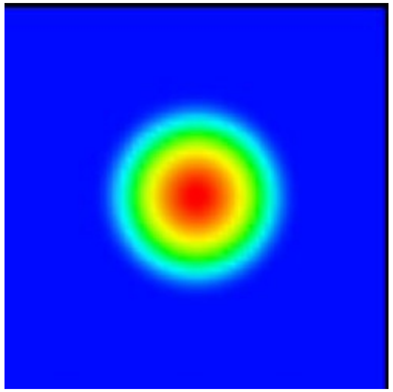
Quantized vortices in rotating superfluid

Makoto TSUBOTA, Osaka Metropolitan University, Japan

1. Quantized vortices compared with usual vortices
2. Vortex lattice formation in rotating BECs
3. Vortex lattice in two-component BECs



4. Direct excitation and observations of Kelvin waves on quantized vortices

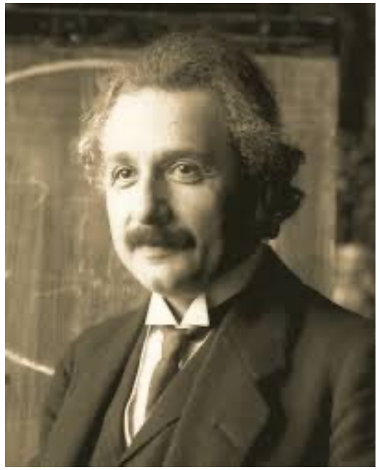
Y. Minowa, Y. Yasui, T. Nakagawa, S. Inui, MT, M. Ashida, Nat. Phys. 1-6(2025)

1. Quantized vortices compared with usual vortices

Quantum hydrodynamics and turbulence

The systems:
Superfluid ^{4}He, Superfluid ^{3}He,
Atomic Bose-Einstein condensates(BECs)

https://youtu.be/shdLjIkRaS8?si=XU703KREqDBHzG2U



A. Einstein

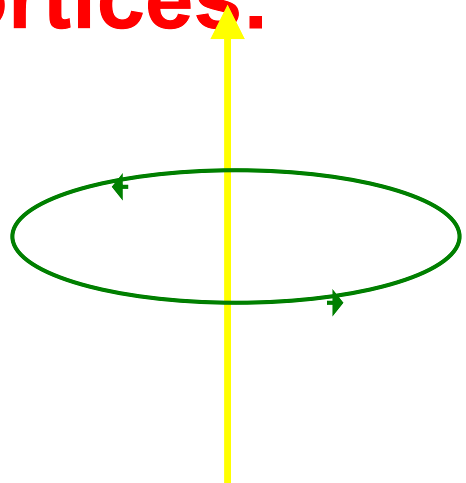
Predicted in 1925 and realized in 1995

A quantized vortex is a vortex of superflow in a BEC.Any rotational motion in superfluid is sustained by quantized vortices.

(i) The circulation is quantized.

$$\oint \mathbf{v}_s \cdot d\mathbf{s} = \kappa n \qquad n = 1, 2, 3, \cdots$$

$$\kappa = h/m$$



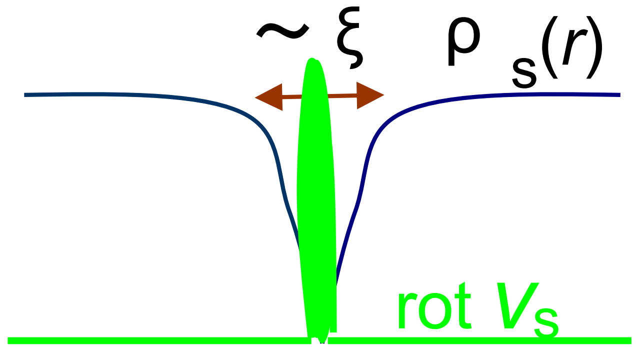
A vortex with $n \geqq 2$ is unstable.

⇨ **Every vortex has the same circulation.**

(ii) Free from the decay mechanism of the viscous diffusion of the vorticity.

⇨ **The vortex is stable.**

(iii) The core size is very small.



⇨ **The order of the coherence length.**

Models available for numerical simulation

Gross-Pitaevskii (GP) model for the macroscopic wave function

$$\Psi(r) = \sqrt{n_0(r)}e^{i\theta(r)}$$

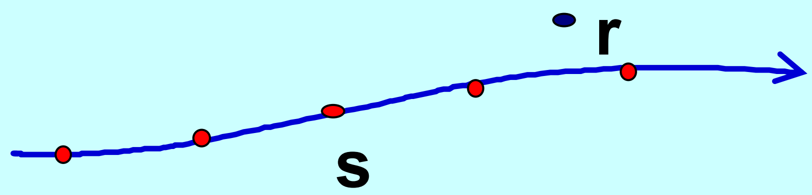
$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(\mathbf{r}) + g\left|\Psi(\mathbf{r},t)\right|^2\right]\Psi(\mathbf{r},t)$$

ATOMIC BECS

Vortex filament model(VFM)

Biot-Savart law

$$\mathbf{v}_s(\mathbf{r}) = \frac{\kappa}{4\pi}\int\frac{(\mathbf{s}-\mathbf{r})\times d\mathbf{s}}{|\mathbf{s}-\mathbf{r}|^3}$$



A vortex makes the superflow of the Biot-Savart law, and moves with this local flow.

SUPERFLUID HELIUM

Realization of atomic gas BEC

1995 ^{87}Rb, ^{7}Li, ^{23}Na

Laser cooling

An atom is subjected to a laser beam whose frequency is tuned to lie just below that of an atomic transition between an excited ... by the ...

Cold atoms are collected at the focus of six laser beams. *T*~ 100μK

Motion of atoms is reduced by laser.

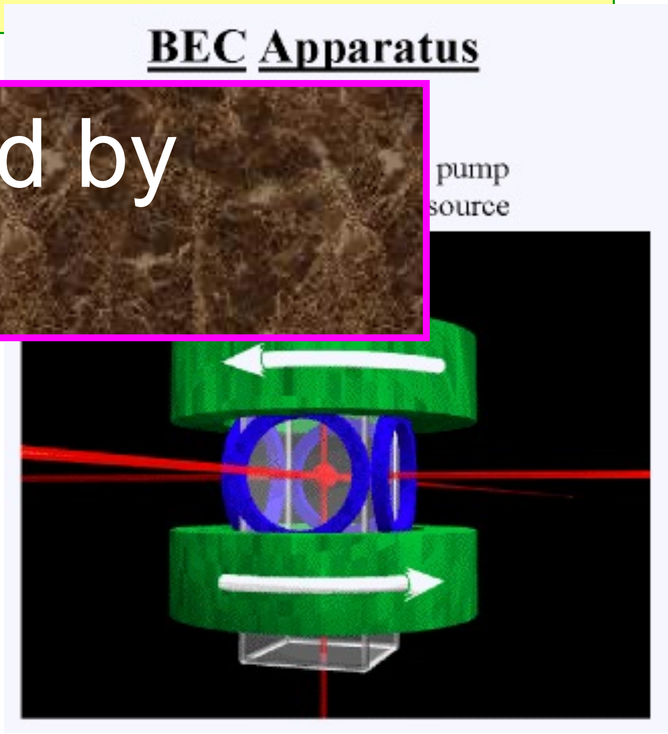
Magnetic trap

The atoms are trapped by potential.

Evapora...

The fast atoms are released out of the trap.

BEC *T*~ 100 nK

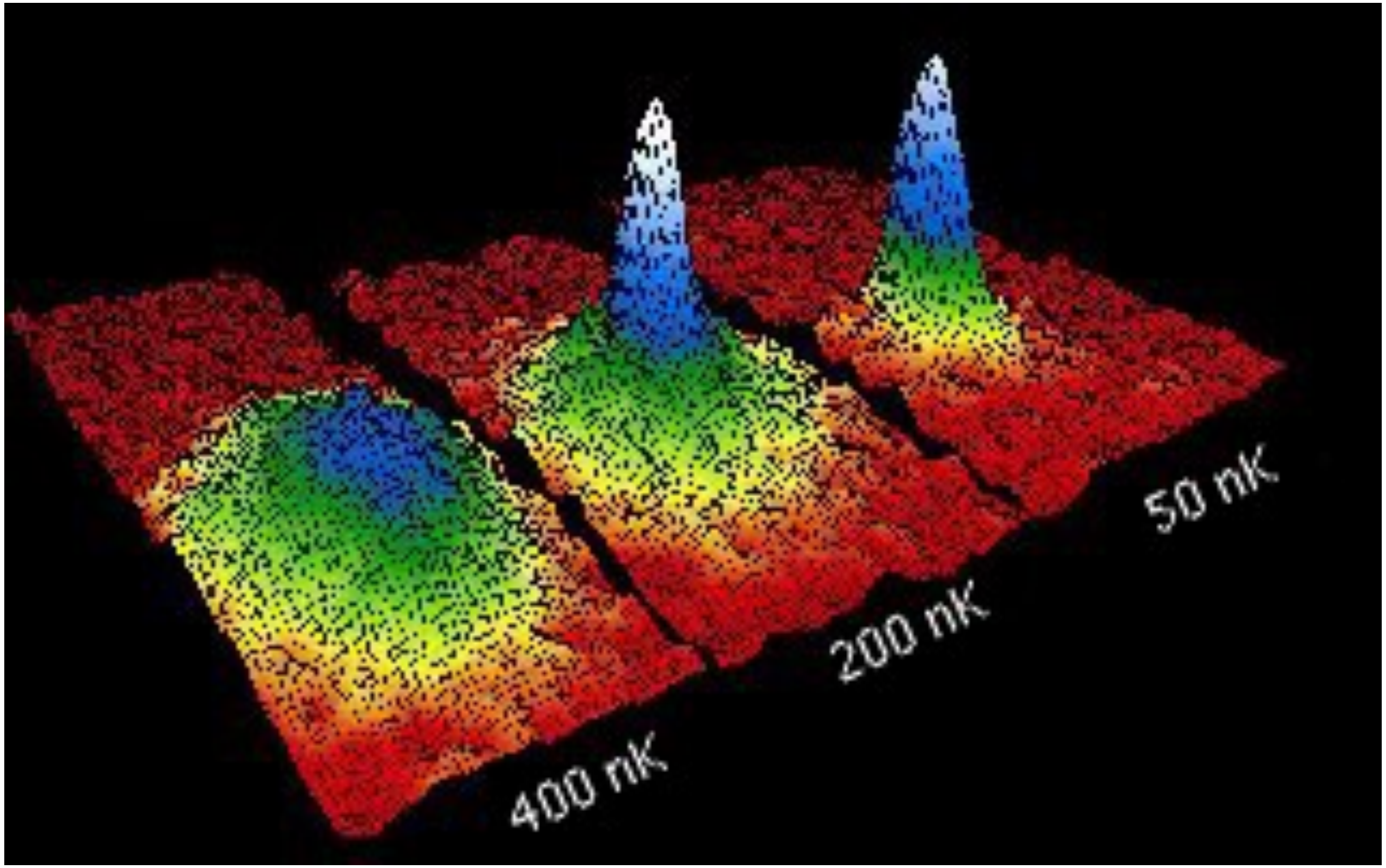


Observation of BEC

Turning off the trapping potential,

→ the gas expands with falling feely.

→ The observation of the position of atoms determines the initial distribution of velocity.

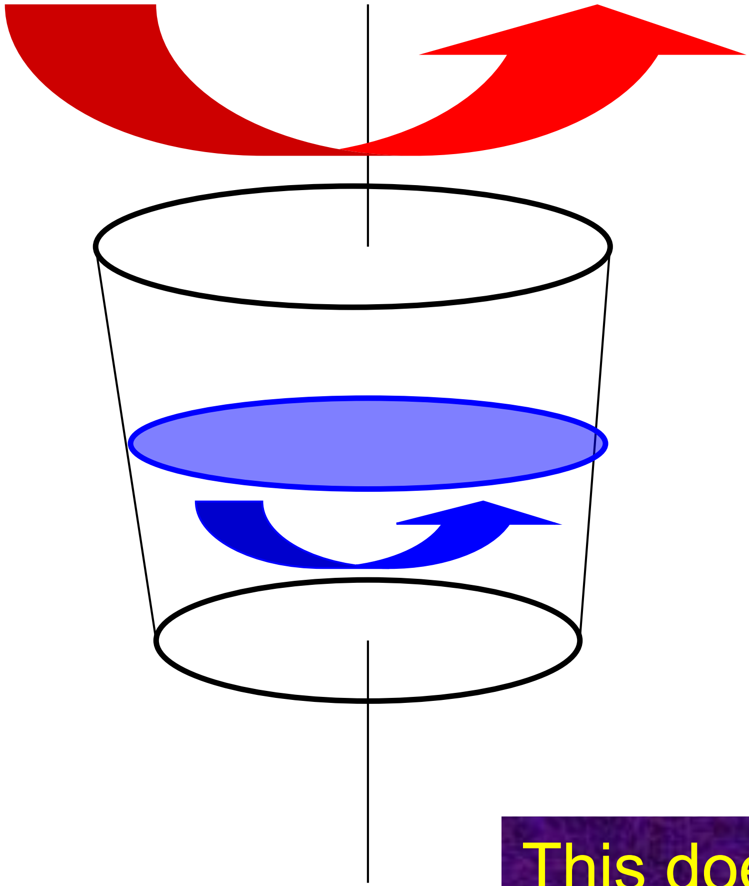


By JILA

The Nobel Prize in Physics 2001!

2. Vortex lattice formation in rotating BEC

What happens if we rotate a vessel having a usual viscous classical fluid inside?



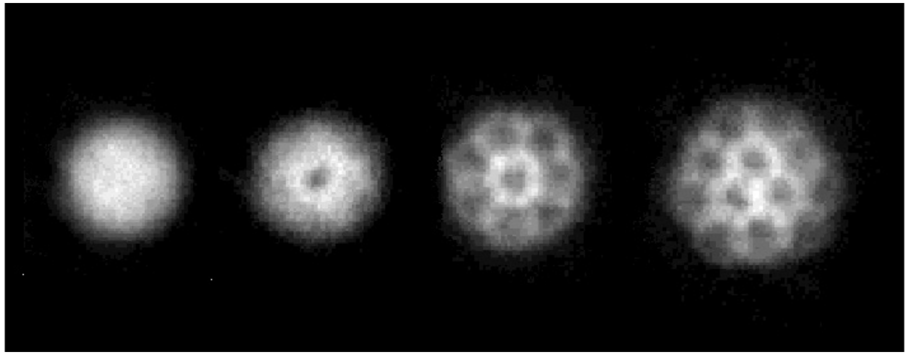
The fluid rotates with the same angular velocity with the vessel.

There appears a uniform vortex making the solid-body rotation with any angular velocity.

This does not occur in quantum fluids! Such experiments were done in atomic BECs.

