ip\sin(kb) - k\cos(kb) \tag{10}$$

$$A(p) = \frac{1}{2}\left[\sin(kb) + i\frac{k}{p}\cos(kb)\right]e^{ipb} \tag{11}$$

and

$$B(p) = -A(p)e^{-2ipb} + \sin(kb)e^{-ipb} \tag{12}$$

$$= -\frac{1}{2}\left[\sin(kb) + i\frac{k}{p}\cos(kb)\right]e^{ipb}e^{-2ipb} + \sin(kb)e^{-ipb} \tag{13}$$

$$= \frac{1}{2}\left[-\sin(kb) - i\frac{k}{p}\cos(kb)\right]e^{-ipb} + \sin(kb)e^{-ipb} \tag{14}$$

$$= \frac{1}{2}\left[\sin(kb) - i\frac{k}{p}\cos(kb)\right]e^{-ipb} \tag{15}$$

If we set $A(p) = 0$, we obtain

$$0 = \frac{1}{2}\left[\sin(kb) + i\frac{k}{p}\cos(kb)\right]e^{ipb} \tag{16}$$

$$0 = \sin(kb) + i\frac{k}{p}\cos(kb) \tag{17}$$

$$\sin(kb) = -i\frac{k}{p}\cos(kb) \tag{18}$$

With $k = \sqrt{p^2 - 2V_0}$, we obtain

$$\tan(\sqrt{p^2 - 2V_0}b) = -i\frac{\sqrt{p^2 - 2V_0}}{p} \tag{19}$$

Ex.3)

Use the completeness relation

$$1 = \int dE \sum_{l,m} |\, E, l, m \,\rangle\langle\, E, l, m \,| \tag{20}$$

in the definition of the scattering amplitude:

$$\langle\, \boldsymbol{p}' \,|(\hat{S} - 1)|\, \boldsymbol{p} \,\rangle = \int dE \sum_{l,m} \langle\, \boldsymbol{p}' \,|(\hat{S} - 1)|\, E, l, m \,\rangle\langle\, E, l, m \,|\, \boldsymbol{p} \,\rangle \tag{21}$$

$$= \int dE \sum_{l,m} [S_l(E) - 1]\langle\, \boldsymbol{p}' \,|\, E, l, m \,\rangle\langle\, E, l, m \,|\, \boldsymbol{p} \,\rangle \tag{22}$$

$$= \int dE \sum_{l,m} [S_l(E) - 1]\frac{1}{\sqrt{\mu p'}}\delta(E_{p'} - E)Y_l^m(\hat{\boldsymbol{p}}')^* \frac{1}{\sqrt{\mu p}}\delta(E_p - E)Y_l^m(\hat{\boldsymbol{p}}) \tag{23}$$

Integrating over E, we obtain $\delta(E_p' - E_p)$, so we can replace p' by p. Using the property of the spherical harmonics

$$\sum_m Y_l^m(\hat{\boldsymbol{p}}')^* Y_l^m(\hat{\boldsymbol{p}}) = \frac{2l+1}{4\pi} P_l(\cos\theta) \tag{24}$$

where P_l is the Legendre polynomial, we obtain

$$\langle\, \boldsymbol{p}' \,|(\hat{S} - 1)|\, \boldsymbol{p} \,\rangle = \frac{1}{\mu p}\delta(E_{p'} - E_p) \sum_{l,m} [S_l(E_p) - 1] Y_l^m(\hat{\boldsymbol{p}}')^* Y_l^m(\hat{\boldsymbol{p}}) \tag{25}$$

$$= \frac{1}{4\pi\mu p}\delta(E_{p'} - E_p) \sum_l (2l+1)[S_l(E_p) - 1]P_l(\cos\theta) \tag{26}$$

