Proposal for simulating quantum spin models with Dzyaloshinskii-Moriya interaction using Rydberg atoms and construction of asymptotic quantum many-body scar states

MK, T. Tomita, H. Katsura, and Y. Kato, Phys. Rev. A **110**, 043312 (2024): DM in 1D and QMBS

H. Kuji, MK, and T. Nikuni, arXiv:2408.04160 (2024): DM in 2D

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- Results (Quantum many-body scars)
  - Quantum many-body scar states of the DH model
  - Asymptotic quantum many-body scar states of the DH model
- Summary

Introduction : Quantum simulation

Quantum simulation

R. P. Feynman, Int. J. Theor. Phys. **21**, 467 (1982).

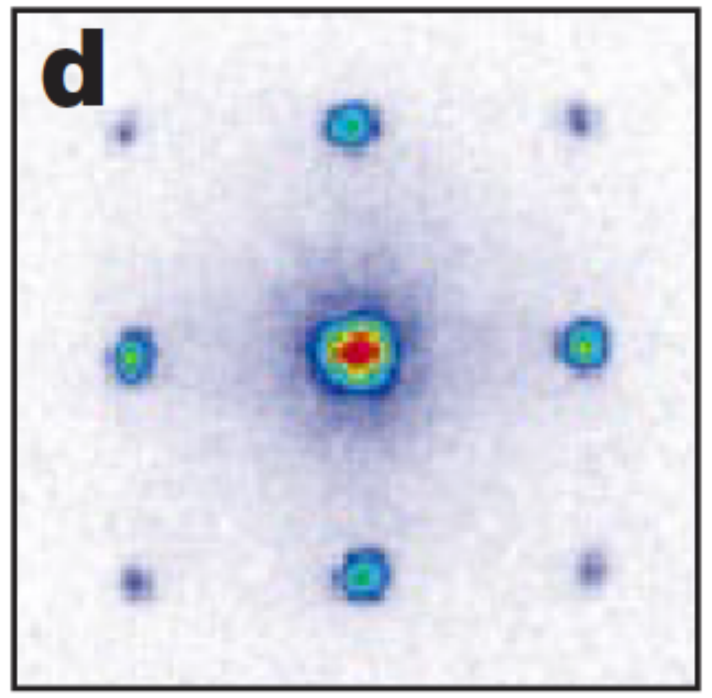
An approach for solving quantum many-body problems using **highly controllable experimental systems**

There are several platforms of quantum simulation:

- **Ultracold neutral atoms**
- **Rydberg atoms array**
- **Trapped ions**
- **Superconducting qubit**

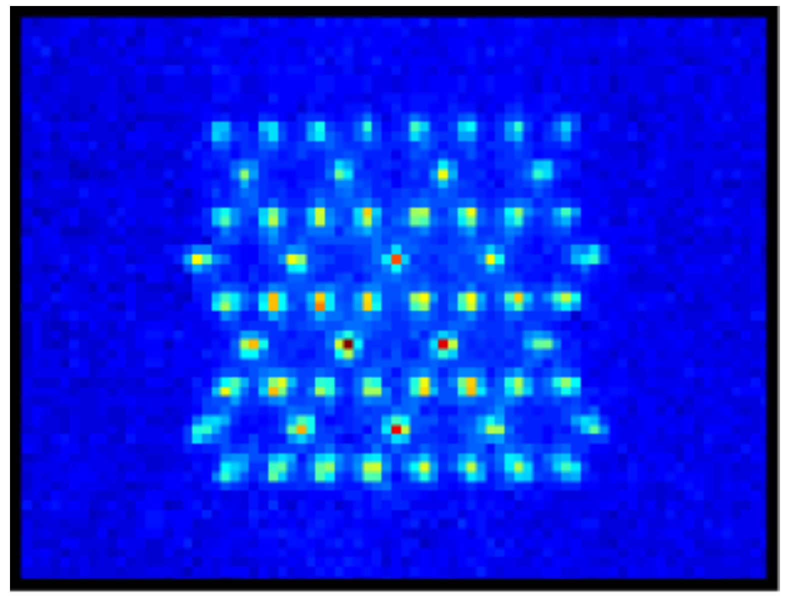
⋮

Neutral atoms



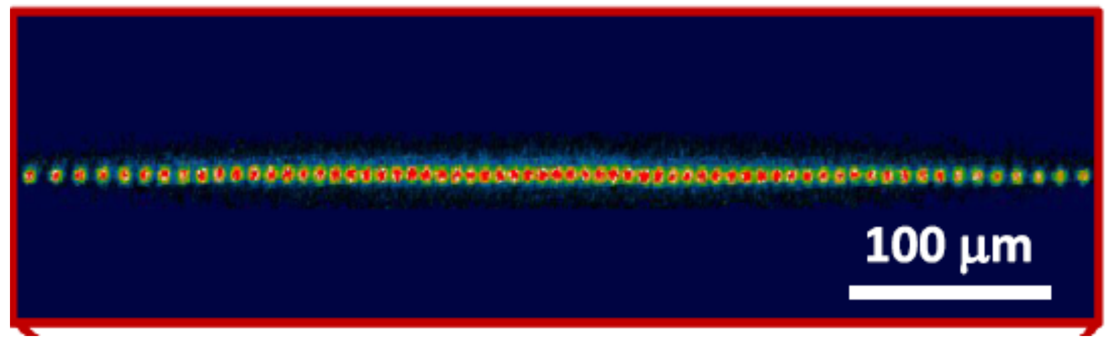
M. Greiner et al., Nature **415**, 39 (2002).

Rydberg atoms



F. Nogrette et al., Phys. Rev. X **4**, 021034 (2014).

Trapped ions



R. Islam et al., Nature Commun. **2**, 377 (2011).

Introduction : Rydberg atom quantum simulator

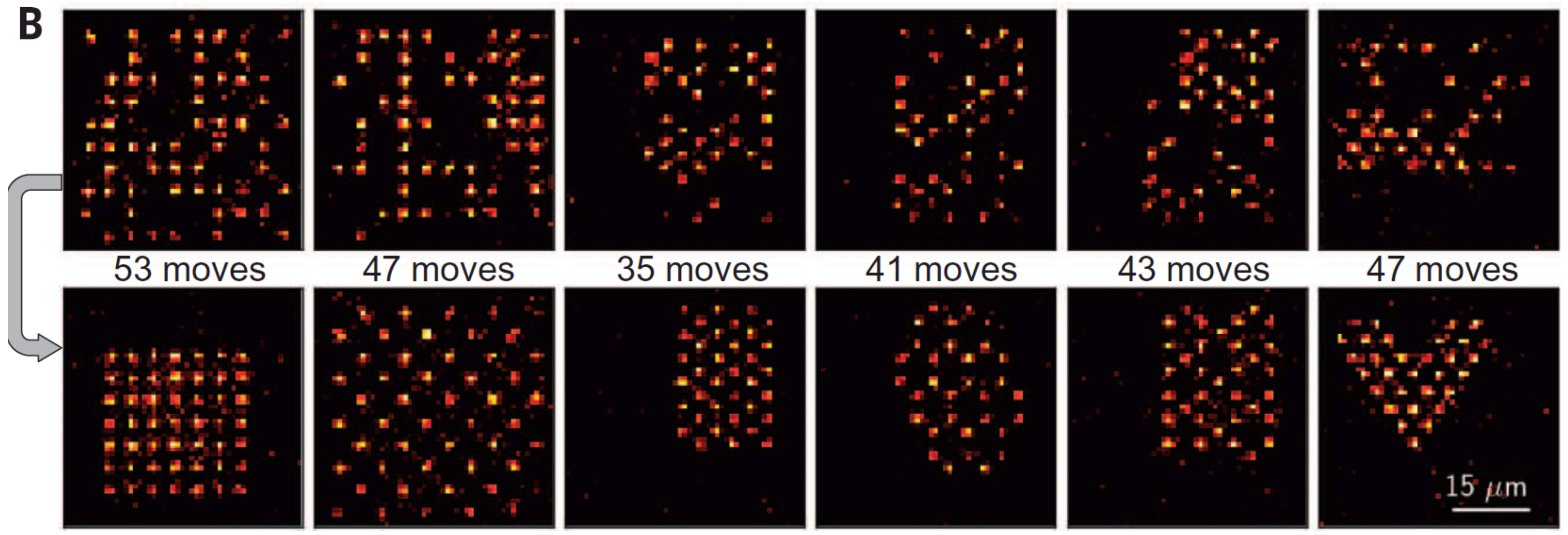
Rydberg atom quantum simulator

Reviews: A. Browaeys and T. Lahaye, Nature Phys. **16**, 132 (2020).
M. Morgado and S. Whitlock, AVS Quantum Sci. **3**, 023501 (2021).

Highly controllable S=1/2 quantum spin systems (1D, 2D) have been realized.

✓Atoms can be arranged in almost arbitrary array by optical tweezers.

✓Global and local spin manipulations are possible.



D. Barredo et al., Science **354**, 1021 (2016).

▶ **Quantum many-body scar (Ising model)**

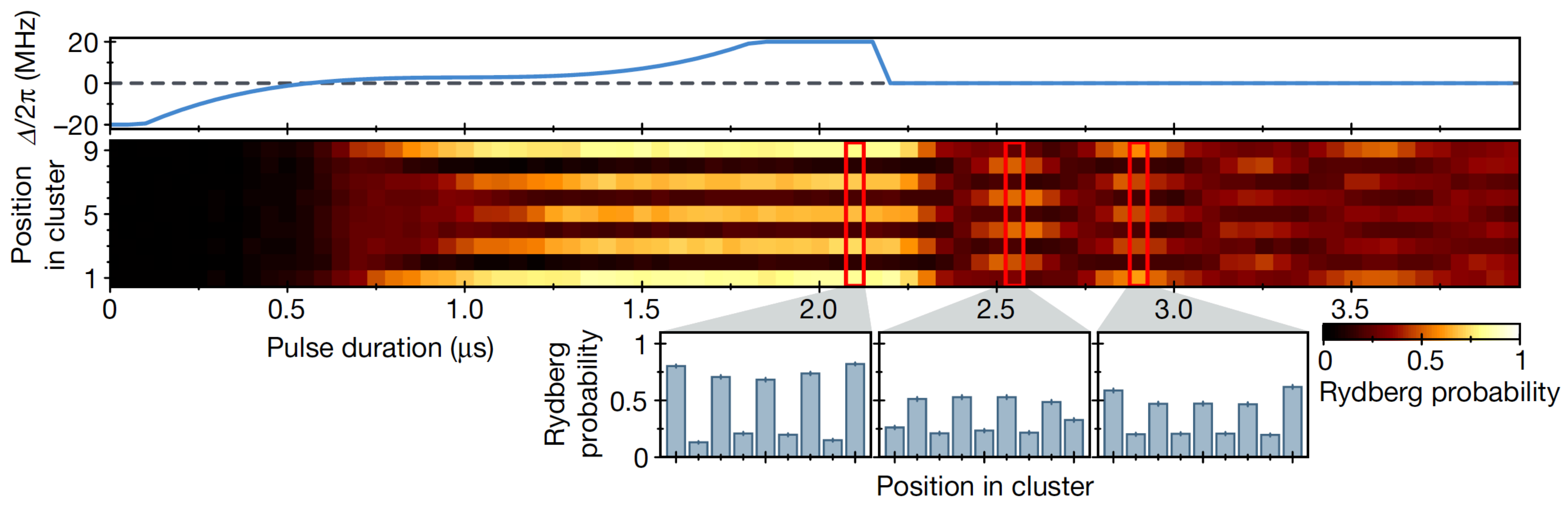
H. Bernien et al., Nature (London) **551**, 579 (2017).

▶ **Topological edge states (XY model)**

S. de Léséleuc et al., Science **365**, 775 (2019).

▶ **Quantum spin liquid state (Ising model)**

G. Semeghini et al., Science. **374**, 1242 (2022).



H. Bernien et al., Nature (London) **551**, 579 (2017).

Introduction : What is the Rydberg atom (state) ?

Rydberg atom (state) **:** Electron in the outermost shell of the alkali or alkali earth atom is highly-excited.

