Walking

with

about half a century

山脇幸一

於:「素粒子論のこの50年、…」
基礎物理学研究所　　2022年3月12-13日

はじめに

今年はKM理論から50年、KM理論に重点を置いた記念シンンポ は既に名古屋大学KMIで開催されました。

ここでは、KM理論以上に益川さんが力を注いだ物理に焦点をあててその過去・現在・未来を俯瞰します。

それはゲージ理論におけるカイラル対称性の力学的破れです。

益川・中島論文（1974）はこれが弱結合で起こるBCS理論と違って、NJL模型と同様強結合でのみ起こるということを示しました。実際、漸近自由のQCDでは赤外部での強結合のため対称性の自発的破れが起こります。ここでもフレーバー数の大きな場合は赤外固定点のため全エネルギー領域で結合定数が臨界値より小さくなり自発的破れは起こりません（conformal window)。ゲージ理論における一般的な結果であると同時に、具体的にはスケール不変な枠組みでの結果であったため、walking gauge theoryの基礎となったものです。私の研究の中心である複合ヒッグス模型の核心です。

会の趣旨から外れてタイトルもabstractも英語なのは``walking”のノリが日本語では表現できないからです。ご理解下さい。

Abstract

Walking gauge theory was first studied back in 1974 by Maskawa and Nakajima who discovered spontaneous chiral symmetry breaking solution for the gauge theory with scale-invariant (non-running) coupling larger than a critical coupling in the ladder Schwinger-Dyson equation. Based on this, I together with Bando and Matumoto proposed a scale-invariant technicolor ("walking technicolor"), where the perturbatively non-running coupling becomes slowly running ("walking") nonperturbatively towards critical coupling (ultraviolet fixed point) in the broken phase, having a large anomalous dimension $\gamma_m \simeq 1$ as a solution for the flavor-changing neutral current problem of the original technicolor, and a pseudo dilaton ("technidilaton"), a pseudo Nambu-Goldstone boson of the scale symmetry, as a candidate for the composite Higgs. In this talk I will describe the walking gauge theory from the original version up to the most recent lattice studies for which Maskawa has been a member of our LatKMI Collaboration at Kobayashi-Maskawa Institute (KMI) at Nagoya University.

個人的年表

個人的年表

益川さん　　　　　　　　　　　　　　山脇

1962年　名古屋大学理学部卒業
名古屋大学大学院（E研）
（牧・中川・坂田理論：ν混合、4元模型）
1967年　名古屋大学助手

1970年　京都大学助手　　　　　　　　京都大学大学院
（湯川博士定年退官、坂田博士逝去）

1972年　小林・益川理論
1974年　益川・中島理論

「くりこみ可能性」

1976年　益川・山脇論文（lightcone zero mode, DLCQ）

1985年　Walking Technicolor（山脇・坂東・松本）
益川・中島に基づく

2010年　名古屋大学素粒子宇宙起源研究機構 (Kobayashi-Maskawa Institute, KMI)
LatKMI 格子理論グループ（Walking Gauge Theoriesの研究）

Progress of Theoretical Physics, Vol. 56, No. 1, July 1976

The Problem of $P^{+}=0$ Mode in the Null-Plane Field Theory and Dirac's Method of Quantization

Toshihide MASKAWA and Koichi YAMAWAKI*

Department of Physics, Kyoto University, Kyoto 606[†)]

**Research Institute for Fundamental Physics, Kyoto University, Kyoto 606*

(Received January 7, 1976)

The null-plane quantization is studied with the emphasis on the $P^{+}=0$ mode, by using Dirac's quantization for constrained systems. This mode is eliminated from the Hilbert space and the physical vacuum can be defined in a kinematical way. It enables us to construct the physical Fock space kinematically. Poincaré invariance is also studied in detail.

§ 1. Introduction

Spontaneous Chiral Symmetry Breaking

益川さんの最も力を注いだ問題　！！

$$\partial^\mu J_\mu = 0$$

Equal time quantization

$$i\dot{Q} = [Q, H] = 0, \qquad Q|0\rangle \neq 0.$$

Complicate Vacuum, no Fock space,
Chiral representation makes no sense

Null-plane (Lightcone) quantization $\longleftrightarrow$ Infinite momentum frame

We proved

$$i\dot{Q} = [Q, H] \neq 0, \qquad Q|0\rangle = 0,$$

Zero Mode carrying SSB dropped out from Q
physical Fock space without Zero mode
Chiral representation makes sense (representation mixing)

Proposing ``Discrete Light-Cone Quantization (DLCQ)"

$$p^+ = \pi n/L \qquad -L \leq x^- \equiv (x^0 - x^3)/\sqrt{2} \leq L \quad \Longleftrightarrow \quad \text{lattice}$$

2008年10月8日　NHK 7時のニュースから

南部・Jona-Lasinio (NJL)模型

素粒子の質量の起源

強結合

核子（陽子・中性子）Nucleon: 当時の``素粒子”

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.

The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

[2] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 106, 162 (1957).

(BCS)

弱結合 (1+1次元 フェルミ面)

対称性の**自発的破れ(SSB)**は

対称性の**力学的破れ(DSB)**

として発見された！

南部-Jona-Lasinio (1960)

質量の起源　（核子）

タキオン ⟺ **強結合**

（ゼロ質量フェルミオンのペアー）

Vol. 41 No. 6 (1969) pp. 1515-1532 : (5)

Single Pion Production in Low Energy Pion-Nucleon Scattering and Chiral Dynamics

Chuichiro Hattori, Makoto Kobayashi, Hiroki Kondo and Toshihide Maskawa

Vol. 43 No. 5 (1970) pp. 1334-1342 : (5)

Nonlinear Realizations of Groups and Chiral Transformations in the Quark Model

Chuichiro Hattori, Makoto Kobayashi, Hiroki Kondo and Toshihide Maskawa

Vol. 44 No. 5 (1970) pp. 1422-1424 : (5)

Chiral Symmetry and η-*X* Mixing

Makoto Kobayashi and Toshihide Maskawa

Vol. 45 No. 6 (1971) pp. 1955-1959 : (5)

Symmetry Breaking of the Chiral $U(3) \otimes U(3)$ and the Quark Model

Makoto Kobayashi, Hiroki Kondo and Toshihide Maskawa

Vol. 46 No. 5 (1971) pp. 1647-1649 : (5)

Fundamental Quartets and Chiral $U(4) \otimes U(4)$

Ziro Maki and Toshihide Maskawa

Vol. 47 No. 3 (1972) pp. 1060-1062 : (5)

A Note on the Leptonic Decays of Charmed Mesons (**Chiral $U(4) \otimes U(4)$**)

Hiroki Kondo, Ziro Maki and Toshihide Maskawa

Vol. 47 No. 5 (1972) pp. 1682-1703 : (5)

Quartet Scheme of Hadrons in Chiral $U(4) \otimes U(4)$

Ziro Maki, Toshihide Maskawa and Isao Umemura

Vol. 48 No. 2 (1972) pp. 596-606 : (5)

A New Approach to $\eta \to 2\gamma$ Decay and Models of Elementary Particles (**Chiral $U(4) \otimes U(4)$**)

Ziro Maki, Toshihide Maskawa and Isao Umemura

Vol. 49 No. 2 (1973) pp. 634-639 : (5)

Symmetry Breaking of Chiral $U(3) \otimes U(3)$ and $X \to \eta\pi\pi$ Decay Amplitde

Makoto Kobayashi, Hiroki Kondo and Toshihide Maskawa

CP Violation in the Renormalizable Theory of Weak Interaction

Makoto Kobayashi (Kyoto U.), Toshihide Maskawa (Kyoto U.) (Feb, 1973)

Published in: *Prog.Theor.Phys.* 49 (1973) 652-657

Chiral $U(4) \otimes U(4)$

Hadron symmetries and gauge theory of weak and electromagnetic interactions

Z. Maki (Kyoto U.), T. Maskawa (Kyoto U.) (Mar, 1973)

Published in: *Prog.Theor.Phys.* 49 (1973) 1007-1013

Chiral $U(4) \otimes U(4)$

Spontaneous Symmetry Breaking in Vector-Gluon Model

Toshihide Maskawa (Kyoto U.), Hideo Nakajima (Kyoto U.) (Feb, 1974)

Published in: *Prog.Theor.Phys.* 52 (1974) 1326-1354

Spontaneous Breaking of Chiral Symmetry in a Vector-Gluon Model. 2.

Toshihide Maskawa (Kyoto U.), Hideo Nakajima (Kyoto U.) (Jan, 1975)

Published in: *Prog.Theor.Phys.* 54 (1975) 860

The Bag Theory with Dirichlet Boundary Conditions and Spontaneous Symmetry Breakdown

Takuo Inoue (Kyoto U.), Toshihide Maskawa (Kyoto U.) (Apr, 1975)

Published in: *Prog.Theor.Phys.* 54 (1975) 1833

1970年

時代背景：主流＝hadronが最終構成要素

bootstrap/S-matrix theory/Veneziano amplitude

・GIM paper：クオーク4元模型によるFCNC解決

＜──＞レプトン（混合）も含めた4元模型（牧・中川・坂田1962）

・湯川先生定年退職

・坂田先生逝去

坂田思想からの脱却：くりこみ可能な理論へ

KM理論（１９７２）

益川・中島理論（１９７４）

。

１９７１年　GSW模型のくりこみ可能性　（'t Hooft−Veltman)

丹生イベント（チャーム）

１９７２年　KM理論

１９７３年　ゲージ理論漸近自由性、

中性カレントの発見（GSW模型の確立）

CP破れの小林益川論文の裏話
＜益川さんから直接聞いた話＞

（小林さん京大助手着任（1972.04）直前
名古屋でGSW模型(当時はWeinberg模型と呼んでいた)のセミナー
（中性カレントに否定的な実験結果で模型は否定されていた（覆されるのは（1973））
GSW模型のくりこみ可能性証明（1971）を重視
<-> 坂田思想からの脱却。）

小林：このセミナーで大貫さんがこの模型でCP violationはどうなるかと
言ってるよ。
益川：そりゃ面白い、調べてみるか。
——＞小林益川論文 1972.09.01 received）

（4元模型は名古屋では常識、ご両人の論文多数）
主要部分は4元模型ではCPの破れ出ないことの証明、6元模型は論文最後の
解決策の候補についてのコメントの一つ（2番目）

その後京都ではご本人たちも含めてKM理論に関する研究無し

Progress of Theoretical Physics, Vol. 52, No. 4, October 1974

Spontaneous Breaking of Chiral Symmetry in a Vector-Gluon Model

Massive(Higgs mechanism assumed), *Confinement unknown*

Toshihide MASKAWA and Hideo NAKAJIMA

Departm ... *ty, Kyoto*

Solutions of the self-c ... or in a vector-gluon model are fully examined. The ... rameters, i.e., the coupling constant g, the bare mass ... . It is proved that with a suitable gauge chosen, the ... only in the case $m_0=0$. It is then shown that, if $g^2/4$ ... on is infinity of continuum. This situation does not come from the freedom of fixing the mass scale since the gluon mass is chosen to be non-vanishing. When the cutoff is introduced, the equation has a unique solution for $g^2/4\pi<\pi/4$. In this case, however, β turns out to be identically zero if we put $m_0=0$, which means that any "superconducting" solutions do not exist for such a value of g irrespective of cutoff momentum; that is, there are no Nambu-Goldstone bosons. When g^2 is large enough, such a "superconducting" solution does exist in the model in which the vector part of the inverse fermion propagator is identically set equal to unity. Furthermore the existence of many "superconducting" solutions is inferred in this model. It is also found that in the region $g^2/4\pi>8\pi$, the "normal-state" solution for the equation without cutoff, even if it existed, should necessarily have an unphysical singularity. This fact implies that the "normal-state" solution becomes unstable for a sufficiently large value of g^2.

Motivation (from Introduction)

Therefore, some other renormalizable models for spontaneous breaking of symmetry would have to be considered instead of the Nambu-Jona-Lasinio model. The vector-gluon model appears to be a candidate,[4] which is renormalizable and has a set of approximations satisfying the Ward-Takahashi identity. Moreover, it is possible, in principle, to discuss the relations between NG bosons and higher spin mesons within this model.

くりこみ可能性についての坂田哲学からの脱皮

to the singlet axial-vector current. However, the ε-term requires, if represented with fermion fields, six-fermion interactions in the triplet model and eight-fermion interactions in the quartet model, respectively. Incidentally, Adler's anomalous term in the presence of a singlet vector-gluon appears only in the divergence of the singlet axial-vector current and, therefore, is capable of playing a role of the ε-term; this means that the vector-gluon model provides a way of resolving π-η degeneracy without introducing the six (or eight)-fermion couplings.

X(η')mesonの問題

't Hooft determinant (1976) <ーー

Prog. Theor. Phys. Vol. 44 (1970), No. 5

Chiral Symmetry and η-X Mixing

Makoto KOBAYASHI and
Toshihide MASKAWA*

Department of Physics
Nagoya University, Nagoya
**Department of Physics*
Kyoto University, Kyoto

August 5, 1970

対称性の自発的破れ(SSB)は

対称性の力学的破れ(DSB)

として発見された！

南部-Jona-Lasinio (1960)

質量の起源　（核子）

タキオン ⟺ **強結合**

（ゼロ質量フェルミオンのペアー）

真空（エネルギー最低状態）

空っぽではない！

ヴァーチャルな粒子・反粒子ペアーの生成消滅の世界

不確定性原理

質量の起源

“質量は無から生じる”

- 通常の真空＝ペアーはランダム
- 質量を生じる真空＝ペアーが方向性をもつ

（真空凝縮）

``対称性の力学的破れ‘’

南部・Jona-Lasinio (NJL)

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
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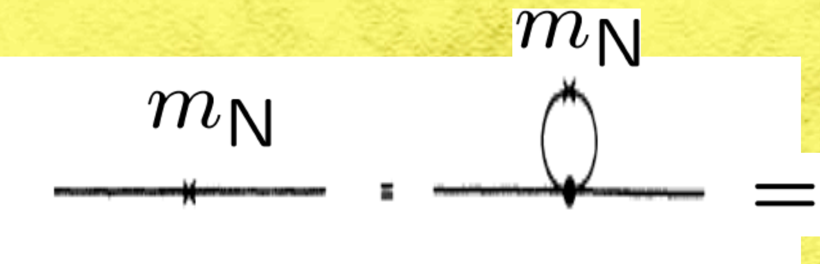
The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

[2] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957). (BCS)

$$\frac{G}{2}\,(\bar{N}N)\,(\bar{N}N)$$



$$= -G\langle \bar{N}N \rangle$$

Gap eq. $G > G_{cr}$

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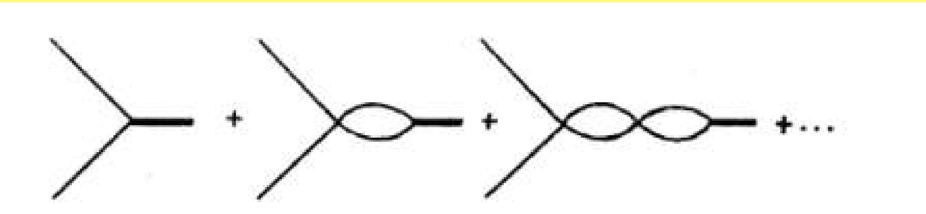
$$\pi \sim \bar{N}N$$

複合南部・Goldstone(NG) ボソン



σ (``Higgs")