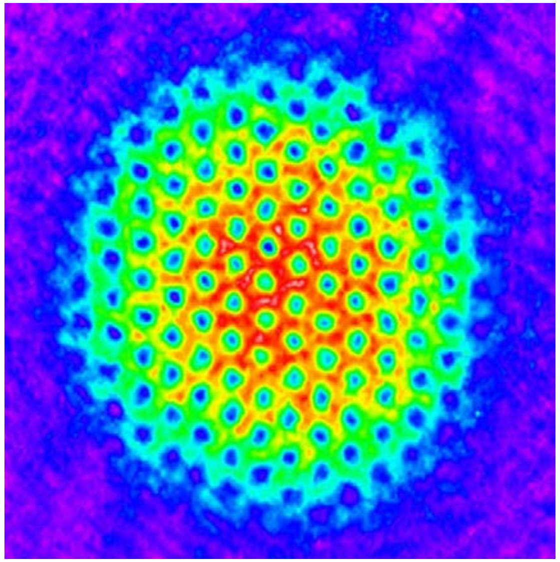
Observation of quantized vortices in atomic BECs

ENS K.W.Madison, et al. PRL **84**, 806 (2000)

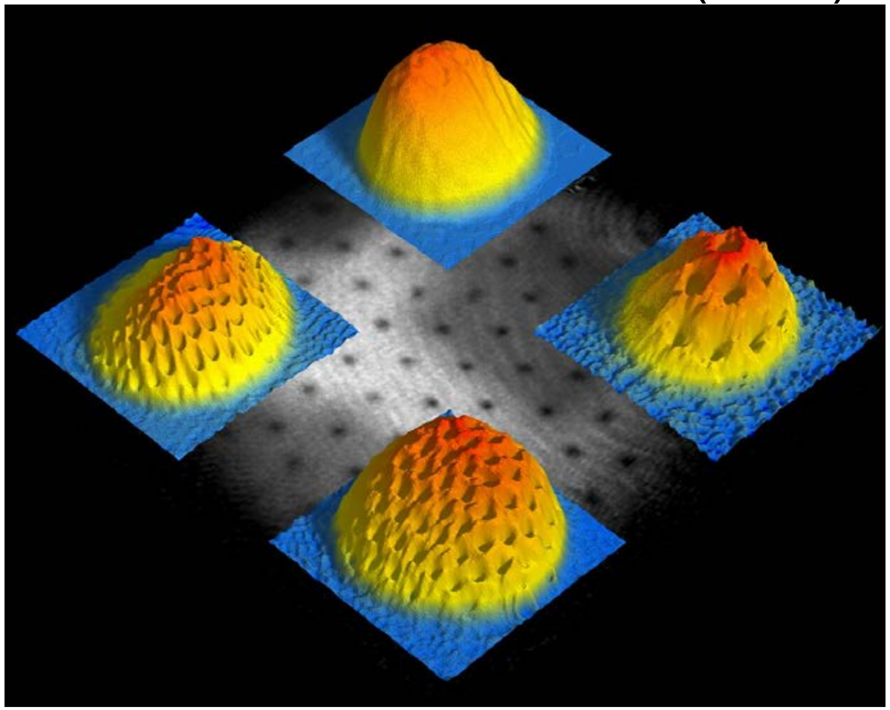


JILA

P. Engels, et al.
PRL **87**, 210403 (2001)

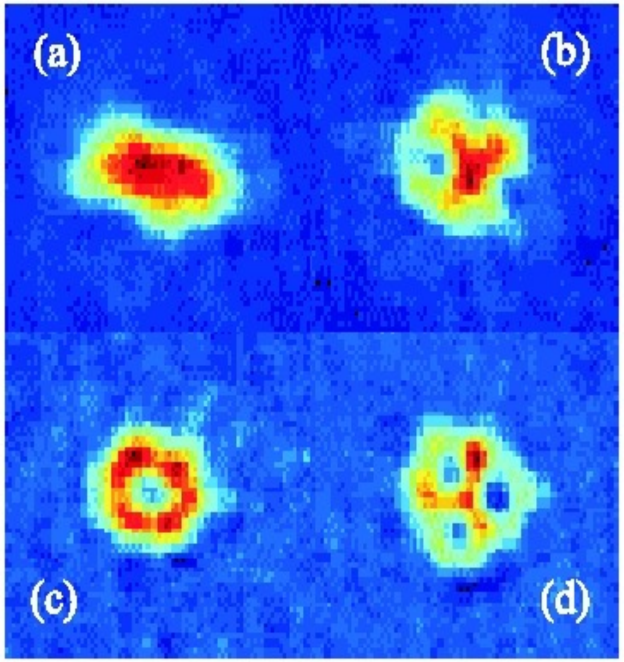


MIT J.R. Abo-Shaeer, et al.
Science **292**, 476 (2001)

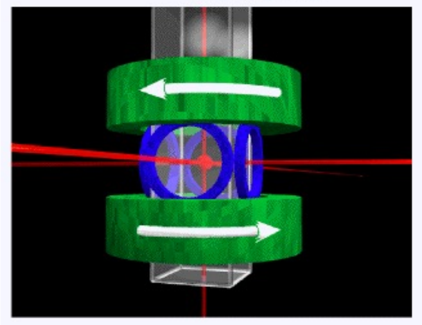


Oxford

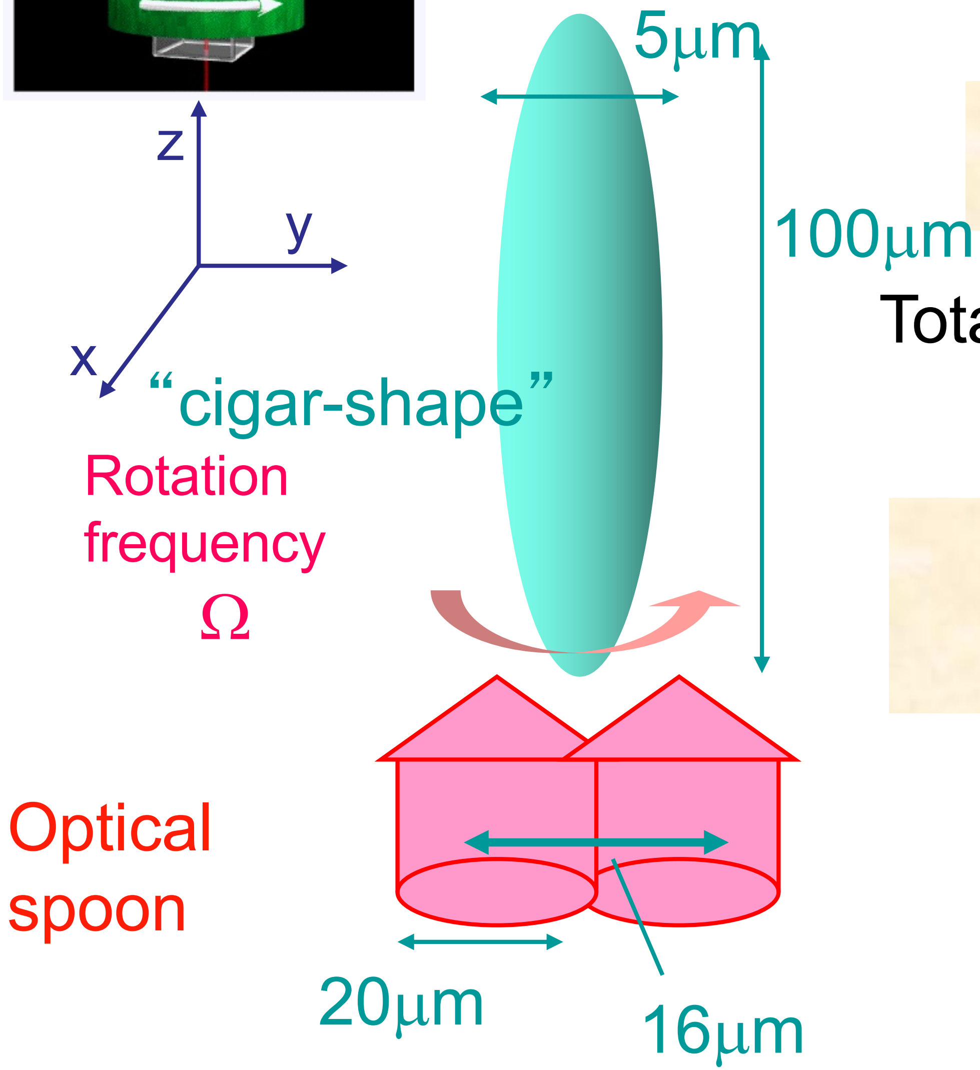
E. Hodby, et al.
PRL **88**, 010405 (2002)



How can we rotate the trapped BEC？



K.W.Madison et al. Phys.Rev Lett **84**, 806 (2000)



Axisymmetric potential

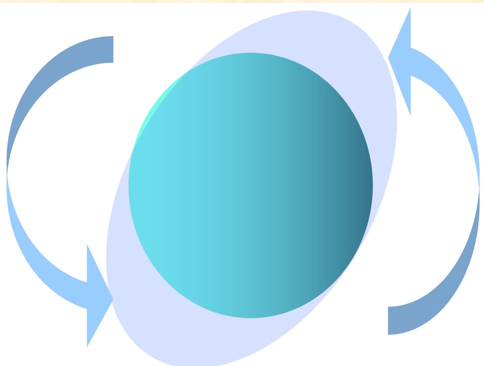
$$V_{ext}(\mathbf{R}) = V_{trap}(\mathbf{R}) + U_{stir}(\mathbf{R})$$

Total potential

Non-axisymmetric poten

$$U_{stir}(\mathbf{R}) = \frac{m}{2}\omega_{\perp}^{2}(\varepsilon_x X^2 + \varepsilon_y Y^2)$$

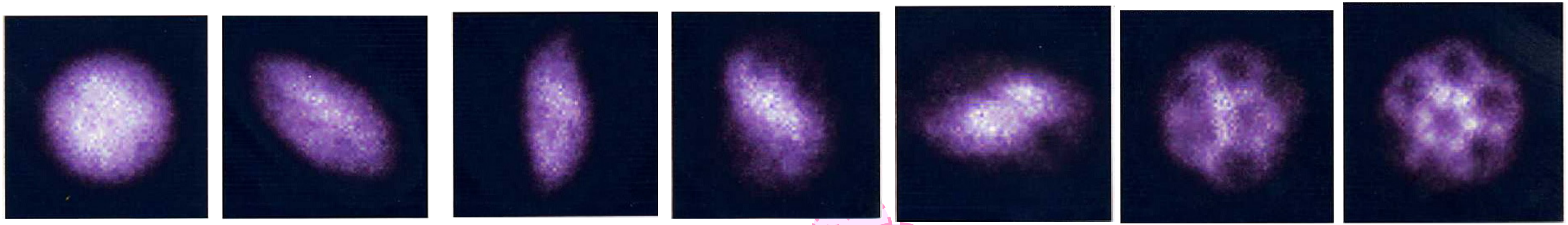
$$\varepsilon_x \neq \varepsilon_y$$



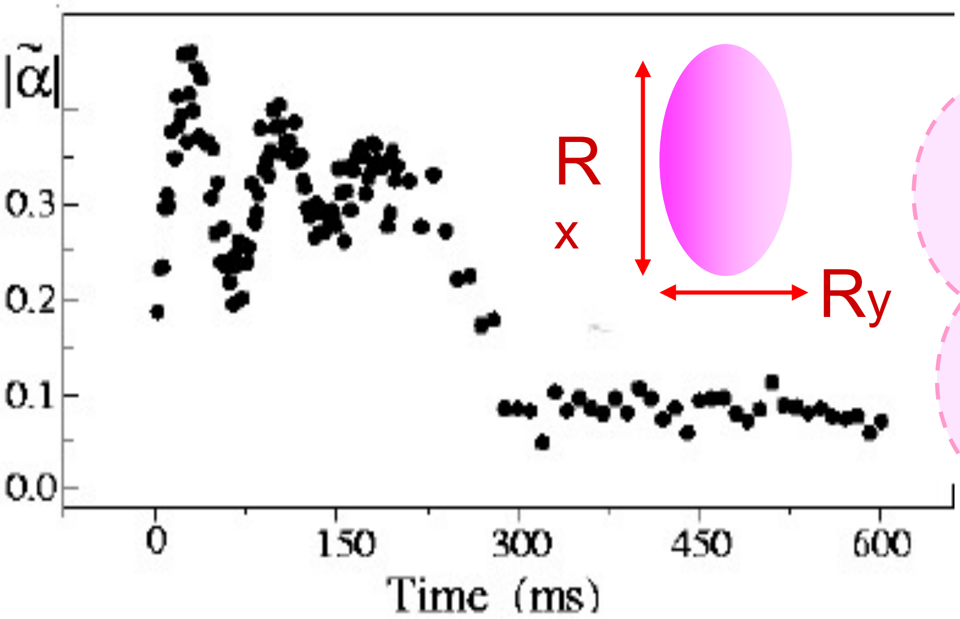
Direct observation of the vortex lattice formation

K.W. Madison *et al.* PRL **86** , 4443 (2001)

Snapshots of the BEC after turning on the rotation



$$\alpha = \Omega \frac{R_x^2 - R_y^2}{R_x^2 + R_y^2}$$



1. The BEC becomes elliptic, then oscillating.
2. The surface becomes unstable.
3. Vortices enter the BEC from the surface.
4. The BEC recovers the axisymmetry, the vortices forming a lattice.

The Gross-Pitaevskii(GP) equation in a rotating frame

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi + V_{\mathrm{trap}}\Psi + g|\Psi|^2\Psi$$

Wave function $\Psi(\mathbf{r},t)$

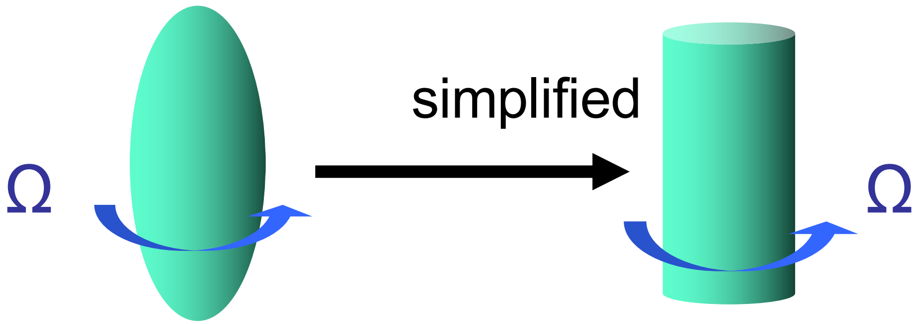
Interaction $g = \frac{4\pi\hbar^2 a_s}{m}$

a_s s-wave scattering length

in a rotating frame

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi + (V_{\mathrm{trap}} + U_{\mathrm{stir}})\Psi + g|\Psi|^2\Psi - \Omega L_z \Psi$$

Two-dimensional $U_{\mathrm{stir}}(\mathbf{R}) = \frac{m}{2}\omega_\perp^2(\varepsilon_x X^2 + \varepsilon_y Y^2)$



The GP equation with a dissipative term

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + (V_{\mathrm{trap}} + U_{\mathrm{stir}}) + g|\Psi|^2 - \mu - \Omega L_z \right] \Psi$$

$$(i - \gamma)\hbar \frac{\partial \Psi}{\partial t}$$

$\gamma = 0.03$: dimensionless parameter

S.Choi, et al. PRA 57, 4057 (1998)

I.Aranson, et al. PRB 54, 13072 (1996)

This dissipation comes microscopically from the interaction between the condensate and the noncondensate.

