Rotating Field Theory: Beyond Landau-Lifshitz-Pitaevski

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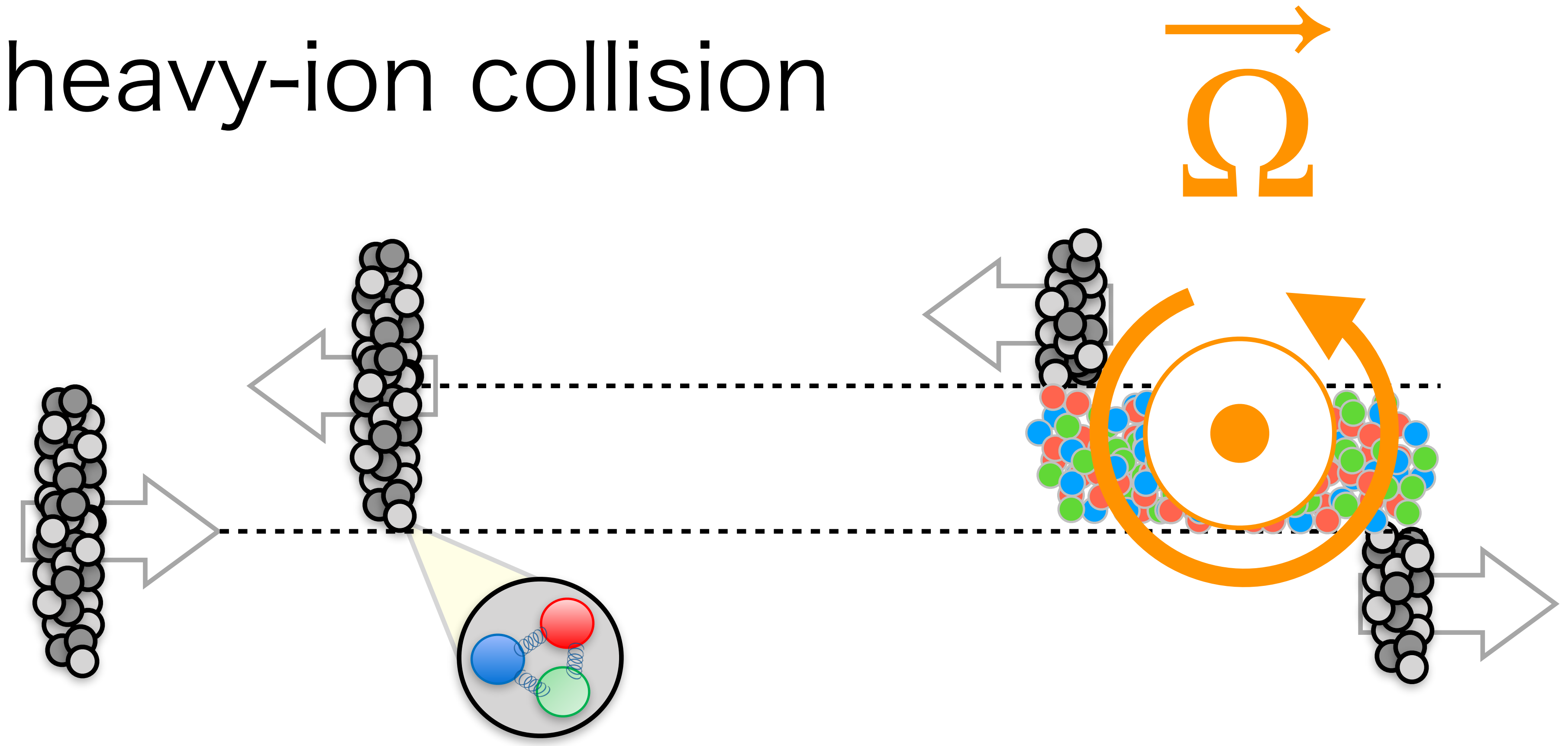
H.-L.-Chen, K. Fukushima, X.-G. Huang, and KM, PRD (2016), PRD (2017)
S. Ebihara, K. Fukushima, and KM, PLB (2017)
KM, PRD (2023),   KM, and K. Takizawa, PLB (2023)
K. Fukushima, K. Hattori, and KM, arXiv:2409.18652

QCD Matter under Rotation

heavy-ion collision

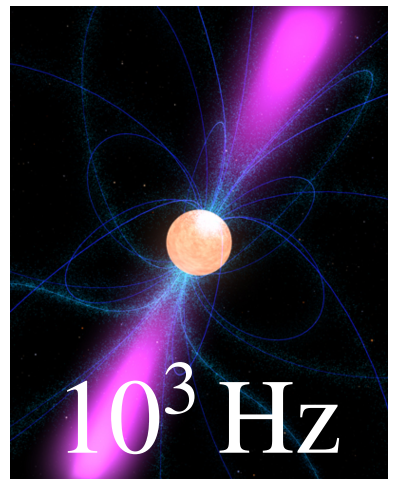


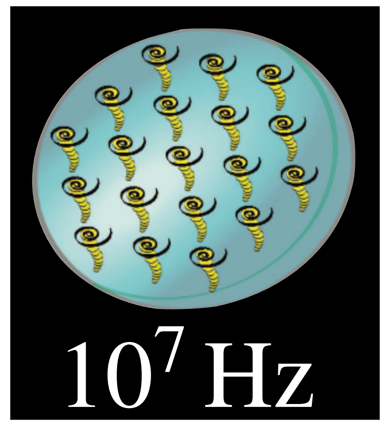
The Fastest Fluid
by Sylvia Morrow

Superhot material spins at an incredible rate.

10^{22} Hz

STAR (2017)

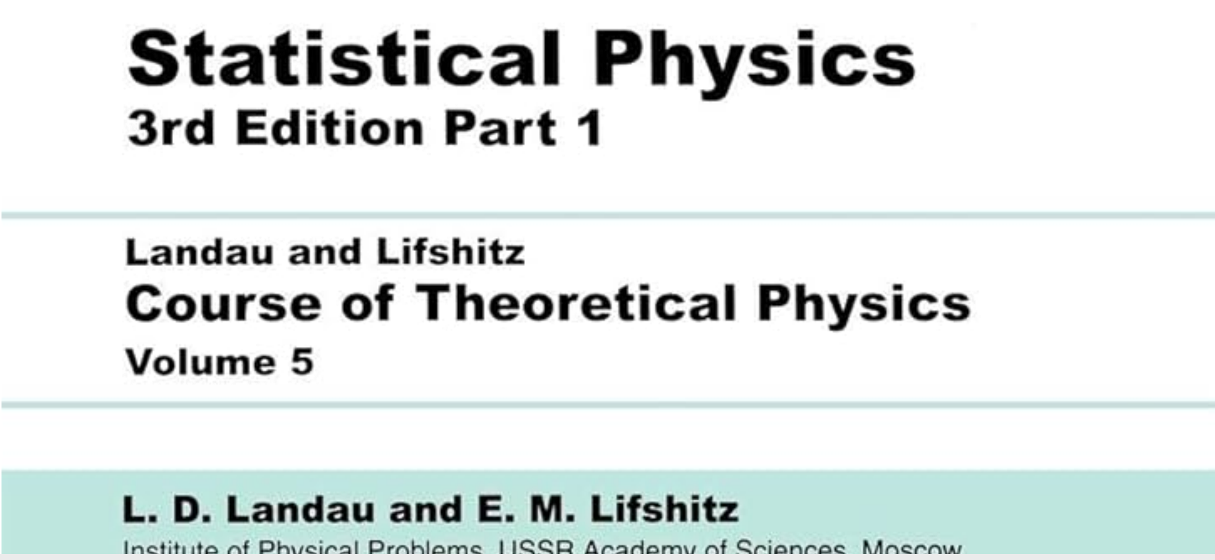


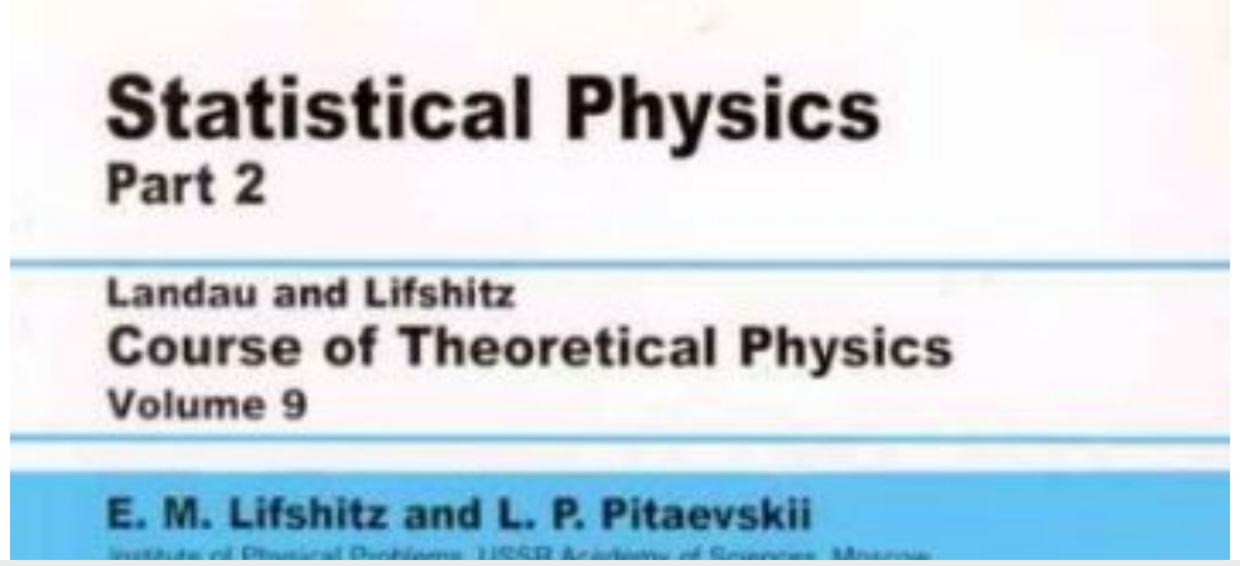


superfluid ^{4}He

- ✓ source of angular momentum
- ✓ simplest nontrivial fluid background
- ✓ simplest nontrivial geometric effect

Contents





Landau, Lifshitz (Pitaevskii) (1969)

$$H \longrightarrow H - \Omega J$$

For relativistic rotating matter, we need the refinement w.r.t.

