International Molecule-type workshop 2025
Topology and Dynamics of Magneto-Vortical Matter

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SPRING GX
Fostering Advanced Human Resources to Lead Green Transformation (GX)

Dynamics of Domain Walls and Quantum Vortices in type-II Superconductors Under Gradients of Temperature/Spin Density

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Acknowledgement
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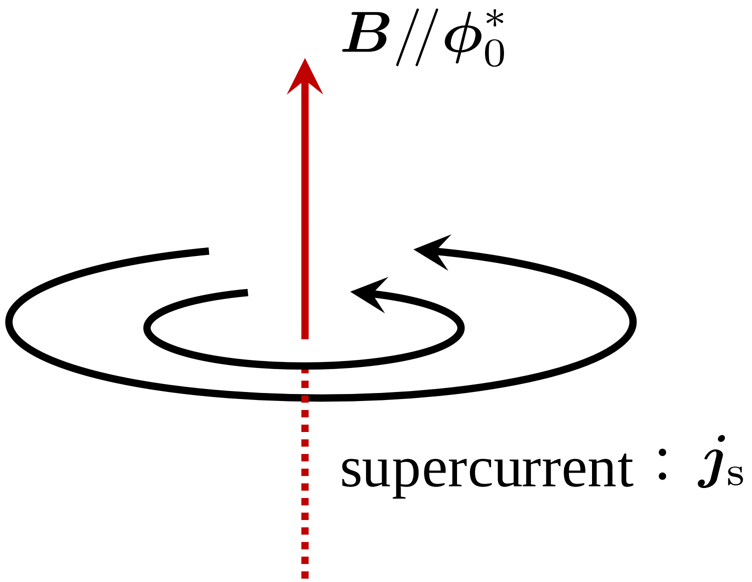
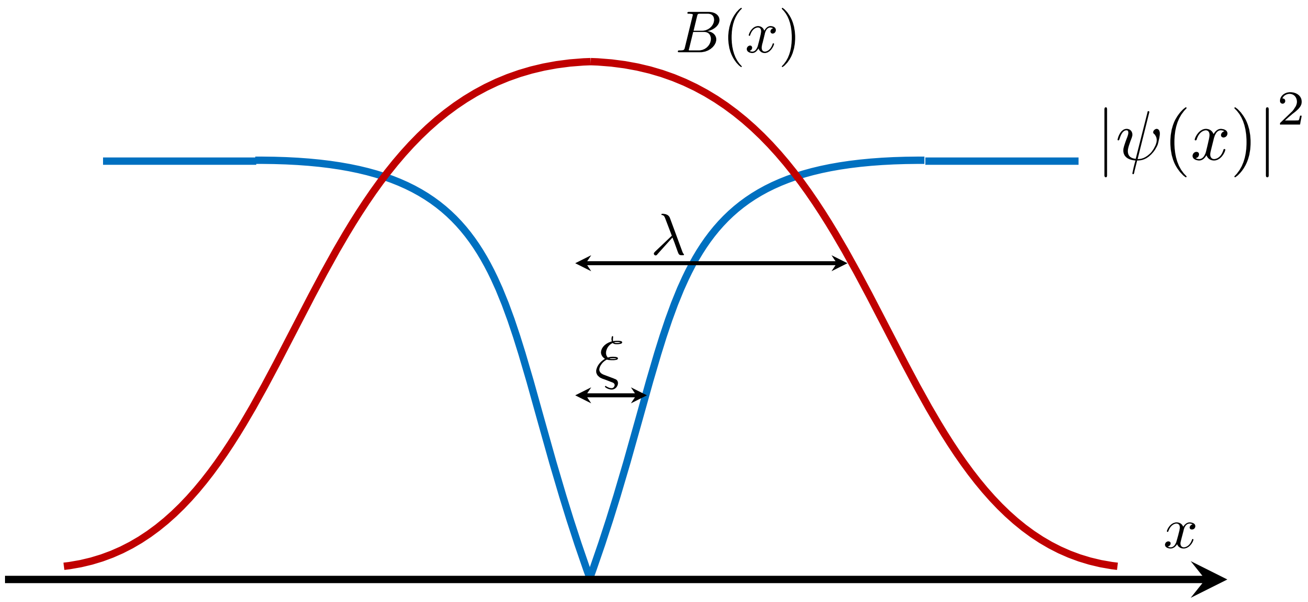
✓ <u>Vortex state in **type-II superconductor**</u>

- Quantized magnetic flux: $\phi_0^*, |\phi_0^*| = h/(2e)$
- Characteristic lengths: penetration length λ and coherence length ξ
- Supercurrent is circulating (radius ~ λ) around a magnetic flux → vortex
- Superconductivity is broken at the center of a vortex (radius ~ ξ)