E.Zaremba, T. Nikuni, and A. Griffin, J. Low Temp. Phys. **116**, 277 (1999)

C.W. Gardiner, J.R. Anglin, and T.I.A. Fudge, J. Phys. B **35**, 1555 (2002)

M. Kobayashi and M. Tsubota, Phys. Rev. Lett. **97**, 145301 (2006)

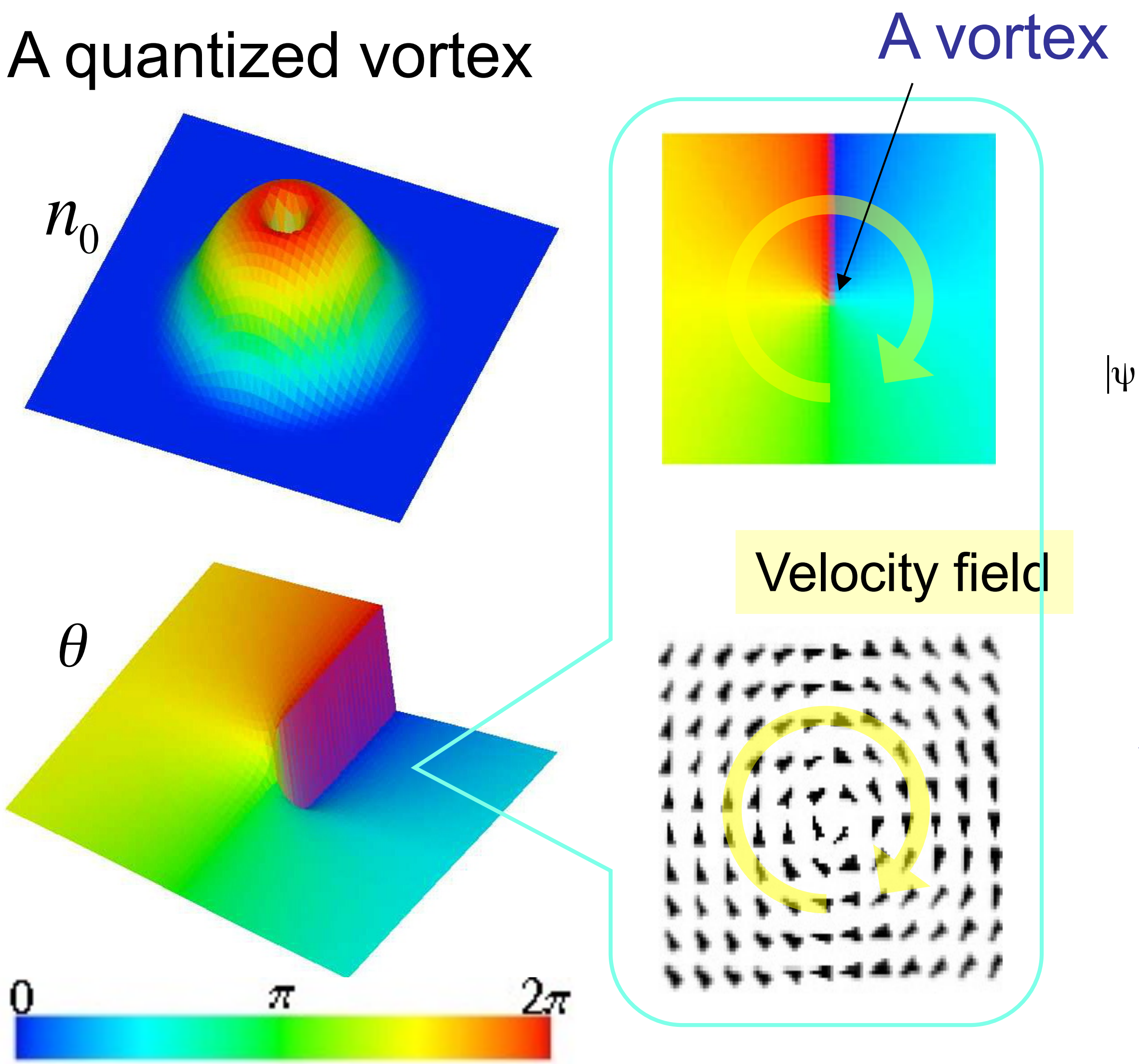
Profile of a single quantized vortex

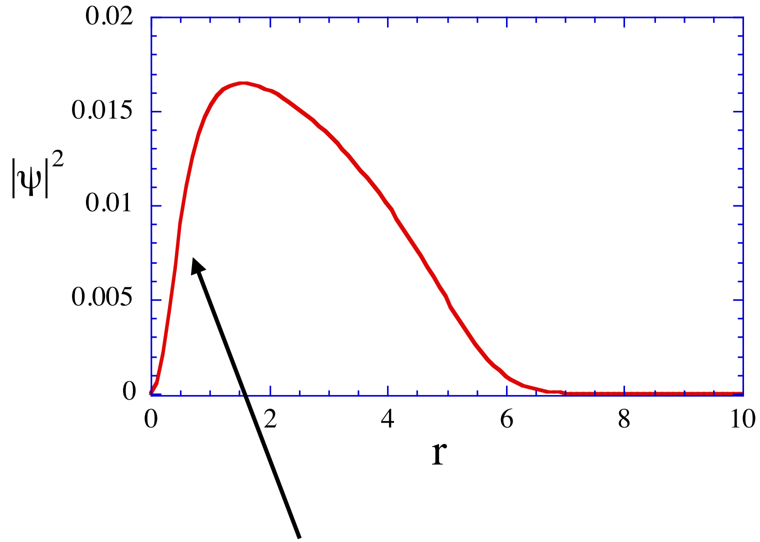
$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V_{\text{trap}}\Psi + g|\Psi|^2\Psi = \mu\Psi$$

$$\Psi(r) = \sqrt{n_0(r)}e^{i\theta(r)}$$

$$\mathbf{v}_s \equiv \frac{\hbar}{m}\nabla\theta$$

A quantized vortex





Vortex core= healing leng

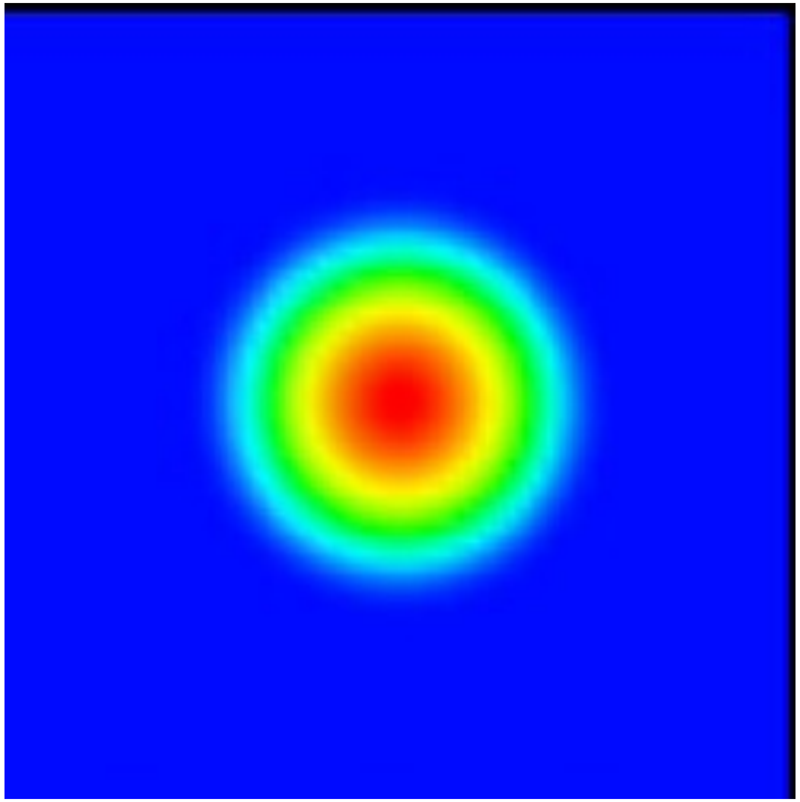
$$\xi \approx \frac{\hbar}{\sqrt{2mgn_0}}$$

Dynamics of the vortex lattice formation (1)

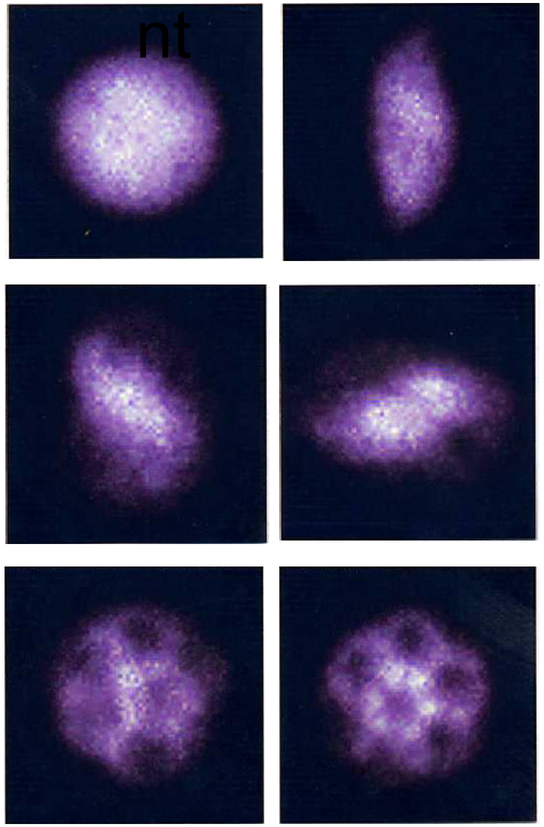
Time development of the condensate density n_0

$$\Omega = 0.7\omega_\perp$$

MT, K. Kasamatsu, M. Ueda, Phys. Rev. A **65**, 023603 (2002)



Experime nt



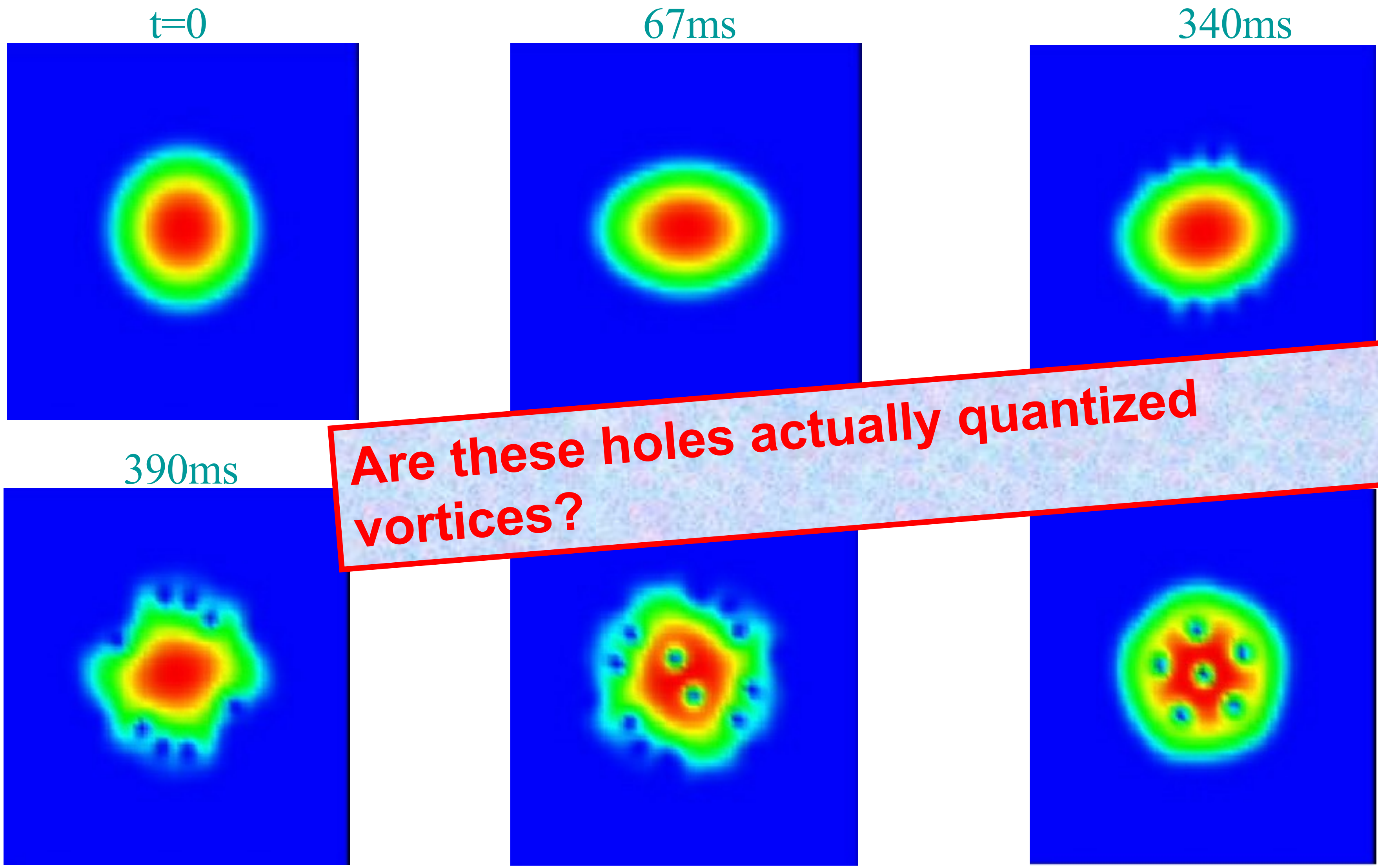
$$V_{\text{trap}}(r) = \frac{1}{2} m\omega_\perp^2 r^2$$

$$\Psi(r) = \sqrt{n_0(r)} e^{i\theta(r)}$$

K.W.Madison *et al*. PRL **86** , 4443 (2001)

Dynamics of the vortex lattice formation (2)

Time-development of the condensate density n_0

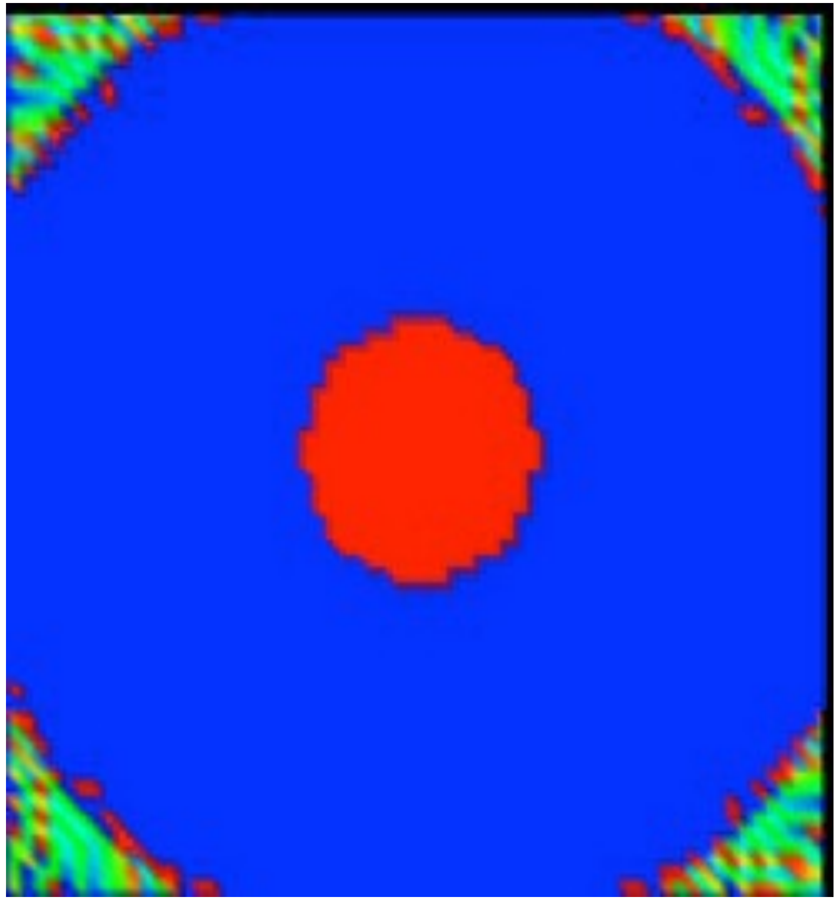


Are these holes actually quantized vortices?