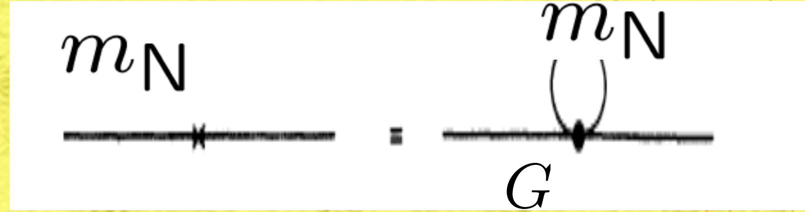
Fermi-Yang Model (1948)

p, n → p, n, Λ

Sakata Model (1956)

NJL II

$$\mathcal{L}_{\rm int} = \frac{G}{2}(\bar{N}N)^2 + \cdots$$



$$\longrightarrow G\langle \bar{N}N\rangle \cdot \bar{N}N = -m_{\rm N}\bar{N}N$$

$\bar{N}N \to \langle \bar{N}N\rangle + \bar{N}N$

$$m_{\rm N} = -G\langle \bar{N}N\rangle = G\cdot {\rm Tr}S_F(p) = G\cdot 4\int \frac{d^4p}{(4\pi)^4 i}\frac{N_c m_{\rm N}}{m_{\rm N}^2 - p^2}$$

ギャップ方程式

$$\overset{1}{\cancel{m_{\rm N}}} = \overset{1}{\cancel{m_{\rm N}}}\cdot\frac{N_c G}{4\pi^2}\left(\Lambda^2 - m_{\rm N}^2\ln\left(\frac{\Lambda^2}{m_{\rm N}^2}\right)\right)$$

$$m_{\rm N} \neq 0 \iff \Lambda^2\left(\frac{1}{g} - \frac{1}{g_{\rm cr}}\right) = -m_{\rm N}^2\ln\left(\frac{\Lambda^2}{m_{\rm N}^2}\right) < 0$$

$$N_c G \equiv g\frac{4\pi^2}{\Lambda^2} > G_{\rm cr} \quad (g_{\rm cr} = 1 \neq 0)$$

強結合！

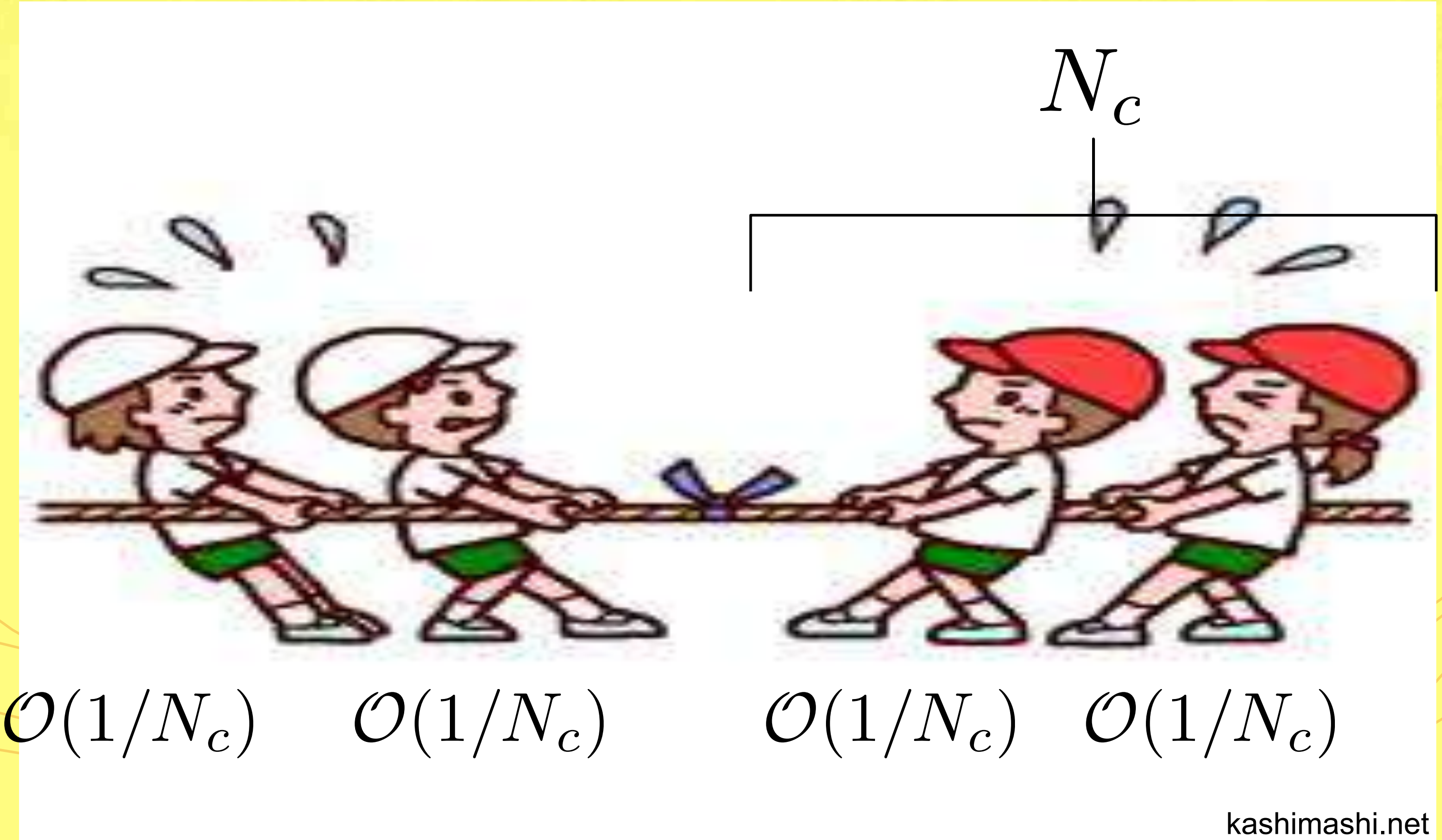
$$m_{\rm N} = 0 \iff G < G_{\rm cr}$$

強結合 = 概念的に「弱結合」(クオークの場合)

$$N_c \times G > N_c \times G_{\rm cr} = \mathcal{O}(1) \cdot \left(\frac{4\pi^2}{\Lambda^2} \right)$$

$$G_{\rm cr} = \mathcal{O}(1/N_c) \to 0 \ (N_c \to \infty)$$

平均場近似
(Large Nc 展開、
はしご近似)



BCS 理論（弱結合）

フェルミ面近傍（幅Debye振動）のみフォノンの引力　ー＞クーパーペア
フェルミオンの有効次元が2次元分落ちる!　(brane fermion)

ボソンの自由度は4次元のまま

$$\int d^4p = \int dp^0 d^3p \longrightarrow \int dp^0 4\pi p^2 dp \simeq 4\pi p_F^2 \int dp^0 dp$$

$$\int dp = \frac{m_e}{p_F}\int_{-\omega_D/2}^{\omega_D/2} dE(p) \qquad \left| E(p) = \frac{p^2}{2m_e} - \frac{p_F^2}{2m_e} \right| < \frac{\omega_D}{2} \ll \frac{p_F^2}{2m_e}$$

$$1 = G\frac{N}{2}\int_0^{\omega_D/2} dE(p) \frac{1}{\sqrt{|\Delta|^2 + E(p)^2}} \sim \frac{GN}{2}\ln\frac{\omega_D}{|\Delta|}$$

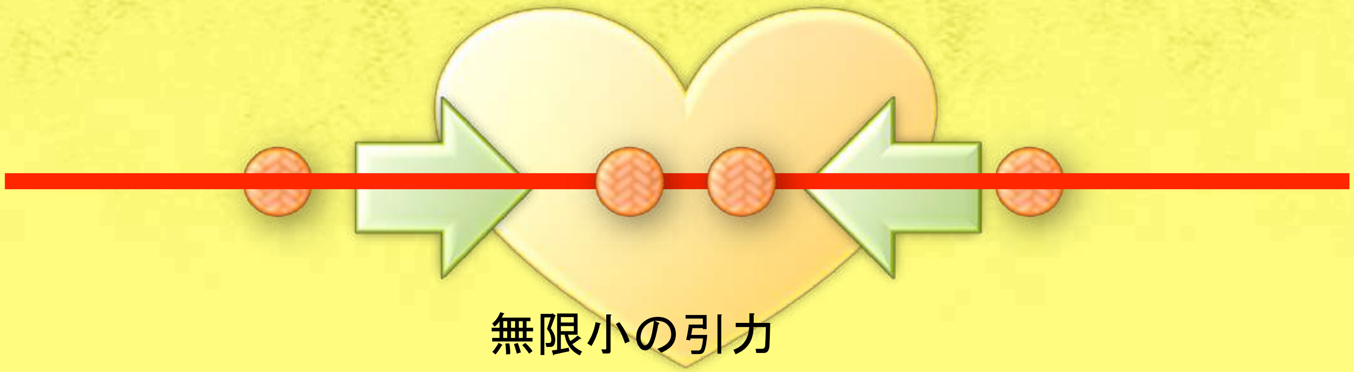
$$|\Delta| \simeq \omega_D\, e^{-\frac{2}{GN}} \ll \omega_D \qquad N = p_F m_e/\pi^2$$

$$\boxed{G_{\rm cr} = 0}$$

相転移がない！！
どんなに小さな引力でも凝縮
T=0

温度による相転移はある
T＞Tc≠0

3次元のボーズ粒子
1次元のフェルミ粒子



「BCS 関係式」

弱結合極限 $G \to G_{\rm cr} = 0$

$$m_\sigma = 2m_\Delta$$

$$m_\pi = 0$$

NJL

$$G\left(\bar{\psi}\psi\right)^2 \sim -\frac{1}{G}\phi^\dagger\phi + \bar{\psi}\psi\phi + h.c.$$

質量の起源

核子 ➔ クォーク

タキオン

$$\mu^2 = \frac{1}{G} - \frac{1}{G_{\rm cr}} = -\frac{N_c\, m_{\rm N}^2}{4\pi^2}\ln\frac{\Lambda^2}{m_{\rm N}^2} < 0$$

$$m_{\rm N} \simeq \Lambda\left(\frac{1}{g_{\rm cr}} - \frac{1}{g}\right)^{1/2}$$

強結合

N_c

$$G > G_{\rm cr} = \frac{4\pi^2}{\Lambda^2} \neq 0$$

内在的スケール Λ

$1/\sqrt{G}$

$m_{\rm N} = 0$ のままだと複合粒子はタキオン

$$m_\sigma = 2m_{\rm N}$$

$N_c \to \infty$

BCS

$$G_{\rm cr} = 0$$

(弱結合：
フェルミ面　2次元)

$$m_\sigma = 2m_\Delta$$

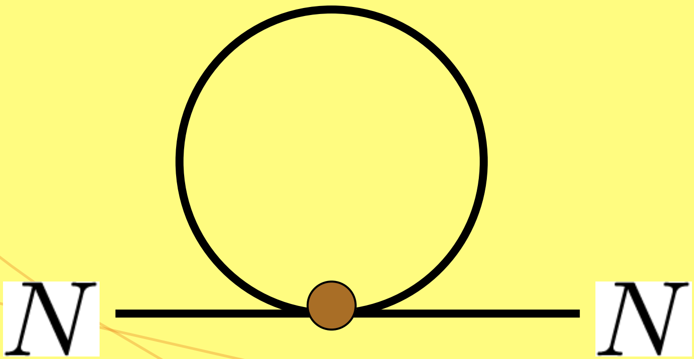
ＱＣＤでも本質的に同じ機構

- 核子　　　　　　　　　　　　クォーク

$$\langle \bar{N} N \rangle \quad \Rightarrow \quad \langle \bar{q} q \rangle$$

$$M_{\mathrm{N}}$$

- ${}_4$体フェルミ相互作用



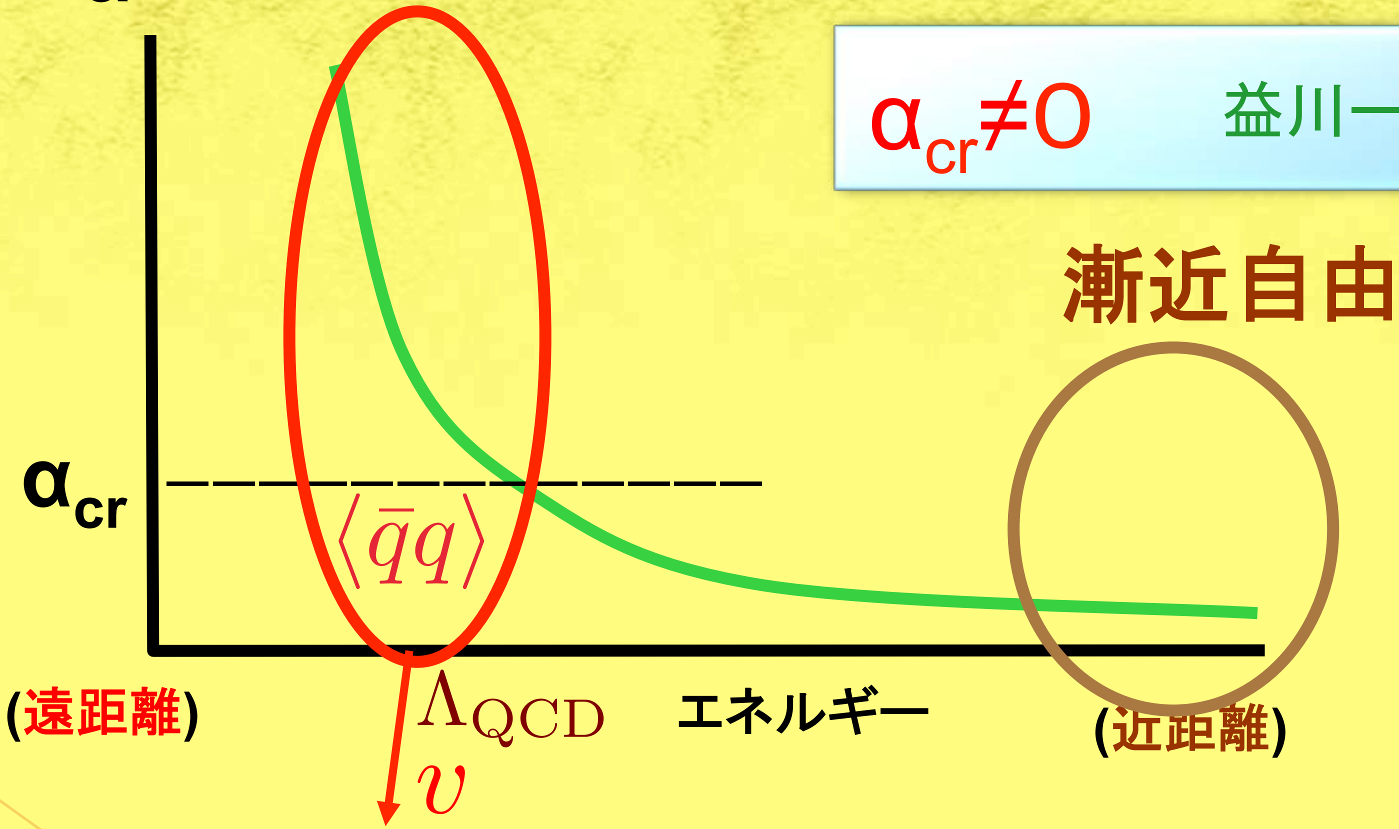
$$M_{\mathrm{N}} \sim \left(\frac{1}{G_{\mathrm{cr}}} - \frac{1}{G} \right)^{\frac{1}{2}}$$

$$m_q^* : \alpha(\mu = m_q^*) > \alpha_{\mathrm{cr}}$$

強結合：QCD（クォークの力学）

α >α$_{cr}$≠0になるスケールが v を決める

α$_{cr}$≠0　　益川ー中島 (1974)