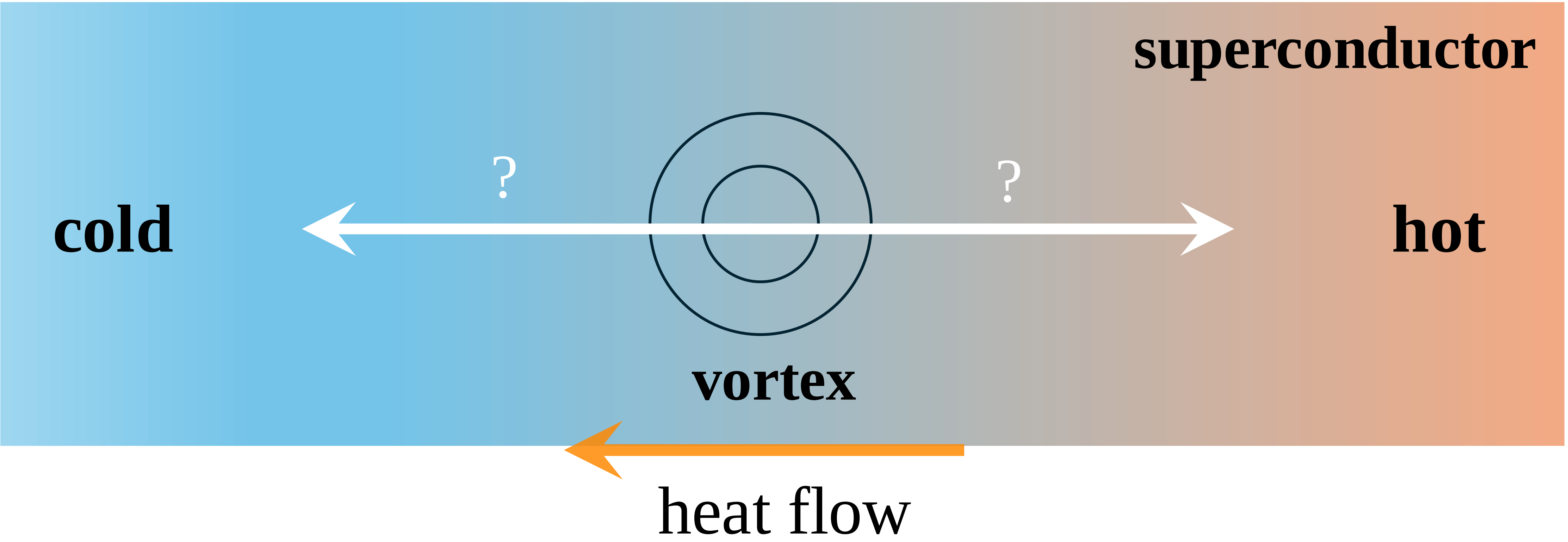




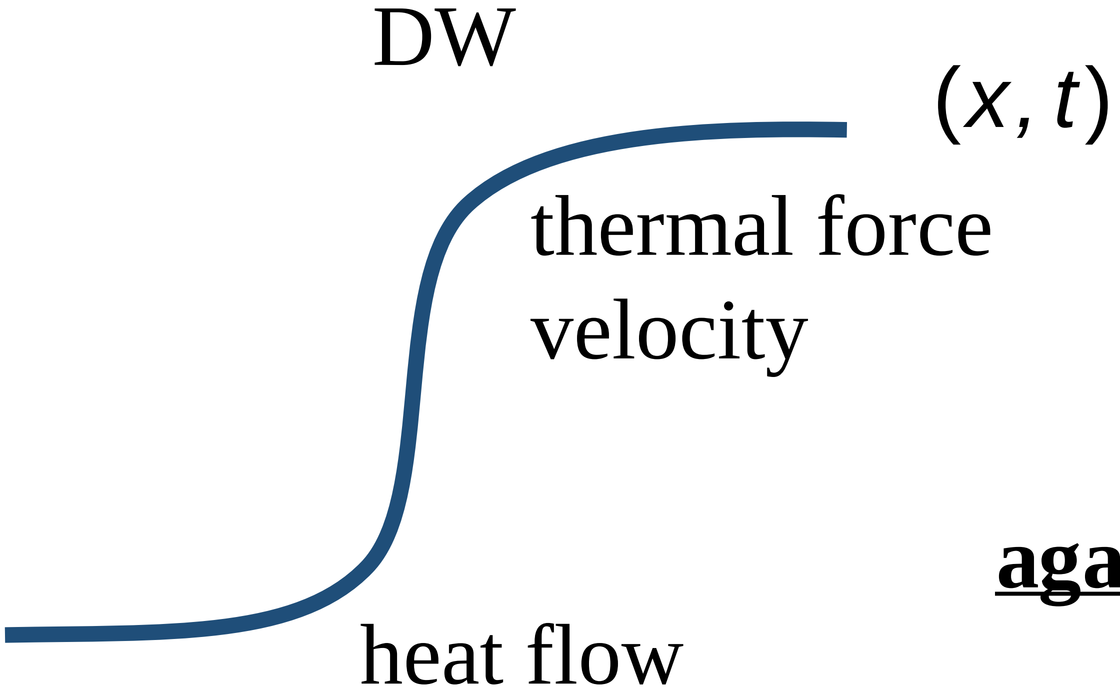
- Topological defect
- Transport phenomena

Dynamics of vortices

- How can we define the force on a vortex **under the temperature gradient**?
- Is the motion along the heat flow or against it?



$$F_{\mathrm{th}} = \frac{1}{2(1-\tau_{\mathrm{L}})} \int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} \mathrm{d}x \frac{\mathrm{d}\tau_1(x)}{\mathrm{d}x} \left(\psi_0^2(x_{\mathrm{R}}) - \psi_0^2(x)\right) \qquad v = \frac{\int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} \mathrm{d}x \frac{\mathrm{d}\tau_1(x)}{\mathrm{d}x} \left(\psi_0^2(x_{\mathrm{R}}) - \psi_0^2(x)\right)}{2(1-\tau_{\mathrm{L}})\tilde{\gamma} \int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} \mathrm{d}x \left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x}\right)^2}$$



against heat flow

- [Theory] M. J. Stephen, Phys. Rev. Lett. **16**, 801 (1966).

GALVANOMAGNETIC AND RELATED EFFECTS IN TYPE-II SUPERCONDUCTORS*

M. J. Stephen

Physics Department, Yale University, New Haven, Connecticut

(Received 30 March 1966)

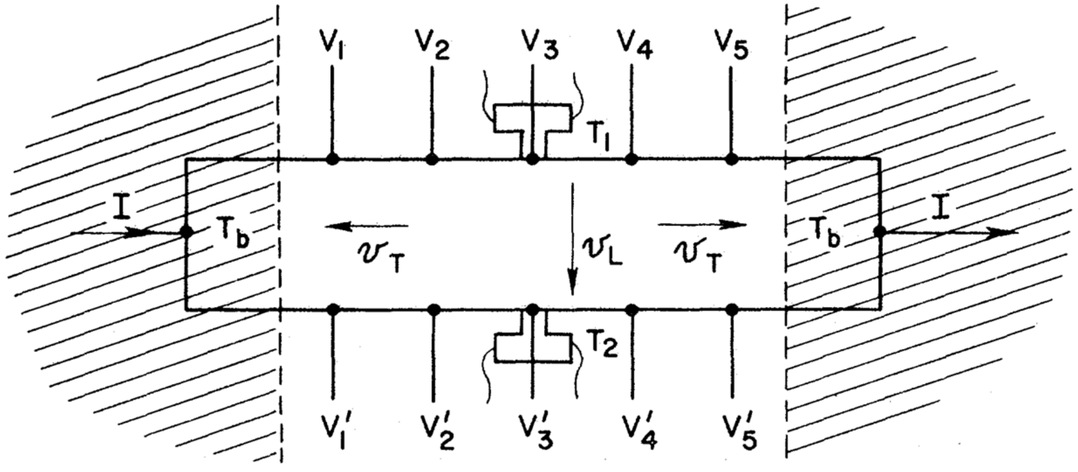
effective thermal force on a vortex line

$$F_{\rm th} = -\frac{\varphi_0 S}{B} \nabla T \qquad : \text{hot} \rightarrow \text{cold}$$

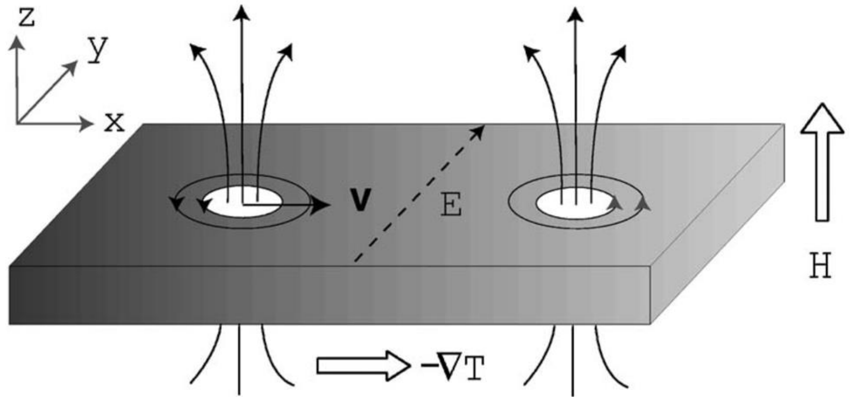
called "the entropy of unit length of a vortex line " $\overset{?}{\longleftrightarrow}$ "transport entropy"

P. Solomon and F. Otter Jr, Phys. Rev. **164**, 608 (1967).

- [Experiment] F. A. Otter, P. R. Solomon, Phys. Rev. Lett. **16**, 681 (1966).