The energy level can be characterized by

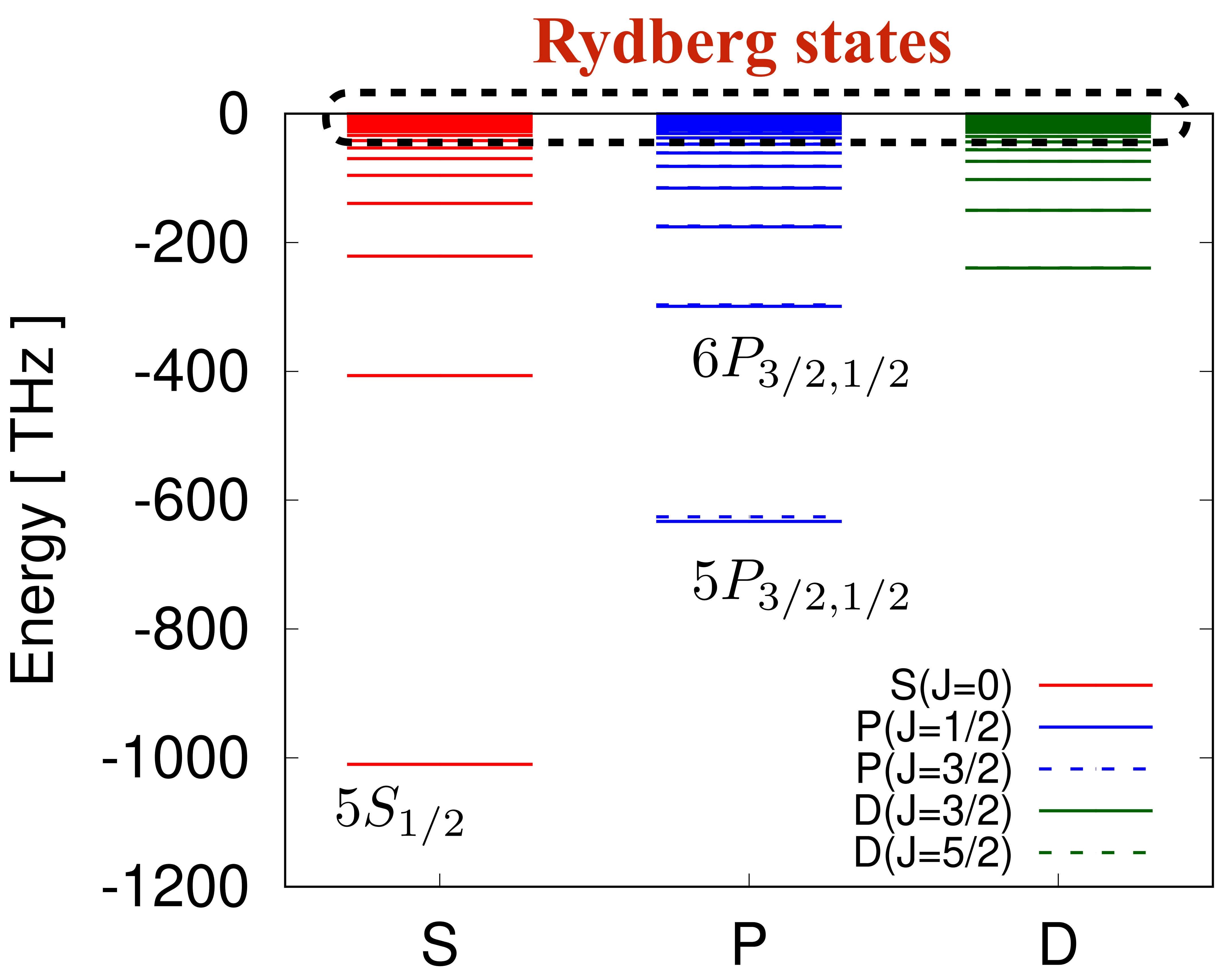
$$|n, J, l, m_J\rangle$$

(We need to consider spin-orbit coupling)

- n : principal quantum number
- J : Total angular momentum $J = l \pm 1/2$
- l : Orbital angular momentum of electron
- m_J : magnetic quantum number

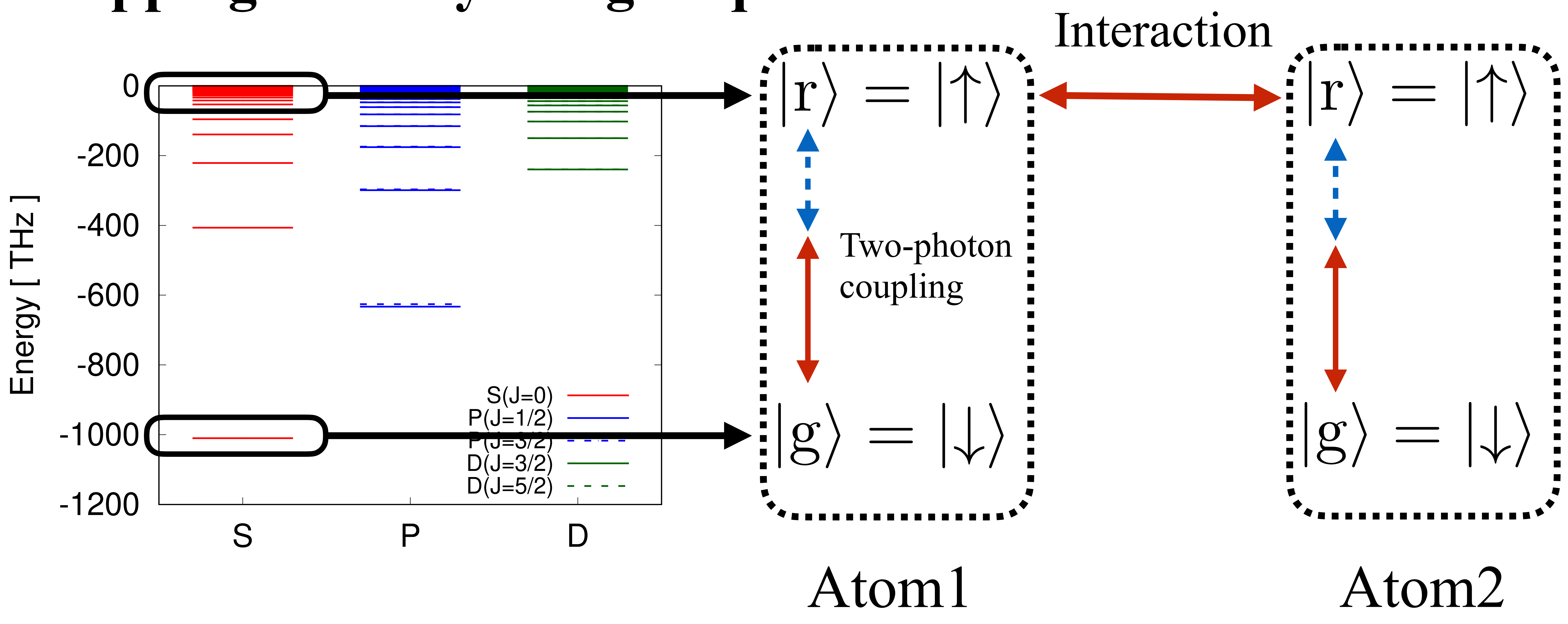
Typically n=20~100 state is used for quantum simulation.

Example : ^{87}Rb atom



Introduction : How to map Rydberg states to quantum spins

Mapping from Rydberg to qubit

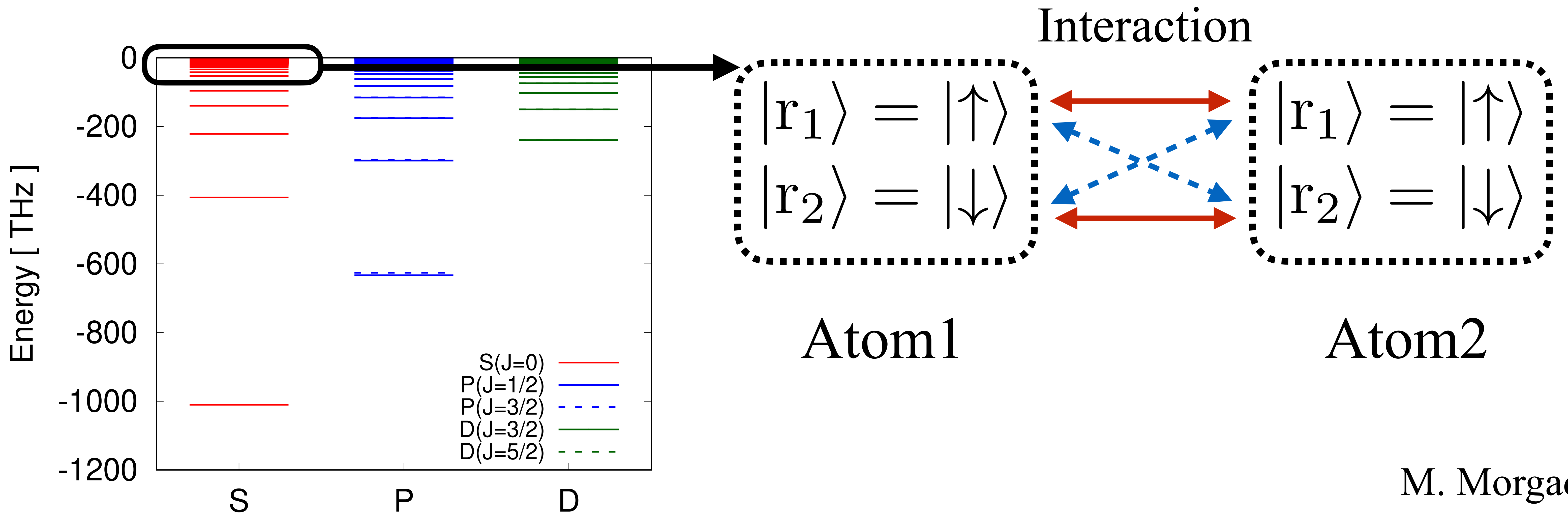


Dipole-dipole interaction works between the Rydberg states due to the strong electric polarization.

$$\hat{H}_{\mathrm{int}} = V\,|rr\rangle\langle rr| \sim \hat{S}^z_1 \hat{S}^z_2$$

$$|r\rangle\langle r| = 1/2 + \hat{S}^z$$

$\Rightarrow$ Ising model



$$\hat{H}_{\mathrm{int}} \sim |r_1 r_2\rangle\langle r_2 r_1| + \mathrm{H.c.}$$

$\Rightarrow$ XY or XXZ model

M. Morgado and S. Whitlock, AVS Quantum Sci. **3**, 023501 (2021).

Introduction : List of realized and proposed Hamiltonians

List of quantum spin models realized in the Rydberg atom quantum simulators

Spin models	References (experiments)
Ising	H. Bernien et al., Nature **551**, 579 (2017). V. Lienhard et al., Phys. Rev. X **8**, 021070 (2018). S. Ebadi et al., Nature **595**, 227 (2021).
XY	A. P. Orioli et al., Phys. Rev. Lett. **120**, 063601 (2018). S. de Léséleuc et al., Science **365**, 775 (2019). Y. Chew et al., Nature Photonics **16**, 724 (2022).
XXZ	A. Signoles et al., Phys. Rev. X **11**, 011011 (2021). P. Scholl et al., PRX QUANTUM **3**, 020302 (2022). T. Tranz et al., arXiv:2207.14216 (2022).

List of theoretically proposed models in the Rydberg atom quantum simulators

Dzyaloshinskii-Moriya(DM)	F. Perciavalle et al., Phys. Rev. A **108**, 023305 (2023). N. Nishad et al., Phys. Rev. A **108**, 053318 (2023). **MK et al., Phys. Rev. A 110, 043312 (2024).** **H. Kuji, MK, and T. Nikuni, arXiv:2408.04160**
Kitaev-Honeycomb	M. Kalinowski et al., Phys. Rev. X **13**, 031008 (2023). N. Nishad et al., Phys. Rev. A **108**, 053318 (2023). Y.-H. Chen et al., Phys. Rev. Res. **6**, L042054 (2024).

⇒Today's topic

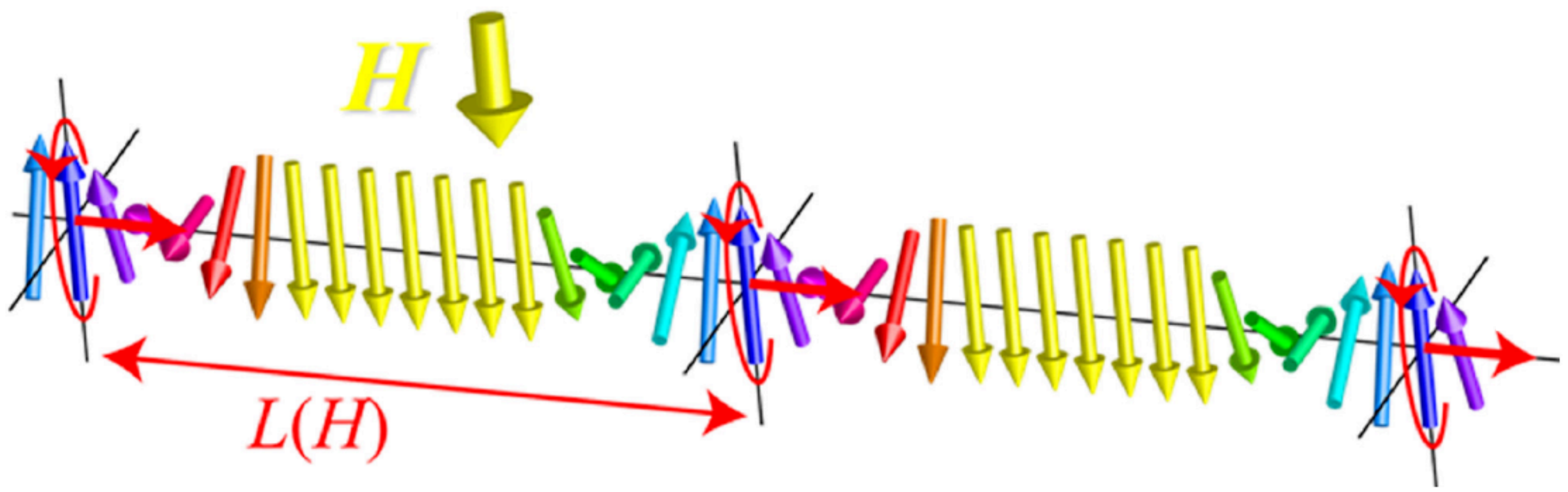
Introduction : Dzyaloshinskii-Moriya (DM) interaction

