Phase transition on a quantum vortex

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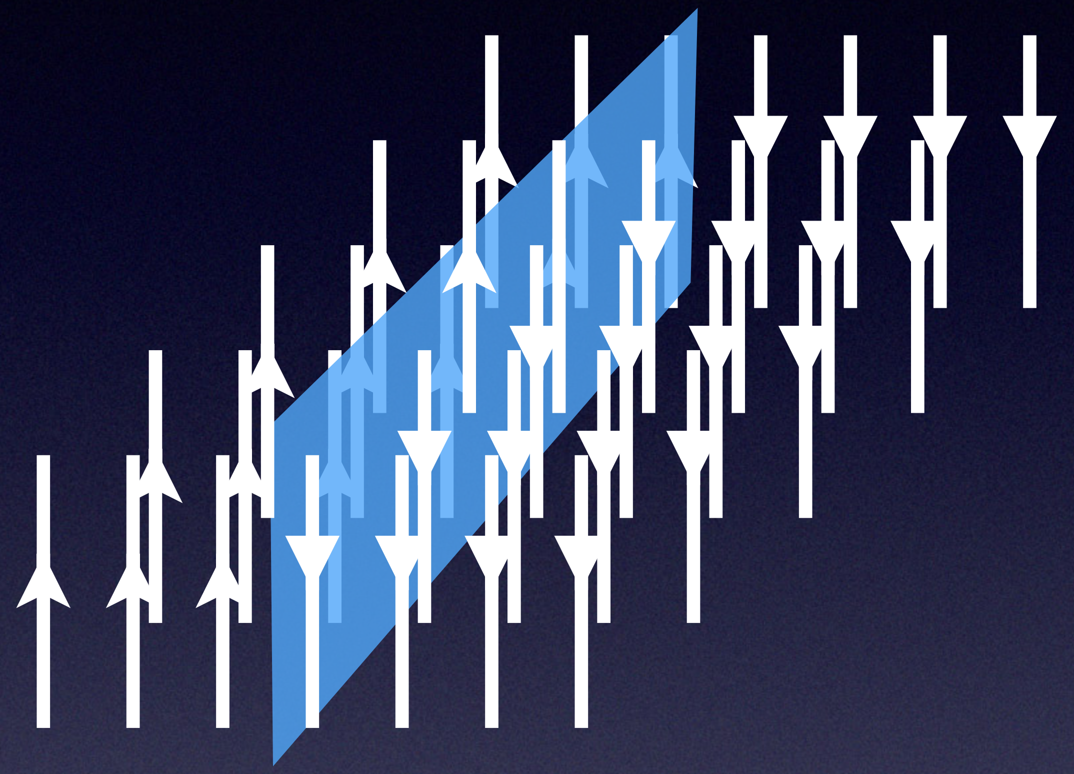
based on arXiv: 2411.03676

Does a phase transition on a topological defect occurs, while the bulk has no phase transition?

Domain wall



vortex

Our answer is YES!

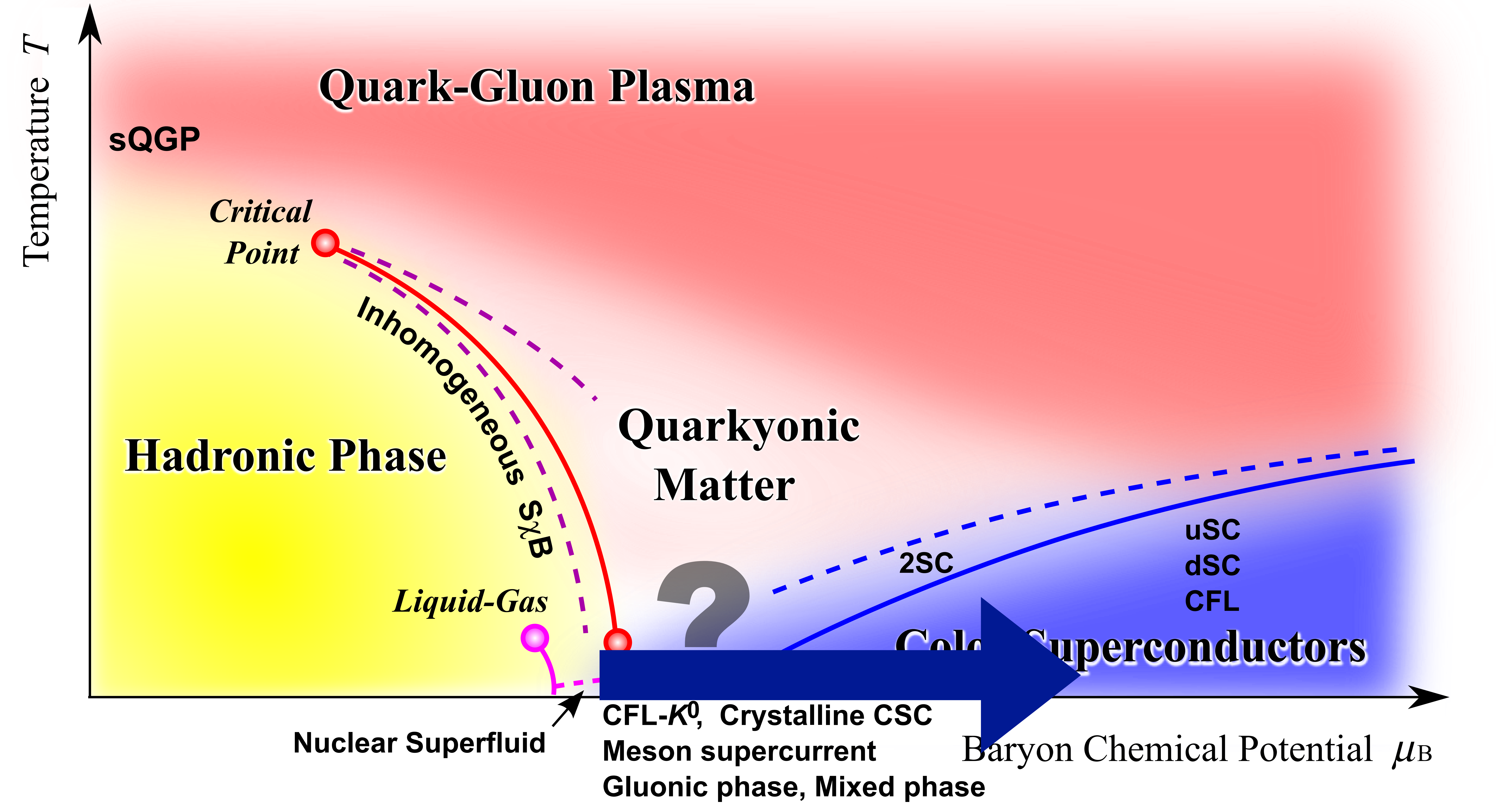
Effective theory on a topological defect=

a lower-dimensional field theory may exhibit phase transition

Phase transitions may occur in quantum vortices.

Motivation: QCD phase diagram

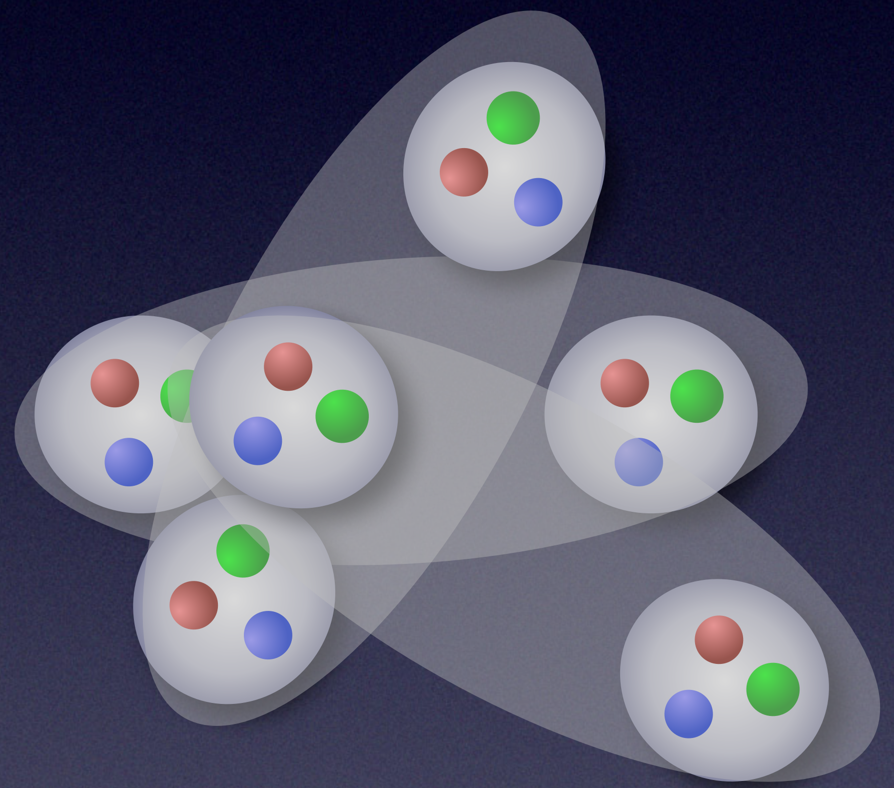
Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011) 014001