Thus, equating with the definition of the scattering amplitude, we obtain

$$\frac{i}{2\pi\mu}\delta(E_{p'} - E_p) f(E_p, \theta) = \frac{1}{4\pi\mu p}\delta(E_{p'} - E_p) \sum_l (2l+1)[S_l(E_p) - 1]P_l(\cos\theta) \tag{27}$$

$$\frac{i}{2\pi\mu} f(E_p, \theta) = \frac{1}{4\pi\mu p} \sum_l (2l+1)[S_l(E_p) - 1]P_l(\cos\theta) \tag{28}$$

$$f(E_p, \theta) = \frac{1}{2ip} \sum_l (2l+1)[S_l(E_p) - 1]P_l(\cos\theta) \tag{29}$$

Ex.4)

$$\sigma(E) = \int d\Omega |f(E,\theta)|^2 \tag{30}$$

$$= \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta \sum_l (2l+1)^2 |f_l(E)|^2 [P_l(\cos\theta)]^2 \tag{31}$$

Now we use the formula

$$\int_{-1}^{1} d\cos\theta P_l(\cos\theta) P_m(\cos\theta) = \frac{2}{2l+1}\delta_{lm} \tag{32}$$

to obtain

$$\sigma(E) = \sum_l 2\pi(2l+1)^2 |f_l(E)|^2 \frac{2}{2l+1} \tag{33}$$

$$= \sum_l 4\pi(2l+1)|f_l(E)|^2 \tag{34}$$

Ex.5)

For $p \in \mathbb{R}$, (ii) leads to

$$[\not{f}_l(p)]^* = \not{f}_l(-p) \tag{35}$$

Thus, we obtain

$$[s_l(p)]^* = \frac{[\not{f}_l(-p)]^*}{[\not{f}_l(p)]^*} \tag{36}$$

$$= \frac{\not{f}_l(p)}{\not{f}_l(-p)} \tag{37}$$

$$= \frac{1}{s_l(p)} \tag{38}$$

for $p \in \mathbb{R}$. We can also write as

$$f_l(p) = \frac{s_l(p) - 1}{2ip} \tag{39}$$

$$= \frac{\not{f}_l(-p)/\not{f}_l(p) - 1}{2ip} \tag{40}$$

$$= \frac{\not{f}_l(-p) - \not{f}_l(p)}{2ip\not{f}_l(p)} \tag{41}$$

Ex.6)

From the definitions of S-matrix and phase shift, we obtain

$$
\begin{aligned}
f_l(p) &= \frac{s_l(p)-1}{2ip} && (42)\\
&= \frac{e^{2i\delta_l(p)}-1}{2ip} && (43)\\
&= \frac{e^{i\delta_l(p)}-e^{-i\delta_l(p)}}{2i}\frac{1}{e^{-i\delta_l(p)}p} && (44)\\
&= \sin\delta_l \frac{1}{(\cos\delta_l(p)-i\sin\delta_l(p))p} && (45)\\
&= \frac{1}{p\cot\delta_l(p)-ip} && (46)\\
&= \frac{p^{2l}}{p^{2l+1}\cot\delta_l(p)-ip^{2l+1}} && (47)
\end{aligned}
$$

Next, denote the Jost function as

$$
\not{f}_l(p) = F(p^2) + ipG(p^2) \qquad (48)
$$

with $F(p^2) \sim \mathcal{O}(p^0)$ and $G(p^2) \sim \mathcal{O}(p^{2l})$ for $p \to 0$. Then

$$
\begin{aligned}
\not{f}_l(-p) - \not{f}_l(p) &= F(p^2) - ipG(p^2) - F(p^2) - ipG(p^2) && (49)\\
&= -2ipG(p^2) && (50)\\
f_l(p) &= \frac{-2ipG(p^2)}{2ip[F(p^2)+ipG(p^2)]} && (51)\\
&= \frac{G(p^2)}{-F(p^2)-ipG(p^2)} && (52)\\
&= \frac{p^{2l}}{-p^{2l}F(p^2)/G(p^2)-ip^{2l+1}} && (53)
\end{aligned}
$$

From Eq. (47), the term $-p^{2l}F(p^2)/G(p^2)$ corresponds to $p^{2l+1}\cot\delta_l(p)$. It is a function of p^2 and of the order of $\mathcal{O}(p^0)$ for $p \to 0$.