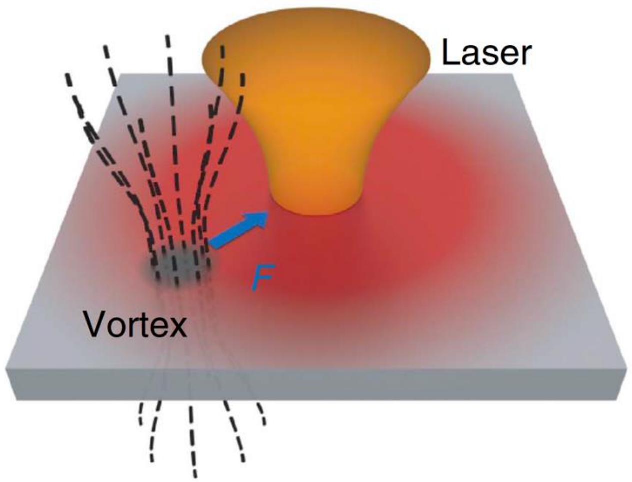


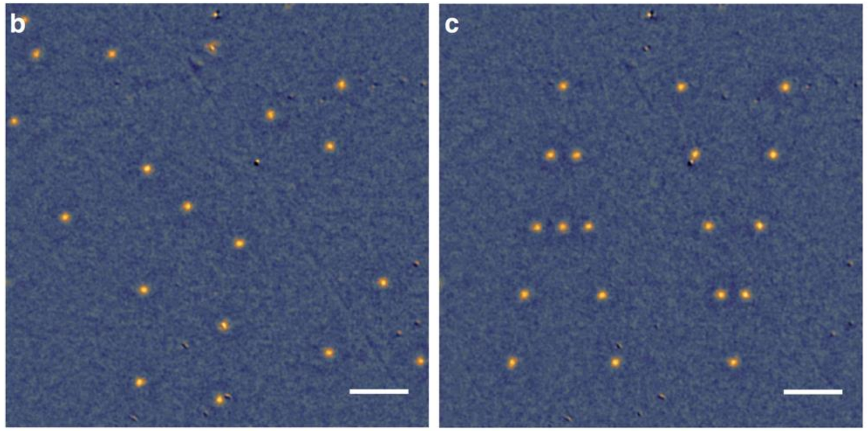
Vortex Nernst Effect



Y. Wang *et al.*, Phys. Rev. B **73**, 024510 (2006).

- [Experiment] I. S. Veshchunov *et al*., Nat. Commun. **7**, 12801 (2016).

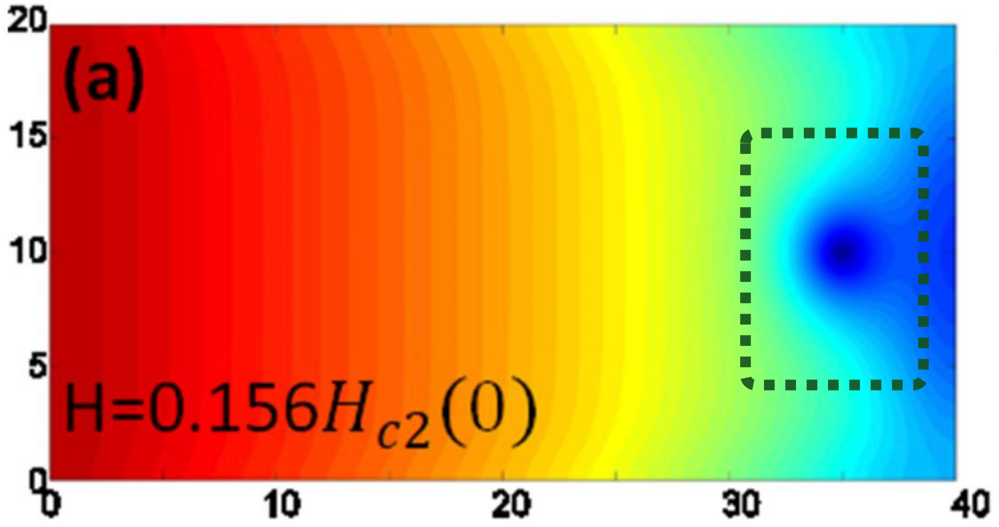




- Vortex manipulation by the local temperature gradient induced by laser.

[Theory] E. C. S. Duarte *et al*., J. Phys.: Condens. Matter **31**, 405901 (2019).

cold



hot

- Numerical simulation of GL Eq. with temperature gradient
- As the magnetic field gets stronger, vortex forms on the hotter side in the stationary state.

Motion of Vortices Under Various Flows

Current	Research Trends	Force
Electric	many studies[1-5]	fluid+magnetic
Heat	vortices move to the colder region[6-7] vs. vortices move to the hotter region[8-10]	?
Spin	recently started[11-15]	?

Remaining issues

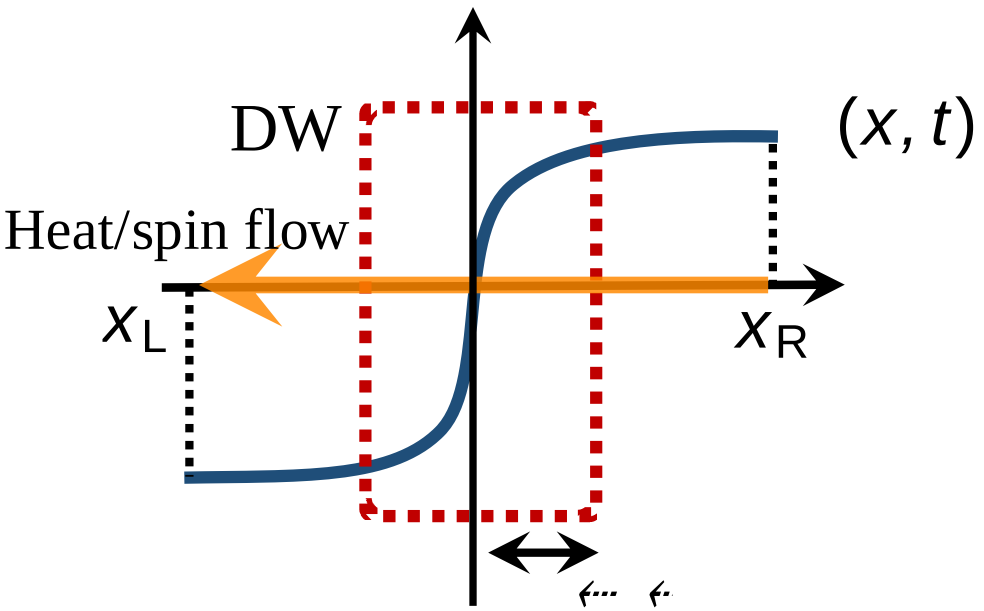
- Controversial statements
- The origin of the thermal force is still unclear

[1] L. P. Gor'kov , N. B. Kopnin, Sov. Phys. Usp. **18**, 496 (1975).
[2] A. T. Dorsey: Phys. Rev. B **46**, 8376 (1992) .
[3] N. B. Kopnin, *Theory of Nonequilibrium Superconductivity* (Oxford Univ. Press, Oxford, 2001).
[4] Y. Kato, C-K Chung, J. Phys. Soc. Jpn. **85**, 033703/1-5 (2016).
[5] S. Sugai, N. Kurosawa, Y. Kato, Phys. Rev. B **104**, 064516 (2021).
[6] M. J. Stephen, Phys. Rev. Lett. **16**, 801 (1966).
[7] F. A. Otter, P. R. Solomon, Phys. Rev. Lett. **16**, 681 (1966).
[8] A. Sergeev *et al.*, Europhys. Lett. **92**, 27003 (2010).
[9] I. S. Veshchunov *et al.*, Nat. Commun. **7**, 12801 (2016).
[10] E. C. S. Duarte *et al.*, J. Phys.: Condens. Matter **31**, 405901 (2019).
[11] Se Kwon Kim *et al.*, Phys. Rev. Lett. **121**, 187203 (2018).
[12] A. Vargunin, M. Silaev, Sci Rep **9**, 5914 (2019).
[13] T. Taira *et al.*, Phys. Rev. B **103**, 134417 (2021).
[14] B. Niedzielski *et al.*, Phys. Rev. Applied **19**, 024073 (2023).
[15] H. Adachi, Y. Kato, J.-i. Ohe, and M. Ichioka, Phys. Rev. B **109**, 174503 (2024).

Today's talk

TK *et al.*, arXiv:2405.10200 (2024).

- Interest : Motion of vortices under temperature gradient
- Issues : (1) Is the motion along the heat/spin flow or against it?
  (2) What is the origin of the force on vortices under heat flow?
- Target: focusing on 1D problem : **<u>Domain Wall (DW)</u>** for mathematical simplicity
- Model: Time-dependent Ginzburg-Landau (TDGL) Eq. & Thermal diffusion Eq.