Dynamics of the vortex lattice formation (3)

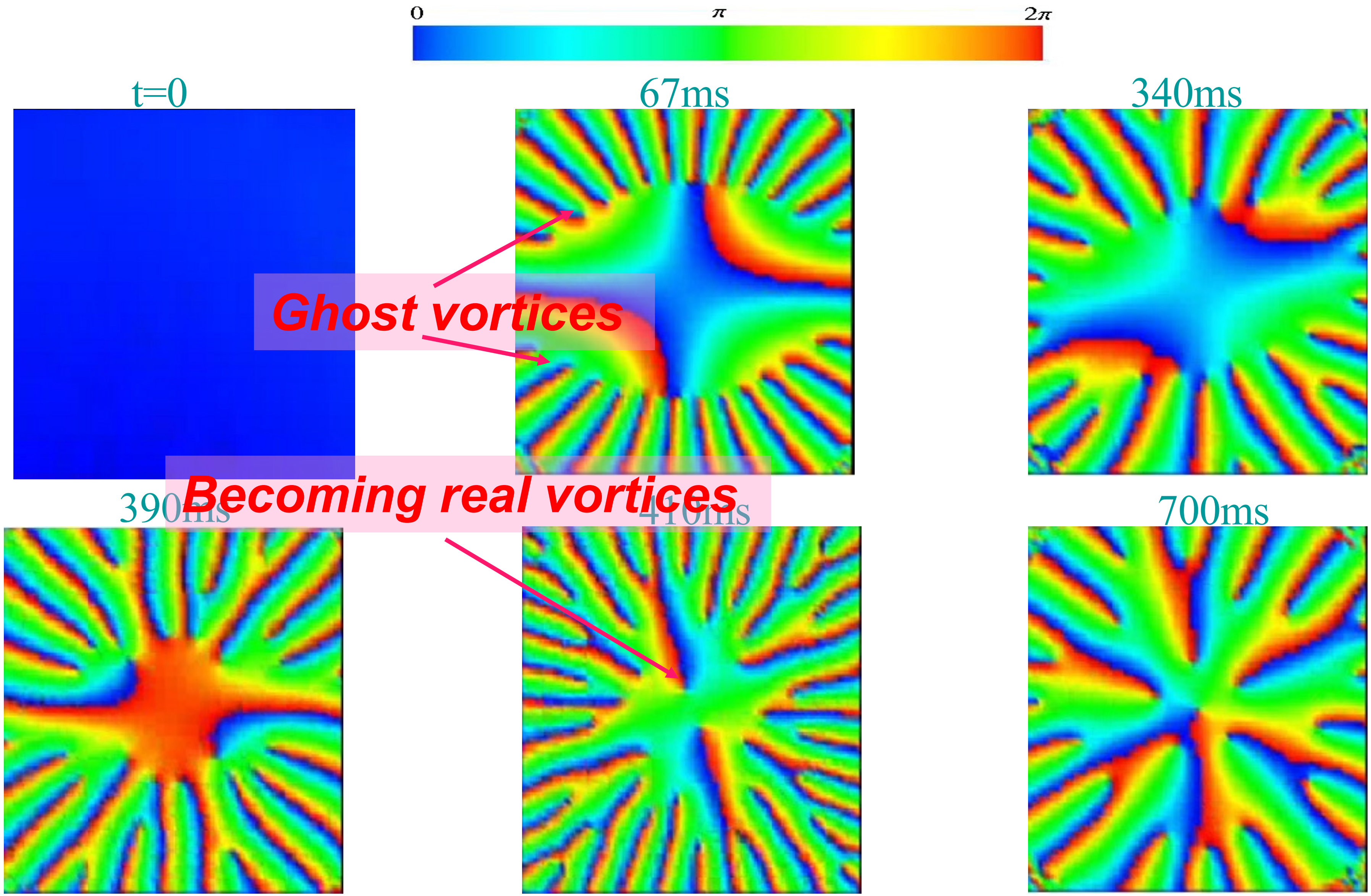
Time-development of the phase θ

$$\Psi(r) = \sqrt{n_0(r)}e^{i\theta(r)}$$

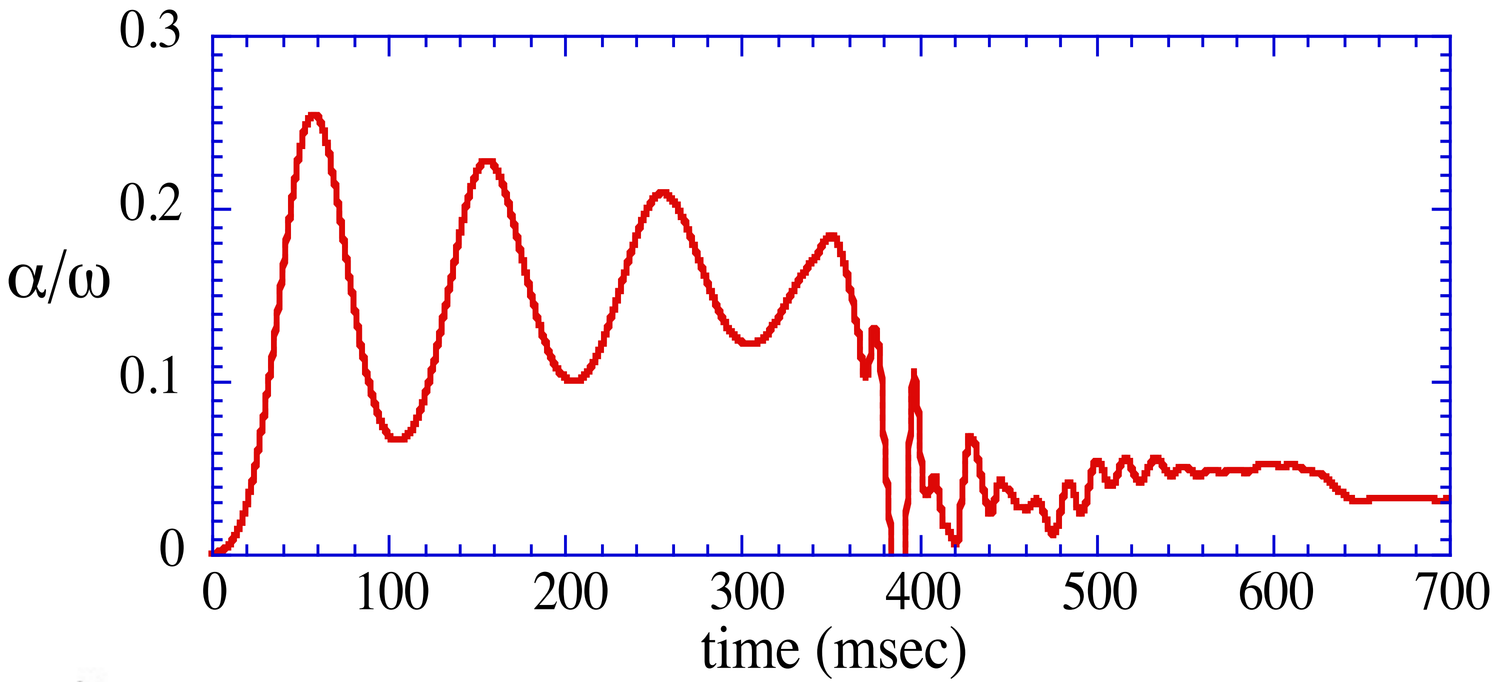


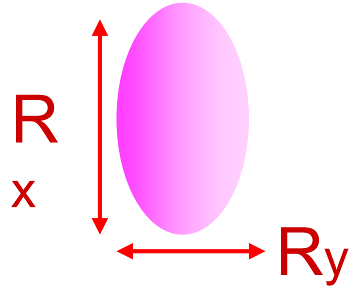
Dynamics of the vortex lattice formation (4)

Time-development of the phase θ

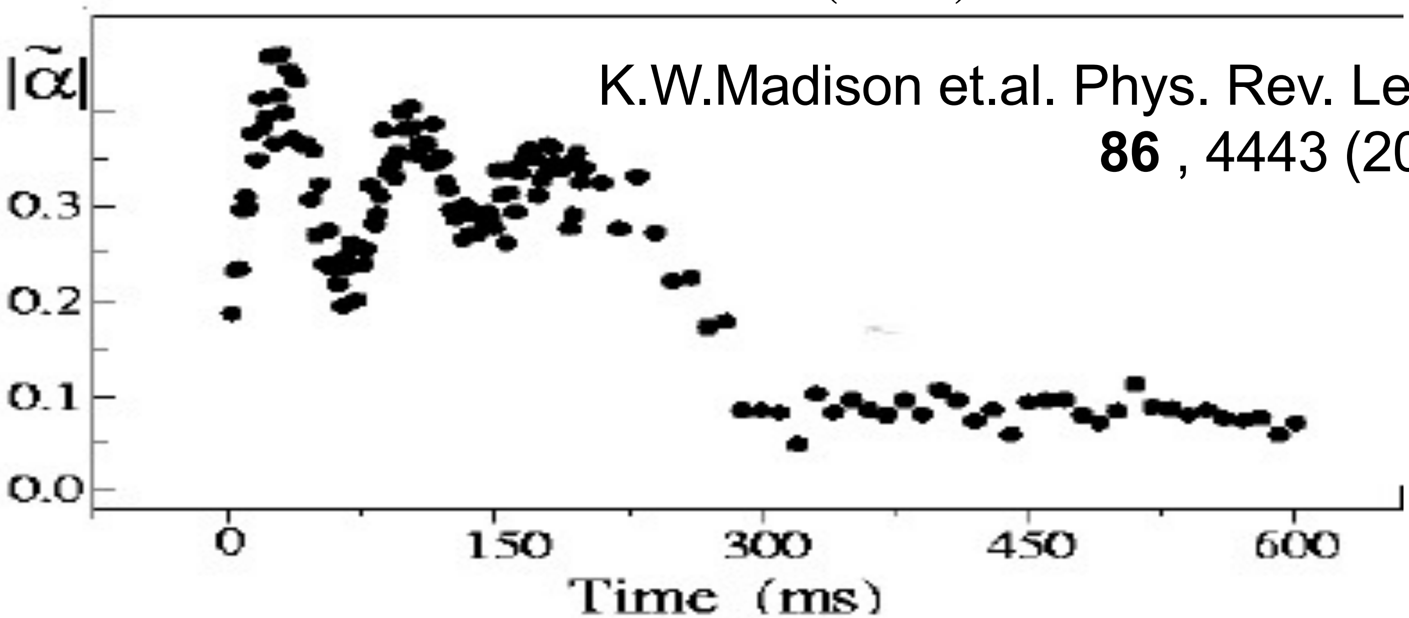


Dynamics of the vortex lattice formation (5)





$$\alpha = \Omega \frac{R_x^2 - R_y^2}{R_x^2 + R_y^2}$$

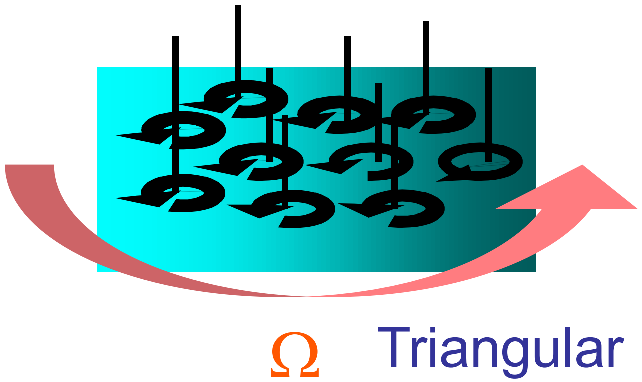


K.W.Madison et.al. Phys. Rev. Lett. **86** , 4443 (2001)

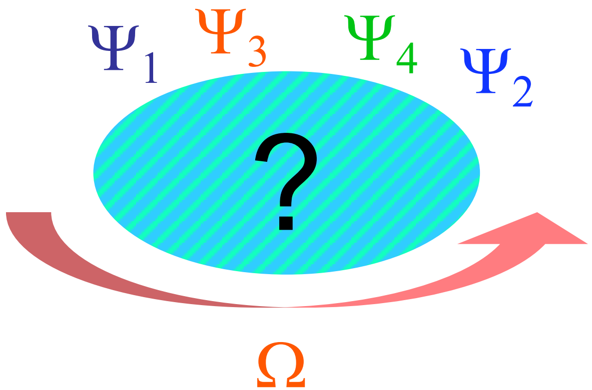
3. Vortex lattice in two-component BECs

Depending on the symmetry, multi-component order parameters can yield various kinds of topological defects.

superfluid ^3He, superconductivity with non-s-wave symmetry (Sr_2RuO_4, UPt_3), bilayer quantum Hall systen, nonlinear optics, nuclear physics, cosmology(Neutron star), …



Triangular (Abrikosv)lattice



Two-component BEC

Two order parameters (macroscopic wave functions) $\Psi_1 \; \Psi_2$

Coupled Gross-Pitaevskii(GP) equations

$$i\hbar\partial_t\Psi_1 = \left(-\frac{\hbar^2}{2m_1}\nabla^2 + U_1 + g_{11}|\Psi_1|^2 + g_{12}|\Psi_2|^2\right)\Psi_1$$

$$i\hbar\partial_t\Psi_2 = \left(-\frac{\hbar^2}{2m_2}\nabla^2 + U_2 + g_{12}|\Psi_1|^2 + g_{22}|\Psi_2|^2\right)\Psi_2$$

$g_{11}, \; g_{22}$: intracomponent interaction

g_{12} : intercomponent interaction

When $g_{11}g_{22} > g_{12}^2$, two BECs are mixed.

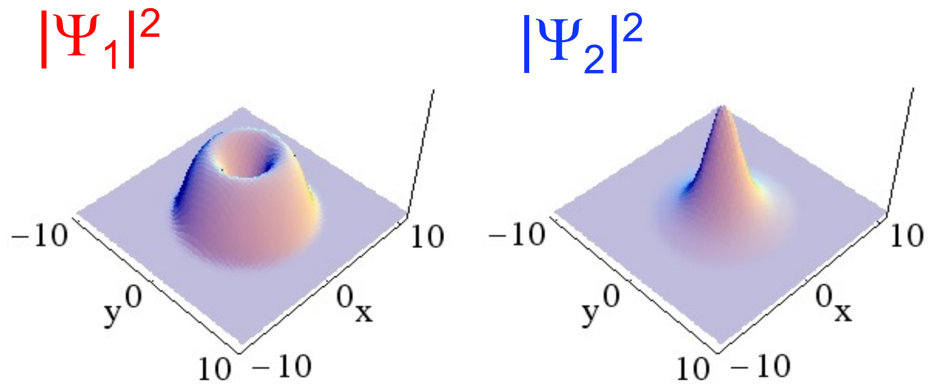
When $g_{11}g_{22} < g_{12}^2$, two BECs are phase-separated.

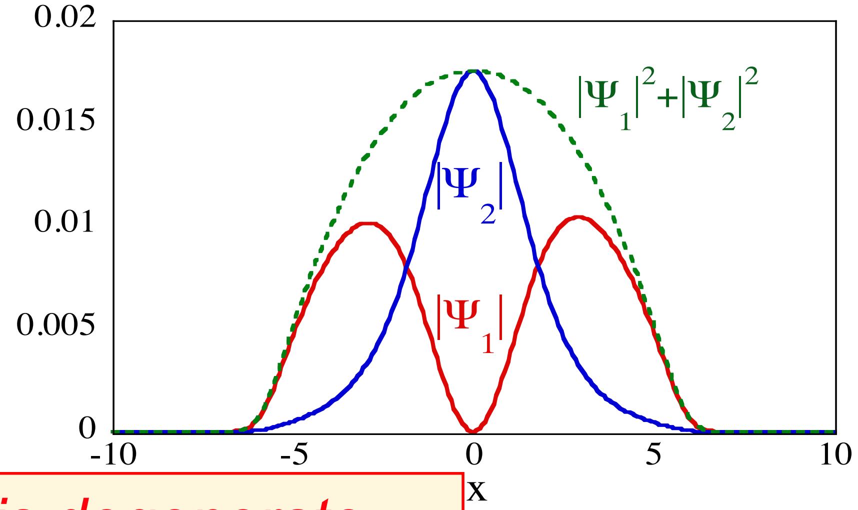
Axisymmetric coreless vortex state ~Skyrmion~

$g_1 = g_2 = g_{12}$ (SU(2) symmetry), $\Omega = 0.15\omega$

Axisymmetric structure

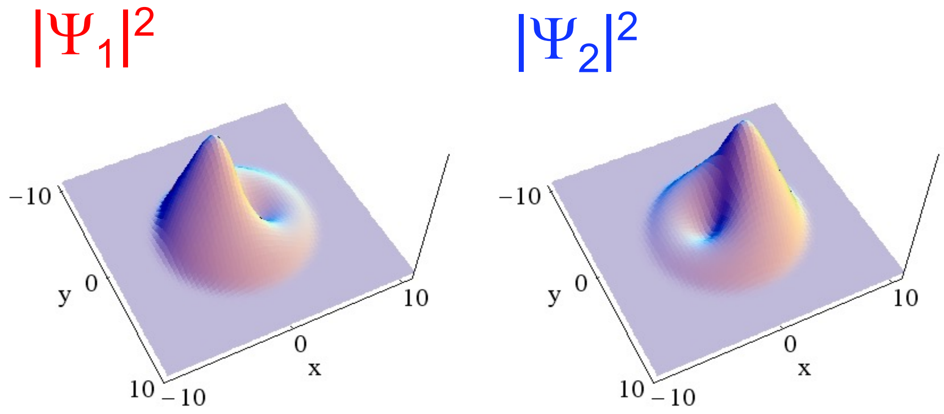
Realized in JILA group (Matthews et al., PRL **83**, 2498 (1999))