DM : $\hat{H}_{\mathrm{DM}} = \boldsymbol{D} \cdot \sum_j (\hat{\boldsymbol{S}}_j \times \hat{\boldsymbol{S}}_{j+1}) \Rightarrow$ Energy is minimized when $\boldsymbol{S}_j \perp \boldsymbol{S}_{j+1}$

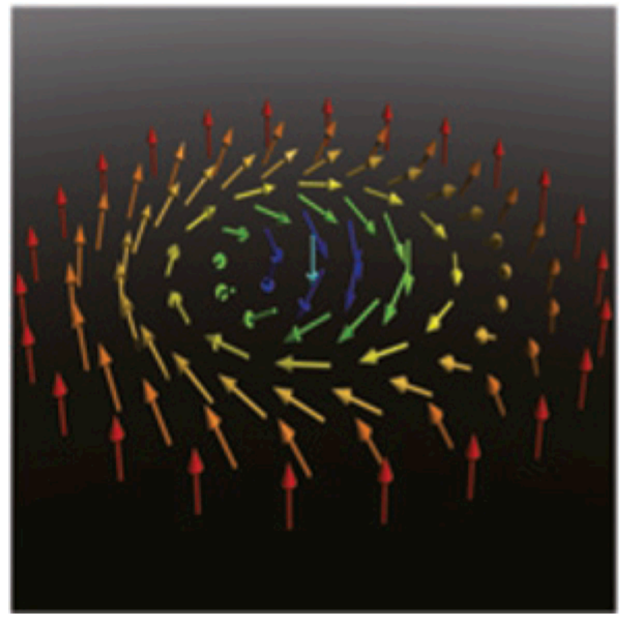
Exchange : $\hat{H}_{\mathrm{ex}} = -J \sum_j \hat{\boldsymbol{S}}_j \cdot \hat{\boldsymbol{S}}_{j+1} \Rightarrow$ Energy is minimized when $\boldsymbol{S}_j \parallel \boldsymbol{S}_{j+1}$

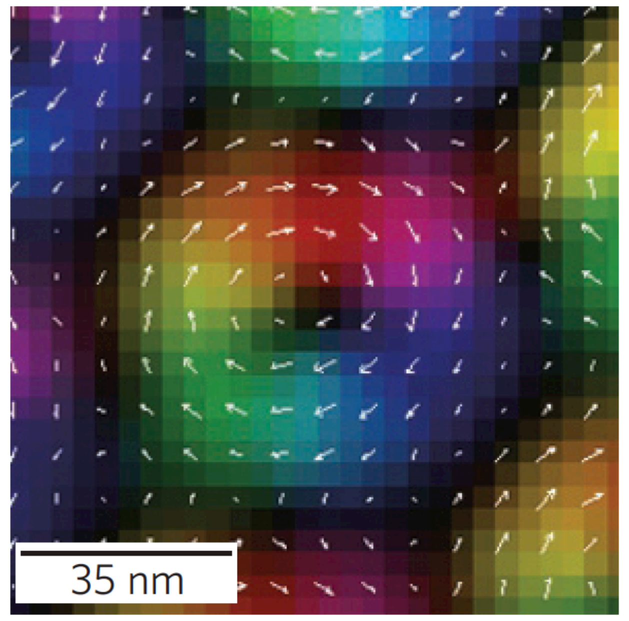
When above two terms exist, various phenomena emerge due to the competition between them.

Example : **Chiral soliton lattice (1D)** $(\boldsymbol{D} \propto \boldsymbol{e}_z)$ **Skyrmion (2D)** (***D*** is bond dependent)



Y. Togawa et al., J. Phys. Soc. Jpn. **85**, 112001 (2016).





X. Z. Yu et al., Nature Materials **10**, 106 (2011).

c.f. Prof. Y. Kato's talk, "Quantum soliton in chiral magnetic chain" (1/24, Friday)

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Summary of our proposals

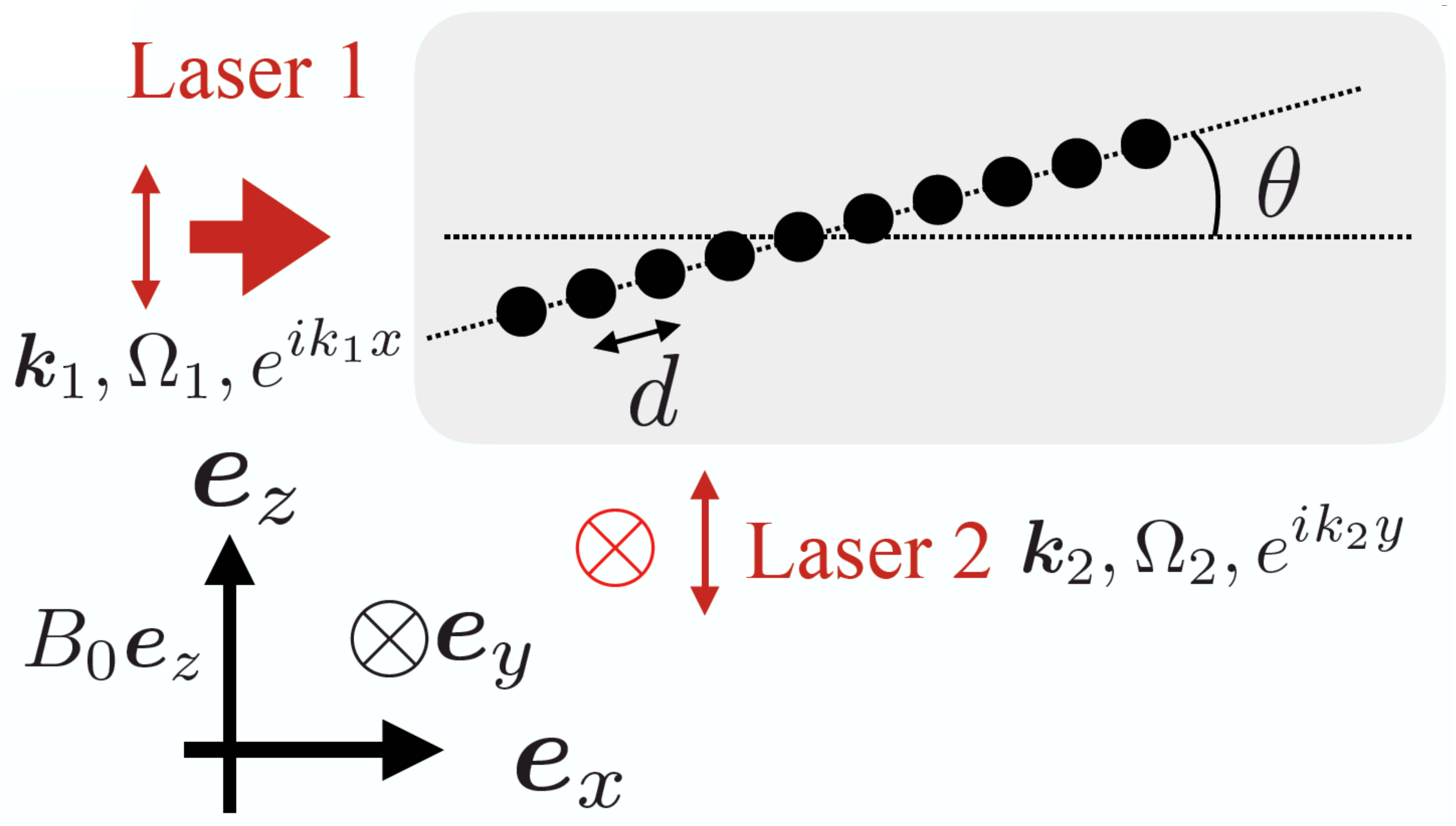
How to create the DM interaction $\boldsymbol{D}_{ij} \cdot (\hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{S}}_j)$ in Rydberg atom quantum simulator ?

One-dimensional system

MK et al, Phys. Rev. A **110**, 043312 (2024).

$\boldsymbol{D}_{ij} \propto \boldsymbol{e}_z$:monoaxial type DM interaction

Using two-photon Raman transition and unitary equivalence, the quantum spin model with the DM interaction can be realized.



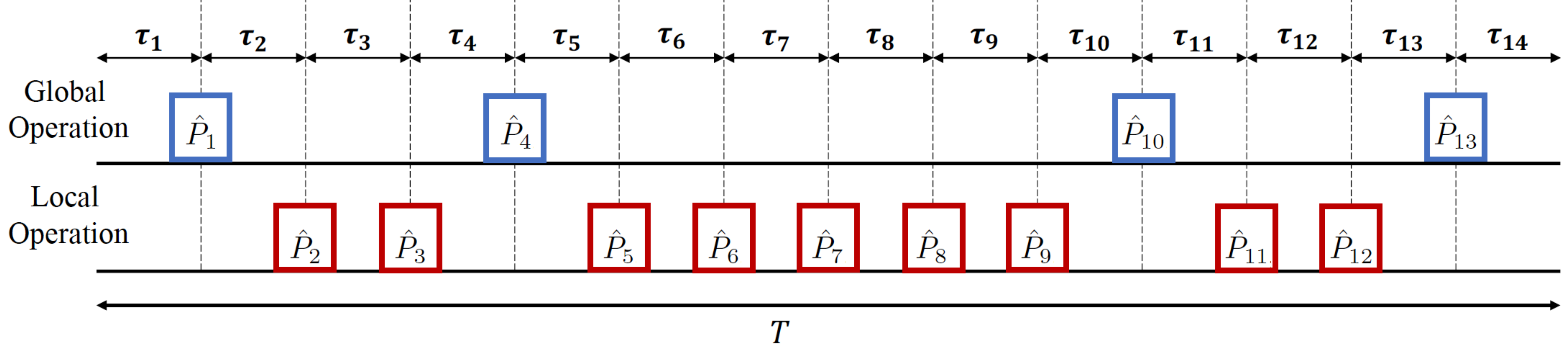
Two-dimensional system

H. Kuji, MK et al., arXiv:2408.04160 (2024).

$\boldsymbol{D}_{ij}$: bond dependent

c.f. $\boldsymbol{D}_{ij} \propto \boldsymbol{e}_x$ for x-bond
$\boldsymbol{D}_{ij} \propto \boldsymbol{e}_y$ for y-bond
(Bloch type DM interaction)

The Hamiltonian with the Heisenberg and bond-dependent DM interactions can be realized by **Floquet engineering** with global and local spin manipulation.



Results : How to create DM interaction in 1D?

It is not easy to create the DM interaction term. $(\hat{\boldsymbol{S}}_j \times \hat{\boldsymbol{S}}_{j+1})_z$

$\Rightarrow$ We use the following unitary equivalence about the DM term:

Hamiltonian in the laboratory frame

$$\hat{H}_{\mathrm{lab}} = J\sum_j \left(\hat{S}^x_j\hat{S}^x_{j+1} + \hat{S}^y_j\hat{S}^y_{j+1} + \delta\hat{S}^z_j\hat{S}^z_{j+1}\right) - h^z\sum_j \hat{S}^z_j - h^x\sum_j [\cos(qj)\hat{S}^x_j + \sin(qj)\hat{S}^y_j]$$