- ✓ **Causality and Vacuum**
- ✓ **Gauge Invariance**

Causality and Vacuum

Thermodynamics of Rotating Systems

conserved quantity			partition function w/ Lagrange multiplier
	Q	entropy maximization →	$Z = Z(\lambda Q)$
energy	H	→	$Z = \mathrm{tr}\exp[-\beta H]$
particle number	N	→	$Z = \mathrm{tr}\exp[-\beta(H - \mu N)]$
angular momentum	J	→	$Z = \mathrm{tr}\exp[-\beta(H - \Omega J)]$

Vilenkin (1980)

Limitation of Nonrelativistic Theory

$$Z = \mathrm{tr}\exp[-\beta(H - \Omega J)]$$

rotation ≃ density

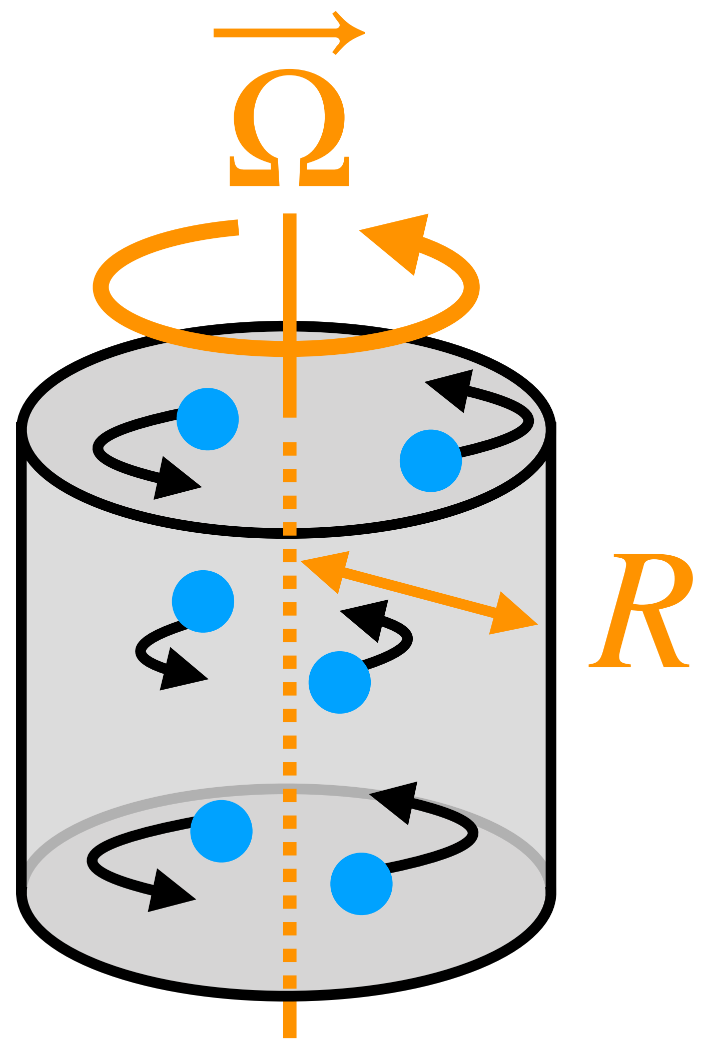
ex.) fermions

$$n_\mathrm{F} = \frac{1}{\mathrm{e}^{\beta(\epsilon - \Omega j)} + 1} \xrightarrow[\beta\to\infty]{} \theta(\Omega j - \epsilon)$$

"Vacuum affected by rotation"?

Answer is No : checked with relativistic theory

Infrared Gap



causality → finite size → **infrared gap**

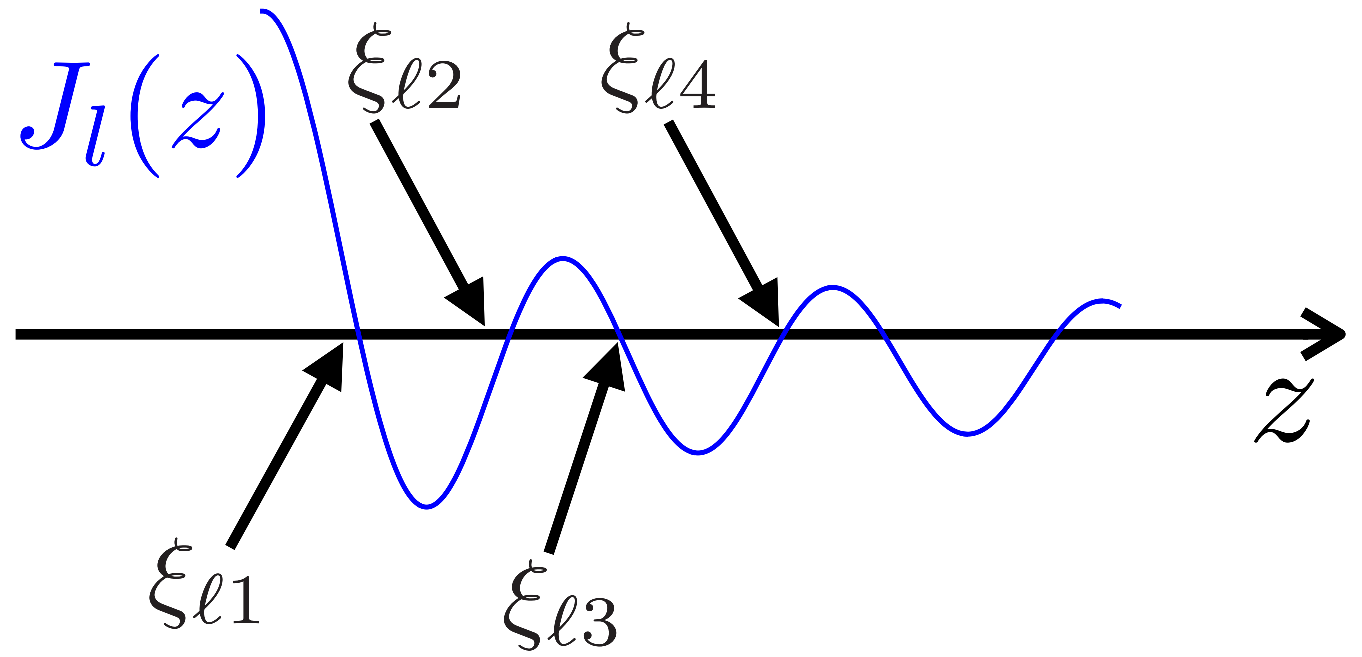
$$\Omega R \leq c = 1 \qquad \psi|_{\text{surf.}} = 0$$

cf) confined inside potential well

$$\sin(px)|_{x=L} = 0 \;\rightarrow\; p = \frac{n\pi}{L} \geq \frac{\pi}{L}$$

confined inside cylinder

$$J_l(p_\perp r)|_{r=R} = 0 \;\rightarrow\; p_\perp = \frac{\xi_{l,k}}{R} \geq \frac{\xi_{l,1}}{R}$$



No Visible Effect on Vacuum

Ebihara, Fukushima, KM (2017)

Bessel root property

$$\xi_{l,1} > l + 1/2$$

causality

$$1/R \geq \Omega$$

$$\epsilon = \sqrt{p_\perp^2 + p_z^2 + m^2} \geq \xi_{l,1}/R \geq \Omega(l + 1/2)$$

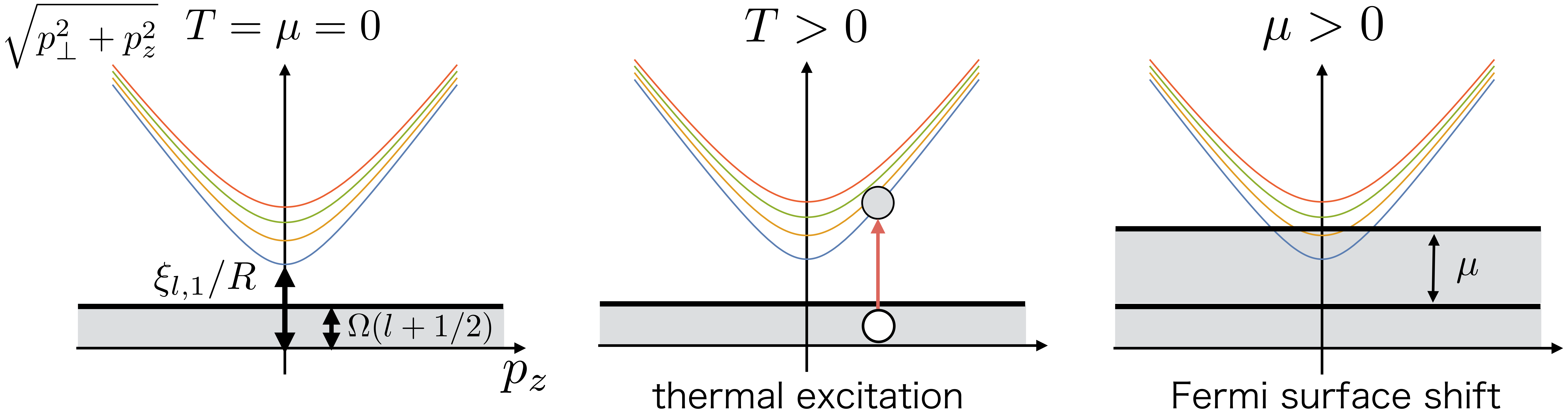
$$n_{\mathrm{F}} = \frac{1}{\mathrm{e}^{\beta\{\epsilon - \Omega(l+1/2)\}} + 1} \underset{\beta\to\infty}{\longrightarrow} \theta\Big(\Omega(l+1/2) - \epsilon\Big) = 0$$ for arbitrary l

Vacuum cannot be rotated

Visible in Thermal Part

$$n_{\mathrm{F}} = \frac{1}{\mathrm{e}^{\beta\{\epsilon - \Omega(l+1/2)\}} + 1}$$

effective Fermi surface

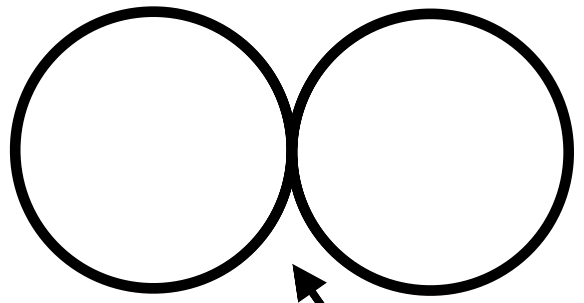
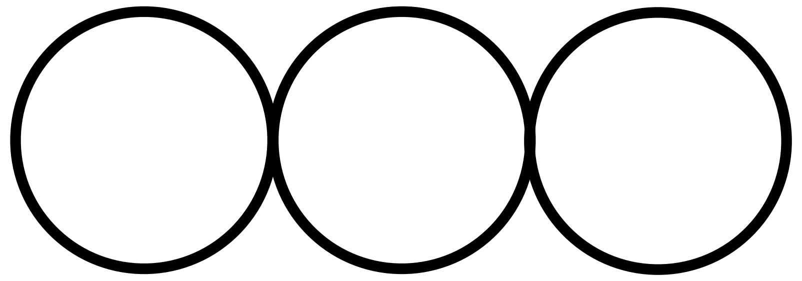
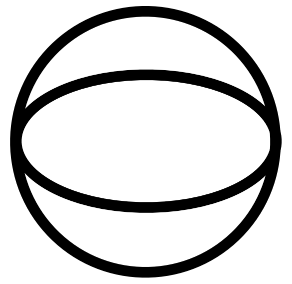


cf.) chiral vortical effect $\vec{J} = \left(\frac{T^2}{12} + \frac{\mu^2}{4\pi}\right)\vec{\Omega}$

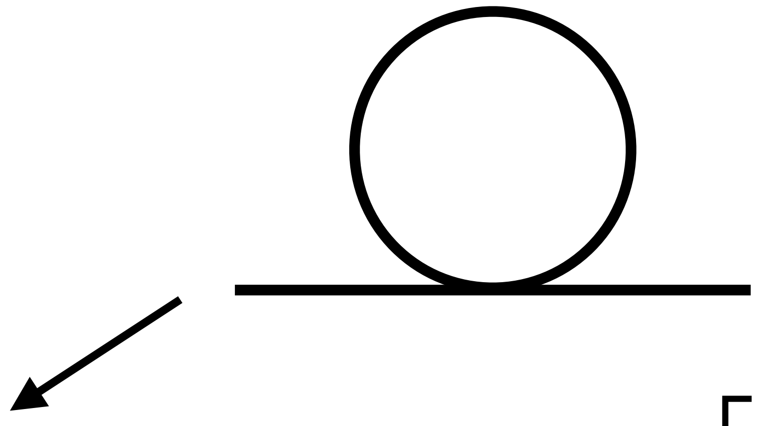
How about Perturbation Theory?