What we know

For 3-flavor QCD : $G = SU(3)_f \times U(1)_B$

- **Superfluid(dilute phase)**

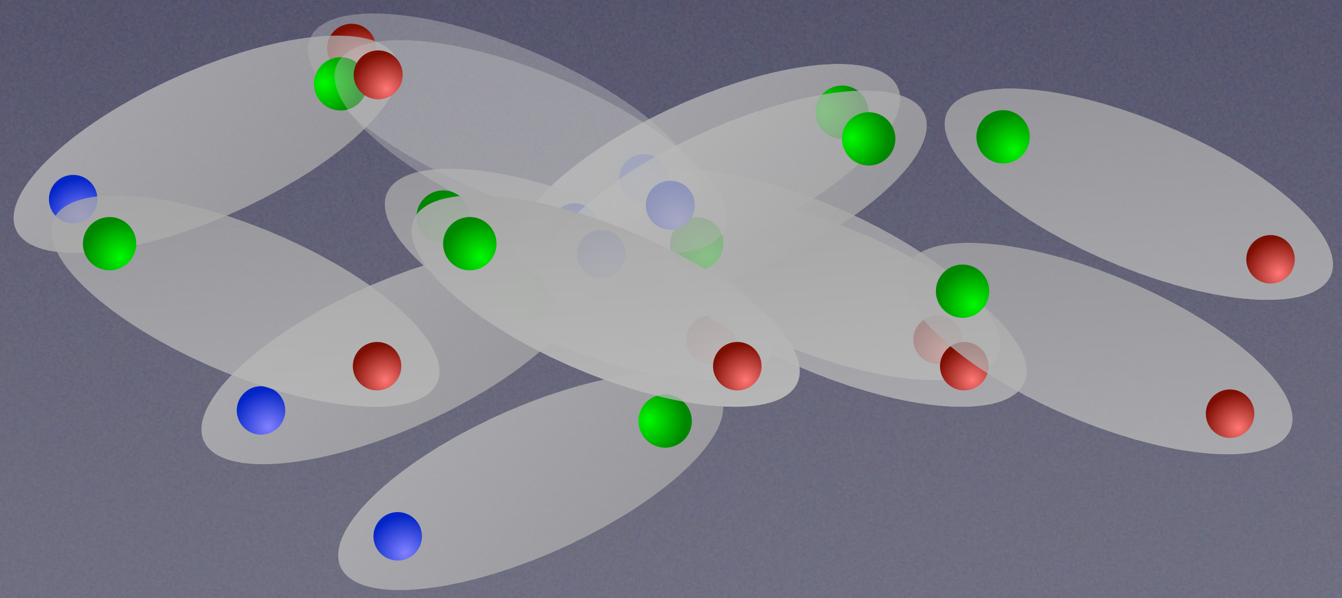


Baryon pair condensation

$$\Delta = \langle \Lambda\Lambda \rangle \neq 0 \qquad \Lambda \sim uds$$

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

- **Color super conductor (dense phase)**



"quark pair condensate"

$$(\Phi_L)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_L)^b_j (Cq_L)^c_k \rangle = -\epsilon^{ijk}\epsilon_{abc}\langle (q_R)^b_j (Cq_R)^c_k \rangle$$

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

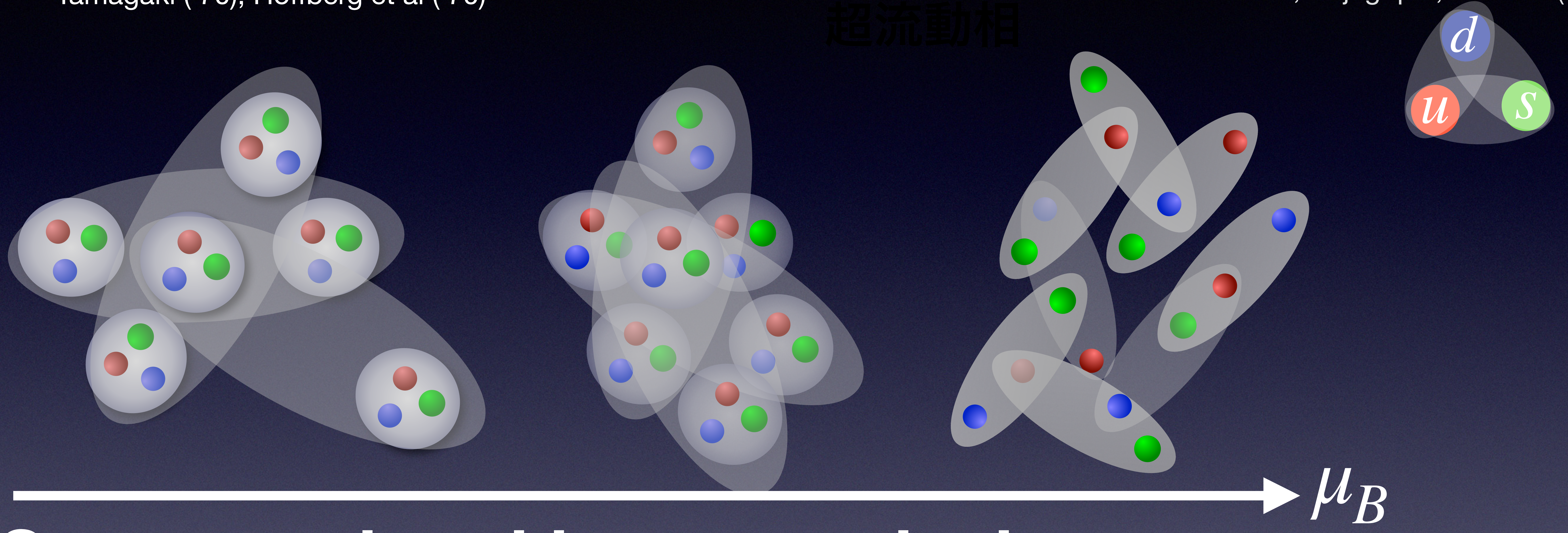
Quark hadron continuity

Hadronic superfluid

Tamagaki ('70), Hoffberg et al ('70)

Color flavor locked phase (CFL phase)

Alford, Rajagopal, Wilczek ('99)



Symmetry breaking patter is the same

⇒Quark hadron continuity

Excitations

Baryons ⇒ Quarks

Vector meson ⇒ Gluons

cf. Hatsuda, Tachibana, Yamamoto, Baym ('06)