Why 1D?

- Analytical results
- Prototype model for vortex system
- Application to FFLO state

(1) DW motion under Temperature Gradient

- Model: Time-dependent Ginzburg-Landau (TDGL) Eq. & Thermal diffusion Eq.
- Results[1]: Numerical Simulation
- Results[2]: Analytical Calculation based on Momentum Balance Relation
- Discussion

(2) DW motion under Spin density Gradient

- Model: Time-dependent Ginzburg-Landau (TDGL) Eq. & Spin diffusion Eq.
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TDGL Eq. & Thermal Diffusion Eq.

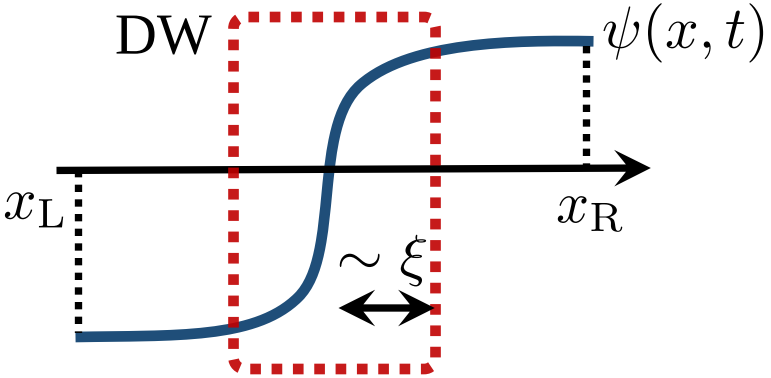
$$\begin{cases} -\xi^2 \dfrac{\partial^2 \psi(x,t)}{\partial x^2} + \dfrac{\tau(x,t) - 1}{1 - \tau_{\mathrm{L}}} \psi(x,t) + \psi^3(x,t) = -\tilde{\gamma} \dfrac{\partial \psi(x,t)}{\partial t} \\ \dfrac{\partial}{\partial x} \left[\left\{ 1 + (k-1)\psi^2(x,t) \right\} \dfrac{\partial \tau(x,t)}{\partial x} \right] = \dfrac{C}{\kappa_{\mathrm{n}}} \dfrac{\partial \tau(x,t)}{\partial t} \end{cases}$$

thermal conductivity

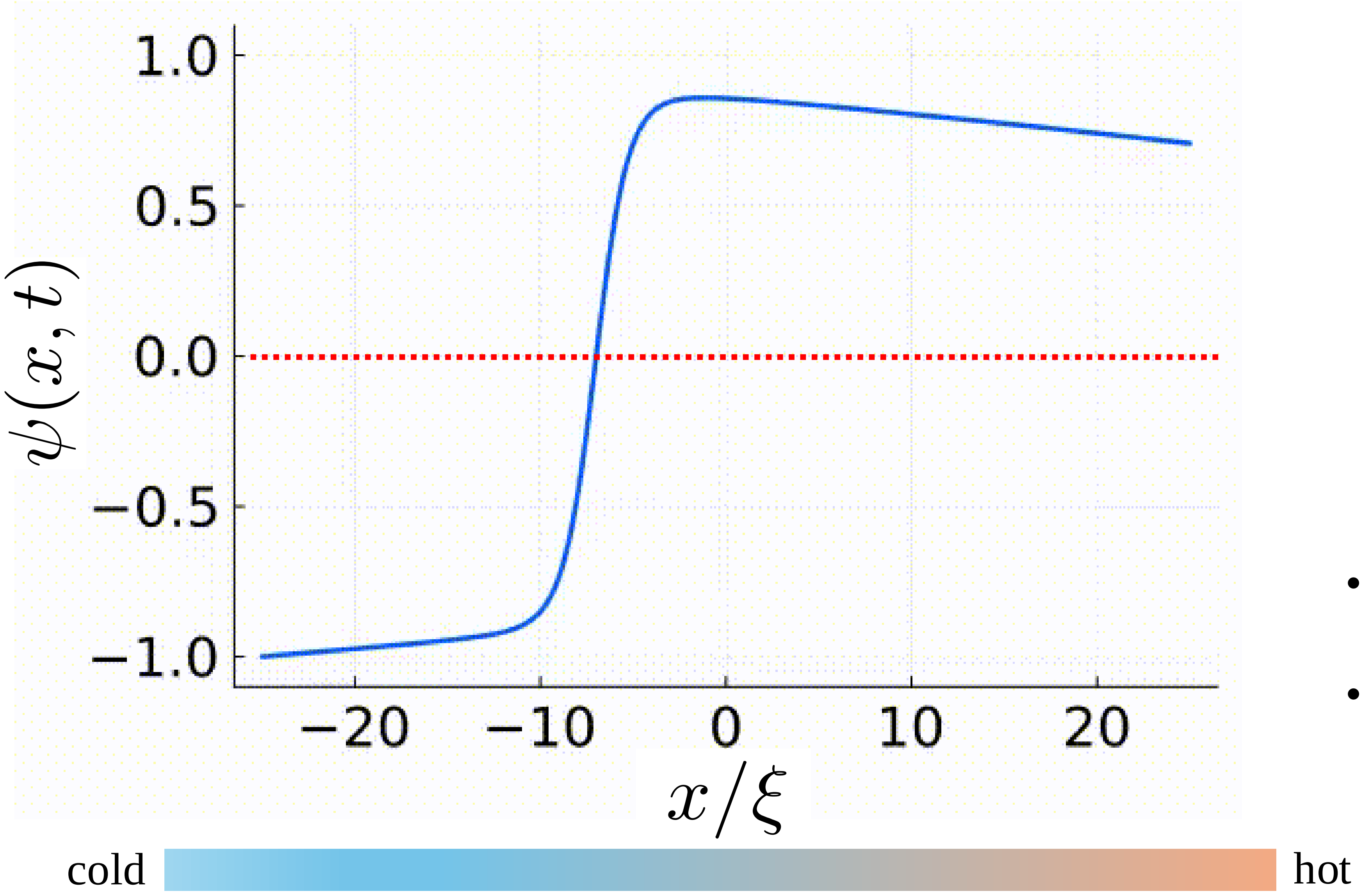
$\psi(x,t)$: order parameter

$\tau(x,t)$: temperature / T_{C}

- One-dimensional <u>finite</u> system defined in $x_{\mathrm{L}} \leq x \leq x_{\mathrm{R}}$
- τ depends on space and time.
- The thermal conductivity depends on $\psi(x,t)$, varying from κ_{s} to κ_{n} $(\kappa_s < \kappa_n)$ $k = \kappa_{\mathrm{s}}/\kappa_{\mathrm{n}}$



Two approaches
(1) Numerical simulation
(2) Analytical calculation



parameters

$\kappa_s/\kappa_n = 1/20$

$\tau_L = 0.990$

$\tau_R = 0.995$

- from colder to hotter region
- consistent with
  the previous research. [10]

[10] E. C. S. Duarte *et al.*, J. Phys.: Condens. Matter **31**, 405901 (2019).

