Quantum Soliton in monoaxial chiral ferromagnetic chain*

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3: QuaRC, Institute of Molecular Science, Japan

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[Software]

Hphi (Kawamura-Yoshimi-Misawa-Yamaji-Todo-Kawashima 2017)

for Exact Diagonalization,

Outline

I: Introduction to **Classical** Chiral magnet

- Chiral magnet, Dzyaloshinskii-Moriya Interaction, Chiral Soliton Lattice
- Sine-Gordon Theory of Chiral Soliton Lattice
- Example of Real Material: $CrNb_3S_6$
- Similarity with Chiral Liquid Crystal, Vortex in type II superconductors,
- Hysteresis in Continuous Phase Transition: Common Property between Chiral magnets and Type II Superconductors Shinozaki et al, 2018

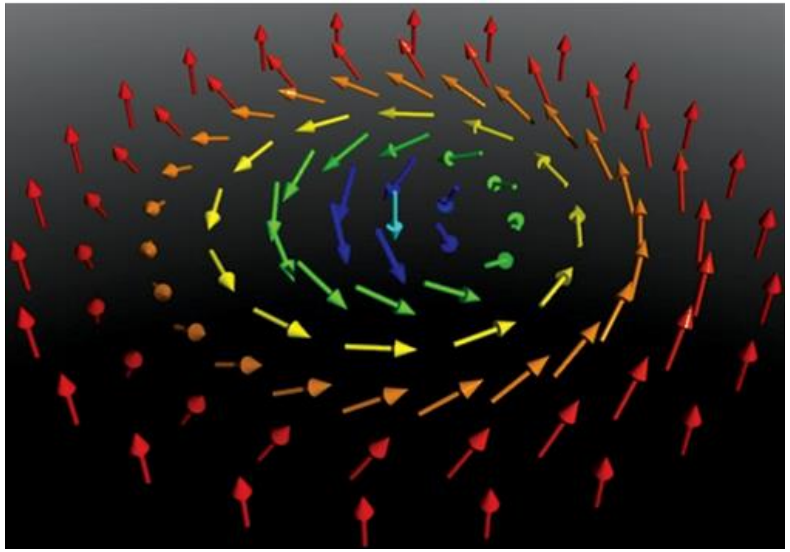
II: **Quantum** Spin Chain of monoaxial Chiral magnet Kodama et al , 2023

- Numerical Result: Difference between Half integer spin and Integer spin in chiral magnet
- Theory for S=1/2
- Theory for higher Spin

Chiral Magnets have

- Crystal structure without inversion and mirror symmetries
- Nontrivial magnetic structure caused by Dzyaloshinskii-Moriya Interaction

Ex. Chiral soliton lattice, Skyrmion,



Reviews on Chiral Soliton Lattice:
Togawa, Kohsaka, Kishine, Inoue,
J. Phys. Soc. Jpn. **85**, 112001 (2016)

http://www.riken.jp/en/research/rikenresearch/highlights/6527

Reviews on Skyrmion: Nagaosa and Tokura 2013, Nat. Nanotechnol. 8 899

Dzyaloshinskii-Moriya Interaction

Which has the form of $\boldsymbol{D}_{ij} \cdot (\boldsymbol{S}_i \times \boldsymbol{S}_j)$

DM vector (Dzyaloshinskii-Moriya vector)

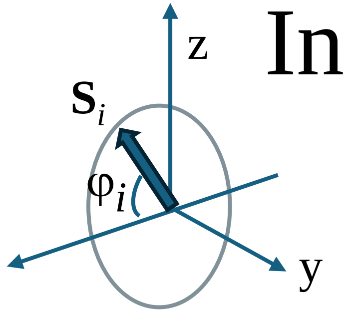
- Interaction between spins inherent to the bond (*i*,*j*) without inversion/reflection symmetry.

- Direction of the DM vector depends on the crystal structure

In this talk, we will focus on a simple case, where the DM vector is parallel to the crystal axis, where <u>the chiral soliton lattice</u> is formed as the ground state.

Reasons: 1. Existence of good material ($CrNb_3S_6$)
2. Common aspects with vortex in Superconductors and Chiral Liquid Crystal
3. Possible simulating of DM vector in Cold Atoms cf. M. Kunimi's talk on Monday
4. Possible realization of CSL in QCD (Higaki, Kamada, Nishimura)

Formation of Helical Spin Structure in Chiral magnets

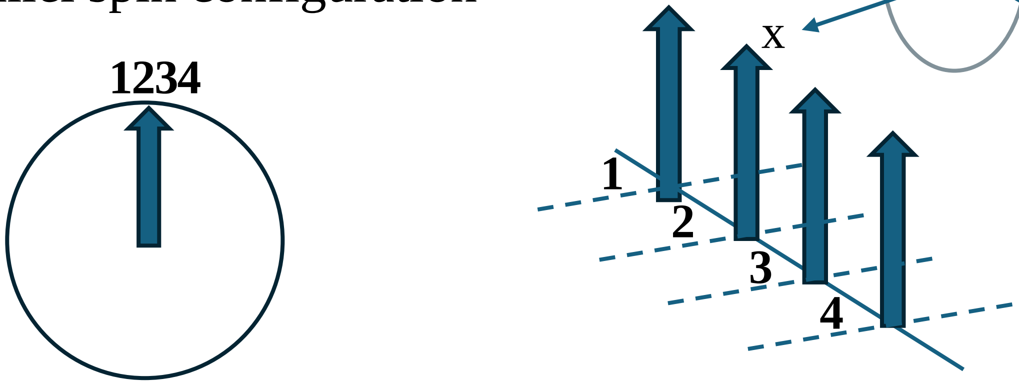


In the absence of magnetic field

Ferromagnetic exchange interaction

$-J\, \mathbf{S}_i \cdot \mathbf{S}_{i+1} = -J \cos(\varphi_i - \varphi_{i+1})$

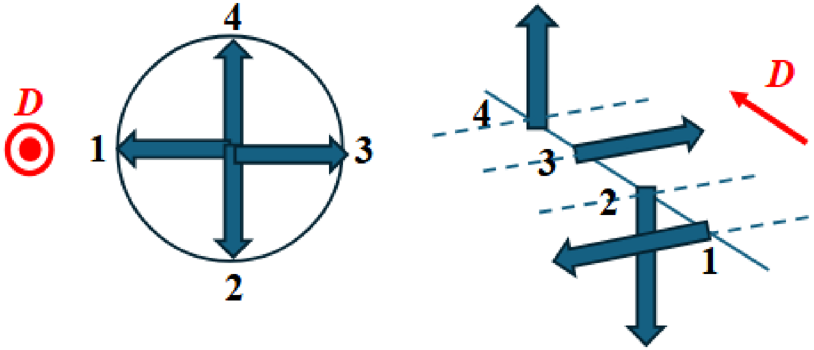
favors parallel spin configuration



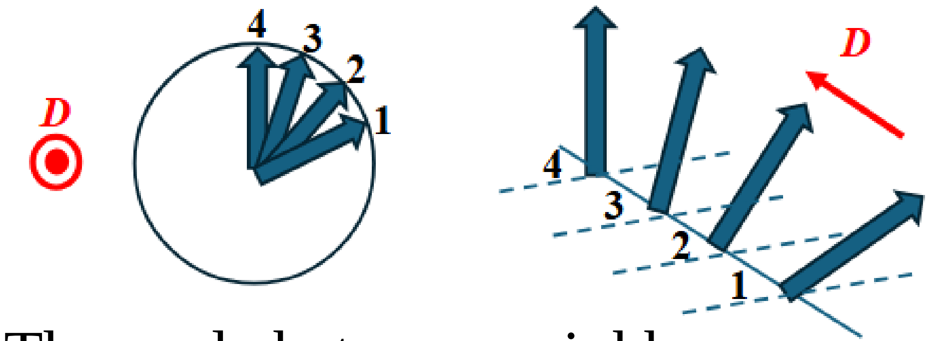
Dzyaloshinskii Moriya Interaction

$-D\, (\mathbf{S}_i \times \mathbf{S}_{i+1})_z = -D\sin(\varphi_i - \varphi_{i+1})$

favors perpendicular configuration



As a competition between the two,
Spins form helical structure



The angle between neighbors
is of the order of D/J

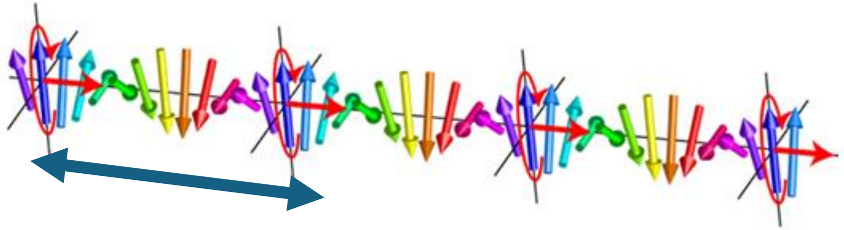
$-J\cos(\varphi_i - \varphi_{i+1}) - D\sin(\varphi_i - \varphi_{i+1})$