2 Effective field theory

Ex.7)

Let $S(t, \boldsymbol{x})$ be the Fourier transform of the propagator $G(\omega, \boldsymbol{p})$. Acting the kinetic term operator to $S(t, \boldsymbol{x})$, we obtain

$$\left[i\partial_t + \frac{\boldsymbol{\nabla}^2}{2m}\right] S(t, \boldsymbol{x}) = \left[i\partial_t + \frac{\boldsymbol{\nabla}^2}{2m}\right] \int \frac{d\omega d^3\boldsymbol{k}}{(2\pi)^4} e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}} G(\omega, \boldsymbol{k}) \tag{54}$$

$$= \int \frac{d\omega d^3\boldsymbol{k}}{(2\pi)^4} \left[i(-i\omega) + \frac{(i\boldsymbol{k})^2}{2m}\right] e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{1}{\omega - \frac{\boldsymbol{k}^2}{2m} + i0^+} \tag{55}$$

$$= \int \frac{d\omega d^3\boldsymbol{k}}{(2\pi)^4} \left[\omega - \frac{\boldsymbol{k}^2}{2m}\right] e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{1}{\omega - \frac{\boldsymbol{k}^2}{2m} + i0^+} \tag{56}$$

$$= \int \frac{d\omega d^3\boldsymbol{k}}{(2\pi)^4} e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}} \tag{57}$$

$$= \delta^4(x) \tag{58}$$

There are 2 ways to contract ψs in $\psi^\dagger\psi\psi^\dagger\psi$ to the final bosons, and 2 ways to contract $\psi^\dagger$s to the initial ones. Altogether, this multiplicity factor 4 cancels with the 1/4 factor in the Lagrangian.

Ex.8)

Define

$$iI_0(E) = \int \frac{d\omega d^3\boldsymbol{k}}{(2\pi)^4} \frac{i}{\omega - \frac{\boldsymbol{k}^2}{2m} + i0^+} \frac{i}{E - \omega - \frac{\boldsymbol{k}^2}{2m} + i0^+} \tag{59}$$

Then the amplitude is calculated as

$$i\mathcal{A}(E) = -i\lambda_0 - i\lambda_0 \frac{1}{2} iI_0(E) i\mathcal{A}(E) \tag{60}$$

$$\mathcal{A}(E) = -\lambda_0 + \frac{\lambda_0}{2} I_0(E)\mathcal{A}(E) \tag{61}$$

$$\mathcal{A}(E) - \frac{\lambda_0}{2} I_0(E)\mathcal{A}(E) = -\lambda_0 \tag{62}$$

$$\mathcal{A}(E)\left[1 - \frac{\lambda_0}{2} I_0(E)\right] = -\lambda_0 \tag{63}$$

$$\mathcal{A}(E) = -\lambda_0 \left[1 - \frac{\lambda_0}{2} I_0(E)\right]^{-1} \tag{64}$$

Now our task is to evaluate $I_0(E)$. We first perform the ω integration. In the complex ω plane, there are poles of the integrand at $\omega = \boldsymbol{k}^2/(2m) - i0^+$ and $\omega = E - \boldsymbol{k}^2/(2m) + i0^+$. The first pole is located in the lower half plane, and the second one in the upper half plane. By closing the integration contour $-\infty \to +\infty$ clockweise in the lower half plane, we pick up the pole at $\boldsymbol{k}^2/(2m) - i0^+$, and the residue theorem leads to

$$iI_0(E) = \frac{1}{2\pi}(-2\pi i) \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{i^2}{E - \frac{\boldsymbol{k}^2}{2m} - \frac{\boldsymbol{k}^2}{2m} + i0^+} \tag{65}$$

$$= i \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{E - \frac{\boldsymbol{k}^2}{m} + i0^+} \tag{66}$$