T. Kanakubo *et al.*, arXiv:2405.10200 (2024).

- TDGL Eq. yields the local momentum balance relation as follows:

$$\frac{\partial}{\partial x}\left[-\xi^2\left(\frac{\partial\psi(x,t)}{\partial x}\right)^2-\frac{\psi^2(x,t)}{1-\tau_{\mathrm{L}}}+\frac{\psi^4(x,t)}{2}\right]$$

Driving force density

$$+2\tilde{\gamma}\frac{\partial\psi(x,t)}{\partial t}\frac{\partial\psi(x,t)}{\partial x}+\frac{\tau(x,t)}{1-\tau_{\mathrm{L}}}\frac{\partial\psi^2(x,t)}{\partial x}=0$$

Viscous force density

Thermal force density

[1] L. P. Gor'kov , N. B. Kopnin, Sov. Phys. Usp. **18**, 496 (1975).
[2] A. T. Dorsey: Phys. Rev. B **46**, 8376 (1992) .
[3] N. B. Kopnin, *Theory of Nonequilibrium Superconductivity* (Oxford Univ. Press, Oxford, 2001).
[4] Y. Kato, C-K Chung, J. Phys. Soc. Jpn. **85**, 033703/1-5 (2016).
[5] S. Sugai, N. Kurosawa, Y. Kato, Phys. Rev. B **104**, 064516 (2021).

Linearization

- Strategy: Linearization w.r.t. the velocity of DW[1-3]

$$X(x,t) = X_0(x - vt) + X_1(x - vt) + \mathcal{O}(v^2)$$

$$X = \psi, \tau$$

X_0 : Equilibrium solution

$$\psi_0 = \tanh \frac{x}{\sqrt{2}\xi}$$

$$\tau_0 = \text{const.} = \tau_{\mathrm{L}}$$

X_1 : Modulation component in $\mathcal{O}(v)$

- **Linearized** local momentum balance relation (or TDGL Eq.)

$$-\xi^2 \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x}\frac{\mathrm{d}\psi_1(x)}{\partial x} - \frac{\mathrm{d}^2\psi_0(x)}{\mathrm{d}x^2}\psi_1(x)\right]$$

Driving force density

$$-\tilde{\gamma}v\left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x}\right)^2 + \frac{\tau_1(x)}{2(1-\tau_{\mathrm{L}})}\frac{\mathrm{d}}{\mathrm{d}x}\psi_0^2(x) = 0.$$

Viscous force density

Thermal force density

Force on DW

$$F \equiv \int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} \mathrm{d}x\ f(x)$$

$f(x)$: local force density

- When the system size is much larger than ξ, we have the force balance relation as

$$F_{\text{vis}} + F_{\text{th}} = 0,$$

“Transport entropy” ?

where

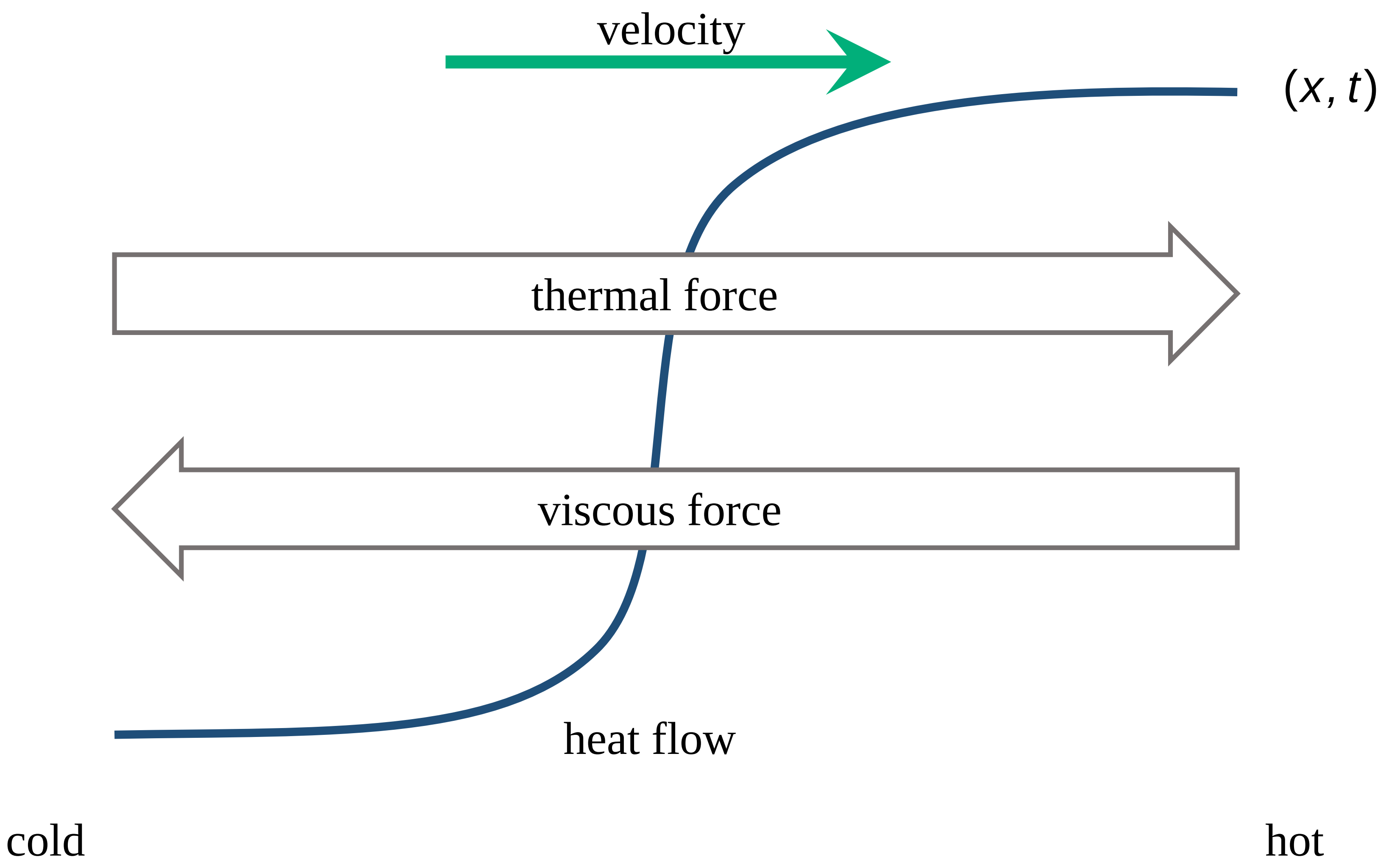
$$F_{\text{vis}} \equiv -\tilde{\gamma} v \int_{x_L}^{x_R} \mathrm{d}x \left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x} \right)^2 \qquad F_{\text{th}} = \frac{1}{2(1-\tau_L)} \int_{x_L}^{x_R} \mathrm{d}x \frac{\mathrm{d}\tau_1(x)}{\mathrm{d}x} \underbrace{(\psi_0^2(x_R) - \psi_0^2(x))}_{\geq 0}$$