ペアー凝縮

タキオン　$\langle\bar{q}q\rangle = \mathcal{O}(v^3)$　s.t.　$\alpha(\mu = v) > \alpha_{\rm cr} = \mathcal{O}(1/N_c)$

質量の起源 (QCD)

$\mathcal{L}_{\rm QCD}(m_q = 0)$ ：スケール不変(古典論)

スケール異常 (量子論):
結合定数がエネルギー的に変化

内在的スケール

$$\Lambda_{\rm QCD} = \mu \exp\left(-\int^{\alpha(\mu)} \frac{d\alpha}{\beta(\alpha)}\right) \simeq \mu\, e^{-\frac{1}{b\alpha(\mu)}}$$

$$\alpha(\mu) \sim \frac{b}{\ln \frac{\mu}{\Lambda_{\rm QCD}}}$$

$1/\sqrt{G}$

NJL機構　強結合 $\alpha(\mu \sim \Lambda_{\rm QCD}) > \alpha_{\rm cr}$

N_c N_c

質量

$$v = f_\pi,\ m_q^*,\ \langle \bar{q} q \rangle^{1/3}, \cdots \quad = \mathcal{O}(\Lambda_{\rm QCD})$$

QCDの相転移（ $\alpha_{\rm cr} \neq 0$ の存在）？

$v \neq 0$ $\alpha > \alpha_{\rm cr}$

$v = 0$ $\alpha < \alpha_{\rm cr}$

高温 $\alpha(\mu > T) < \alpha_{\rm cr}$ クォーク・グルーオン プラズマ
格子理論、RHIC実験

高密度 $\alpha(\mu = E_F) < \alpha_{\rm cr}$ フェルミ面 BCS

カラー超伝導

QCDにおけるBCS理論
フェルミ面

$$\alpha_{\rm cr}^{(\bar{q}q)} \neq 0 \qquad \alpha_{\rm cr}^{(qq)} = 0$$

高密度核物質 (中性子星内部？)；
クォークのみフェルミ面
（フェルミ面上のエネルギー） 大

$$E_F \gg \Lambda_{\rm QCD}$$

$$\alpha(\mu \simeq E_F) < \alpha_{\rm cr} \Rightarrow \langle \bar{q}q \rangle = 0$$

反クォークの
フェルミ面なし

弱結合　！

$$\langle qq \rangle \neq 0$$

フェルミ面

$$3 \times 3 \to 3^*$$

引力
無限小の引力でも凝縮

「クォーク」の種類 N_f を多くする

u、d、s、c、b、t、……. 全部ゼロ質量

$$m_u,\ m_d,\ m_s,\ m_c,\ m_b,\ m_t \cdots = 0$$

「ウォーキングテクニカラー」
（スケール不変な複合ヒッグス模型）

テクニディラトン ϕ

NGボソン（スケール不変性の自発的破れ）

山脇－坂東－松本（1986）

$$m_\phi \ll v \qquad m_\sigma \to \infty$$

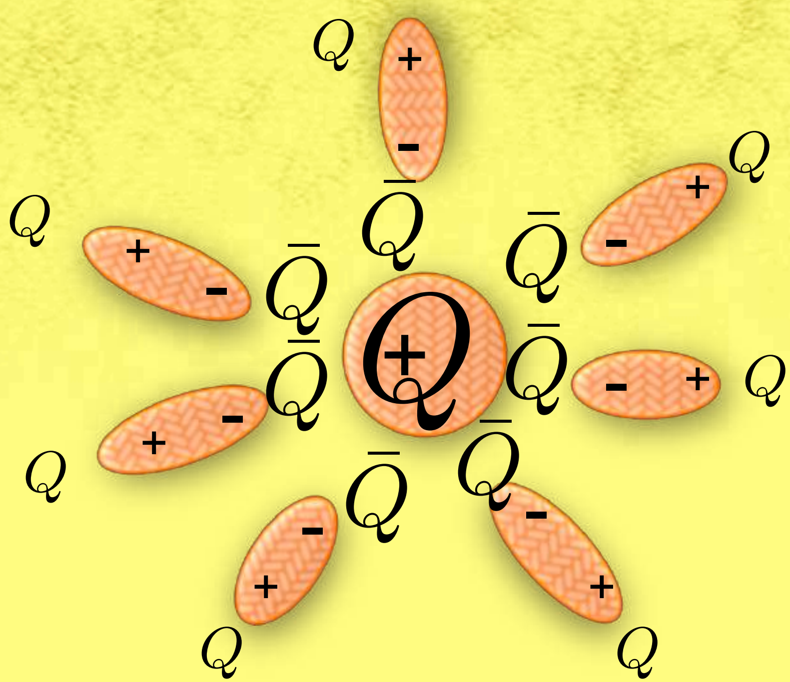
（電荷gの）遮蔽効果

Q クォークもどき
「テクニクォーク」

クォークの数

$N_f \nearrow$

$Q \ \bar{Q}$ ペアの効果増大



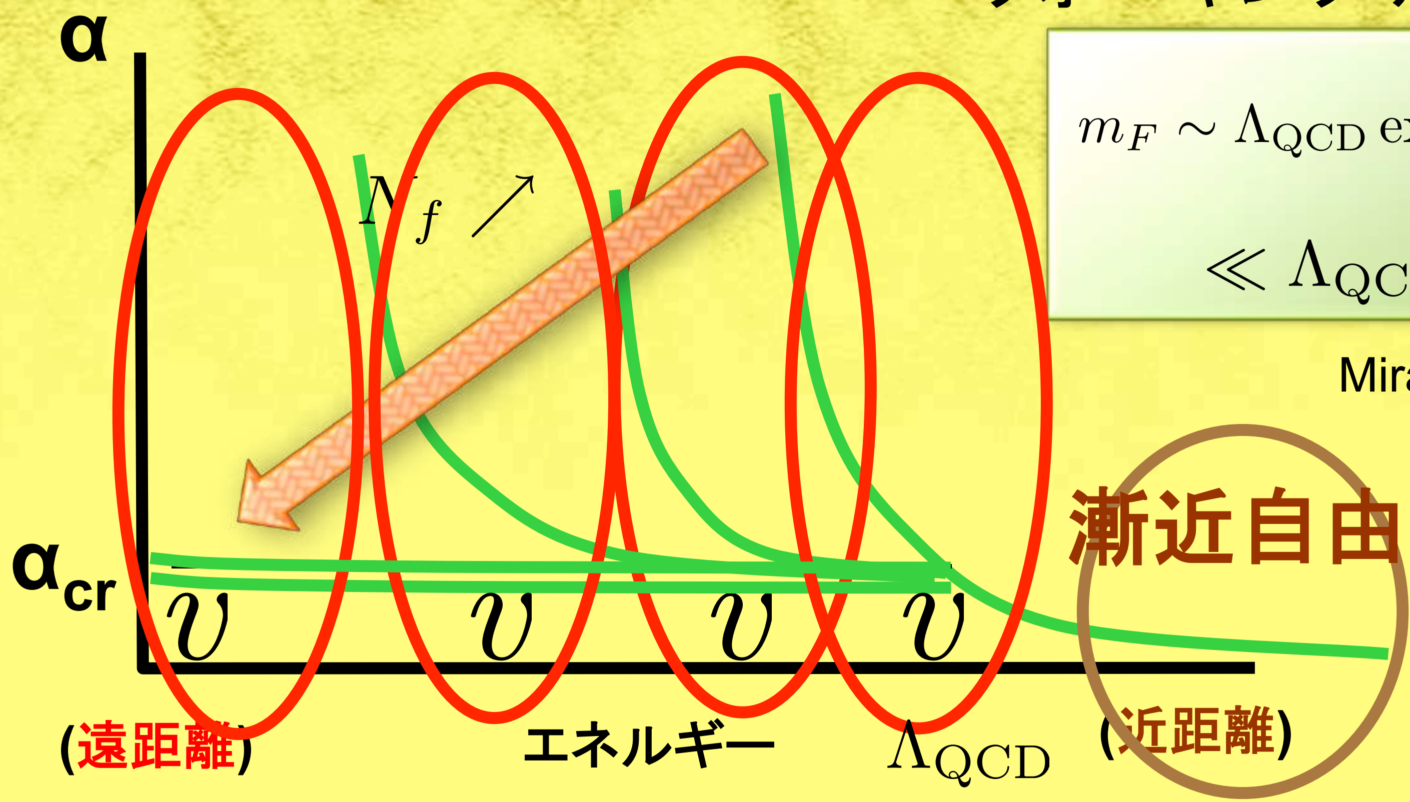
遠距離（低エネルギー）で結合が小さく見える

QCDの結合定数 $N_f \gg N_c$

``ウォーキングテクニカラー''

$$m_F \sim \Lambda_{\rm QCD} \exp\left(-\frac{\pi}{\sqrt{\frac{\alpha}{\alpha_{\rm cr}} - 1}}\right) \ll \Lambda_{\rm QCD} \quad (\alpha \to \alpha_{\rm cr})$$

Miransky スケーリング



格子理論で計算機シミュレーション検証

ペアー凝縮が発生するスケール v $\alpha(\mu^2 < v^2) > \alpha_{\rm cr}$

$$v = 0 \Leftrightarrow \alpha(\forall \mu) < \alpha_{\rm cr} \neq 0$$

質量の起源？

ヒッグスとの結合定数
g_{Yukawa}，g_{gauge}

$$m_{W/Z} \sim g_{\mathrm{gauge}}\, v$$

$$m_{\mathrm{quark/lepton}} \sim g_{\mathrm{Yukawa}}\, v$$

ヒッグス粒子

真空凝縮 v

$v = 246\,\mathrm{GeV}$

より基本的な理論？

ヒッグス粒子は素粒子か？

http://www.sci.nagoya-u.ac.jp/kouhou/15/p3.html

Technicolor: a Scale-Up of QCD

S. Weinberg (1976)
L. Susskind (1979)

$$\text{Composite } \pi \Rightarrow \text{Composite } \pi_{\mathrm{TC}}$$

$v = f_\pi$ $\quad$ $v = F_\pi$ $\quad \longrightarrow m_{W,Z}$

$H \sim \bar{F}F \qquad F_\pi = 246 \text{ GeV}$

$\langle \bar{F}F \rangle \sim (700 \text{ GeV})^3$

$\frac{N_{\mathrm{TC}}}{N_{\mathrm{C}}}$ $\qquad$ $\sqrt{\frac{N_{\mathrm{C}}}{N_{\mathrm{TC}} N_{\mathrm{D}}}}$

X 2600

$\sigma \sim \bar{q}q \qquad f_\pi = 93 \text{ MeV}$

$\langle \bar{q}q \rangle \sim (250 \text{ MeV})^3$

Solution to $M_\phi \ll 10^{19}\,\text{GeV}$ not $M_\phi \ll 4\pi v$

TC was killed 3 times

- FCNC

$$m_{q,l} \ll m_{q,l}^{(\mathrm{exp})}$$

Walking TC

$$\boxed{\gamma_m \simeq 1}$$

- S,T,U parameters

$$S/(N_{\mathrm{TC}} N_D) \sim S_{\mathrm{QCD}}/3 = \mathcal{O}(0.1)$$

$$S^{(\mathrm{exp})} < 0.1$$

(Holographic) Walking TC [and/or ETC effects]

- 125 GeV Higgs

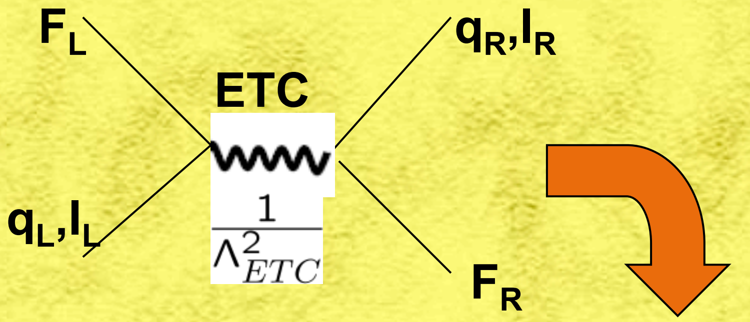
$$125\ \mathrm{GeV} \ll \Lambda_{\mathrm{TC}} = \mathcal{O}(\mathrm{TeV})$$

$$4\pi v$$

Walking TC

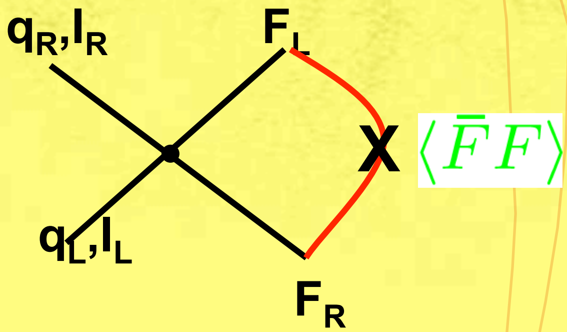
scale inv.

FCNC Problems:



Mass of Quarks/Leptons

$$m_{q/l} \sim \frac{1}{\Lambda_{ETC}^2} \langle \bar{F} F \rangle$$



FCNC

$$\frac{1}{\Lambda_{ETC}^2} \; \bar{s} d \bar{s} d \; < (10^3 \, \text{TeV})^{-2}$$

$$m_s < (10^3 \, \text{TeV})^{-2} \times (0.7 \, \text{TeV})^3 \sim 10^{-1} \, \text{MeV}$$

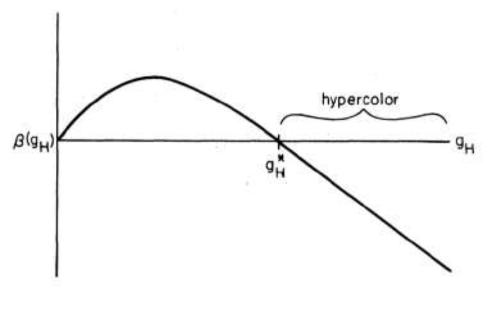
Needs 10^3 enhancement

By Large Anomalous Dimension γ_m

Holdom (1981)

Pure Assumption of
Existence of Large γ_m
No Concrete Dynamics
No Concrete Value γ_m

$$m_{q/l} = \frac{1}{\Lambda_{\rm ETC}^2} \langle \bar{F} F \rangle_{\Lambda_{\rm ETC}}$$



$$\langle \bar{F} F \rangle|_{\Lambda_{ETC}} = Z_m^{-1} \cdot \langle \bar{F} F \rangle|_{\Lambda_{EW}}$$

$$Z_m^{-1} = (\Lambda_{\rm ETC}/\Lambda_{\rm EW})^{\gamma_m} \simeq (10^3)^{\gamma_m}$$

Iff $\gamma_m > 1$ $\Longrightarrow$ $> 10^3$

Walking Technicolor

K.Y., Bando, Matumoto (Dec. 24, 1985)

Ladder Schwinger-Dyson Equation

Maskawa-Nakajima Solution (1974)

Scale Invariance $\Leftarrow$ $(\alpha(p) = \text{constant})$

$\gamma_m = 1$ $\Longleftrightarrow$ FCNC Sol.

