YITP Workshop

Linearly Polarized Electromagnetic Waves in Pair Plasmas

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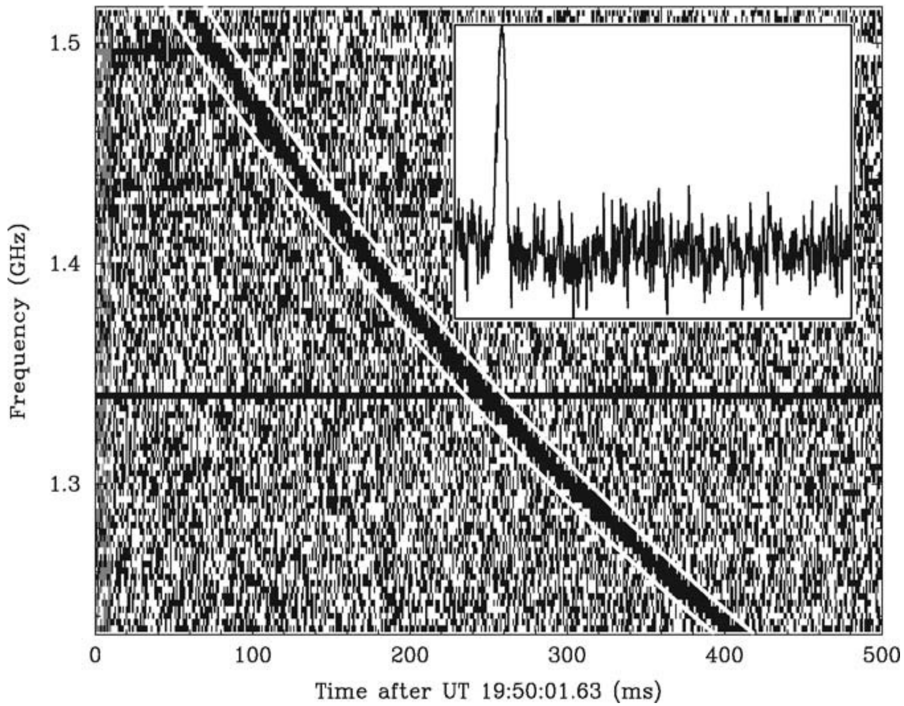
Fast Radio Bursts (FRB)

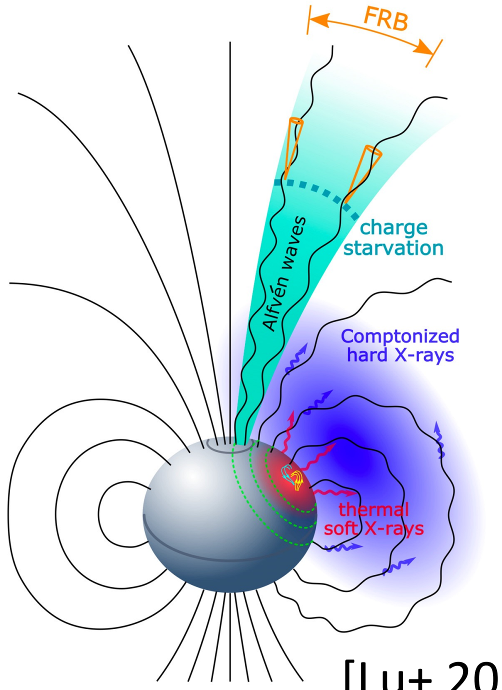
- ✓ Millisecond-duration, radio pulse(Lorimer+ 2007)
- ✓ Radio busts from SGR 1935+2154
  →Magnetar origin?(Bochenek+2020; CHIME/FRB Collaboration+ 2020)

[Lorimer+ 2007]



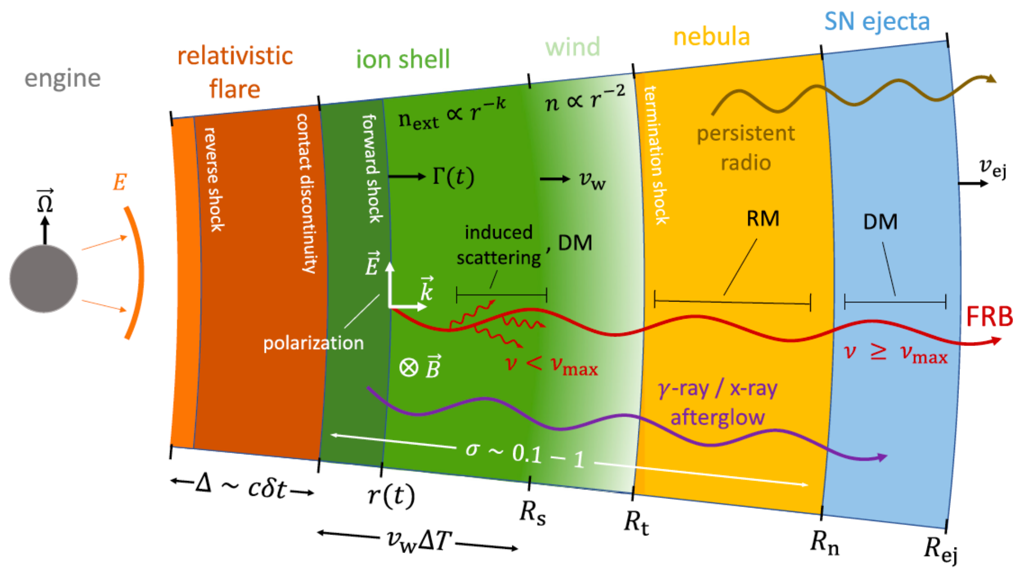
Emission Mechanism of FRBs

Pulsar-like Model
Curvature radiation from charge bunches inside magnetosphere



[Lu+ 2020]

GRB-like model
Synchrotron maser instability in relativistic shocks away from magnetar



[Metzger+ 2019]

Intense electromagnetic waves propagate through magnetar wind

→ Nonlinear interaction with pair plasmas
(induced Raman/Brillouin/Compton scattering, filamentation instability, etc.)