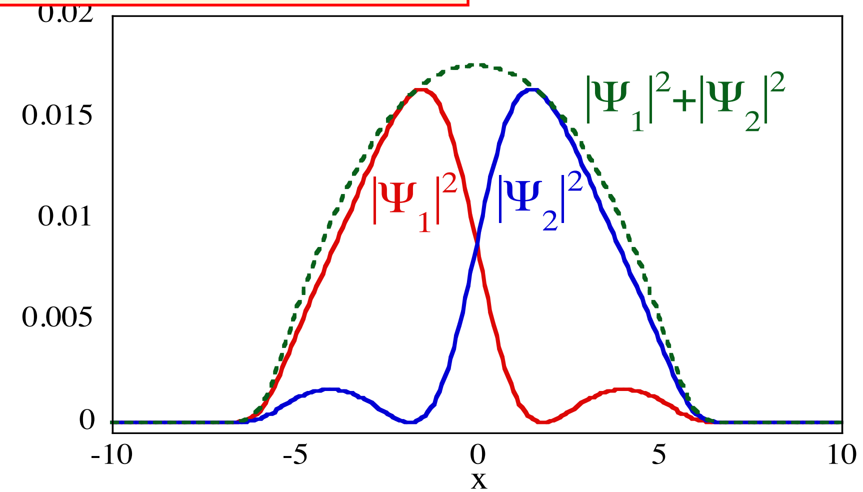




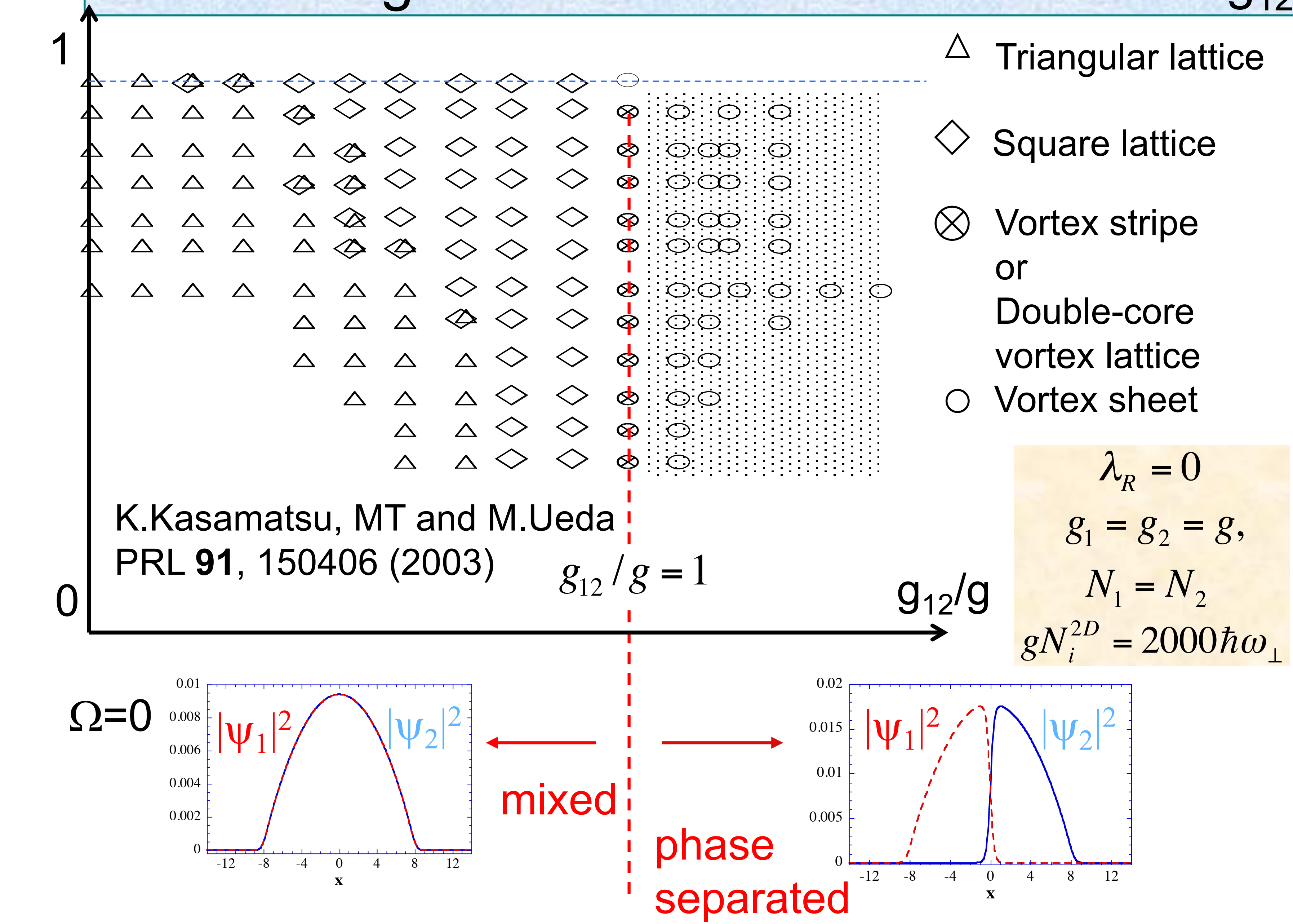
The energy is degenerate.

Nonaxisymmetric structure

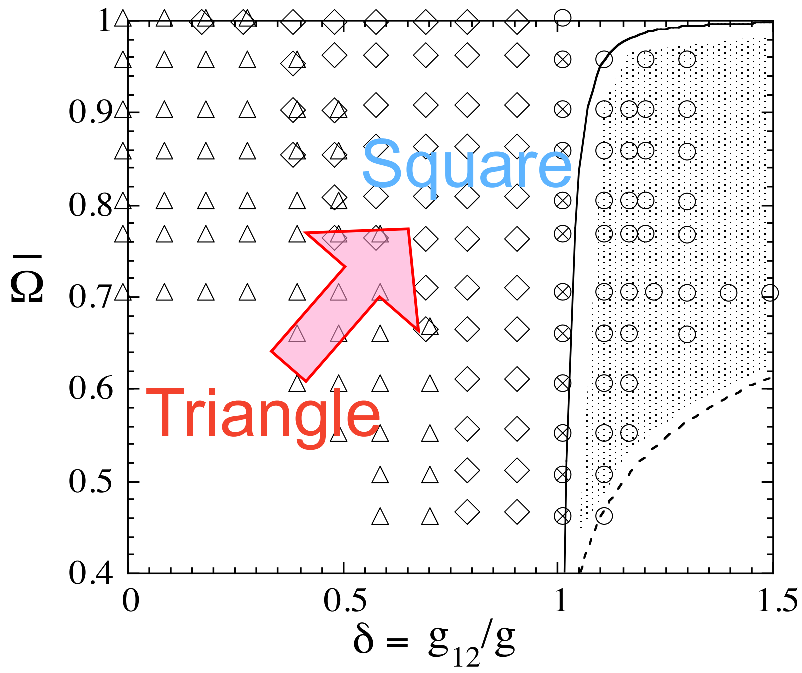




Phase diagram of vortex states ~rotation vs g_{12} ~

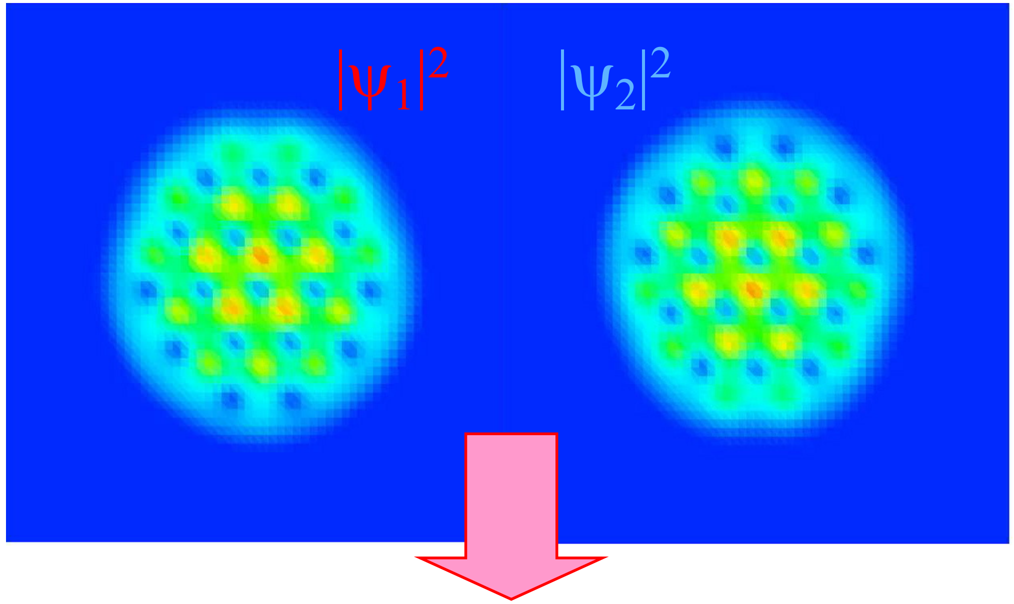


Vortex lattices in rotating two-component BECs

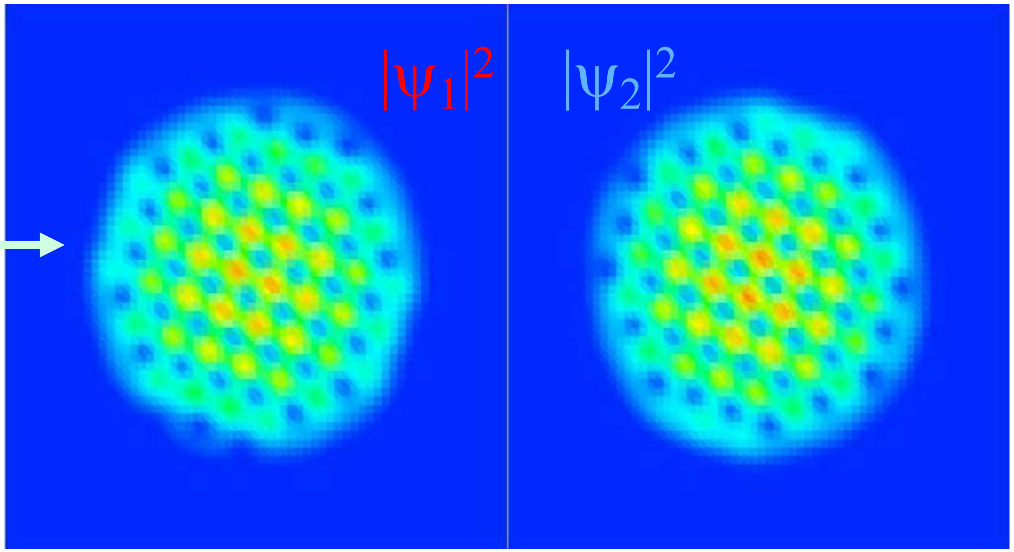


K. Kasamatsu, MT, M. Ueda, PRL91, 150406 (2003)

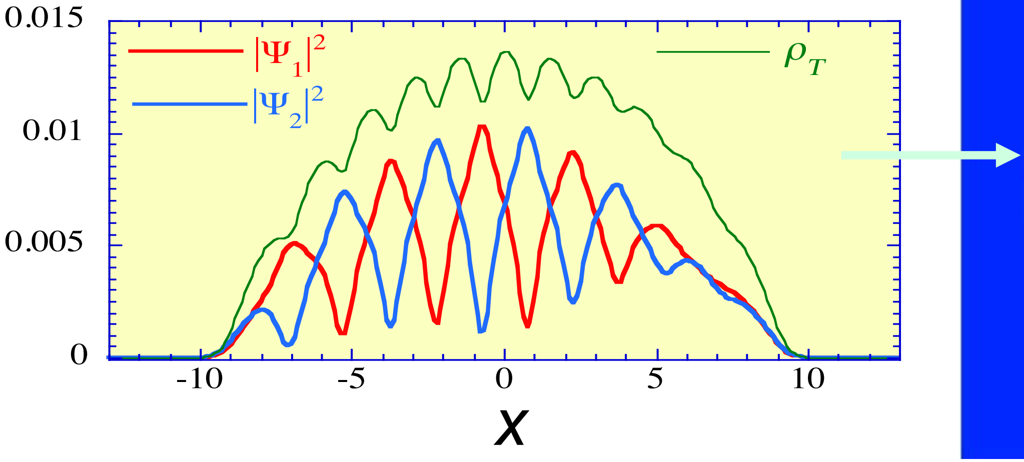
Triangular lattices



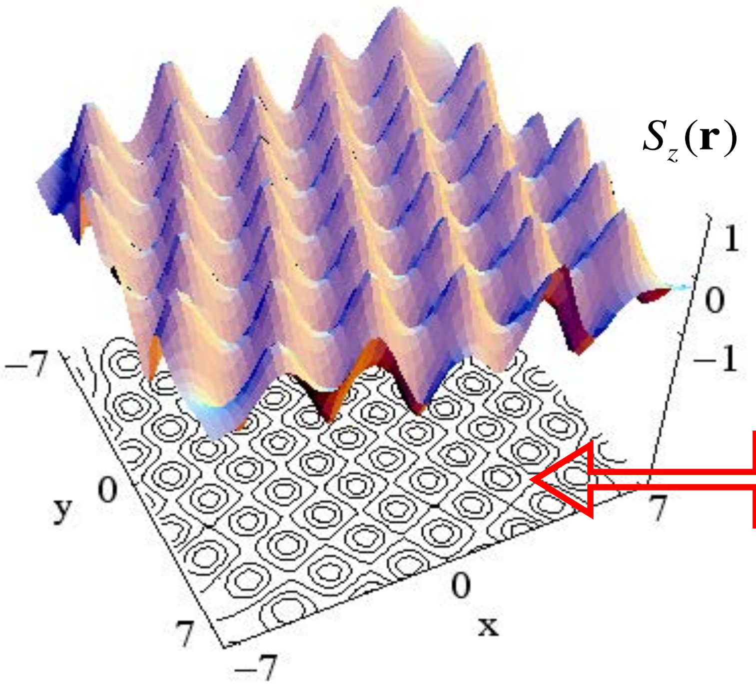
Square lattices



cross section



What likes the square lattice?



Interaction energy

$$E_{\text{int}} = \int d\mathbf{r}\left[\frac{g_{11}}{2}|\psi_1|^4 + \frac{g_{22}}{2}|\psi_2|^4 + g_{12}|\psi_1|^2|\psi_1|^2\right]$$

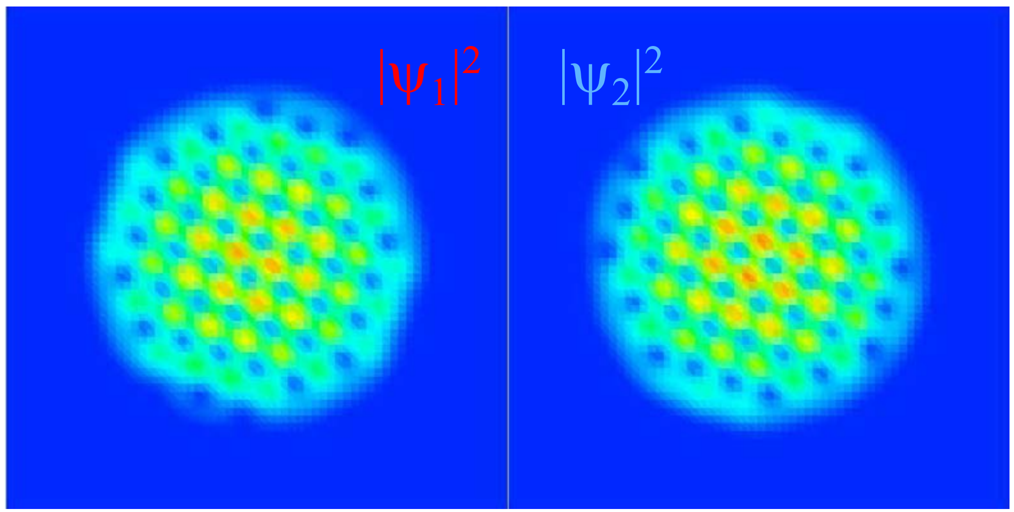
$$= \int d\mathbf{r}\,\frac{1}{4}\ [\ (g_{11} + g_{12})\left(|\psi_1|^2 + |\psi_2|^2\right)^2$$

$$+ (g_{11} - g_{12})\left(|\psi_1|^2 - |\psi_2|^2\right)^2\ ]$$

$(g - g_{12})S_z^2(\mathbf{r})$

In the core of ψ_2, $|\psi_1| = 1, |\psi_2| = 0 \Rightarrow S_z = 1$

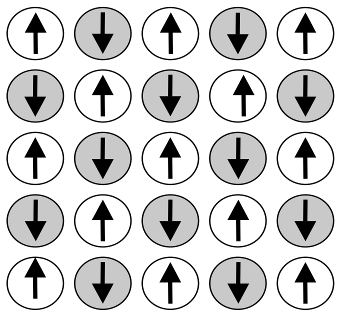
In the core of ψ_1, $|\psi_1| = 0, |\psi_2| = 1 \Rightarrow S_z = -1$



