A discussion on the effects of QGP's electric conductivity on observables in high-energy heavy-ion collisions

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KOMAZAWA UNIVERSITY

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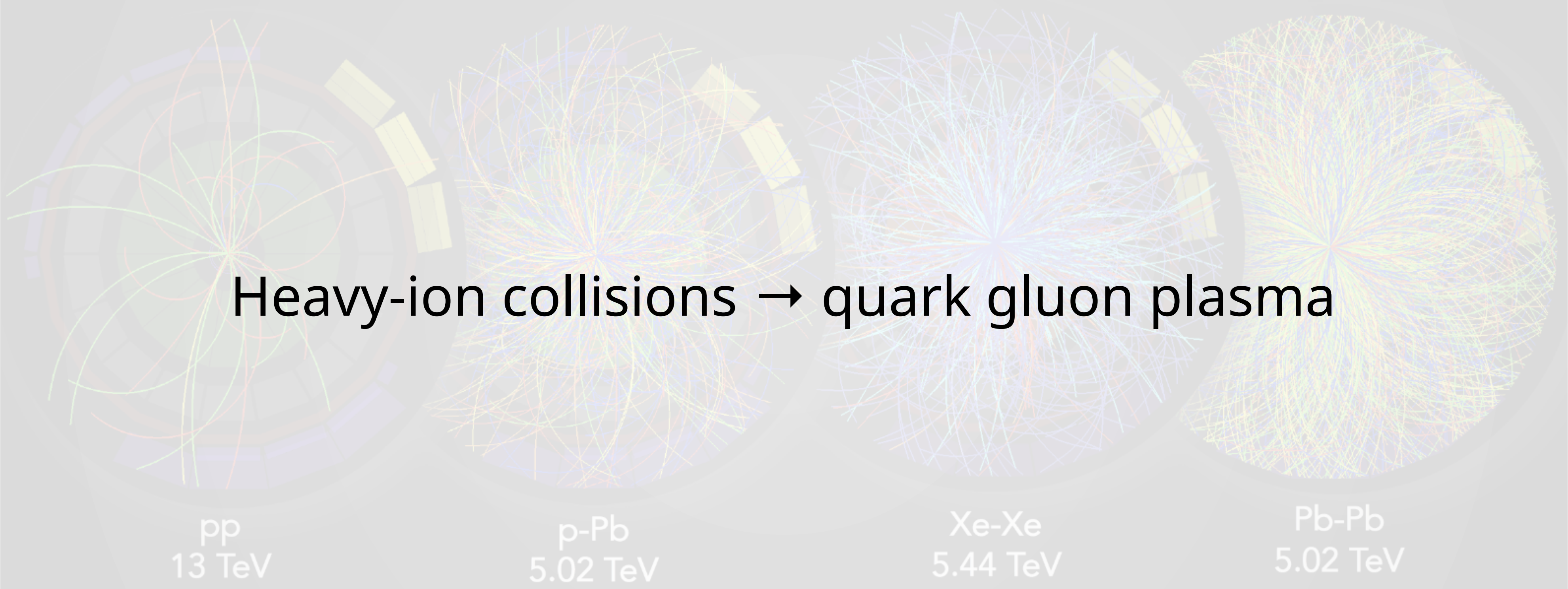
素粒子宇宙起源研究所 Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

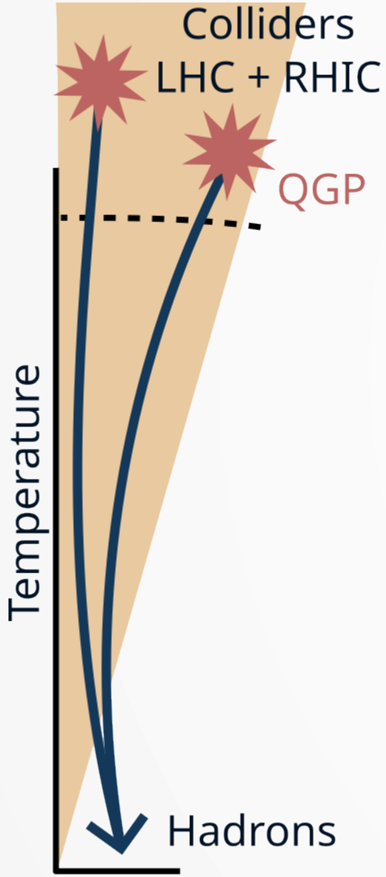
Collaborators [paper soon to be released]:
T. Miyoshi (HU), C. Nonaka (HU, SKCM2, KMI), A. Sakai (HU), and H. R. Takahashi (KU)

Topology and Dynamics of Magneto-Vortical Matter @ YITP 2025/01/20

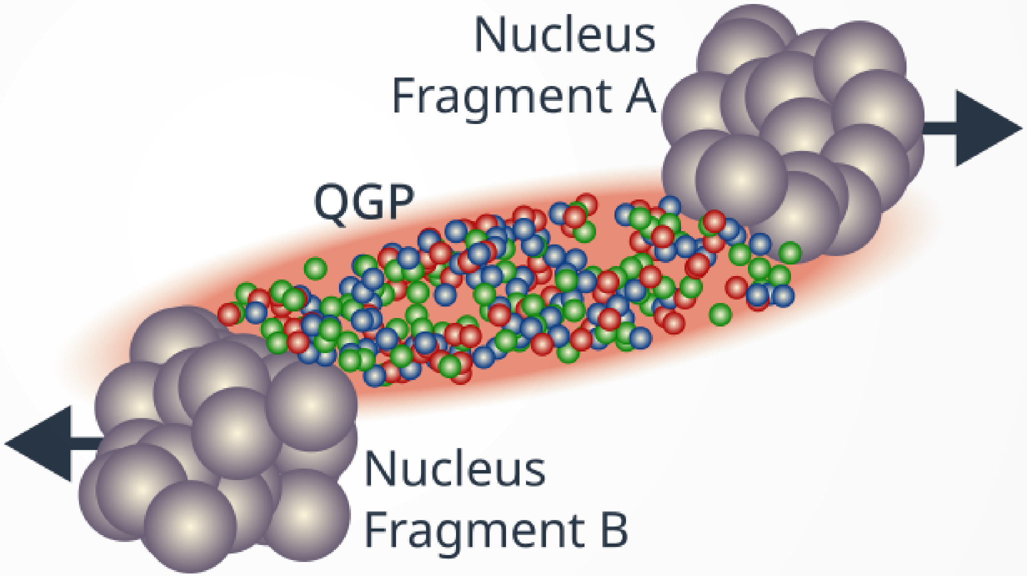


Heavy-ion collisions → quark gluon plasma

How to make quark gluon plasma



- How is quark gluon plasma (QGP) created in heavy-ion collisions?



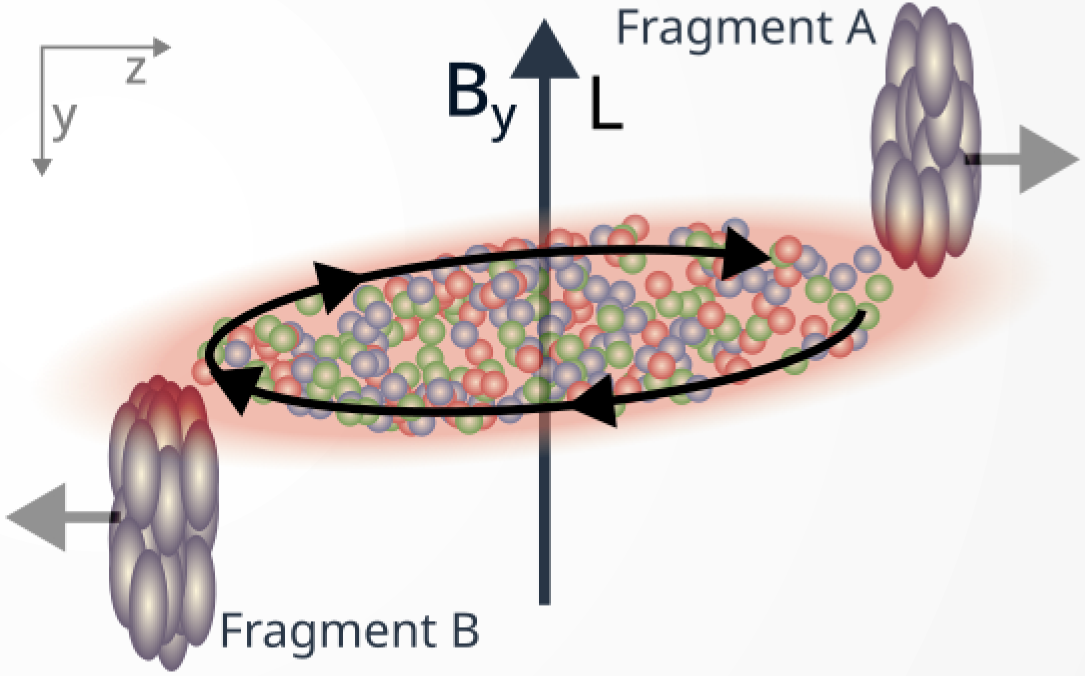
- Collide heavy nuclei like Au, Cu, Xe, Pb, etc. at relativistic energies

Non-central collisions and global polarization

Non-central collisions have many interesting properties,

- large orbital angular momentum
- strong magnetic field
- etc.

These result in a “global” polarization for particles

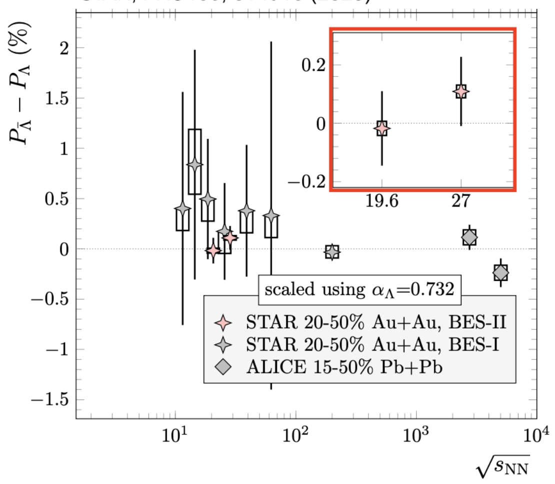


Non-central collisions and global polarization

$P_{\bar{\Lambda}}$ - P_{Λ} difference?

- B-field lifetime is likely very short (<0.5 fm/c) but could be sustained by QGP electric conductivity (unknown)
- Can we put constraint on the late-stage B-field by the polarization difference?

STAR, PRC108, 014910 (2023)



$$P_\Lambda \simeq \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T}$$

$$P_{\bar{\Lambda}} \simeq \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

μ_Λ: Λ magnetic moment

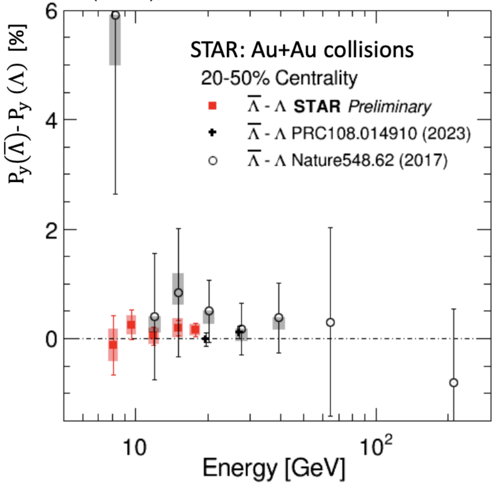
$$B = (P_{\bar{\Lambda}} - P_\Lambda)T/(-2\mu_\Lambda)$$

$$\approx 30\Delta P_\Lambda \times 10^{14}\ [T]$$

$$eB/m_\pi^2 \sim 10\Delta P_\Lambda$$

(T=120 MeV)

Q. Hu (STAR), SQM2024



New BES-II results with better precision do not show significant difference.
An upper limit of the late-stage B-field: $B \lesssim 10^{13}$ T

T. Niida, West Lake Workshop NP2024

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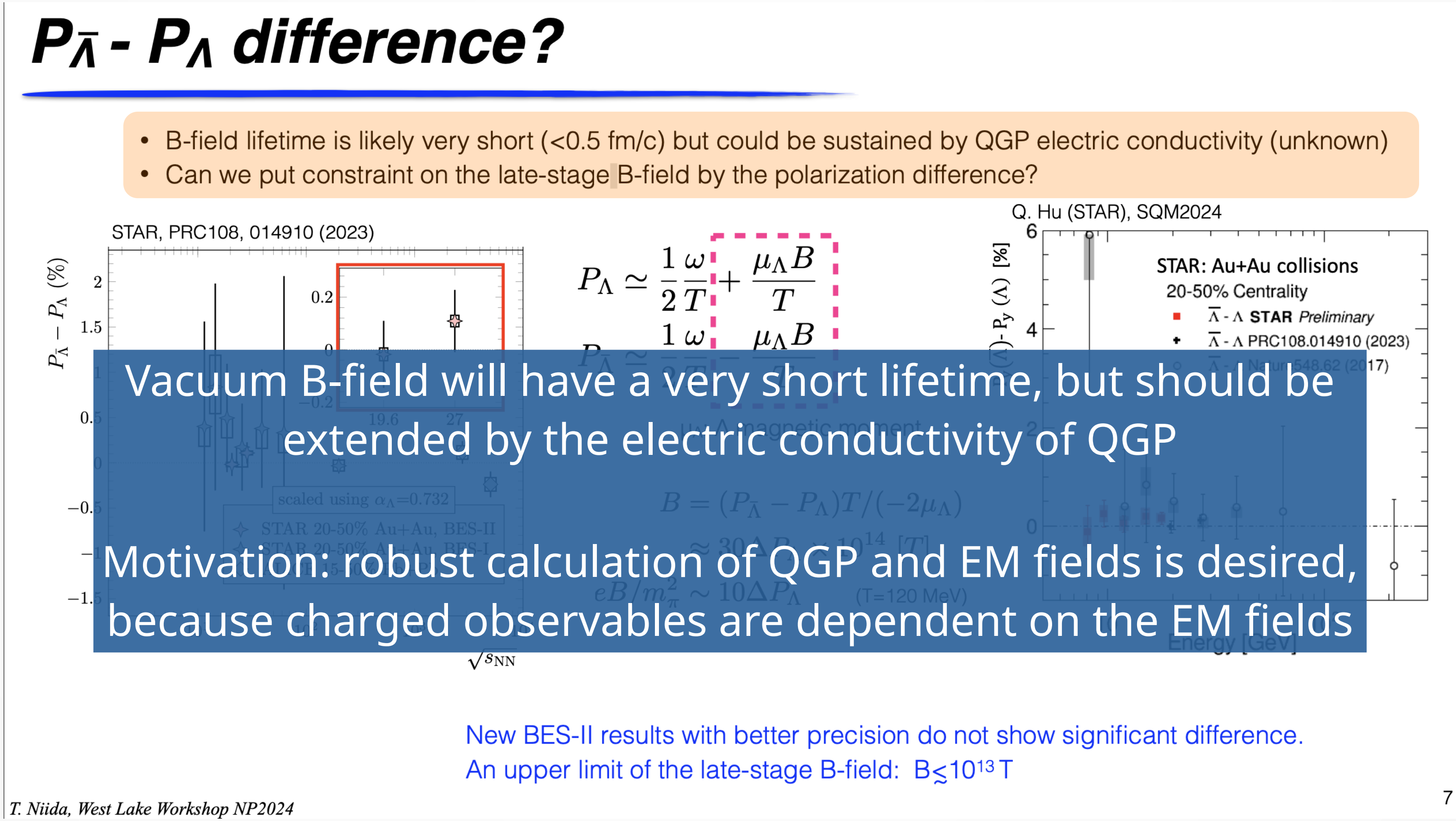
Non-central collisions and global polarization