Techni-dilaton

Similar FCNC Sol. Without γ_m, Scale Invariance, Techni-dilaton:

Akiba, Yanagida (Jan. 3, 1986)

Appelquist, Karabali, Wijewardhana (June 2, 1986)

(Holdom (Oct. 12, 1984), purely numerical)

Folklore:

~~Technicolor = "Higgsless" Model~~

S. Weinberg (1976)
L. Susskind (1979)

~~(No light scalar)~~

Walking Technicolor KY-Bando-Matumoto (1986)

= Composite Higgs Model

Approx. Scale Symmetry

$\Rightarrow$ **Techni-dilaton**

"Conformal Higgs"

$\Leftrightarrow$ **125 GeV Composite Higgs**

Ladder **Scale-Invariant Hypercolor Model and a Dilaton**

Koichi Yamawaki, Masako Bando,[a] and Ken-iti Matumoto[b]
Department of Physics, Nagoya University, Nagoya 464, Japan
(Received 24 December 1985)

We propose a scale-invariant hypercolor model with a nontrivial ultraviolet fixed point having large anomalous dimension, which resolves the notorious flavor-changing neutral-current problem in hypercolor models, and at the same time predicts a $J^{PC}=0^{++}$ Nambu-Goldstone boson (dilaton) associated with the spontaneous breakdown of the scale invariance.

INSPIRE

```
%\cite{Yamawaki:1985zg}
\bibitem{Yamawaki:1985zg}
  K.~Yamawaki, M.~Bando and K.~-i.~Matumoto,
  %``Scale Invariant Technicolor Model and a Technidilaton,''
  Phys.\ Rev.\ Lett.\  {\bf 56}, 1335 (1986).
  %%CITATION = PRLTA,56,1335;%%
```

$$\gamma^* \equiv \gamma(\alpha^{(HC)} = \alpha_c^{(HC)}) = 1, \tag{10}$$

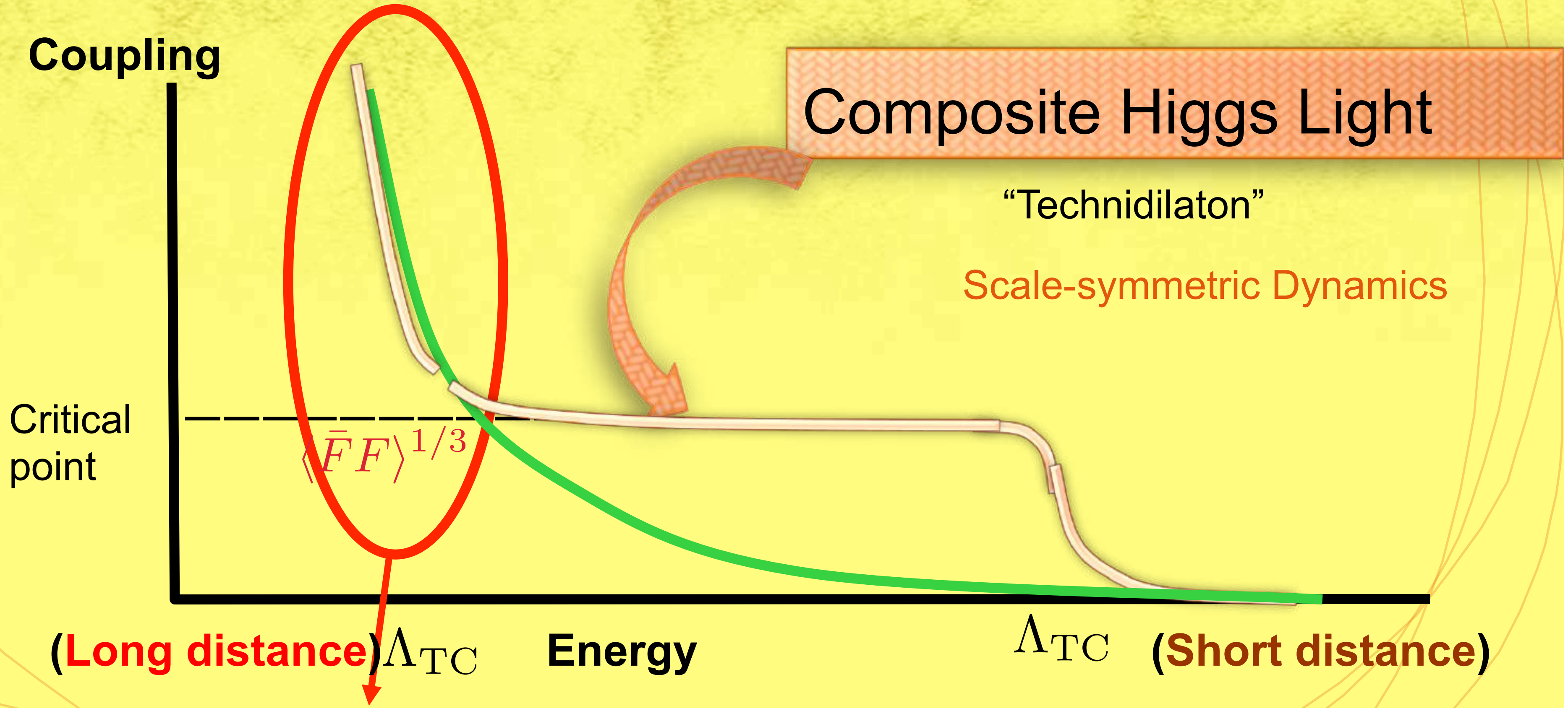
$$\Sigma(p) \sim \Lambda_{HC}^2/p, \quad \Leftrightarrow \Sigma_{\rm QCD}(p) \sim \Lambda_{\rm QCD}^3/p^2 \tag{11}$$

the hyperdilaton is expected to be quite similar to that of the neutral Higgs boson, both of which possess

Walking Technicolor

Technicolor

Strong Coupling ⟺ Condensate



Pair Condensate Generated

$$\langle \bar{F}F \rangle^{1/3} = \mathcal{O}(v) \ll \Lambda_{\rm TC} \ll 10^{19}\,\mathrm{GeV}$$

テクニディラトン質量・崩壊定数：PCDC (はしご近似)

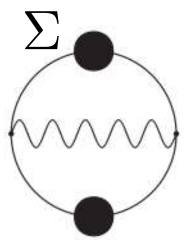
Bando-Matumoto-KY, PLB178 (1986) 308
Hashimoto-KY, PRD83 (2011) 015008
Kurachi-Matsuzaki-KY, PRD90 (2014) 095013; Matsuzaki-KY, JHEP12(2015)053

$$\boxed{M_\phi^2 F_\phi^2} = -F_\phi \langle 0|\partial_\mu D^\mu|\phi\rangle = -4\underline{\langle 0|\theta_\mu^\mu|0\rangle} = -\frac{\underline{\beta(\alpha)}}{\alpha}\underline{\langle G_{\mu\nu}^2\rangle}$$

$$\langle \theta_0^0 \rangle = E = V(\Sigma_{\text{solution}})$$

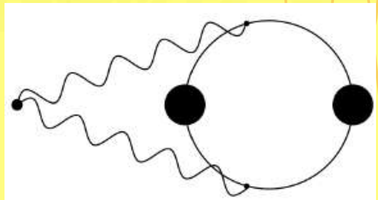
非摂動β関数

$$\Gamma_{\text{CJT}} = -i\,\text{Tr}\,\text{Ln} S_f^{-1} - i\,\text{Tr} S_f S_0^{-1} +$$



$$\Gamma_{\text{CJT}} = -V_{\text{CJT}} d^4 x$$

独立な3つの計算が一致 !!



Miransky-Gusynin (1989)

$$\frac{1}{\ln^3(\frac{4\Lambda}{m_F})} \times \ln^3(\frac{4\Lambda}{m_F})$$

$$\boxed{\simeq 0.154 \cdot N_f N_c \cdot m_F^4}$$

$$\boxed{F_\phi^2 \sim N_f N_c m_F^2}$$

$$\boxed{v_{\text{EW}}^2 = \frac{N_f}{2} F_\pi^2 \simeq 0.028 N_f N_c m_F^2}$$

Pagels-Stokar

$$M_\phi^2 F_\phi^2 \simeq (2.5 \cdot v_{\rm EW}^2)^2 \cdot \left[\frac{8}{N_f}\frac{4}{N_c}\right]$$

$$\frac{M_\phi}{v_{\rm EW}} \cdot \frac{F_\phi}{v_{\rm EW}} \simeq 0.5 \cdot 5 \cdot \sqrt{\frac{8}{N_f}\frac{4}{N_c}}$$

$\mathcal{O}(N_f^0, N_c^0)$

Anti-Veneziano limit
= Walking

$$\xrightarrow[N_c \to \infty]{N_f/N_c = \text{fixed}(\gg 1)} 0$$

Light TD !

One-family model $(N_f = 8\,, N_c = 4)$

Benchmark

軽質量 = 弱結合

$(\propto 1/F_\phi)$

$$M_\phi \simeq 125\,\text{GeV}$$

$$\frac{v_{\rm EW}}{F\phi} \simeq 0.2$$

$\boxed{\text{LHC Higgs のデータと整合 !!}}$

複合ディラトン　ポテンシャル

$$V(\phi) = \frac{m_\phi^2 F_\phi^2}{4}\chi^4\left(\ln\chi - \frac{1}{4}\right)$$

$$\langle\chi\rangle = \langle e^{\frac{\phi}{F_\phi}}\rangle = 1$$

スケール変換 $$\delta_D\chi = \chi + x^\mu\partial_\mu\chi$$

$$\langle\theta_\mu^\mu\rangle = \langle\partial^\mu D_\mu\rangle = -\delta_D V = -\frac{m_\phi^2 F_\phi^2}{4}\langle\chi\rangle = -\frac{m_\phi^2 F_\phi^2}{4}$$ PCDCと一致

$$V(\phi) = -\frac{M_\phi^2 F_\phi^2}{16} + \frac{1}{2}M_\phi^2\phi^2 + \frac{4}{3}\frac{M_\phi^2}{F_\phi}\phi^3 + 2\frac{M_\phi^2}{F_\phi^2}\phi^4 + \cdots .$$

$$\left.\frac{g_{\phi^3}}{g_{h^3_{\rm SM}}}\right|_{M_\phi=m_h} = \left.\frac{\frac{4M_\phi^2}{3F_\phi}}{\frac{m_h^2}{2v_{\rm EW}}}\right|_{M_\phi=m_h} \simeq \frac{8}{3}\left(\frac{v_{\rm EW}}{F_\phi}\right) \simeq 0.5\,,$$

$$\left.\frac{g_{\phi^4}}{g_{h^4_{\rm SM}}}\right|_{M_\phi=m_h} = \left.\frac{\frac{2M_\phi^2}{F_\phi^2}}{\frac{m_h^2}{8v_{\rm EW}^2}}\right|_{M_\phi=m_h} = 16\left(\frac{v_{\rm EW}}{F_\phi}\right)^2 \simeq 0.6$$

複合ディラトンは
弱結合！！

S. Matsuzaki-KY,
JHEP 1512 (2015) 053

Discovering the Walking Technicolor at the LHC

1. 125 GeV Higgs as a Technidilaton at LHC Run I
   Testing Technidilaton at Run II
   (Precise measurements)
2. Searching Technipions at LHC Run II
   (Discovery)
3. Searching Techirho at LHC Run II (Discovery)

Benchmark model: One family model

$$N_{\mathrm{TC}} = 3, 4, 5$$

TF_{EW}	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L$	3	2	1/6
$L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L$	1	2	-1/2
U_R	3	1	2/3
D_R	3	1	-1/3
N_R	1	1	0
E_R	1	1	-1

$$\boxed{N_f = 2N_D = 8}$$

$$\langle \bar{Q}_i^c Q_i^c \rangle = \langle \bar{L}_i L_i \rangle \neq 0$$

$c = R, G, B$

$i = u, d$

$$G/H = SU(8)_L \times SU(8)_R / SU(8)_V$$

The signal strength fit to the LHC-Run I full data

Matsuzaki-KY, PLB 719(2013)378;
JHEP 1512 (2015) 053

One-parameter fit (Fφ)

$$M_\phi^2 \simeq \left(\frac{v_{\rm EW}}{2}\right)^2 \cdot \left(\frac{5 v_{\rm EW}}{F_\phi}\right)^2 \cdot \left(\frac{8}{N_{\rm TF}} \frac{4}{N_{\rm TC}}\right)$$

$$\chi^2 = \sum_{i \in \text{events}} \left(\frac{\mu_i - \mu_i^{\rm exp}}{\sigma_i}\right)^2$$

N_{TC}	$[v_{EW}/F_\Phi]$best	χ^2 min /d.o.f.
4	0.23	16/17 = 0.92

Compared w/ SM Higgs χ^2/d.o.f = 8/18 = 0.44

- best-fit v_{EW}/F_Φ ~ 0.2: F_Φ ~ 5 v_{EW}
  agreement w/ ladder PCDC for 1FM w/ N_{TC}=4 !!

N_C	$[v_{\rm EW}/F_\phi]_{\rm best}$	$\chi^2_{\rm min}$/d.o.f.
3	0.27	$25/17 \simeq 1.5$
4	0.23	$16/17 \simeq 0.92$
5	0.17	$32/17 \simeq 2.0$
0 [SM Higgs]	1	$8.0/18 \simeq 0.44$

Theoretical Issues

- Walking Dynamics beyond Ladder/Holography ?
- More Precise Quantitative Predictions?

$$F_\pi,\ F_\phi,\ M_\phi,\ M_\rho, M_{a_1}, M_{\text{baryon}},\ etc.$$
$$S, T, U - \text{Parameters}$$

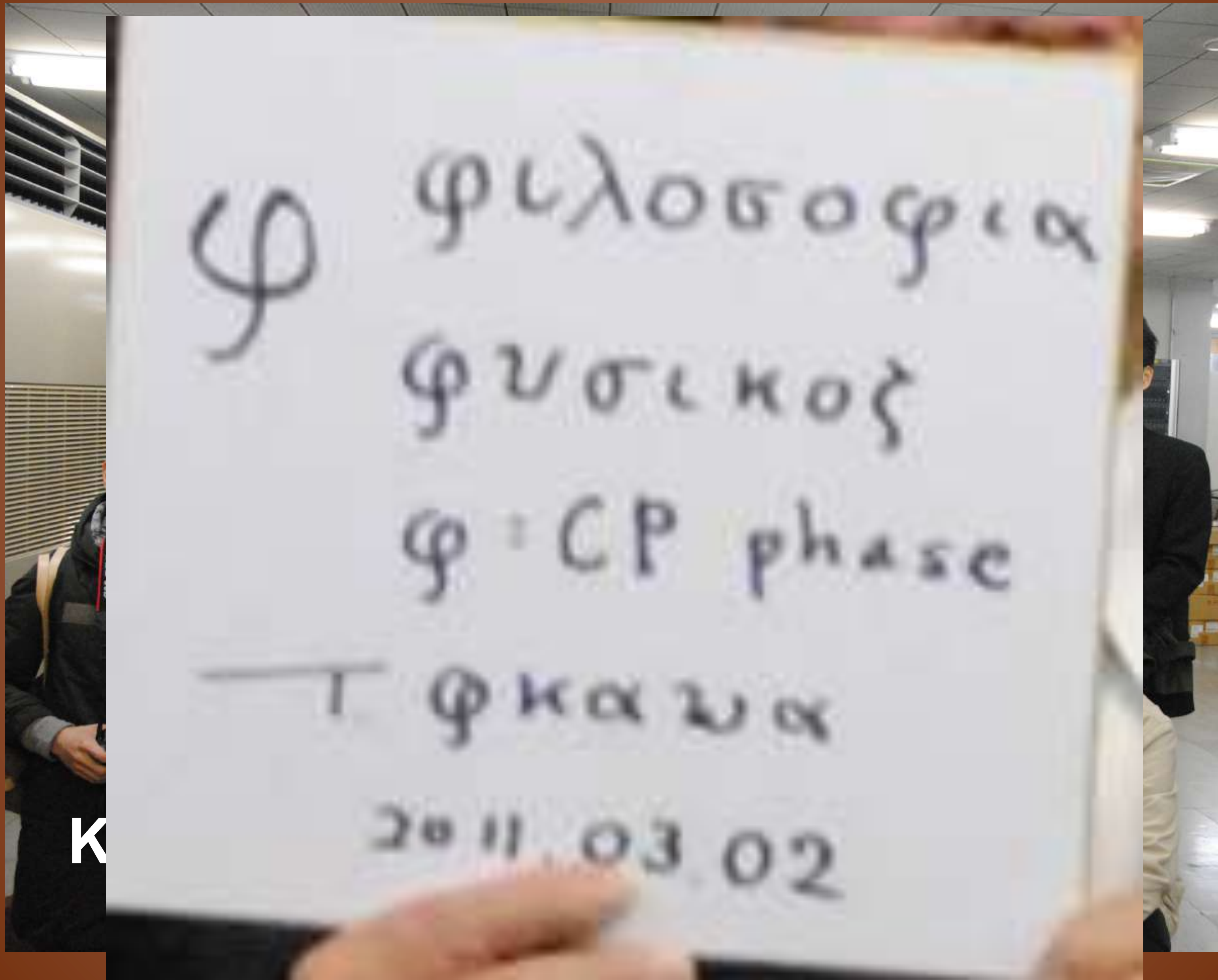
Lattice !