2D Ising model

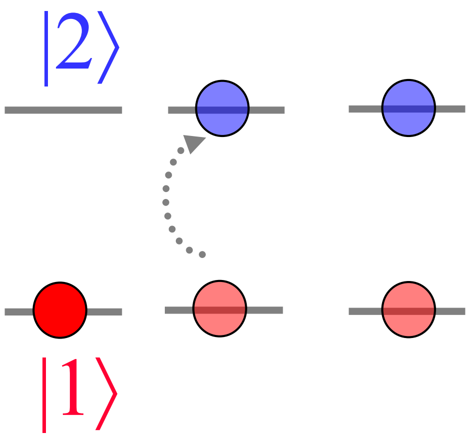
Antiferro *J>0*

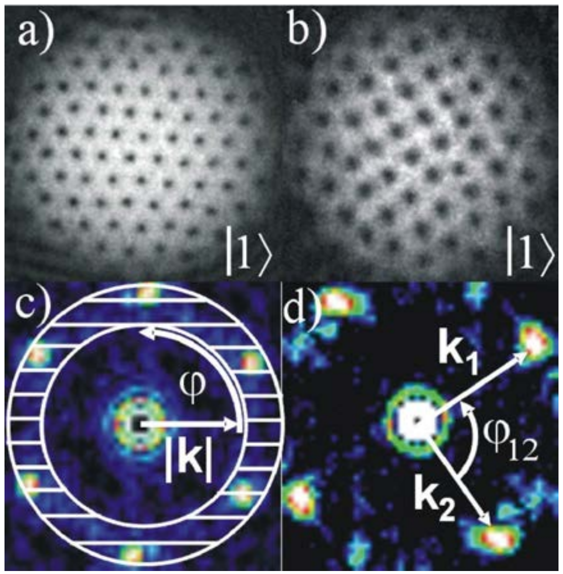
$$H = J\sum S_{zi}S_{zj}$$

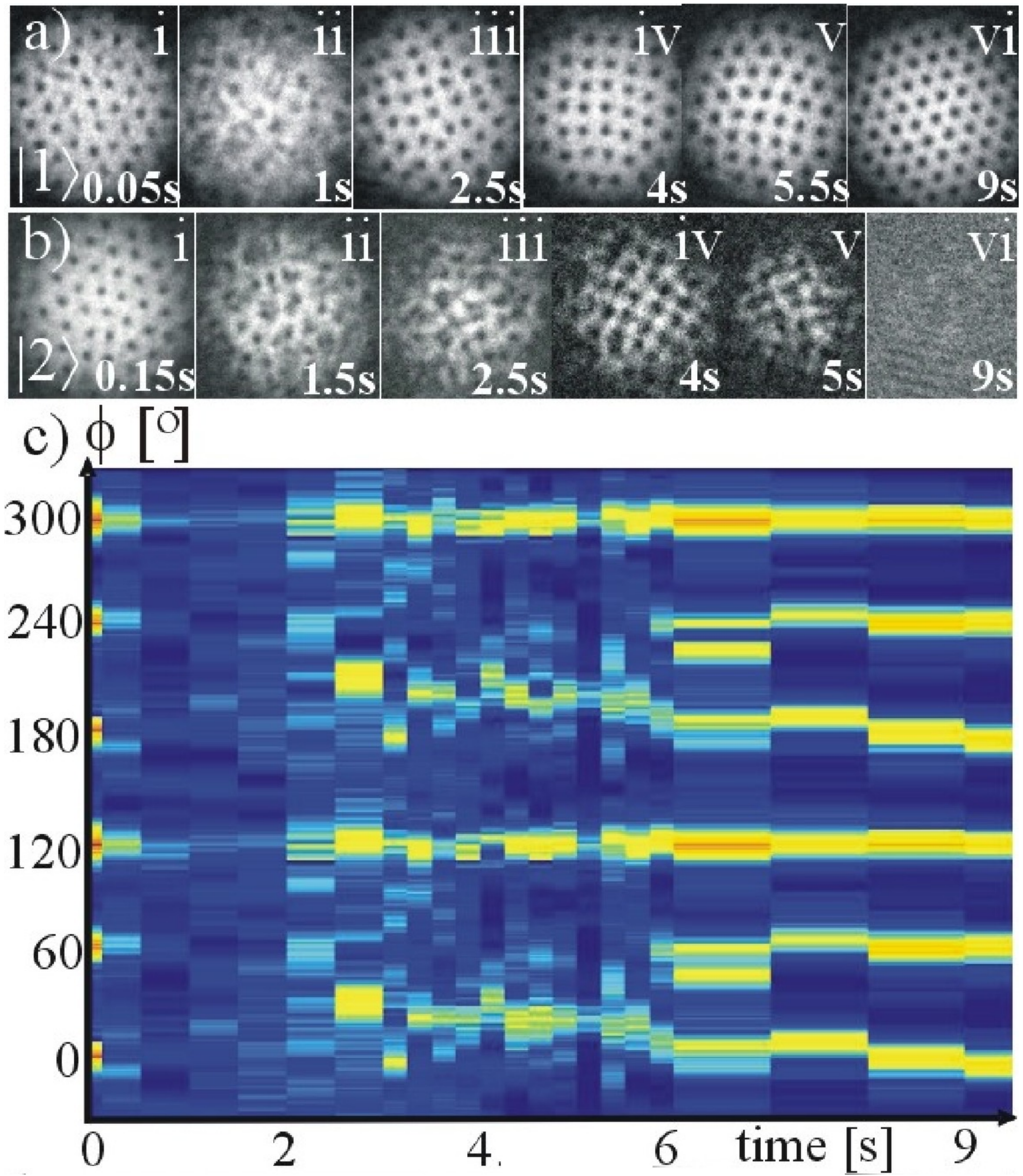


Observation of triangular and square lattices

V. Schweikhard, et al., PRL **93**, 210403 (2004)



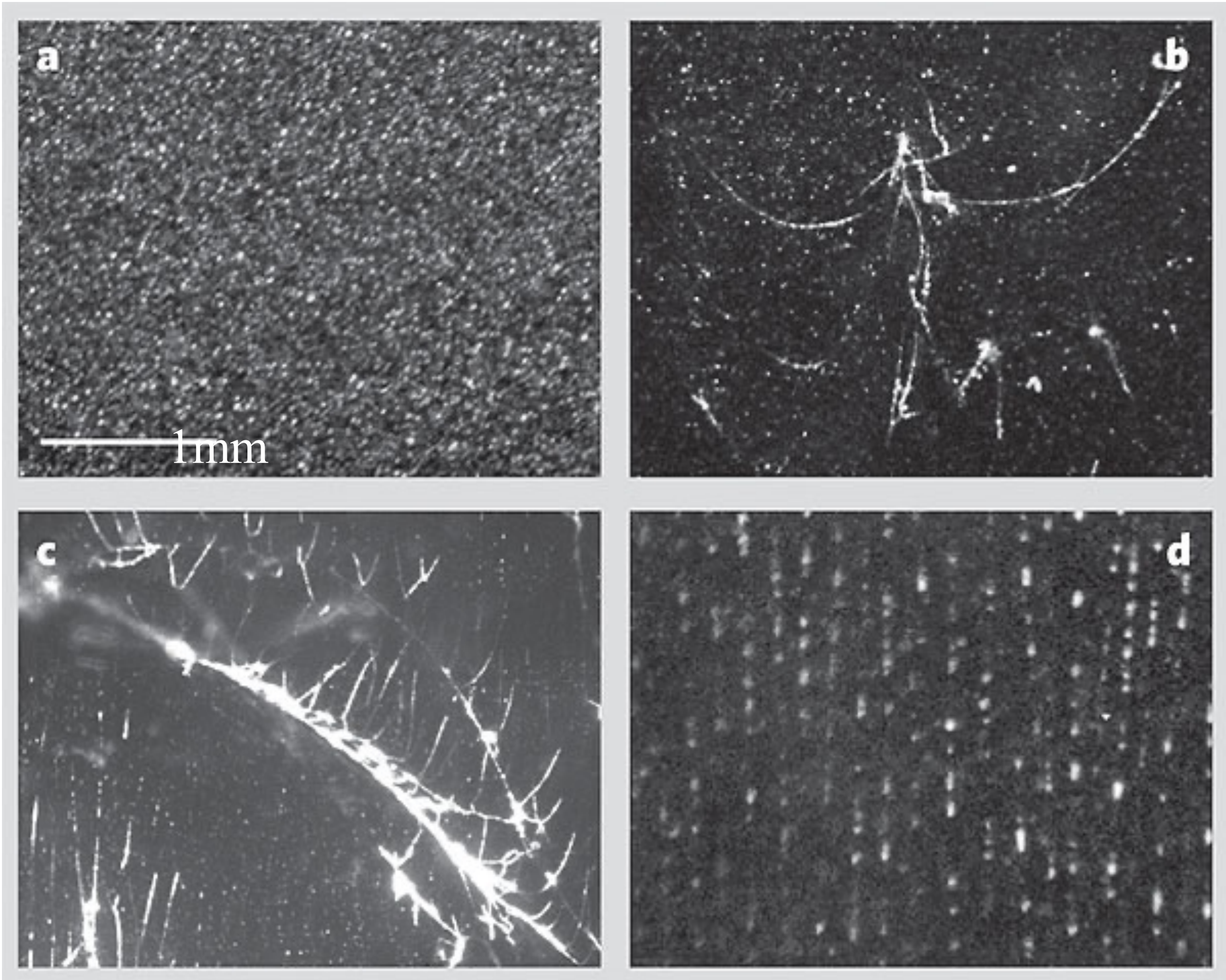




Visualization of quantized vortices in superfluid ^4He

G. P. Bewley, D. P. Lathrop, K. R. Sreenivasan, Nature 441, 588(2006)

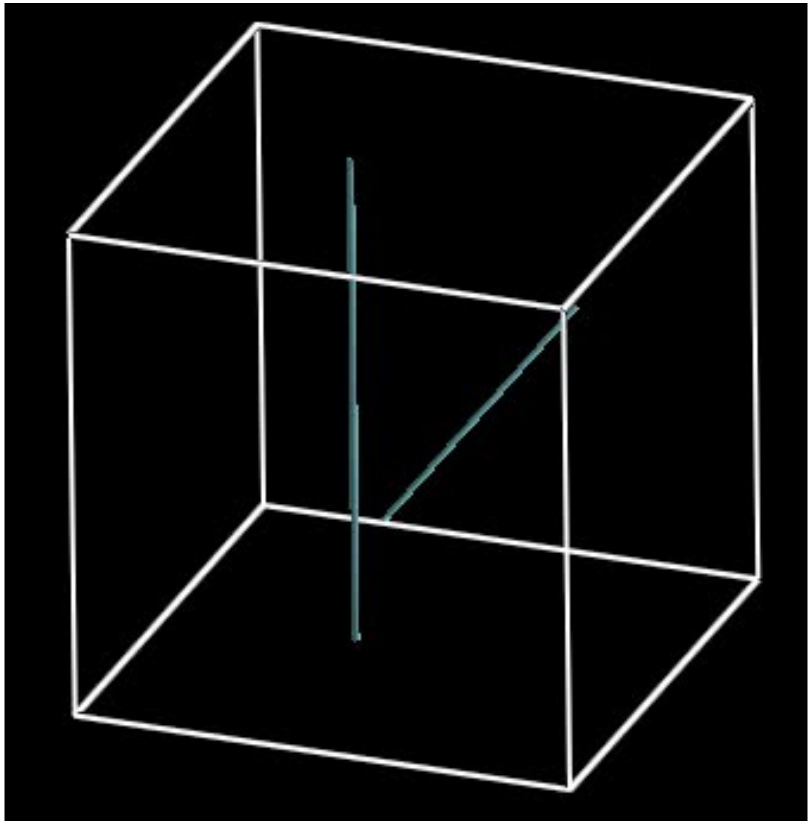
$T > T_\lambda$

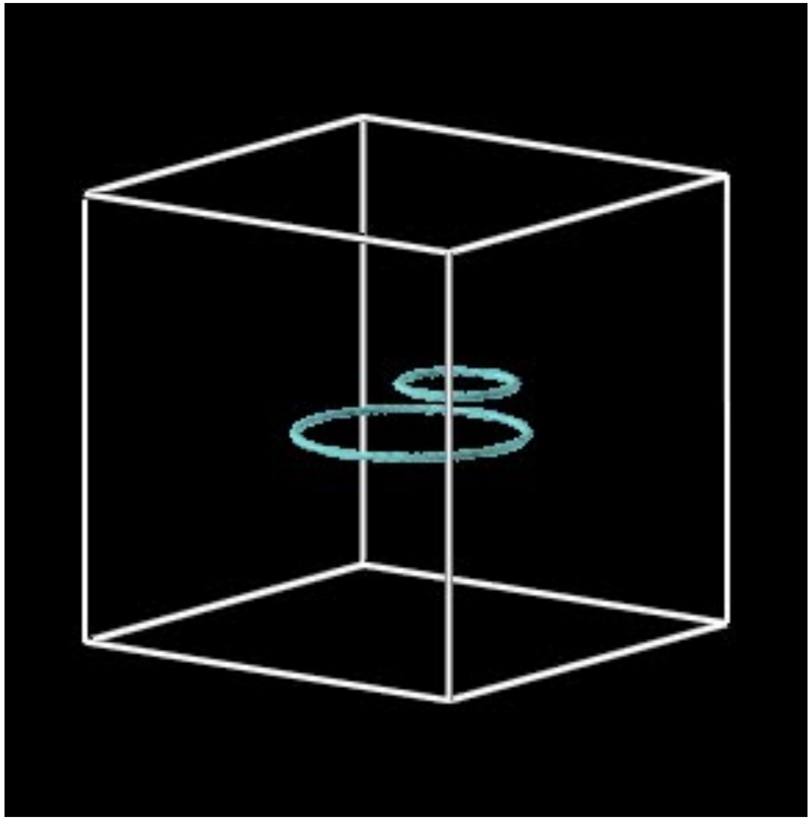


$T < T_\lambda$

Solid hydrogen particles are trapped by quantized vortices.

Vortex reconnection by the GP model



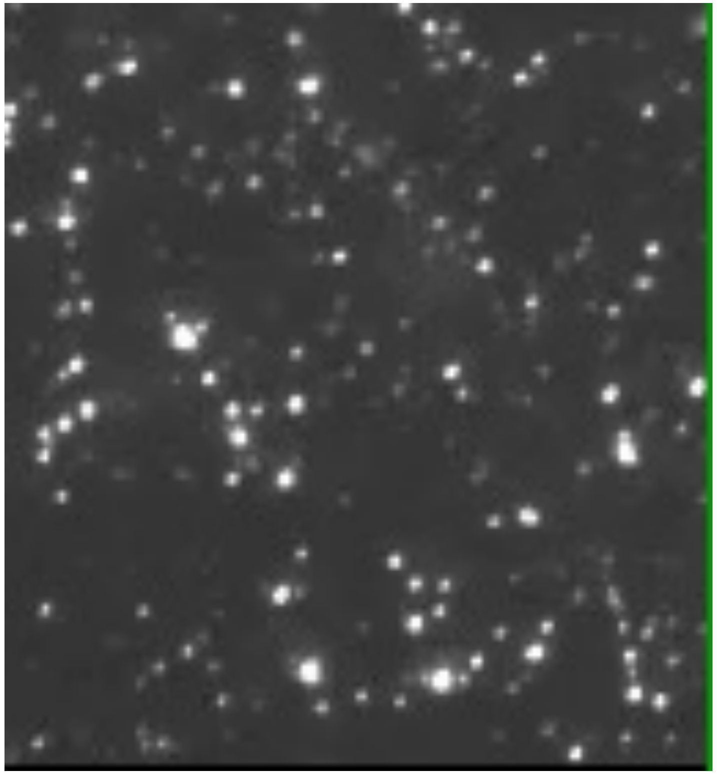


Race of two vortex rings

Visualization of vortex reconnections

8 mm

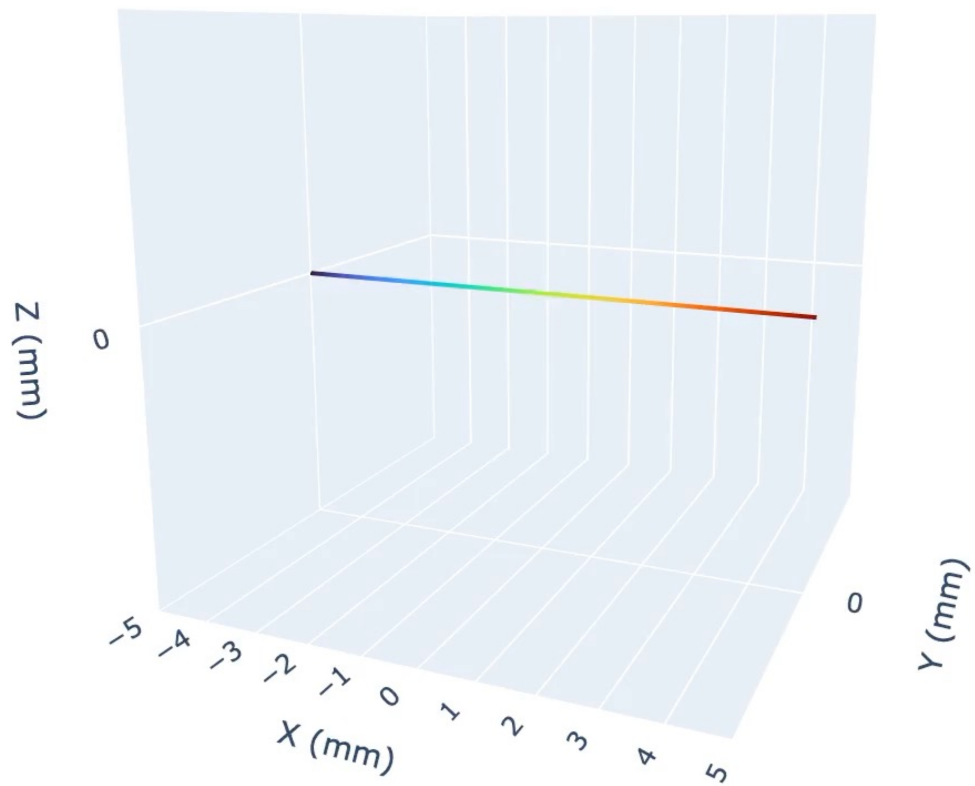
Real time movie



M. S. Paoletti, M. E. Fisher, D. P. Lathrop, Physica D (2010)

What are Kelvin waves?