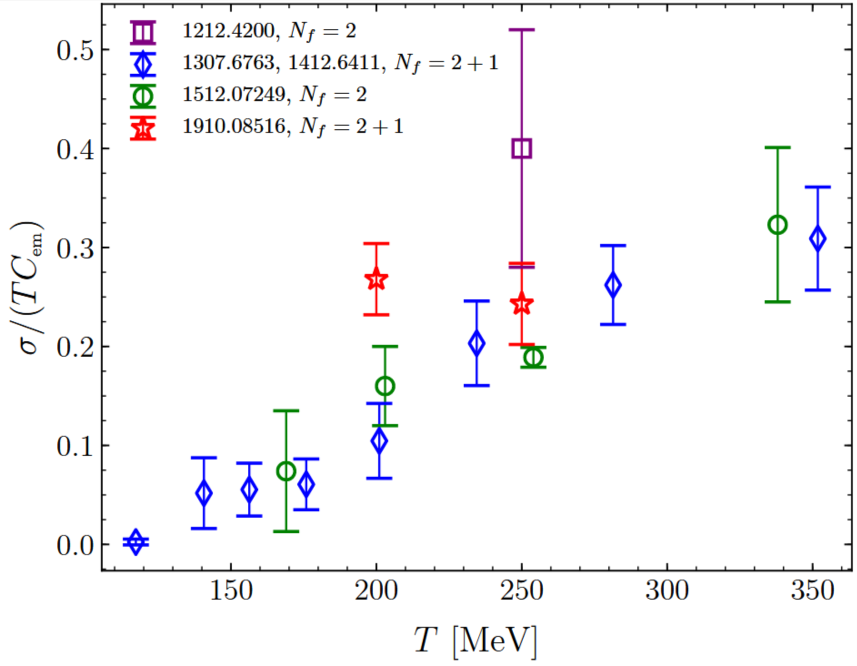
Vacuum B-field will have a very short lifetime, but should be extended by the electric conductivity of QGP

Motivation: robust calculation of QGP and EM fields is desired, because charged observables are dependent on the EM fields

Quark-gluon plasma (QGP) electric conductivity

- Studied by lattice calculations (~10 papers), pQCD, and kinetic transport theories

  Review: G. Aarts and A. Nikolaev; EPJ. A 57, 118 (2021) arXiv:2008.12326 [hep-lat]



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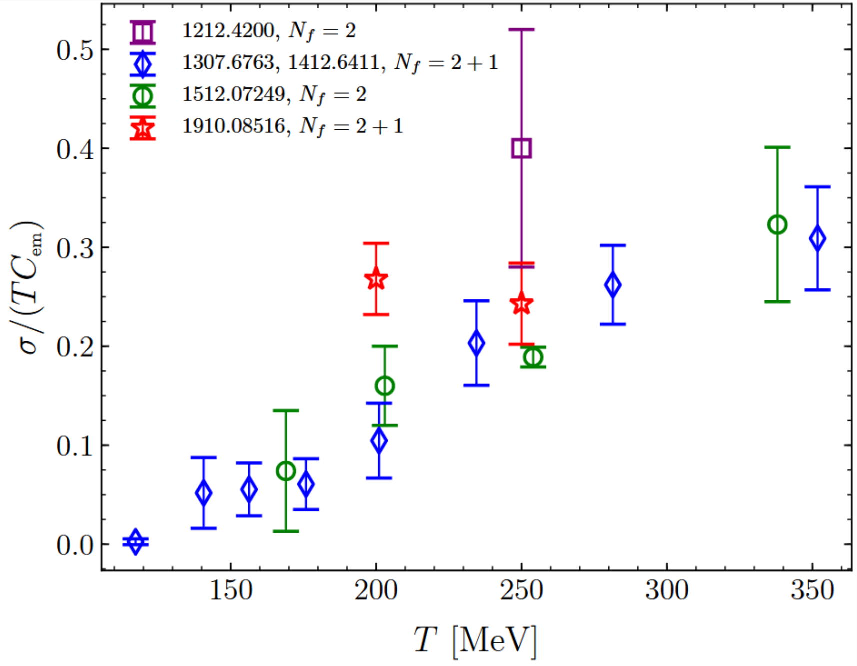

- Uses linear-response theory (Kubo formula)
- Low energy limit - electromagnetic spectral function

$$\sigma = \frac{1}{6}\frac{\partial}{\omega}\left(\int d^4x e^{i\omega t}\langle[j^{\rm em}_\mu(t,x), j^{\rm em}_\mu(0,0)]\rangle\right)|_{\omega=0}$$

where the EM current depends on # of quark flavors

$$j^{\rm em}_\mu(x) = \sum_{f=1}^{N_f}(eq_f)\overline{\psi}^f(x)\gamma_\mu\psi^f(x)$$

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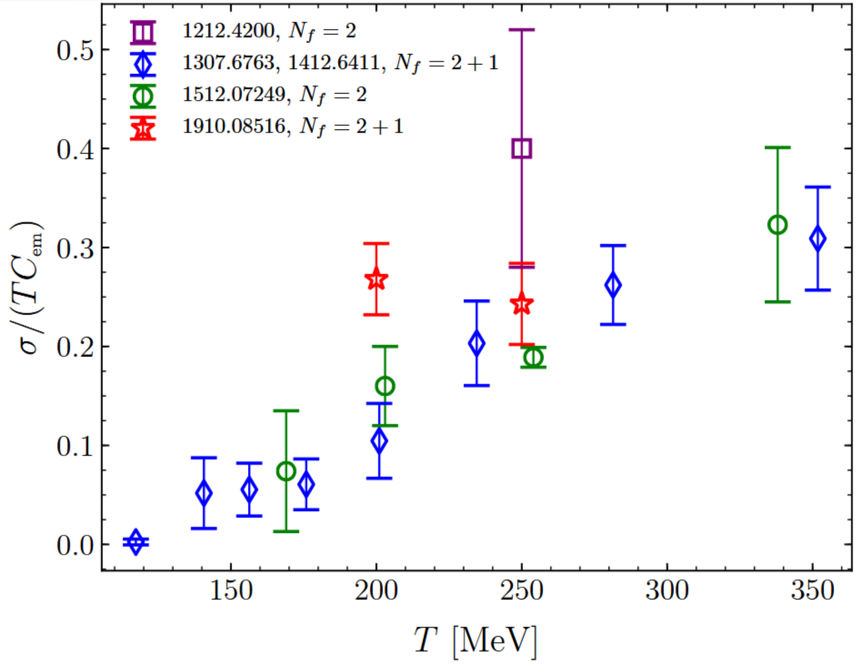

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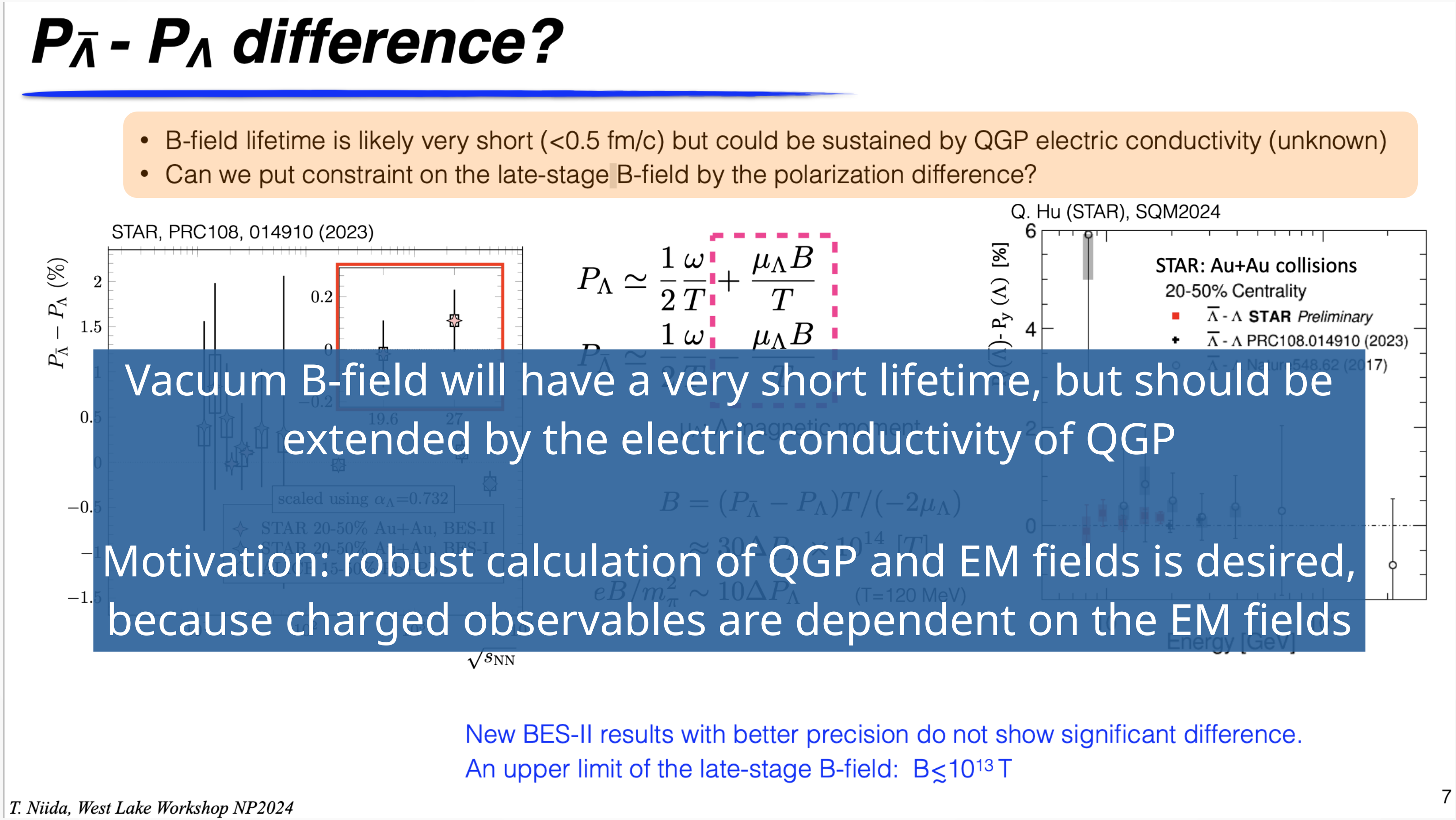
$$j^{\text{em}}_\mu(x) = \sum_{f=1}^{N_f}(eq_f)\overline{\psi}^f(x)\gamma_\mu\psi^f(x)$$

- Does not include external magnetic field effects
- Uses approximately realistic pion mass
- General agreement among results using a variety of methods and parameters (see paper)

Non-central collisions and global polarization



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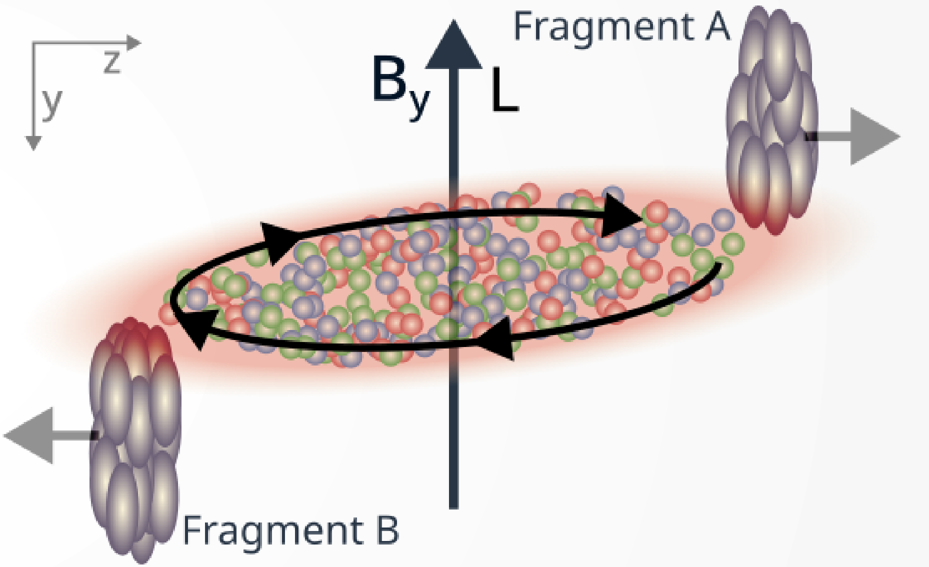
Motivation: robust calculation of QGP and EM fields is desired, because charged observables are dependent on the EM fields

What observables can we use?

Charged leptons and thermal photons
Dileptons → Y. Akamatsu, H. Hamagaki, T. Hatsuda, and T. Hirano, PRC.85.054903 (2012).
Photons → J.-A. Sun and L. Yan, PRC.109.034917 (2024)

Charge dependent directed flow (v_1)
Asymmetic collisions → Y. Hirono, M. Hongo, and T. Hirano, PRC.90.021903 (2014)
Symmetric collisions → U, Gürsoy, D. Kharzeev, and K. Rajagopal PRC.89.054905 (2014)

As some example papers.



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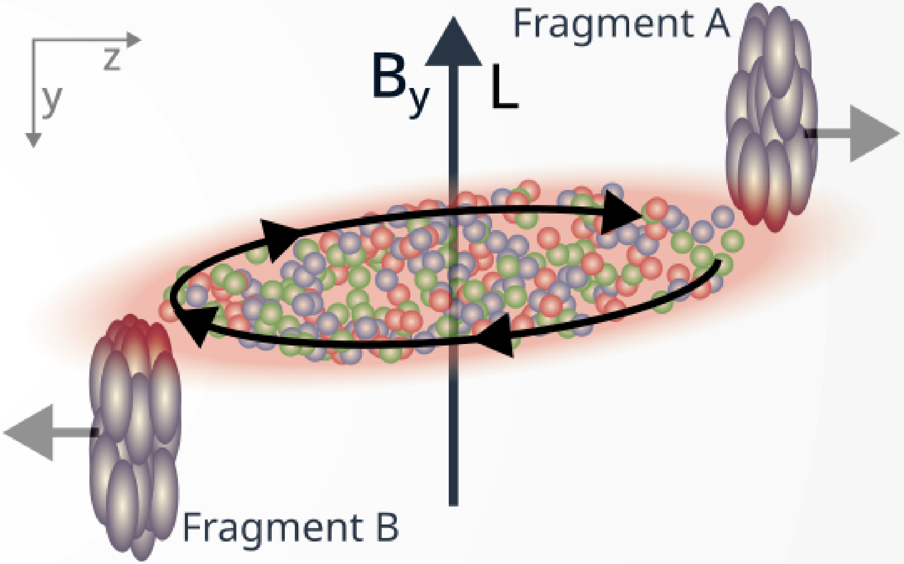
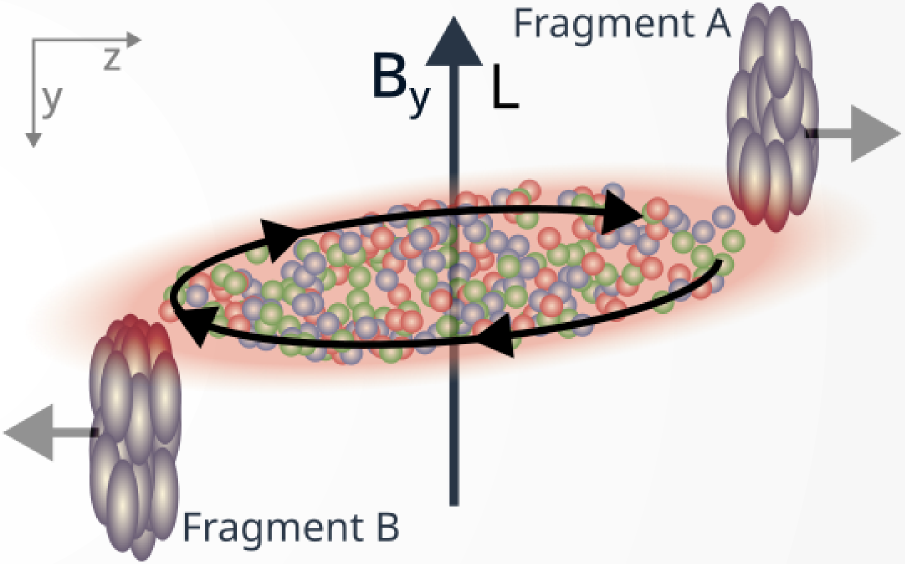


This works focuses on these two

How can we model such a system?