Kuboniwa, KM (in prep.)

$\lambda\phi^4$ theory under rotation

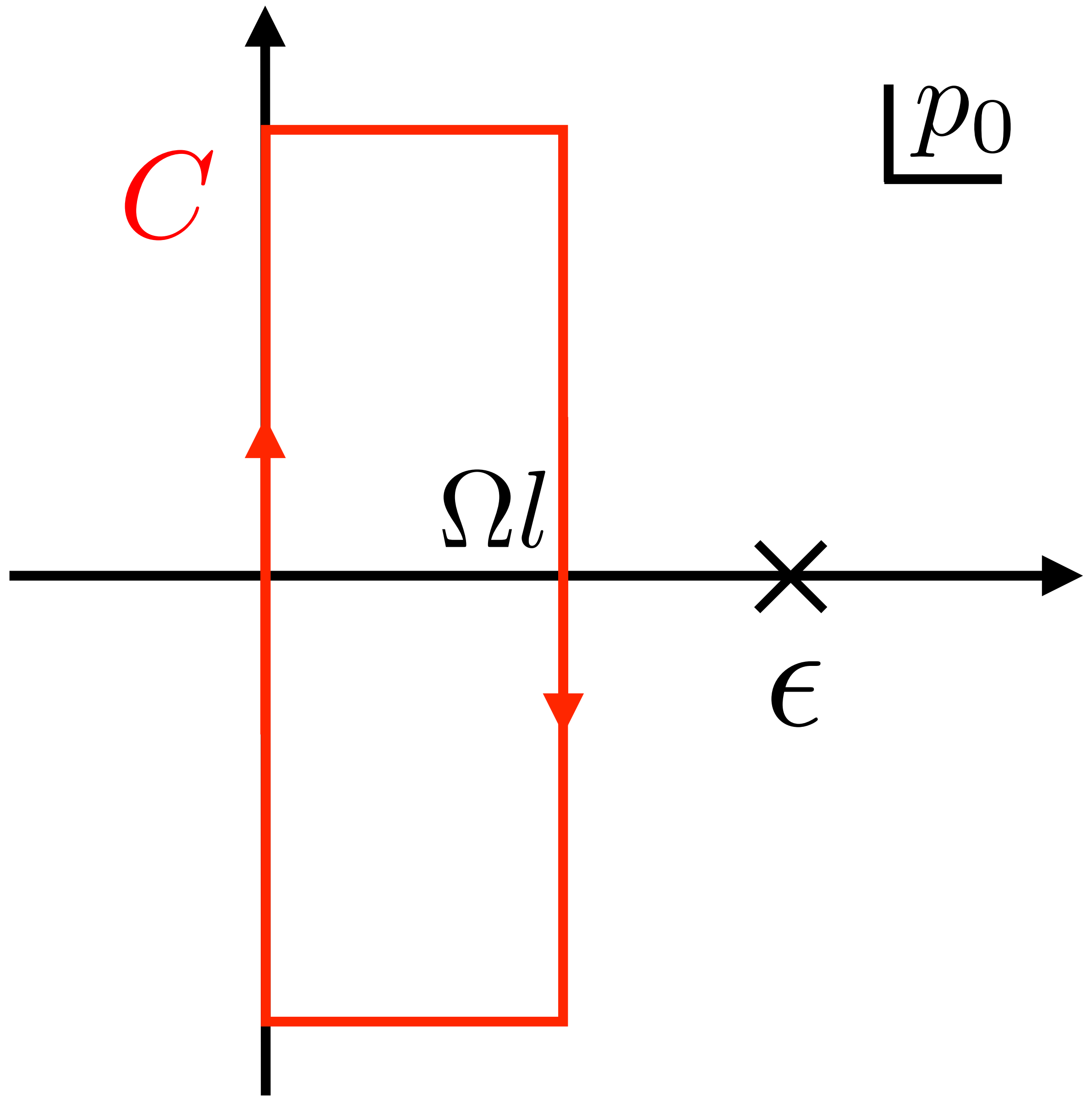
$\ln Z = \ln Z_0 + \#$  $+ \#$  $+ \#$  $+ \cdots$

$\lambda \times I(\nu_1, \nu_2)$ integral with four $J_l(p_{l,k} r)$'s



$$\Pi(\bar{\nu}) \propto \sum_{\nu, p_z} I(\bar{\nu}, \nu) \left[\frac{n_{\mathrm{B}}(\epsilon - \Omega l)}{2\epsilon} + \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}p_0}{2\pi\mathrm{i}} \frac{1}{\epsilon^2 - p_0^2} + \int_C \frac{\mathrm{d}p_0}{2\pi\mathrm{i}} \frac{1}{\epsilon^2 - p_0^2} \right]$$

$= 0$



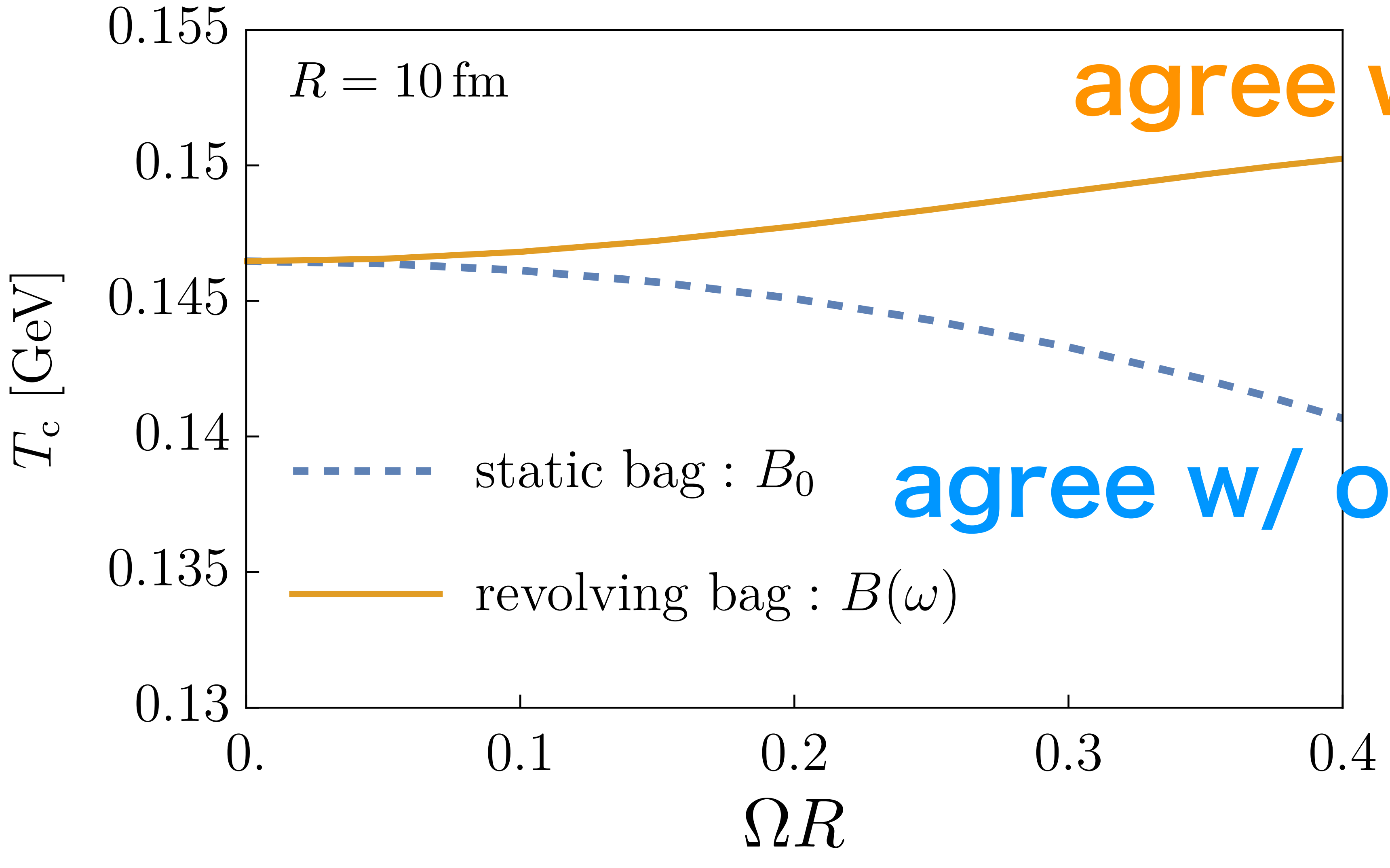
We can prove this to all orders

QCD Vacuum Affected by Rotation

✔ bag model analysis KM, Takizawa (2023)