Discovering Walking Technicolor on the Lattice

LatKMI Collaboration

- Finding a candidate for WTC on the Lattice
- Finding a light scalar composite on the Lattice
- Calculating the composite spectra on the Lattice

K



LatKMI collaboration members

Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

Y. Aoki T. Aoyama M. Kurachi T. Maskawa K. Nagai K. Yamawaki T. Yamazaki H. Ohki K. Miura









名古屋大学

理化学研究所 計算科学研究センター R-CCS RIKEN Center for Computational Science

Keio University

奈良女子大学 Nara Women's University

筑波大学 University of Tsukuba

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

E. Rinaldi

The University of Edinburgh

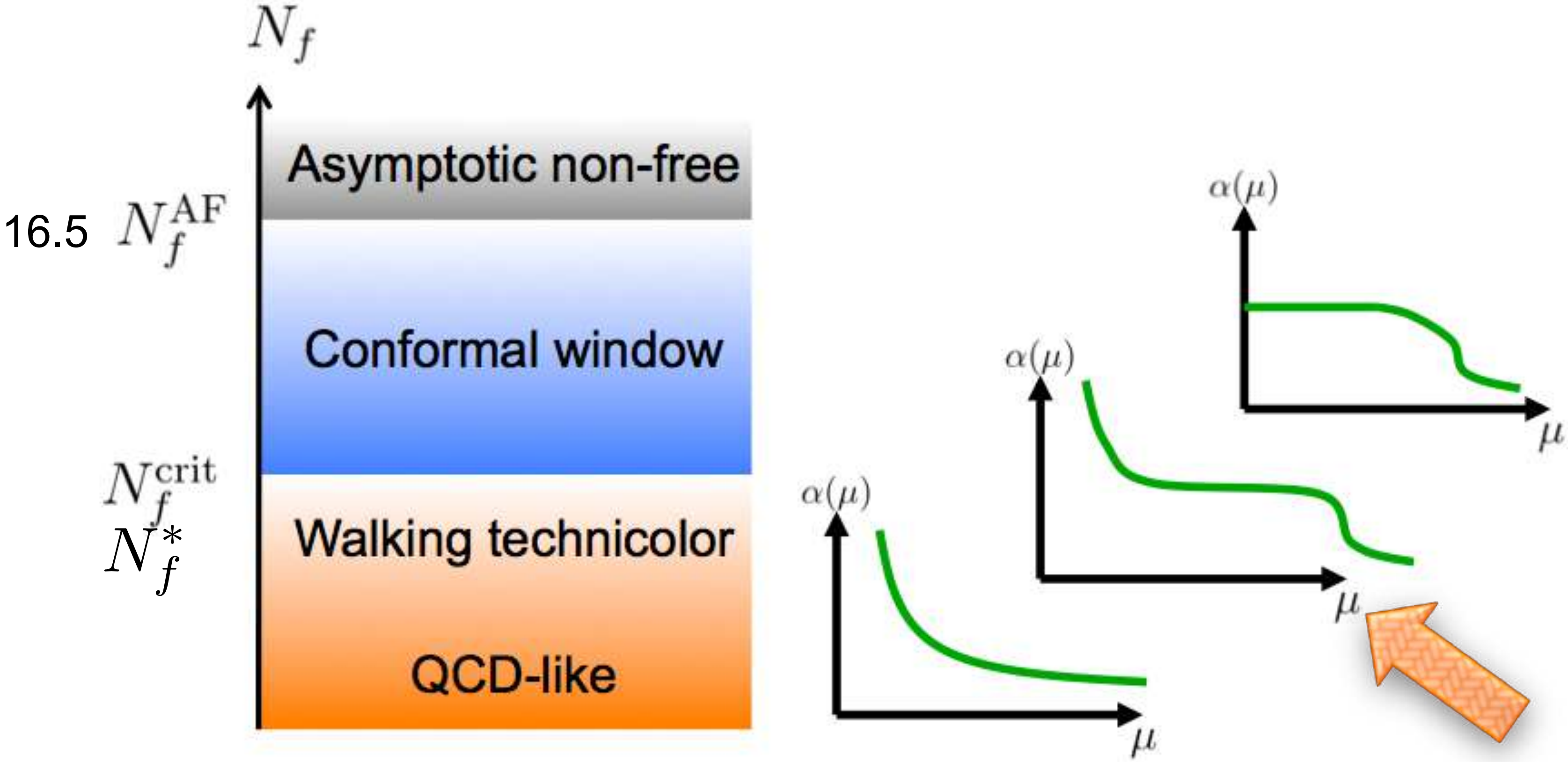
A. Shibata

Ed Bennett

MICHIGAN STATE UNIVERSITY 理化学研究所 KEK Swansea University

$$SU(3);\ N_c = 3$$

$$N_f = 4, 8, 12, 16 (< N_f^{\mathrm{AF}} = 11N_c/2 = 16.5)$$



$$2-\mathrm{loop}:\ \ N_f^* = 8.05$$ (would-be IRFP)

$$2-\mathrm{loop} +\ \mathrm{ladder\,SD\,equation}:\ N_f^{\mathrm{crit}} = 11.9$$

Walking candidate & Light Scalar

- Nf=12 : Conformal $\gamma_m = 0.4 - 0.5$

  LatKMI Collaboration, Phys. Rev. D86, 054506 (2012)
  consistent with many other groups

- Nf=8: Walking $\gamma_m =$ 0.97

  LatKMI Collaboration,Phys. Rev. D 87, 094511 (2013); D96 (2017) 014508
  confirmed by LSD,PRD90(2014)114502; Hasenfratz et al JHEP1506(2015)143)

- Light flavor-singlet scalar (& scalar glueball)

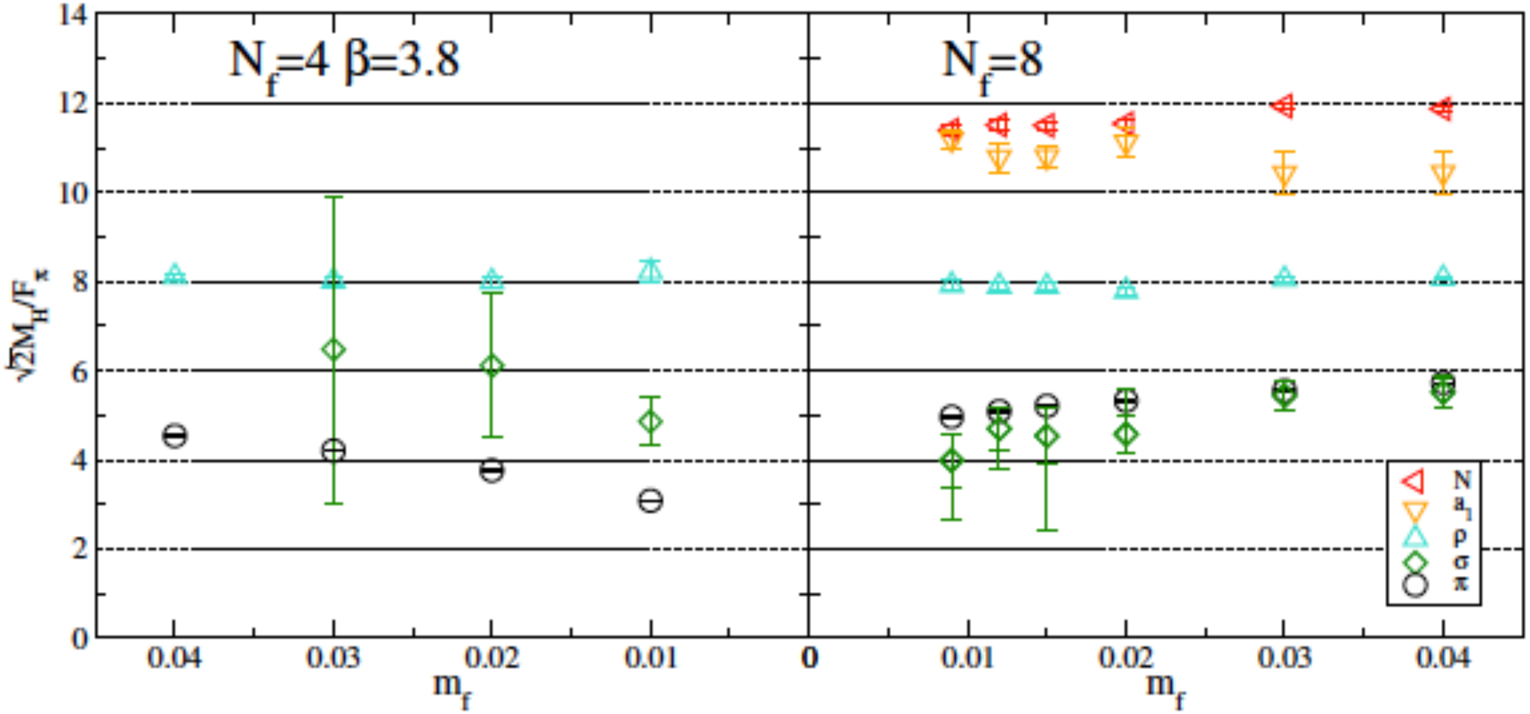
  in Nf=12 $M_\sigma < M_\pi$

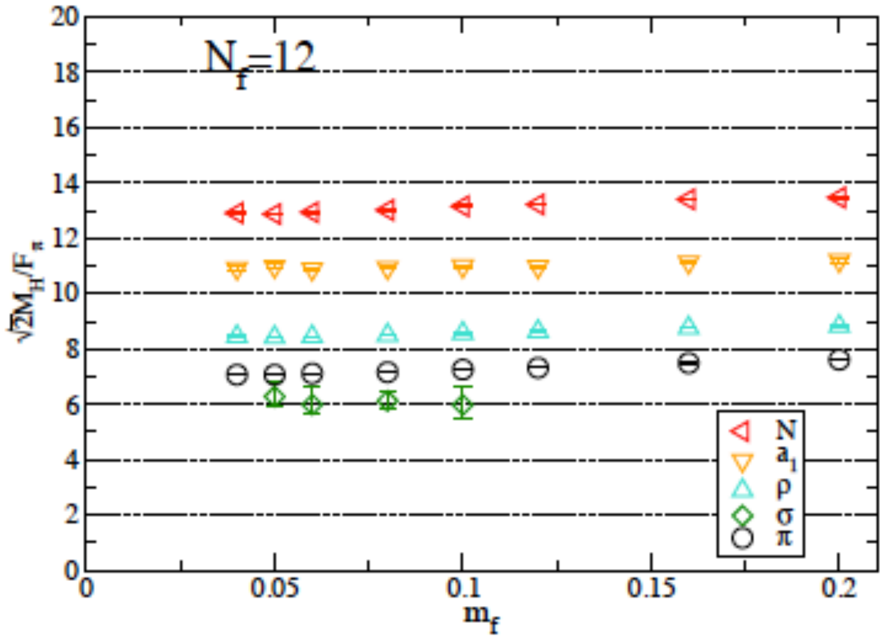
  LatKMI Collaboration, Phys. Rev. Lett. 111 (2013) 162001
  confirmed by LHC Coll (1401.2176); Brower et al (1411.3243)

- Light flavor-singlet scalar $M_\sigma \sim M_\pi$

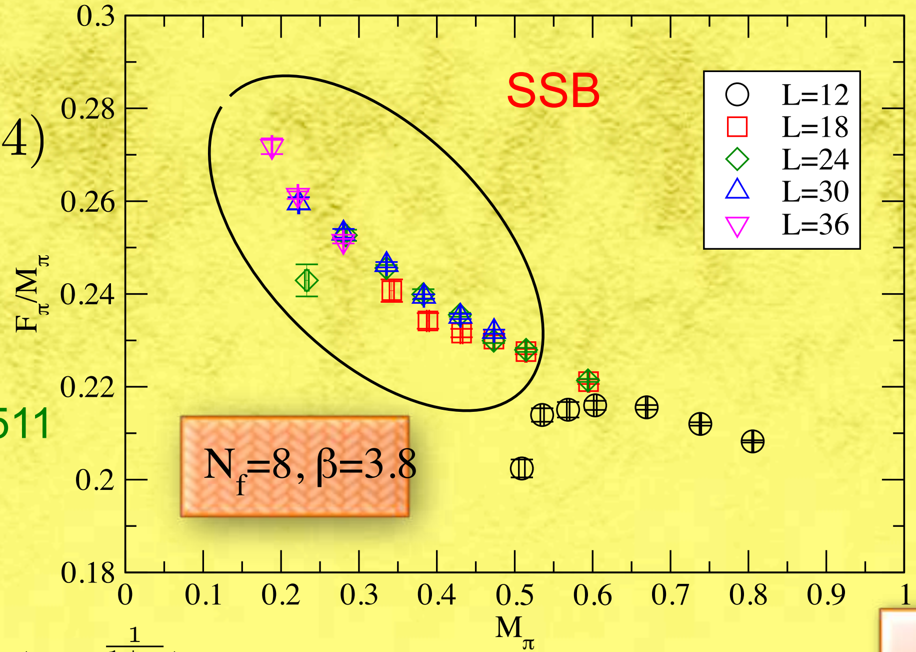
  in Nf=8

  LatKMI Collaboration, Phys. Rev. D 89 (2014) 111502(R)
  PR D96 (2017) 014508
  Confirmed by LSD (PRD93(2016)114514; 1807.08411)