Thought experiment : rotating neutron stars

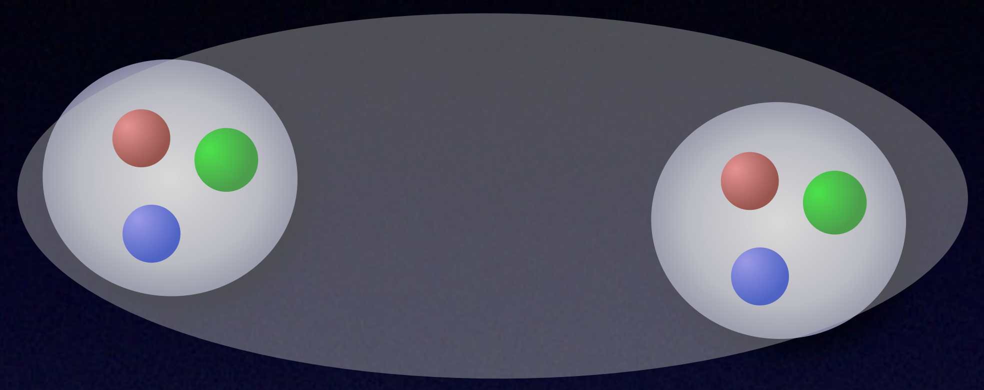
Quantum vortex

Consider continuity of vortices

- Circulation
- Emergent symmetry

Hadronic superfluid phase

di-baryons condense



$$\Delta = \langle \Lambda\Lambda \rangle \neq 0 \quad \Lambda \sim uds$$

Symmetry breaking pattern

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

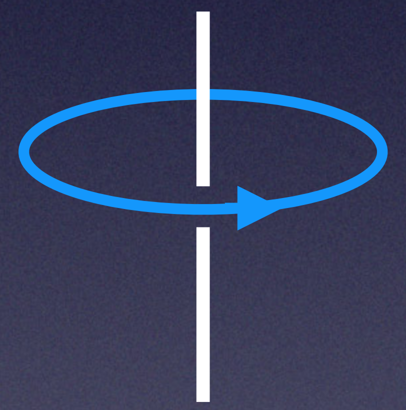
Topological excitation: U(1) vortex $\pi_1(U(1)_B) = \mathbb{Z}$

$$\phi = \Delta f(r) e^{i\theta} \quad \int \frac{d\theta}{2\pi} \in \mathbb{Z} \quad f \xrightarrow{r \to 0} 0 \quad f \xrightarrow{r \to \infty} 1$$

Quantum number in Hadronic superfluid phase

Global $U(1)_B$ symmetry is broken

U(1) vortex: topological defect $\Delta e^{i\theta}$

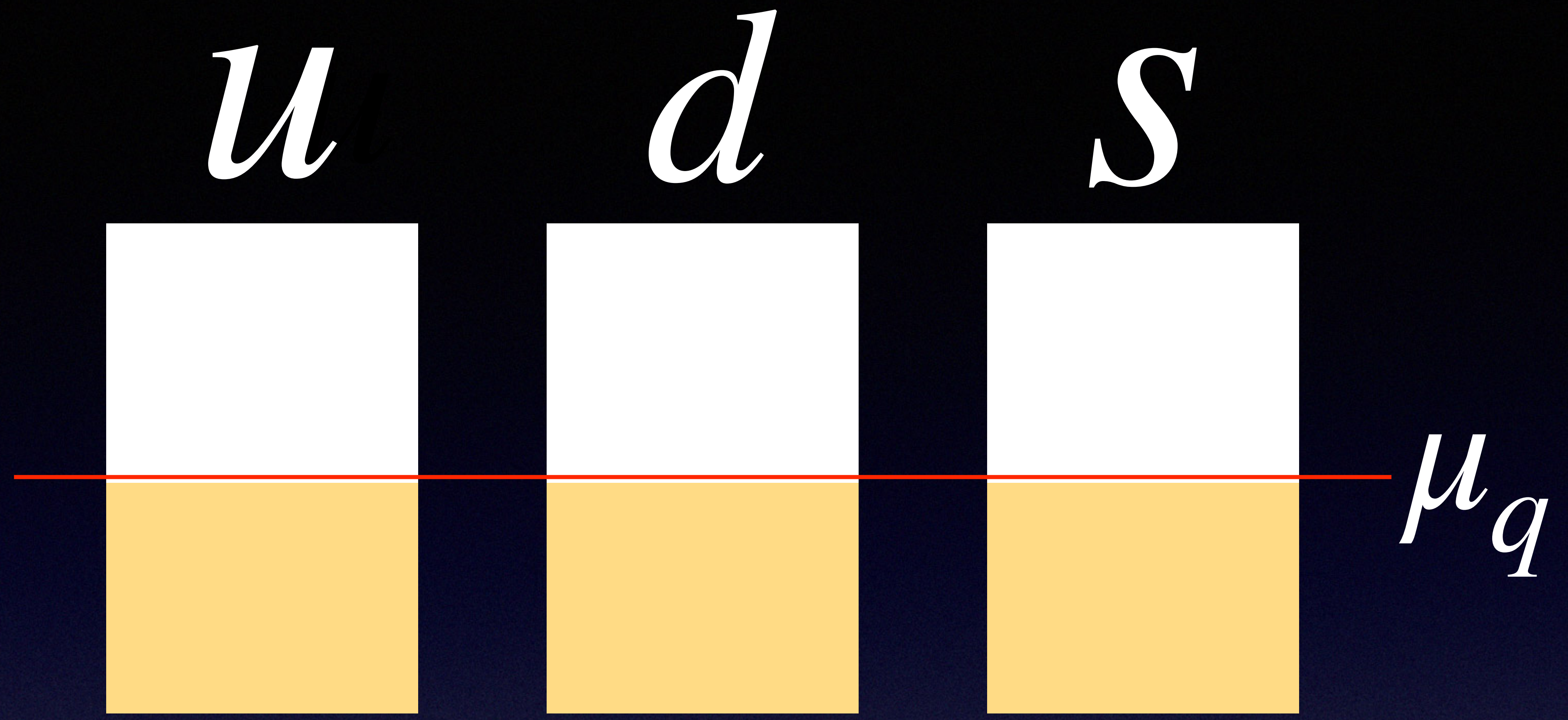


Circulation: $\int v = \int \frac{d\theta}{2\mu_B} = \frac{2\pi\nu_B}{2\mu_B}$

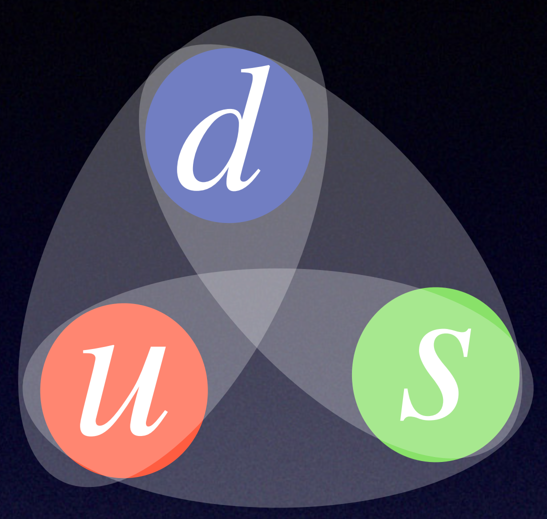
$\nu_B = \int \frac{d\theta}{2\pi}$: **Winding number**

$2\mu_B$: **Baryon chemical potential of order parameter**

Color-flavor locking phase



quark pair



$$(\Phi_L)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_L)^b_j (Cq_L)^c_k \rangle \quad (\Phi_R)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_R)^b_j (Cq_R)^c_k \rangle$$

$$\Phi := \Phi_L = -\Phi_R = \begin{pmatrix} \Delta_{\mathrm{CFL}} & 0 & 0 \\ 0 & \Delta_{\mathrm{CFL}} & 0 \\ 0 & 0 & \Delta_{\mathrm{CFL}} \end{pmatrix}$$

Topological excitations

cf. Eto, Hirono, Nitta & Yasui, PTEP 2014, 012D01 (2014)

order parameter space $G/H \simeq \frac{SU(3)_c \times U(1)_B}{\mathbb{Z}_3} \simeq U(3)$

U(1) vortex

$$\Phi := \Delta_{\mathrm{CFL}} \begin{pmatrix} e^{i\theta} f(r) & 0 & 0 \\ 0 & e^{i\theta} f(r) & 0 \\ 0 & 0 & e^{i\theta} f(r) \end{pmatrix}$$

Non-abelian CFL vortex

Balachandran, Digal, Matsuura, PRD73, 074009 (2006)

$$\Phi := \Delta_{\mathrm{CFL}} \begin{pmatrix} e^{i\theta} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix} = \Delta_{\mathrm{CFL}} e^{i\frac{\theta}{3}} \begin{pmatrix} e^{i\frac{2\theta}{3}} f(r) & 0 & 0 \\ 0 & e^{-i\frac{\theta}{3}} g(r) & 0 \\ 0 & 0 & e^{-i\frac{\theta}{3}} g(r) \end{pmatrix}$$