$$B(\Omega) = B_0 + \frac{1}{2V}\Omega^2 I_{\mathrm{b}}$$

moment of inertia of QCD vacuum

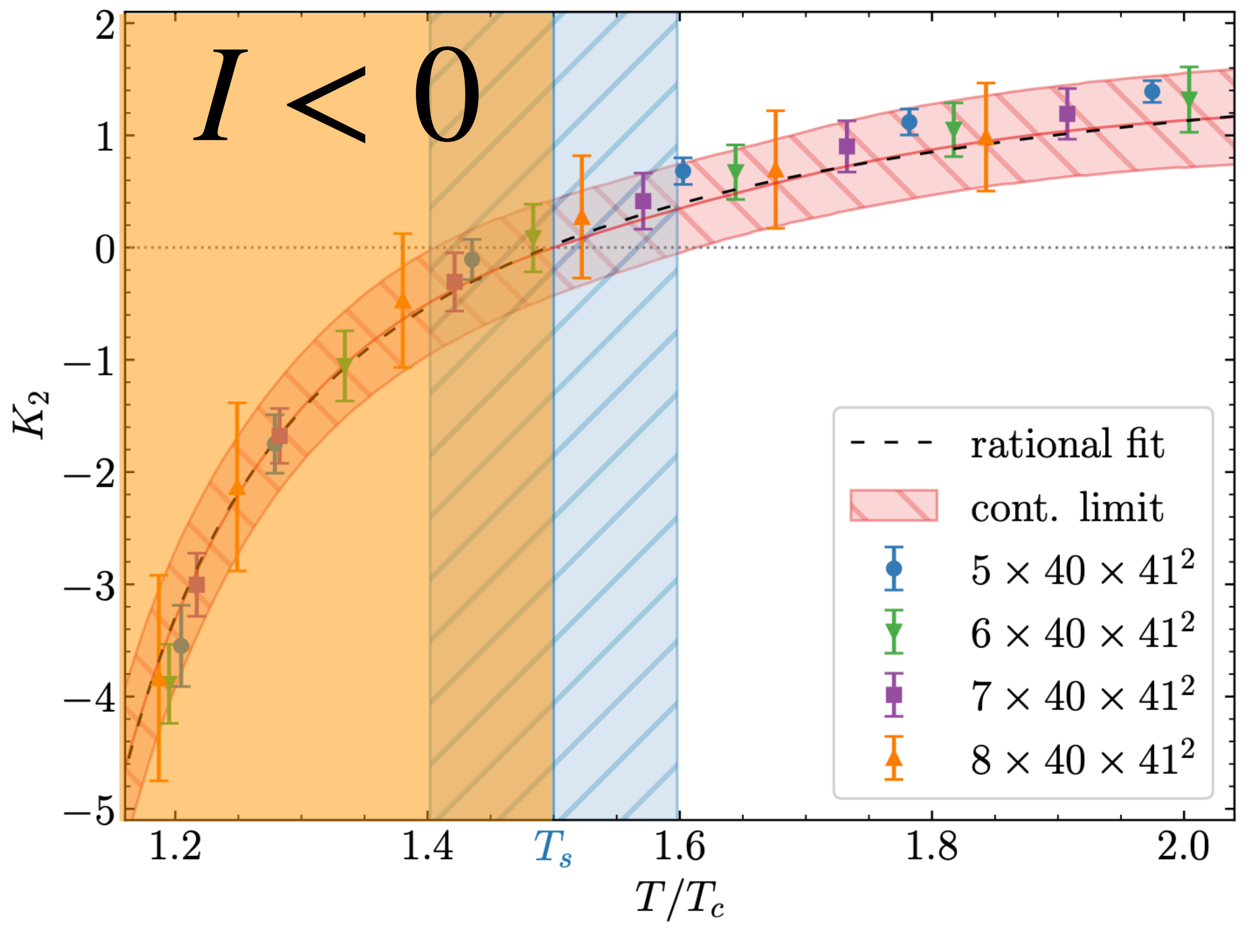


agree w/ lattice

agree w/ other models

✔ MoI from lattice pure YM Braguta-Chernodub-Roenko-Sychev (2023)

$$I = \left.\frac{\partial F}{\partial \Omega^2}\right|_{\Omega=0}$$



unstable result due to

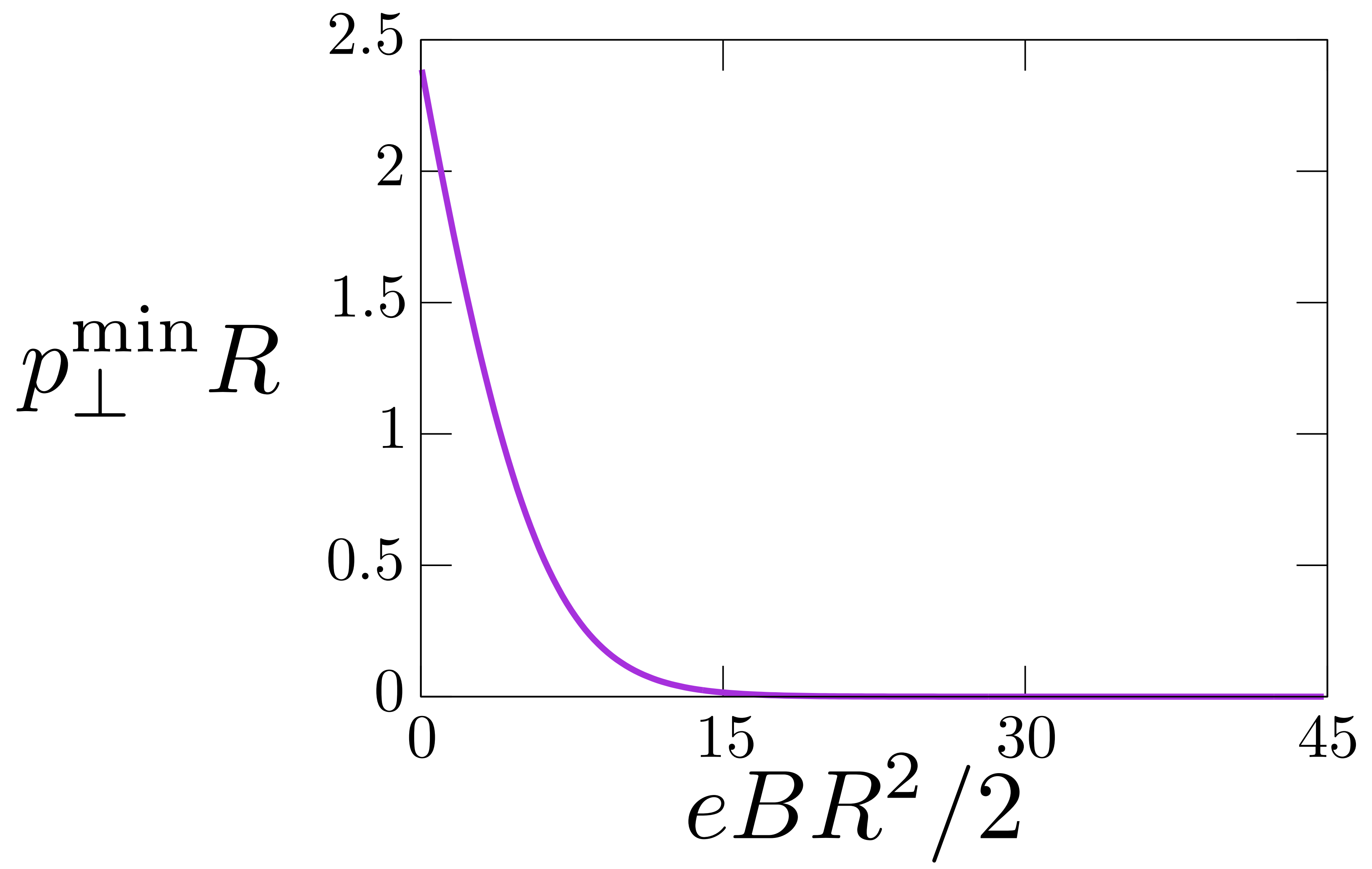
$$\langle\langle \mathcal{O} \rangle\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$$

vacuum contribution

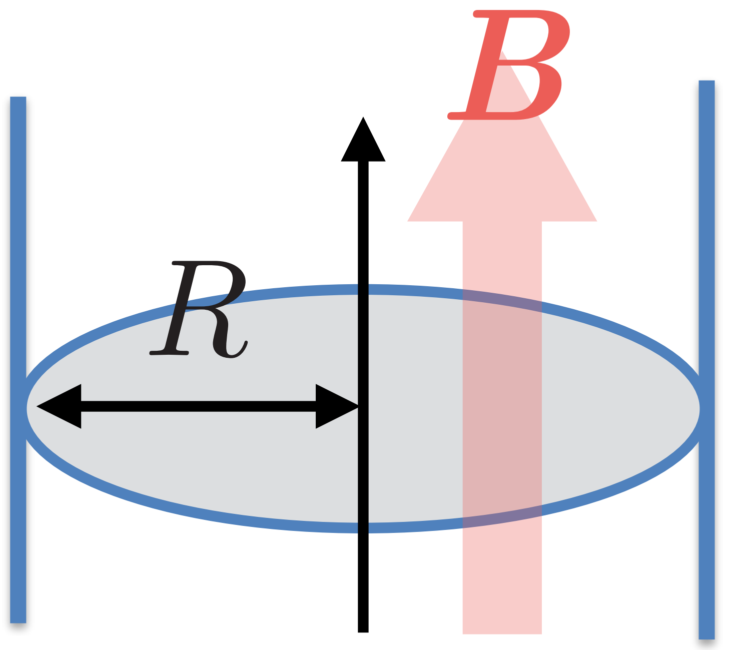
Gauge Invariance
(Magneto-Vortical Matter)

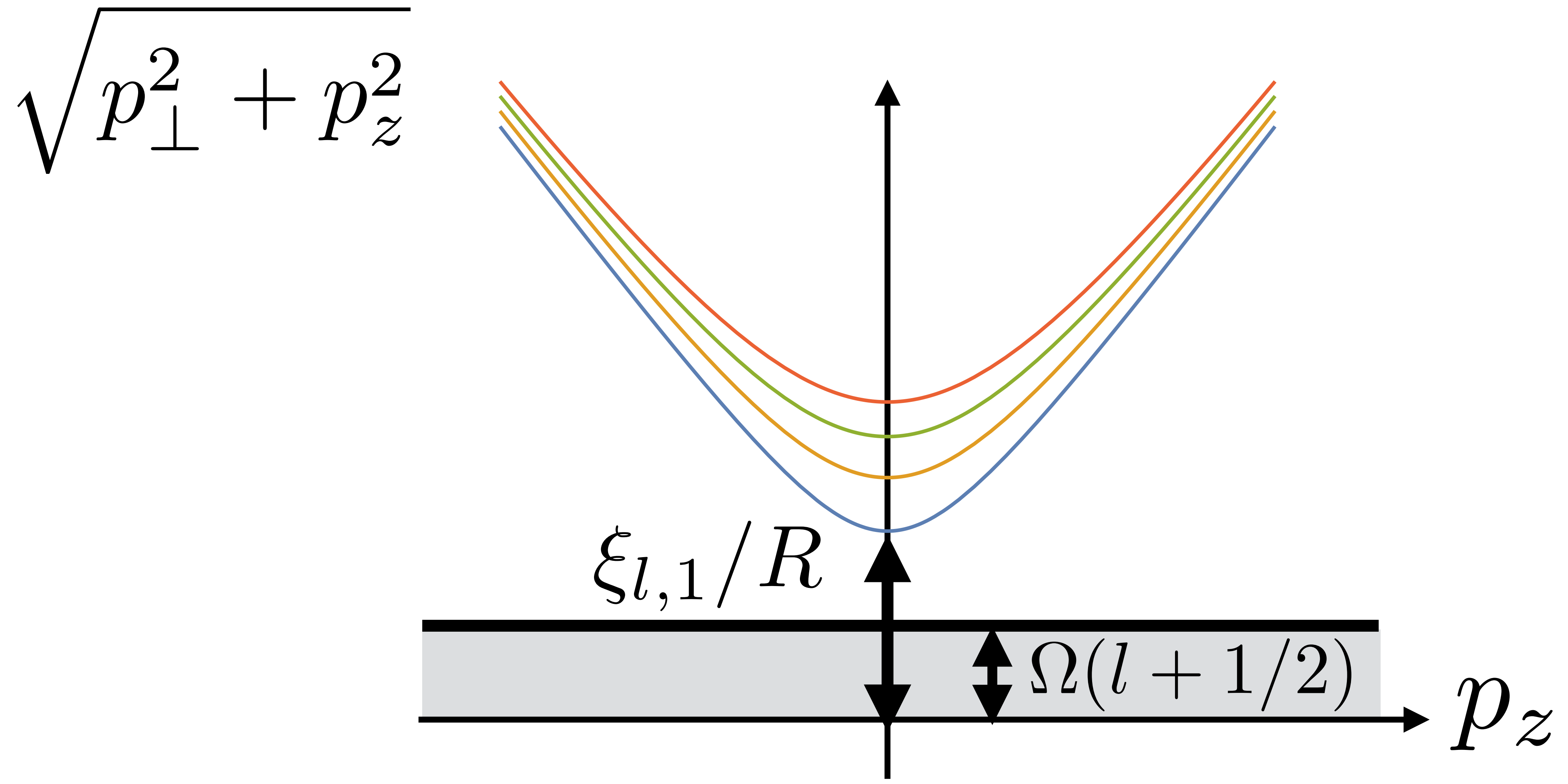
Visible under Magnetic Field?