$= -\sqrt{J^2 + D^2}\, \cos(\varphi_i - \varphi_{i+1} - \alpha)$

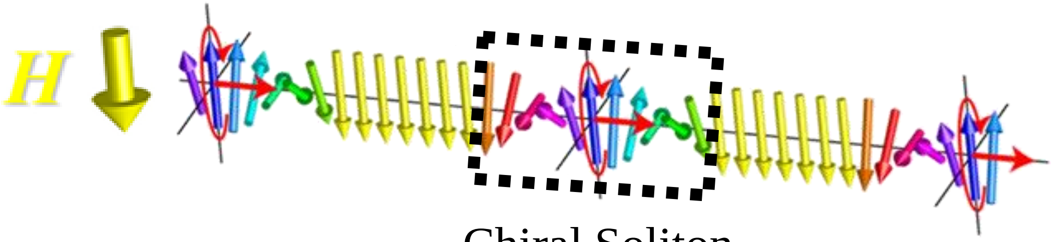
$\alpha = \arctan(D/J)$

in magnetic fields

$$E=-\sum_i J\cos(\varphi_i - \varphi_{i+1}) + D \sin(\varphi_i - \varphi_{i+1}) - H \cos(\varphi_i)$$

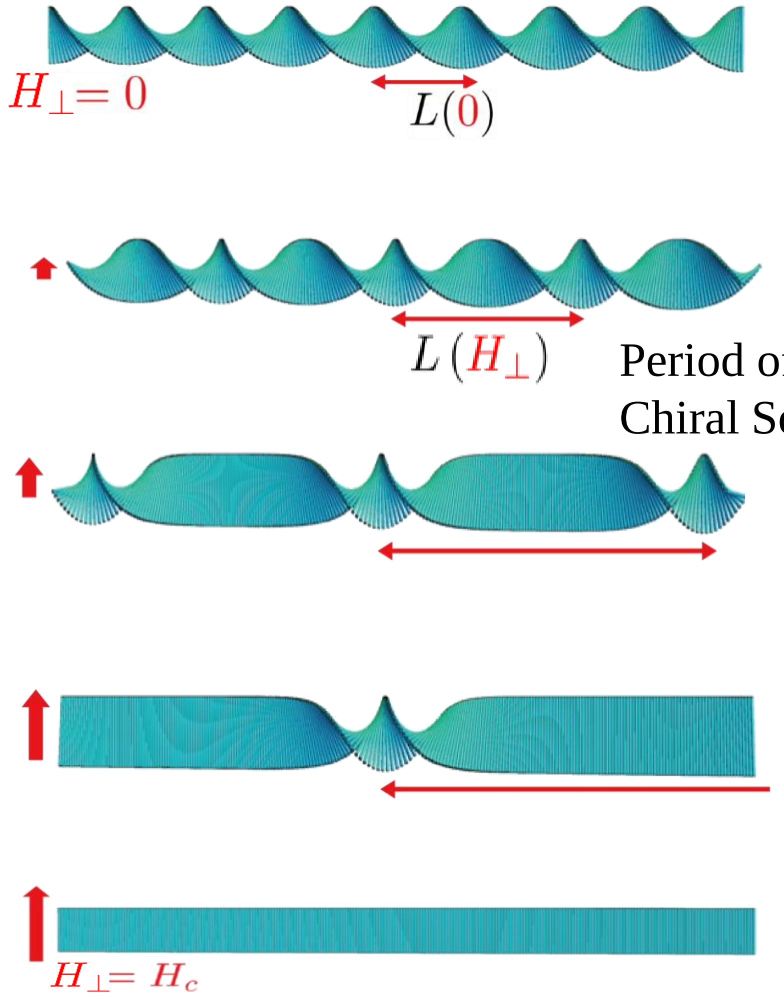


Period of Chiral Soliton Lattice $\dfrac{2\pi}{\arctan(D/J)}$ $\sim J/D$ for $J \gg D$



Chiral Soliton

Figures: Togawa *et al.*, J. Phys. Soc. Jpn. **85**, 112001 (2016)



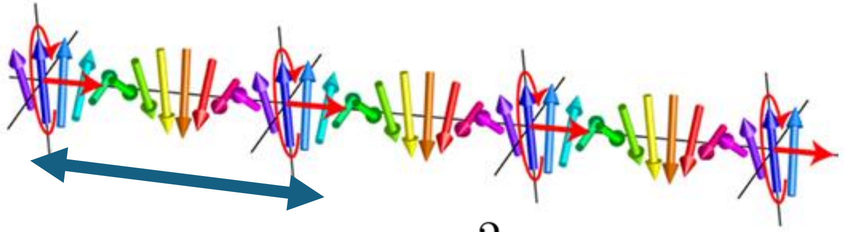
Period of Chiral Soliton Lattice

Continuum approximation：Sine-Gordon Equation

$$E=\sum_i\left[-JS^2\cos\left(\varphi_{i+1}-\varphi_i\right)-DS^2\sin\left(\varphi_{i+1}-\varphi_i\right)-HS\cos\varphi_i\right]$$

Continuum approximation

$$\sim\frac{JS^2}{a}\int dz\left[\left(\frac{d\varphi(z)}{dz}\right)^2-\frac{2\pi}{L(0)}\frac{d\varphi(z)}{dz}-\left(\frac{m}{L(0)}\right)^2\cos\varphi(z)\right]$$



Period $L(0)$ $\frac{2\pi}{\arctan(D/J)}$ $\sim J/D$ for J » D

Stationary condition $m=\frac{\pi^2}{2}\left(\frac{H}{H_c}\right)^{\frac{1}{2}}$, $H_c=\frac{\pi^2}{16}\frac{D^2S}{J}$

Chiral Soliton

$$\frac{d^2\varphi(z)}{dz^2}-\left(\frac{m}{L(0)}\right)^2\sin\varphi(z)=0$$ Sine-Gordon equation

$$\varphi(z)=\pi+2\,\mathrm{am}\left(\frac{m(z-z_s)}{\kappa L(0)};\kappa\right)$$

Elliptic function

○ (red) determins the period

○ (blue) center of mass coordinate

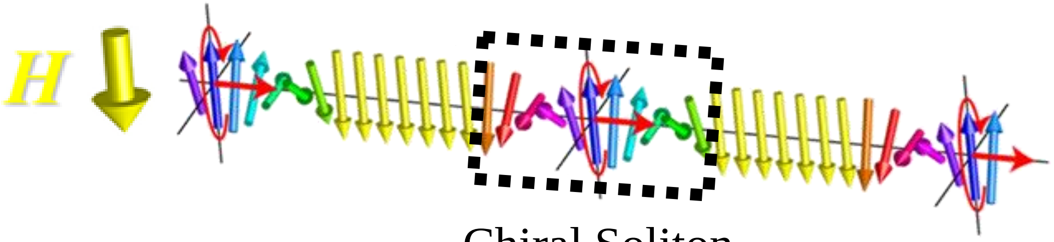
Figure: Togawa *et al*., J. Phys. Soc. Jpn. **85**, 112001 (2016)

κ is determined by minimizing E[φ]