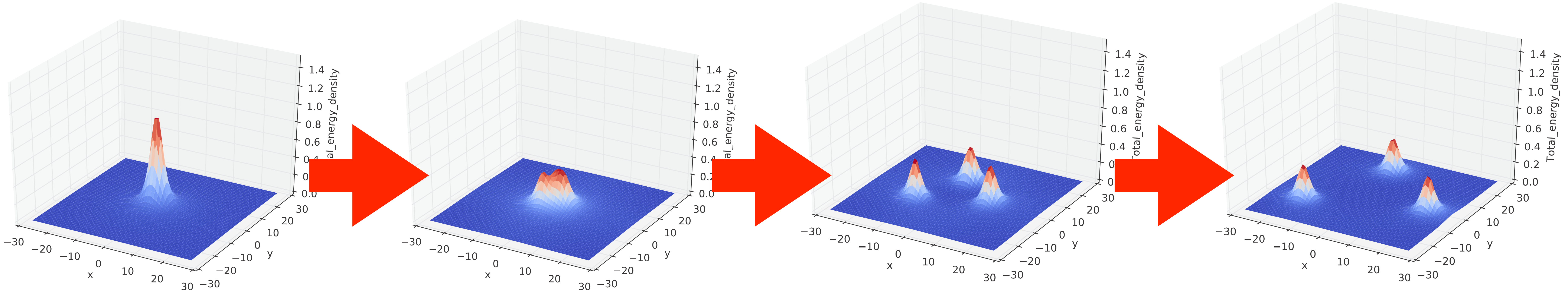
$$A_i = -\frac{\epsilon_{ij} x^j}{g_s^2 r^2}(1 - h(r))\mathrm{diag}\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$$ **both superfluidity and superconductivity**

Numerical Simulation

Alford, Mallavarapu, Vachaspati, Windisch, PRC 93, 045801 (2016)

U(1) vortex

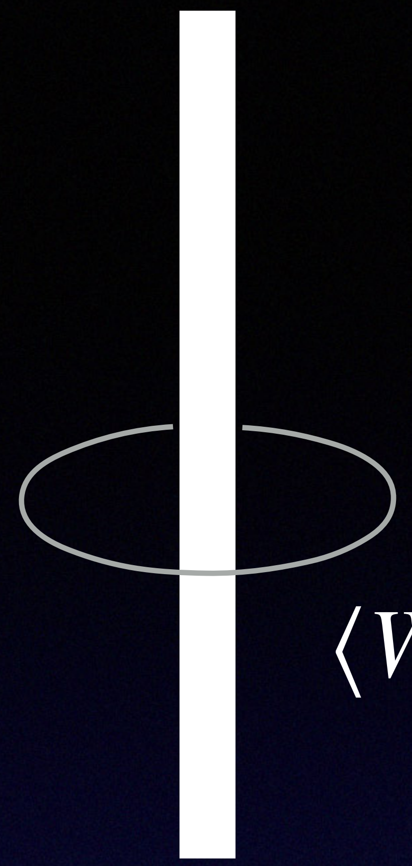
non-abelian vortices



(a) (b) (c) (d)

U(1) vortex decays into three non-abelian vortices

U(1) vortex in Hadronic phase



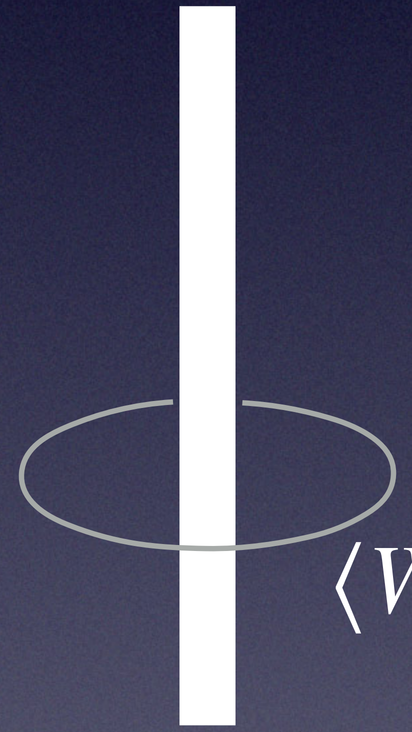
$\langle W \rangle = |\langle W \rangle|$

Alford, Baym, Fukushima, Hatsuda, Tachibana ('19)

Circulation $\boxed{2\pi \frac{\nu_B}{2\mu_B}}$

ν_B: Winding number

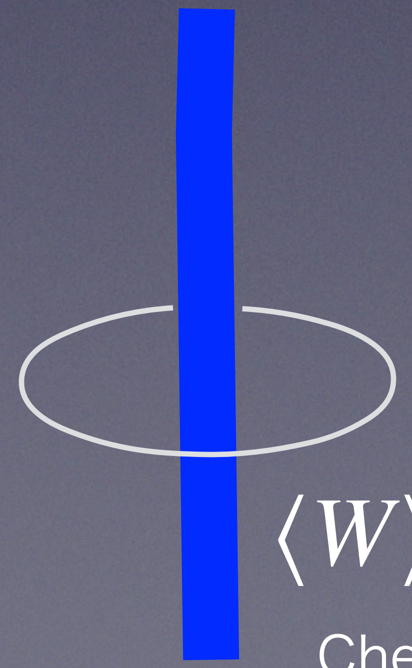
U(1) vortex in CFL



$\langle W \rangle = |\langle W \rangle|$

Circulation $2\pi \frac{\nu_A}{2\mu_q} = 2\pi \frac{3\nu_A}{2\mu_B}$

Non-abelian vortex in CFL



$\langle W \rangle = e^{i\frac{2\pi\nu_A}{3}} |\langle W \rangle|$

Cherman, Sen, Yaffe, PRD 100, 034015 (2019)

Circulation $\frac{2\pi\nu_A/3}{2\mu_q} = \boxed{2\pi \frac{\nu_A}{2\mu_B}}$

Topological ordered phase?

CFL vortex: emergent $\mathbb{Z}_3^{[2]}$ symmetry

However, it is not unbroken, i.e. not topological order

Hirono, Tanizaki ('19)

What is the fate of $e^{\frac{2\pi}{3}i}$?

The magnetic flux will not penetrate through the vortices
in the hadronic phase
$\Rightarrow$ This allow us to distinguish the phases.

Cherman, Jacobson, Sen, Yaffe ('20), ('24)

Magnetic flux may penetrate through the vortices
in the hadronic phase or dissipate during the transition.

Hayashi ('23)