The integrand does not depend on the angular variables, so the angle integragion can be pefrormed trivially. Introducing the momentum cutoff Λ, we obtain

$$I_0(E) = \frac{4\pi}{8\pi^3} m \int_0^\Lambda d|\boldsymbol{k}| \frac{|\boldsymbol{k}|^2}{mE - |\boldsymbol{k}|^2 + i0^+} \tag{67}$$

$$= -\frac{m}{2\pi^2} \int_0^\Lambda d|\boldsymbol{k}| \frac{|\boldsymbol{k}|^2}{|\boldsymbol{k}|^2 + (-mE - i0^+)} \tag{68}$$

$$= -\frac{m}{2\pi^2} \left[|\boldsymbol{k}| - \sqrt{-mE - i0^+} \arctan \frac{|\boldsymbol{k}|}{\sqrt{-mE - i0^+}} \right]_0^\Lambda \tag{69}$$

$$= -\frac{m}{2\pi^2} \left(\Lambda - \sqrt{-mE - i0^+} \arctan \frac{\Lambda}{\sqrt{-mE - i0^+}} \right) \tag{70}$$

Substituging this into Eq. (64), we obtain

$$\mathcal{A}(E) = -\lambda_0 \left[1 + \frac{\lambda_0}{2} \frac{m}{2\pi^2} \left(\Lambda - \sqrt{-mE - i0^+} \arctan \frac{\Lambda}{\sqrt{-mE - i0^+}} \right) \right]^{-1} \tag{71}$$

$$= -\left[\frac{1}{\lambda_0} + \frac{m}{4\pi^2} \left(\Lambda - \sqrt{-mE - i0^+} \arctan \frac{\Lambda}{\sqrt{-mE - i0^+}} \right) \right]^{-1} \tag{72}$$

Ex.9)

In s-wave ($l = 0$), S-matrix and scattering amplitude are related as

$$f(p) = \frac{s(p) - 1}{2ip} \tag{73}$$

$$2ipf(p) = s(p) - 1 \tag{74}$$

$$s(p) = 2ipf(p) + 1 \tag{75}$$

The nonperturbative amplitude leads to

$$s(p) = 2ip \frac{1}{-\frac{1}{a_0} - ip} + 1 \tag{76}$$

$$= \frac{2ip - \frac{1}{a_0} - ip}{-\frac{1}{a_0} - ip} \tag{77}$$

$$= \frac{-\frac{1}{a_0} + ip}{-\frac{1}{a_0} - ip} \tag{78}$$

Thus, for $p \in \mathbb{R}$, we have

$$s(p)s^*(p) = \frac{-\frac{1}{a_0} + ip}{-\frac{1}{a_0} - ip} \frac{-\frac{1}{a_0} - ip}{-\frac{1}{a_0} + ip} = 1 \tag{79}$$

and the unitarity is satisfied. On the other hand, the perturbative amplitude leads to

$$s(p) = 2ipC + 1 \tag{80}$$

$$s(p)s^*(p) = (2ipC + 1)(-2ipC + 1) \tag{81}$$

$$= 1 + 4C^2p^2 \tag{82}$$

and the unitarity is not satisfied.

3 Chiral dynamics

Ex.10)

The relevant part of the vector current is

$$V_\mu \sim \begin{pmatrix} & K^+ \\ K^- & \end{pmatrix} \begin{pmatrix} & \partial_\mu K^+ \\ \partial_\mu K^- & \end{pmatrix} - \begin{pmatrix} & \partial_\mu K^+ \\ \partial_\mu K^- & \end{pmatrix} \begin{pmatrix} & K^+ \\ K^- & \end{pmatrix} \tag{83}$$

$$= \begin{pmatrix} K^+\partial_\mu K^- & \\ & K^-\partial_\mu K^+ \end{pmatrix} - \begin{pmatrix} (\partial_\mu K^+)K^- & \\ & (\partial_\mu K^-)K^+ \end{pmatrix} \tag{84}$$

$$= \begin{pmatrix} K^+\partial_\mu K^- - (\partial_\mu K^+)K^- & \\ & K^-\partial_\mu K^+ - (\partial_\mu K^-)K^+ \end{pmatrix} \tag{85}$$