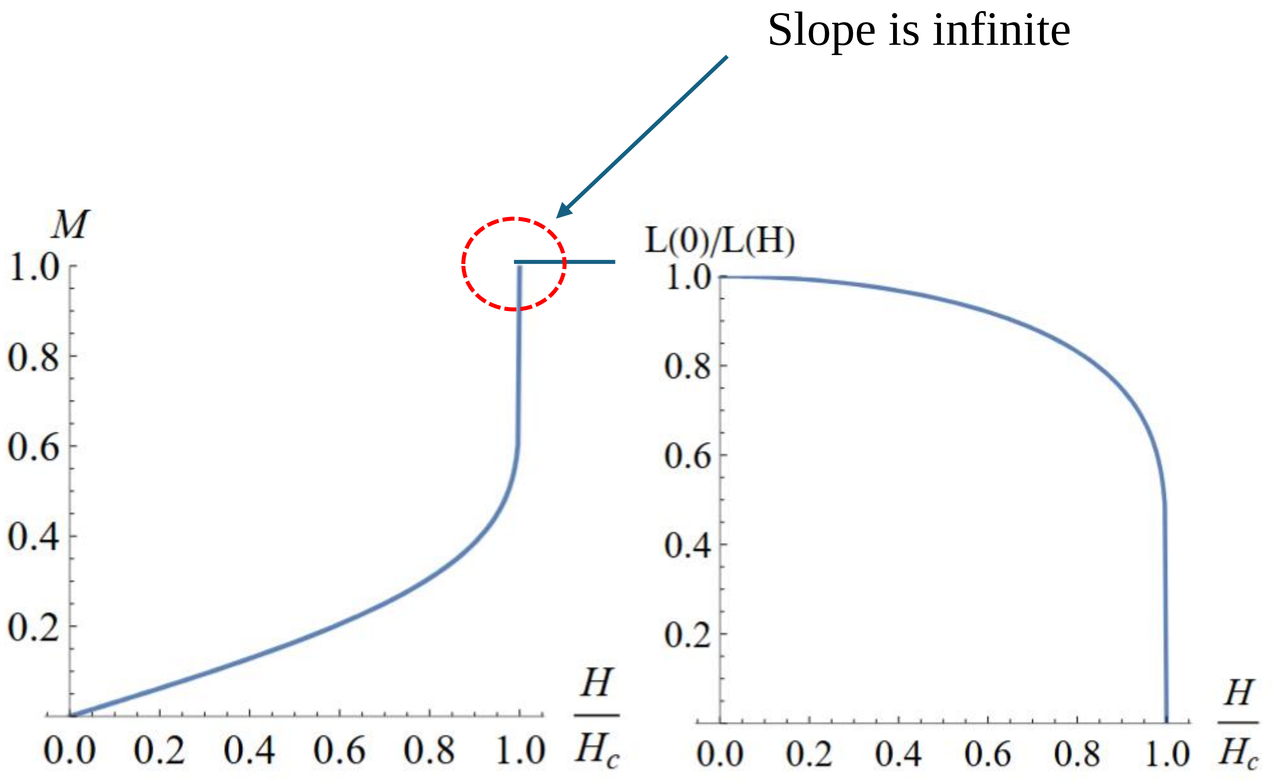
Helimagnet in magnetic fields

Period of
Chiral soliton Lattice

$$\frac{2\pi}{\arctan(D/J)} \sim J/D \text{ for } J \gg D$$



Chiral Soliton

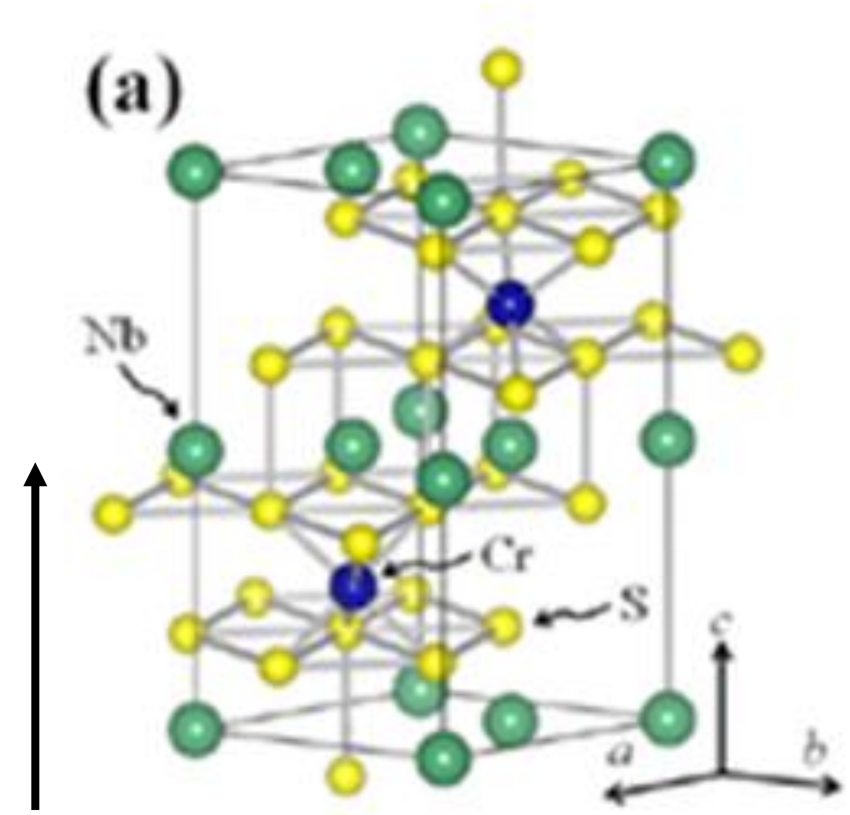


$$\frac{M(H^z)}{M(H^z_c)} \simeq 1 + \frac{1}{\log(H^z_c - H^z)}$$

Review: Togawa *et al.*, J. Phys. Soc. Jpn. **85**, 112001 (2016)
Kishine-Ovchinnikov, Solid State Physics(2015)

Dzyaloshinskii (1965)
Chiral Soliton Lattice.

Material: $CrNb_3S_6$

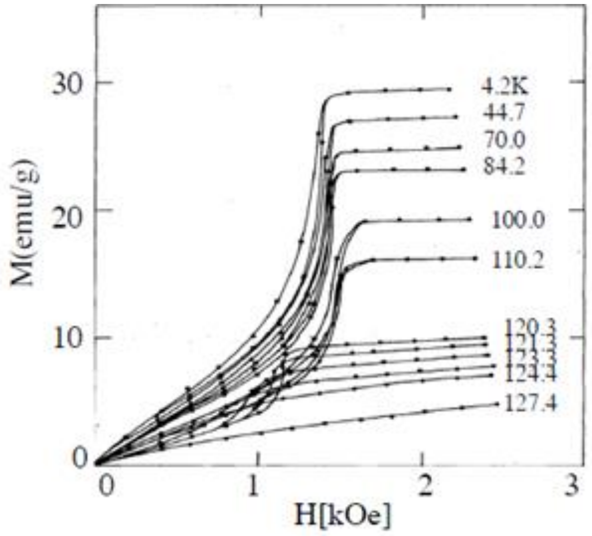


C axis
= helical axis

Hexagonal, $P6_322$, which has chiral axis.
Metallic
Cr spin 3/2, $M \sim 3\mu_B$
T_c=127K

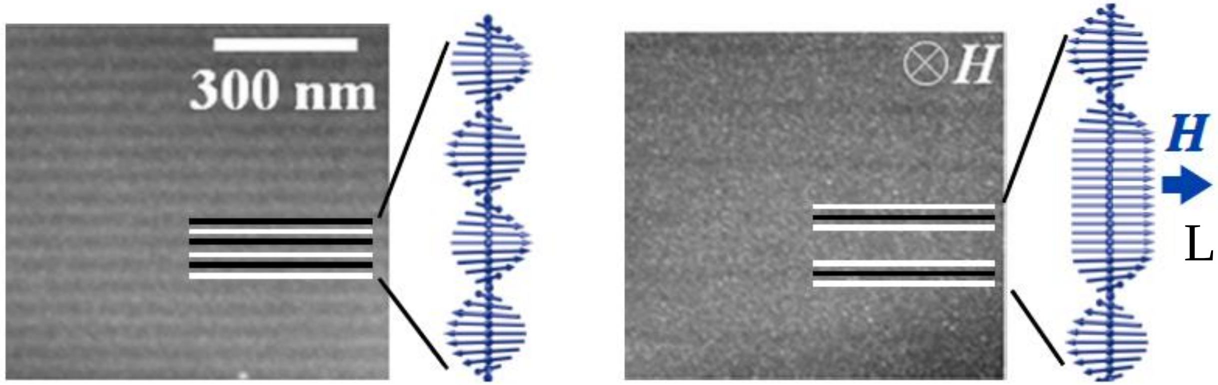
Review article
・Kishine-Ovchinnikov 2015
・Togawa, Kohsaka, Kishine, Inoue. 2016

Magnetic properties



(a)

Magnetization measurements
of $CrNb_3S_6$
Moriya-Miyadai 1982,
Miyadai et al 1983.



Direct observation
of “chiral soliton lattice”

Lorentz-transmission electron microscopy
Togawa et al 2012,

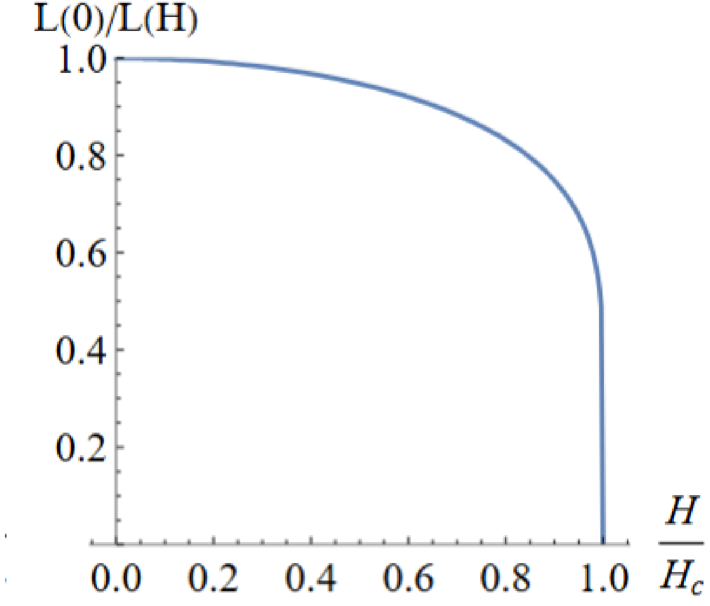
L(0)=48nm: period of helix at zero field

cf: Direct observation
of Skyrmion, Yu et al. 2010

Similarity with the Freedericksz transition in chiral liquid crystal

V. Freedericksz and V. Zolina,
'Forces causing the orientation of an anisotropic liquid.'
Trans. Farad. Soc. **29**, 919–930 (1933).