Outline

- Phase transition on a vortices
- Summary

Phase transition on a vortices

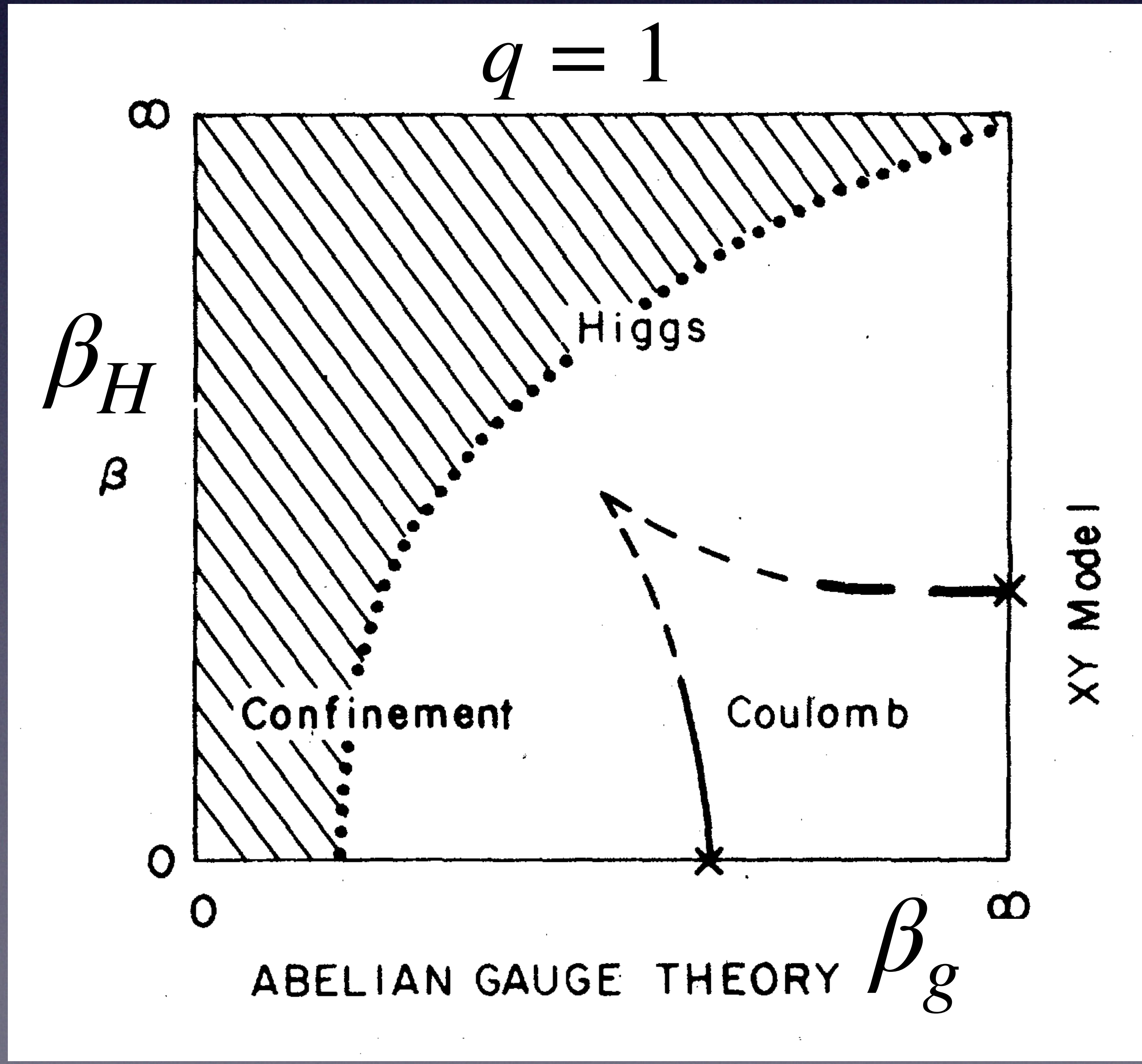
Abelian Higgs model in (3+1) dimensions

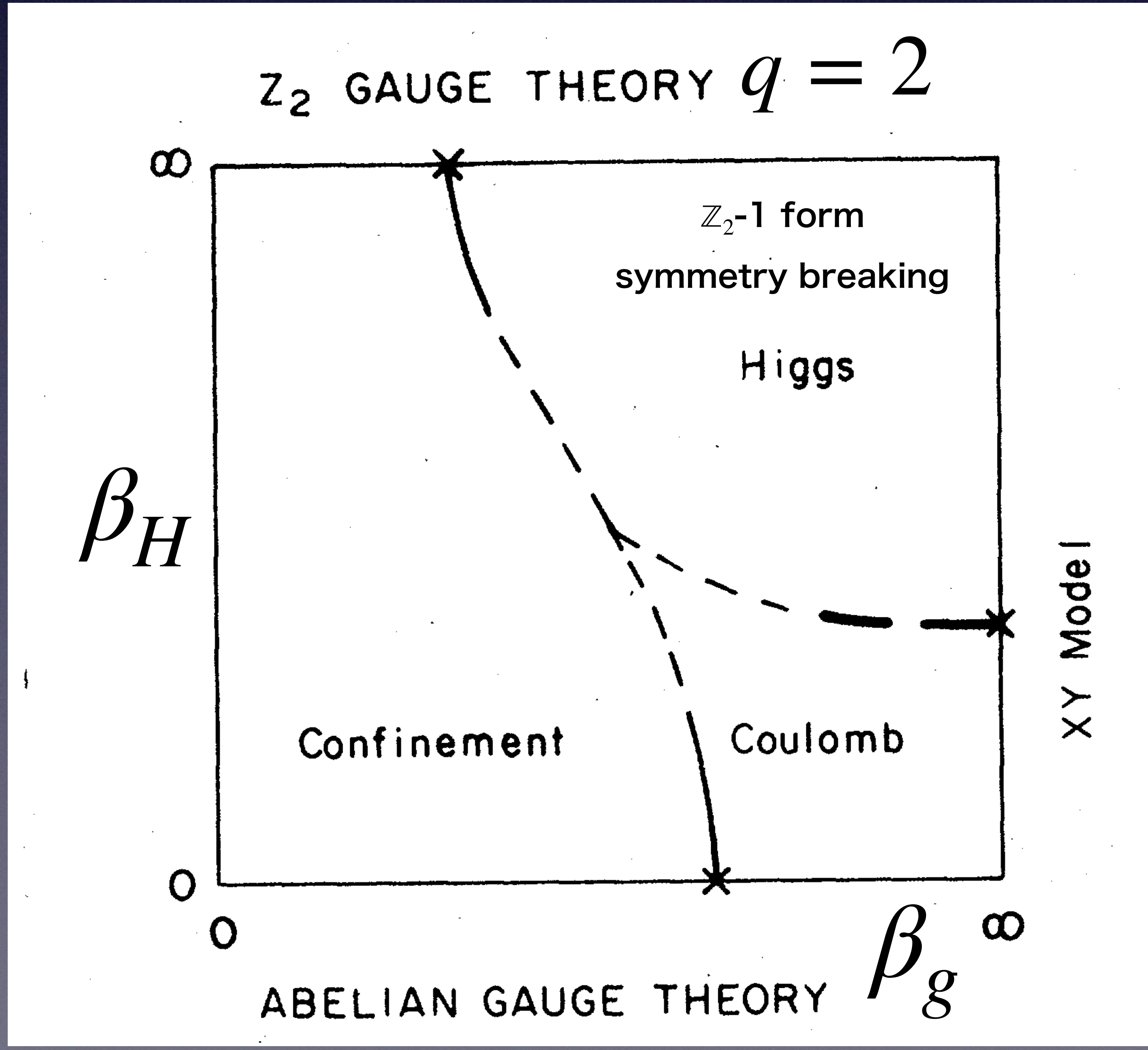
$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \cos\left(\Delta_\mu \varphi(x) - qA_\mu(x)\right)$$

Field strength **Scalar field** **(phase dof)** **Gauge field**

$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$

Fradkin-Schenker Phys. Rev. D 19, 3682 ('79)





$\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ lattice model

cf. Motrunich, Senthil ('05)

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Field strength

Scalar field (phase dof)