- In medium EM fields evolve at a similar rate as the QGP expansion (QGP is a conductive medium)
- EM fields could modify the collective behavior of charged particles
- Charged currents in the plasma should influence the EM fields



Use relativistic resistive magneto-hydrodynamics

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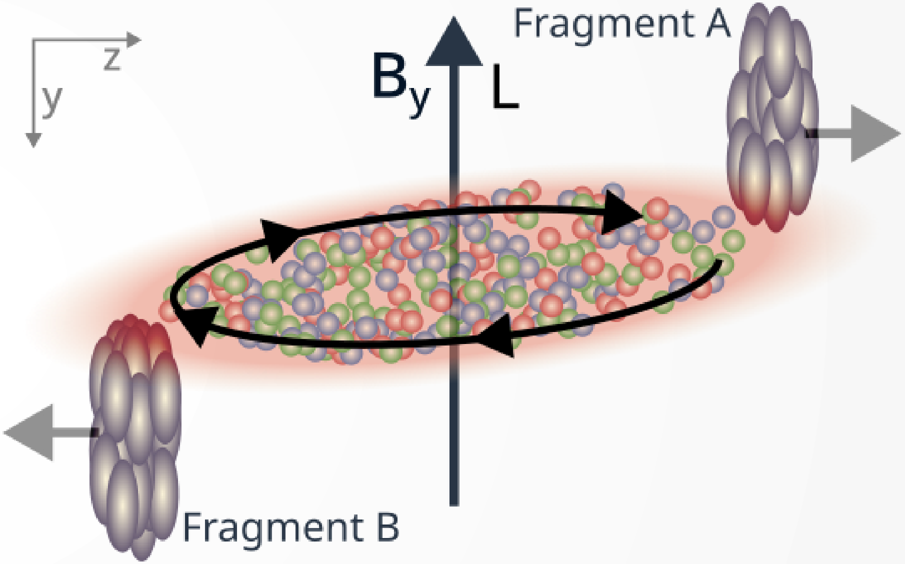

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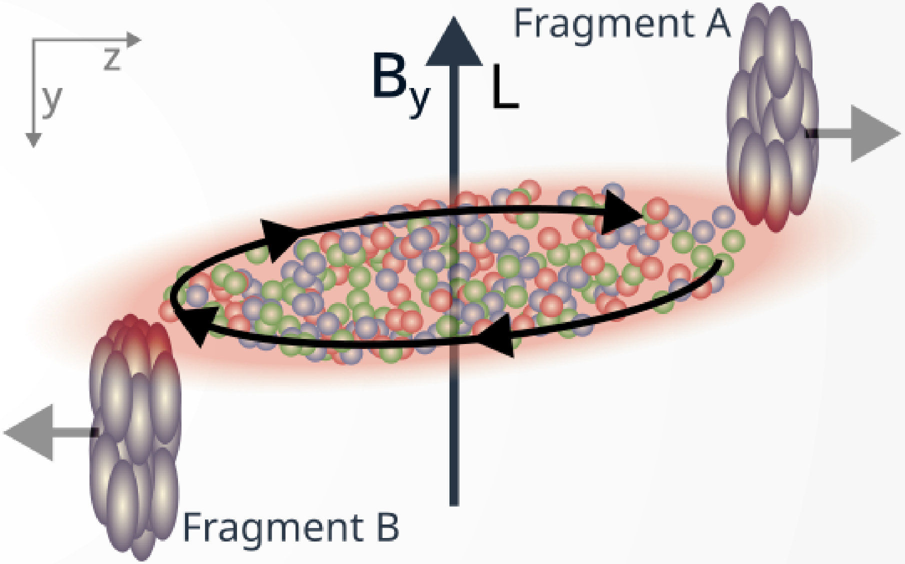


Use relativistic resistive magneto-hydrodynamics

Details of our RRMHD model

- Start from ideal-hydrodynamics + Maxwell equations

$\nabla_\mu N^\mu = 0$ Continuity equation (i.e., net-baryon current)

$\nabla_\mu T^{\mu\nu} = 0$ Total energy-momentum

$\nabla_\mu S^\mu \geq 0$ 2nd law of thermodynamics

Maxwell's equations

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$

$$\nabla_\mu {}^*F^{\mu\nu} = 0$$

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{m}} + T^{\mu\nu}_{\mathrm{f}}$$

$$T^{\mu\nu}_{\mathrm{m}} = (\epsilon + p_{\mathrm{gas}})\, u^\mu u^\nu + p_{\mathrm{gas}} g^{\mu\nu}$$

fluid matter part

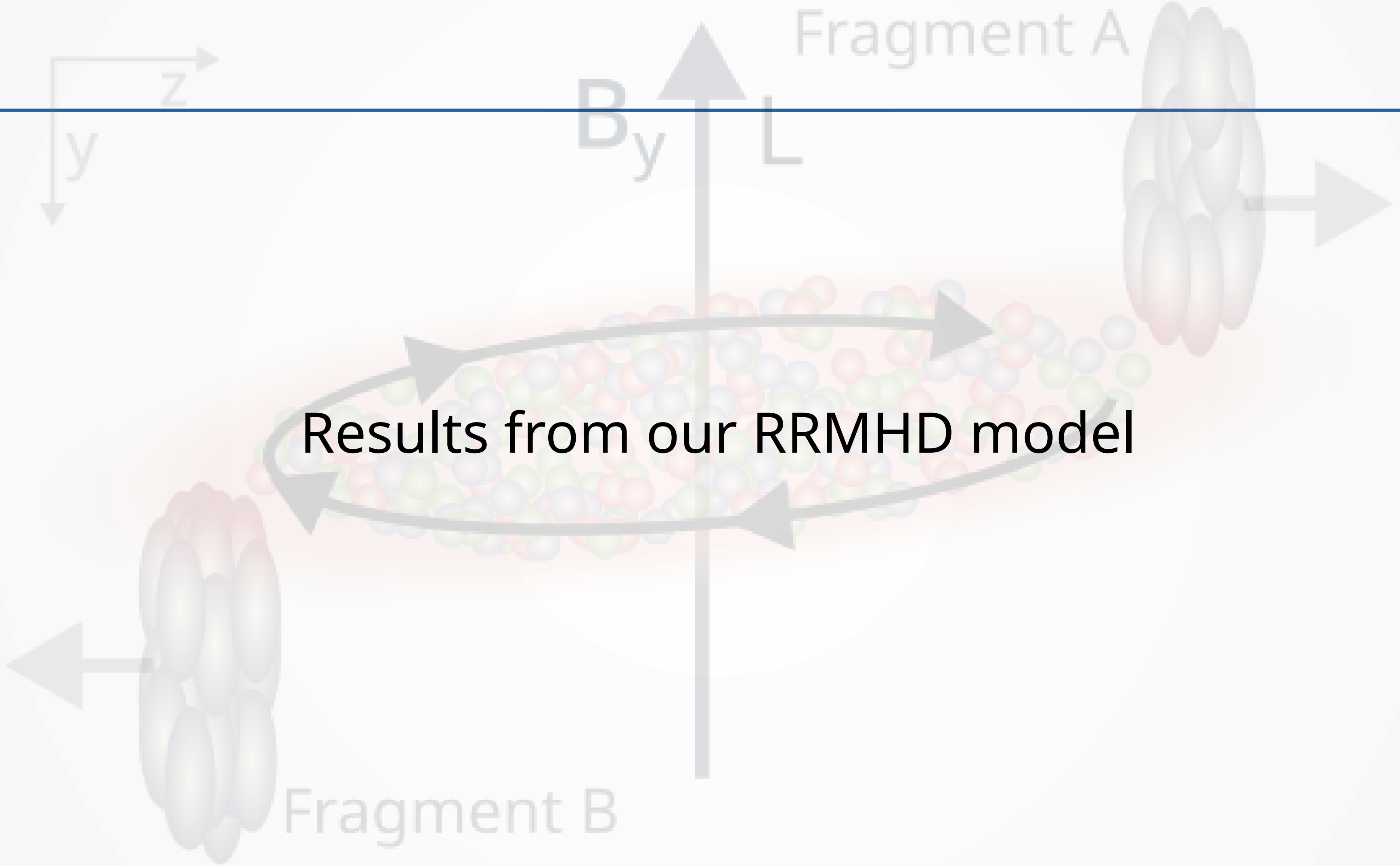
$$T^{\mu\nu}_{\mathrm{f}} = F^{\mu\lambda} F^\nu_\lambda - \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta}$$

EM fields part

$$\nabla_\mu T^{\mu\nu}_{\mathrm{m}} = -J_\mu F^{\mu\nu}$$

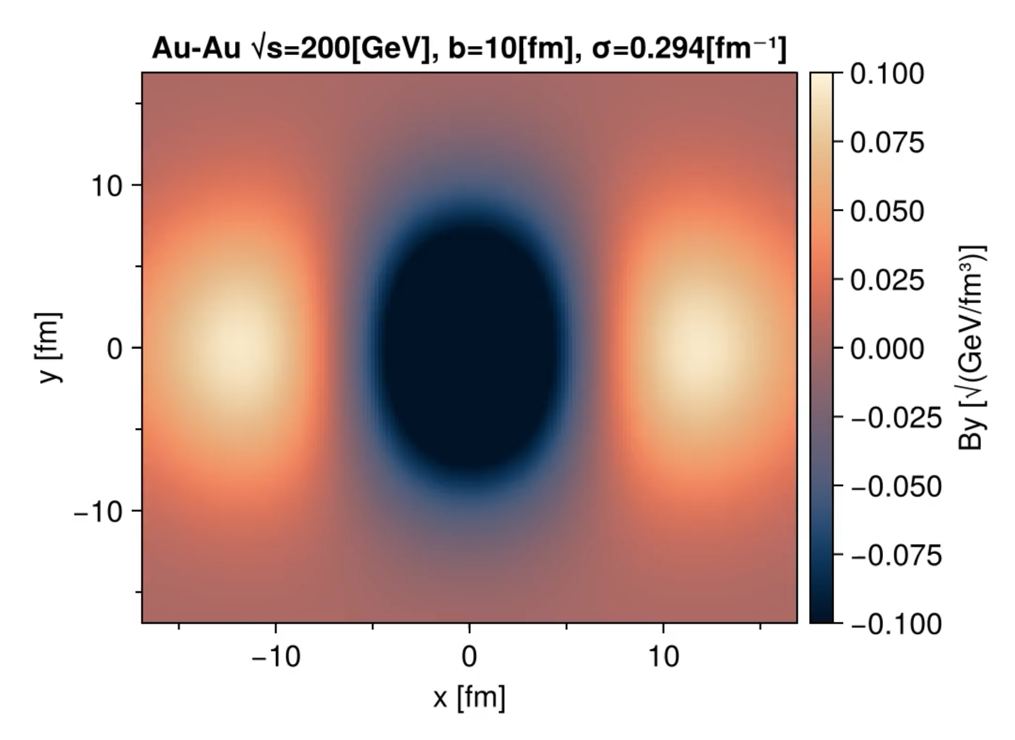
EM fields acting on the plasma and vice-versa

Results from our RRMHD model

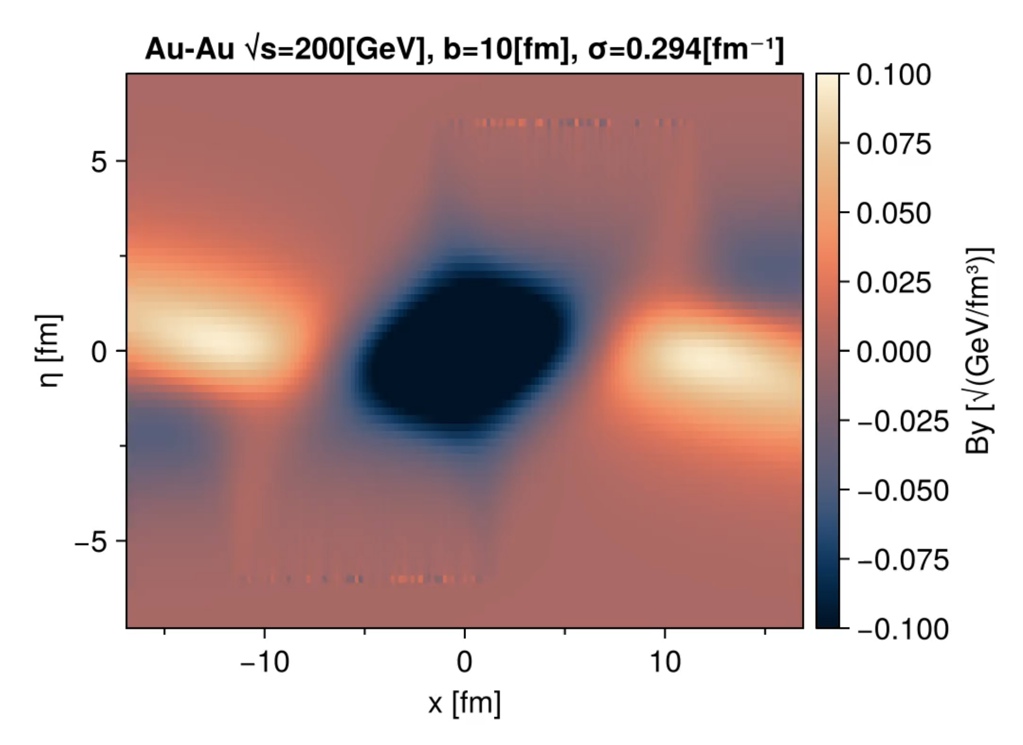


Time evolution of the magnetic field

Transverse Plane (η=0)



Collision Plane (y=0)



Time evolution of the magnetic field

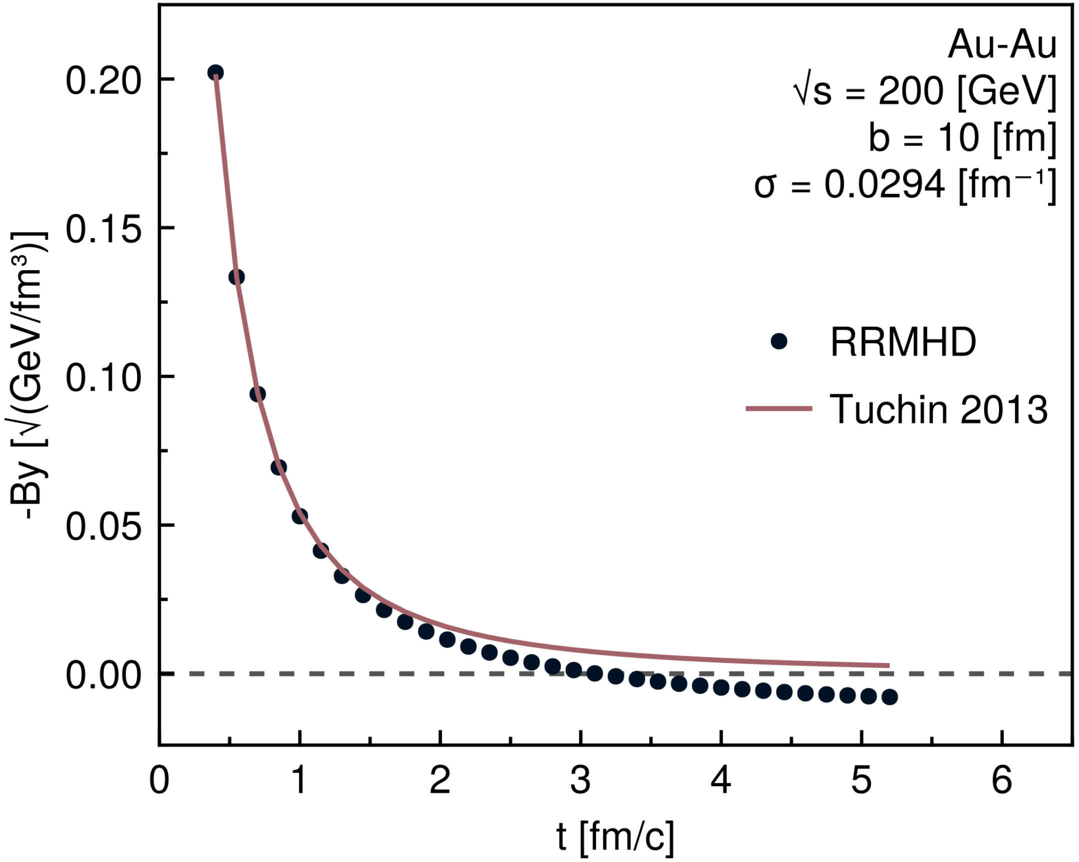
Our RRMHD model vs. an analytic estimation

Tuchin; PRC.88.024911 (2013), arXiv:1305.5806 [hep-ph]

- Captured @ center of grid (collision)
- At early times they agree because the initial conditions are the same
- Late time is different because of different source term in calculation