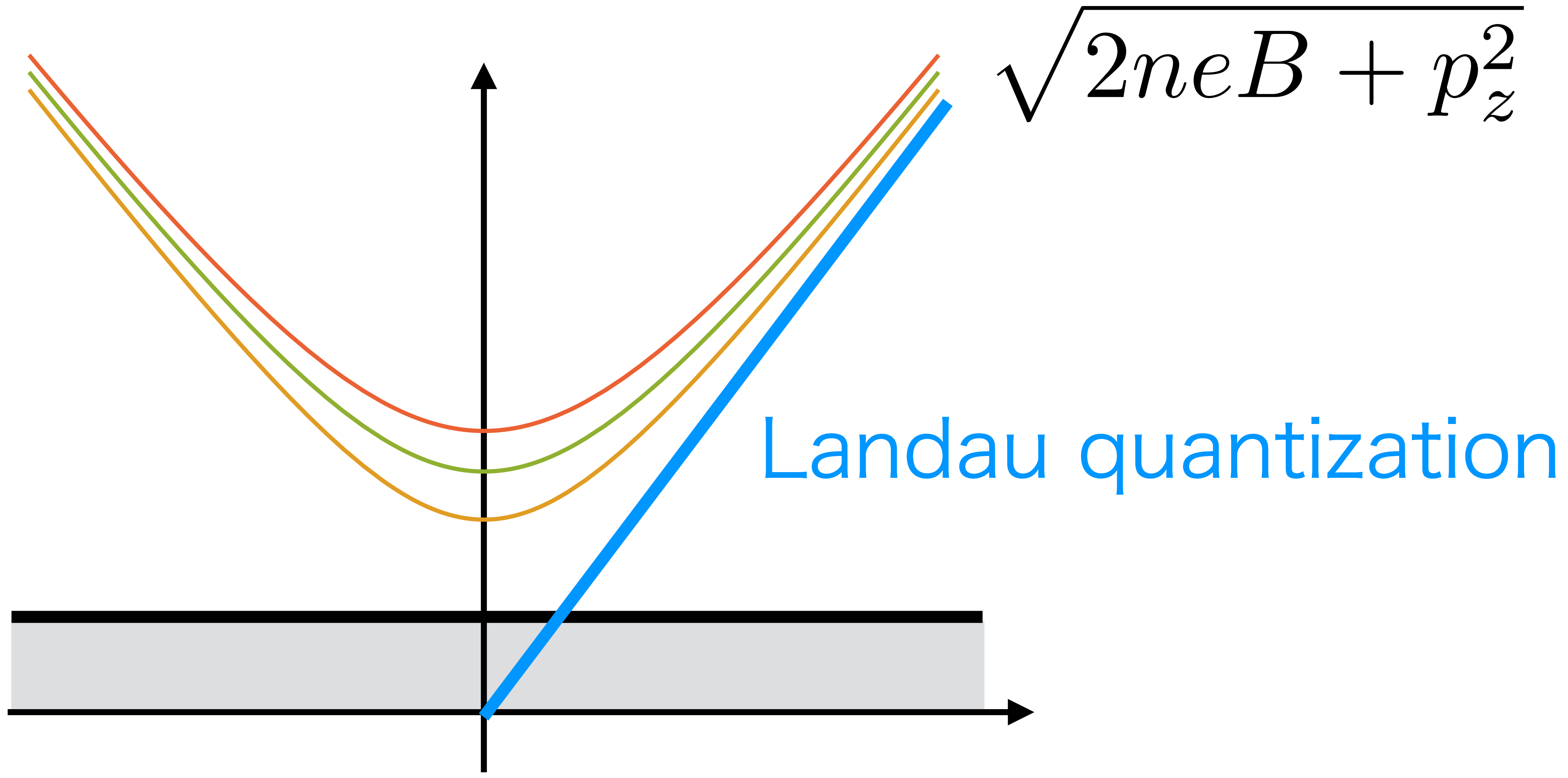
Chen, Fukushima, Huang, KM (2017)



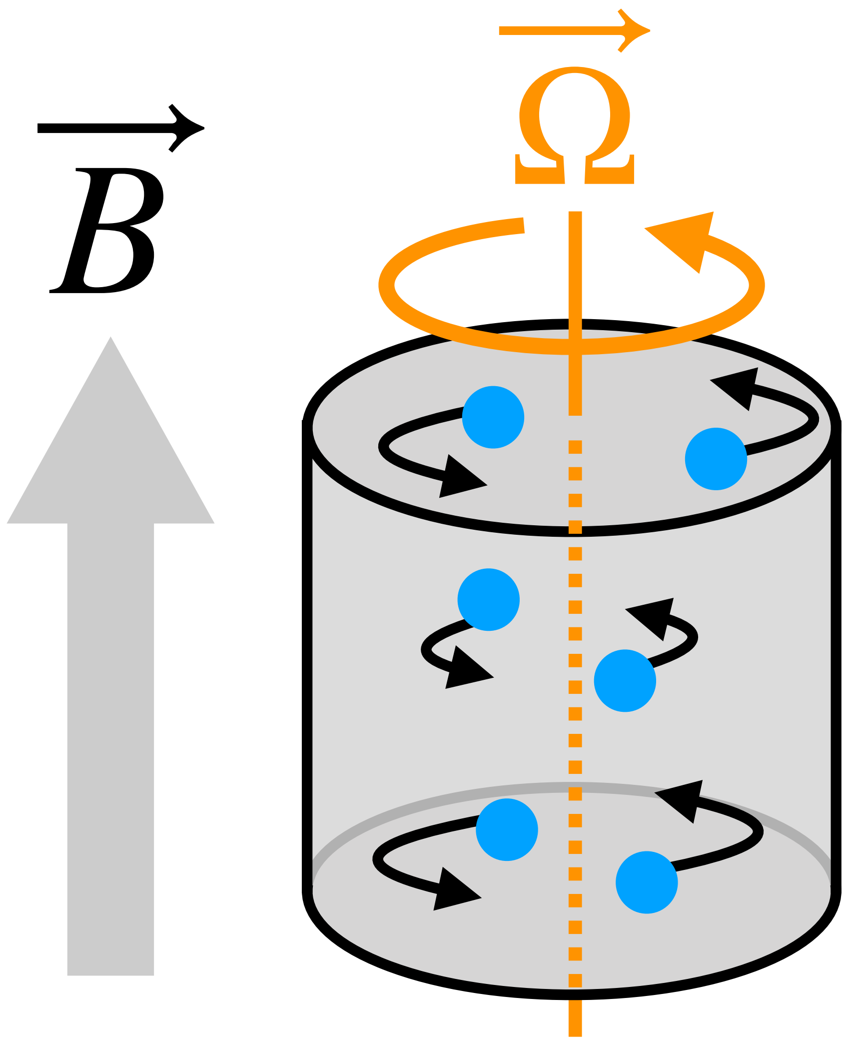
Dirac eq. in







Two Angular Momenta



$$Z = \mathrm{tr}\left[\mathrm{e}^{-\beta(H-\Omega\mathcal{J})}\right]$$
$$= \det\left[-\mathrm{i}\gamma^\mu D_\mu + m - \gamma^0\Omega(L+S)\right]$$

Chen-Fukushima-Huang-Mameda (2016)

$$L_{\mathrm{can}} = -\mathrm{i}(x\partial_y - y\partial_x)$$

conserved AM

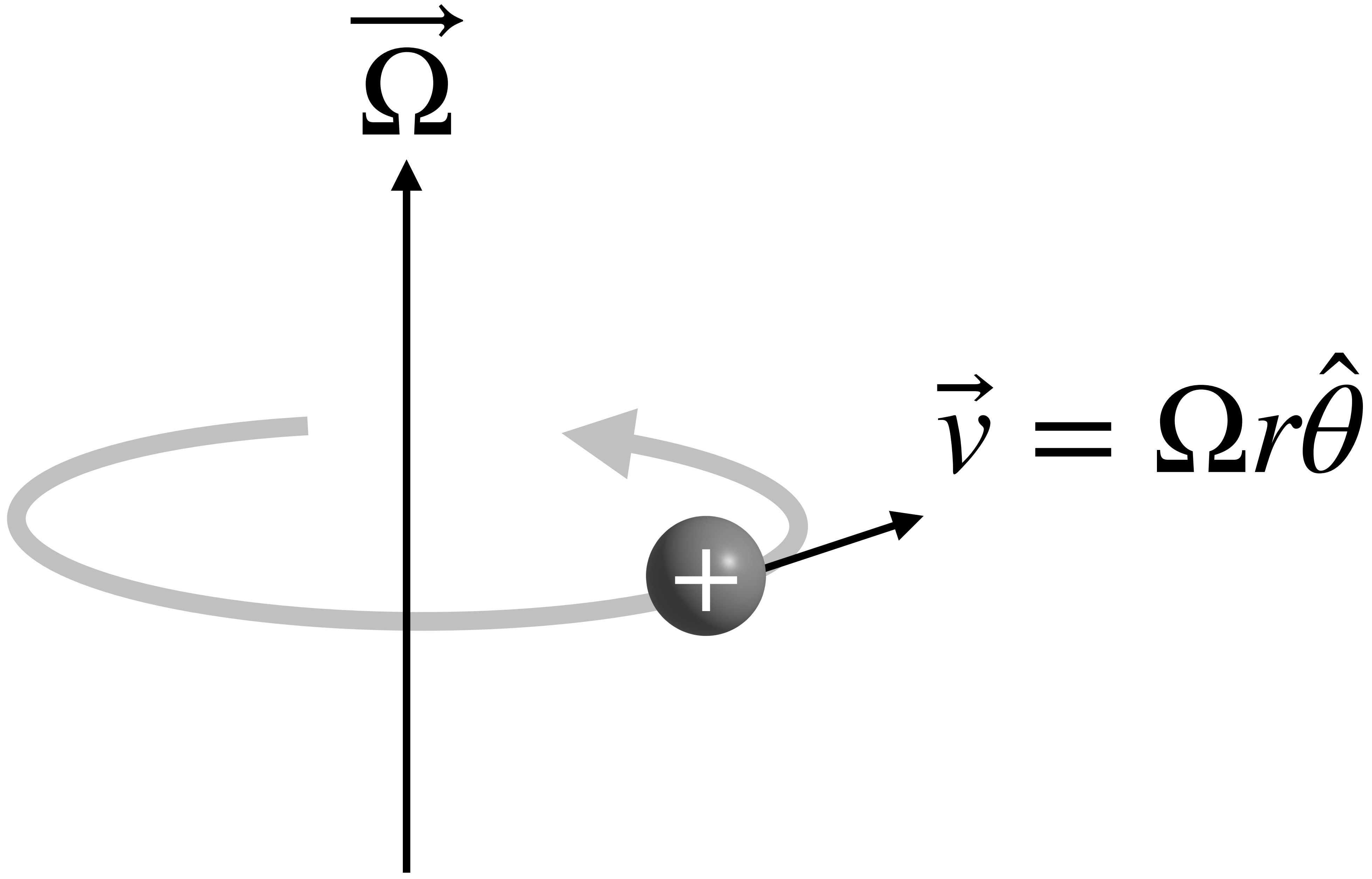
Fukushima-Hattori-Mameda (2024)