Unitary transformation (spatially dependent spin rotation around z-axis)

$$\hat{U} \equiv \prod_j e^{-i\hat{S}^z_j qj} \qquad q \in \mathbb{R}$$

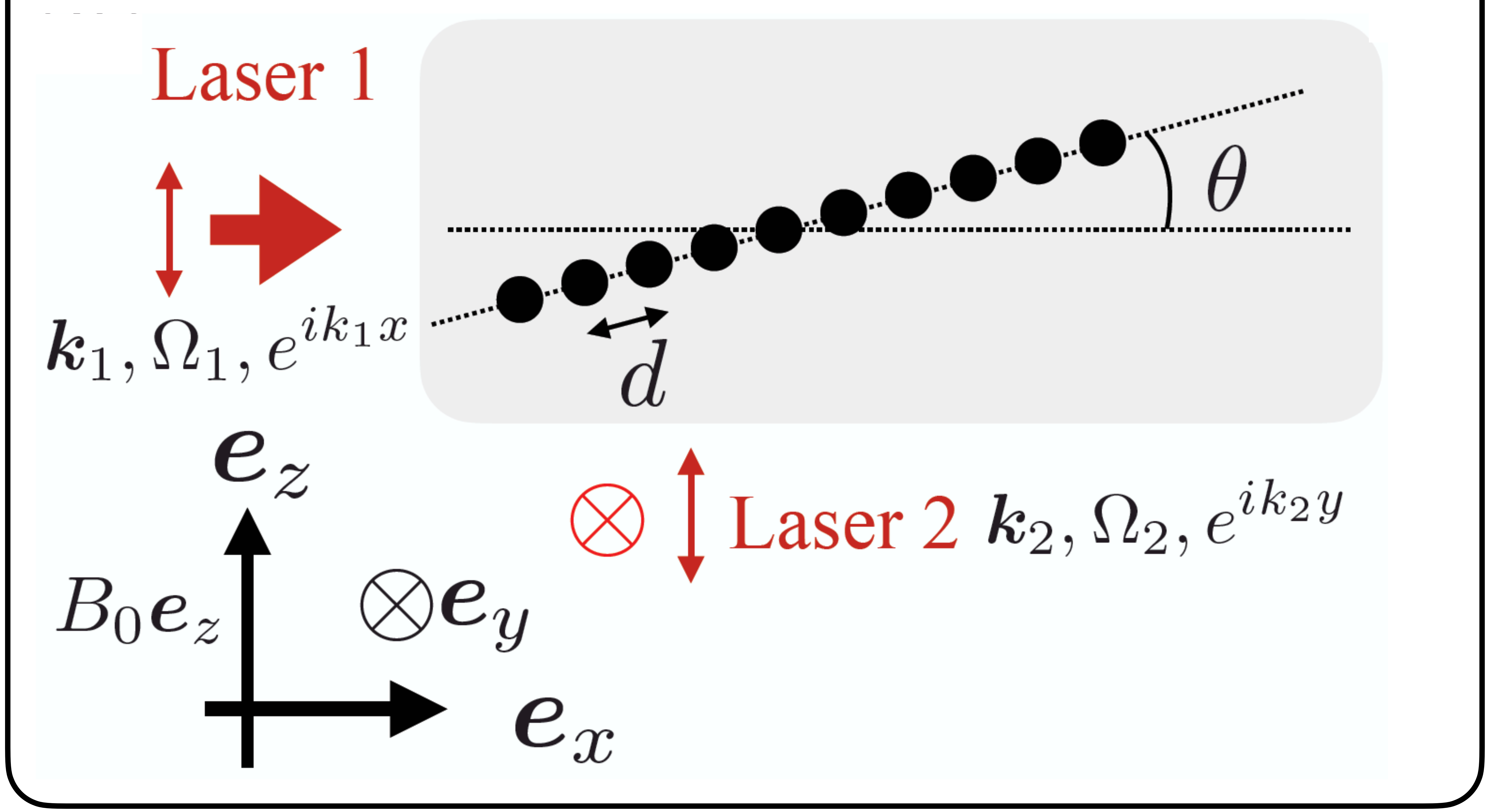
L. Shekhtman et al., Phys. Rev. Lett. **69**, 836 (1992).
T. Nikuni and N. Shiba, J. Phys. Soc. Jpn. **62**, 3268 (1993).
M. Oshikawa and I. Affleck, Phys. Rev. Lett. **79**, 2883 (1997).

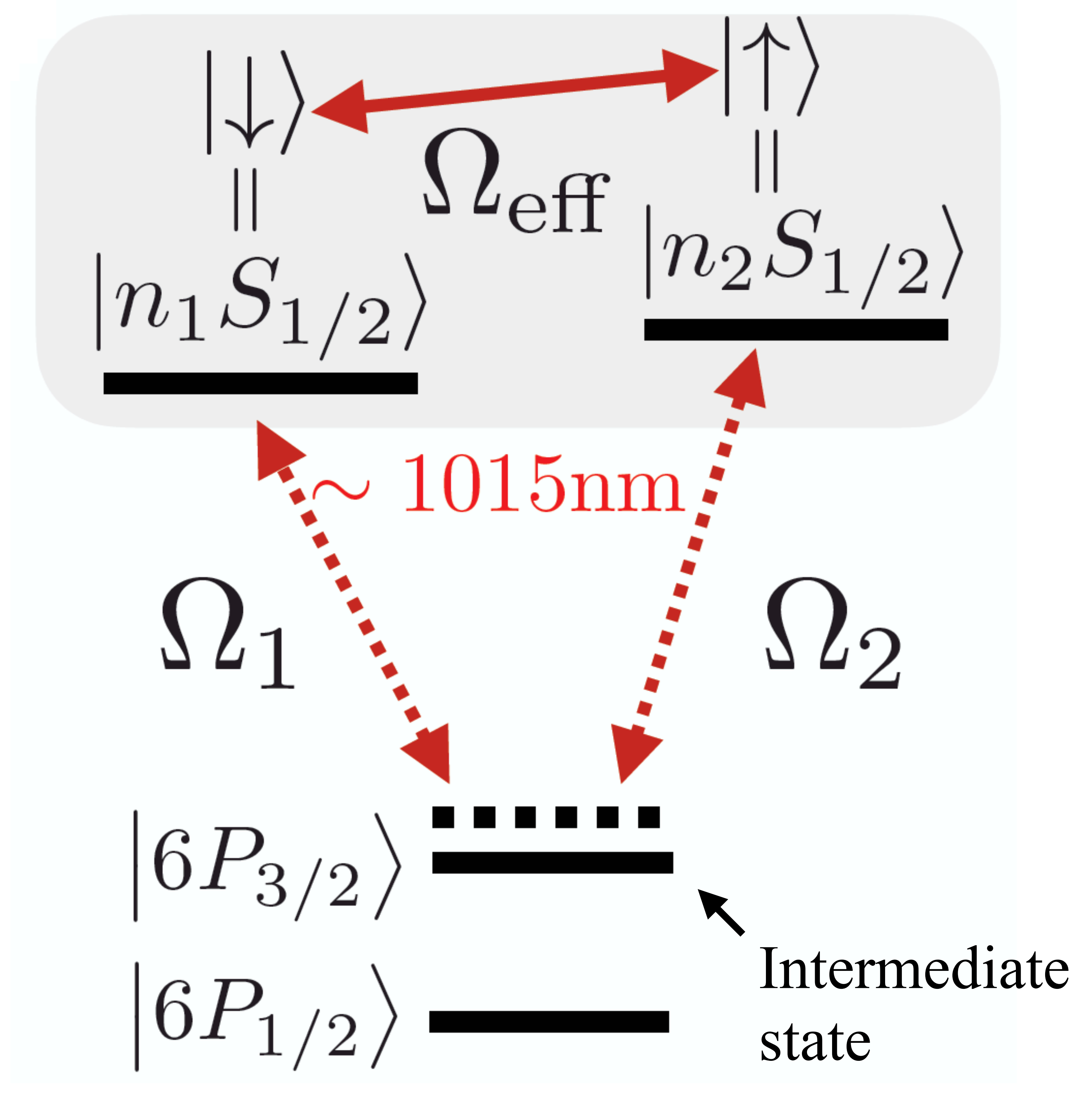
Hamiltonian in the rotating frame

$(\hat{\boldsymbol{S}}_j \times \hat{\boldsymbol{S}}_{j+1})_z$: DM interaction term

$$\hat{H}_{\mathrm{rot}} = \hat{U}^\dagger \hat{H}_{\mathrm{lab}} \hat{U}$$
$$= J\sum_j \left[\cos(q)\left(\hat{S}^x_j\hat{S}^x_{j+1} + \hat{S}^y_j\hat{S}^y_{j+1}\right) + \delta\hat{S}^z_j\hat{S}^z_{j+1} - \sin(q)\left(\hat{S}^x_j\hat{S}^y_{j+1} - \hat{S}^y_j\hat{S}^x_{j+1}\right)\right] + \sum_j(-h^x\hat{S}^x_j - h^z\hat{S}^z_j)$$

Results : Rydberg atom array with rotating transverse field





Spatially varying phase of laser 1 $\Rightarrow$ Phase of the Rabi coupling varies spatially.

$$\frac{\hbar\Omega_{\mathrm{eff}}}{2}e^{ik_1dj\cos\theta}|\downarrow_j\rangle\langle\uparrow_j| + \mathrm{h.c.} = \hbar\Omega_{\mathrm{eff}}[\cos(k_1dj\cos\theta)\hat{S}_j^x + \sin(k_1dj\cos\theta)\hat{S}_j^y]$$ $\Rightarrow$ **Rotating transverse field!**

Results : Realizing DH Hamiltonian

Hamiltonian in the rotating frame

$$\hat{H}_{\mathrm{rot}} = J\sum_j \left[\underline{\cos(k_1 d\cos\theta)}\left(\hat{S}^x_j\hat{S}^x_{j+1}+\hat{S}^y_j\hat{S}^y_{j+1}\right)+\delta\hat{S}^z_j\hat{S}^z_{j+1}-\underline{\sin(k_1 d\cos\theta)}\left(\hat{S}^x_j\hat{S}^y_{j+1}-\hat{S}^y_j\hat{S}^x_{j+1}\right)\right] - \sum_j\left(h^x\hat{S}^x_j+h^z\hat{S}^z_j\right)$$

θ : Angle between laser 1 and array d : Lattice spacing

Experimentally, the **lattice spacing *d* and angle *θ* can be tuned easily using optical tweezer**.

Using Rydberg atoms, we can create a model whose dominant term is the DM interaction.

DM int. > Exchange int.

The DH model can be realized in our proposed scheme!

DH model: $\hat{H}_{\mathrm{DH}} = D\sum_j\left(\hat{S}^x_j\hat{S}^y_{j+1}-\hat{S}^y_j\hat{S}^x_{j+1}\right) - h\sum_j\hat{S}^x_j$ $k_1 d\cos\theta = \pi/2$ and $\delta = 0$

S. Kodama et al., Phys. Rev. B **107**, 024403 (2023).

The anisotropy parameter δ can be tuned by appropriately choosing the Rydberg state and applying static electric field and/or magnetic field.

Introduction: Floquet engineering

System Hamiltonian：$\hat{H}(t) = \hat{H}_0 + \hat{H}_{\mathrm{drive}}(t)$ Time-periodic field：$\hat{H}_{\mathrm{drive}}(t+T) = \hat{H}_{\mathrm{drive}}(t)$

Time evolution operator for single period: $\hat{U}(T) = \hat{\mathcal{T}} \exp\left[-\frac{i}{\hbar}\int_0^T dt \hat{H}(t)\right] \equiv e^{-i\hat{H}_{\mathrm{F}}T/\hbar}$

$\hat{H}_{\mathrm{F}}$:**Floquet Hamiltonian, effectively describe the dynamics of the system.**

Floquet Hamiltonian can be obtained by Magnus expansion: M. Bukov et al., Adv. Phys. 64, 139 (2015).

$$\hat{H}_{\mathrm{F}} \simeq \hat{H}_{\mathrm{eff}}^{(0)} \equiv \frac{1}{T}\int_0^T dt \hat{H}_{\mathrm{rot}}(t)$$

$$\hat{H}_{\mathrm{rot}}(t) \equiv \hat{U}_{\mathrm{drive}}^{\dagger}(t)\hat{H}_0\hat{U}_{\mathrm{drive}}(t)$$

$$\hat{U}_{\mathrm{drive}}(t) \equiv \hat{\mathcal{T}} \exp\left[-\frac{i}{\hbar}\int_0^t dt' \hat{H}_{\mathrm{drive}}(t')\right]$$

In the Floquet engineering, we choose appropriate external fields to create the desired Floquet Hamiltonian.

Results : Setup

Spin-1/2 XY model in two-dimensional square lattice: $\hat{H}_{XY} = J \sum_{\langle j,k \rangle} (\hat{S}^x_{\boldsymbol{R}_j} \hat{S}^x_{\boldsymbol{R}_k} + \hat{S}^y_{\boldsymbol{R}_j} \hat{S}^y_{\boldsymbol{R}_k})$

Correspondence between spin states and Rydberg states

$$|nS\rangle \to |\downarrow\rangle \quad |nP\rangle \to |\uparrow\rangle$$

S. de Léséleuc et al., Science **365**, 775 (2019).

S. Geier et al., Science **374**, 1149 (2021).

P. Scholl et al., PRX QUANTUM **3**, 020303 (2022).

Total Hamiltonian: $\hat{H}(t) = \hat{H}_{XY} + \hat{H}_{\mathrm{drive}}(t)$

Time-periodic field : $\hat{H}_{\mathrm{drive}}(t) = \hat{H}_{\mathrm{G}}(t) + \hat{H}_{\mathrm{L}}(t)$

$\hat{H}_{\mathrm{G}}(t)$:Spatially uniform pulse (microwave)

$\hat{H}_{\mathrm{L}}(t)$:Spatially nonuniform pulse (laser)