Gauge field

$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$

Symmetry

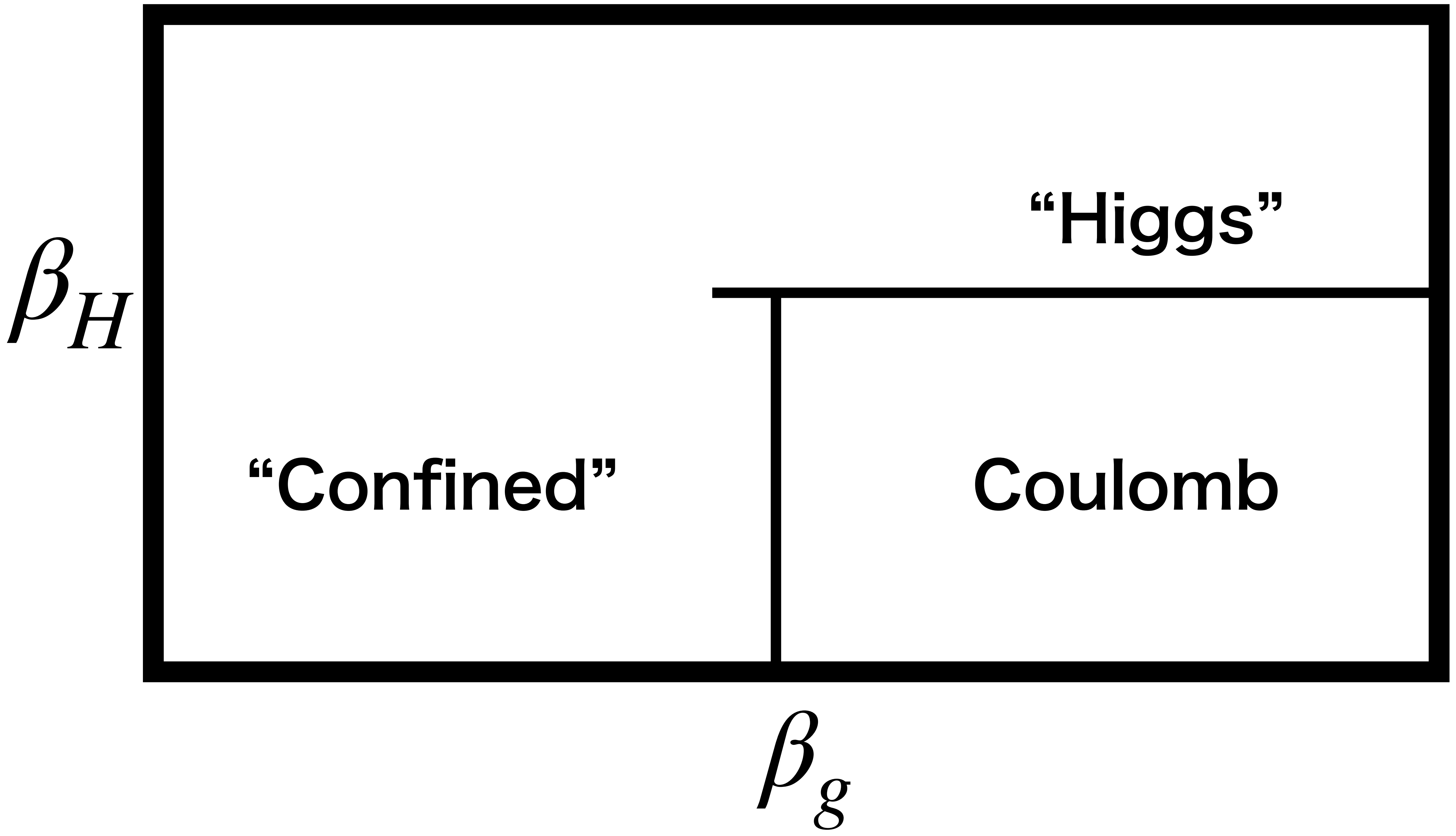
$$\mathrm{U}(1)_{\mathrm{gauge}} : \begin{array}{l} \varphi_1 \to \varphi_1 - \lambda \\ \varphi_2 \to \varphi_2 - \lambda \\ A_\mu \to A_\mu + \Delta_\mu \lambda \end{array}$$

$$\mathrm{U}(1)_{\mathrm{global}} : \begin{array}{l} \varphi_1 \to \varphi_1 + \theta \\ \varphi_2 \to \varphi_2 - \theta \end{array}$$

$$\mathbb{Z}_{2F} : \begin{array}{l} \varphi_1 \to \varphi_2 \\ \varphi_2 \to \varphi_1 \end{array}$$

Phase diagram

Fradkin-Schenker



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Field strength — **Scalar field (phase dof)** — **Gauge field**

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Symmetry

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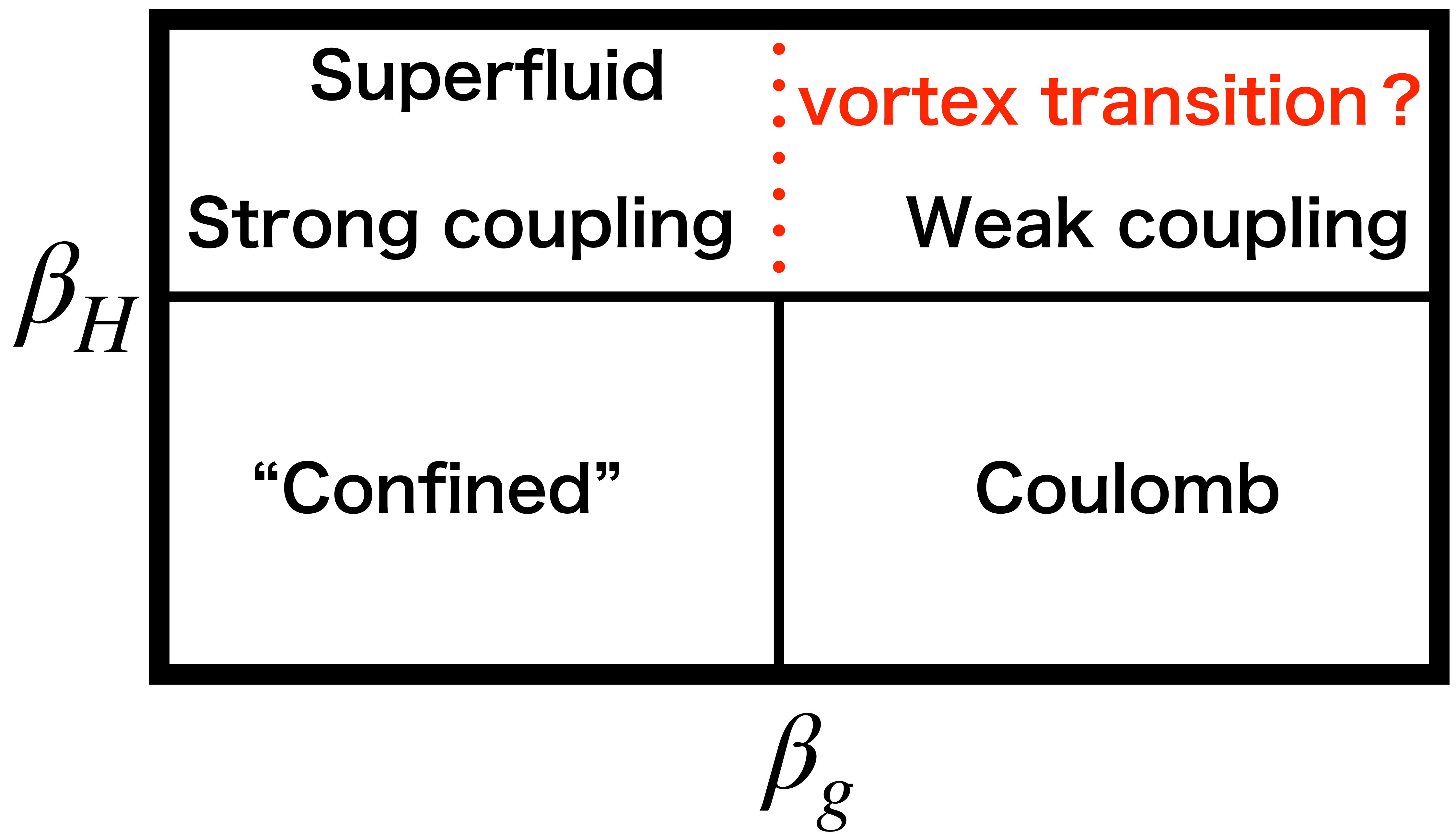
Phase diagram



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Field strength **Scalar field (phase dof)** **Gauge field**

$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$

Emergent symmetry at large β_H (SSB of $U(1)_{\mathrm{global}}$)

YH, Kondo ('22)

Emergent	$U(1)^{[2]}$	$\mathbb{Z}_2^{[2]}$
Symmetry operator	$e^{i\frac{\theta}{2\pi}\int_C (d\varphi_1 - d\varphi_2)}$	$e^{i\frac{1}{2}\int_C (d\varphi_1 + d\varphi_2)}$

Example: $\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ lattice model

Strong coupling $\beta_g \ll 1$

Integrating over gauge fields

$$S_{\mathrm{eff}} = -\sum_{x,\mu} \ln I_0 \left[2\beta_H \cos\left(\frac{\Delta_\mu \varphi_1(x) - \Delta_\mu \varphi_2(x)}{2} \right) \right]$$

$I_0(z)$:Modified Bessel

Essential d.o.f. is $\varphi_1 - \varphi_2$

i.e., one d.o.f.