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu {}^*F^{\mu\nu} = 0$$

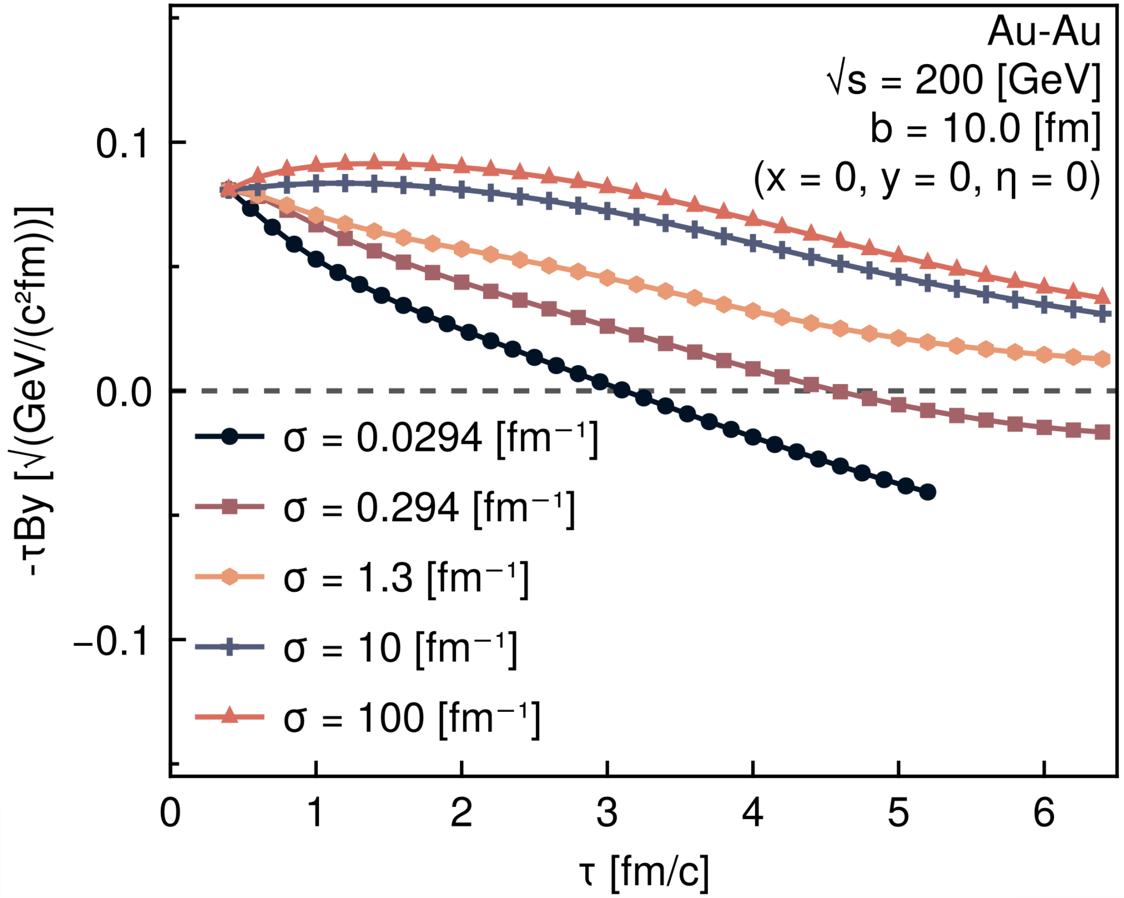
RRMHD = only QGP sources
Analytic = only collision spectators



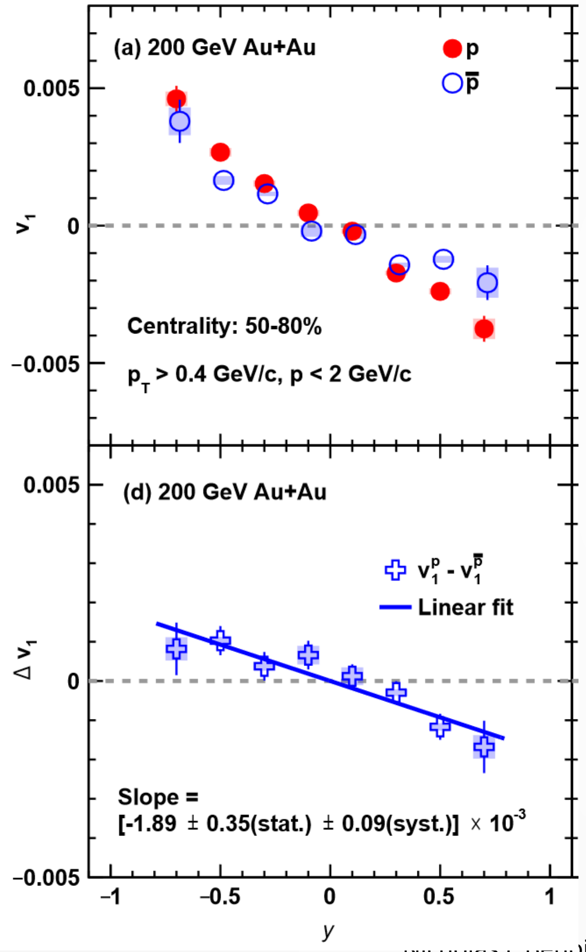
Time evolution of the magnetic field

- Captured @ center of grid (collision) starting from the same initial condition

- A larger electric conductivity means longer field lifetime

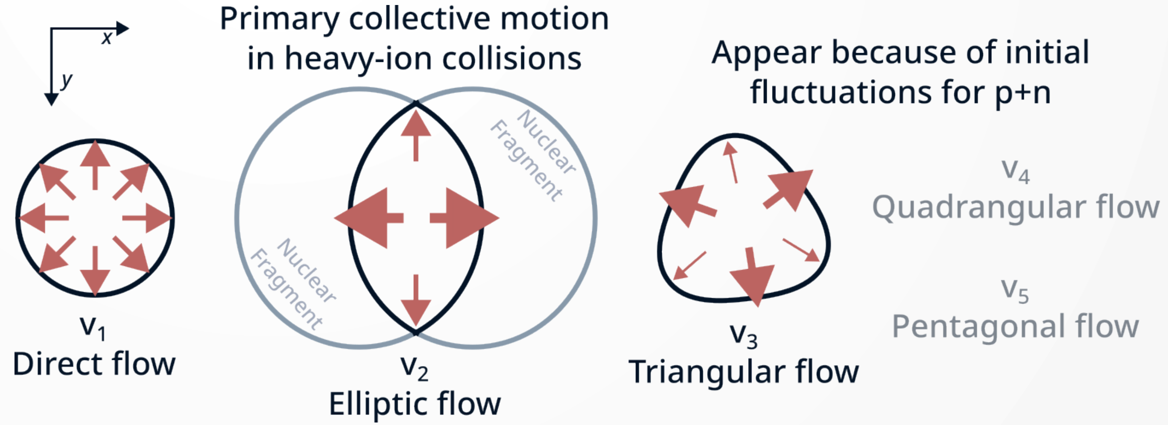


Recent STAR experimental result



- New 2024 results from STAR experiment for RHIC @ Phys. Rev X 14, 011028 (2024) arXiv:2304.03430 [hep-ex]

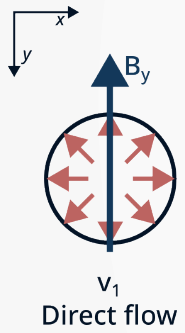
- They measure direct flow of charged hadrons like protons, kaons, and pions
- And they calculate the difference in flow between negatively – positively charged hadrons

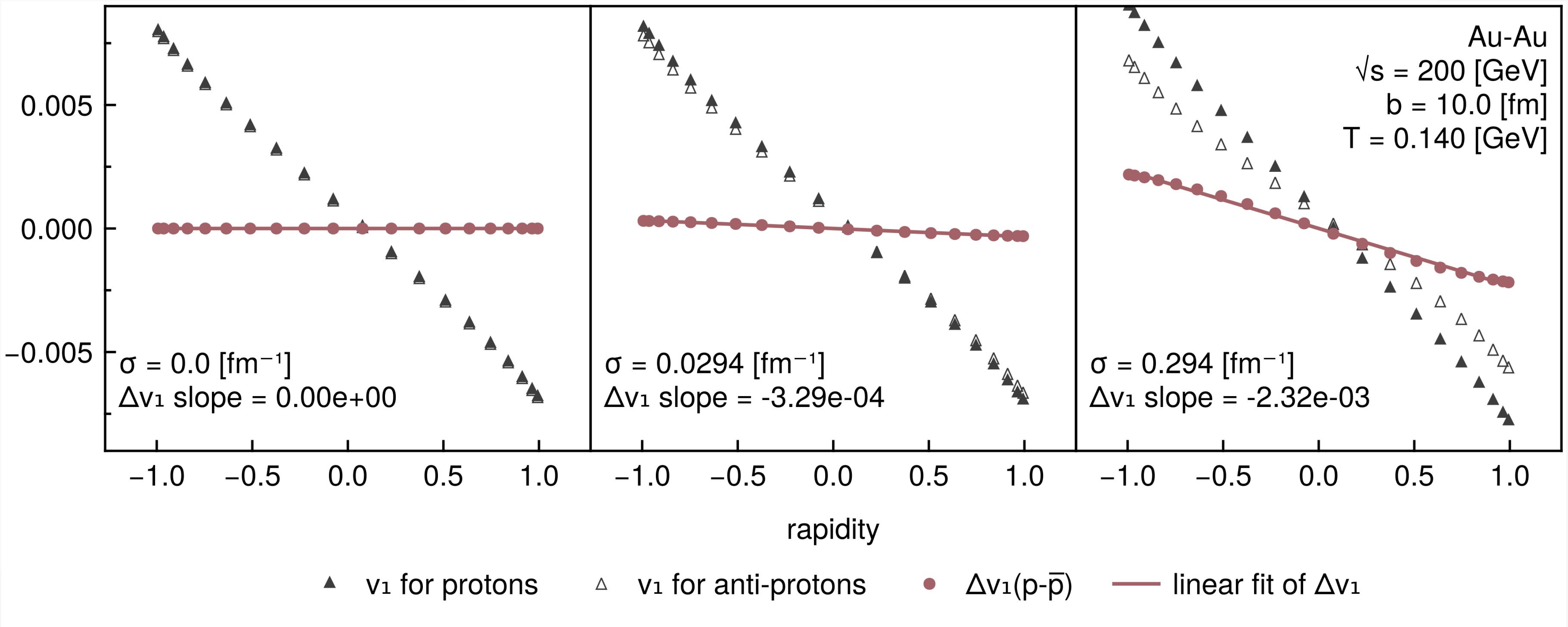


RRMHD Δv_1 results

- Our RRMHD model can reproduce the negative slopes from STAR results

STAR; PRX 14, 011028 (2024), arXiv:2304.03430 [hep-ex]





RRMHD Δv_1 results

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- Value of the negative slope depends on the electric conductivity of QGP

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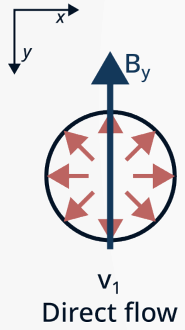