N. Nishad et al., Phys. Rev. A **108**, 053318 (2023).

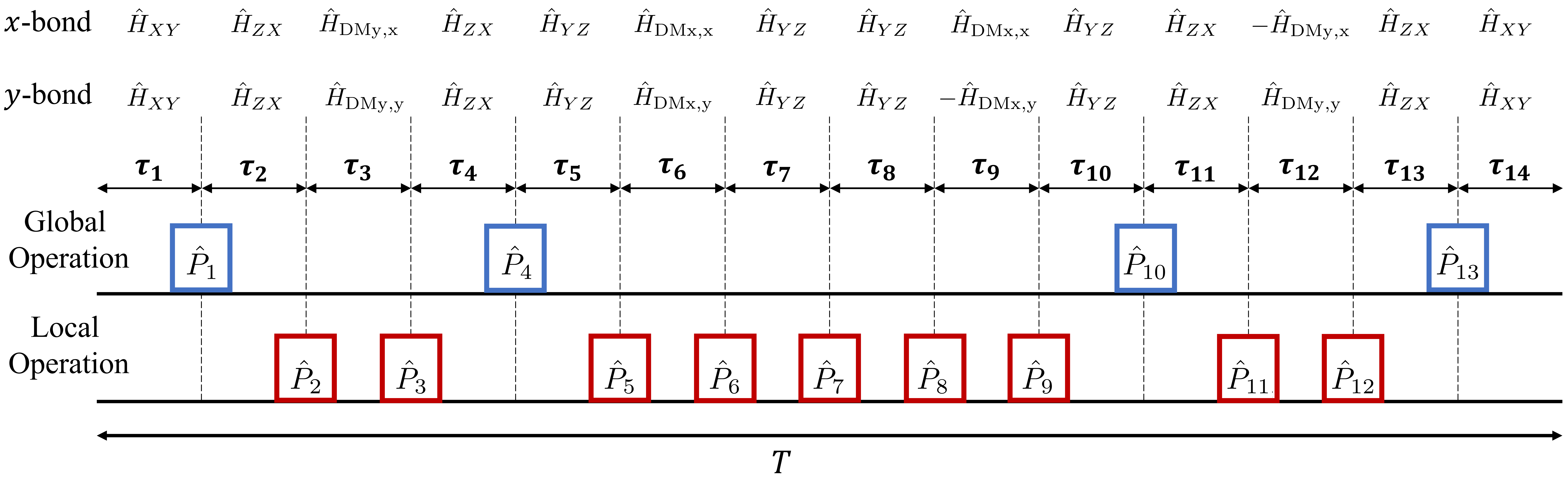
Target Hamiltonian

x-bond y-bond

$$\hat{H}_{\mathrm{target}} = J_{\mathrm{F}} \sum_{\langle i,j \rangle} (\hat{\boldsymbol{S}}_{\boldsymbol{R}_i} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{R}_j}) + D_{\mathrm{F}} \sum_i \left[(\hat{\boldsymbol{S}}_{\boldsymbol{R}_i} \times \hat{\boldsymbol{S}}_{\boldsymbol{R}_i + a\boldsymbol{e}_x})_x + (\hat{\boldsymbol{S}}_{\boldsymbol{R}_i} \times \hat{\boldsymbol{S}}_{\boldsymbol{R}_i + a\boldsymbol{e}_y})_y \right]$$

Results : Heisenberg + DM interaction

Proposed pulse sequence:



Tuning the pulse intervals, we can control the interaction coefficients.

The Heisenberg model with the bond-dependent DM interaction can be realized.

$$\hat{H}_{\mathrm{target}} = J_{\mathrm{F}} \sum_{\langle i,j \rangle} (\hat{\boldsymbol{S}}_{\boldsymbol{R}_i} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{R}_j}) + D_{\mathrm{F}} \sum_i \left[(\hat{\boldsymbol{S}}_{\boldsymbol{R}_i} \times \hat{\boldsymbol{S}}_{\boldsymbol{R}_i + a\boldsymbol{e}_x})_x + (\hat{\boldsymbol{S}}_{\boldsymbol{R}_i} \times \hat{\boldsymbol{S}}_{\boldsymbol{R}_i + a\boldsymbol{e}_y})_y \right]$$

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Introduction : Quantum many-body scar states

Eigenstate thermalization hypothesis (ETH) : $\langle E_n | \hat{A} | E_n \rangle = \langle \hat{A} \rangle_{\mathrm{MC}}(E_n)$ holds for all eigenstates.

$$\hat{H} |E_n\rangle = E_n |E_n\rangle, \ \langle \cdots \rangle_{\mathrm{MC}}(E) \equiv \mathrm{Tr}[\hat{\rho}_{\mathrm{MC}}(E) \cdots]$$

If the ETH holds, we can show $\lim_{T\to\infty} \frac{1}{T} \int_0^T dt \langle \psi(t) | \hat{A} | \psi(t) \rangle = \langle \hat{A} \rangle_{\mathrm{MC}}(E_0)$. $\quad E_0 \equiv \langle \psi(0) | \hat{H} | \psi(0) \rangle$

$\Rightarrow$ **Long-Time average = Microcanonical average.**

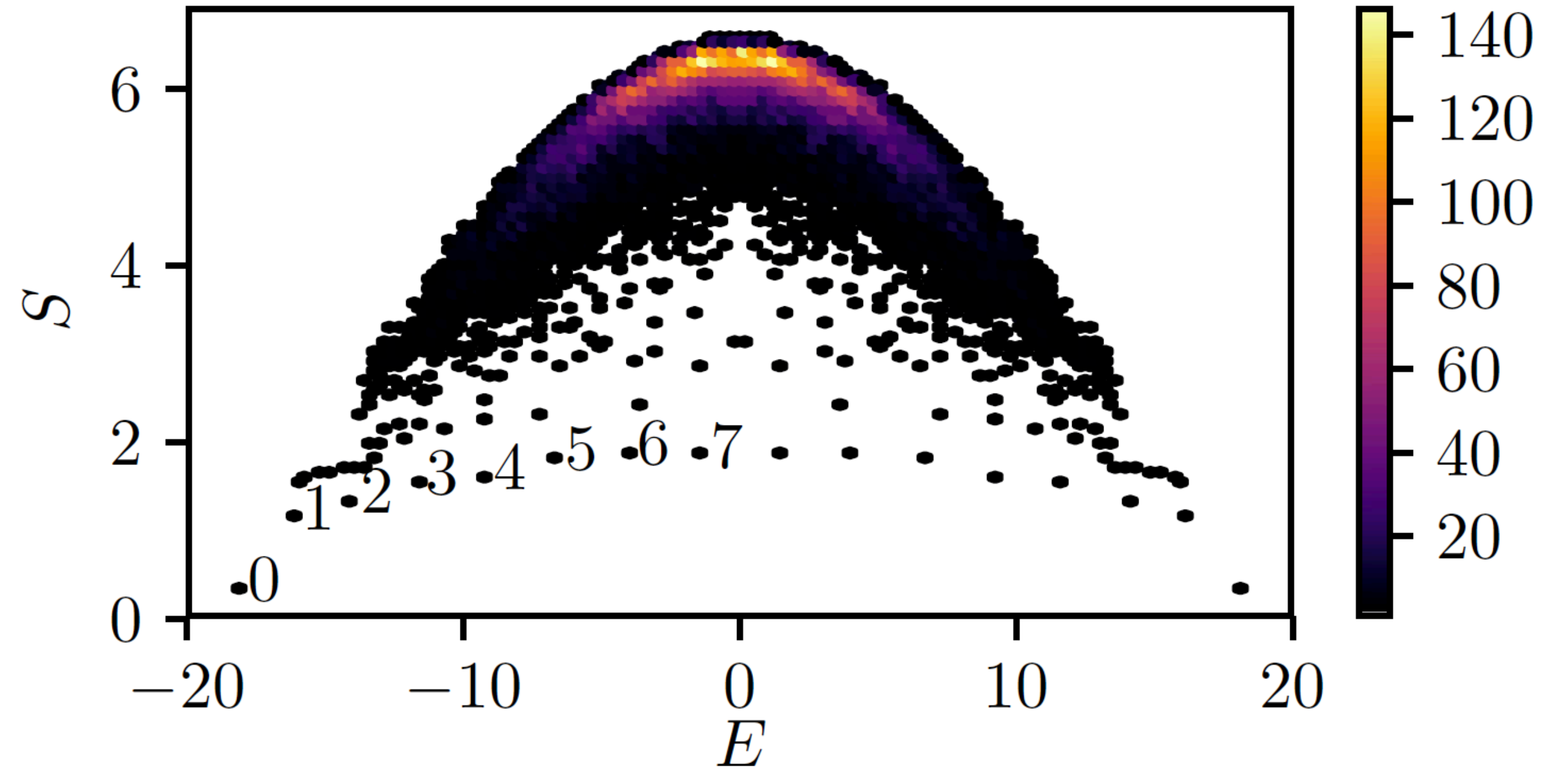
J. M. Deutsch, Phys. Rev. A **43**, 2046 (1991).
M. Srednicki, Phys. Rev. E **50**, 888 (1994).
M. Rigol, et al., Nature **452**, 854 (2008).

It is believed that almost nonintegrable systems satisfy the ETH.

However, some nonintegrable systems do not satisfy the ETH.

One example is the **quantum many-body scar state.**

- High-energy but low-entangled states (sub-volume law scaling)
- Equally spaced spectrum (tower states) $E_n = E_0 + n\mathcal{E}$



C. J. Turner et al., Phys. Rev. B **98**, 155134 (2018).

Introduction : Quantum many-body scar states

- Spin-1/2 Ising+strong interaction (PXP model)

  C. J. Turner et al., Phys. Rev. B **98**, 155134 (2018).

- Spin-1 XY model

  M. Schecter and T. Iadecola, Phys. Rev. Lett. **123**, 147201 (2019).

- Fermi-Hubbard model+some modification

  D. K. Mark and O. I. Motrunich, Phys. Rev. B **102**, 075132 (2020).

  M. Nakagawa et al., Phys. Rev. Res. **6**, 043259 (2024).

- Bose-Hubbard model+constraint

  R. Kaneko, MK, and I. Danshita, Phys. Rev. A **109**, L011301 (2024).

- Lattice gauge theory

  F. M. Surace et al., Phys. Rev. X **10**, 021041 (2020).

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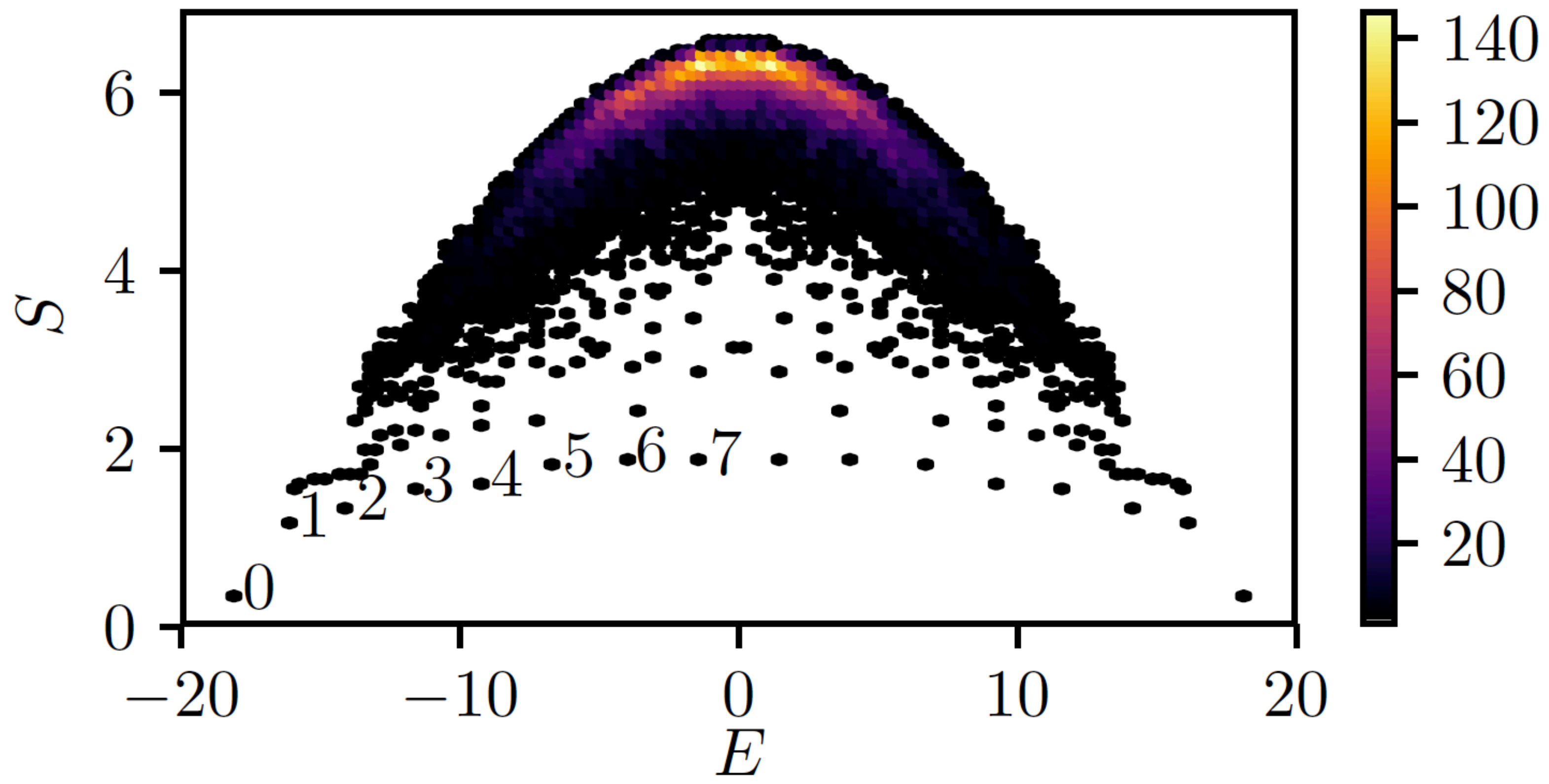
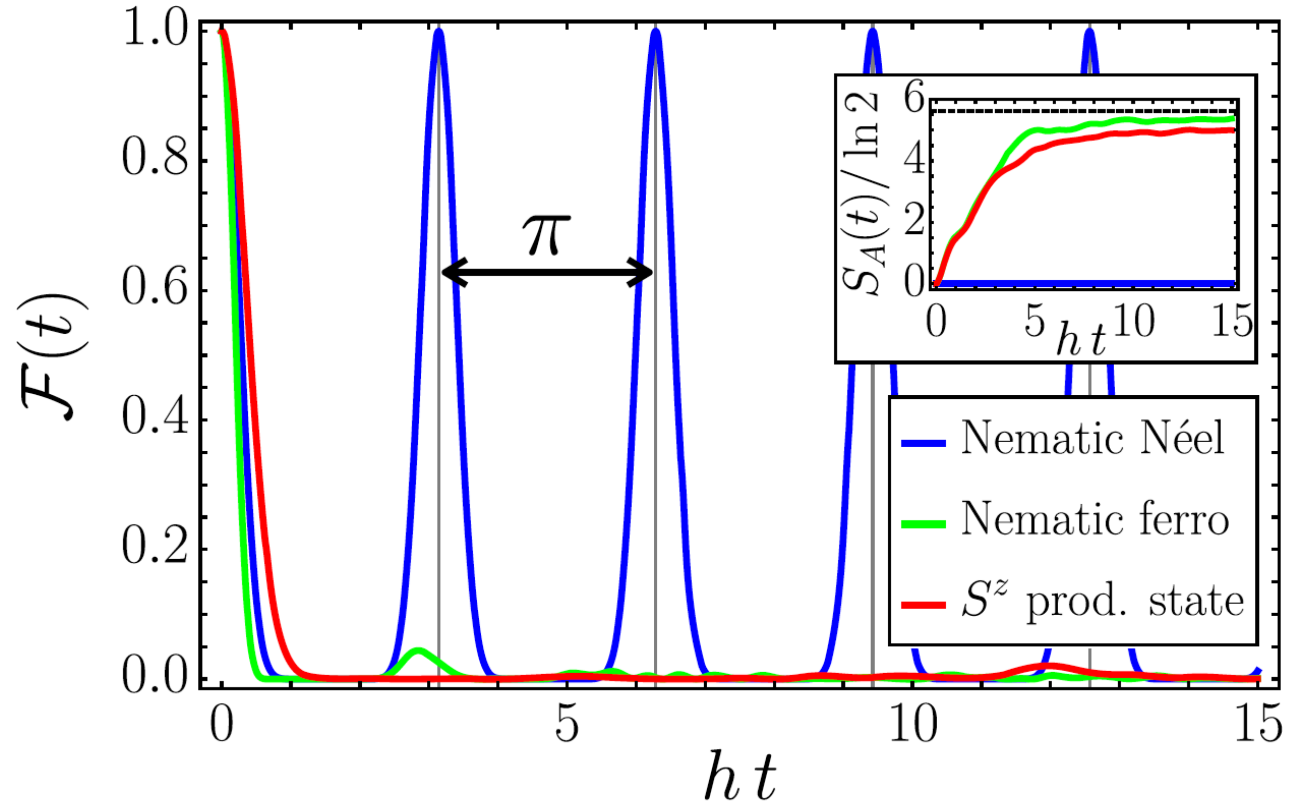