N_f=4 β=3.8
N_f=8
$\sqrt{2}M_H/F_\pi$
m_f
N
a_1
ρ
σ
π


N_f=12
$\sqrt{2}M_H/F_\pi$
m_f
N
a_1
ρ
σ
π

$$(m_f = 0.015 - 0.04)$$



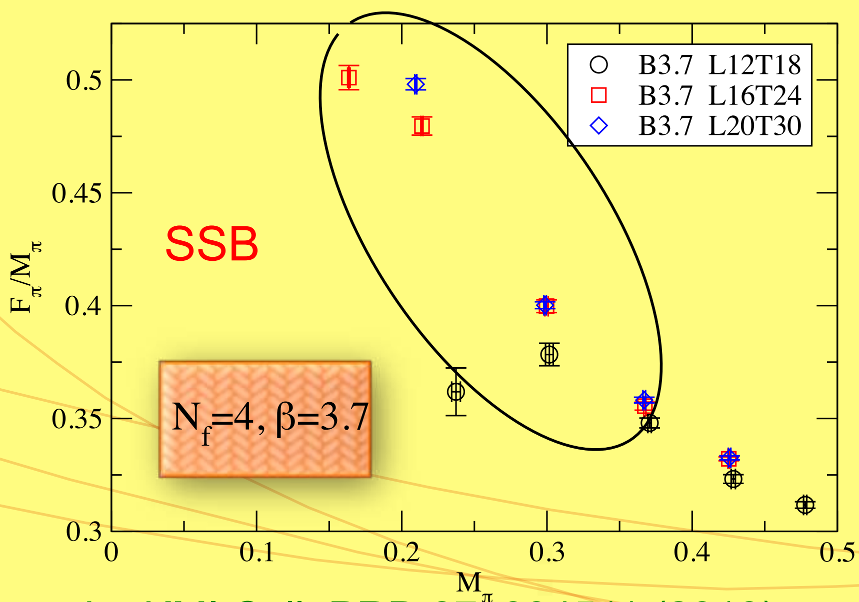
$$\xi_H \equiv LM_H = f_H(Lm_f^{\frac{1}{1+\gamma}})$$

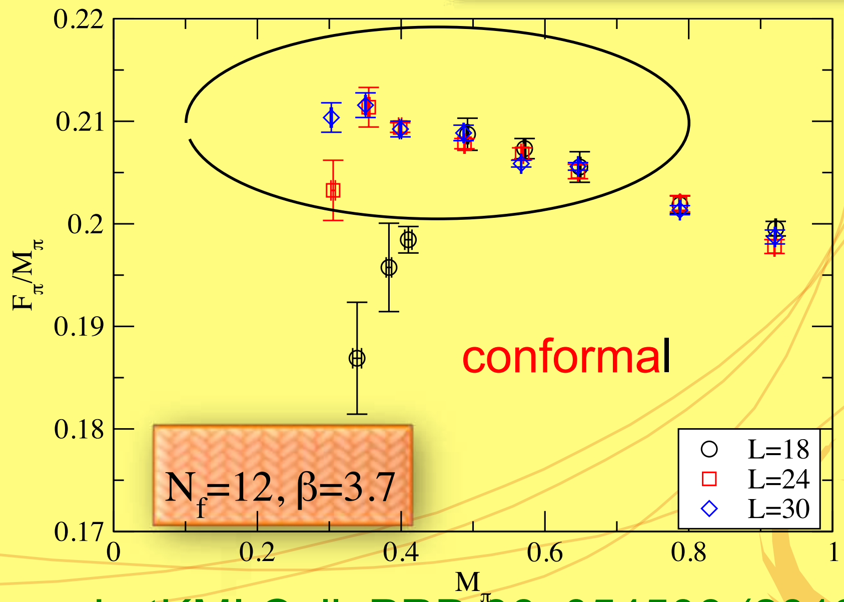
$$(m_f = 0.05 - 0.16)$$

LatKMI Coll, PRD 87, 094511 (2013)

$$\xi_H \equiv LM_H = f_H(Lm_f^{\frac{1}{1+\gamma}})$$

Similarly for M_ρ/M_π



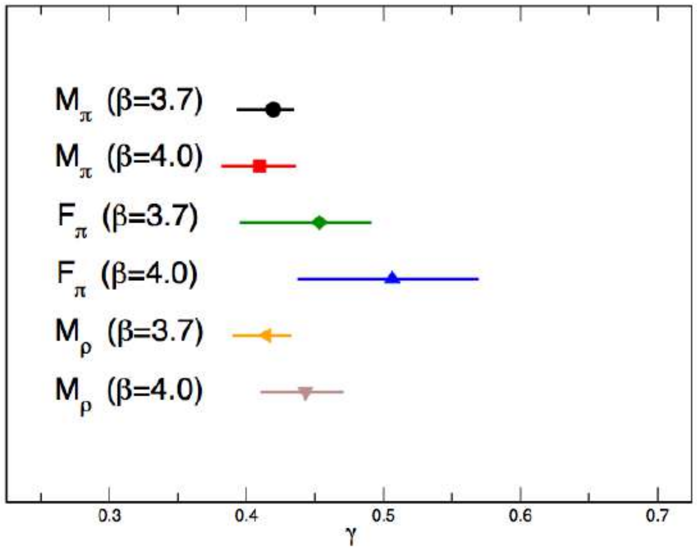


LatKMI Coll, PRD 87, 094511 (2013)

LatKMI Coll, PRD 86, 054506 (2012)

Nf=8 vs Nf=12

$$M_H \sim m_R = Z_m^{-1} \cdot m_0,\ Z_m^{-1} = (\Lambda/m_R)^{\gamma_m}$$
$$M_H \sim m_0^{1/(1+\gamma_m)}$$



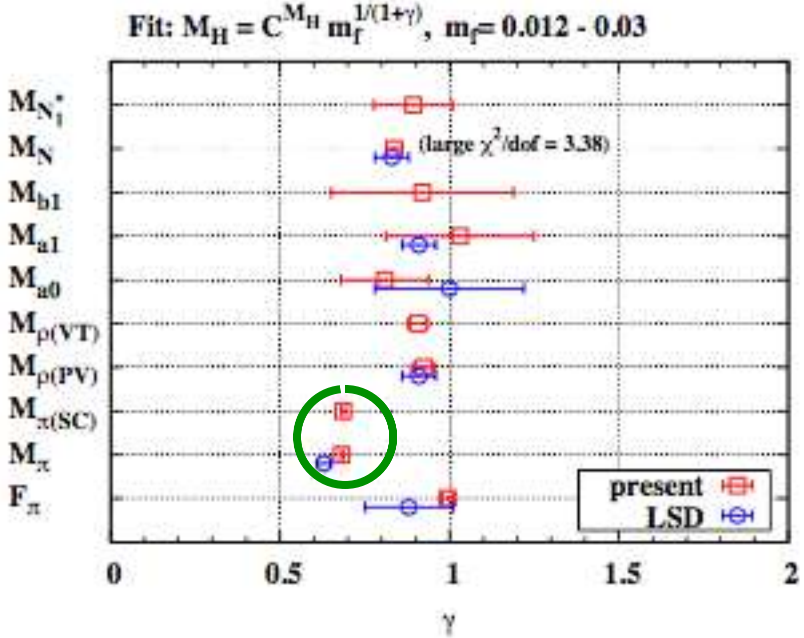


FIG. 24: γ obtained from the Individual fit of the finite size hyper scaling without corrections. Left: $N_f = 12$, Right: $N_f = 8$. (Lattice 2014 poster, proceedings, for $N_f = 12$)

Universal ~ conformal

$$\gamma_m \simeq 0.4 - 0.5$$

Non-universal

$$\gamma_m \simeq 1$$

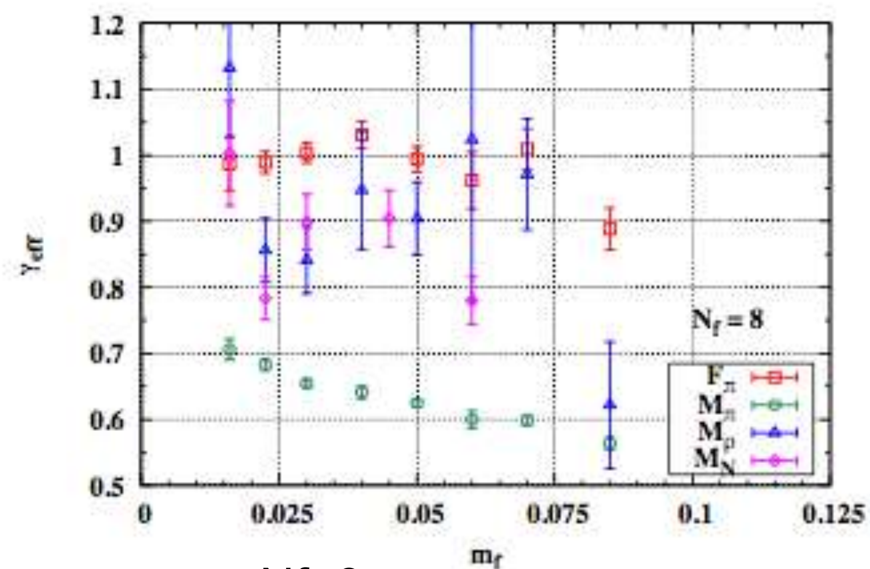
Walking characteristic for Nf=8

$$M \sim m_D + m_f^{\frac{1}{1+\gamma}}$$

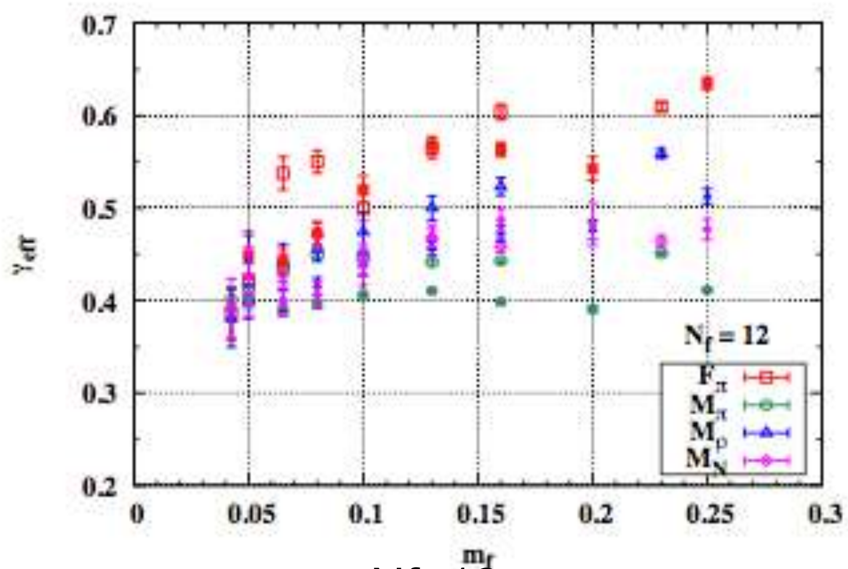
$$\gamma \sim \infty \,(m_f \to 0)$$
$$\gamma \sim 1 \,(m_f \gg m_D)$$

$$M_\pi^2 \sim m_f m_D + m_f^2$$

$$\gamma \sim 1 \,(m_f \to 0)$$
$$\gamma \sim 0.5 \,(m_f \gg m_D)$$



Nf=8



Nf=12

$N_f = 8$ LatKMI Collaboration Phys. Rev. D 89 (2014) 111502(R) Scalar

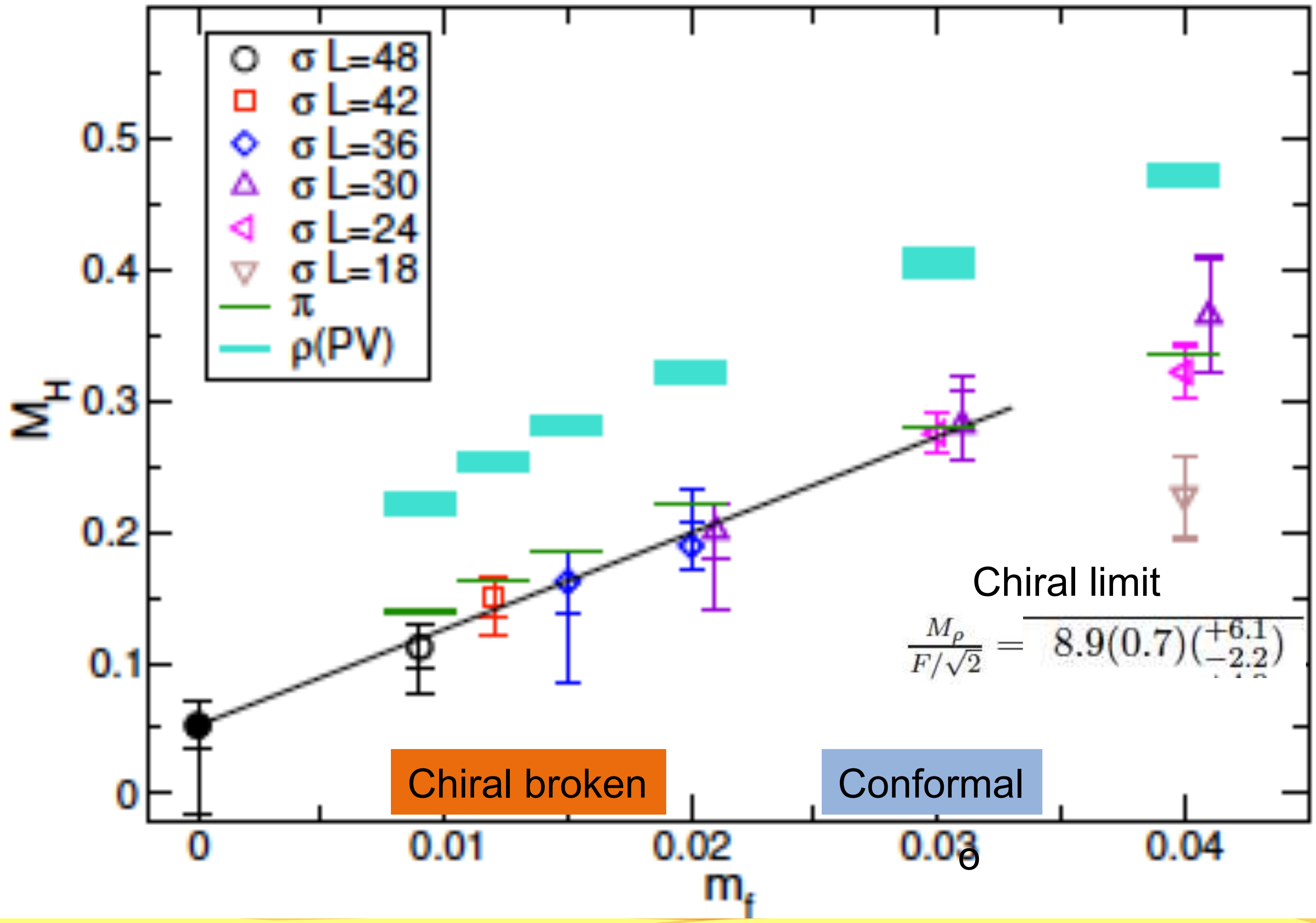
m_f	$L^3 \times T$	$N_{\rm cf}[N_{\rm st}]$	m_σ	m_π	F_π
0.015	$36^3 \times 48$	3200[2]	$0.155(21)\left(^{0}_{41}\right)$	$0.1861(4)^*$	$0.0503(2)^*$
0.02	$36^3 \times 48$	5000[1]	$0.190(17)\left(^{39}_{0}\right)$	$0.2205(3)^*$	$0.0585(1)^*$
0.02	$30^3 \times 40$	8000[1]	$0.201(21)\left(^{0}_{60}\right)$	0.2227(9)	0.0578(2)
0.03	$30^3 \times 40$	16500[1]	$0.282(27)\left(^{24}_{0}\right)$	$0.2812(2)^*$	$0.07140(9)^*$
0.03	$24^3 \times 32$	36000[2]	$0.276(15)\left(^{6}_{0}\right)$	0.2832(14)	0.0715(4)
0.04	$30^3 \times 40$	12900[3]	$0.365(43)\left(^{17}_{0}\right)$	$0.3349(3)^*$	$0.0826(1)^*$
0.04	$24^3 \times 32$	50000[2]	$0.322(19)\left(^{8}_{0}\right)$	0.3353(7)	0.0823(2)
0.04	$18^3 \times 24$	9000[1]	$0.228(30)\left(^{0}_{16}\right)$	0.3421(29)	0.0823(5)
0.06	$24^3 \times 32$	18000[1]	$0.46(7)\left(^{12}_{0}\right)$	0.4295(6)	0.1012(3)
0.06	$18^3 \times 24$	9000[1]	$0.386(77)\left(^{12}_{0}\right)$	0.4317(15)	0.0999(5)

TABLE I: Simulation parameters for $N_f = 8$ QCD at $\beta = 3.8$. $N_{\rm cf}(N_{\rm st})$ is the total number of gauge configurations (Markov chain streams). The second error of m_σ is a systematic error coming from the fit range. The values for m_π and F_π are from Ref. [10], but the ones with (*) have been updated.