1/Period of Chiral Soliton for chiral magnetc lattice



Period of Chiral soliton in Liquid Crystal （$L(H)$）

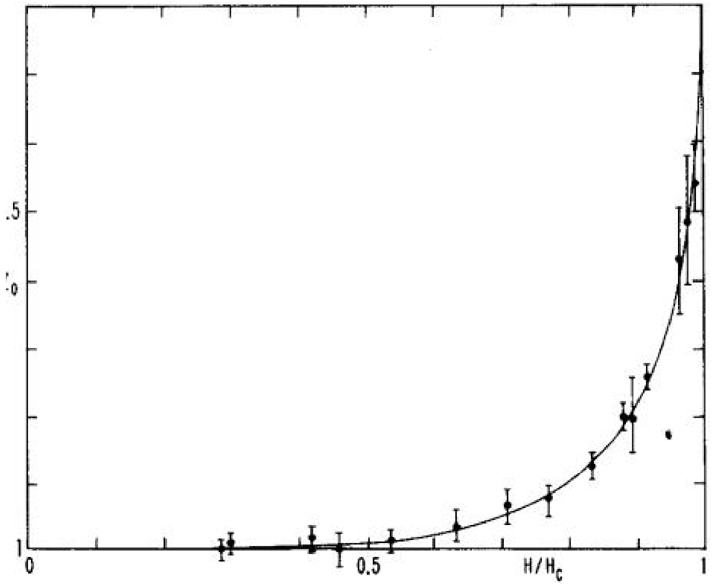


Fig. 2. Dependence of helix pitch Z on field strength H; theoretical curve after De Gennes (see Ref. 4).

CALCUL DE LA DISTORSION D'UNE STRUCTURE CHOLESTERIQUE PAR UN CHAMP MAGNETIQUE

P. G. De Gennes

Physique des Solides,* Faculté des Sciences, 91 Orsay, France

(Received 5 January 1968)

En champ magnétique nul un cristal liquide cholestérique a une structure hélicoïdale.[1] En présence d'un champ H la structure est distordue et la période spatiale augmente. Finalement, pour H supérieur à une valeur critique H_c (~ 2.10^4 Oe) il y a alignement complet (passage à une phase nématique).

Volume 14, Number 7 APPLIED PHYSICS LETTERS 1 April 1969

DISTORTION OF A CHOLESTERIC STRUCTURE BY A MAGNETIC FIELD*

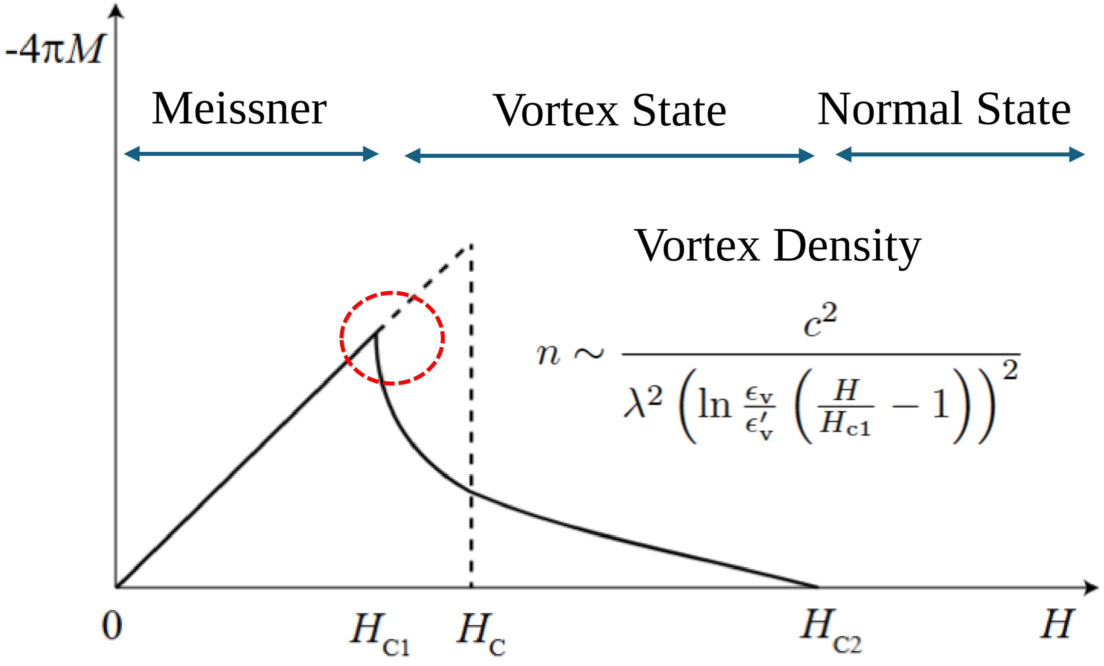
Robert B. Meyer
Gordon McKay Laboratory, Harvard University
Cambridge, Massachusetts 02138

(Received 3 February 1969)

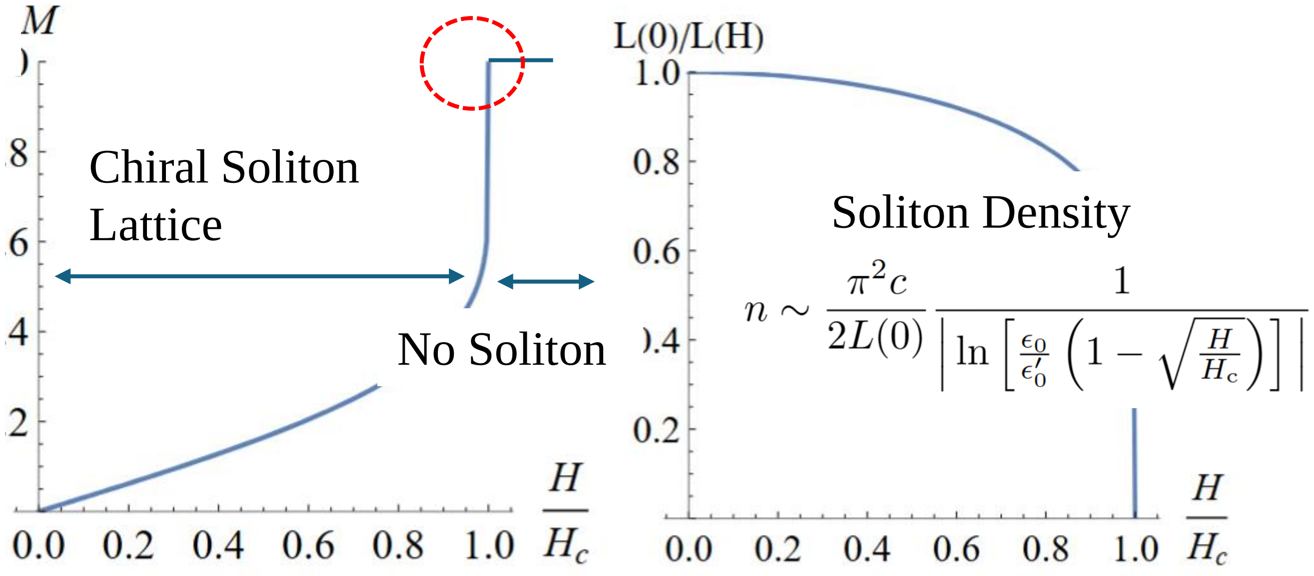
An experiment is described which confirms the theory of the distortion of a cholesteric structure by a magnetic field. Field effects in a sample of p-azoxyanisole doped with cholesteryl acetate were viewed directly with a microscope, and the pitch of the helical structure was measured as a function of field strength.