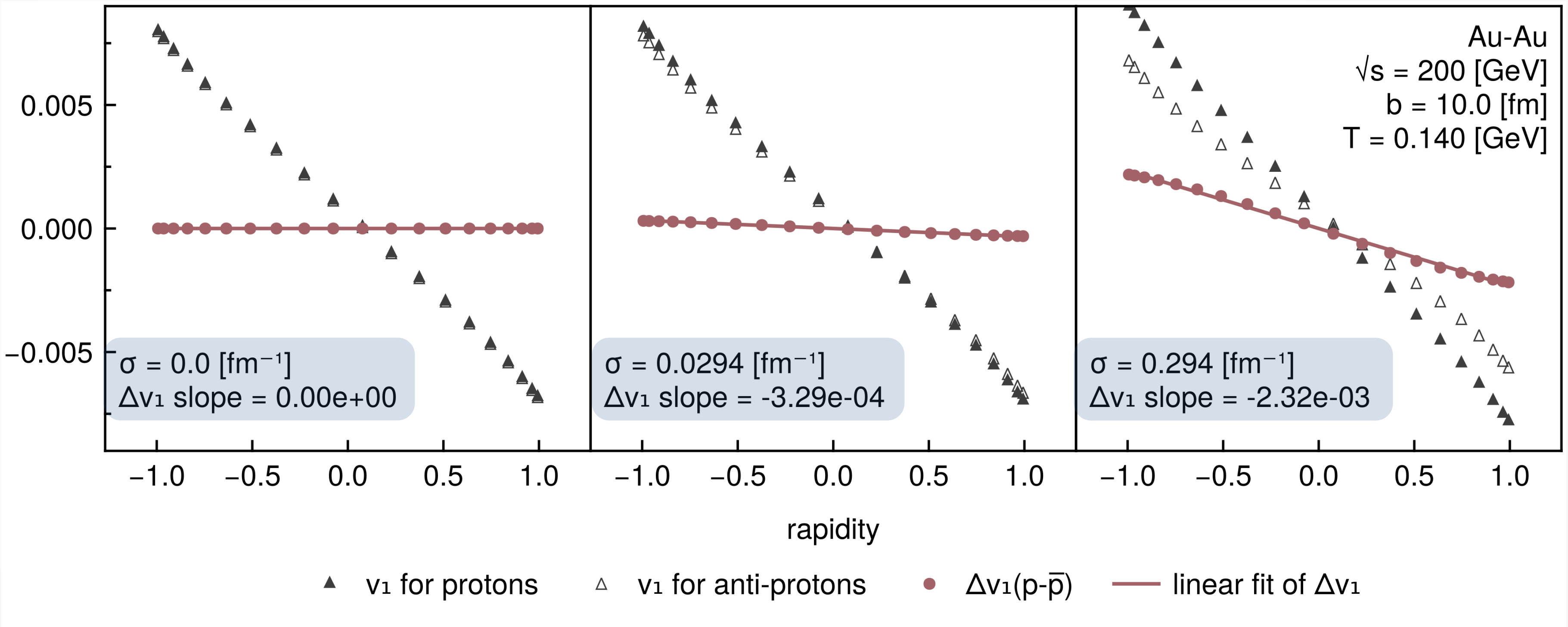

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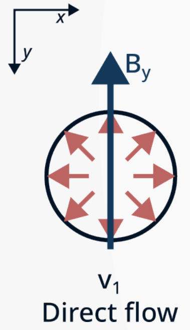
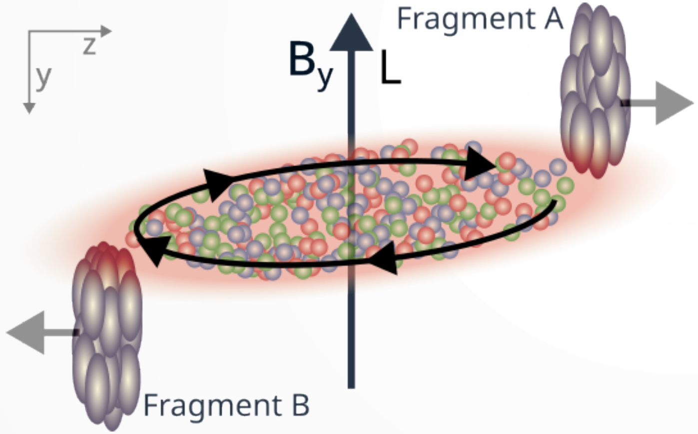
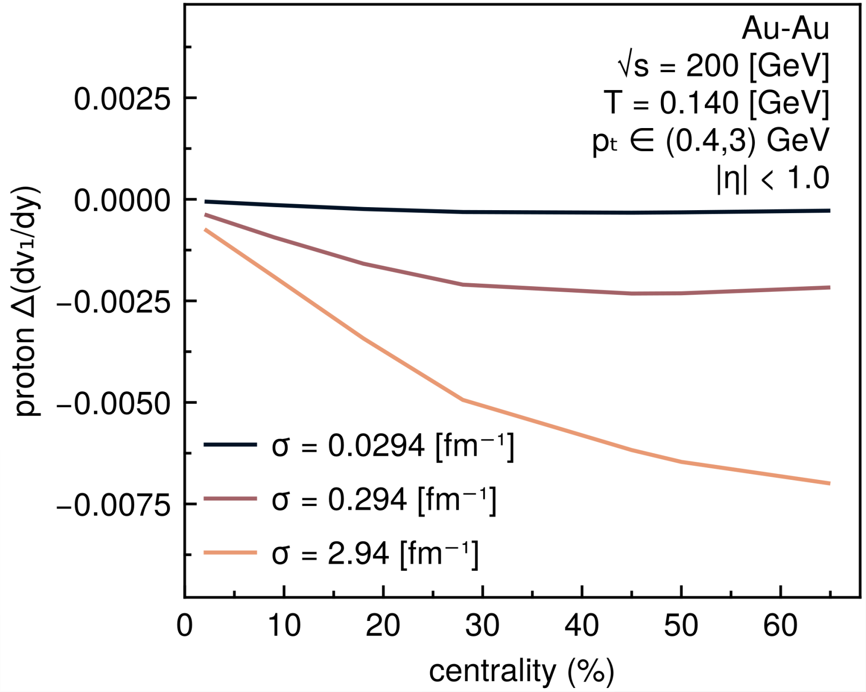
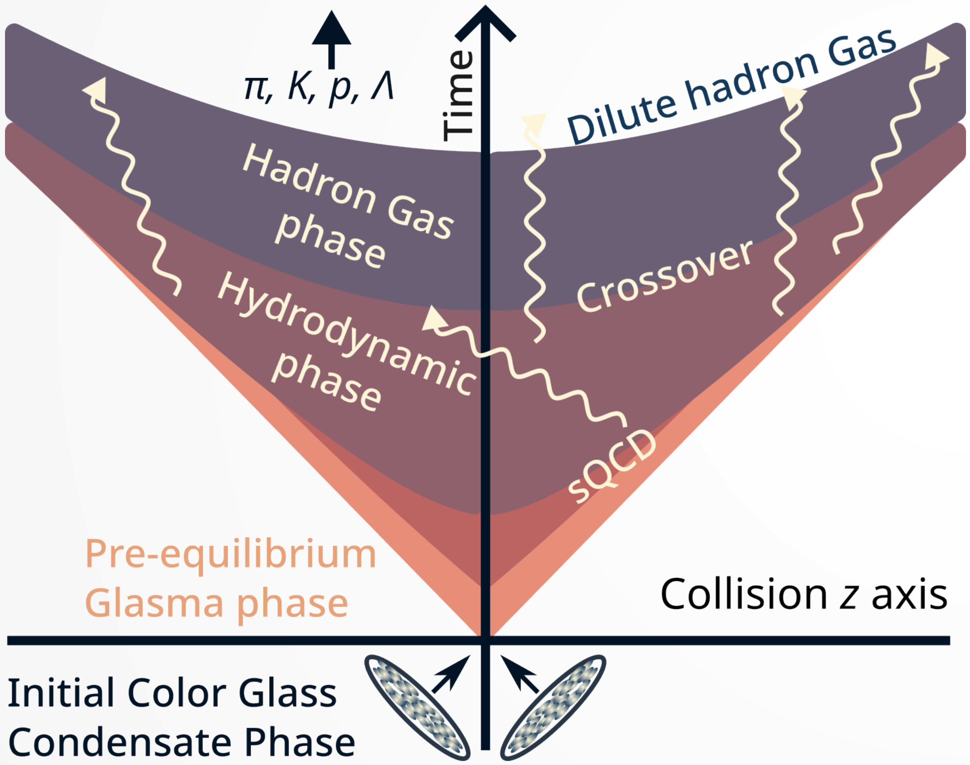


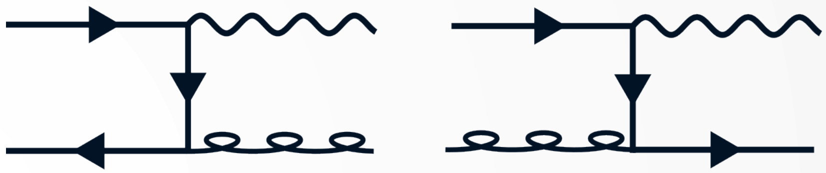




Thermal Photons from quark-gluon plasma

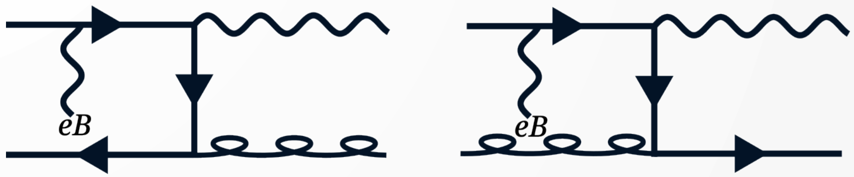


- Quark-gluon plasma will radiate photons as it expands and cools



Example of some leading order processes

- EM fields can increase the thermal photon production



Thermal Photons from quark-gluon plasma

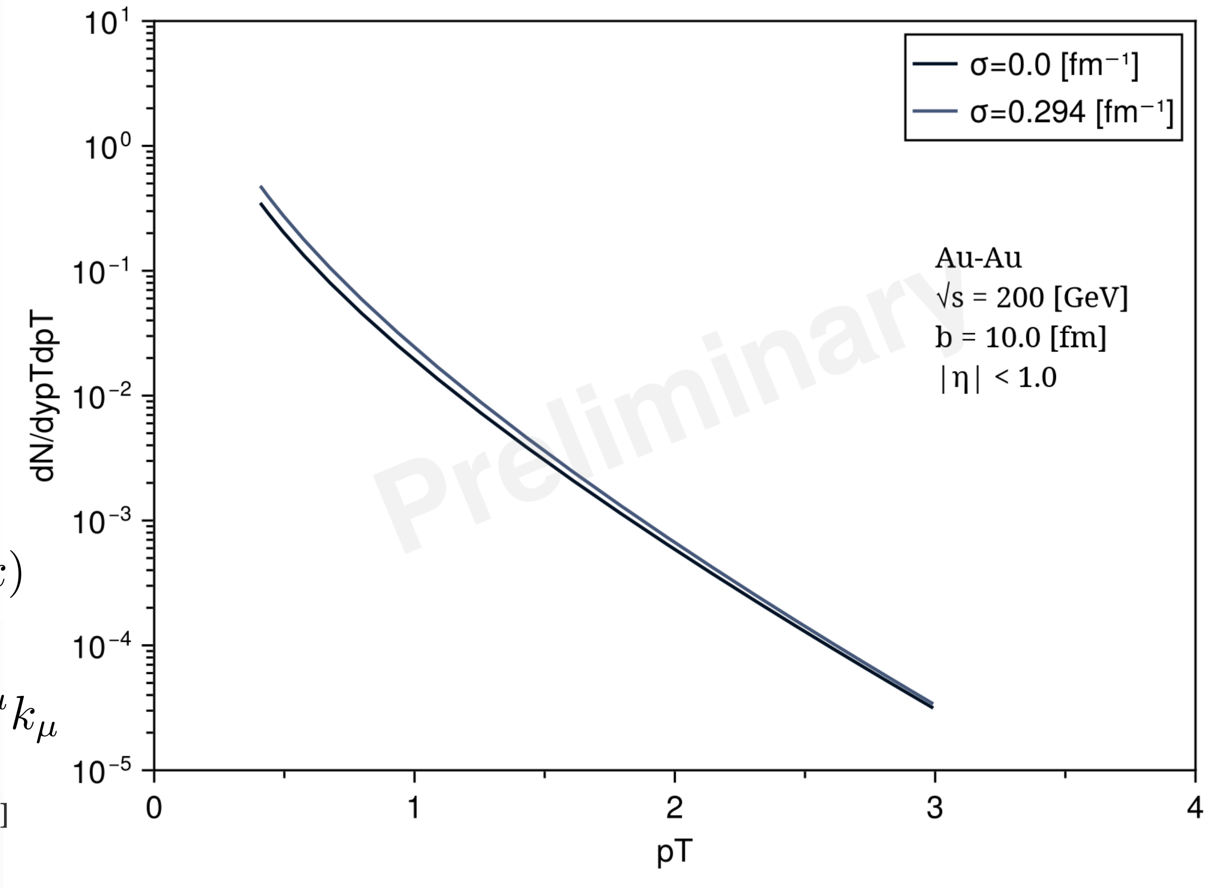
- Small number of photons produced, because EM fields are "weak" compared to QGP

$$E_k \frac{d\mathcal{R}}{d^3\vec{k}} = E_k \frac{d\mathcal{R}^{\mathrm{QGP}}}{d^3\vec{k}} + E_k \frac{d\mathcal{R}^{\mathrm{EM}}}{d^3\vec{k}}$$

$$E_k \frac{d\mathcal{R}^{\mathrm{EM}}}{d^3\vec{k}} \simeq C\alpha_s\alpha_{\mathrm{EM}}\mathcal{I}\mathcal{L}_c \sum_a \delta f^{(1)}_{a,\mathrm{EM}}(X,k)$$

$$\delta f^{(1)}_{a,\mathrm{EM}}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el}k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

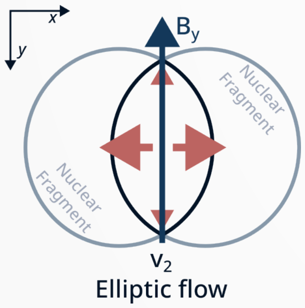
J.A. Sun and L. Yan; PRC.109.034917 arXiv:2311.03929 [hep-ph]

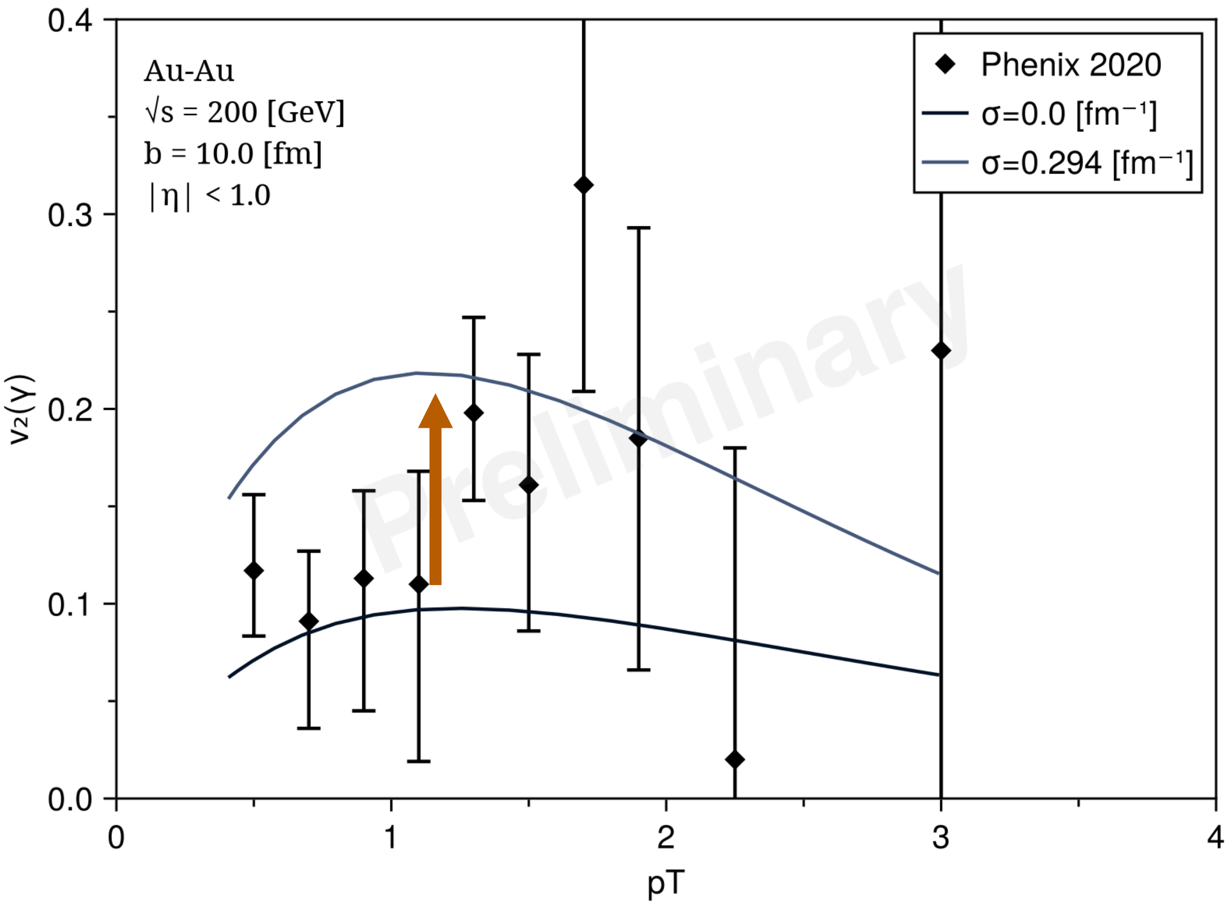


Thermal Photons from quark-gluon plasma

- Enhancement of the photon elliptic flow when the EM fields are included

$$v_2(\gamma) \equiv \frac{v_0 v_2 + v_0^{\mathrm{EM}} v_2^{\mathrm{EM}}}{v_0 + v_0^{\mathrm{EM}}}$$

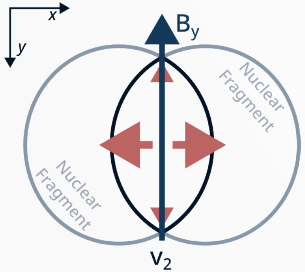


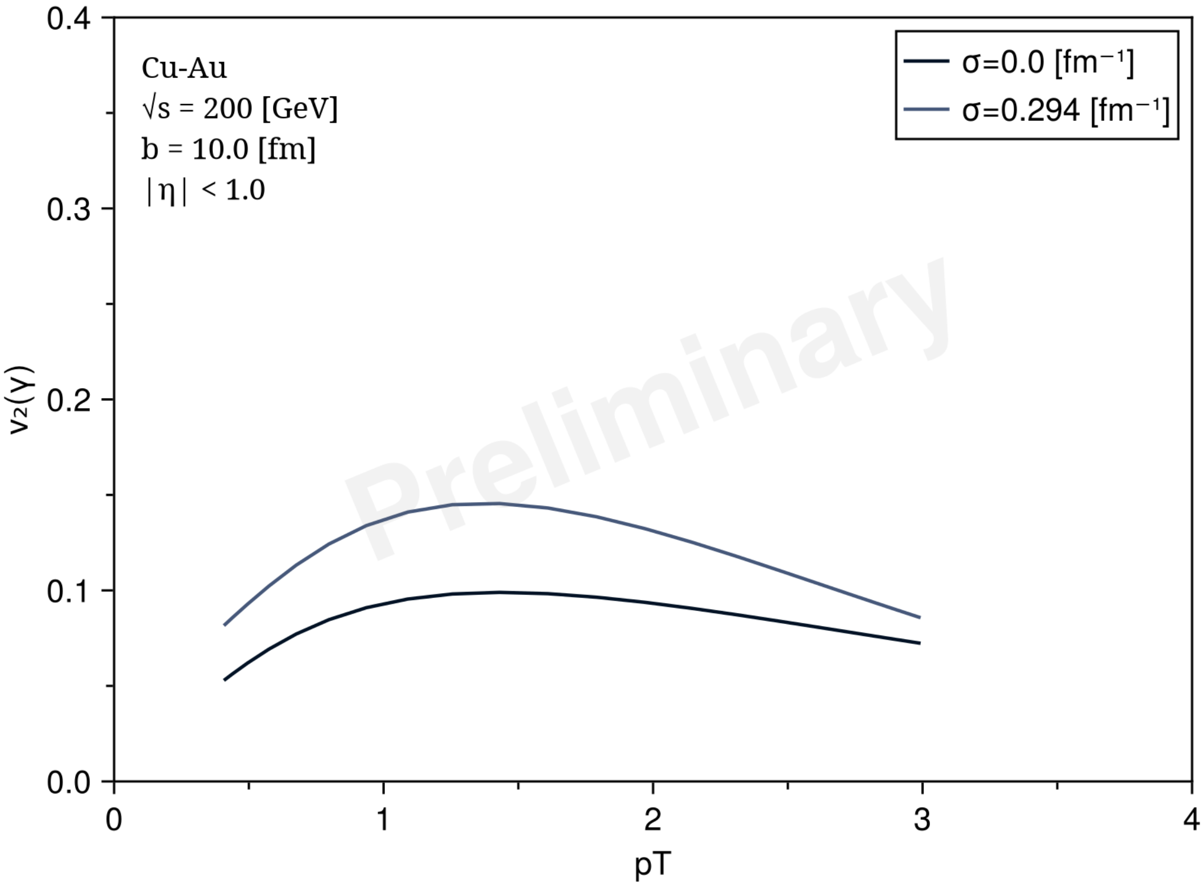


Thermal Photons from quark-gluon plasma

- Asymmetric collision the QGP thermal photon is similar to symmetric collision
- But, enhancement from the EM fields is much smaller