C. J. Turner et al., Phys. Rev. B **98**, 155134 (2018).



M. Schecter and T. Iadecola, Phys. Rev. Lett. **123**, 147201 (2019).

Introduction : Quantum many-body scar states

There are several types of scar states.
One of them can be characterized by the following relation:

Reviews: M. Serbyn et al., Nat. Phys. **17**, 675 (2021).
A. Chandran et al., Annu. Rev. Condens. Matter Phys. **14** 443 (2023).
S. Moudgalya et al., Rep. Prog. Phys. **85**, 086501 (2022).

Restricted spectrum generating algebra (RSGA) S. Moudgalya et al., Phys. Rev. B **102**, 085140 (2020).

If there exist a state $|\psi_0\rangle$ and operator $\hat{Q}^\dagger$,
these satisfy the relations

$$\hat{H}|\psi_0\rangle = E_0|\psi_0\rangle$$
$$[\hat{H}, \hat{Q}^\dagger]|\psi_0\rangle = \mathcal{E}\hat{Q}^\dagger|\psi_0\rangle$$
$$[[\hat{H}, \hat{Q}^\dagger], \hat{Q}^\dagger] = 0$$

If the left relations hold, we can show

$$\hat{H}(\hat{Q}^\dagger)^n|\psi_0\rangle = (E_0 + n\mathcal{E})(\hat{Q}^\dagger)^n|\psi_0\rangle$$

$(\hat{Q}^\dagger)^n|\psi_0\rangle$: **scar state**

Hilbert space splits into two parts: $\mathcal{H}_{\mathrm{tot}} = \mathcal{H}_{\mathrm{thermal}} \oplus \mathcal{H}_{\mathrm{scar}}$

Scar subspace: $\mathcal{H}_{\mathrm{scar}} = \mathrm{Span}\{|\psi_0\rangle, \hat{Q}^\dagger|\psi_0\rangle, (\hat{Q}^\dagger)^2|\psi_0\rangle \ldots\}$

Results : Quantum many-body scar states in the DH model

We can analytically show that the scar states of the DH model with periodic boundary conditions and open boundary conditions. In this talk, we focus on the OBC case.

DH model： $\hat{H}_{\mathrm{DH}} = D\sum_{j=1}^{M-1}(\hat{S}_j^z\hat{S}_{j+1}^x - \hat{S}_j^x\hat{S}_{j+1}^z) - H\sum_{j=1}^{M}\hat{S}_j^z - \frac{D}{2}(\hat{S}_1^x - \hat{S}_M^x)$ (OBC)

$$= D\sum_{j=1}^{M}\hat{S}_j^x(\hat{S}_{j-1}^z - \hat{S}_{j+1}^z) - H\sum_{j=1}^{M}\hat{S}_j^z$$

Edge magnetic field

$$\hat{S}_0^z = \hat{S}_{M+1}^z \equiv -1/2$$

We define $\hat{Q}^\dagger \equiv \sum_{j=1}^{M}\hat{P}_{j-1}\hat{S}_j^+\hat{P}_{j+1}$, $\hat{P}_0 = 1$, $\hat{P}_{M+1} = 1$, $\hat{P}_j \equiv \frac{1}{2} - \hat{S}_j^z$

$$\hat{P}_j\,|{\uparrow_j}\rangle = 0$$

$$\hat{P}_j\,|{\downarrow_j}\rangle = |{\downarrow_j}\rangle$$

We can show the RSGA relation when we choose $|\psi_0\rangle = |\downarrow\downarrow\cdots\downarrow\rangle$.

$$\hat{Q}^\dagger\,|{\downarrow\downarrow\downarrow}\rangle = |{\uparrow\downarrow\downarrow}\rangle + |{\downarrow\uparrow\downarrow}\rangle + |{\downarrow\downarrow\uparrow}\rangle$$

Scar states: $\hat{H}_{\mathrm{DH}}\,|S_n\rangle = (E_0 - nH)\,|S_n\rangle$, $|S_n\rangle \propto (\hat{Q}^\dagger)^n\,|\downarrow\downarrow\cdots\downarrow\rangle$

Results : Quantum many-body scar states in the DH model

Scar states: $\hat{H}_{\mathrm{DH}}\left|S_n\right\rangle=(E_0-nH)\left|S_n\right\rangle, \quad \left|S_n\right\rangle \propto (\hat{Q}^\dagger)^n \left|\downarrow\downarrow\cdots\downarrow\right\rangle, \; \hat{Q}^\dagger \equiv \sum_{j=1}^{M} \hat{P}_{j-1}\hat{S}_j^+\hat{P}_{j+1}$

Explicit expression of scar states: (6 sites)

$$\left|S_1\right\rangle \propto \left|\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\right\rangle+\left|\downarrow\uparrow\downarrow\downarrow\downarrow\downarrow\right\rangle+\left|\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\right\rangle+\left|\downarrow\downarrow\downarrow\uparrow\downarrow\downarrow\right\rangle+\left|\downarrow\downarrow\downarrow\downarrow\uparrow\downarrow\right\rangle+\left|\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\right\rangle$$

$$\left|S_2\right\rangle \propto \left|\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow\right\rangle+\left|\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\right\rangle+\left|\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\right\rangle+\left|\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\right\rangle+\left|\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\right\rangle+\left|\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\right\rangle$$
$$+\left|\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\right\rangle+\left|\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\right\rangle+\left|\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\right\rangle+\left|\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\right\rangle$$

$$\left|S_3\right\rangle \propto \left|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\right\rangle+\left|\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\right\rangle+\left|\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\right\rangle+\left|\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\right\rangle$$

❌ $\uparrow\uparrow$ is prohibited

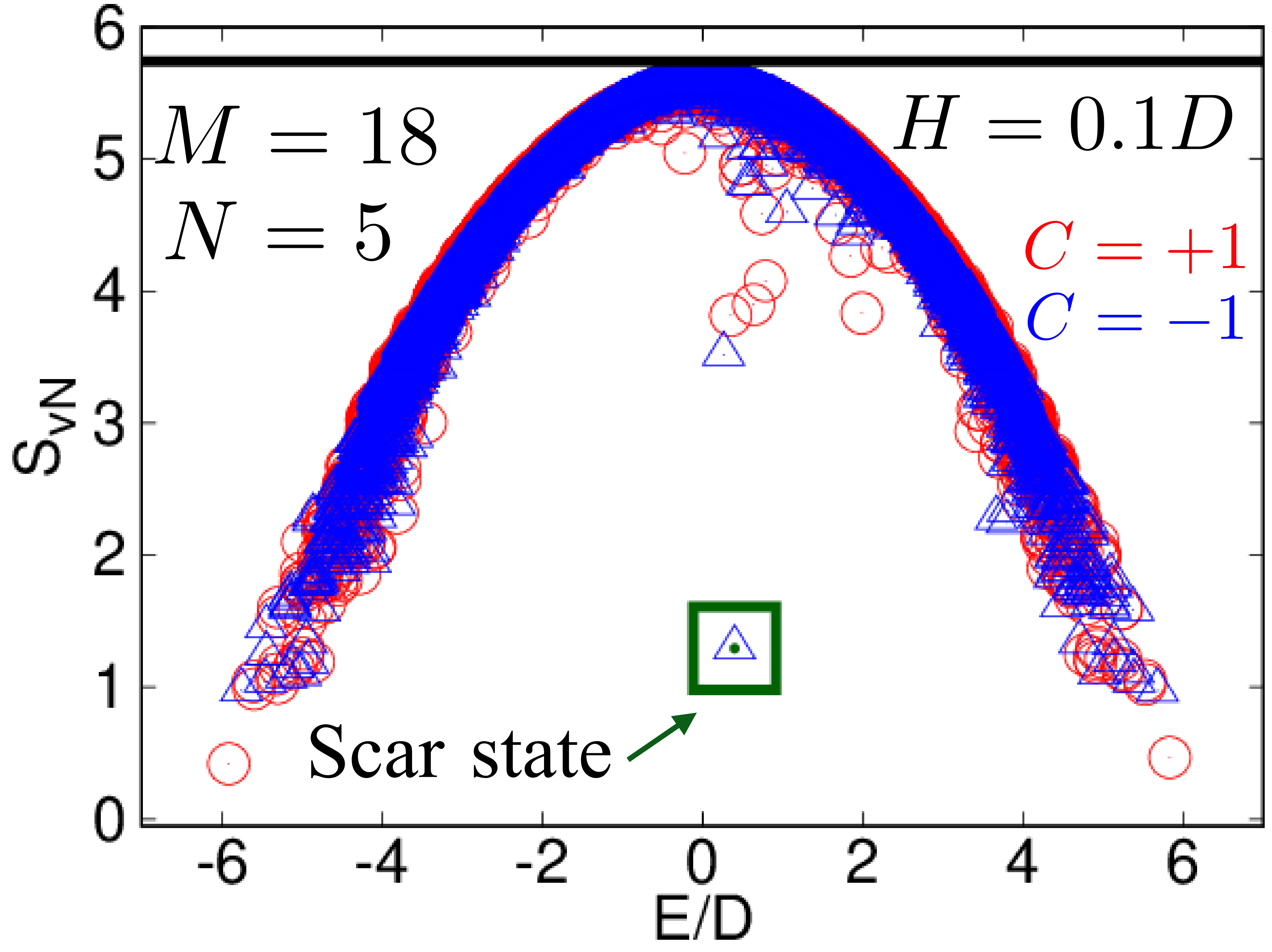
n: number of up spins.

Equal-weight superposition of the states that do not contain $\uparrow\uparrow$.