Similarity with Type II Superconductors

Magnetization Curve of Type II Superconductors

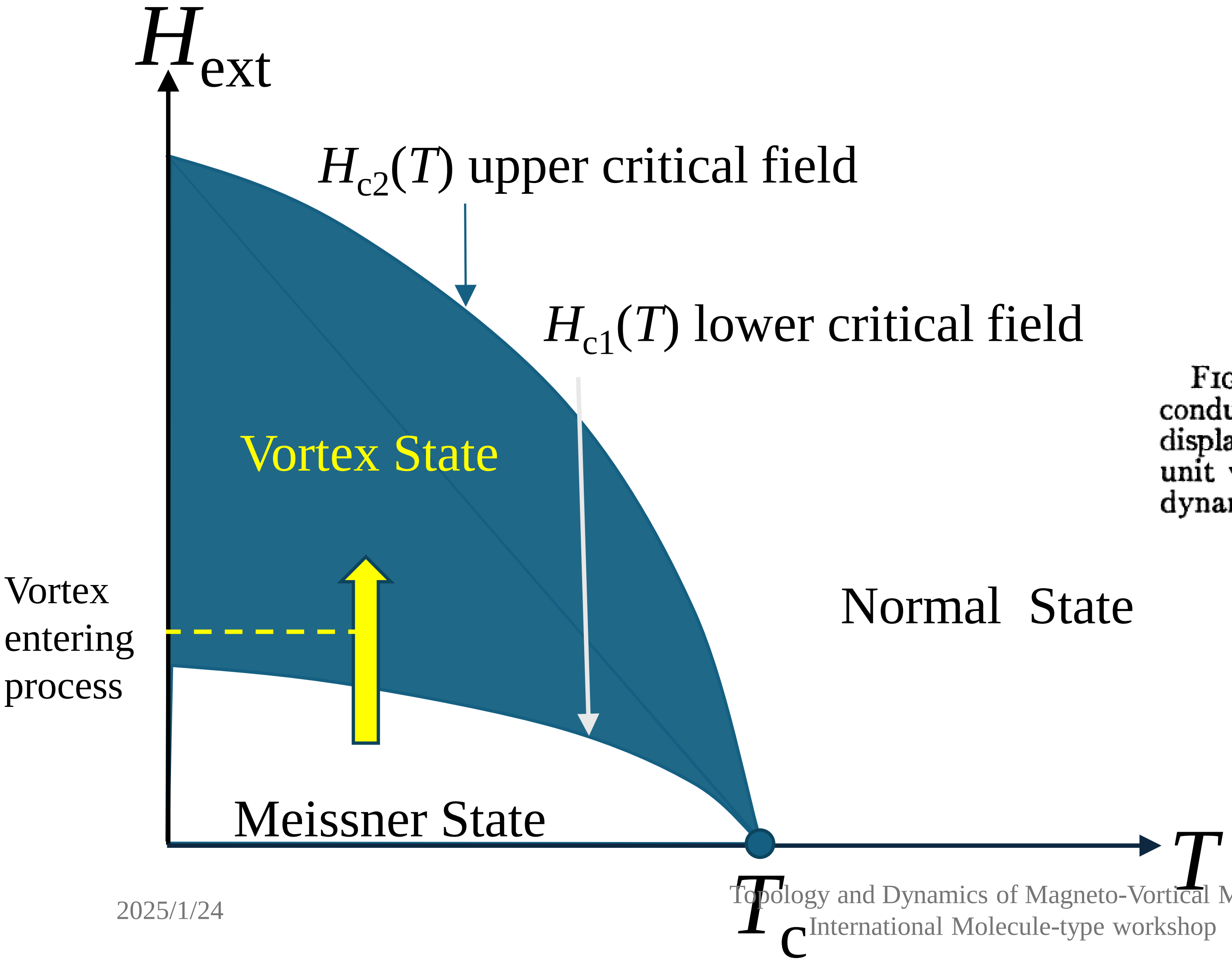


Magnetization Curve of Chiral Magnet



This singularity is common for the condensation of topological defects

Another similarity:
Hysteresis in continuous transition



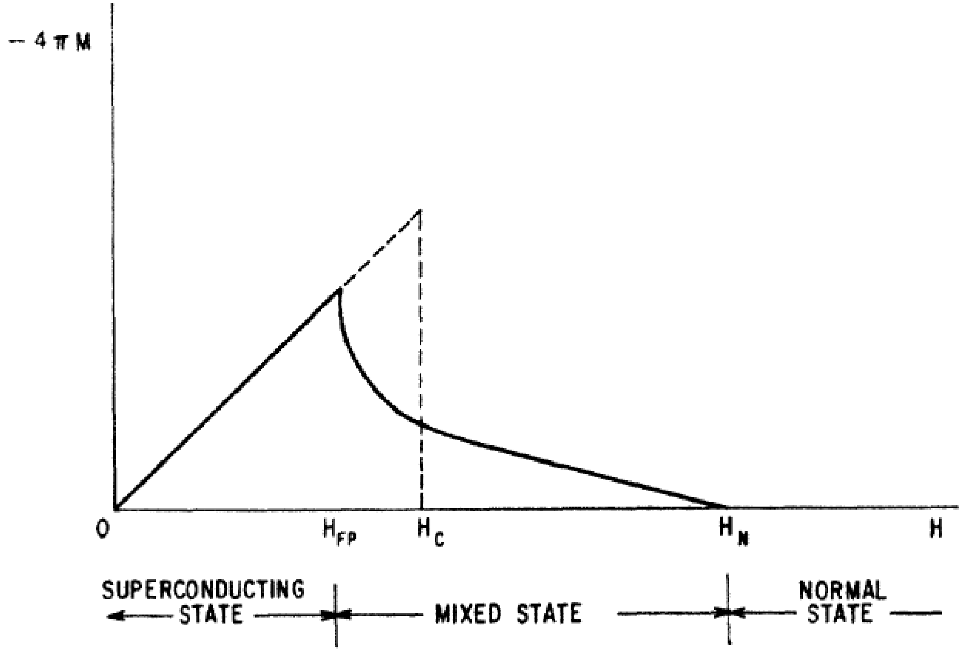
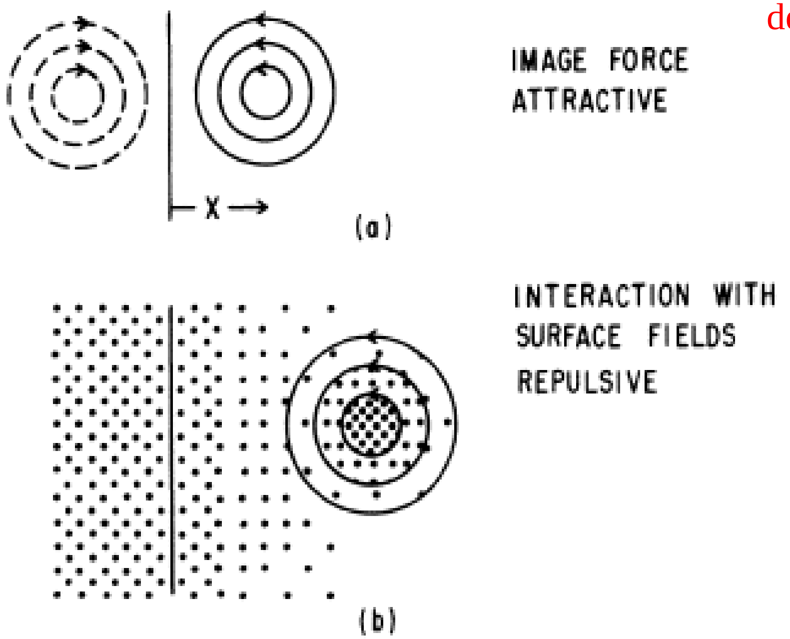
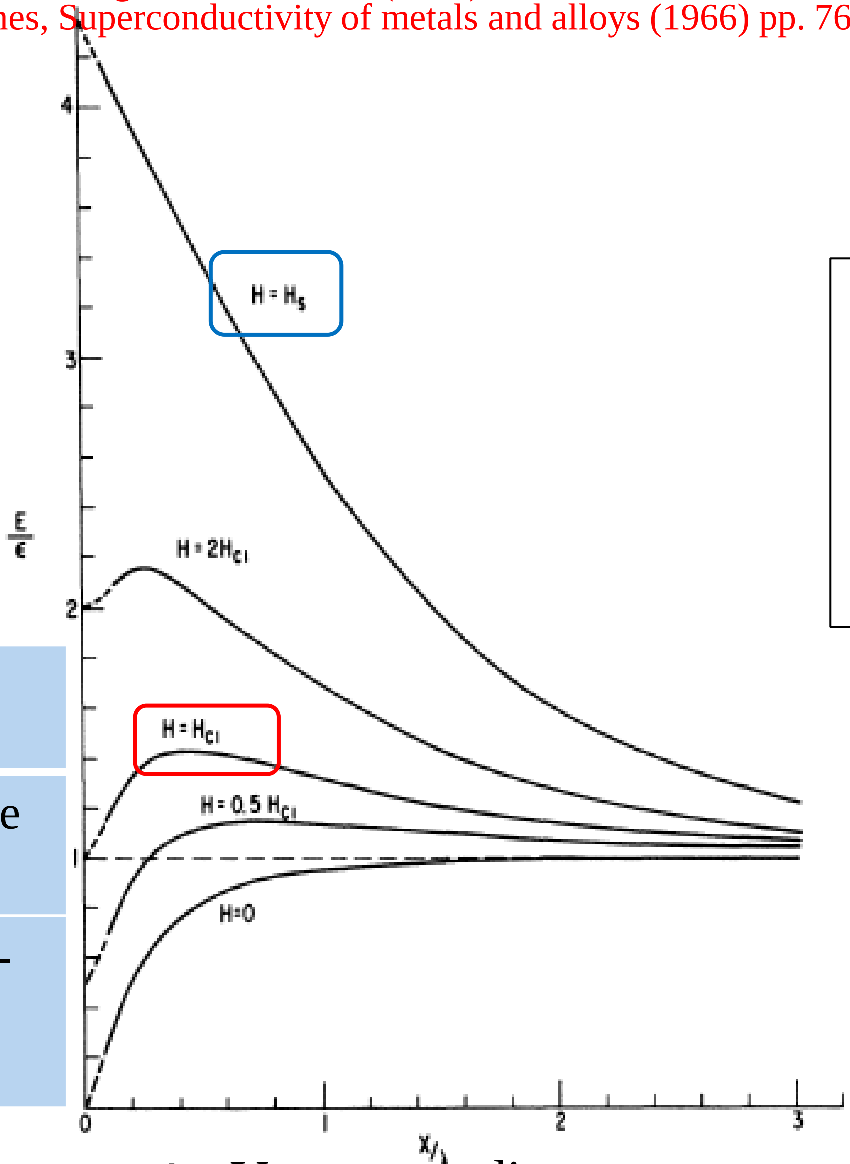


FIG. 1. Dashed curve: Magnetization curve for a soft superconductor. Solid curve: magnetization curve for a superconductor displaying "negative surface energy." M is the magnetization per unit volume, H is the applied magnetic field, H_C is the thermodynamic critical field.