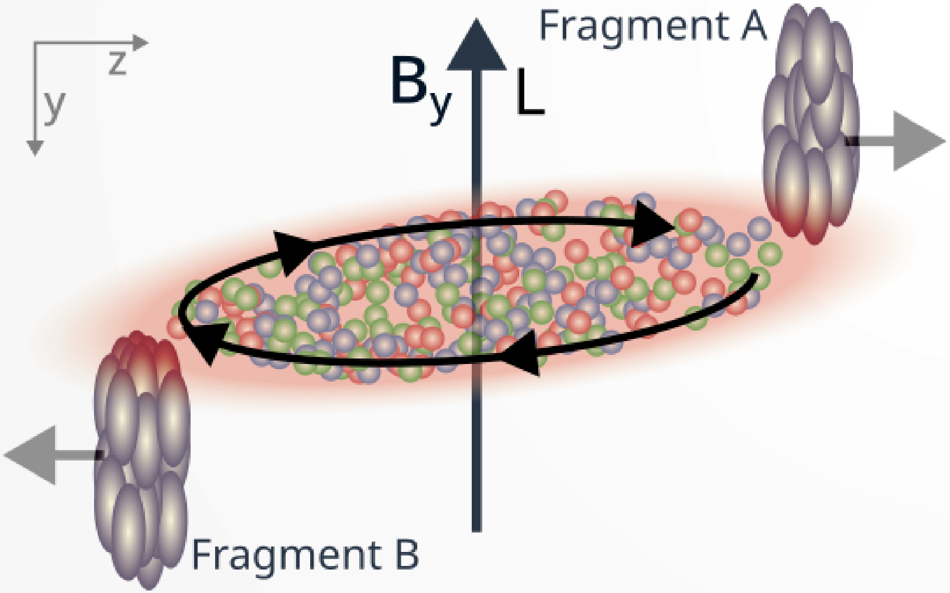
$$v_2(\gamma) \equiv \frac{v_0 v_2 + v_0^{\mathrm{EM}} v_2^{\mathrm{EM}}}{v_0 + v_0^{\mathrm{EM}}}$$





Summary and Outlook

- We have applied relativistic resistive magneto-hydrodynamics (RRMHD) to studying charged observables in heavy-ion collisions
- Our results show how important EM fields are to those charged observables
- Progress toward a robust calculation of QGP+EM fields



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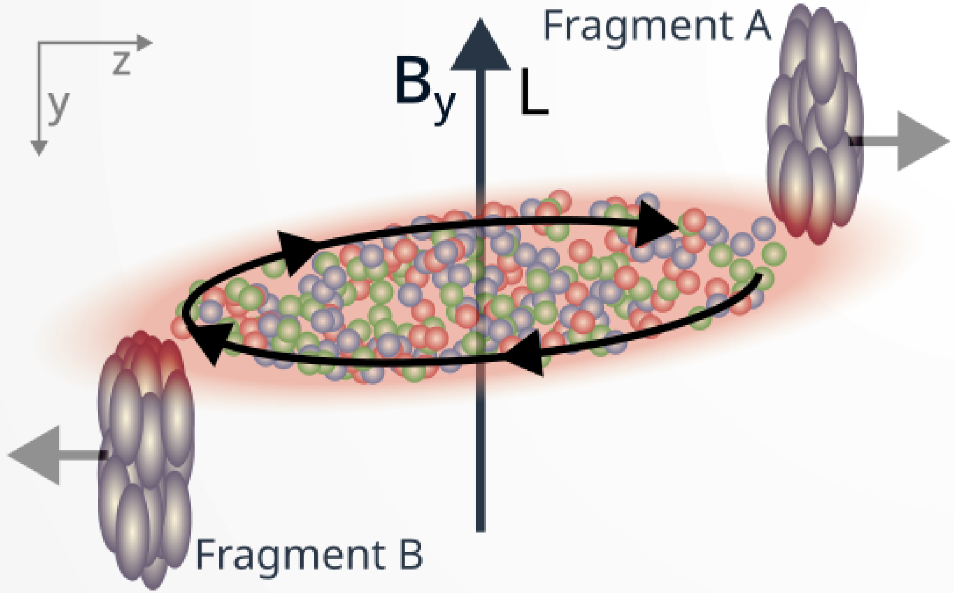
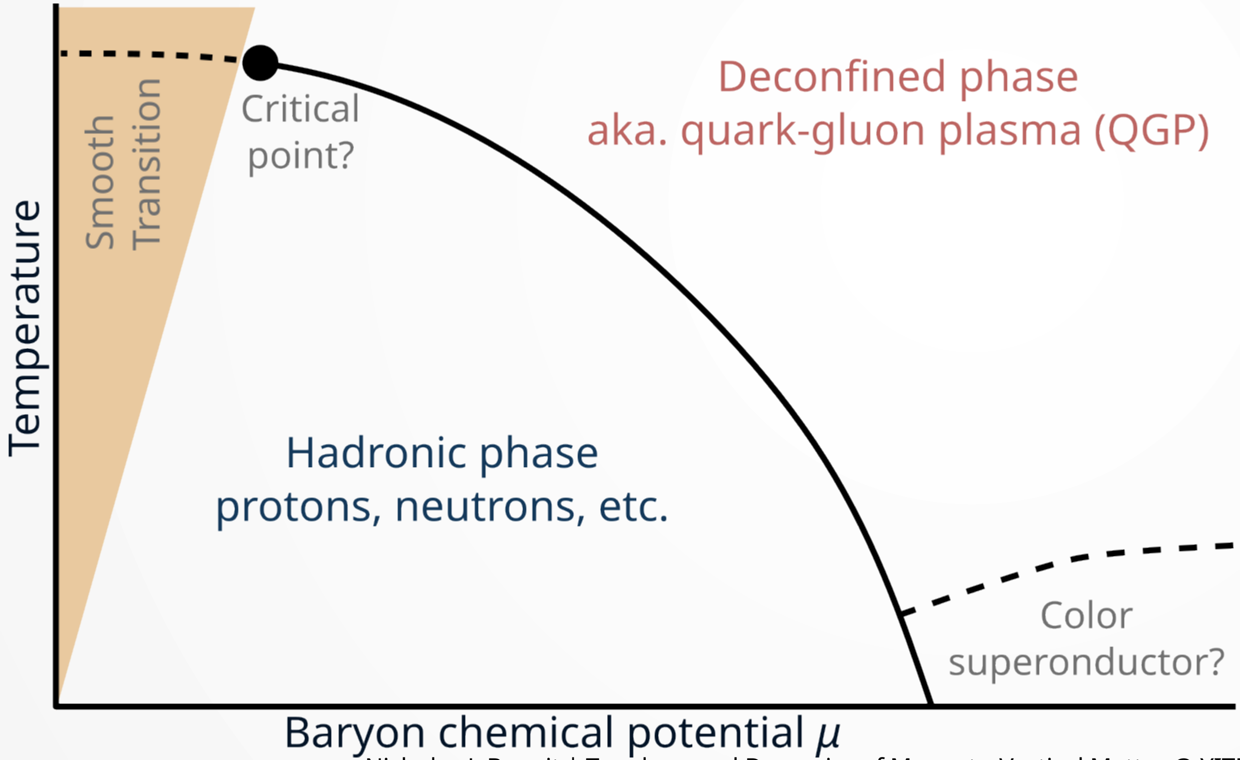


- In the future,
  - Non-equilibrium initial state
  - Hadron Gas + EM fields phase
  - Hydrodynamic fluctuations
  - Vortexes + EM fields
  - etc.

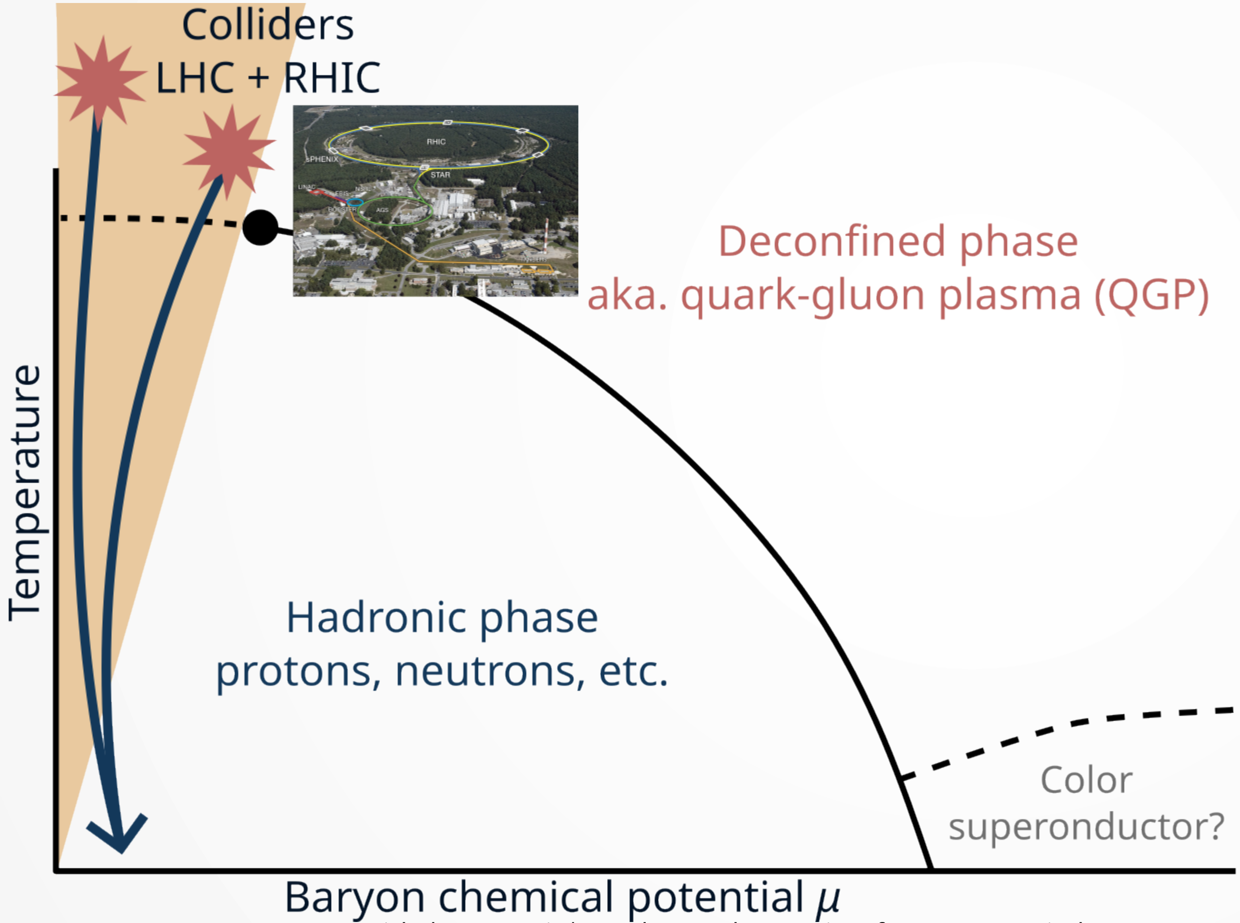
- Lots of work and results to come!

Heavy-ion collisions

- Heavy-ion collisions study ordinary matter under extreme conditions



Heavy-ion collisions

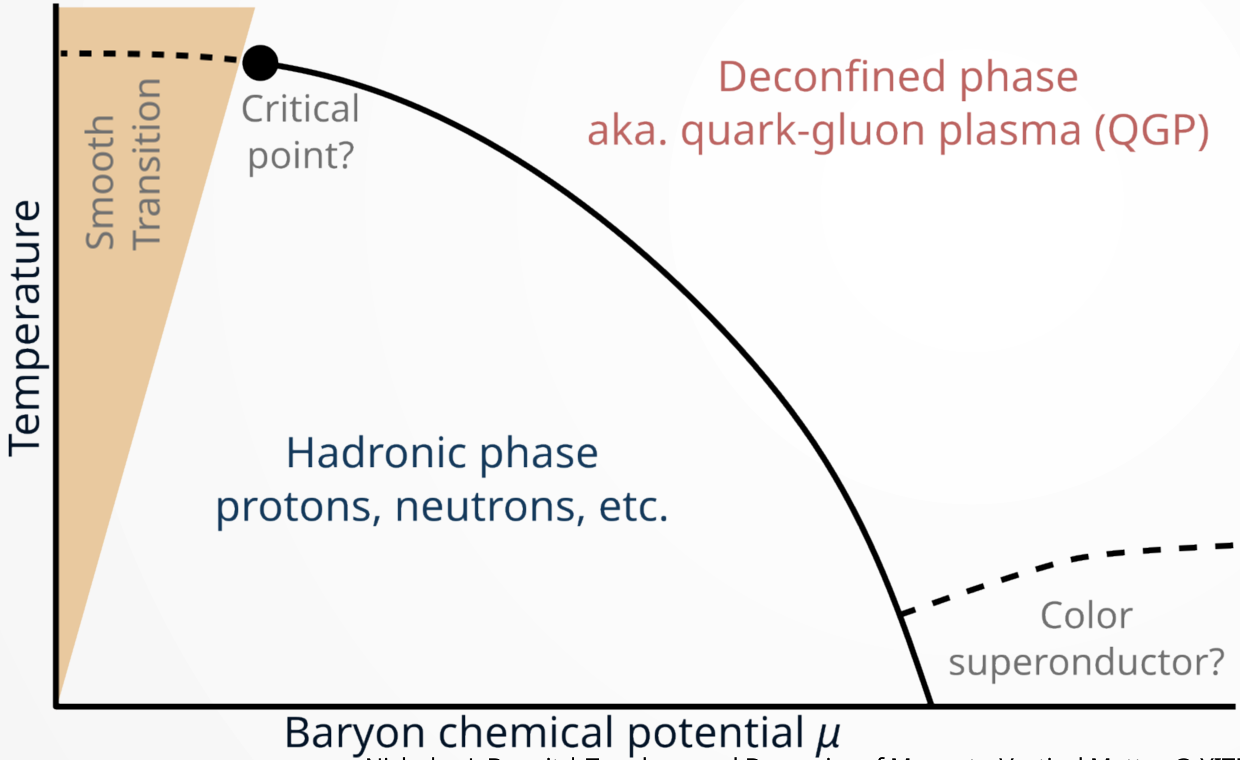


We use high-energy colliders to study the deconfined phase of QCD matter called quark-gluon plasma (QGP)

SPS @ CERN (1982~2001?)
RHIC @ BNL (2001~2025)
LHC @ CERN (2010~)

Why study quark-gluon plasma?