Thus

$$\mathcal{L} \sim \mathrm{Tr}\left[\begin{pmatrix} & \\ \bar{p} & \end{pmatrix} \left[V_\mu, \begin{pmatrix} & p \\ & \end{pmatrix} \right] \right] \tag{86}$$

$$= \mathrm{Tr}\left[\begin{pmatrix} & \\ \bar{p} & \end{pmatrix} V_\mu \begin{pmatrix} & p \\ & \end{pmatrix} - \begin{pmatrix} & \\ \bar{p} & \end{pmatrix} \begin{pmatrix} & p \\ & \end{pmatrix} V_\mu \right] \tag{87}$$

$$= \mathrm{Tr}\left[V_\mu \begin{pmatrix} & p \\ & \end{pmatrix} \begin{pmatrix} & \\ \bar{p} & \end{pmatrix} - \begin{pmatrix} & \\ & \bar{p}p \end{pmatrix} V_\mu \right] \tag{88}$$

$$= \mathrm{Tr}\left[\begin{pmatrix} \bar{p}p & \\ & -\bar{p}p \end{pmatrix} V_\mu \right] \tag{89}$$

$$= \mathrm{Tr} \begin{pmatrix} \bar{p}(K^+\partial_\mu K^- - (\partial_\mu K^+)K^-)p & \\ & -\bar{p}(K^-\partial_\mu K^+ - (\partial_\mu K^-)K^+)p \end{pmatrix} \tag{90}$$

$$= -2[\bar{p}\partial_\mu K^+ K^- p - \bar{p}(\partial_\mu K^-)K^+ p] \tag{91}$$

and we obtain $C_{K^-pK^-p} = 2$. Other coefficients are obtained in Similarly. One may use Mathematica (see Appendix).

Ex.11)

Denoting the NG boson (baryon) state with isospin I and I_3 as $|\,\Phi, I, I_3\,\rangle$ ($|\,B, I, I_3\,\rangle$), we obtain

$$
\begin{aligned}
|\,\bar{K}N(I=0)\,\rangle &= |\,\Phi B, 0, 0\,\rangle && (92)\\
&= C(\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2};0,0)|\,\Phi,\tfrac{1}{2},\tfrac{1}{2}\,\rangle|\,B,\tfrac{1}{2},-\tfrac{1}{2}\,\rangle \\
&\quad + C(\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2};0,0)|\,\Phi,\tfrac{1}{2},-\tfrac{1}{2}\,\rangle|\,B,\tfrac{1}{2},\tfrac{1}{2}\,\rangle && (93)\\
&= \frac{1}{\sqrt{2}}|\,\bar{K}^0\,\rangle|\,n\,\rangle + (-\frac{1}{\sqrt{2}})(-|\,K^-\,\rangle)|\,p\,\rangle && (94)\\
&= \frac{1}{\sqrt{2}}(|\,\bar{K}^0 n\,\rangle + |\,K^- p\,\rangle) && (95)\\
|\,\pi\Sigma(I=0)\,\rangle &= |\,\Phi B, 0, 0\,\rangle && (96)\\
&= C(1,1,1,-1;0,0)|\,\Phi,1,1\,\rangle|\,B,1,-1\,\rangle \\
&\quad + C(1,0,1,0;0,0)|\,\Phi,1,0\,\rangle|\,B,1,0\,\rangle \\
&\quad + C(1,-1,1,1;0,0)|\,\Phi,1,-1\,\rangle|\,B,1,1\,\rangle && (97)\\
&= \frac{1}{\sqrt{3}}(-|\,\pi^+\,\rangle|\,\Sigma^-\,\rangle + (-\frac{1}{\sqrt{3}})|\,\pi^0\,\rangle|\,\Sigma^0\,\rangle + \frac{1}{\sqrt{3}}|\,\pi^-\,\rangle(-|\,\Sigma^+\,\rangle) && (98)\\
&= -\frac{1}{\sqrt{3}}(|\,\pi^+\Sigma^-\,\rangle + |\,\pi^0\Sigma^0\,\rangle + |\,\pi^-\Sigma^+\,\rangle) && (99)
\end{aligned}
$$

where $C(j_1,\mu_1,j_2,\mu_2;J,\mu_1+\mu_2)$ are the Clebsch-Gordan coefficients. Then the coefficients in the isospin basis is obtained as