Surface barrier of vortex in superconductors

Bean and Livingston, PRL 12 14 (1964).
de Gennes, Superconductivity of metals and alloys (1966) pp. 76.





Vortex coordinate

	Outward force	Inward force
Vortex in SC	Force with image vortex	Int. with surface current
Chiral soliton	Zeeman energy Exchange energy	Dzyaloshinskii-Moriya int

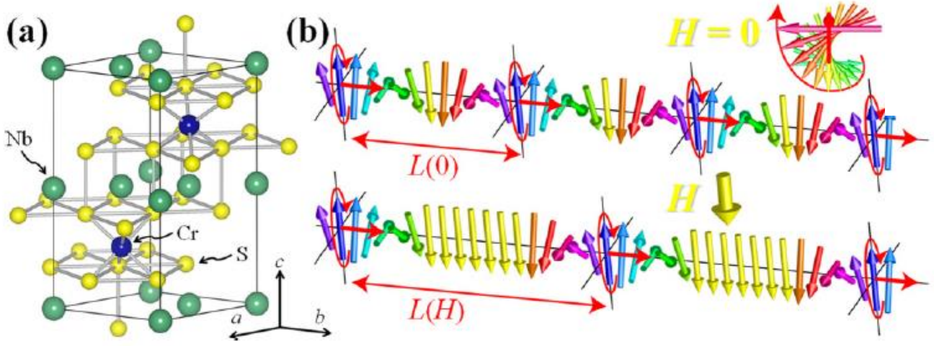
Cf: Surface barrier of Skyrmion

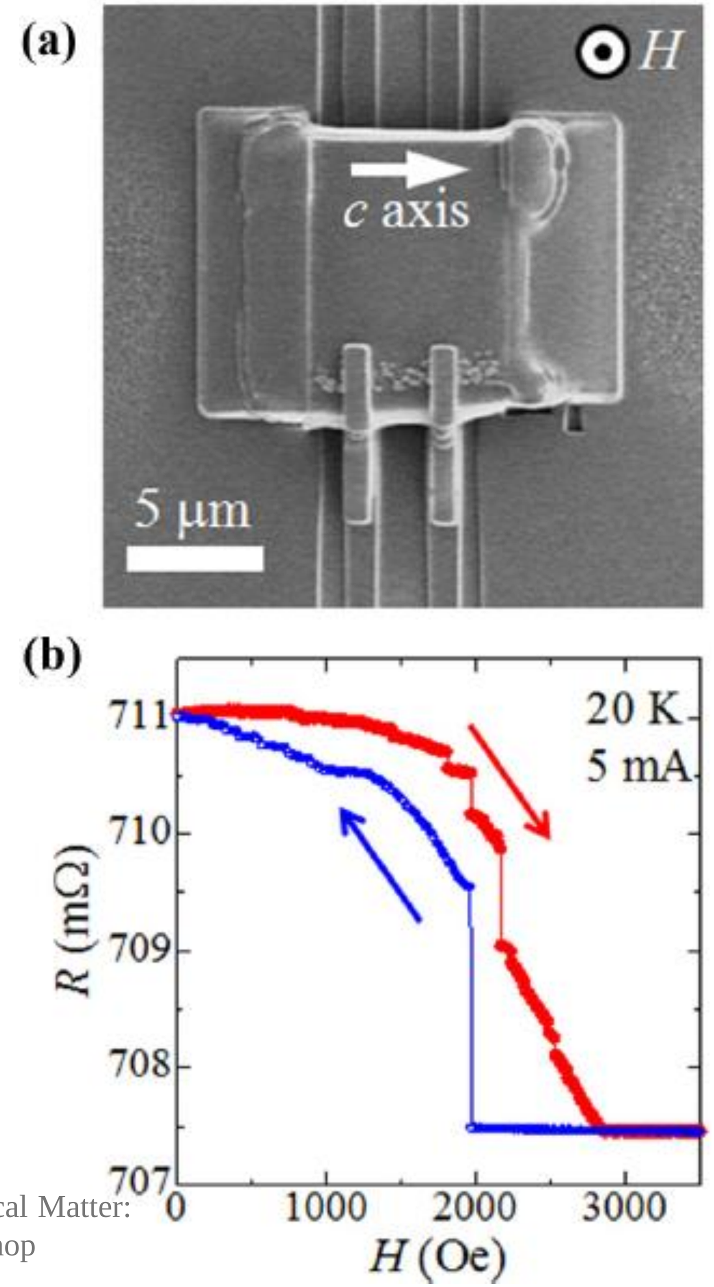
Iwasaki, Mochizuki and Nagaosa 2013
Mueller et al. 2016

Hysteresis in micron-sized samples

Togawa et al. PRB **92**, 220412(R) (2015)

CrNb3S6 Magneto Resistance

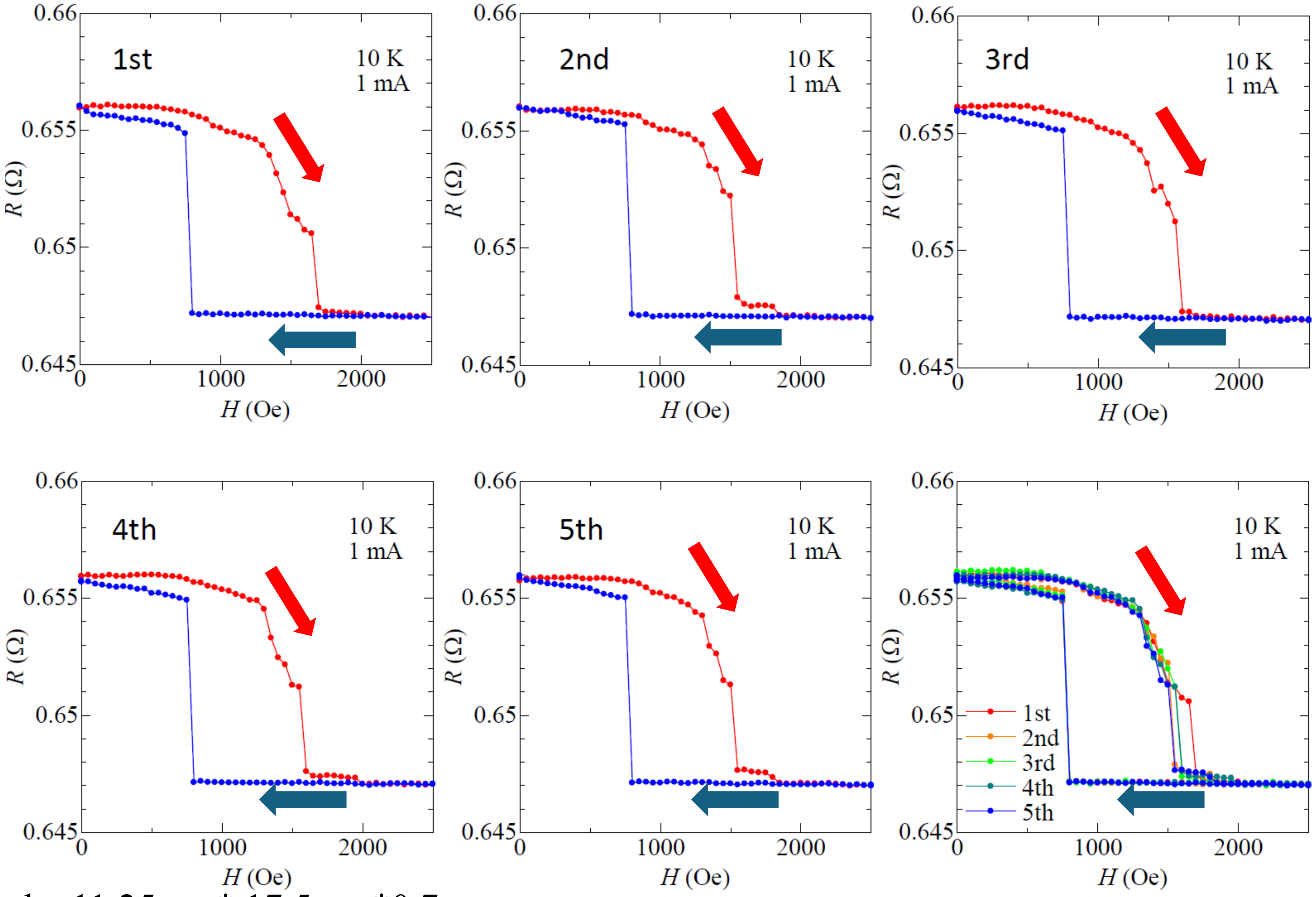




Hysteresis: Increasing process（red） and decreasing process（blue）

Sample: 10μm * 10μm *1μm

Reproducibility(based on five runs) of Magneto-Resistance of $CrNb_3S_6$ in demagnetization-free configuration