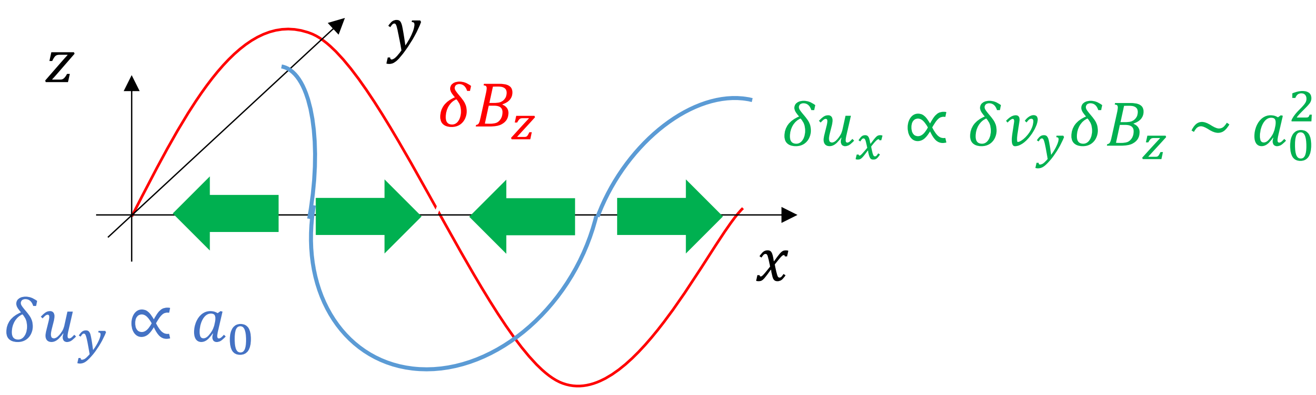
Nonlinearity of EM Waves

Strength parameter

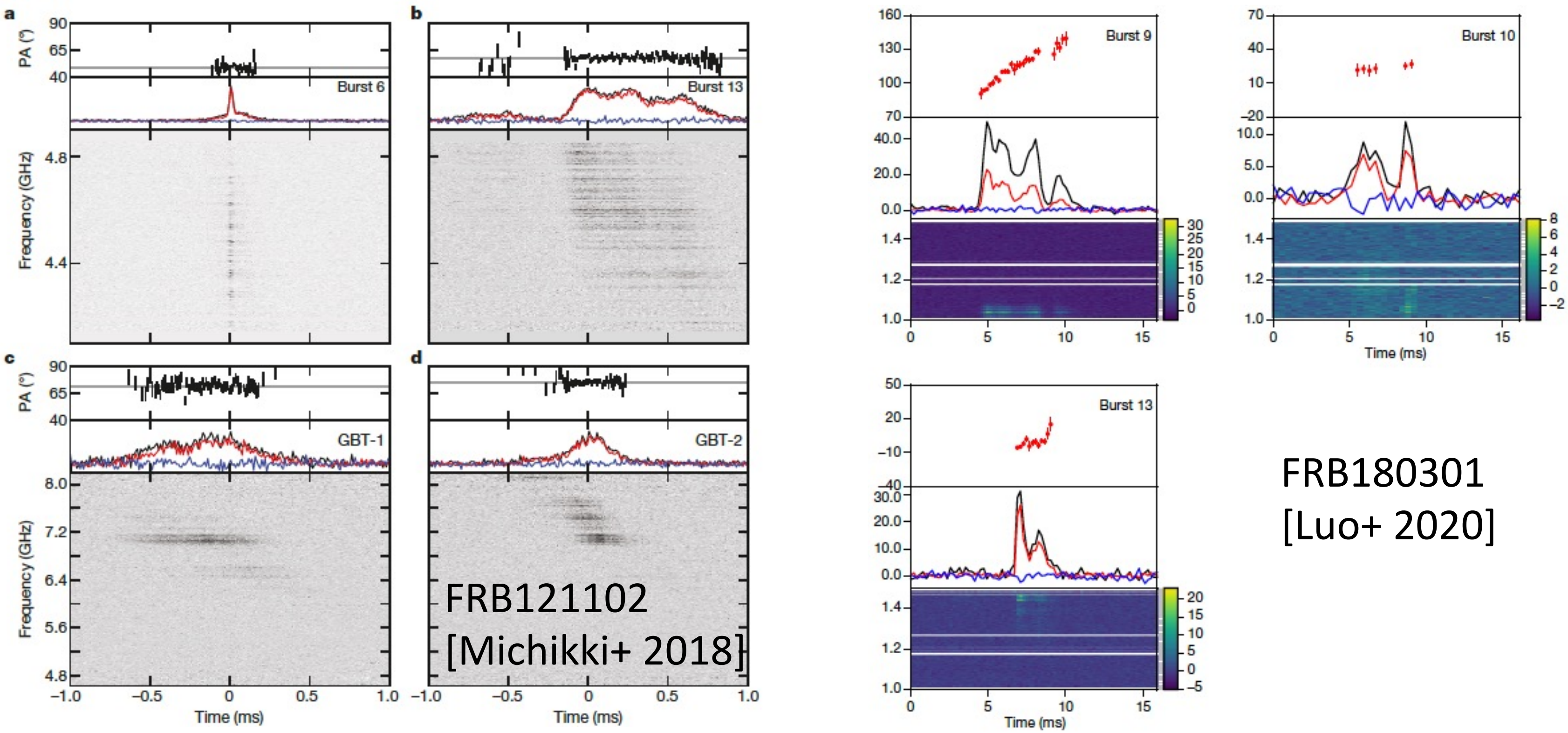
Wave amplitude is well-characterized by

$$a_0 = \frac{eE_0}{m_e c \omega_0}$$

- ✓ Corresponds to oscillation velocity (conservation of transverse canonical momentum)→relativistic for $a_0 \geq 1$
- ✓ Standard theory of nonlinear wave interaction assumes perturbation expansion in $a_0 \ll 1$ and keeps the second order a_0^2
- ✓ $a_0 > 1$ for $R < 10^{13-14}$cm in FRB (Luan & Goldreich 2014)
  →need to study the wave propagation in the regime $a_0 > 1$