$$
\begin{aligned}
&\langle\,\bar{K}N(I=0)\,|\hat{C}|\,\bar{K}N(I=0)\,\rangle && (100)\\
&= \frac{1}{\sqrt{2}}\big(\langle\,\bar{K}^0 n\,| + \langle\,K^- p\,|\big)\hat{C}\frac{1}{\sqrt{2}}\big(|\,\bar{K}^0 n\,\rangle + |\,K^- p\,\rangle\big) && (101)\\
&= \frac{1}{2}\big(\langle\,\bar{K}^0 n\,|\hat{C}|\,\bar{K}^0 n\,\rangle + \langle\,\bar{K}^0 n\,|\hat{C}|\,K^- p\,\rangle + \langle\,K^- p\,|\hat{C}|\,\bar{K}^0 n\,\rangle + \langle\,K^- p\,|\hat{C}|\,K^- p\,\rangle\big) && (102)\\
&= \frac{1}{2}\big(2+1+1+2\big) && (103)\\
&= 3 && (104)\\
&\langle\,\bar{K}N(I=0)\,|\hat{C}|\,\pi\Sigma(I=0)\,\rangle && (105)\\
&= \frac{1}{\sqrt{2}}\big(\langle\,\bar{K}^0 n\,| + \langle\,K^- p\,|\big)\hat{C}(-\frac{1}{\sqrt{3}})\big(|\,\pi^+\Sigma^-\,\rangle + |\,\pi^0\Sigma^0\,\rangle + |\,\pi^-\Sigma^+\,\rangle\big) && (106)\\
&= -\frac{1}{\sqrt{6}}\big(\langle\,\bar{K}^0 n\,|\hat{C}|\,\pi^+\Sigma^-\,\rangle + \langle\,\bar{K}^0 n\,|\hat{C}|\,\pi^0\Sigma^0\,\rangle + \langle\,\bar{K}^0 n\,|\hat{C}|\,\pi^-\Sigma^+\,\rangle \\
&\quad + \langle\,K^- p\,|\hat{C}|\,\pi^+\Sigma^-\,\rangle + \langle\,K^- p\,|\hat{C}|\,\pi^0\Sigma^0\,\rangle + \langle\,K^- p\,|\hat{C}|\,\pi^-\Sigma^+\,\rangle\big) && (107)\\
&= -\frac{1}{\sqrt{6}}\big(1+\tfrac{1}{2}+0+0+\tfrac{1}{2}+1\big) && (108)\\
&= -\frac{3}{\sqrt{6}} && (109)\\
&= -\sqrt{\frac{3}{2}} && (110)\\
&\langle\,\pi\Sigma(I=0)\,|\hat{C}|\,\pi\Sigma(I=0)\,\rangle && (111)
\end{aligned}
$$

$$= (-\frac{1}{\sqrt{3}})\big(\langle\, \pi^+\Sigma^-\, | + \langle\, \pi^0\Sigma^0\, | + \langle\, \pi^-\Sigma^+\, |\big)\hat{C}(-\frac{1}{\sqrt{3}})\big(|\, \pi^+\Sigma^-\,\rangle + |\, \pi^0\Sigma^0\,\rangle + |\, \pi^-\Sigma^+\,\rangle\big) \tag{112}$$

$$\begin{aligned}
&= \frac{1}{3}\big(\langle\, \pi^+\Sigma^-\, |\hat{C}|\, \pi^+\Sigma^-\,\rangle + \langle\, \pi^+\Sigma^-\, |\hat{C}|\, \pi^0\Sigma^0\,\rangle + \langle\, \pi^+\Sigma^-\, |\hat{C}|\, \pi^-\Sigma^+\,\rangle \\
&\quad + \langle\, \pi^0\Sigma^0\, |\hat{C}|\, \pi^+\Sigma^-\,\rangle + \langle\, \pi^0\Sigma^0\, |\hat{C}|\, \pi^0\Sigma^0\,\rangle + \langle\, \pi^0\Sigma^0\, |\hat{C}|\, \pi^-\Sigma^+\,\rangle \\
&\quad + \langle\, \pi^-\Sigma^+\, |\hat{C}|\, \pi^+\Sigma^-\,\rangle + \langle\, \pi^-\Sigma^+\, |\hat{C}|\, \pi^0\Sigma^0\,\rangle + \langle\, \pi^-\Sigma^+\, |\hat{C}|\, \pi^-\Sigma^+\,\rangle\big)
\end{aligned} \tag{113}$$