Sample: 11.25μm * 17.5μm *0.7μm

Shinozaki, Masaki, Aoki Togawa Kato, PRB (2018)

Surface barrier of chiral soliton in chiral magnet

Sine-Gordon equation for helical configuration

$$\boldsymbol{S}(z) = S(\cos\varphi(z), \sin\varphi(z), 0)$$

$$\mathcal{H}[\varphi] = J^{\parallel} S^2 a_0 N_{2\mathrm{d}} \int_I \mathrm{d}z \left(\frac{1}{2}\left(\frac{\partial\varphi}{\partial z}\right)^2 - \frac{2\pi}{L(0)}\left(\frac{\partial\varphi}{\partial z}\right) - \left(\frac{m}{L(0)}\right)^2 \cos\varphi \right)$$

$I = [0, \infty)$: semi-infinite system with a boundary at z=0

$$m = \frac{\pi^2}{2}\left(\frac{H}{H_c}\right)^{\frac{1}{2}}$$

$$L(0) = 2\pi a_0 J^{\parallel}/D$$

$N_{2\mathrm{d}}$: ♯ of spin in each layer

Shinozaki, Masaki, Aoki Togawa Kato. PRB (2018)

Single soliton solution

$$\varphi_0(z; z_{\mathrm{s}}, m) = 4\arctan(\mathrm{e}^{m(z-z_{\mathrm{s}})/L(0)})$$

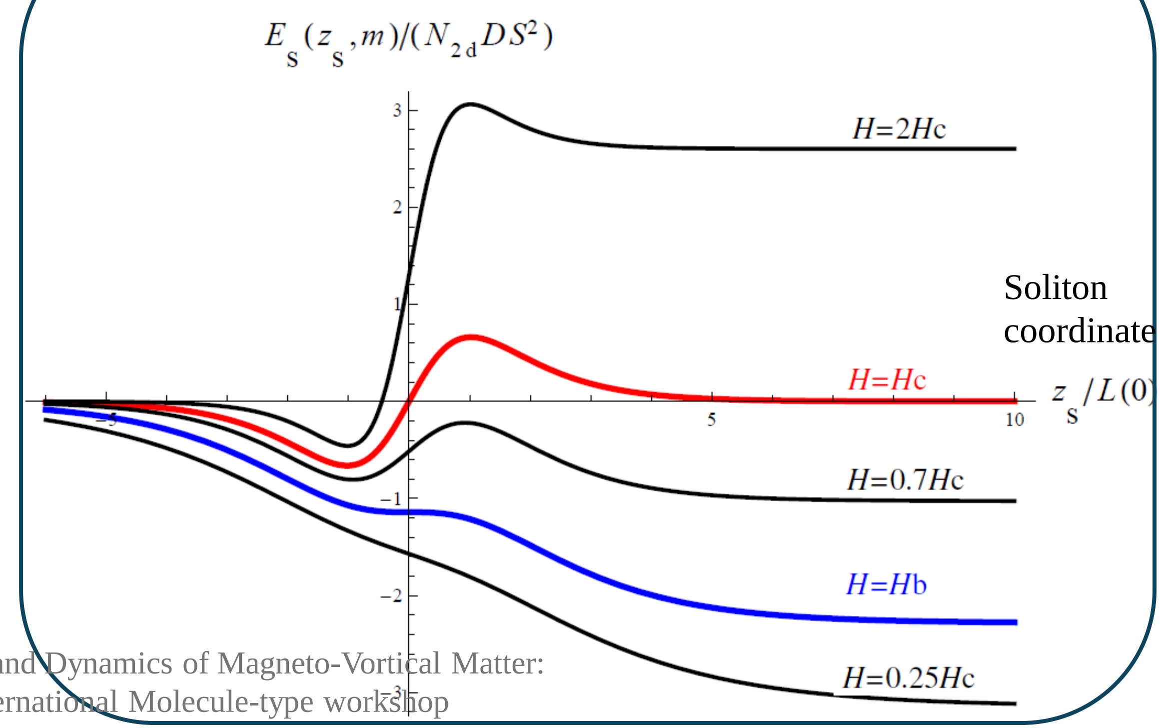
z_{s}: Soliton coordinate

z_{s} >0, inside of sample
z_{s} <0, outside of sample

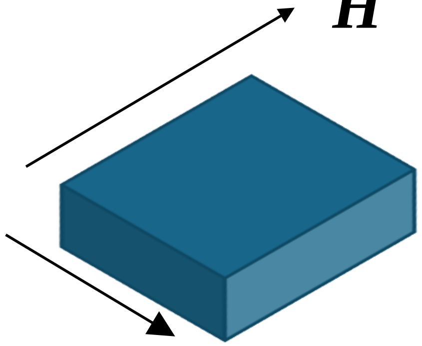
Energy barrier exists at fields higher than

$$H_{\mathrm{b}} = \frac{4}{\pi^2} H_{\mathrm{c}} \simeq 0.405285 H_{\mathrm{c}}$$

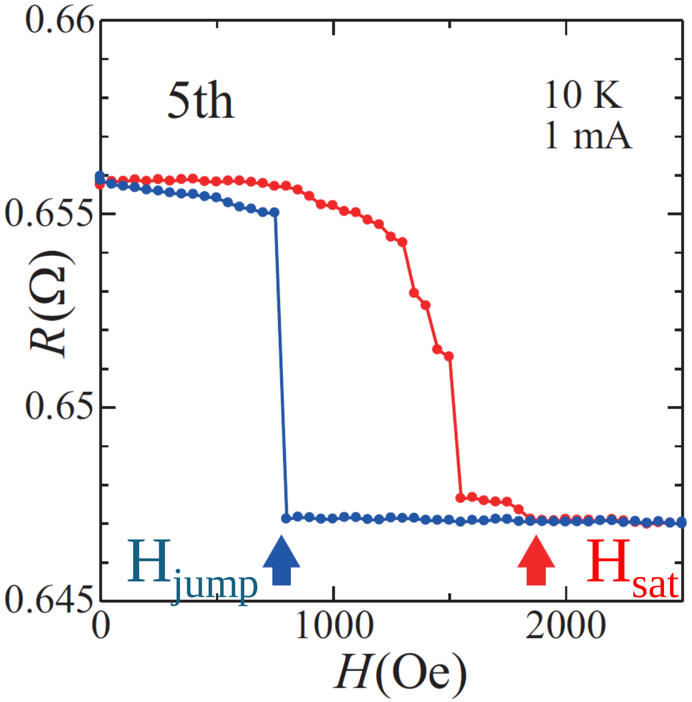
Energy as a function of soliton coordinate



Comparison with experiments
(in demagnetization-free configuration)



Theoretical result: $H_b = \frac{4}{\pi^2} H_c \simeq 0.405285 H_c$



sample	H_{jump}/Oe	H_{sat}/Oe	H_{jump}/H_{sat}
A	775±25	2025±25	0.382
	775±25	1875±25	0.413
	775±25	1725±25	0.449
	775±25	1975±25	0.392
	775±25	1875±25	0.413
B	892.5±2.5	2147.5±2.5	0.415
	892.5±2.5	2202.5±2.5	0.405
	892.5±2.5	2187.5±2.5	0.408
C	710±10	1770±10	0.401

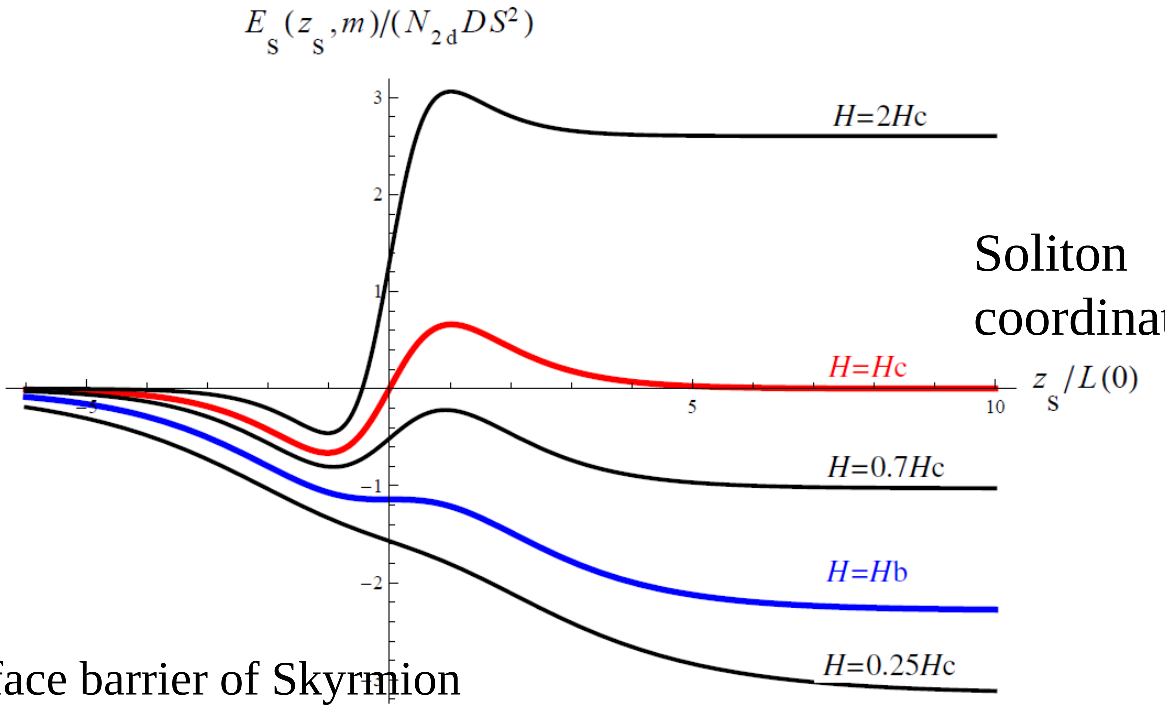
Size(a axis, b axis, c axis) sample A (11µm, 0.7µm, 17.5µm)
B (10µm, 1µm , 10µm)
C (13µm, 0.5µm , 13µm)