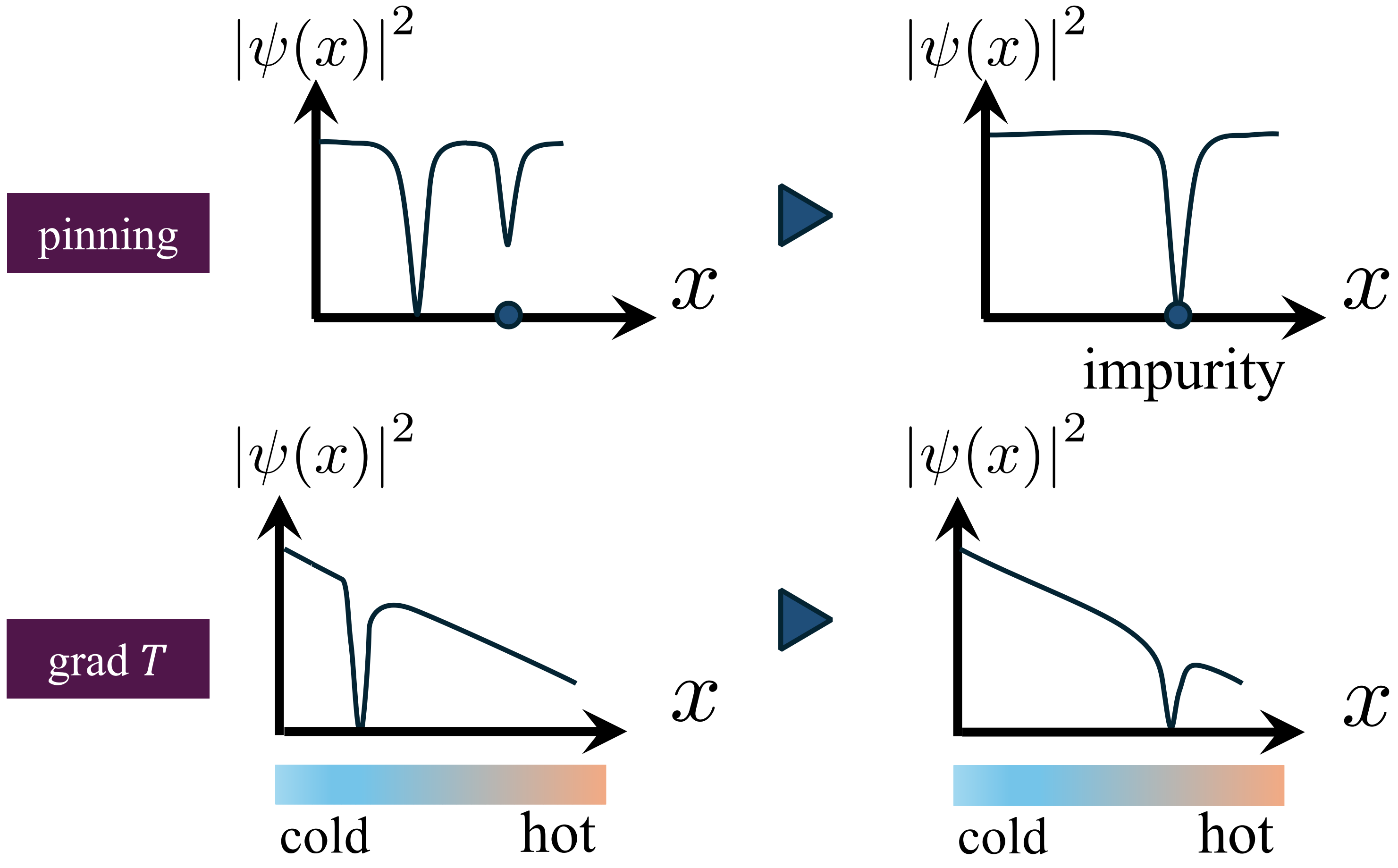
- Analyticl expression for DW velocity

$$v = \frac{\int_{x_L}^{x_R} \mathrm{d}x \frac{\mathrm{d}\tau_1(x)}{\mathrm{d}x} (\psi_0^2(x_R) - \psi_0^2(x))}{2(1-\tau_L)\tilde{\gamma} \int_{x_L}^{x_R} \mathrm{d}x \left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x} \right)^2}$$

- This relation shows the velocity and the thermal force is **against the heat flow**.
- Our thermal force is antiparallel to the thermal force in [6]

[6] M. J. Stephen, Phys. Rev. Lett. **16**, 801 (1966).


velocity
(x, t)
thermal force
viscous force
heat flow
cold
hot



- An energetically preferable distribution (cf. pinning)

(1) DW motion under Temperature Gradient

- Model: Time-dependent Ginzburg-Landau (TDGL) Eq. & Thermal diffusion Eq.
- Results[1]: Numerical Simulation
- Results[2]: Analytical Calculation based on Momentum Balance Relation
- Discussion

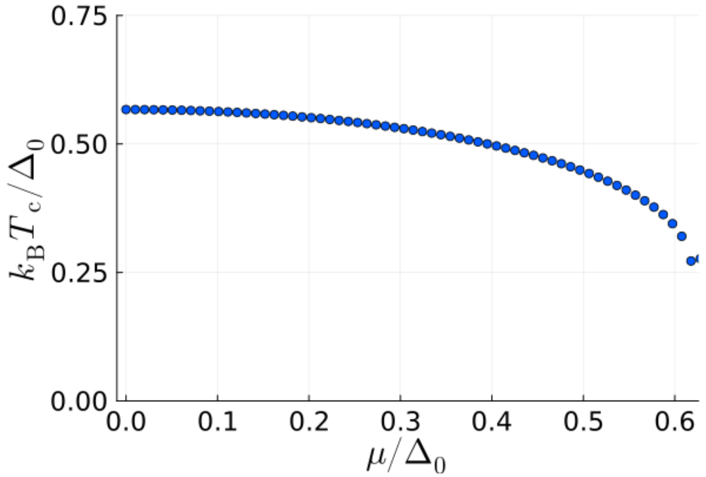
(2) DW motion under Spin density Gradient

- Model: Time-dependent Ginzburg-Landau (TDGL) Eq. & Spin diffusion Eq.
- Results[1]: Numerical Simulation
- Results[2]: Analytical Calculation based on Momentum Balance Relation
- Discussion

$$\begin{cases} -\xi^2 \dfrac{d^2\psi(x)}{dx^2} + \dfrac{T - T_c\left(\mu^2(x,t)\right)}{T_c\left(\mu^2(x,t)\right)} \dfrac{T_c(0)}{T_c(0) - T}\psi(x) + \psi^3(x) = -\tilde{\gamma}\dfrac{\partial \psi(x,t)}{\partial t} \\ \dfrac{\partial}{\partial x}\left[\sigma(x,t)\dfrac{\partial \mu(x,t)}{\partial x}\right] - \dfrac{\mu(x,t)}{\tau^{sp}(x,t)} = \dfrac{\partial \mu(x,t)}{\partial t} \end{cases}$$

Spin relaxation term

$\mu(x,t)$: spin accumulation



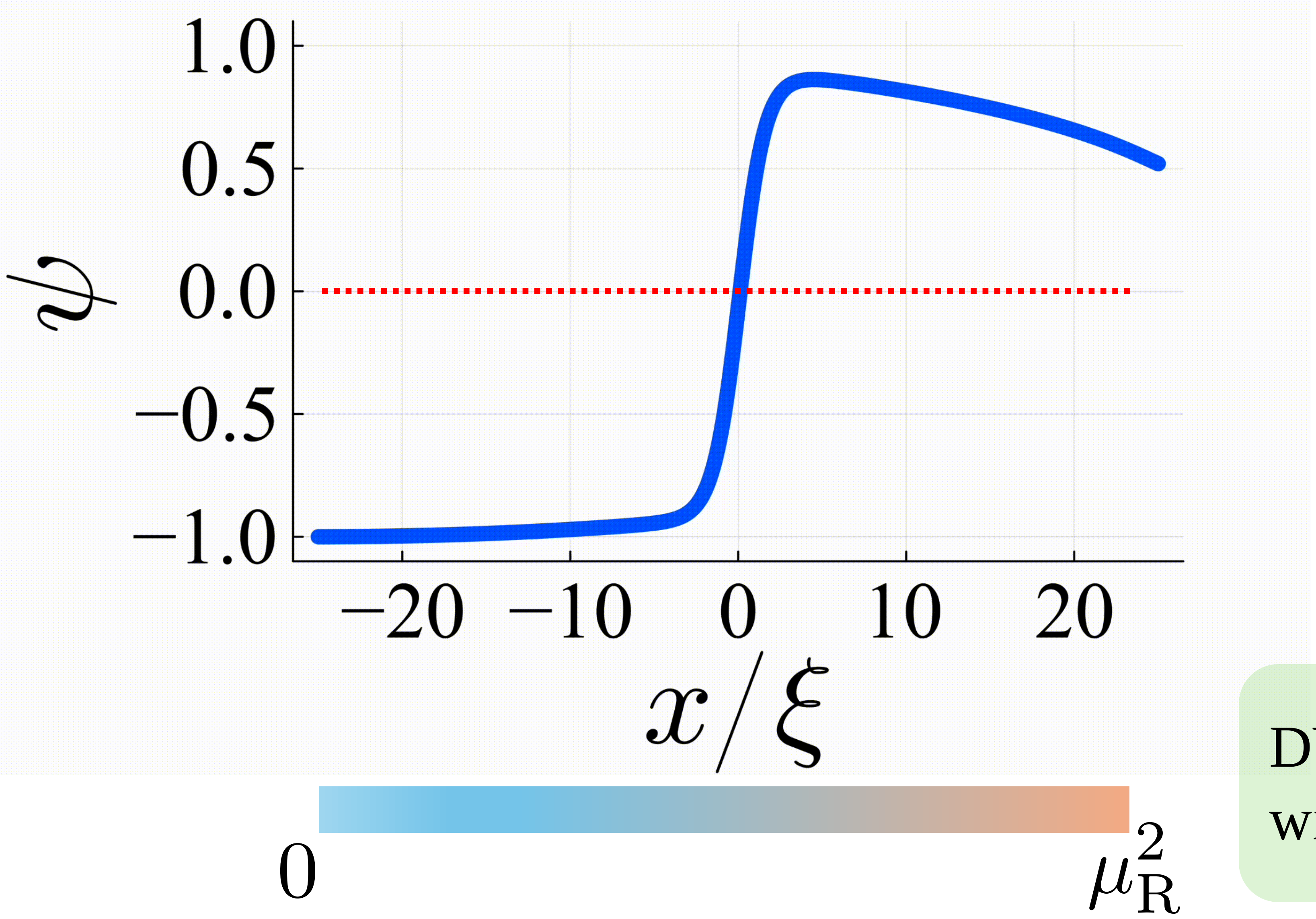
$\sigma(x,t) = \sigma_n + (\sigma_s - \sigma_n)\psi^2(x,t)$: spin conductivity