$$= \frac{1}{3}\big(2 + 2 + 0 + 2 + 0 + 2 + 2 + 2 + 0\big) \tag{114}$$

$$= 4 \tag{115}$$

Calculation of Cij coefficients

Set up

Fields

$$\textbf{Baryon} = \begin{pmatrix} \frac{\Sigma_0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma_+ & p \\ \Sigma_- & \frac{-\Sigma_0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi_- & \Xi_0 & \frac{-2\,\Lambda}{\sqrt{6}} \end{pmatrix};$$

$$\textbf{BBar} = \begin{pmatrix} \frac{B\Sigma_0}{\sqrt{2}} + \frac{B\Lambda}{\sqrt{6}} & B\Sigma_- & B\Xi_- \\ B\Sigma_+ & \frac{-B\Sigma_0}{\sqrt{2}} + \frac{B\Lambda}{\sqrt{6}} & B\Xi_0 \\ Bp & Bn & \frac{-2\,B\Lambda}{\sqrt{6}} \end{pmatrix};$$

$$\textbf{Meson} = \begin{pmatrix} \frac{\pi_0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi_+ & K_+ \\ \pi_- & \frac{-\pi_0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K_0 \\ K_- & \overline{K}_0 & \frac{-2\,\eta}{\sqrt{6}} \end{pmatrix};$$

$$\textbf{DMeson} = \begin{pmatrix} \frac{D\pi_0}{\sqrt{2}} + \frac{D\eta}{\sqrt{6}} & D\pi_+ & DK_+ \\ D\pi_- & \frac{-D\pi_0}{\sqrt{2}} + \frac{D\eta}{\sqrt{6}} & DK_0 \\ DK_- & \overline{DK}_0 & \frac{-2\,D\eta}{\sqrt{6}} \end{pmatrix};$$

Rules

$$\text{rulesKN} = \{\Sigma_+ \to 0,\ \Sigma_- \to 0,\ \Sigma_0 \to 0,\ \Lambda \to 0,\ \Xi_0 \to 0,\ \Xi_- \to 0,\ B\Sigma_+ \to 0,\ B\Sigma_- \to 0,\ B\Sigma_0 \to 0,\ B\Lambda \to 0,\ B\Xi_0 \to 0,\ B\Xi_- \to 0,\ \pi_0 \to 0,\ \pi_+ \to 0,\ \pi_- \to 0,\ \eta \to 0\};$$

$$\text{rulespS} = \{n \to 0,\ p \to 0,\ \Lambda \to 0,\ \Xi_0 \to 0,\ \Xi_- \to 0,\ Bn \to 0,\ Bp \to 0,\ B\Lambda \to 0,\ B\Xi_0 \to 0,\ B\Xi_- \to 0,\ \eta \to 0,\ K_+ \to 0,\ K_0 \to 0,\ K_- \to 0,\ \overline{K}_0 \to 0\};$$

$$\text{rulesKNpS} = \{n \to 0,\ p \to 0,\ \Lambda \to 0,\ \Xi_0 \to 0,\ \Xi_- \to 0,\ B\Sigma_+ \to 0,\ B\Sigma_- \to 0,\ B\Sigma_0 \to 0,\ B\Lambda \to 0,\ B\Xi_0 \to 0,\ B\Xi_- \to 0,\ \eta \to 0,\ D\eta \to 0\};$$

Results

Lagrangian

```
Lag = Tr[BBar.(Meson.DMeson - DMeson.Meson).Baryon
     - BBar.Baryon.(Meson.DMeson - DMeson.Meson)];
```