$$L_{\mathrm{kin}} = -\mathrm{i}(xD_y - yD_x)$$

gauge invariant AM

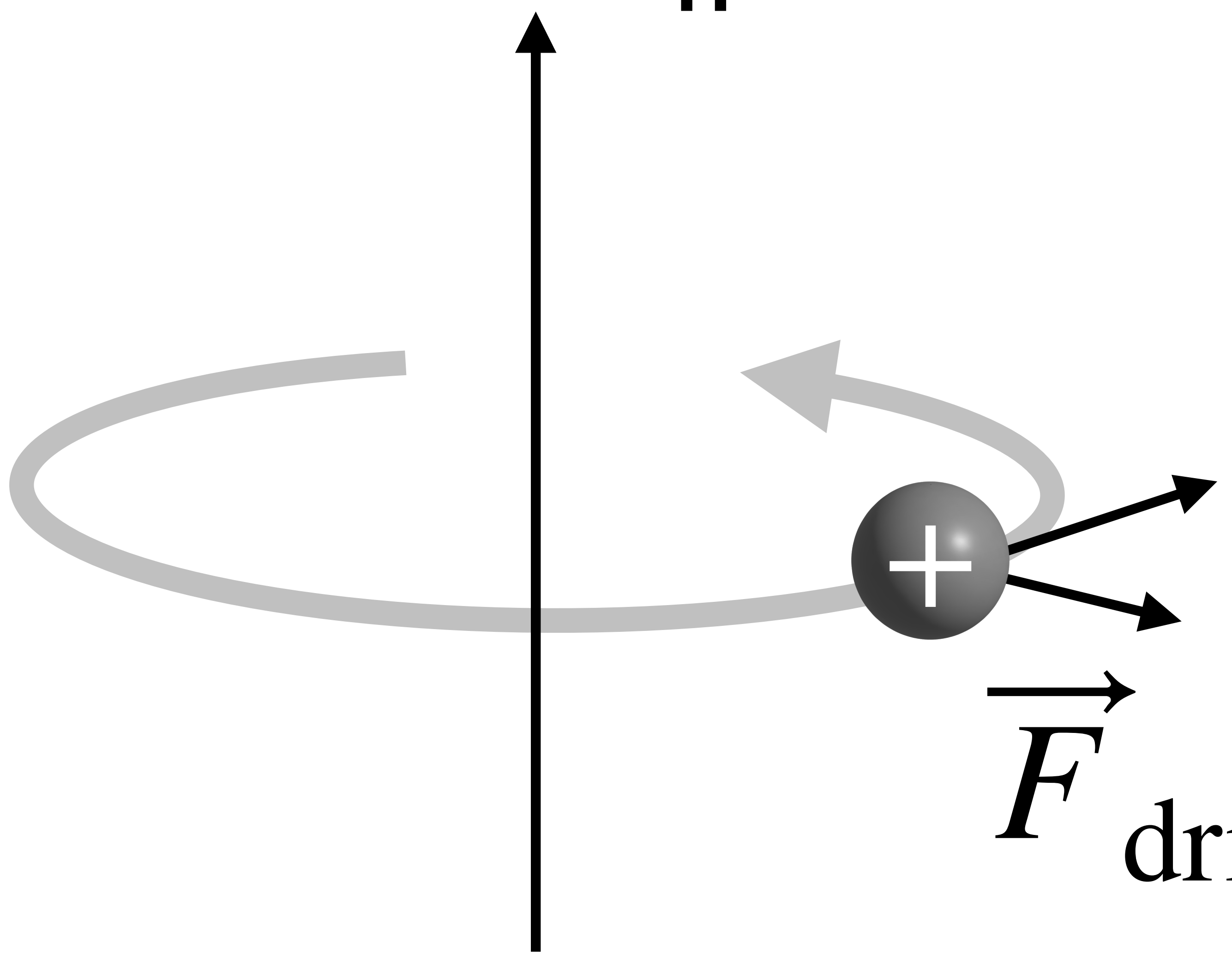
But why thermodynamically?

Classical Argument



Classical Argument

$\overrightarrow{\Omega} \parallel \overrightarrow{B}$



$\vec{v} = \Omega r \hat{\theta}$

$\overrightarrow{F}_{\text{drift}} = eB\Omega\vec{r}$

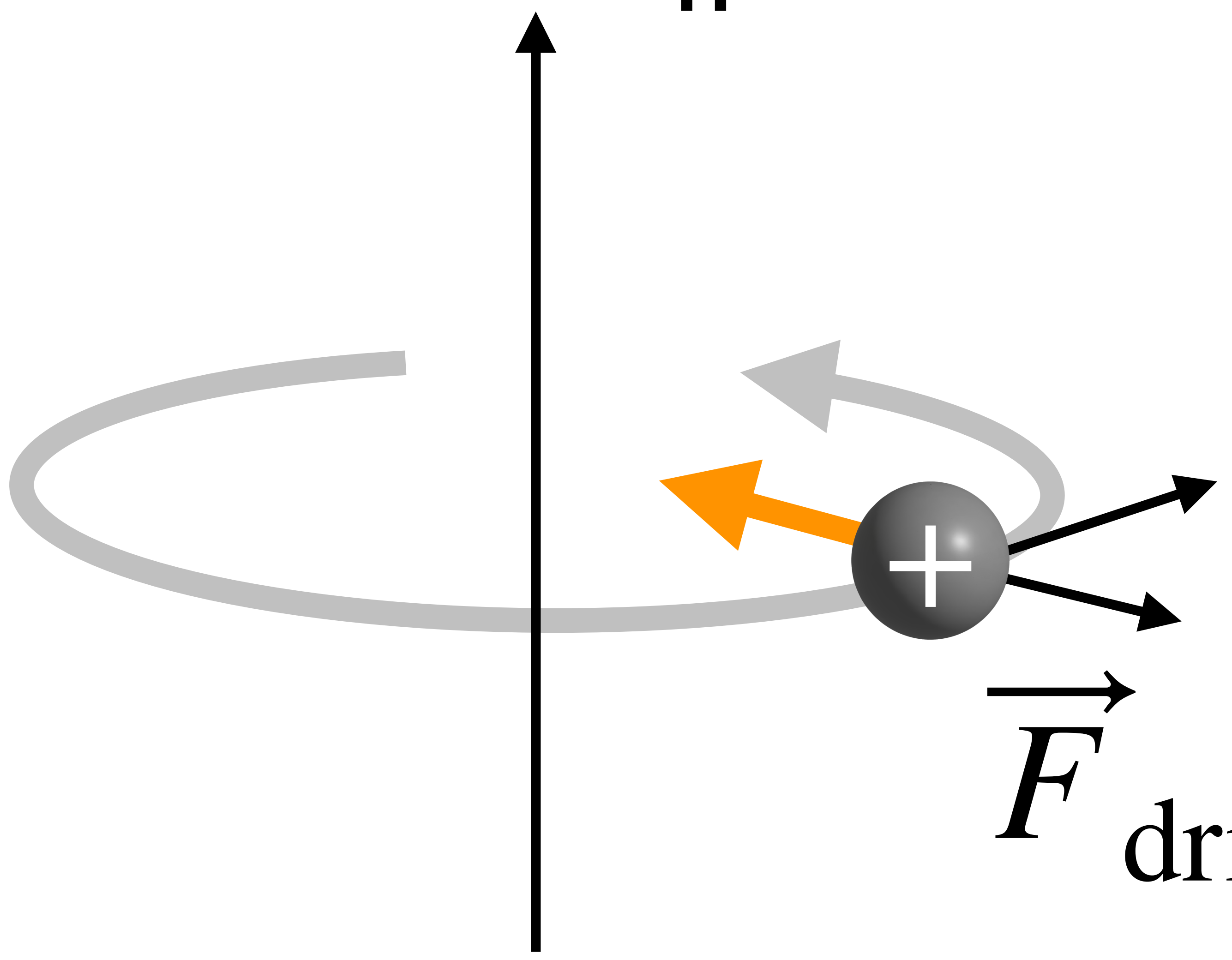
no longer circular

$H - \Omega L_{\text{can}}$

unstable

Classical Argument

$\vec{\Omega} \parallel \vec{B}$



$\vec{v} = \Omega r \hat{\theta}$

$\vec{F}_{\text{drift}} = eB\Omega\vec{r}$

$$e\vec{E} = -eB\Omega\vec{r}$$
$$= -\vec{\nabla}\left[\Omega(L_{\text{can}} - L_{\text{kin}})\right]$$

$$H + \Omega(L_{\text{can}} - L_{\text{kin}}) - \Omega L_{\text{can}} = H - \Omega L_{\text{kin}}$$ stable

cf. Buzzegoli (2020)

gauge invariance ⟷ thermodynamic stability

Total Angular Momentum

Fukushima-Hattori-Mameda (2024)