$N_f = 8$

Scalar

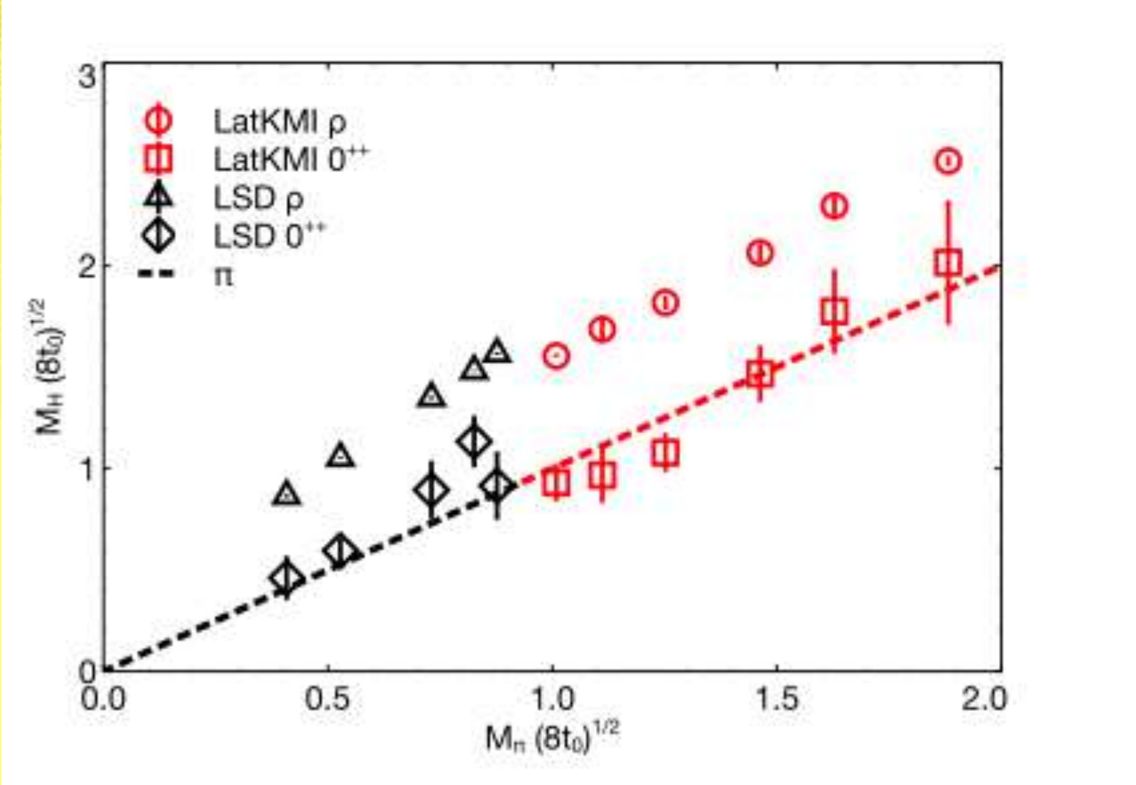
LatKMI Collaboration Phys. Rev. D 89 (2014) 111502(R)
D96 (2017) 014508,
and in preparation



$N_f = 8$

LatKMI vs LSD (Appelquist et al)

LatKMI, arXiv:1710.06549
(EPJ Web Conf. 175(2018) 0823)



LatKMI Collaboration, Phys. Rev. D 89 (2014) 111502(R)
D96 (2017) 014508
LSD Collaboration: PRD93(2016)114514; arXiv:1807.08411

$N_f = 12$ Scalar

PRL 111, 162001 (2013) PHYSICAL REVIEW LETTERS week ending 18 OCTOBER 2013

Light Composite Scalar in Twelve-Flavor QCD on the Lattice

Yasumichi Aoki,[1] Tatsumi Aoyama,[1] Masafumi Kurachi,[1] Toshihide Maskawa,[1] Kei-ichi Nagai,[1] Hiroshi Ohki,[1] Enrico Rinaldi,[1,2] Akihiro Shibata,[3] Koichi Yamawaki,[1] and Takeshi Yamazaki[1]

(LatKMI Collaboration)

Nf=12, β=4. 0

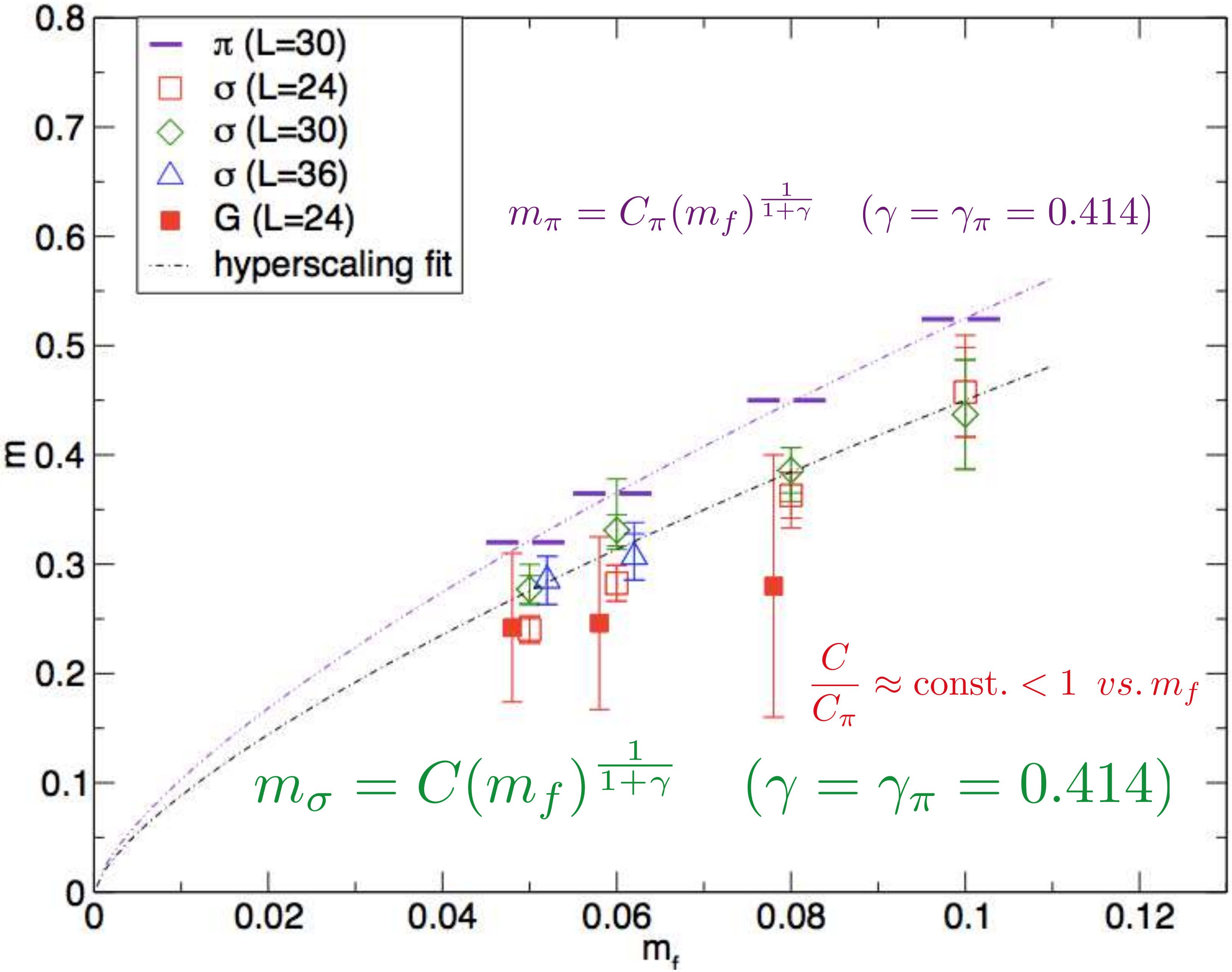
Noise reduction method with Nr=64

$L^3 \times T$	m_f	N_{cfgs}	m_σ	m_π	m_σ/m_π
$24^3 \times 32$	0.05	8800	$0.250(15)\binom{00}{01}$	$0.3273(19)^*$	$0.76(5)\binom{0}{0}$
$24^3 \times 32$	0.06	14000	$0.283(16)\binom{04}{01}$	$0.3646(16)^*$	$0.78(4)\binom{1}{0}$
$24^3 \times 32$	0.08	15000	$0.363(21)\binom{02}{22}$	$0.4459(11)$	$0.81(5)\binom{0}{5}$
$24^3 \times 32$	0.10	9000	$0.458(41)\binom{32}{06}$	$0.5210(7)$	$0.88(8)\binom{6}{1}$
$30^3 \times 40$	0.05	8000	$0.284(15)\binom{24}{09}$	$0.3201(16)^*$	$0.89(5)\binom{7}{3}$
$30^3 \times 40$	0.06	14000	$0.337(15)\binom{51}{12}$	$0.3648(9)^*$	$0.92(4)\binom{14}{3}$
$30^3 \times 40$	0.08	15000	$0.386(21)\binom{00}{20}$	$0.4499(8)$	$0.86(5)\binom{0}{4}$
$30^3 \times 40$	0.10	4000	$0.437(50)\binom{07}{09}$	$0.5243(7)$	$0.83(9)\binom{1}{2}$
$36^3 \times 48$	0.05	5000	$0.285(22)\binom{00}{03}$	$0.3204(7)^*$	$0.89(7)\binom{0}{1}$
$36^3 \times 48$	0.06	6000	$0.307(21)\binom{23}{04}$	$0.3636(9)^*$	$0.84(6)\binom{6}{1}$

$N_f = 12$

LatKMI Collaboration, PRL 111 (2013) 162001

Scalar



$$m_\pi = C_\pi (m_f)^{\frac{1}{1+\gamma}} \quad (\gamma = \gamma_\pi = 0.414)$$

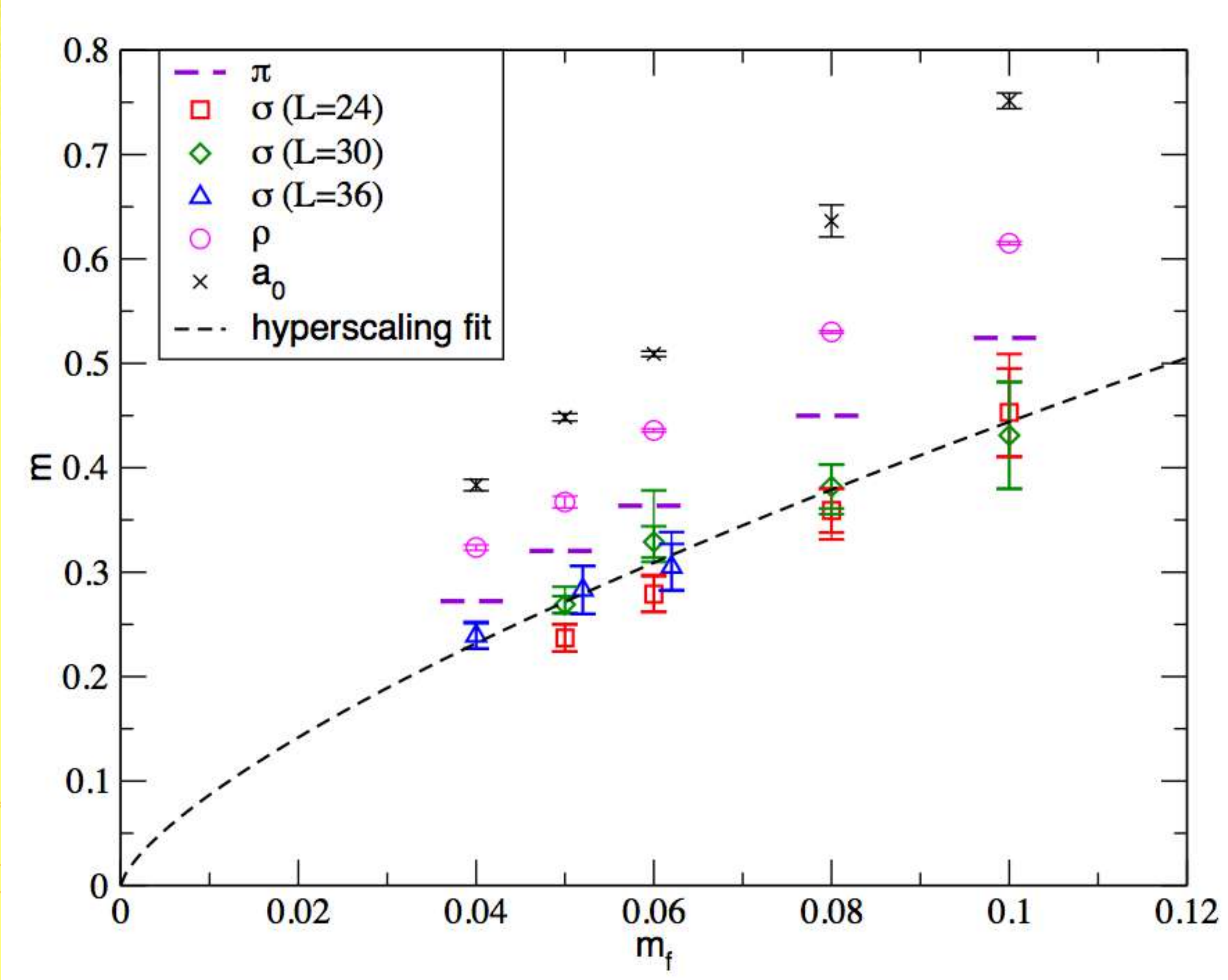
$$\frac{C}{C_\pi} \approx \text{const.} < 1 \ \ vs. \, m_f$$

$$m_\sigma = C (m_f)^{\frac{1}{1+\gamma}} \quad (\gamma = \gamma_\pi = 0.414)$$

$N_f = 12$

LatKMI Collaboration, PRL 111 (2013) 162001

Scalar



Scalar

$$\theta^\mu_\mu|^{\rm NP}_{m_f\neq 0} = \frac{\beta_{\rm NP}(\alpha)G^2_{\mu\nu}}{4\alpha} + (1+\gamma_m)N_f m_f \bar\psi\psi$$