Polarization Properties



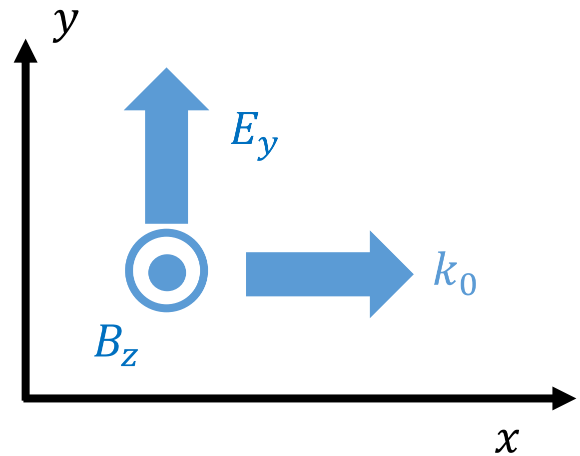
- ✓ Some FRBs are known to repeat, and repeating FRBs often show high linear polarization (e.g., Michikki+ 2018; Luo+ 2020)
- ✓ Analytical solution for linearly polarized wave is required
  → such solution for $a_0 \geq 1$ is not fully understood

Analytical Solution

Akhiezer & Polovin 1954; Max 1973; Clemmow 1974; Kennel & Pellat 1976

Assumptions

- ✓ Unmagnetized, cold electron-positron fluid
- ✓ Linearly polarized, monochromatic plane wave
- ✓ All physical quantities are expressed as a function of $\phi_0 = k_0 x - \omega_0 t$



$$\frac{\alpha^2 a_0^2}{\gamma_g^2}\left(\frac{\mathrm{d}y}{\mathrm{d}\phi_0}\right)^2 = \frac{(1-y^2)(1-y^2+q)}{(1-y^2+q/2)^2}$$

$$\frac{\alpha a_0}{\gamma_g}\frac{2E(m)-(1-m)K(m)}{2\sqrt{m}} = \frac{\pi}{2}$$

$$\gamma = 1 + \frac{\alpha a_0^2}{2}(1-y^2)$$

$$u_x = \frac{\alpha \beta_g a_0^2}{2}(1-y^2)$$

$$u_y = \pm a_0 \int y \mathrm{d}\phi_0$$

Where

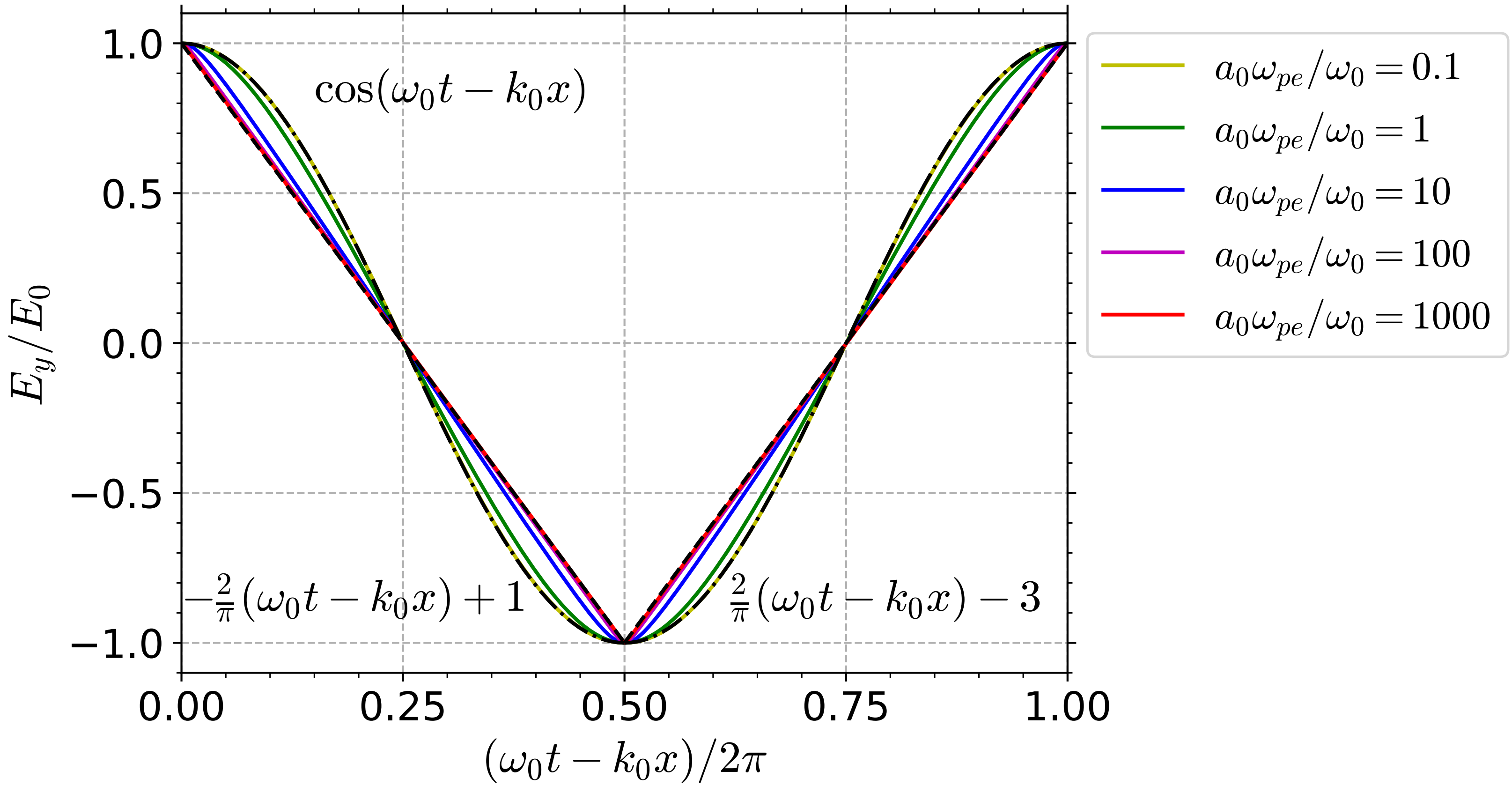
$$y = \frac{E_y}{E_0}, a_0 = \frac{eE_0}{mc\omega_0}, \alpha = \frac{\omega_0^2 - c^2 k_0^2}{2\omega_{pe}^2}$$