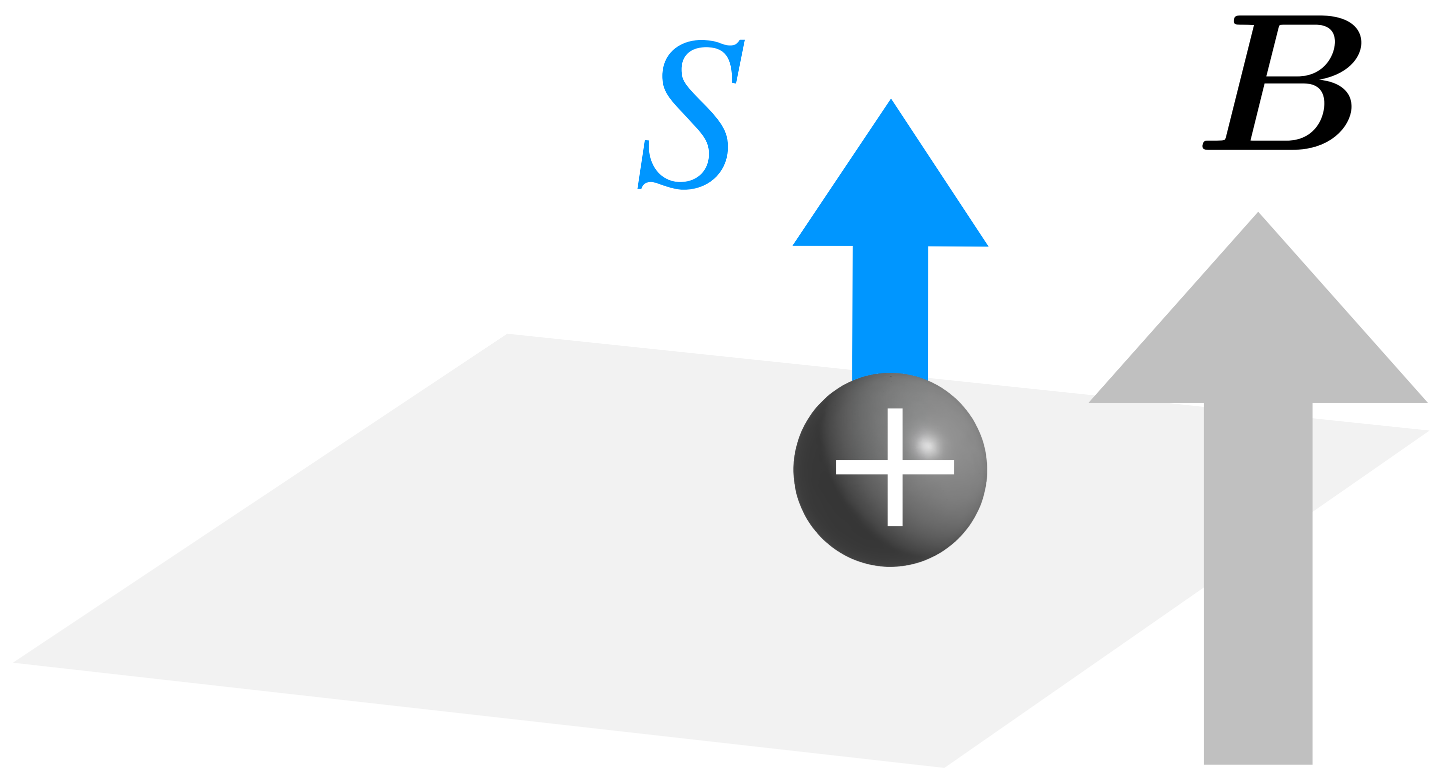
$$Z = \det\left[-\mathrm{i}\gamma^\mu D_\mu + m - \gamma^0\Omega(L + S)\right] \qquad L = -\mathrm{i}(xD_y - yD_x)$$

Not calculable analytically due to AM

except for the lowest Landau level ($B \to \infty$) limit

massless limit
$$J = \left.\frac{\partial P_{\mathrm{LLL}}}{\partial \Omega}\right|_{\Omega=0} = -\frac{eB\mu}{4\pi^2}$$

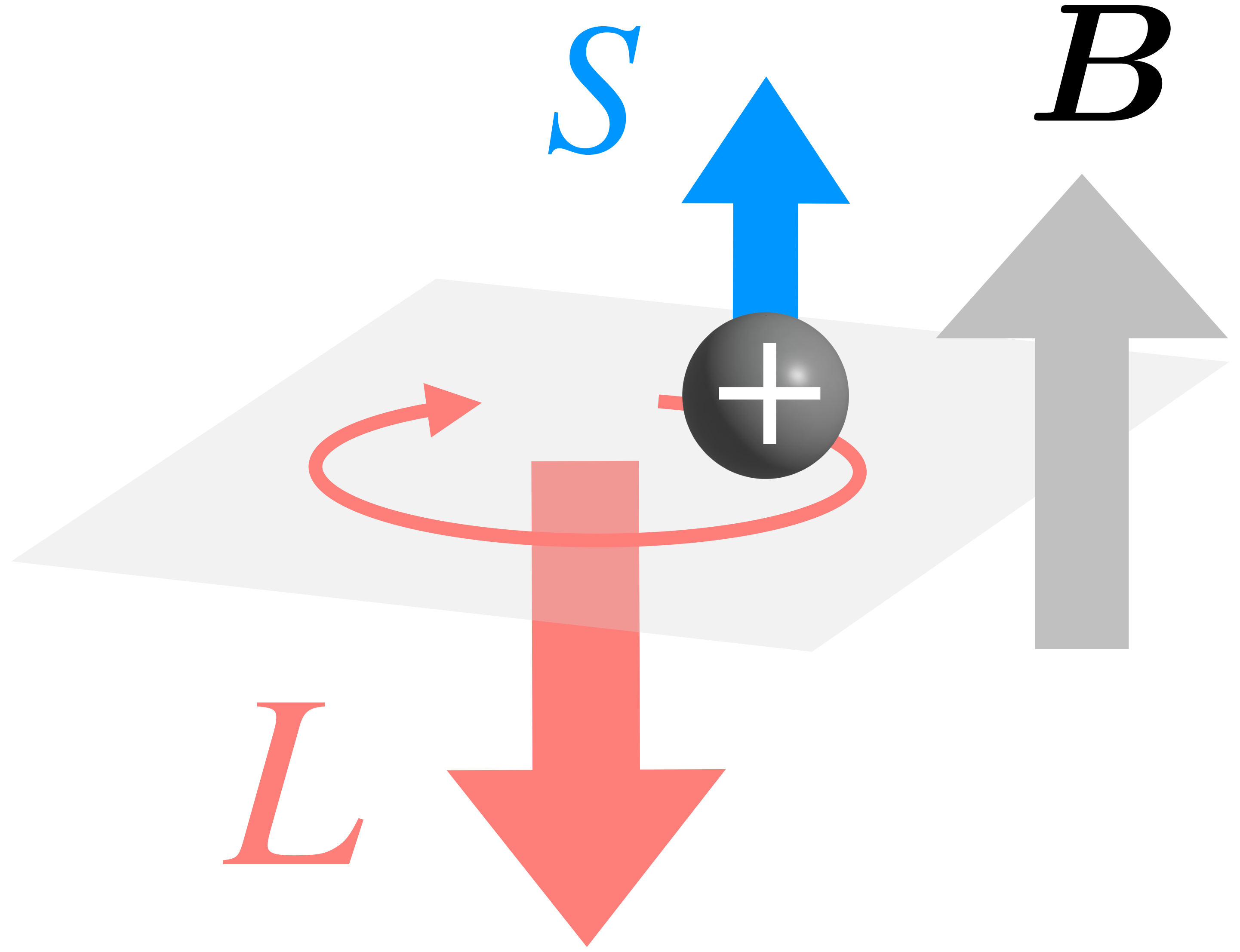
Why Negative?



$\langle S \rangle_{\mathrm{LLL}}$

$+1/2$

Why Negative?

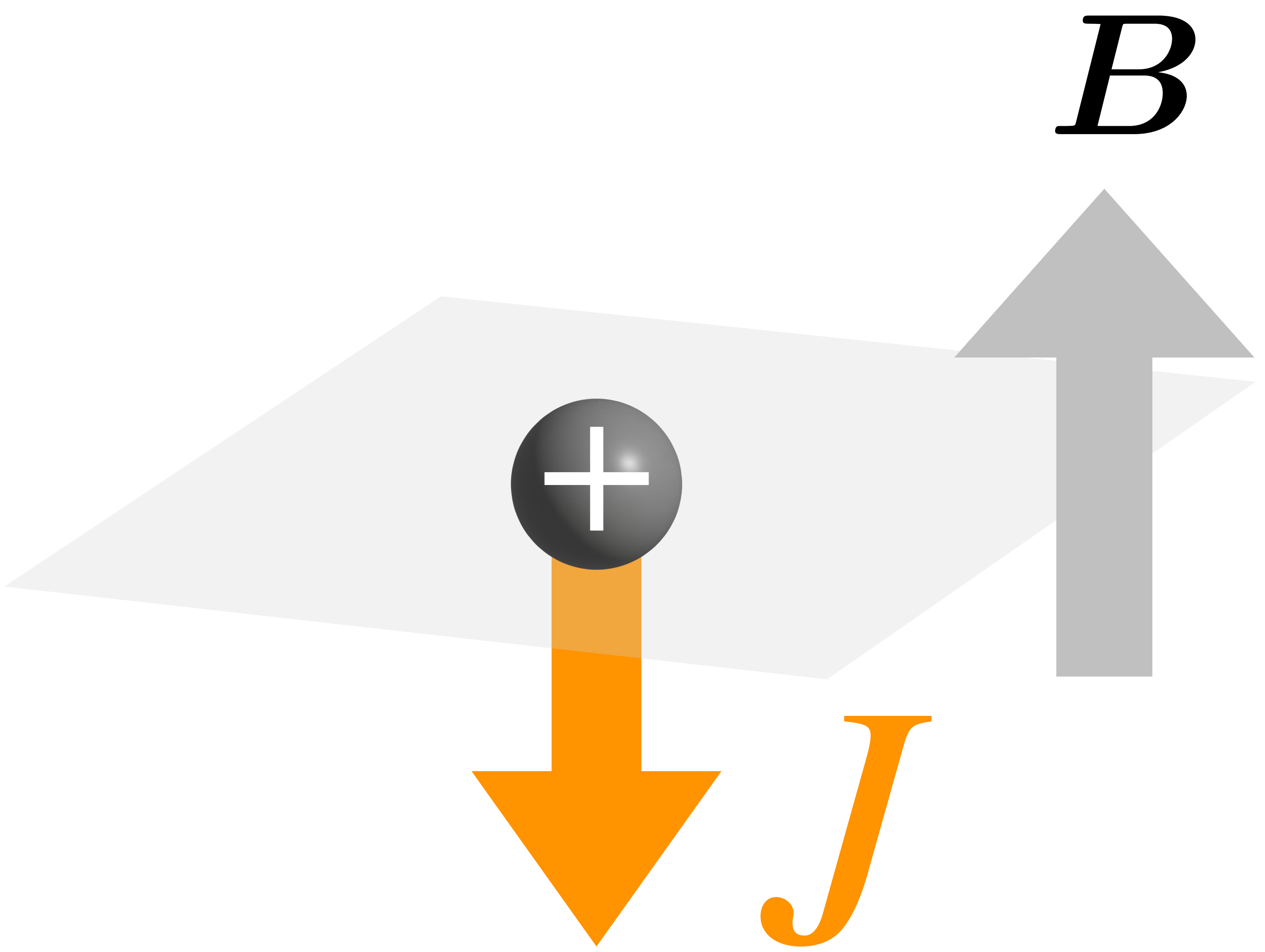


$\langle S \rangle_{\mathrm{LLL}}$ $\quad$ $\langle L \rangle_{\mathrm{LLL}}$

$+1/2$ $\quad$ -1

$$L = x\Pi_y - y\Pi_x = -\left(2a^\dagger a + 1\right) + [\text{off-diagonal}]$$

Why Negative?