- Better understanding of the Strong force and confinement
- Possible similar conditions as the early universe (high temperature)



- Medium energy collisions can study the QCD phase transition and critical point
- Low energy collisions can be connected to studies of neutron stars (large baryon chemical potential)

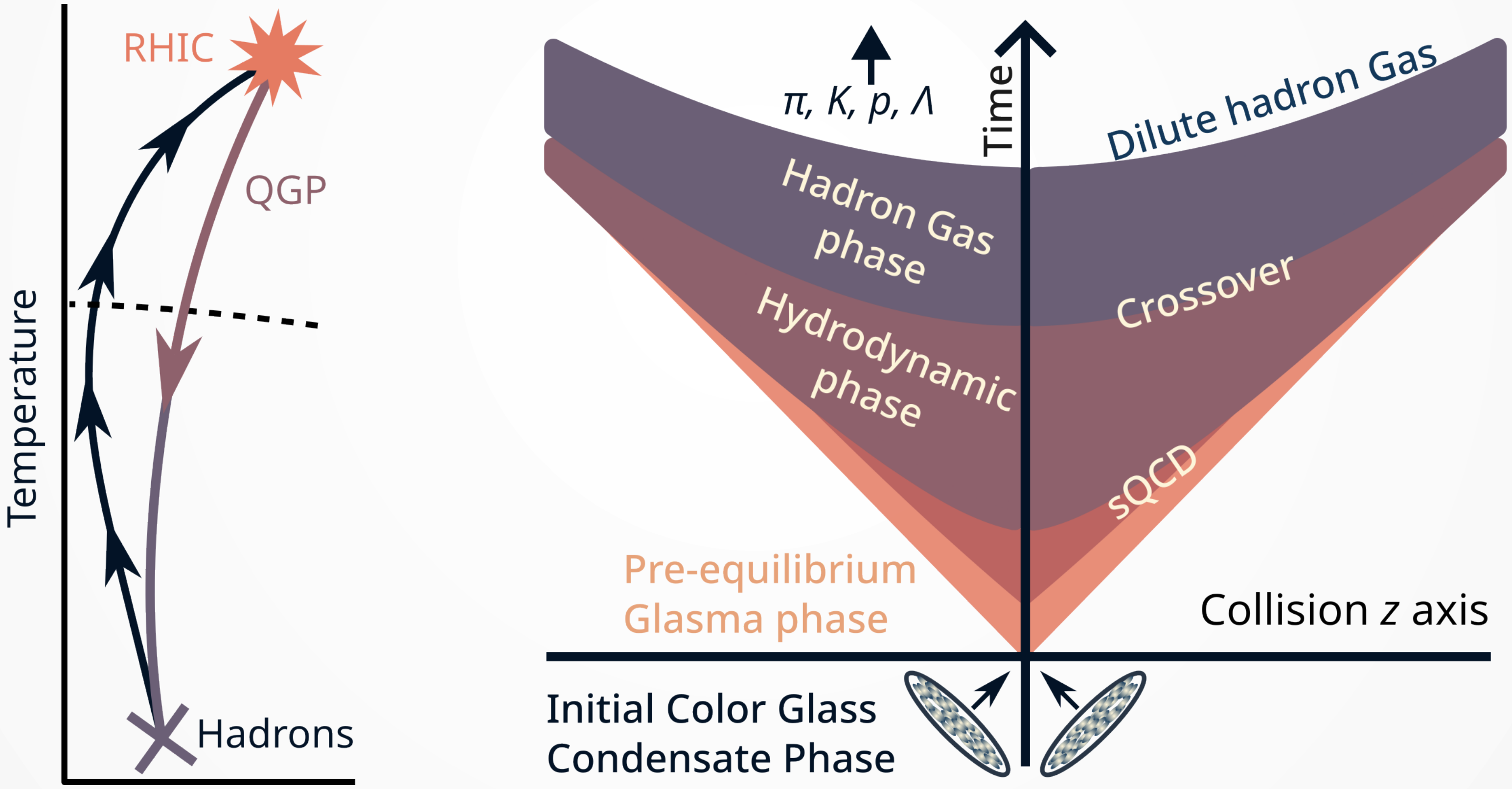
What kind of collective motion appears?

- Details of the collective motion are captured in transport parameters
- For QGP we have,

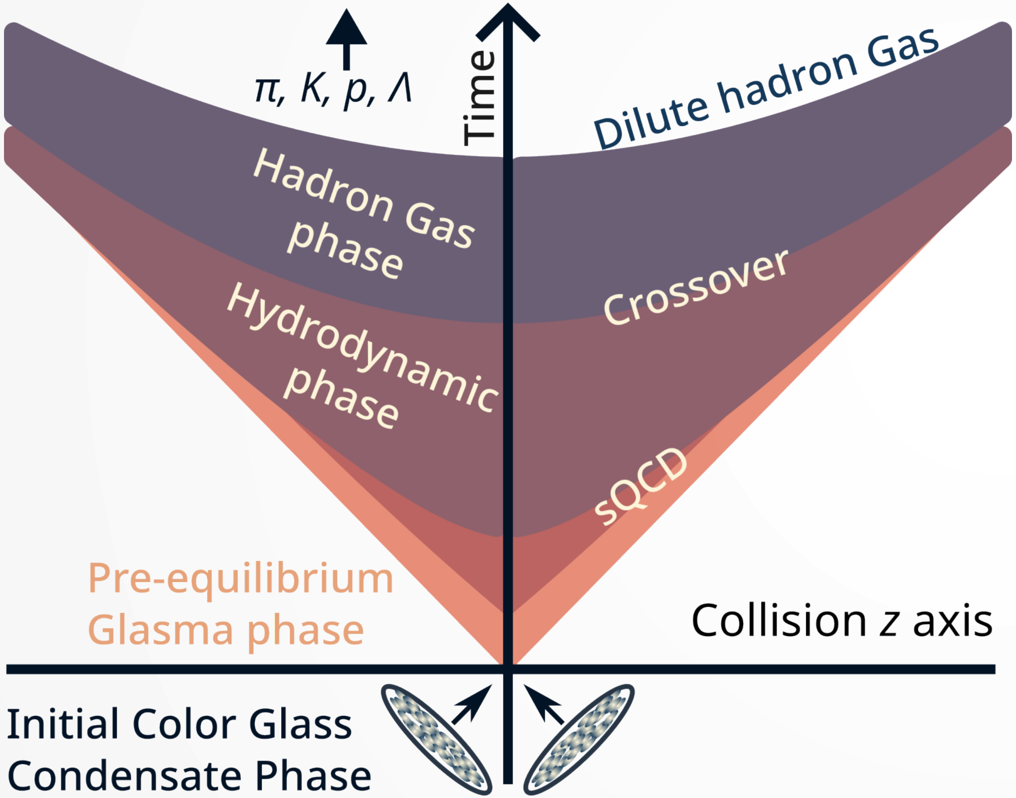
Shear viscosity	η	Energy-momentum
Bulk viscosity	ζ	
Charm-diffusion coefficient	D	Heavy-flavor quantum numbers
Thermal conductivity	κ	Heat via baryon current
Electric conductivity	σ	Electrical charges via the electric current

Stages in relativistic heavy-ion collisions

- Studying QGP is not simple, many phases before experimental measurement



Why model using hydrodynamics?



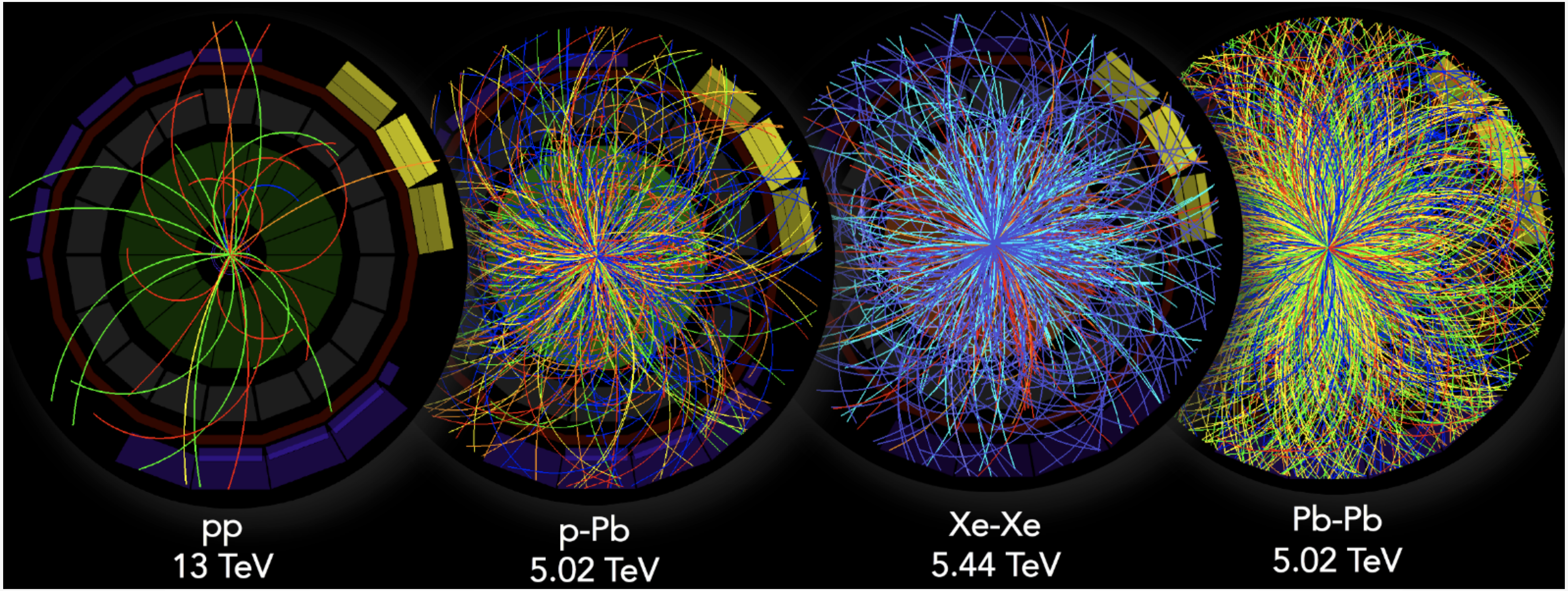
- Why the hydrodynamics phase?
- Many liberated quarks and gluons, so it becomes a many-body problem

What about kinetic transport models like the Boltzman equation?

- Let's look to experiments for an answer

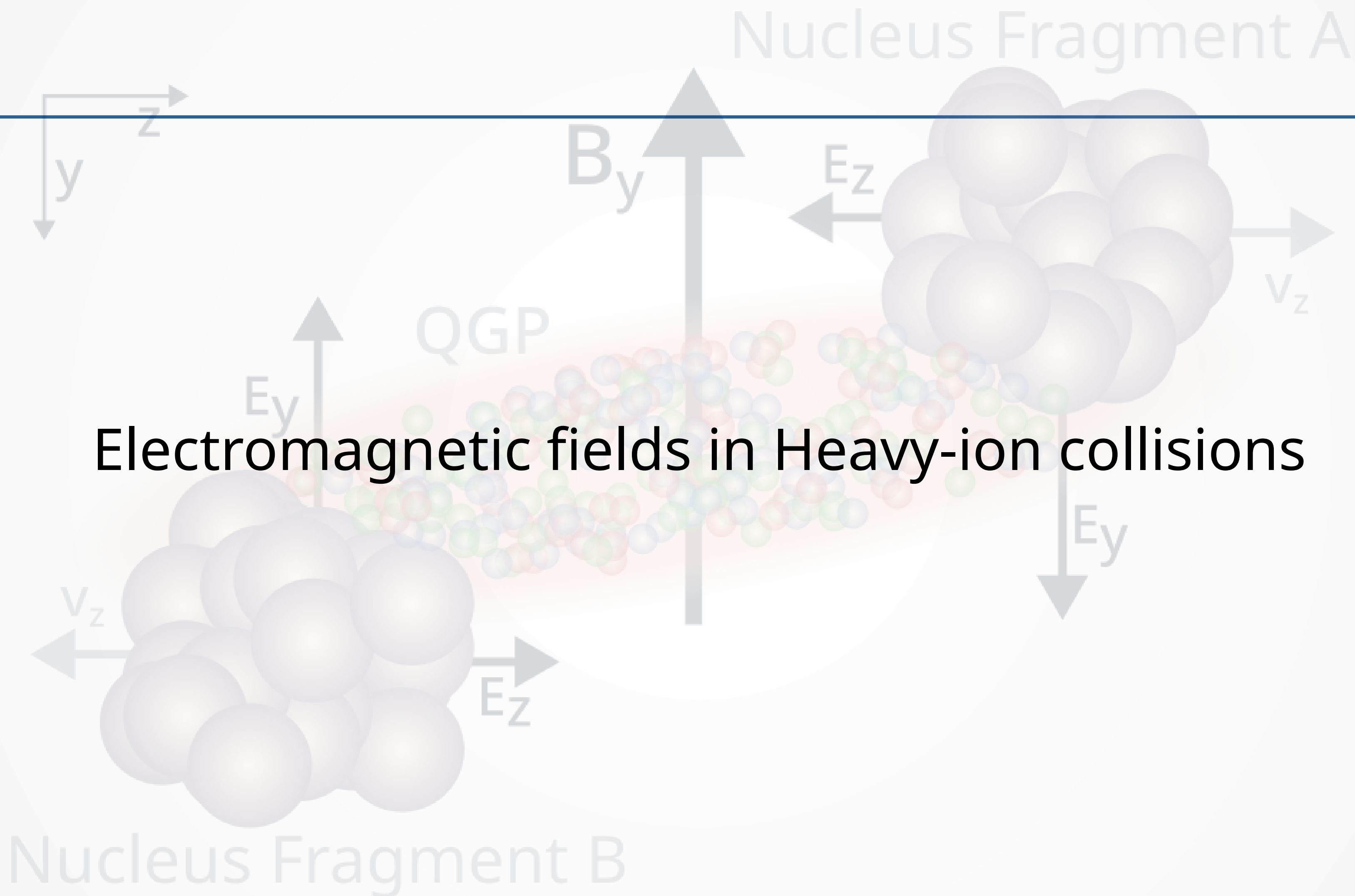
Why model using hydrodynamics?

- Heavy-ion collisions produce a large amount of particles



From: A.P. KALWEIT, PoS (EPS-HEP2023) 027

Electromagnetic fields in Heavy-ion collisions



What about the protons?

Resources: Jackson (1975), Feynman (2010)

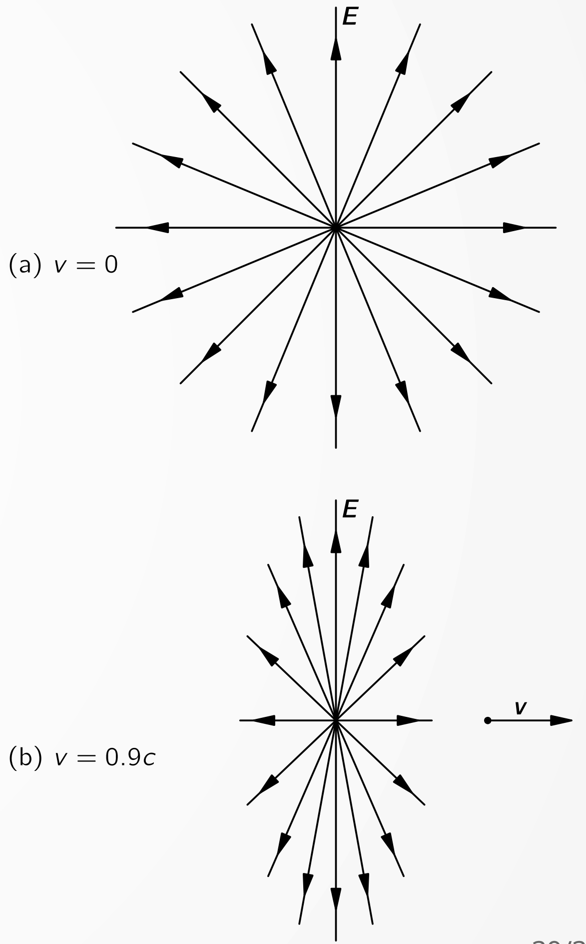
- Using a simple picture, of a point charge moving at a constant velocity in the z-direction

$$E_{x,y} = q\gamma \frac{\vec{x}}{\left[\vec{x}^2 + \gamma^2 (z - vt)^2\right]^{3/2}},$$

$$E_z = q\gamma \frac{z - vt}{\left[\vec{x}^2 + \gamma^2 (z - vt)^2\right]^{3/2}}$$

$$\vec{B} = \vec{v} \times \vec{E}$$

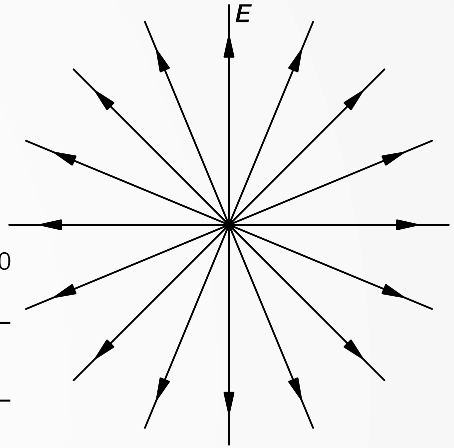
aka: Lienard-Wiechert Fields for a constant velocity



What about the protons?