- Helical waves excited along a vortex



Direct observation of Kelvin waves excited by quantized vortex reconnection

PNAS111(Suppl.1) 4707(2014)

Enrico Fonda[a,b,c], David P. Meichle[a,d], Nicholas T. Ouellette[a,e], Sahand Hormoz[f], and Daniel P. Lathrop[a,d,1]

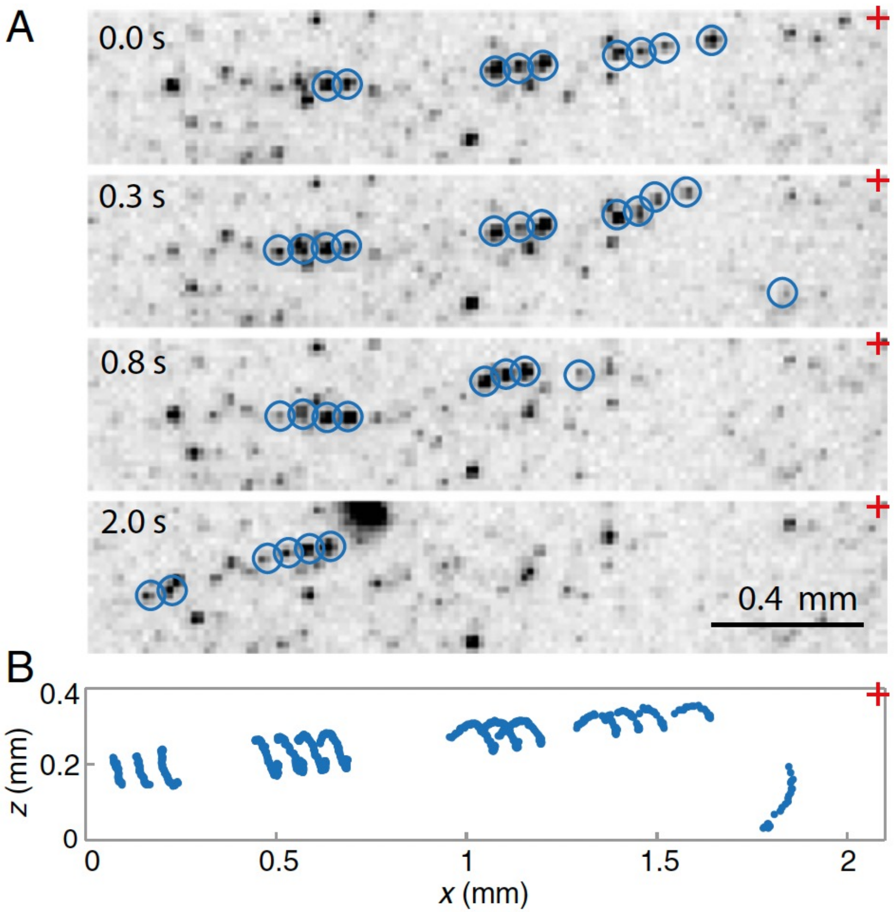


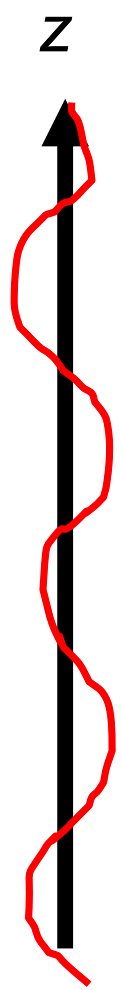
Fig. 2. (*A*) Four frames of our movie sequence (see Movie S1) along with circled particles used in the tracking analysis. (*B*) The positions of the particle tracks on the upper branch show oscillatory behavior after the reconnection event. The cross is the estimated location of the reconnection event.

- ✓ Limited research on Kelvin waves
- ✓ Passive observational research
- ✓ 2D visualization

Solution of Kelvin waves by the Vortex filament model

$$\frac{d\boldsymbol{s}}{dt} = \beta \boldsymbol{s}' \times \boldsymbol{s}'' \qquad \beta \equiv \frac{\kappa}{4\pi} \log\left(\frac{R}{a}\right)$$

A Straight vortex along the z axis $\boldsymbol{s} = (0, 0, z)$



A helical vortex with the small amplitude ε

$$\boldsymbol{s} = (\epsilon \cos\phi, \epsilon \sin\phi, z), \qquad \phi = kz - \omega t$$

$$\boldsymbol{s}' = \frac{d\boldsymbol{s}}{d\xi} \simeq \frac{d\boldsymbol{s}}{dz} = (-k\epsilon \sin\phi, k\epsilon \cos\phi, 1)$$

$$\boldsymbol{s}'' = \frac{d^2\boldsymbol{s}}{d\xi^2} \simeq \frac{d^2\boldsymbol{s}}{dz^2} = (-k^2\epsilon \cos\phi, -k^2\epsilon \sin\phi, 0)$$

$$\boldsymbol{s}' \times \boldsymbol{s}'' = (k^2\epsilon \sin\phi, -k^2\epsilon \cos\phi, 0)$$

$$\frac{d\boldsymbol{s}}{dt} = (\omega\epsilon \sin\phi, -\omega\epsilon \cos\phi, 0)$$

Dispersion relation of Kelvin waves

$$\omega = \beta k^2$$

If we know Chirality and Propagation direction, we know the direction of vorticity.
-> Minowa’s experiment!

$$\frac{d\boldsymbol{s}}{dt} = \beta \boldsymbol{s}' \times \boldsymbol{s}'' \qquad \beta \equiv \frac{\quad}{4\pi} \log\left(\frac{\quad}{a}\right) \qquad \omega = \quad \beta k^2$$

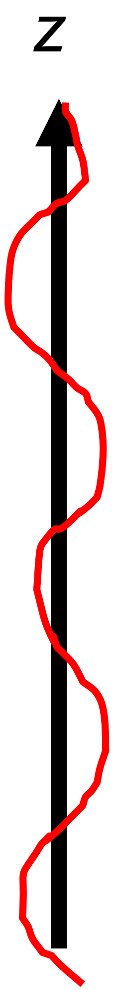
$$\boldsymbol{s} = (\epsilon \cos\phi, \epsilon \sin\phi, z), \qquad \phi = kz - \omega t$$

1. k>0
The Kelvin wave propagates towards +z with right-handed.

2. k<0
The Kelvin wave propagates towards -z with left-handed.

Chirality

Direction of vorticity		Right-handed	Left-handed
	+z	Propagating towards z	Propagating towards -z
	-z	Propagating towards -z	Propagating towards z



nature physics (2025)

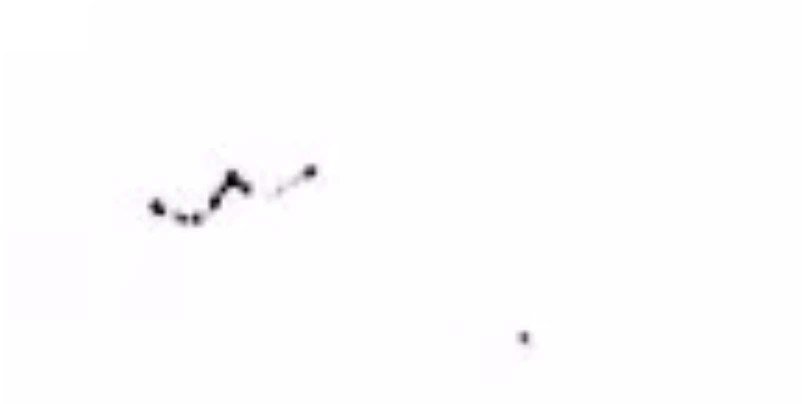
Article https://doi.org/10.1038/s41567-024-02720-9

Direct excitation of Kelvin waves on quantized vortices

Received: 30 March 2024

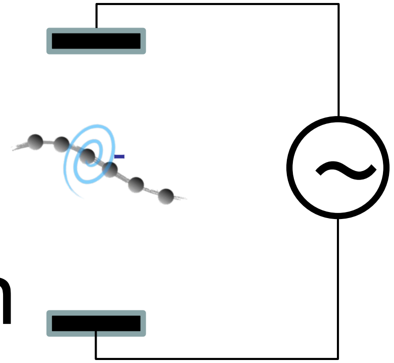
Accepted: 29 October 2024

Yosuke Minowa[1,2,3], Yuki Yasui[1], Tomo Nakagawa[4], Sosuke Inui[5,6], Makoto Tsubota[7,8] & Masaaki Ashida[1]

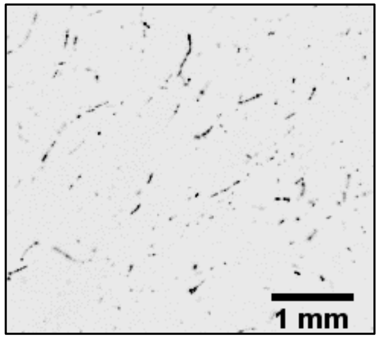


- Visualizing a quantized vortex by charged nano particles.
- Exciting Kelvin waves by an oscillating electric field.
- Revealing the helical stricture by 3D image reconstruction.
- Confirming the dispersion relation, phase velocity, direction of the vorticity.
- Comparison with numerical simulation.

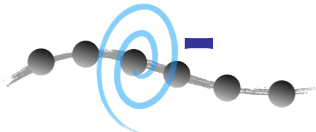
Charged nanoparticles via laser ablation



- Many Si nanoparticles fabricated via in-situ laser ablation
- Clear visualization