Weak coupling $\beta_g \gg 1$

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Distinguishable φ_1 and φ_2

$\mathbb{Z}_{2F}$ is spontaneously broken on the vortices

Criterion of symmetry breaking:

When discrete symmetry is broken: twisting the boundary conditions by the symmetry causes the formation of domain walls

Example: Ising model

$\mathbb{Z}_2$ **broken phase**

↑↑↑↑↑↑|↓↓↓↓↓↓

domain wall

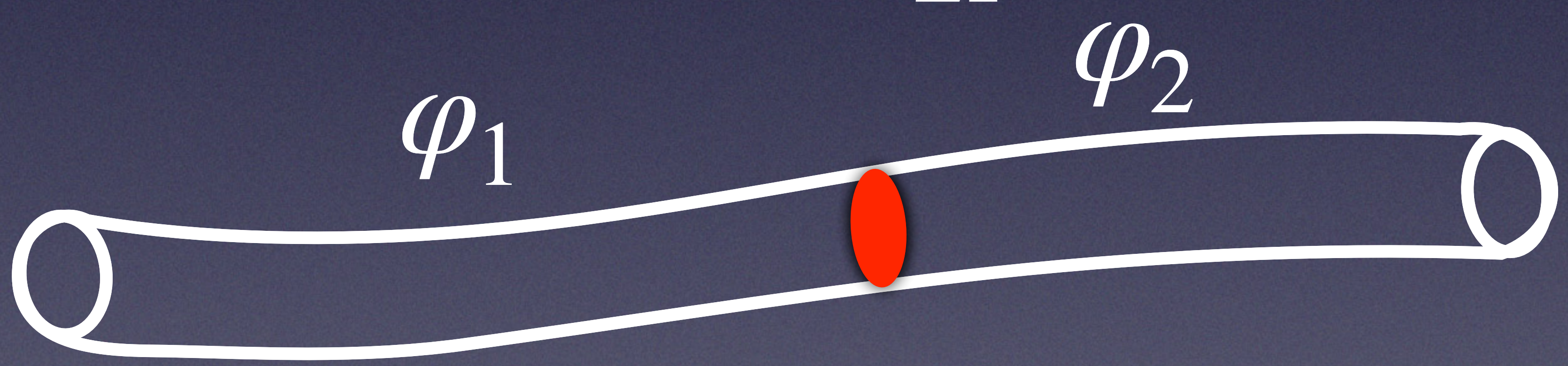
$\mathbb{Z}_2$ **unbroken phase**

random configuration

↑↓↑↑↓↑|↑↓↑↓↓↓

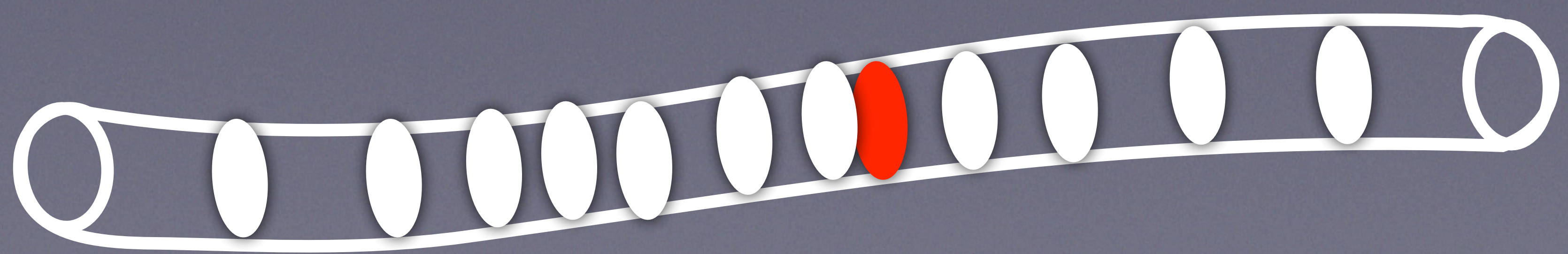
$\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ **model**

Weak coupling ($\mathbb{Z}_{2F}$ broken)

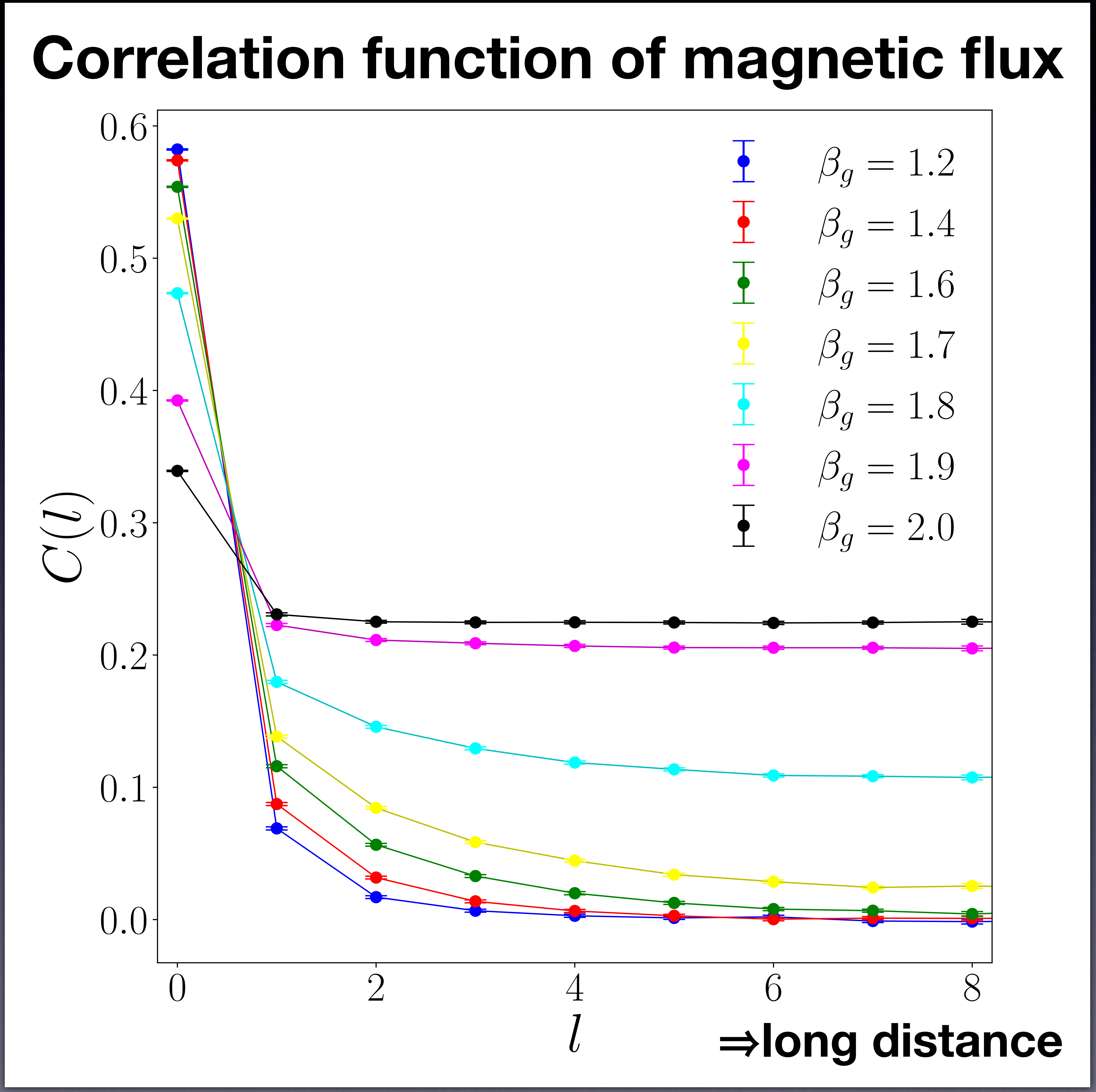


Strong coupling ($\mathbb{Z}_{2F}$ unbroken)