$$\beta_g = \frac{ck_0}{\omega_0}, \gamma_g = \frac{1}{\sqrt{1-\beta_g^2}},$$

$$q = \frac{8\omega_{pe}^2\gamma_g^4}{\omega_0^2 a_0^2}, m = \frac{1}{1+q},$$

K&E:Incomplete elliptic integral of the first/second kind

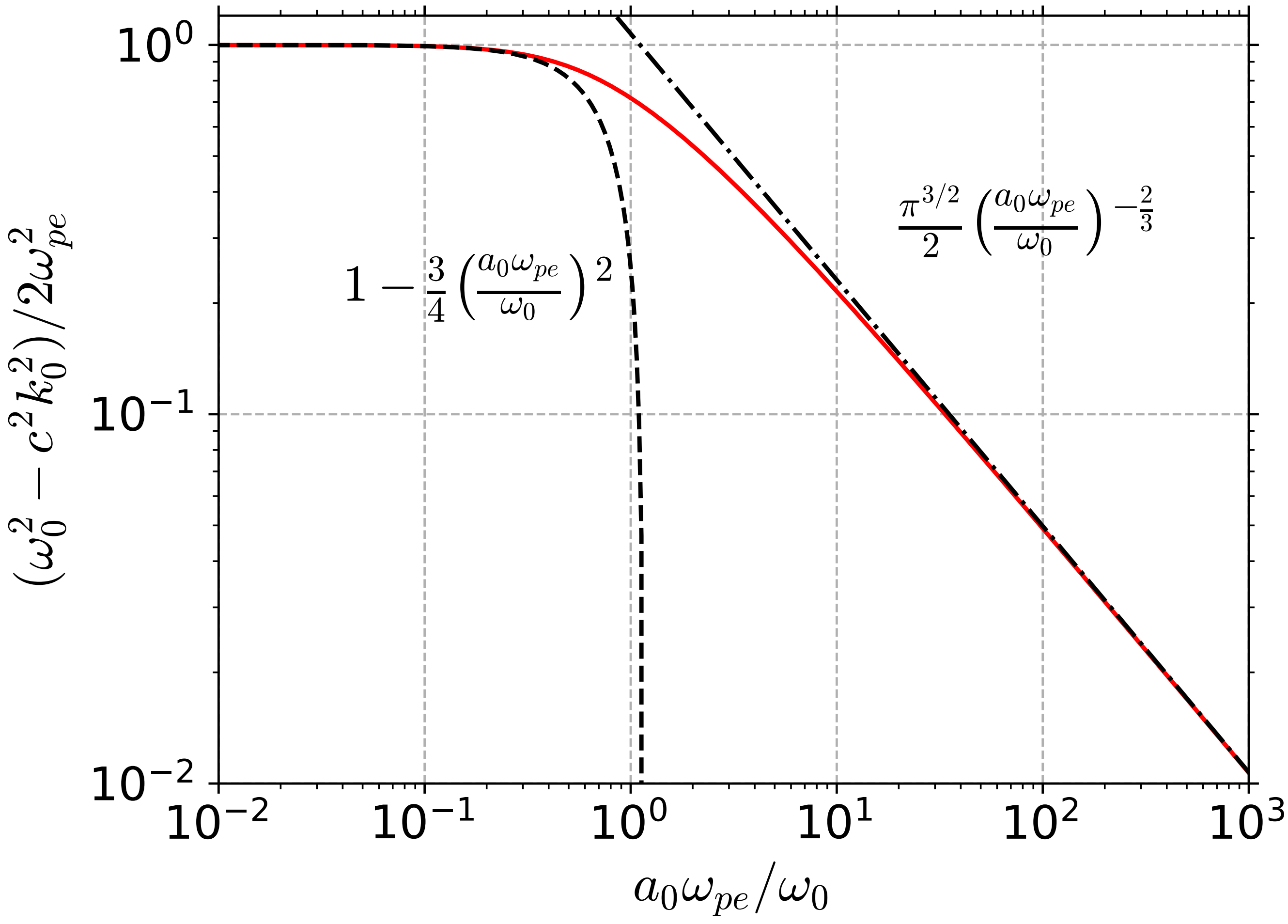
Wave Form



- ✓ Well characterized by $\frac{a_0\omega_{pe}}{\omega_0}$ rather than a_0
- ✓ $\frac{a_0\omega_{pe}}{\omega_0} \ll 1 \rightarrow$ linear solution (=sinusoidal) even if $a_0 > 1$

Dispersion Relation

also well-characterized by $\frac{a_0\omega_{pe}}{\omega_0}$ and same as linear solution for $\frac{a_0\omega_{pe}}{\omega_0} \ll 1$