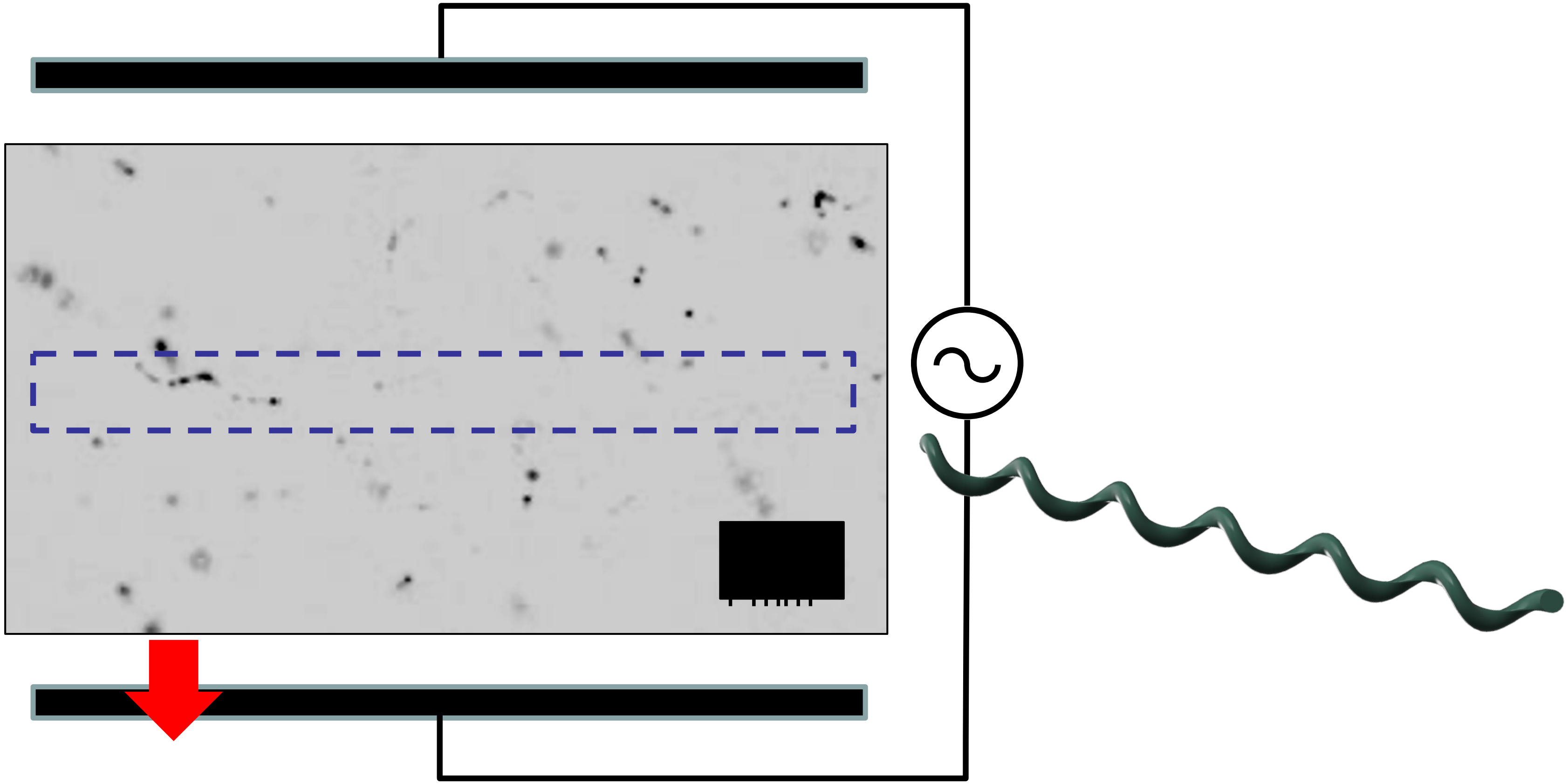


Y. Minowa, *et al.*,
Science Advances, **8**, eabn1143 (2022)



- Some of nanoparticles are charged

Manipulation of quantized vortex via a charged Si nanoparticle

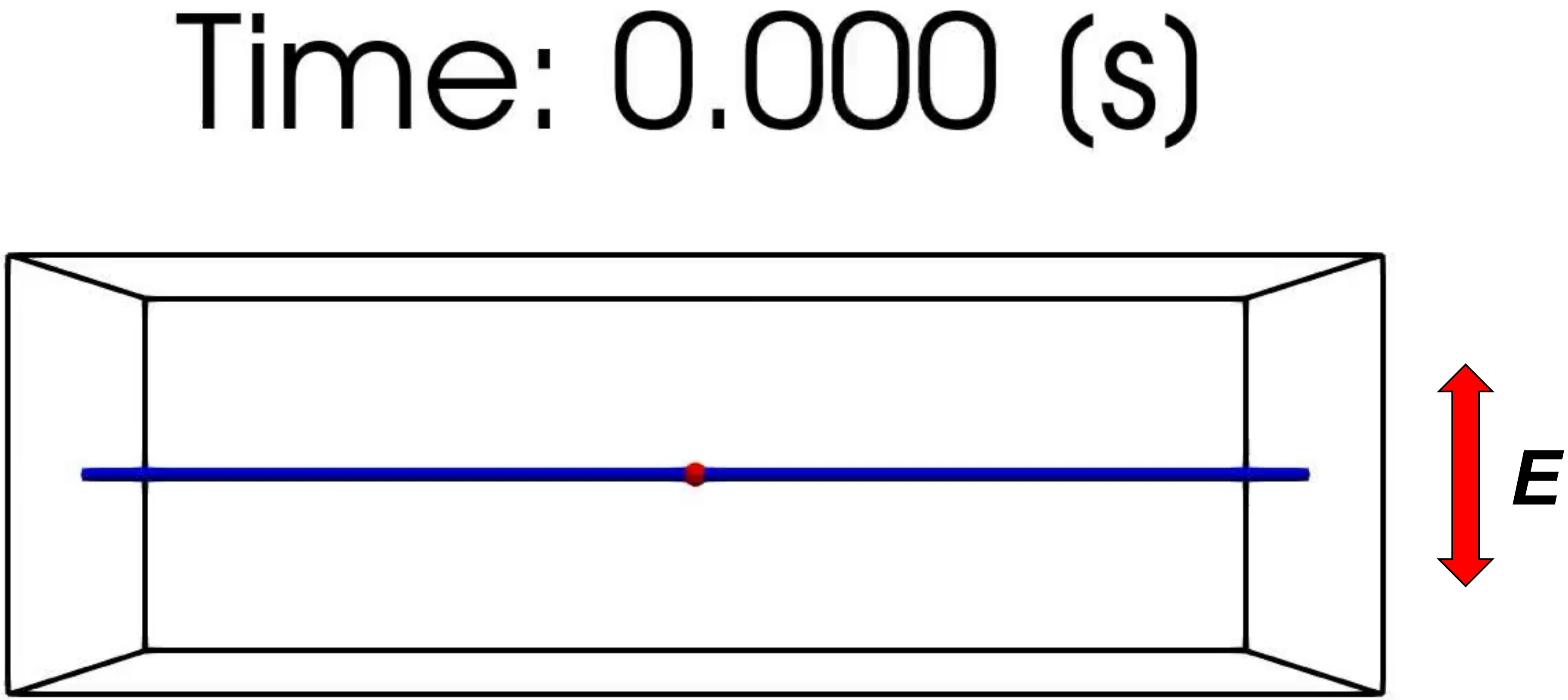


Kelvin wave

Simulation by the VFM under an oscillating electric field

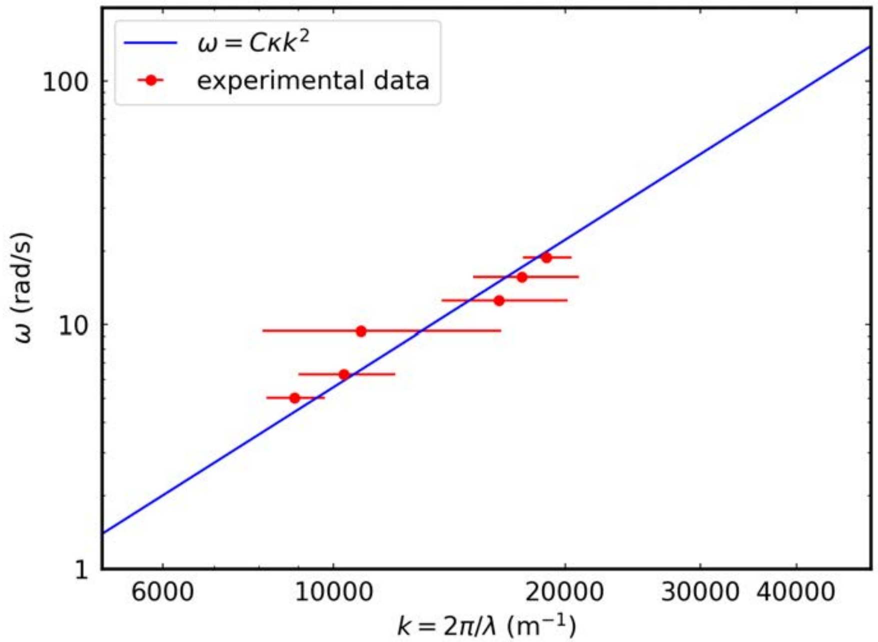
Y. Mineda, MT, Y. Sergeev, C. F. Barneghi, W. F. Vinen, Phys. Rev. B 87, 174508(2013)

The red point refers to a charged particle. Other parts are usual vortex filament.

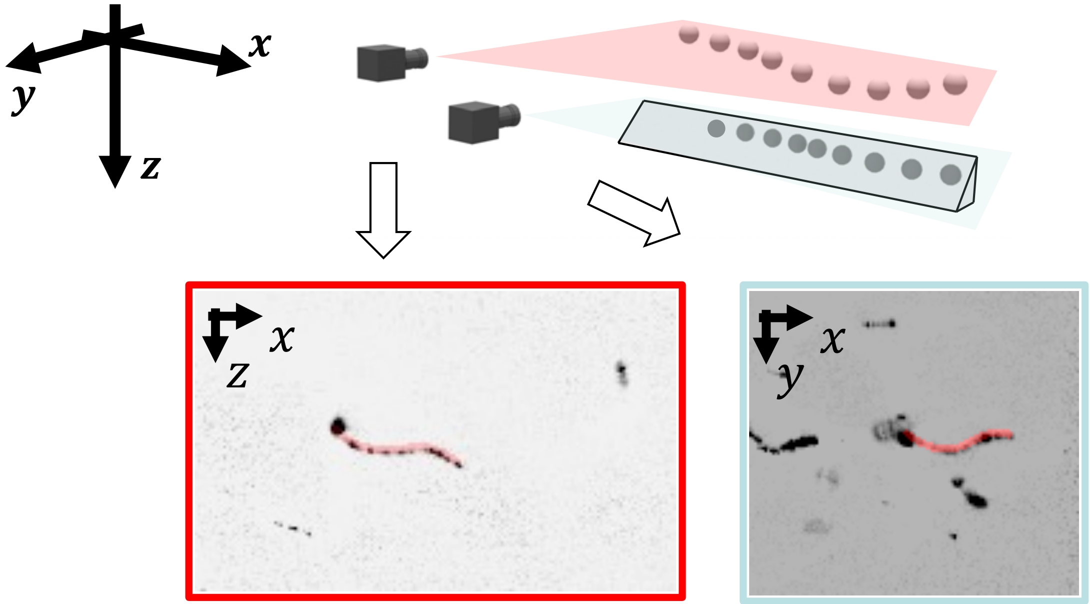


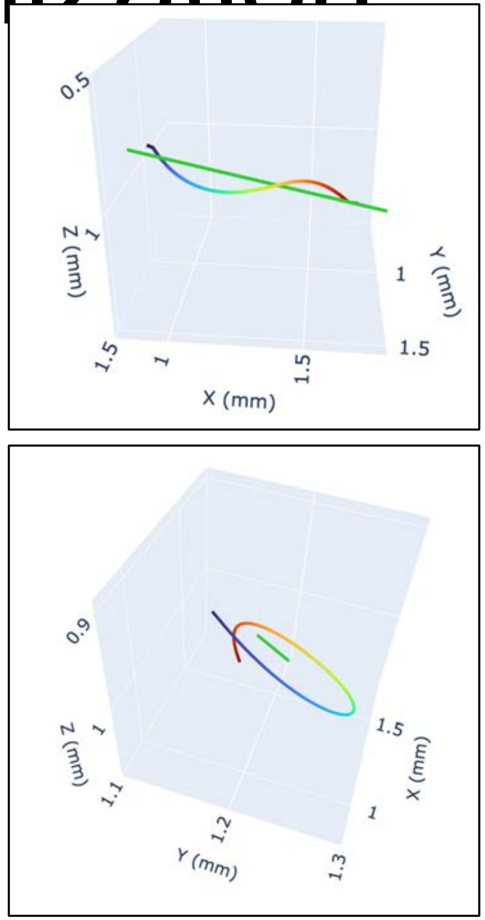
Confirmation of dispersion relation

$\omega = C\kappa k^2$



Experimental 3D visualization

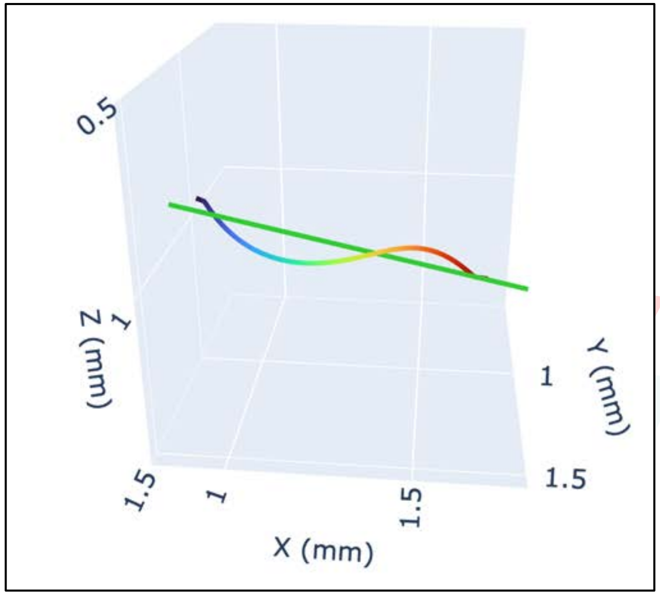


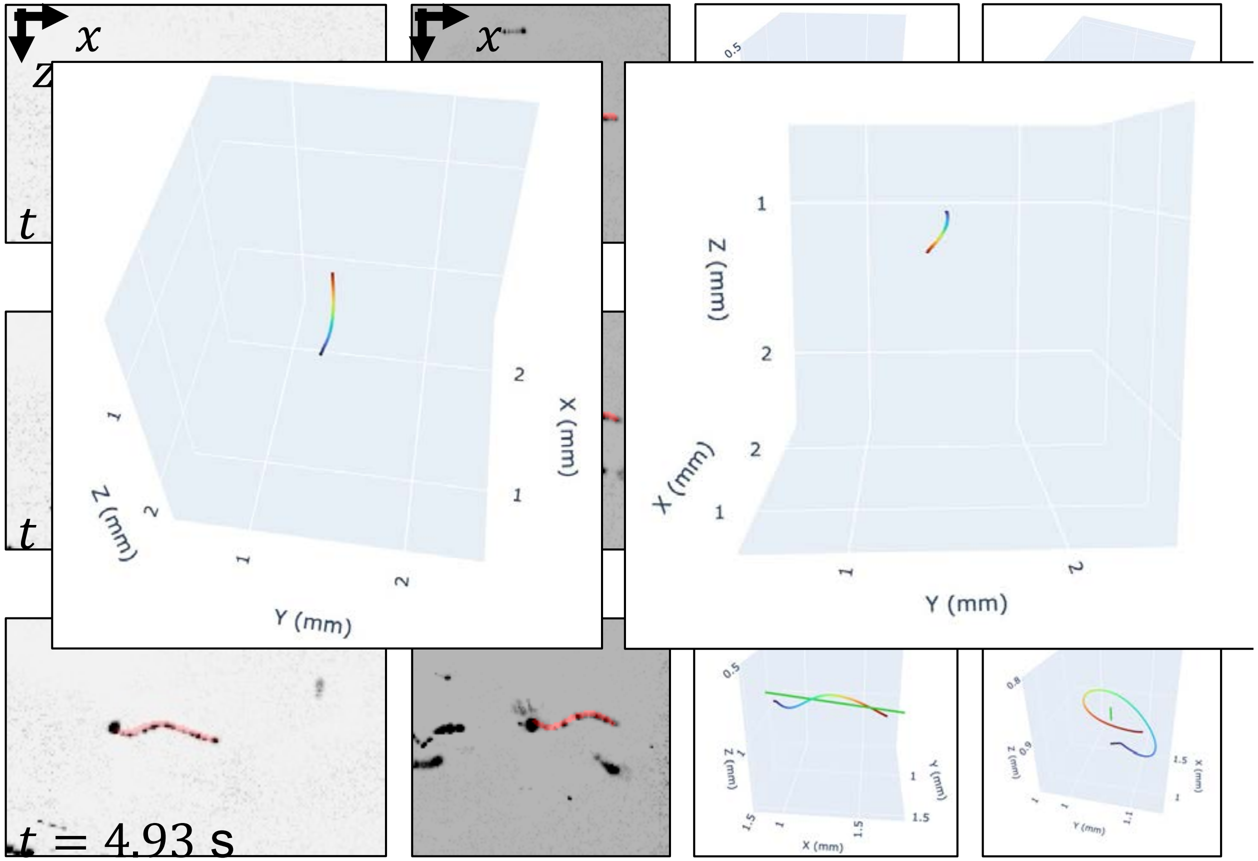


1. Calibrating absolute coordinates
2. (cubic) Spline fitting
3. Reconstructing 3D shape of the curve

Helical!

Experimental 3D visualization





Left-handed helix

Relation between propagation direction, vorticity and chirality

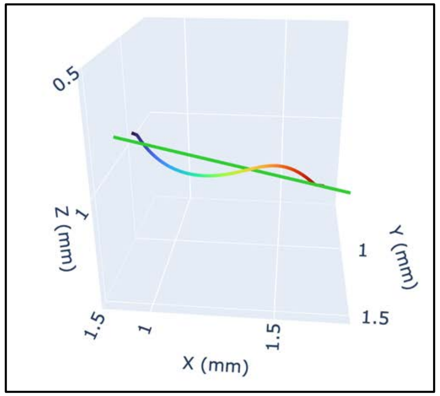
Chirality

Direction of vorticity

	Right-handed	Left-handed
+z	Propagating towards z	Propagating towards -z
-z	Propagating towards -z	Propagating towards z

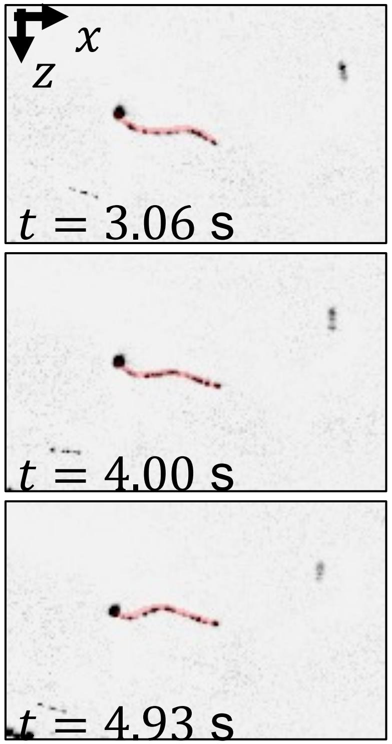
We know experimentally the propagation direction and the chirality. Then we can fix the direction of vorticity.

Vorticity direction can be determined

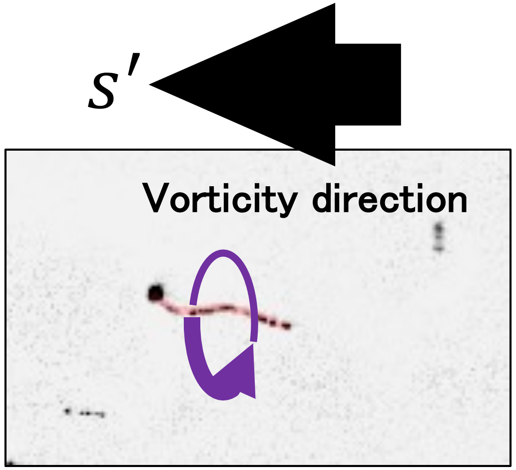


Left handed

&



Propagating direction



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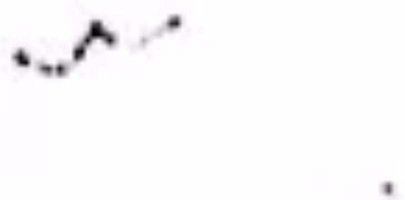

- Exciting Kelvin waves by an oscillating electric field.
- Revealing the **helical** stricture by 3D image reconstruction.
- Determining the **direction of vorticity**

Summary

1. Quantized vortices compared with usual vortices
2. Vortex lattice formation in rotating BECs
3. Vortex lattice in two-component BECs

4. Direct excitation and observations of Kelvin waves on quantized vortices