randomized junctions



Numerical simulation



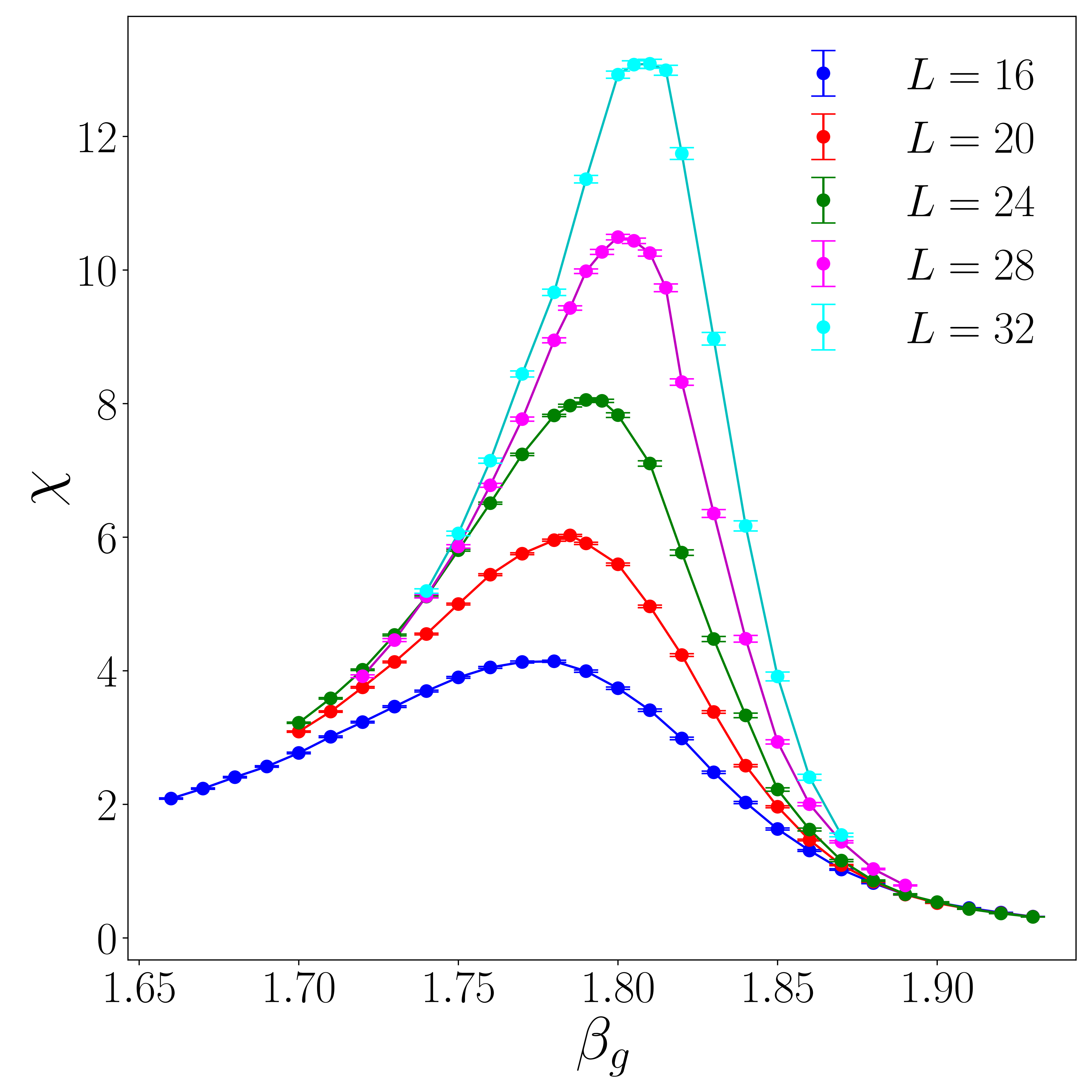
vortex

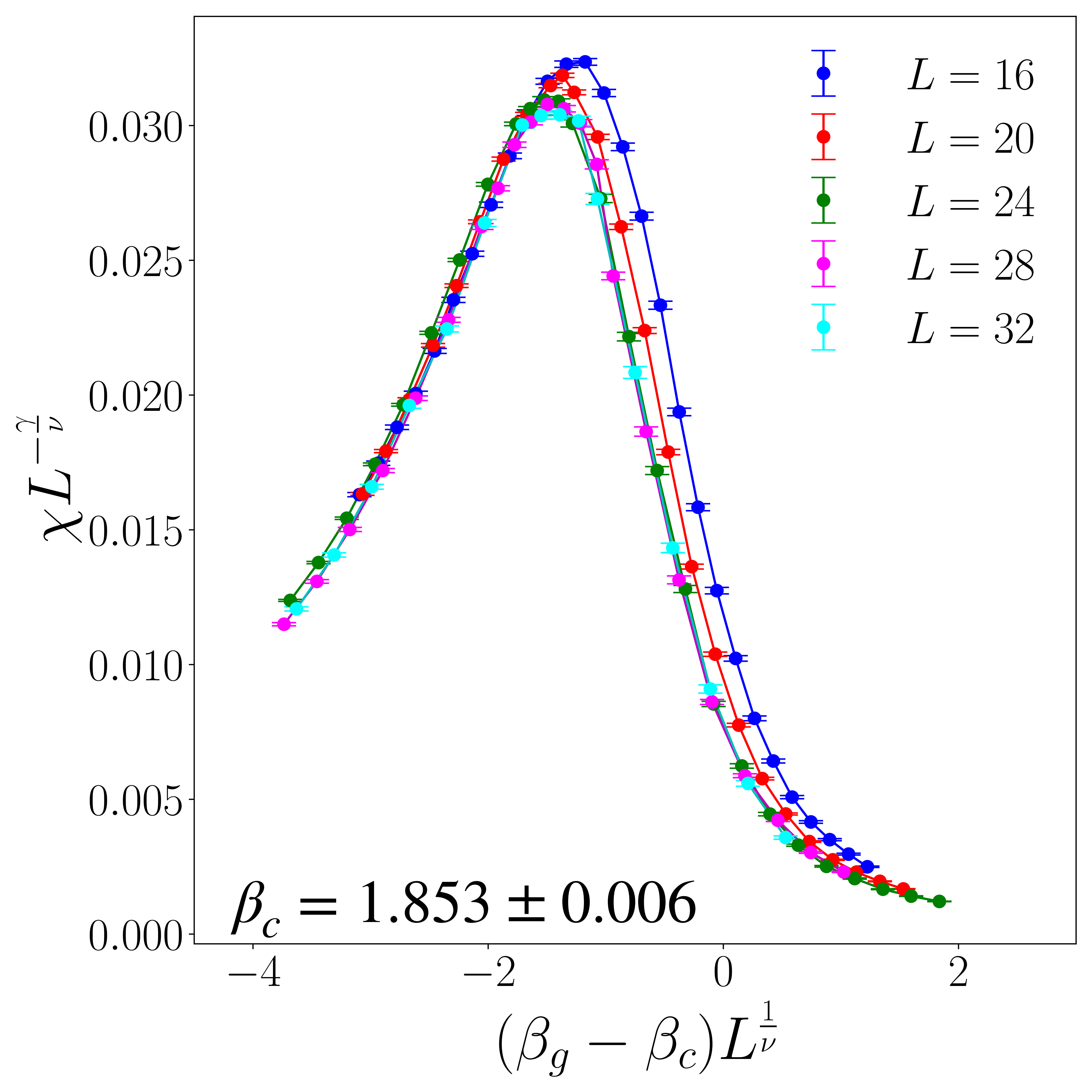
At weak coupling long-range correlation

Spontaneous symmetry breaking

⇒ **Phase transition on a vortex**

Critical point





Ising universality class $\nu = 1, \gamma = 7/4$

predicted in Motrunich, Senthil ('05)

Summary

We found the phase transition on a vortex between strong and weak gauge couplings in superfluid phase

More generally, there can be phase transitions of various phase defects

Codimension 1: transition on a domain wall
Codimension 2: transition on a vortex
Codimension 3: Level crossing

Phase transitions on domain wall junctions are also possible

Outlook

EFT on $U(1) \times U(1)$ model～Ising model

EFT of CFL phase $\sim CP(2)$ model

Ground state of $CP(2)$ model
Gapped phase, no flavor breaking
⇒ continuously connects to the hadronic phase？

What happens if fermion d.o.f. is included？