Stability Analysis

- ✓ In unmagnetized pair plasmas, linearly polarized EM waves are subject to induced Compton scattering and filamentation instability
- ✓ For $a_0 \ll 1$, linear growth rate of Induced Compton scattering and filamentation instability (Ghosh+ 2022; Iwamoto+ 2023):

$$\frac{\Gamma_{max}^{ICS}}{\omega_0} \sim \begin{cases} \sqrt{\frac{\pi}{8e}} \frac{1}{\beta_{th0}^2} \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^2, \beta_{th0} \gg \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^{\frac{2}{3}} \\ \frac{\sqrt{3}}{2} \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^{\frac{2}{3}}, \beta_{th0} \ll \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^{\frac{2}{3}} \end{cases}$$

$$\frac{\Gamma_{max}^{FI}}{\omega_0} \sim \begin{cases} \frac{1}{4\beta_{th0}^2} \left(a_0 \frac{\omega_{pe}}{\omega_0}\right)^2, \beta_{th0} \gg \sqrt{a_0 \frac{\omega_{pe}}{\omega_0}} \\ a_0 \frac{\omega_{pe}}{\omega_0}, \quad \beta_{th0} \ll \sqrt{a_0 \frac{\omega_{pe}}{\omega_0}} \end{cases}$$

→characterized by $\frac{a_0 \omega_{pe}}{\omega_0}$

Motivation

- ✓ Analytical solution shows that linear approximation is valid for $\frac{a_0\omega_{pe}}{\omega_0} \ll 1$ even if $a_0 \geq 1$
- ✓ Linear analysis for $a_0 \ll 1$ indicates that linear growth rates is determined by $\frac{a_0\omega_{pe}}{\omega_0}$
- ✓ FRB propagation is also characterized by $\frac{a_0\omega_{pe}}{\omega_0}$?

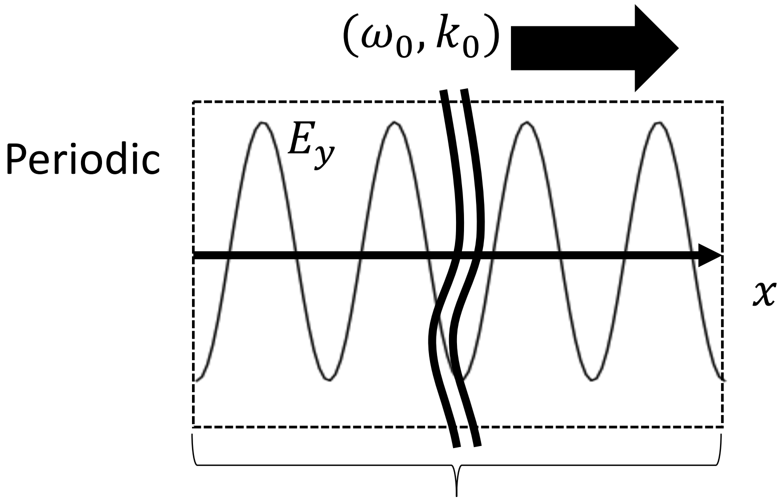
$$a_0 \sim 2\left(\frac{10^{13}\mathrm{cm}}{R}\right), \frac{\omega_{pe}}{\omega_0} \sim 10^{-3}\left(\frac{10^{13}\mathrm{cm}}{R}\right)$$

$$\rightarrow \frac{a_0\omega_{pe}}{\omega_0} \sim 2\times10^{-3}\left(\frac{10^{13}\mathrm{cm}}{R}\right)^2 \text{ (Beloborodov 2020)}$$

Analytical approach is difficult due to relativistic effects

→numerical simulation

Numerical Settings



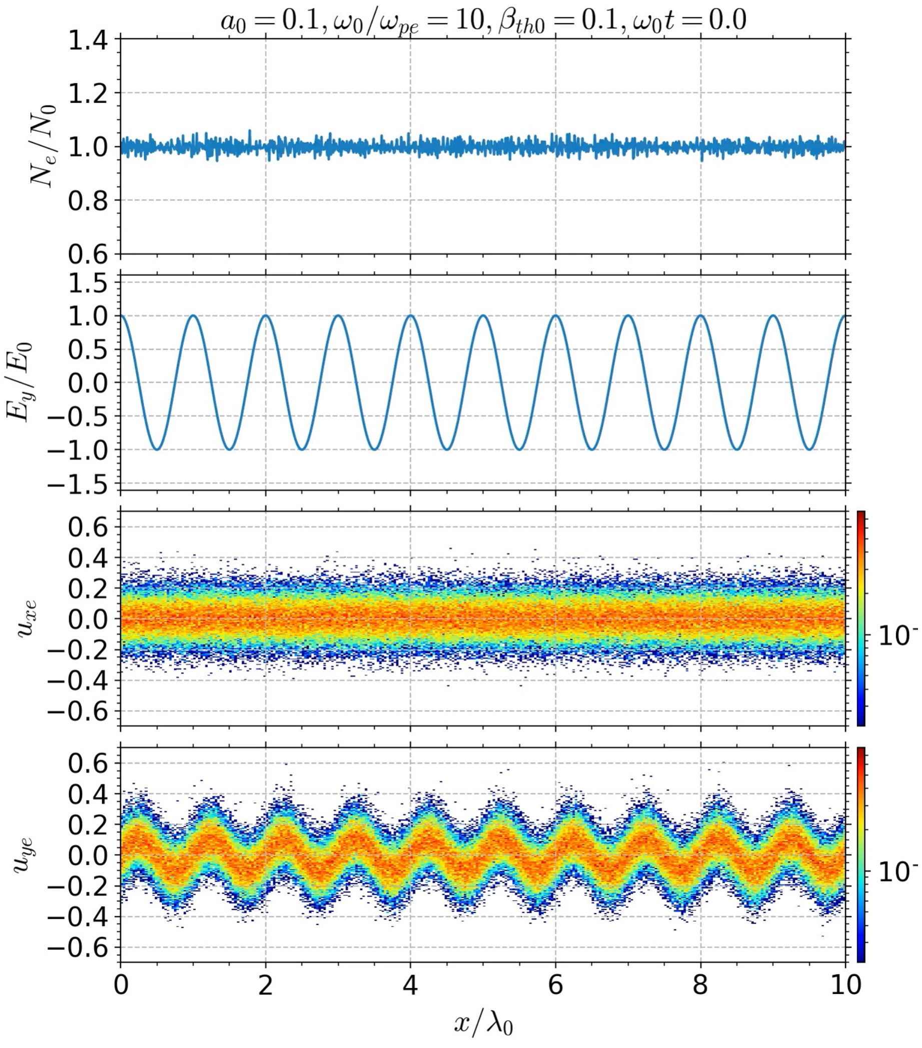
$$100\lambda_0 = 10{,}000\Delta x$$
$$\Delta t = \Delta x / c$$