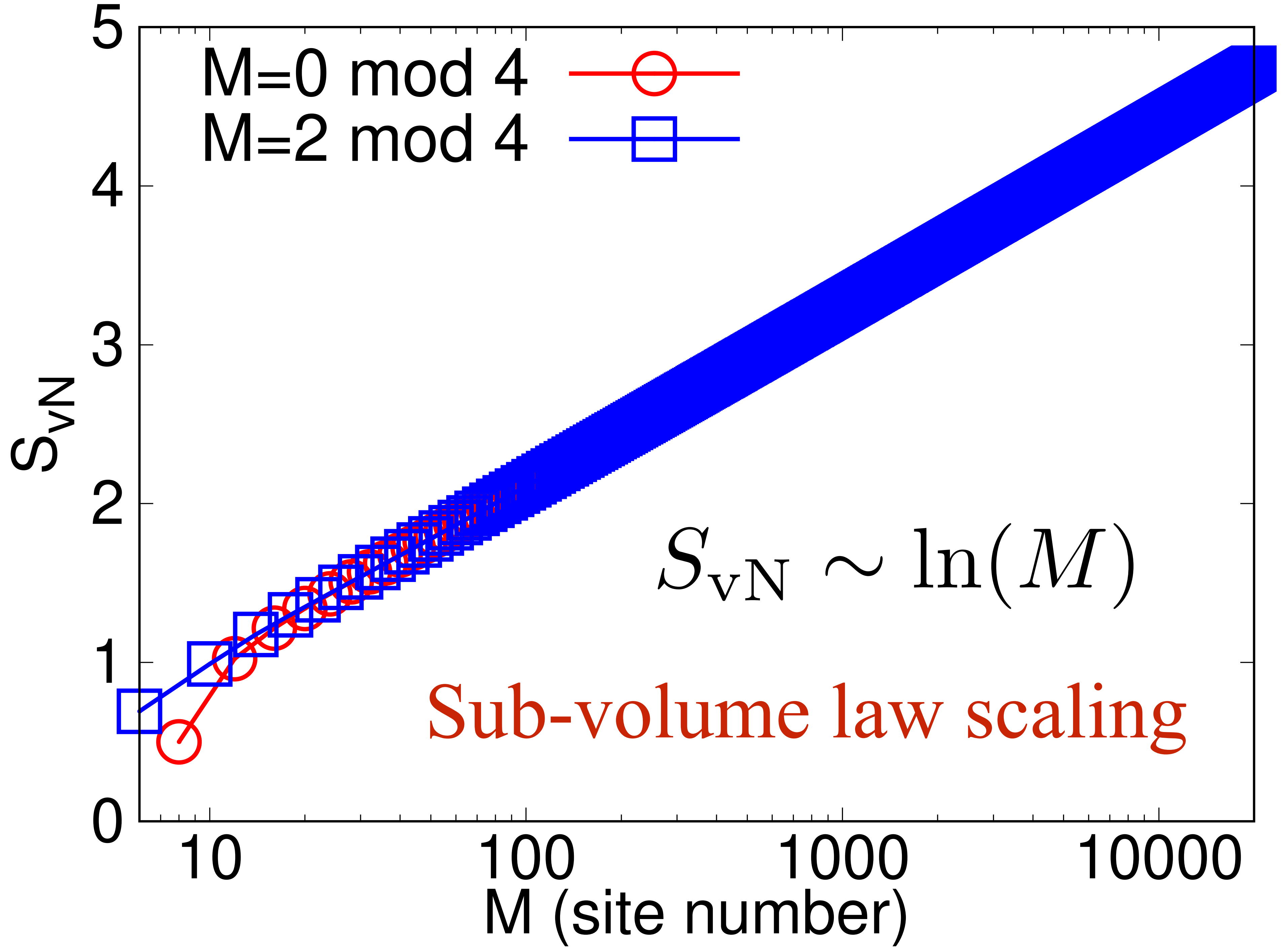
Similar scar states appear in another model: T. Iadecola and M. Schecter, Phys. Rev. B **101**, 024306 (2020).

Results : Quantum many-body scar states in the DH model

Half-chain entanglement entropy



Size dependence of the EE of the scar state



DH model has conserved quantities:

$$\hat{C} \equiv \hat{\mathcal{I}}\hat{C}_z \quad \hat{C}_z \equiv \prod_{j=1}^{M} \hat{\sigma}_j^z \quad \hat{C}^2 = \hat{1}$$

$$\hat{N}_{\mathrm{sol}} = \sum_{j=0}^{M} \left(\frac{1}{4} - \hat{S}_j^z \hat{S}_{j+1}^z \right)$$: soliton number operator

Level spacing ratio $\langle r \rangle_{C=+1} \simeq 0.532$

$\langle r \rangle_{C=-1} \simeq 0.529$

⇒ Consistent with Wigner-Dyson distribution

Introduction : AQMBS in the DH model

Recently, **asymptotic QMBS(AQMBS) has been proposed.** L. Gotta et al., Phys. Rev. Lett. **131**, 190401 (2023).

Properties of AQMBS: $|AS_n\rangle$

(i) Orthogonal to all scar states $\langle S_m|AS_n\rangle = 0$

(ii) Low-entangled states

(iii) The energy variance goes to zero in the thermodynamic limit

$$\Delta E^2 \equiv \langle AS_n|\,\hat{H}^2\,|AS_n\rangle - (\langle AS_n|\,\hat{H}\,|AS_n\rangle)^2 \to 0$$

From (iii) and short-time limit of the fidelity $|\,\langle AS_n|\,e^{-i\hat{H}t/\hbar}\,|AS_n\rangle\,|^2 \simeq 1 - (\Delta E t/\hbar)^2$

the thermalization does not happen in the thermodynamic limit: $\tau_{\mathrm{relax}} \sim \hbar/\Delta E \to \infty$

This work: Construct the AQMBS that satisfies (i), (ii), (iii) in the DH model.

Results : Construction of AQMBS states in the DH model

The previous work constructs the AQMBS in the spin-1 XY model: L. Gotta et al., Phys. Rev. Lett. **131**, 190401 (2023).

Spin-1 XY model

QMBS state

$\hat{\tau}^z \left|-\right\rangle = -\left|-\right\rangle$

QMBS:

M. Schecter and T. Iadecola, Phys. Rev. Lett. **123**, 147201 (2019).

$$\left|S_n\right\rangle \propto (\hat{Q}^\dagger)^n \left|-,-,\ldots,-\right\rangle$$

$$\hat{Q}^\dagger = \frac{1}{2}\sum_j (-1)^j (\hat{\tau}_j^+)^2$$

AQMBS state

$$\left|AS_n\right\rangle \propto \hat{Q}^\dagger(k) \left|S_{n-1}\right\rangle$$

$$\hat{Q}^\dagger(k) = \frac{1}{2}\sum_j \underline{e^{ikj}} (\hat{\tau}_j^+)^2$$

DH model

(OBC
+edge magnetic field)

QMBS state

$$\left|S_n\right\rangle \propto (\hat{Q}^\dagger)^n \left|\downarrow\ldots\downarrow\right\rangle$$

$$\hat{Q}^\dagger \equiv \sum_{j=1}^{M} \hat{P}_{j-1}\hat{S}_j^+\hat{P}_{j+1}$$

AQMBS state

$$\left|AS_n\right\rangle \propto \hat{A}^\dagger \left|S_{n-1}\right\rangle$$

$$\hat{A}^\dagger \equiv \sum_{j=1}^{M} \underline{f_j} \hat{P}\hat{S}_j^+\hat{P}_{j+1}$$

$f_j \equiv \cos[\pi j/(M+1)]$ From the space-inversion symmetry, $\left\langle S_m | AS_n \right\rangle = 0$.

Results : Construction of AQMBS states in the DH model

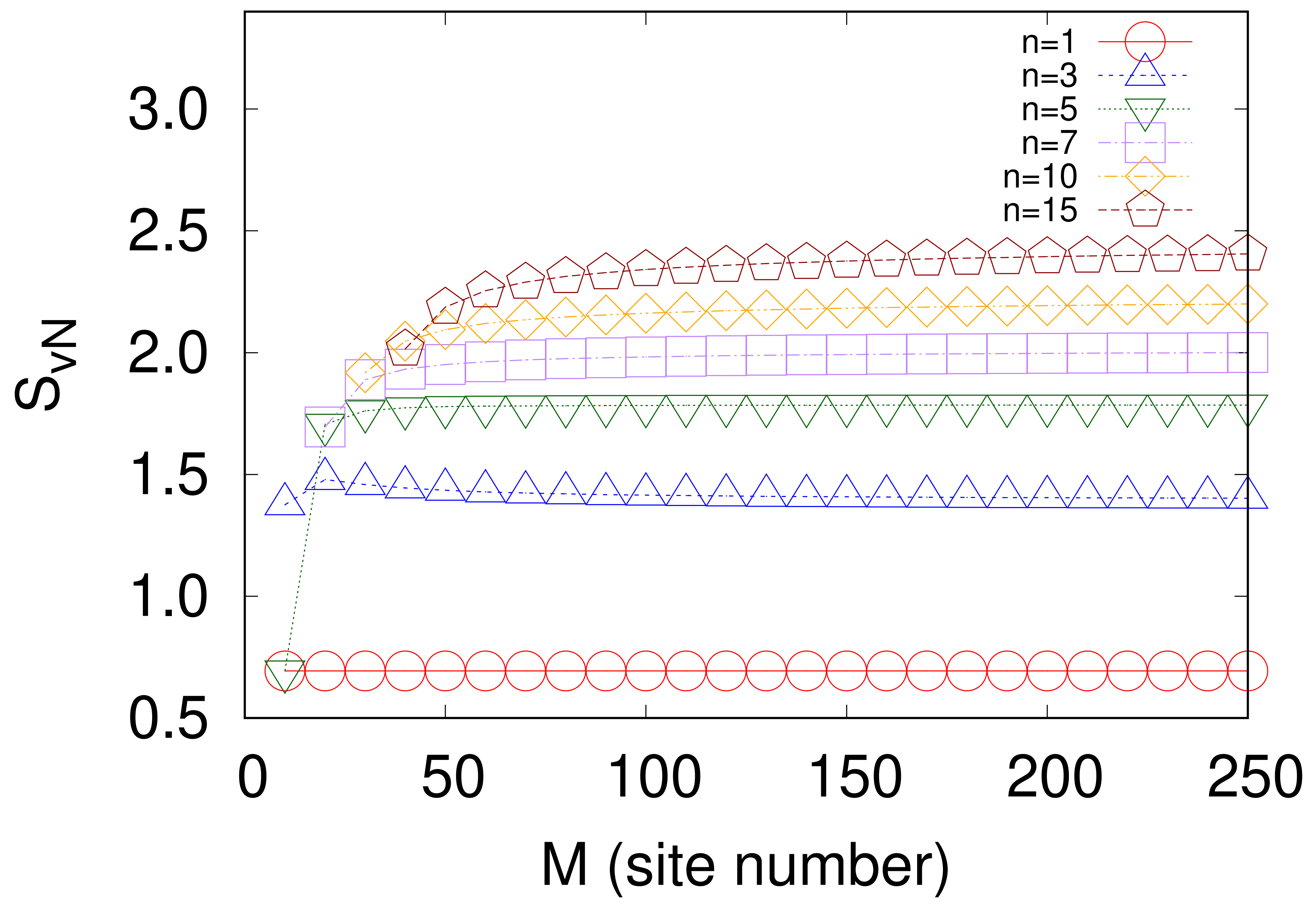
Using the matrix product representation, we can show $|AS_n\rangle$ is low-entangled state.

$$|AS_n\rangle \propto \hat{A}^\dagger(\hat{Q}^\dagger)^{n-1}|\downarrow,\ldots,\downarrow\rangle$$

$\Rightarrow$ We show that the bond dimension of $\hat{A}^\dagger(\hat{Q}^\dagger)^{n-1}$ is at most $\chi_n = n(3n+1)$.

$\Rightarrow$ The bond dimension of $|AS_n\rangle$ is bounded by $\chi_n = n(3n+1)$.