Kbar N diagonal

```
Lag /. rulesKN // Expand
```

$$-\mathrm{Bn}\,\mathrm{n}\,K_-\,DK_+ - 2\,\mathrm{Bp}\,\mathrm{p}\,K_-\,DK_+ + \mathrm{Bn}\,\mathrm{n}\,DK_-\,K_+ + 2\,\mathrm{Bp}\,\mathrm{p}\,DK_-\,K_+ - \mathrm{Bn}\,\mathrm{p}\,K_-\,DK_0 + \mathrm{Bn}\,\mathrm{p}\,DK_-\,K_0 + \mathrm{Bp}\,\mathrm{n}\,K_+\,\overline{DK_0} + 2\,\mathrm{Bn}\,\mathrm{n}\,K_0\,\overline{DK_0} + \mathrm{Bp}\,\mathrm{p}\,K_0\,\overline{DK_0} - \mathrm{Bp}\,\mathrm{n}\,DK_+\,\overline{K_0} - 2\,\mathrm{Bn}\,\mathrm{n}\,DK_0\,\overline{K_0} - \mathrm{Bp}\,\mathrm{p}\,DK_0\,\overline{K_0}$$

pi Sigma diagonal

```
Lag /. rulespS // Expand
```

$$2\,B\Sigma_-\,\pi_-\,\Sigma_-\,D\pi_+ - 2\,B\Sigma_-\,D\pi_-\,\Sigma_-\,\pi_+ - 2\,\pi_-\,B\Sigma_+\,D\pi_+\,\Sigma_+ + 2\,D\pi_-\,B\Sigma_+\,\pi_+\,\Sigma_+ - 2\,\Sigma_-\,\pi_+\,B\Sigma_0\,D\pi_0 - 2\,\pi_-\,\Sigma_+\,B\Sigma_0\,D\pi_0 + 2\,\Sigma_-\,D\pi_+\,B\Sigma_0\,\pi_0 + 2\,D\pi_-\,\Sigma_+\,B\Sigma_0\,\pi_0 + 2\,B\Sigma_-\,\pi_-\,D\pi_0\,\Sigma_0 + 2\,B\Sigma_+\,\pi_+\,D\pi_0\,\Sigma_0 - 2\,B\Sigma_-\,D\pi_-\,\pi_0\,\Sigma_0 - 2\,B\Sigma_+\,D\pi_+\,\pi_0\,\Sigma_0$$

off diagonal

```
Lag /. rulesKNpS // Expand
```

$$-\mathrm{Bp}\,\pi_-\,DK_+\,\Sigma_+ + \mathrm{Bp}\,D\pi_-\,K_+\,\Sigma_+ - \mathrm{Bn}\,\Sigma_-\,\pi_+\,DK_0 + \frac{\mathrm{Bn}\,\Sigma_-\,K_+\,D\pi_0}{\sqrt{2}} + \mathrm{Bn}\,\Sigma_-\,D\pi_+\,K_0 - \frac{\mathrm{Bp}\,\Sigma_+\,D\pi_0\,K_0}{\sqrt{2}} - \frac{\mathrm{Bn}\,\Sigma_-\,DK_+\,\pi_0}{\sqrt{2}} + \frac{\mathrm{Bp}\,\Sigma_+\,DK_0\,\pi_0}{\sqrt{2}} + \frac{\mathrm{Bn}\,\pi_-\,DK_+\,\Sigma_0}{\sqrt{2}} - \frac{\mathrm{Bn}\,D\pi_-\,K_+\,\Sigma_0}{\sqrt{2}} - \frac{\mathrm{Bp}\,\pi_+\,DK_0\,\Sigma_0}{\sqrt{2}} + \frac{1}{2}\,\mathrm{Bp}\,K_+\,D\pi_0\,\Sigma_0 + \frac{\mathrm{Bp}\,D\pi_+\,K_0\,\Sigma_0}{\sqrt{2}} + \frac{1}{2}\,\mathrm{Bn}\,D\pi_0\,K_0\,\Sigma_0 - \frac{1}{2}\,\mathrm{Bp}\,DK_+\,\pi_0\,\Sigma_0 - \frac{1}{2}\,\mathrm{Bn}\,DK_0\,\pi_0\,\Sigma_0$$