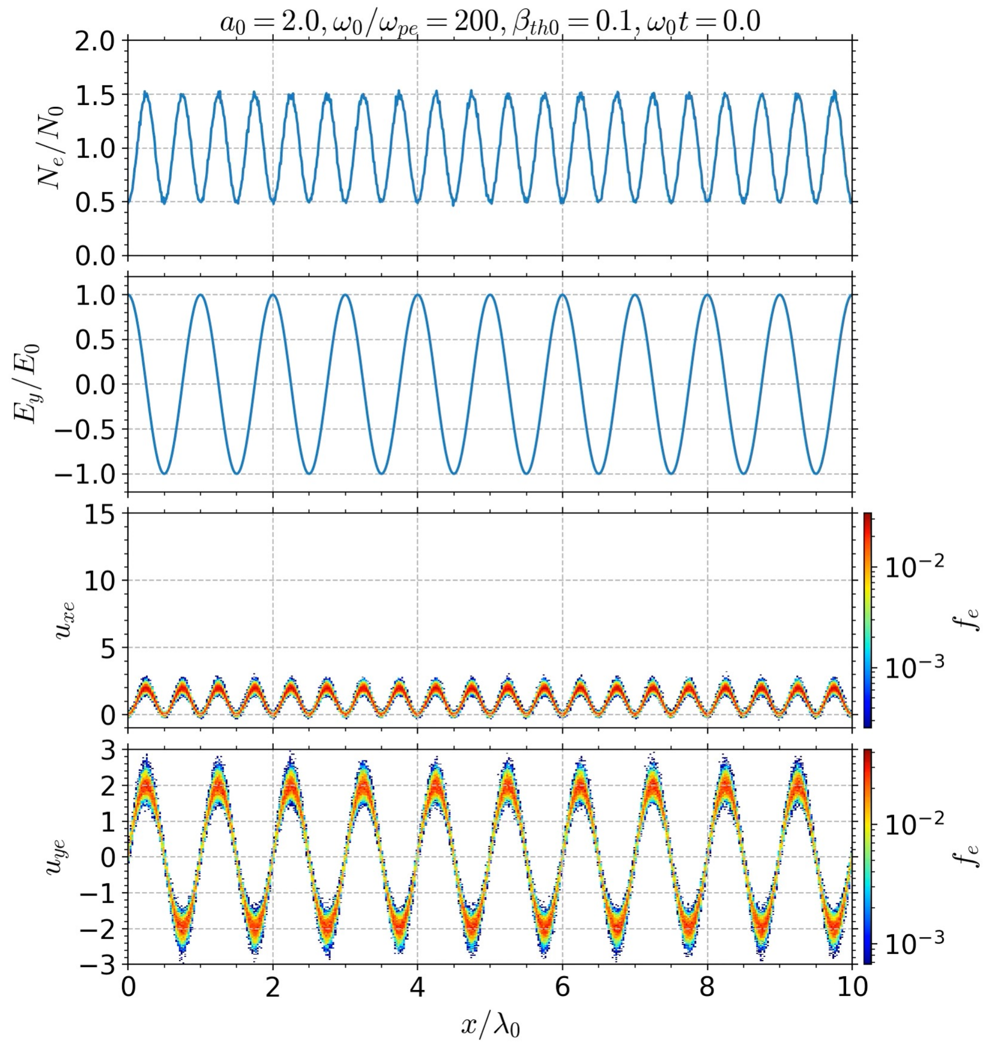
Parameters

$$\left(a_0\,, \frac{\omega_0}{\omega_{pe}}\right) = (0.1,10), (1,100), (2,200)$$
$$\left(a_0 \frac{\omega_{pe}}{\omega_0} = 0.01 \;\text{ is fixed}\right)$$
$$\beta_{th0} = 0.1$$