Half-chain EE



Since $n = 1, 2, \ldots, M/2$ (M : site number),

the half-chain EE is bounded by

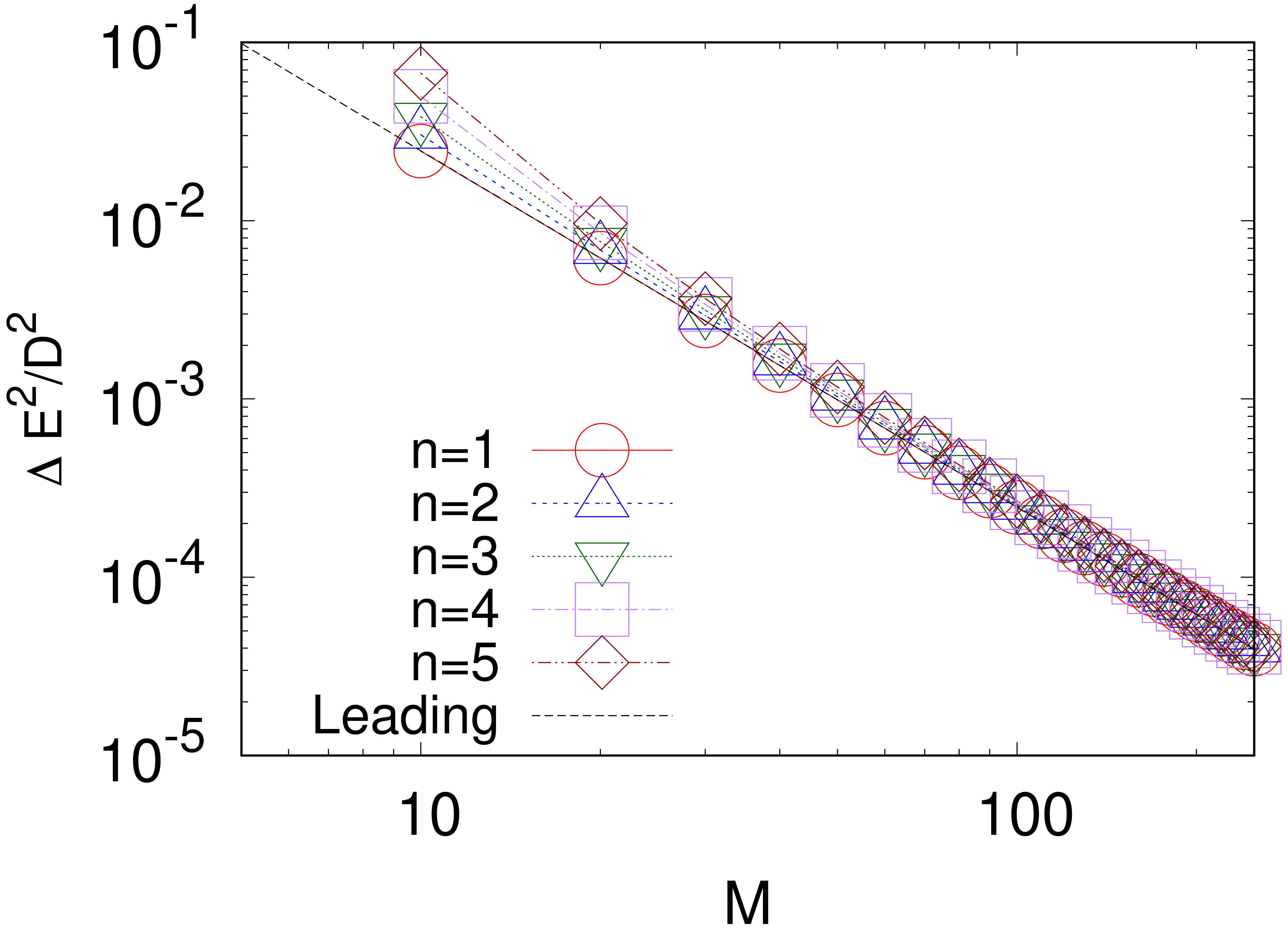
$$S_{\mathrm{vN}} \leq \ln \chi_n = \ln[n(3n+1)] \sim \ln M$$

$\Rightarrow$sub-volume law scaling

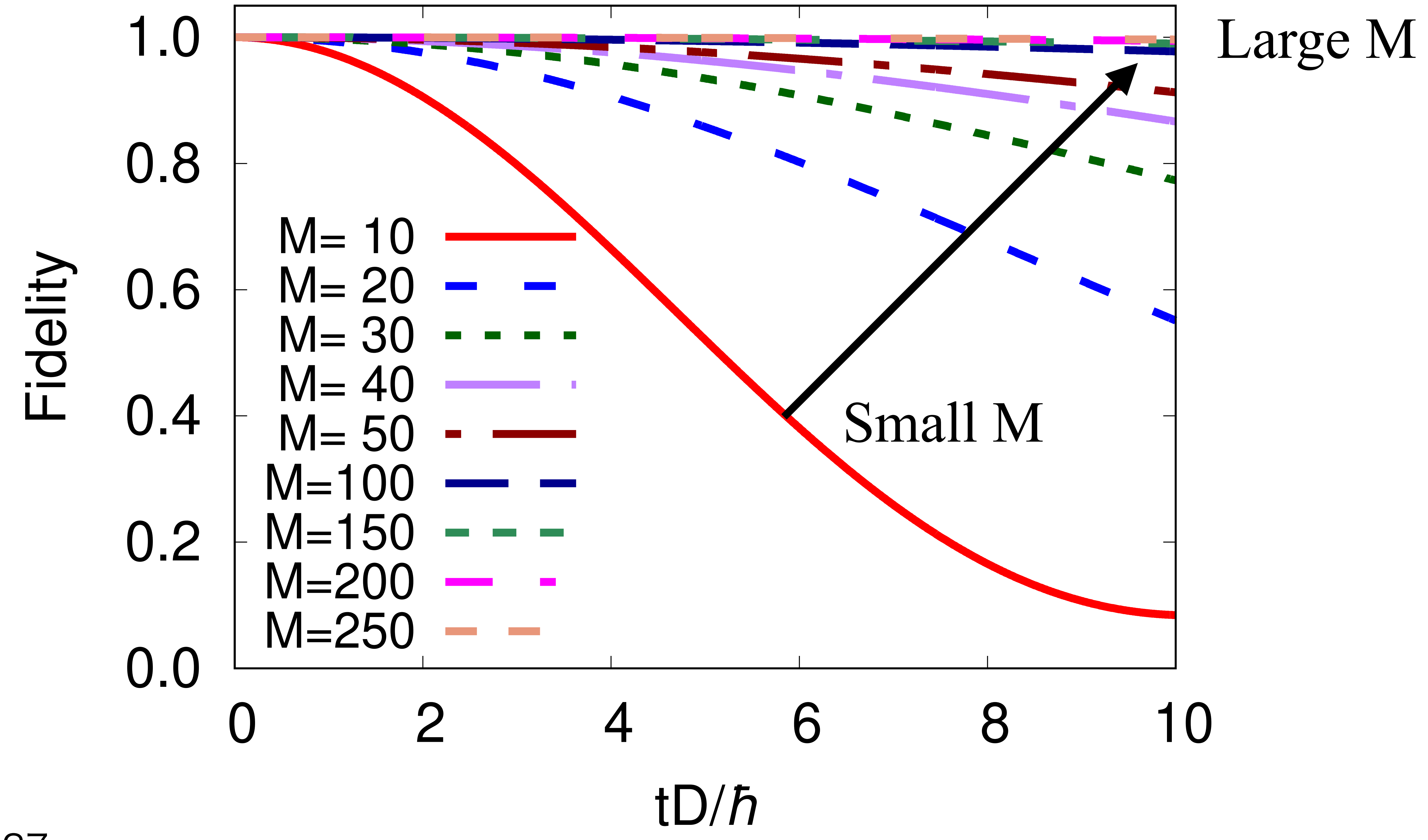
Results : Construction of AQMBS states in the DH model

Energy variance of $|AS_n\rangle$: $\Delta E^2 = \dfrac{\langle S_{n-1}|\,[\hat{A},\hat{H}_{\mathrm{DM}}][\hat{H}_{\mathrm{DM}},\hat{A}^\dagger]\,|S_{n-1}\rangle}{\langle S_{n-1}|\,\hat{A}\hat{A}^\dagger\,|S_{n-1}\rangle} = \dfrac{\pi^2 D^2}{4M^2} + O(1/M^3)$

Numerical results of the energy variance

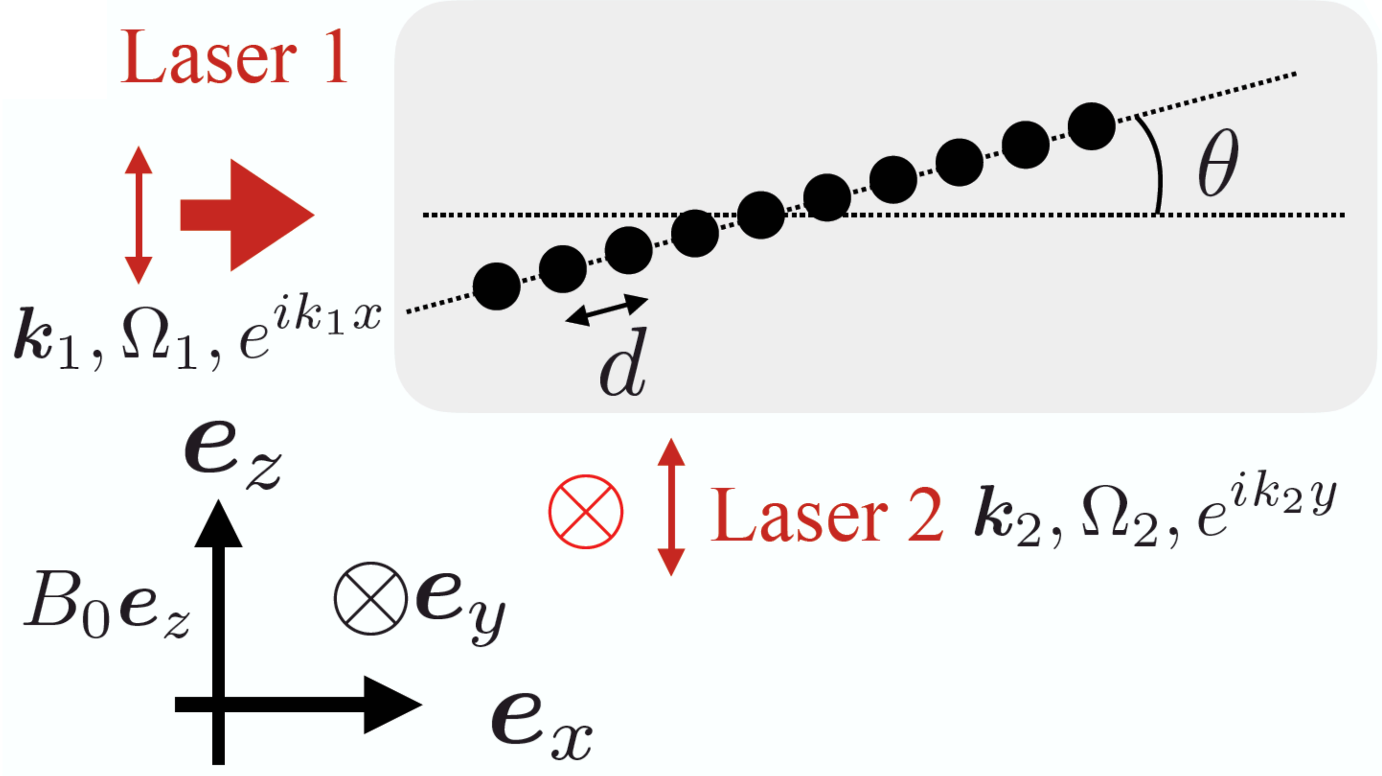


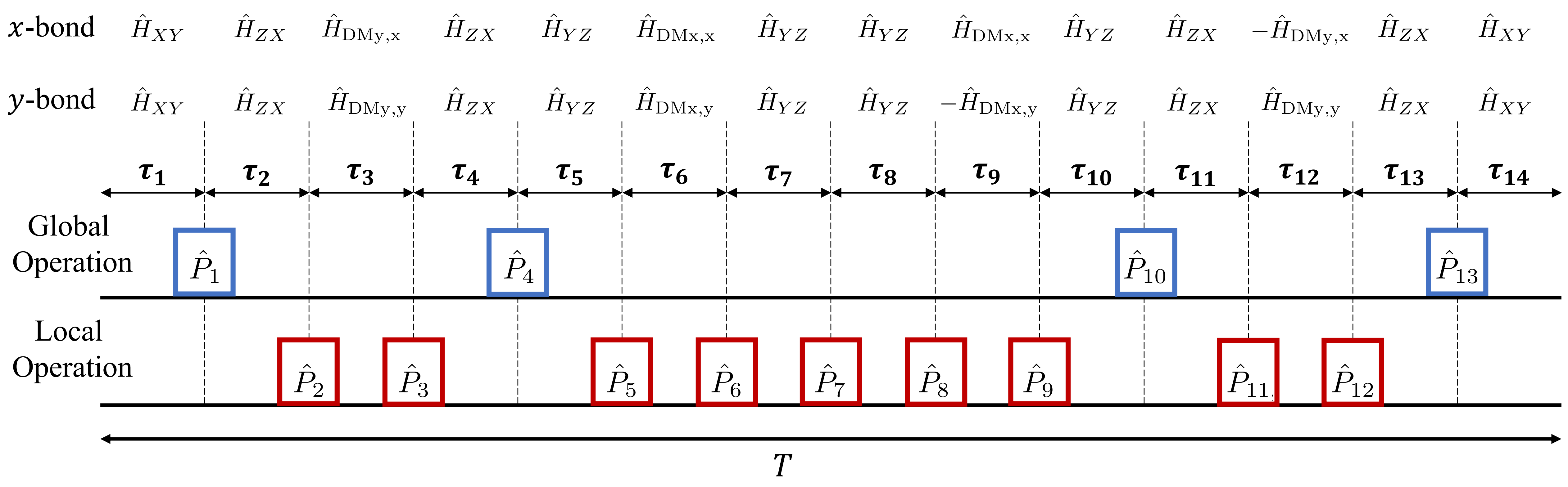
Numerical results of the fidelity (TEBD method)



Summary and future prospect

- We proposed a method to realize the DM interactions using Rydberg atom quantum simulator in 1D and 2D systems.
- Using the Raman transition and unitary equivalence, the monoaxial DM interaction can be simulated in 1D systems.
- The bond-dependent DM interactions can be realized by the Floquet engineering.
- We analytically constructed QMBS and AQMBS states in the DH model.





MK, T. Tomita, H. Katsura, and Y. Kato, Phys. Rev. A **110**, 043312 (2024).

H. Kuji, MK, and T. Nikuni, arXiv:2408.04160 (2024).

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