W-T identity

$$\langle 0|\mathcal{O}|\phi\rangle = \frac{d_{\mathcal{O}}}{F_\phi}\langle\mathcal{O}\rangle$$

$F_\phi M^2_\phi|_{m_f\neq 0}$ $\quad$ $F_\phi M^2_\phi$ $\quad$ $d_{\bar\psi\psi}(1+\gamma_m)N_f m_f\langle\bar\psi\psi\rangle$

$\langle|\partial_\mu D^\mu|\phi\rangle$

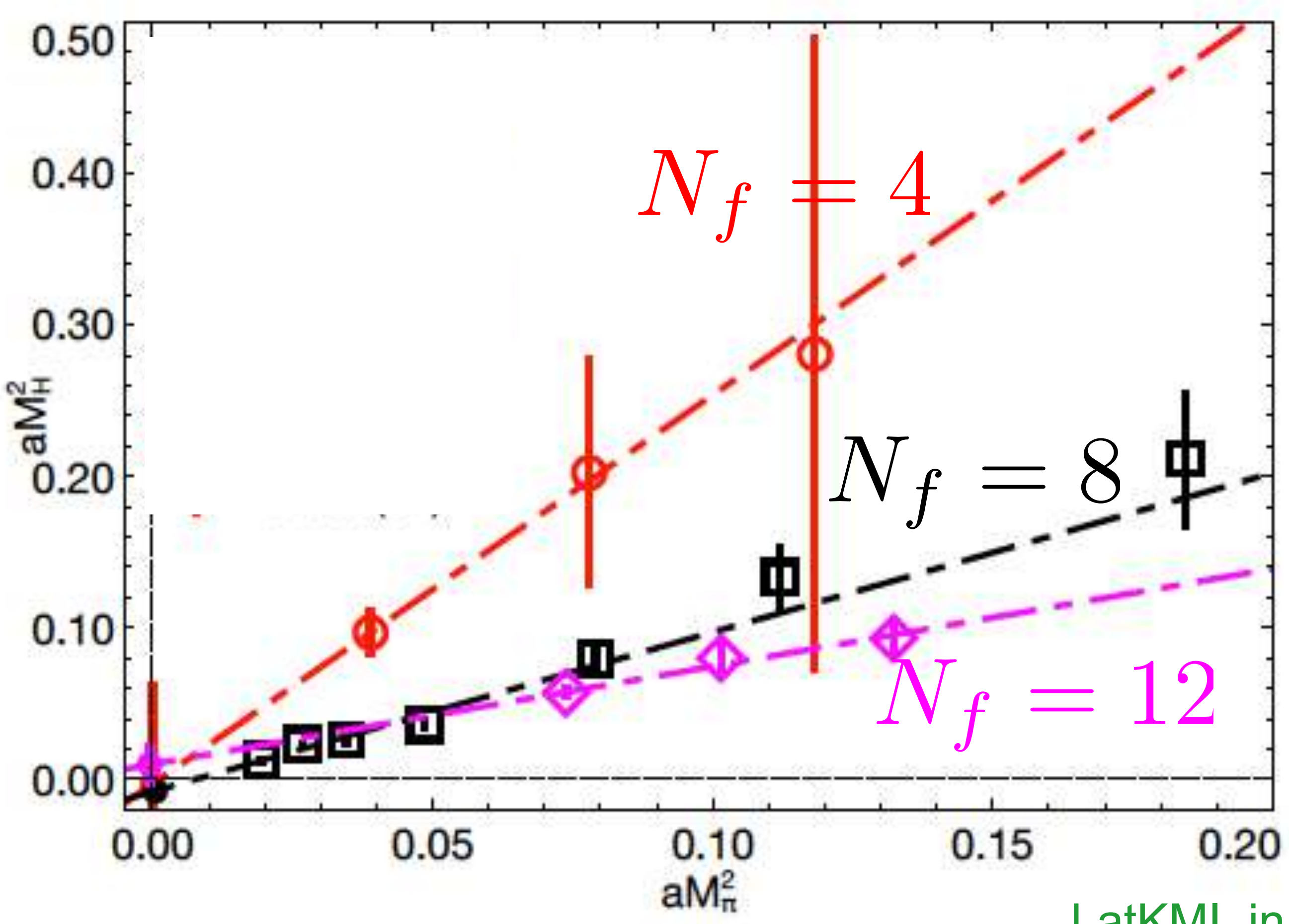
$$m_f\langle\bar\psi\psi\rangle = F^2_\pi m^2_\pi/2$$

$$M^2_\phi|_{m_f\neq 0} = M^2_\phi + \frac{(3-\gamma_m)(1+\gamma_m)}{4}\cdot\frac{2N_f F^2_\pi}{F^2_\phi}\cdot m^2_\pi$$

Lattice observables

Matsuzaki-KY, PRL 113 (2014) 082002

$$M_\sigma^2 = M_\sigma^{2(anomaly)} + \frac{(3-\gamma_m)\cdot(1+\gamma_m)(N_f/2)(F_\pi/\sqrt{2})^2}{F_\sigma^2}\cdot M_\pi^2 \equiv d_0 + d_1\cdot M_\pi^2,$$



LatKMI, in preparation

$$d_0 = -0.005(96)(N_f=4),\ -0.0063(51)\binom{84}{134}(N_f=8), 0.002(11)\binom{8}{4}(N_f=12),$$
$$d_1 = 2.6(2.3)(N_f=4),\ 0.99(14)\binom{50}{32}(N_f=8),\ 0.70(11)\binom{8}{7}(N_f=12).$$

$$M_\sigma^2 \simeq d_1 M_\pi^2 (\gg d_0) \qquad \frac{F_\sigma}{F_\pi/\sqrt{2}} = \sqrt{\frac{(1+\gamma_m)(3-\gamma_m)}{d_1}\frac{N_f}{2}}$$

Induced 4-fermi ?

N_f=4 $\sim 3\, M_\pi^2$ $\simeq 1.5\ (\gamma_m \sim 2\,?)$

Nf=8 $\sim M_\pi^2$ $\simeq 4\ (\gamma_m \simeq 1)$

Nf=12 $\sim 0.7\, M_\pi^2$ $\simeq 5.6\ (\gamma_m \simeq 0.4 - 0.5)$

$F_\sigma^2 = (3-\gamma_m)^2 (N_f/2)(F_\pi/\sqrt{2})^2$ (linear sigma 模型)

$\Rightarrow$ $d_1 = (1+\gamma_m)/(3-\gamma_m)$

$\simeq 1\,(\gamma_m \simeq 1) \quad \simeq 0.6\,(\gamma_m \simeq 0.4 - 0.5)$

N_f=2 $d_1 = 3\,(\gamma_m = 2) \quad F_\sigma = F_\pi/\sqrt{2}$

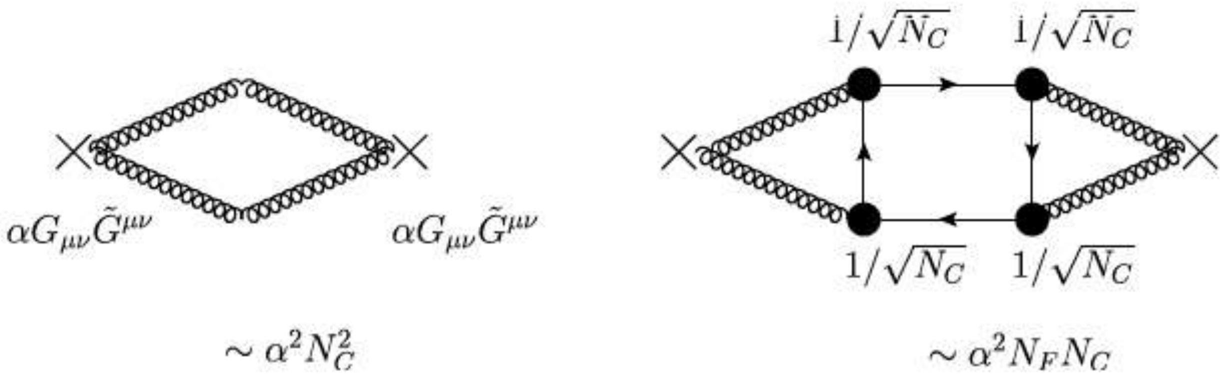
Induced 4-fermi ? Gauged NJL ?

$$\eta'$$

Maskawa-Nakajimaのmotivation

$$\partial^\mu A^0_\mu(x) = 2N_f \frac{\alpha}{8\pi} G^{\mu\nu}\tilde{G}_{\mu\nu}(x) + 2m_f \sum_{i=1}^{N_f} \bar{\psi}_i i\gamma_5 \psi_i,$$

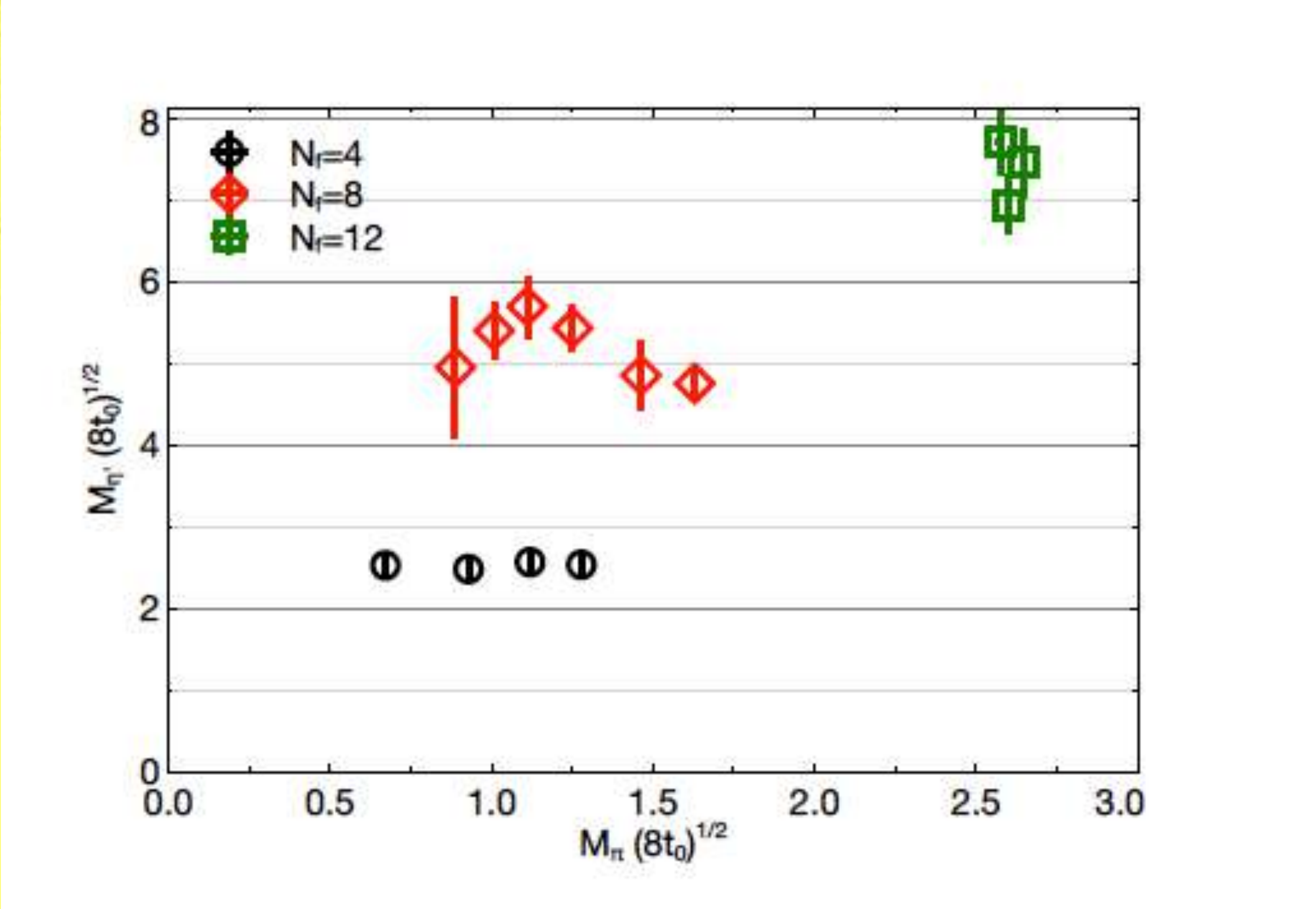
$$(M^2_{\eta'})^{(anomalous)} = M^2_{\eta'} - M^2_\pi = \frac{1}{2N_f f^2_\pi} \mathcal{F.T.} \left\langle T\left(2N_f \frac{\alpha}{8\pi} G^{\mu\nu}\tilde{G}_{\mu\nu}(x) \cdot 2N_f \frac{\alpha}{8\pi} G^{\mu\nu}\tilde{G}_{\mu\nu}(0)\right)\right\rangle \Big|_{q_\mu \to 0}$$



$N_c \to \infty$ with fixed $N_c\alpha$ and N_f/N_c, $\quad N_f/N_c \gg 1$

$$(M^2_{\eta'})^{(anomalous)} \sim \frac{N_f \alpha^2}{f^2_\pi} (N_c^3 N_f \alpha^2 \Lambda^2_{\rm IR}) \sim \left(\frac{N_f}{N_c}\right)^2 \Lambda^2_{\rm IR}, \quad (N_f/N_c \gg 1).$$

$$M_{\eta'}{}^2 \cdot 8t_0 \simeq M_{\eta'}^{2\,(anomaly)} \cdot 8t_0 \simeq (2.5)^2,\ (5.0)^2,\ (7.5)^2,\ \text{for } N_f = 4, 8, 12\,,$$



LatKMI, EPJ Web Conf. 175 (2018) 08023 • Contribution to: Lattice 2017 , also in preparation

まとめ

- 益川さんの主要な研究課題:対称性の力学的破れ
- Maskawa-Nakajimaの結果、カイラル対称性の力学的破れは強結合でのみ起こる、ゲージ理論全般に該当 ➔conformal windowの存在
- スケール不変性は質量生成により(Bardeen et al)

  自発的破れ➔dilaton

  非摂動な露わな破れ➔dilaton mass
- N_c=3, N_f=8 Lattice: $\gamma_m \simeq 1$ walking TC候補
- Walking Technicolor の実験的検証

  Higgs=Technidilaton ?

  他の束縛状態 Technirho etc.?

  今後のlattice, 実験に期待

たまねぎの
皮むくたびの
涙かな

ヒッグス粒子の
皮むける日は

二〇〇八年盛夏

幸一

益川さんに捧ぐ

愛（は）しけやし
君の開きし
扉より
歩み歩みて
我が道尽きず

幸一

語り合ひ
夢追ふ日々や
今日もかも
君亡き里に
梅は匂ひて

合掌

幸一

ご静聴ありがとうございました

君逝きて　想ひを知るや　シジュウカラ
ピースピースと　鳴きてし止まず

幸一