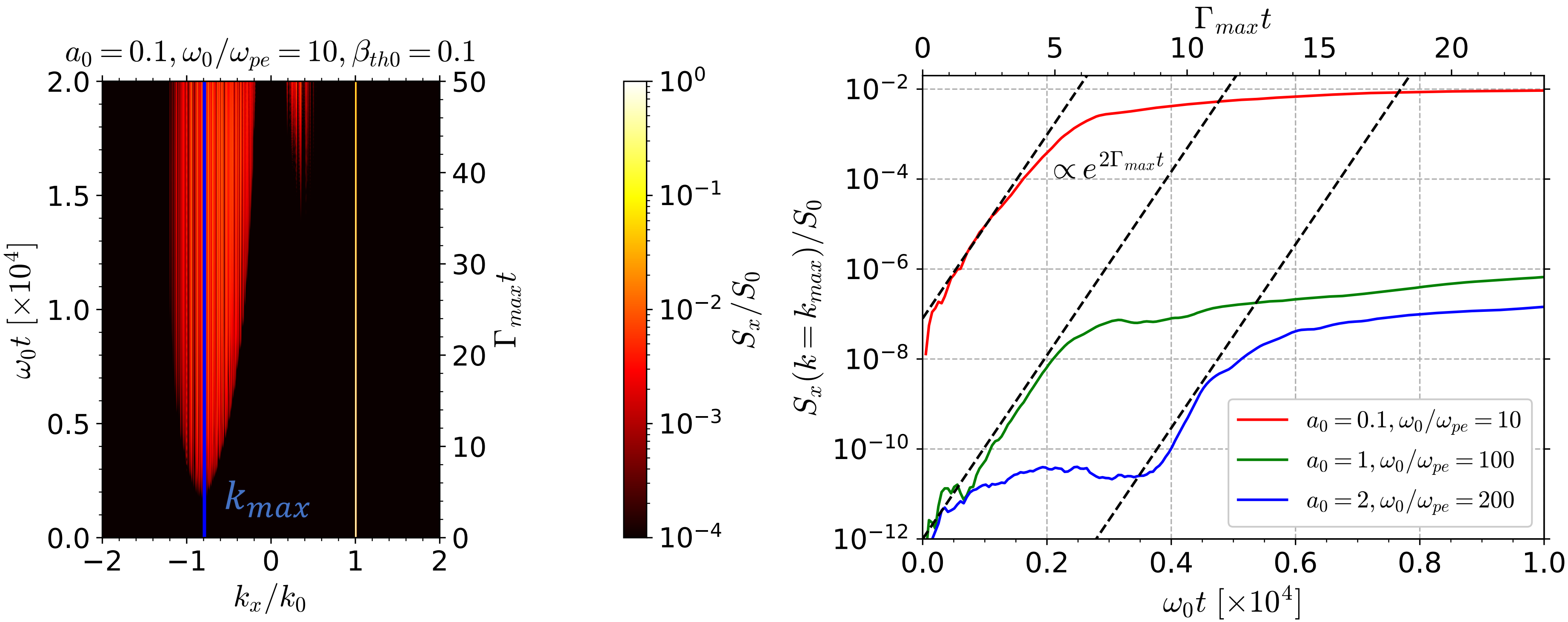
- ✓ Simulation code: Wuming (open source PIC code; Matsumoto+ 2024)
- ✓ One-dimensional systems (only induced Compton scattering works)
- ✓ k_0 is determined by dispersion relation
- ✓ β_{th0} is defined in the proper frame