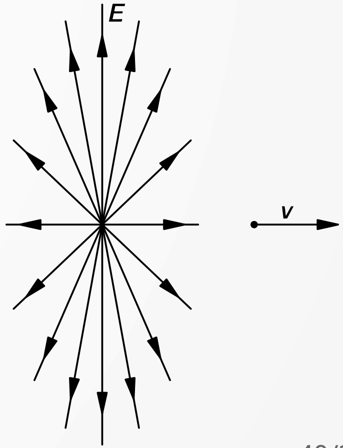
Resources: Jackson (1975), Feynman (2010)

- Summing over all of the charges in a nuclei, produces the strongest EM fields ever created



(a) $v = 0$

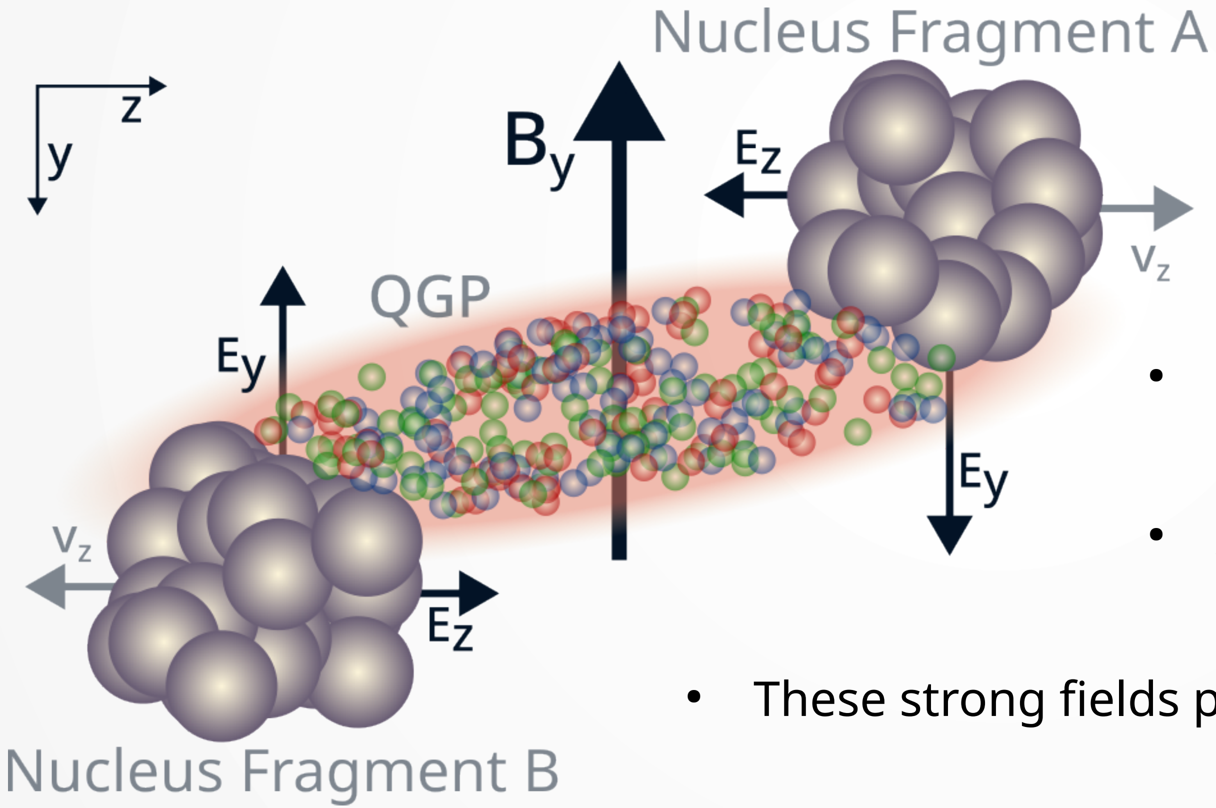
~ 10 T	MRI
~ 45 T	Strongest continuous magnetic fields in labs (China & USA)
~10^9 T	EM fields become nonlinear (Schwinger limit)
~10^{13} T	Strongest magnetic pulsars
~10^{14} T	Possible field strength at 200 GeV RHIC collisions



(b) $v = 0.9c$

Role of the electromagnetic (EM) fields

- Fragments of the collision play an important role as a source of the EM fields



- E-fields from fragmented protons are squished and stretched
- A strong B-field follows in the wakes of the moving nuclei

- These strong fields penetrate the quark-gluon plasma

Role of the electromagnetic (EM) fields

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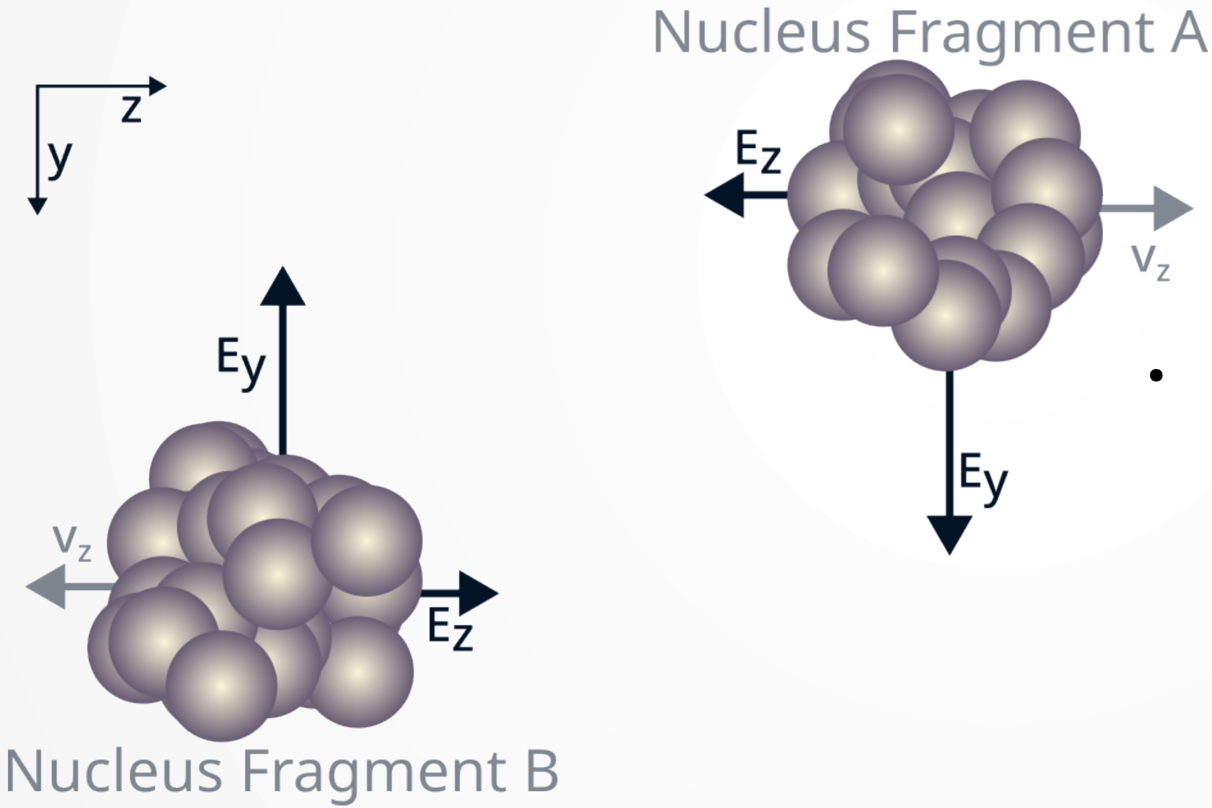

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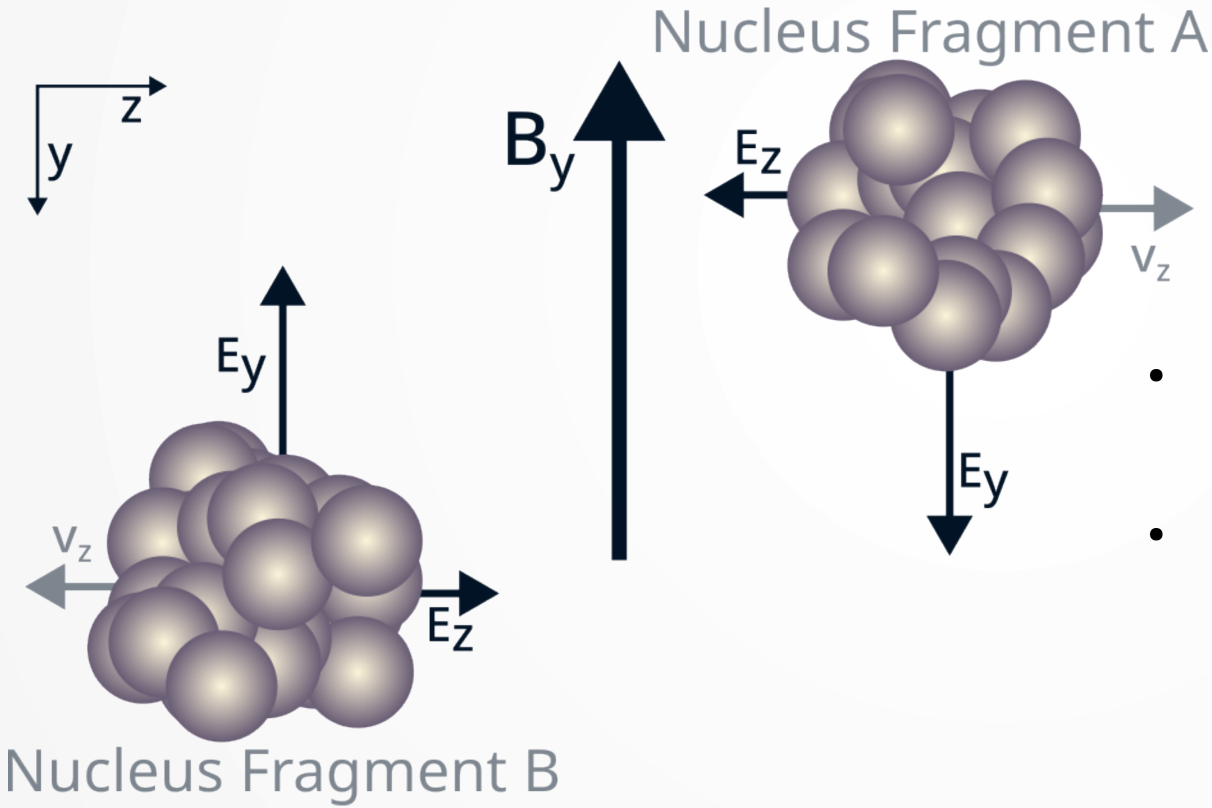

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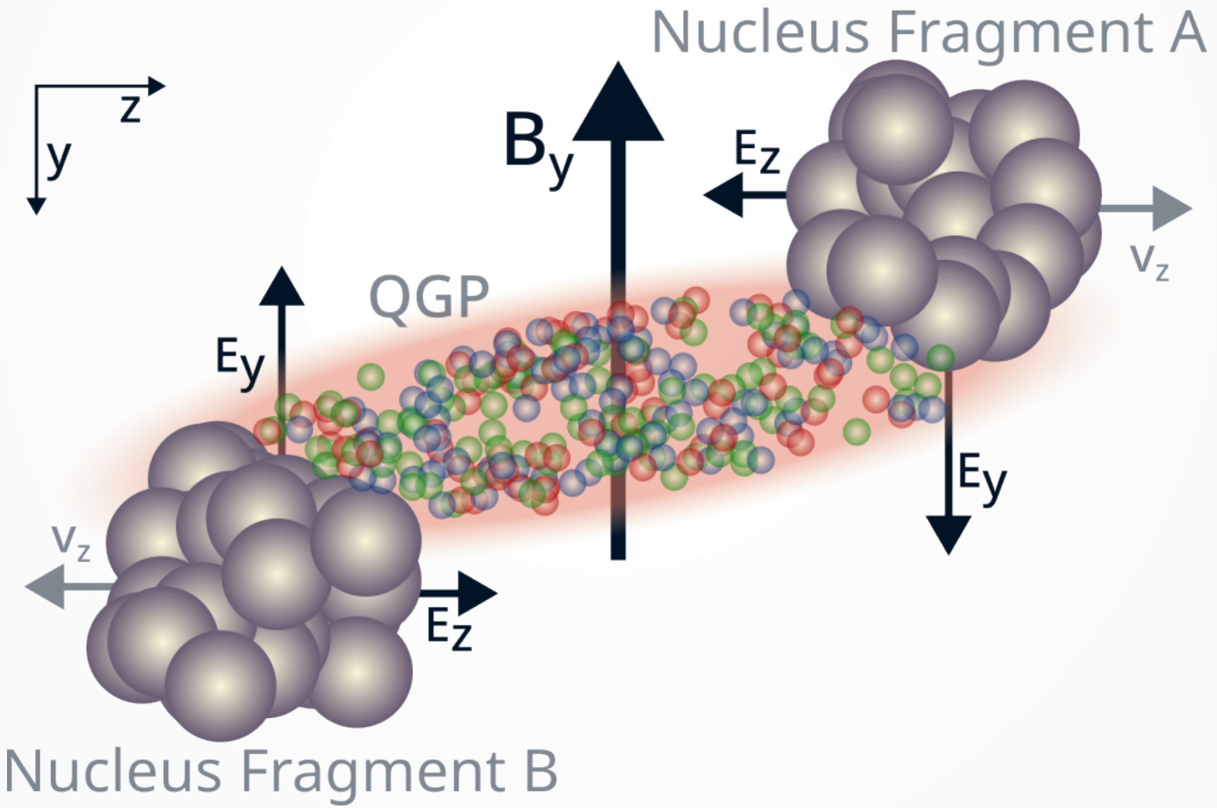
History of relativistic resistive magneto-hydrodynamics

- We are the first group to apply RRMHD to the study of QGP



Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- EM fields penetrating QGP drive charge carriers out-of-equilibrium

$$J^\mu = q u^\mu + \sigma F^{\mu\nu} u_\nu$$

EM current in the QGP medium

First order dissipation from the EM fields

- Taking the Boltzmann equation in the relaxation time approximation focus on 2→2 processes,

$$k^\mu \partial_\mu f_a + e Q_a F^{\mu\nu} k_\mu \frac{\partial f_a}{\partial k^\nu} = -\frac{k^\mu u_\mu}{\tau_R} \delta f^{(n)}_{a,EM}$$

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Vlasov term for the external EM fields

Order "n" corrections to the quark distribution function because of dissipation from the EM fields

Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- For this calculation we focus on 1st order corrections,

$$k^\mu \partial_\mu f_a + eQ_a F^{\mu\nu} k_\mu \frac{\partial f_a}{\partial k^\nu} = -\frac{k^\mu u_\mu}{\tau_R} \delta f_{a,EM}^{(n)}$$

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$$f_a = f_{a,eq} + \delta f_{a,EM}^{(1)} + \delta f_{a,EM}^{(2)} + \delta f_{a,EM}^{(3)} + \cdots$$

Ordered by the EM field strength

$$\delta f_{a,\mathrm{EM}}^{(1)}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el} k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

Electric conductivity of QGP from Landau matching with the current

$$J^\mu = qu^\mu + \sigma F^{\mu\nu} u_\nu$$

EM fields in the fluid rest frame

$$e^\mu = \left(\gamma v_k E^k, \quad \gamma E^i + \gamma \epsilon^{ijk} v_j B_k\right)$$

Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- What we do is calculate the fluid + EM field contributions using hydrodynamics

Temperature and four-velocity

Electric conductivity value

Values that come from a hydrodynamic calculation

$$\delta f^{(1)}_{a,\mathrm{EM}}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el}k^{\mu}u_{\mu}} e\sigma Q_a e^{\mu} k_{\mu}$$

$$e^{\mu} = \left(\gamma v_k E^k, \quad \gamma E^i + \gamma \epsilon^{ijk} v_j B_k\right)$$

Spacetime dependent EM fields in QGP medium

Electric susceptibility of QGP

$$\chi_{a,el} = -\frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3 E_p}(p^{\sigma}p^{\nu}\Delta_{\sigma\nu})\frac{-f_{a,eq}(1-f_{a,eq})}{p^{\mu}u_{\mu}}$$

Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- What we do is calculate the fluid + EM field contributions using hydrodynamics
- All of those values can be calculated self-consistently using relativistic resistive magneto-hydrodynamics (RRMHD)

Temperature and four-velocity

Electric conductivity value

Values that come from a hydrodynamic calculation

$$\delta f^{(1)}_{a,\mathrm{EM}}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el}k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

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Thermal Photons from quark-gluon plasma

- Enhancement of the photon elliptic flow when the EM fields are included