$\tau^{sp}(x,t) = \tau_n^{sp} + (\tau_s^{sp} - \tau_n^{sp})\psi^2(x,t)$: relaxation time

Boundary Conditions

$$\begin{cases} \mu(x_L,t) = 0 \\ \mu(x_R,t) = \mu_R \end{cases}$$

- $\sigma(x,t)$ and $\tau^{sp}(x,t)$ depend on $\psi(x,t)$ (same as $\kappa(x,t)$)
- T_C depends on $\mu^2(x,t)$
  →The linearization method is insufficient



parameters

$$T < T_{\rm c}(\mu_{\rm R}^2) < T_{\rm c}(0)$$

$$T = 0.990 T_{\rm c}(\mu_{\rm R}^2)$$

$$T_{\rm c}(\mu_{\rm R}^2) \simeq 0.973 T_{\rm c}(0)$$

$$\sigma_{\rm s}/\sigma_{\rm n} = \tau_{\rm s}/\tau_{\rm n} = 2$$

DW moves to the region with larger spin accumulation

Linearized Eqs.

- Linearized local momentum balance relation

$$b \equiv \frac{T}{T_c(0)(T_c(0)-T)} \left. \frac{dT_c(\mu^2)}{d\mu^2} \right|_{\mu^2=0} < 0$$

$$-\xi^2 \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x}\frac{\mathrm{d}\psi_1(x)}{\mathrm{d}x} - \frac{\mathrm{d}^2\psi_0(x)}{\mathrm{d}x^2}\psi_1(x)\right]$$

driving force

$$-\tilde{\gamma}v\left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x}\right)^2 - \frac{b}{2}\mu_1^2(x)\frac{\mathrm{d}}{\mathrm{d}x}\psi_0^2(x) = 0$$

viscous force

“spin density gradient force”

cf. temperature gradient case

$$-\xi^2 \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x}\frac{\mathrm{d}\psi_1(x)}{\partial x} - \frac{\mathrm{d}^2\psi_0(x)}{\mathrm{d}x^2}\psi_1(x)\right]$$

$$-\tilde{\gamma}v\left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x}\right)^2 + \frac{\tau_1(x)}{2(1-\tau_{\mathrm{L}})}\frac{\mathrm{d}}{\mathrm{d}x}\psi_0^2(x) = 0.$$

Forces on DW

- When the system size is much larger than ξ, we have the force balance relation as

$$F_{\text{vis}} + F_{\text{spin}} = 0,$$

* We can prove $0 \leq \frac{\mathrm{d}\mu_1^2(x)}{\mathrm{d}x}$

where

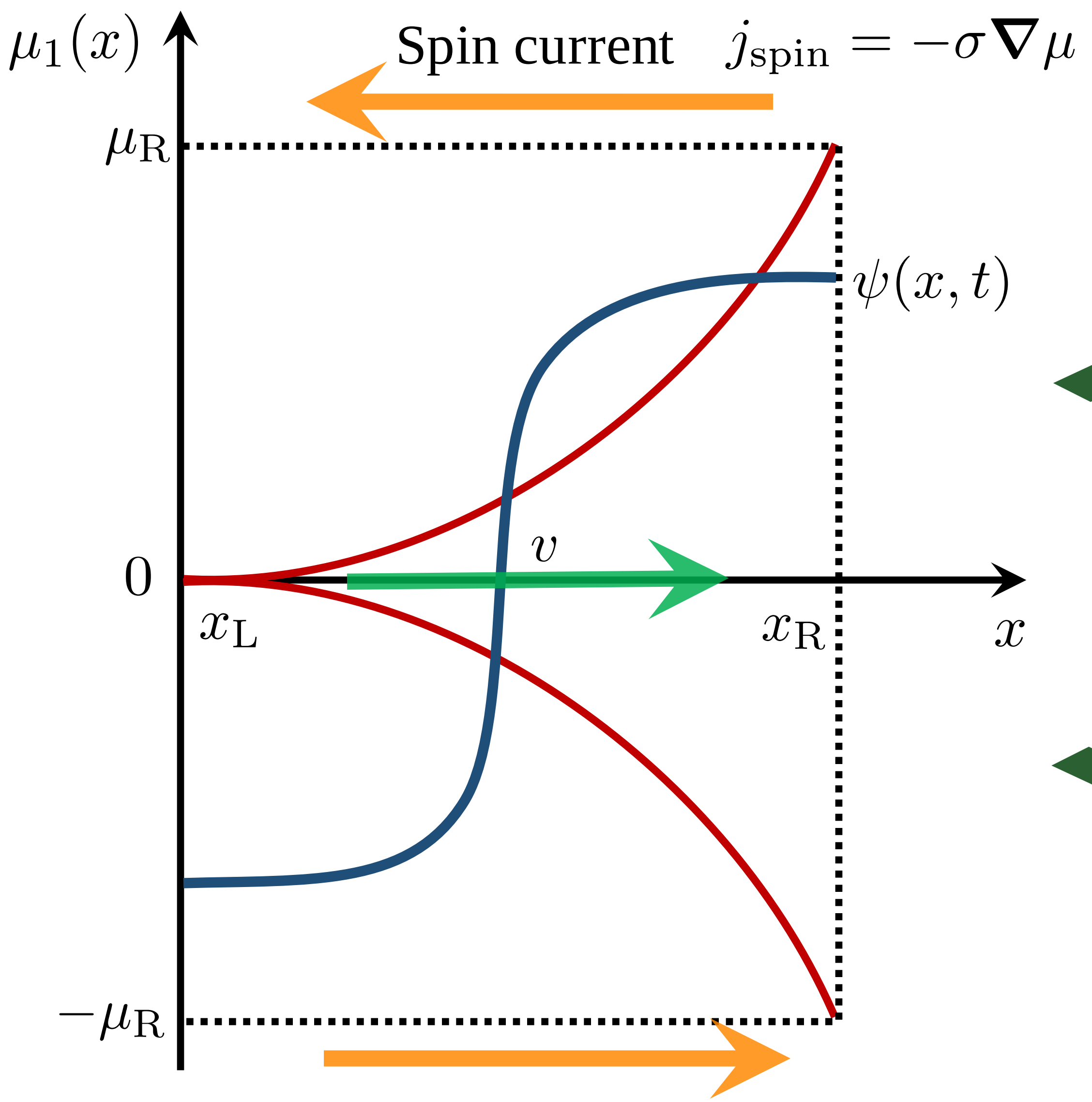
$$F_{\text{vis}} \equiv -\tilde{\gamma} v \int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathrm{d}x \left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x} \right)^2 \qquad F_{\text{spin}} = -\frac{b}{2} \int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathrm{d}x \ \left(\psi_0^2(x_\mathrm{R}) - \psi_0^2(x)\right) \frac{\mathrm{d}\mu_1^2(x)}{\mathrm{d}x}$$