Simulation Results



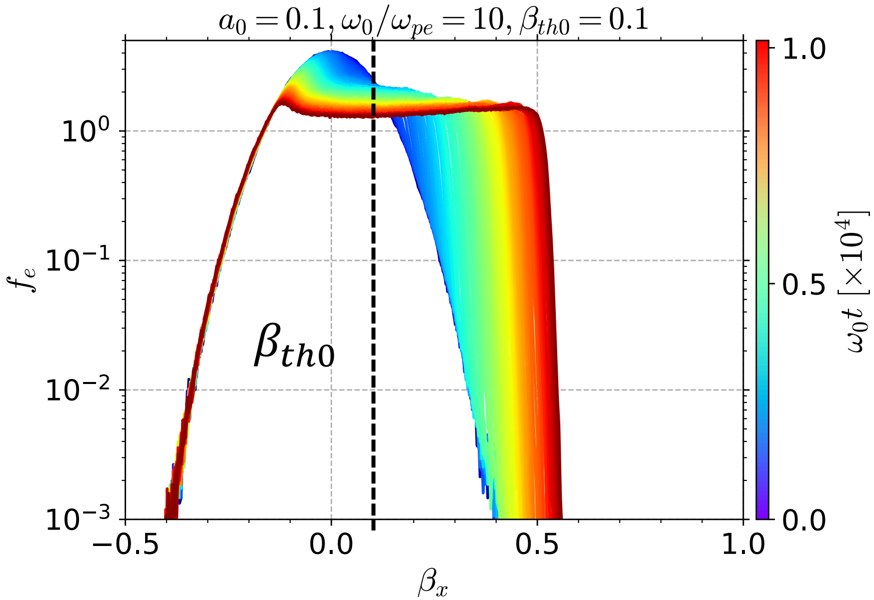


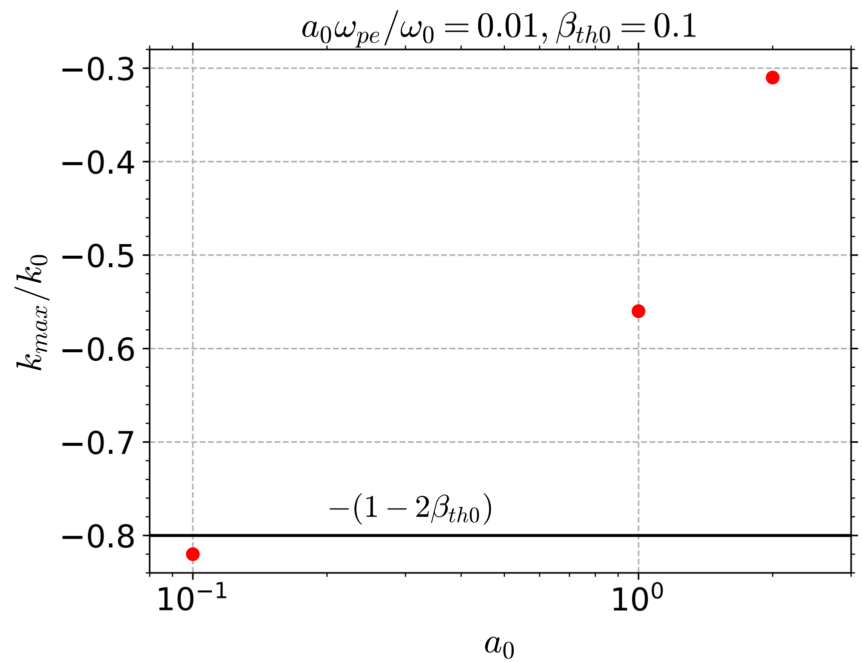
Linear Growth Rate

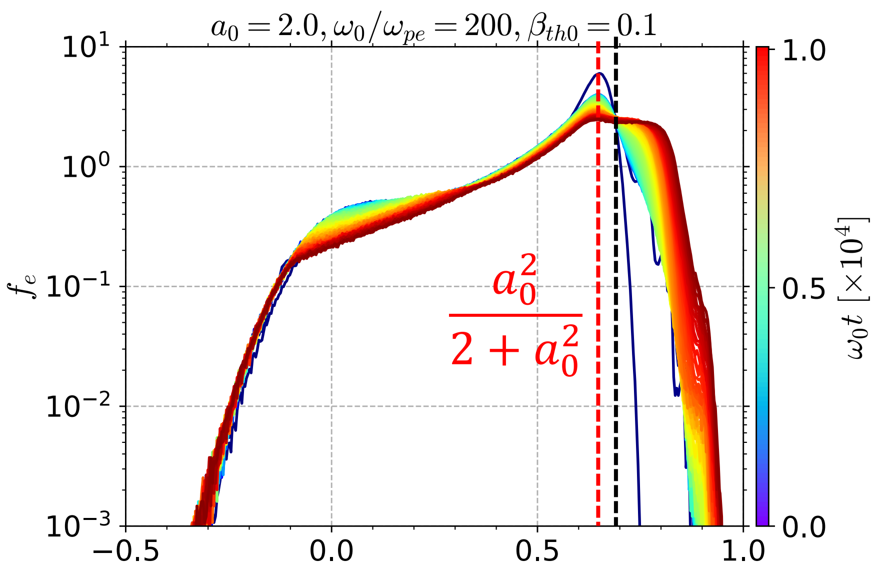


- ✓ FRB: $\omega_0 t_{pulse} \sim 1\text{GHZ} \times 1\text{ms} = 10^6$
  → sufficiently longer than simulation time $\omega_0 t_{max} = 2 \times 10^4$
- ✓ Linear growth rate is well-characterized by $\frac{a_0 \omega_{pe}}{\omega_0}$ even for $a_0 \geq 1$
- ✓ Saturation level depends on $\frac{a_0 \omega_0}{\omega_{pe}} = \sqrt{\frac{E_0^2/4\pi}{n_0 mc^2}}$

Fastest-Growing Mode







- ✓ Landau resonance with beat wave
- ✓ Longitudinal oscillation is not negligible for $a_0 > 1$
  → resonant velocity is deviated from linear analysis

Summary & Future Work

Summary

FRB propagation is controlled by

$$\frac{a_0\omega_{pe}}{\omega_0} \ \& \ \frac{a_0\omega_0}{\omega_{pe}}$$

$\frac{a_0\omega_{pe}}{\omega_0} \ll 1$→ linear analysis is valid even if $a_0 \geq 1$

$\frac{a_0\omega_0}{\omega_{pe}} \gg 1$→ induced Compton scattering is negligible

Future Work

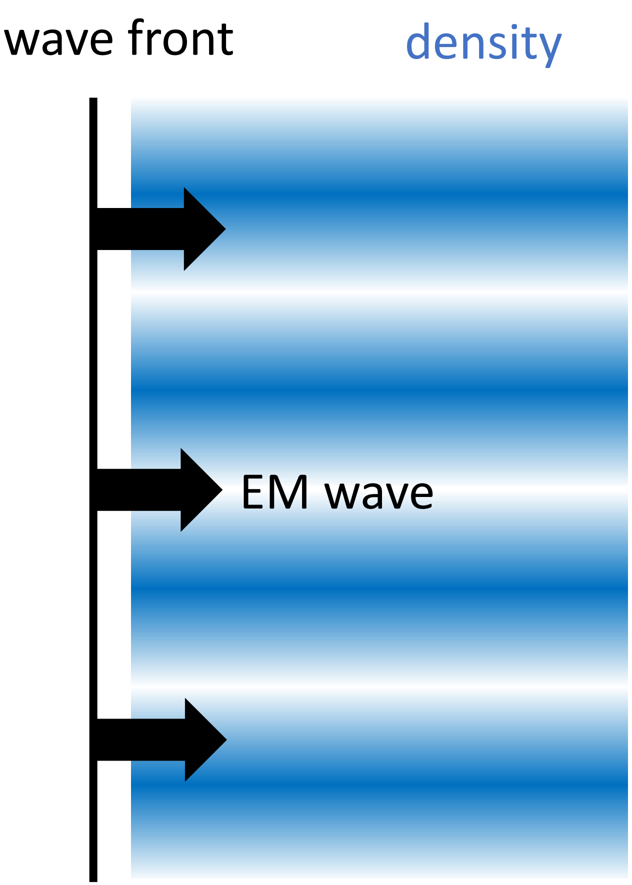
- ✓ Multi-dimensional sysytem
- ✓ Strongly nonlinear regime $\frac{a_0\omega_{pe}}{\omega_0} \gg 1$
- ✓ Background magnetic field
  - ➢ For $\sigma_e \gg 1, \frac{a_0\omega_{ce}}{\omega_0} \ll 1$ → linear (Sobacchi, MI+ 2024, submitted)

Backup

Filamentation Instability

Transverse modulation instability (four-wave coupling)

(Kaw+ 1973; Sobacchi+ 2020;2022;2023)



$$\frac{\omega}{k} = \frac{c}{\sqrt{1 - \omega_{pe}^2/\omega^2}}$$

High density

→High phase velocity