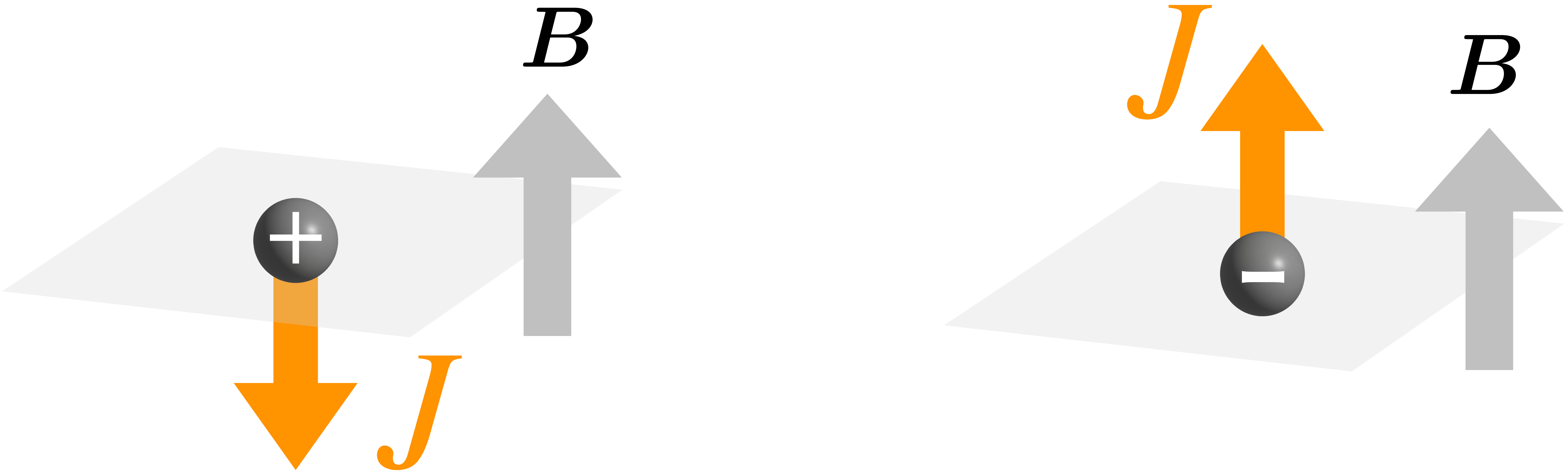


$$\langle S\rangle_{\mathrm{LLL}} + \langle L\rangle_{\mathrm{LLL}} = \langle J\rangle_{\mathrm{LLL}}$$

$+1/2$ -1 $-1/2$

$$L = x\Pi_y - y\Pi_x = -\left(2a^\dagger a + 1\right) + [\text{off-diagonal}]$$

Why Negative?



If $\mu > 0$, then $J = -\frac{eB\mu}{4\pi^2}$

Charge Density

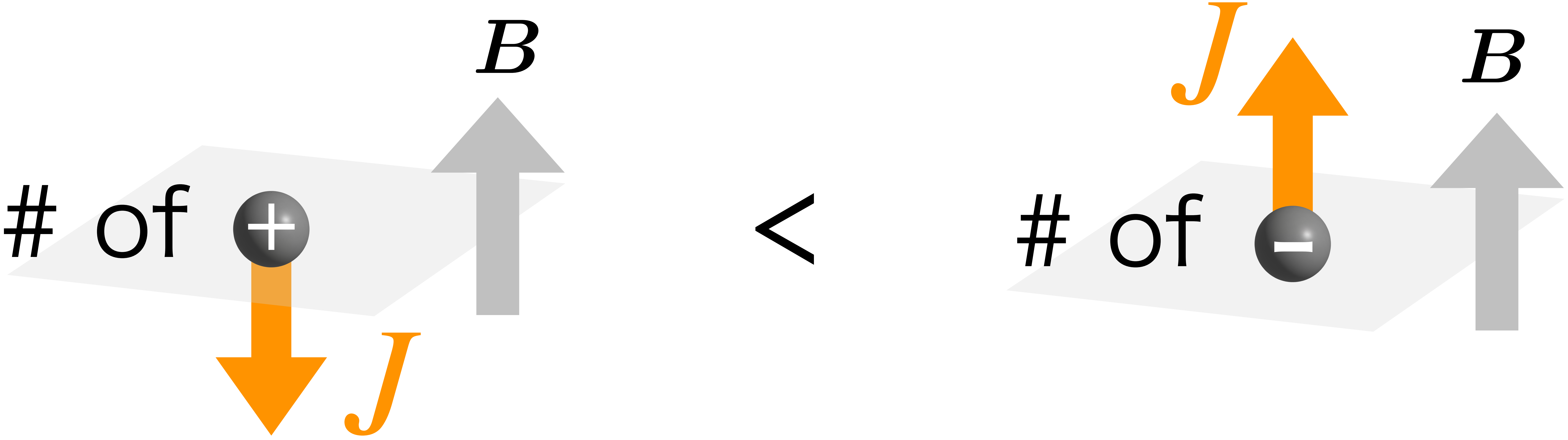
massless limit

$$J = \left.\frac{\partial P_{\mathrm{LLL}}}{\partial \Omega}\right|_{\Omega=0} = -\frac{eB\mu}{4\pi^2}$$

$$\rho = \left.\frac{\partial P_{\mathrm{LLL}}}{\partial \mu}\right|_{\mu=0} = -\frac{eB\Omega}{4\pi^2}$$

Why Negative?

rotation coupling $E = E_0 - \Omega J$



If $\Omega > 0$, then $\rho = -\frac{eB\Omega}{4\pi^2}$

Relation to Chiral Anomaly

charge $$\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \mu} = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2} + \frac{eB\mu}{2\pi^2}$$

angular momentum $$J = \frac{\partial P_{\mathrm{LLL}}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\mathrm{LLL}}}{\partial \mu \, \partial \Omega}$$

→ same coefficients shared

$$\frac{eB}{4\pi^2} \qquad -\frac{eB}{2\pi^2}$$

Relation to Chiral Anomaly

charge $\rho = \dfrac{\partial P_{\rm LLL}}{\partial \mu} = \dfrac{eB\Omega}{4\pi^2} - \dfrac{eB\Omega}{2\pi^2} + \dfrac{eB\mu}{2\pi^2}$

angular momentum $J = \dfrac{\partial P_{\rm LLL}}{\partial \Omega} = \dfrac{eB\mu}{4\pi^2} - \dfrac{eB\mu}{2\pi^2} + \dfrac{eB\Omega}{8\pi^2}$

$= S = j^5_{\rm CSE}/2$

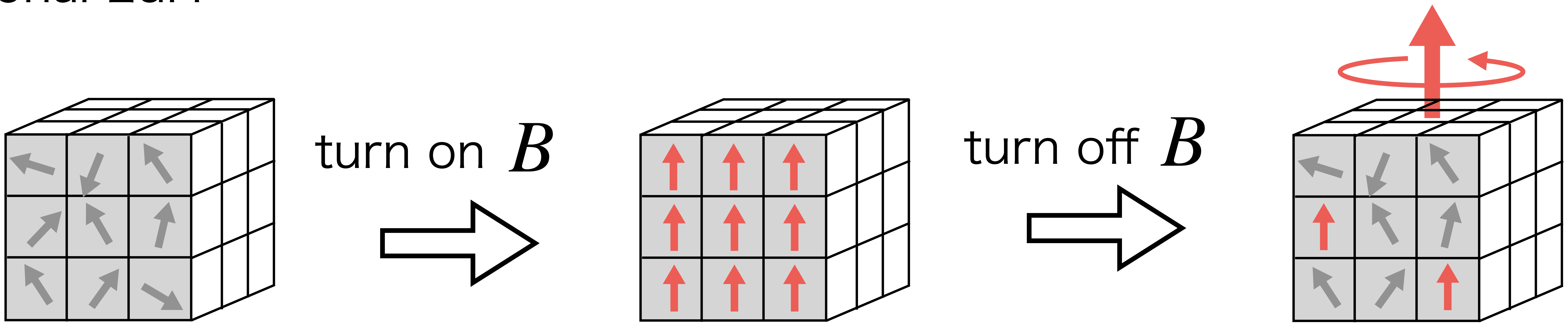
$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\rm LLL}}{\partial \mu\, \partial \Omega}$$

→ Since $j^5_{\rm CSE}$ is anomaly-related, so is ρ

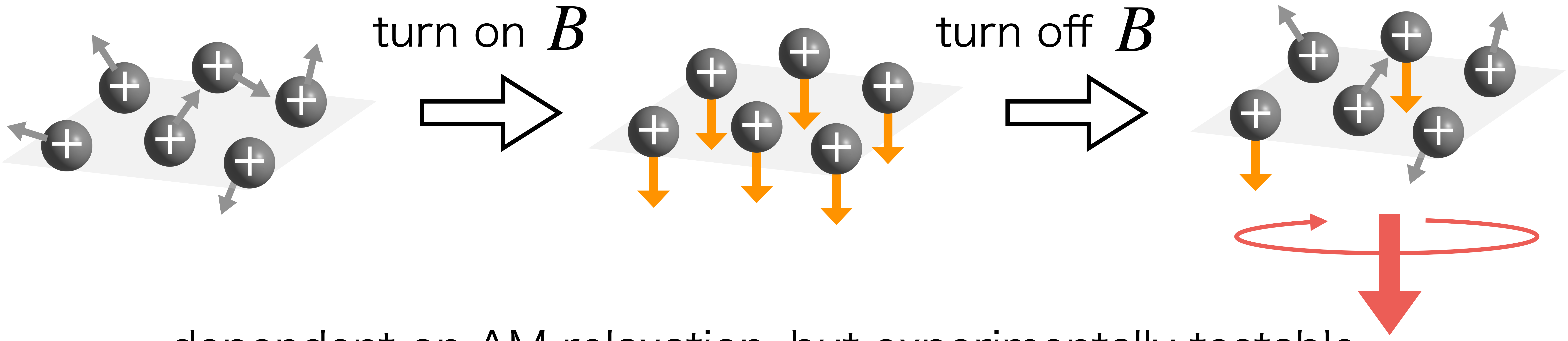
cf. Yang-Yamamoto (2021)

Einstein-de Haas Effect

conventional EdH



opposite EdH (Weyl fermions)



dependent on AM relaxation, but experimentally testable

Summary

- ✓ Landau-Lifshitz treatment for rotating matter is not enough

  causality, vacuum properties, and gauge invariance

- ✓ Relativistic rotating matter/Magneto-Vortical matter are more complicated but interesting

Comparisons

Fukushima-Hattori-Mameda (2024)
partition function (LLL)

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}}$$

Ebihara-Fukushima-Mameda (2017)
partition function (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

due to $\vec{F}_{\text{drift}} = eB\Omega\vec{r}$

Hattori-Yin (2016)
Kubo formula (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2}$$

sign-mistake

Yang et. al (2020) Mameda(2023)
chiral kinetic theory
correct

$$\rho = \frac{eB\Omega}{4\pi^2}$$

no Landau level formed by weak B