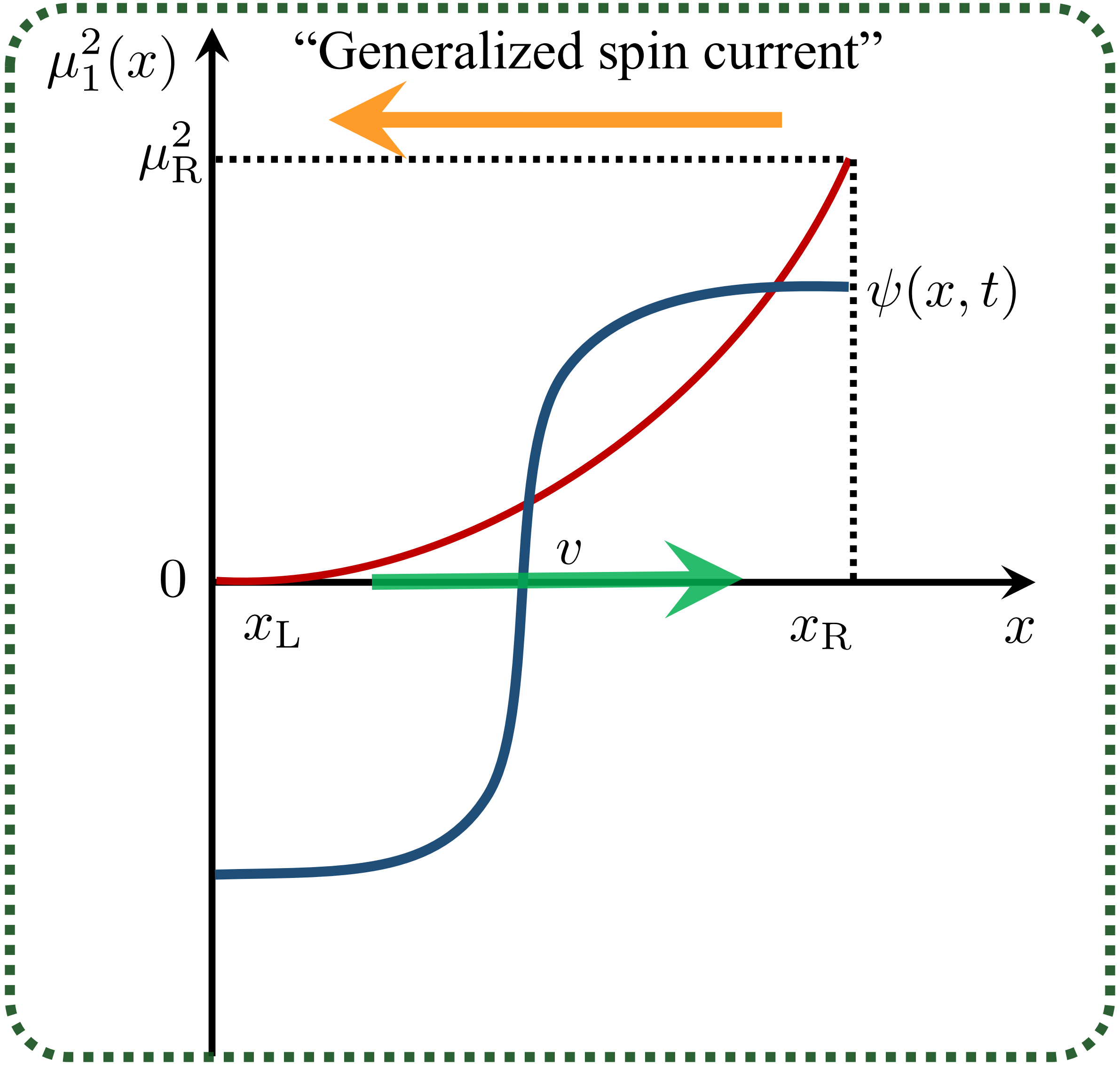
- Analyticl expression for DW velocity

$$v = \frac{-b \int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathrm{d}x \ \frac{\mathrm{d}\mu_1^2(x)}{\mathrm{d}x} \left(\psi_0^2(x_\mathrm{R}) - \psi_0^2(x)\right)}{2\tilde{\gamma} \int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathrm{d}x \left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x} \right)^2}$$

cf. temperature gradient case

$$v = \frac{\int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathrm{d}x \frac{\mathrm{d}\tau_1(x)}{\mathrm{d}x} \left(\psi_0^2(x_\mathrm{R}) - \psi_0^2(x)\right)}{2(1-\tau_\mathrm{L})\tilde{\gamma} \int_{x_\mathrm{L}}^{x_\mathrm{R}} \mathrm{d}x \left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x} \right)^2}$$





Spin current does not interact with DW

(1) DW motion under Temperature Gradient

- Model: Time-dependent Ginzburg-Landau (TDGL) Eq. & Thermal diffusion Eq.
- Results[1]: Numerical Simulation
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- Discussion

(2) DW motion under Spin density Gradient

- Model: Time-dependent Ginzburg-Landau (TDGL) Eq. & Spin diffusion Eq.
- Results[1]: Numerical Simulation
- Results[2]: Analytical Calculation based on Momentum Balance Relation
- Discussion

(3) Vortex motion under Temperature Gradient

- Comparison between the expression of the velocity for the vortex and DW

$$v = \frac{\int_{\mathrm{S}} \mathrm{d}\boldsymbol{r}\ \frac{\partial \tau_1(\boldsymbol{r})}{\partial x}\left(f_0^2(\boldsymbol{R}) - f_0^2(\boldsymbol{r})\right)}{2(1-\tau_{\mathrm{L}})\int_{\mathrm{S}} \mathrm{d}\boldsymbol{r}\left(\gamma_1\left(\frac{\partial f_0(\boldsymbol{r})}{\partial x}\right)^2 + \sigma_{\mathrm{n}}\left|\frac{\partial}{\partial x}\boldsymbol{Q}_0(\boldsymbol{r})\right|^2\right)}$$

$$v_{\mathrm{DW}} = \frac{\int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} \mathrm{d}x \frac{\mathrm{d}\tau_1(x)}{\mathrm{d}x}\left(\psi_0^2(x_{\mathrm{R}}) - \psi_0^2(x)\right)}{2(1-\tau_{\mathrm{L}})\tilde{\gamma}\int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} \mathrm{d}x\ \left(\frac{\mathrm{d}\psi_0(x)}{\mathrm{d}x}\right)^2}$$

T. Kanakubo *et al.*, arXiv:2405.10200 (2024).

- Model: TDGL Eq. and thermal/spin diffusion Eq.
- Analytical expression of the velocity and the thermal force was derived.
- DW moves to the higher temperature/spin-accumulation region.
- Physical picture: pinning phenomenon.