Shinozaki, Masaki, Aoki Togawa Kato. PRB (2018)

Comparison with chiral soliton and vortex in superconductors

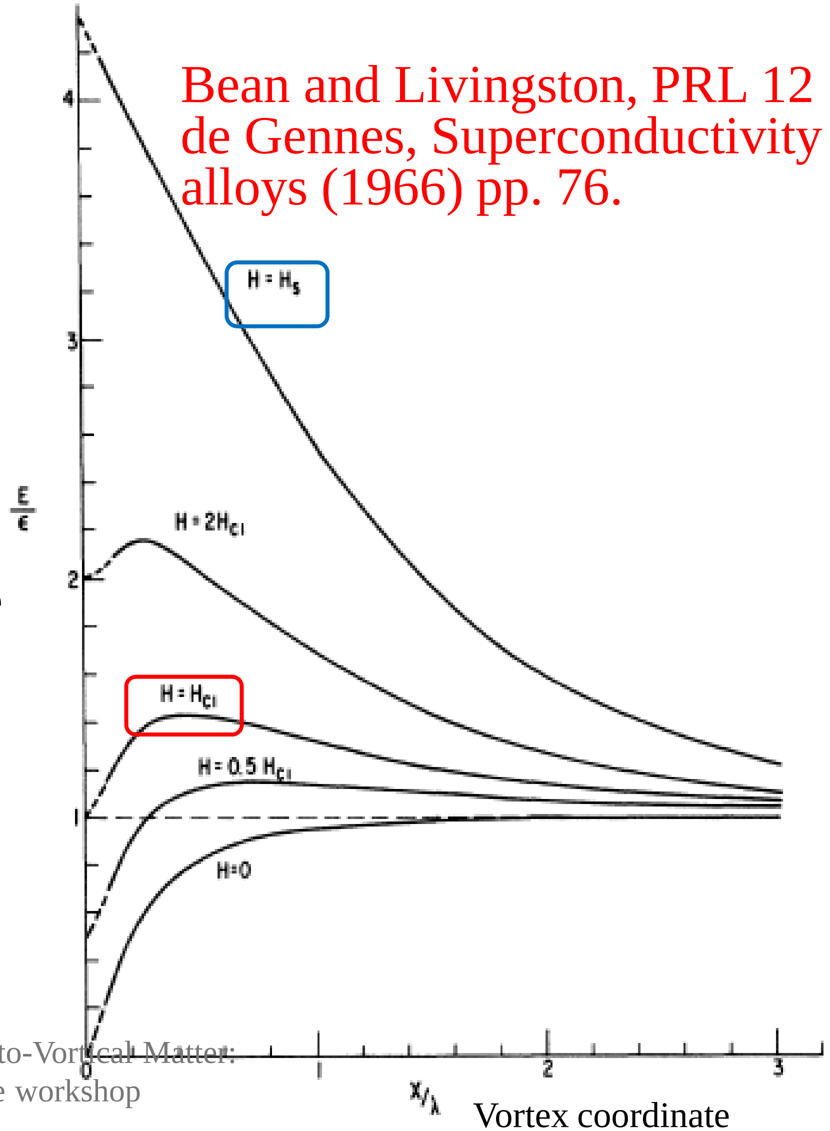
Shinozaki, Masaki, Aoki Togawa Kato. PRB (2018)

Energy as a function of soliton coordinate



Soliton coordinate

Bean and Livingston, PRL 12 14 (1964).
de Gennes, Superconductivity of metals and alloys (1966) pp. 76.



Vortex coordinate

Cf: Surface barrier of Skyrmion
Iwasaki, Mochizuki, and Nagaosa 2013
Mueller et al. 2016

Short Summary: Introduction to Classical Chiral magnet

- Sine-Gordon Theory describes the magnetic properties of Chiral Soliton Lattice

- Good material exists: $CrNb_3S_6$

- Similarity with Chiral Liquid Crystal, Vortex in type II superconductors,

- Hysteresis in Continuous Phase Transition: Common Property between Chiral magnets and Type II Superconductors

- Good Agreement on the hysteresis between theory and experiments
  Shinozaki et al, 2018

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- Sine-Gordon Theory of Chiral Soliton Lattice
- Example of Real Material: CrNbS
- Similarity with Chiral Liquid Crystal, Vortex in type II superconductors,
- Hysteresis in Continuous Phase Transition: Common Property between Chiral magnets and Type II Superconductors Shinozaki et al, 2018

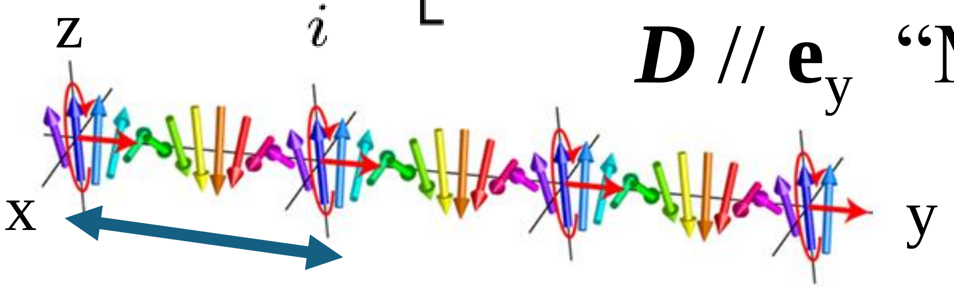
II: Quantum Spin Chain of monoaxial Chiral magnet Kodama et al , 2023

- Numerical Result: Difference between Half integer spin and Integer spin in chiral magnet
- Theory for S=1/2
- Theory for higher Spin

Monoaxial chiral ferromagnets

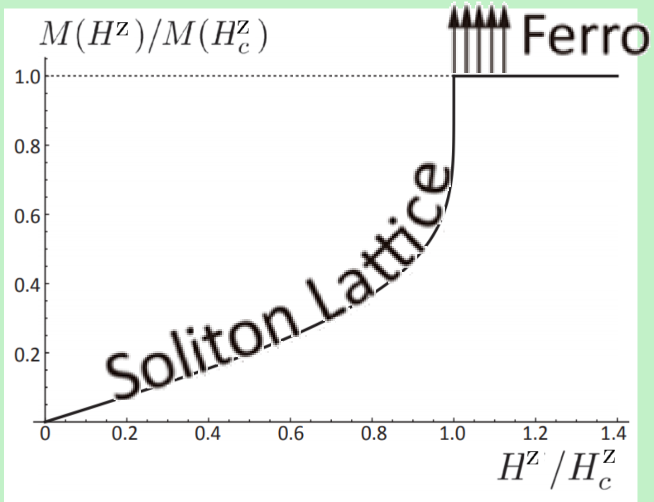
$$\hat{H}_{\text{chiral}} = \sum_i \left[-J \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} - D \left(\hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{S}}_{i+1} \right)_y - H \hat{S}_i^z + K \left(\hat{S}_i^y \right)^2 \right]$$

$\boldsymbol{D} \,//\, \mathbf{e}_y$ "Monoaxial"



Period of
Chiral Soliton Lattice $\dfrac{2\pi}{\arctan(D/J)}$ $\sim J/D$ for $J \gg D$

Chiral Soliton

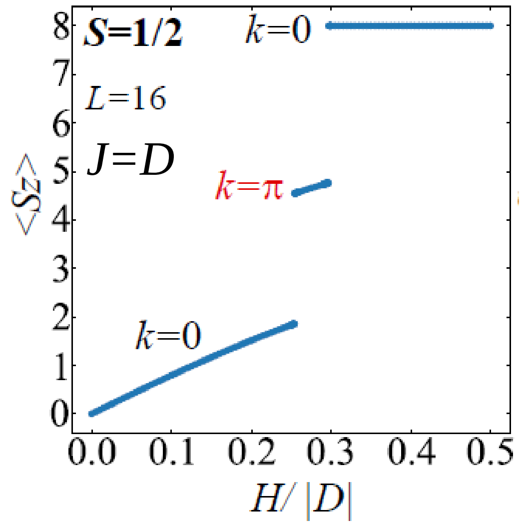


$$\frac{M(H^z)}{M(H_c^z)} \simeq 1 + \frac{1}{\log(H_c^z - H^z)}$$

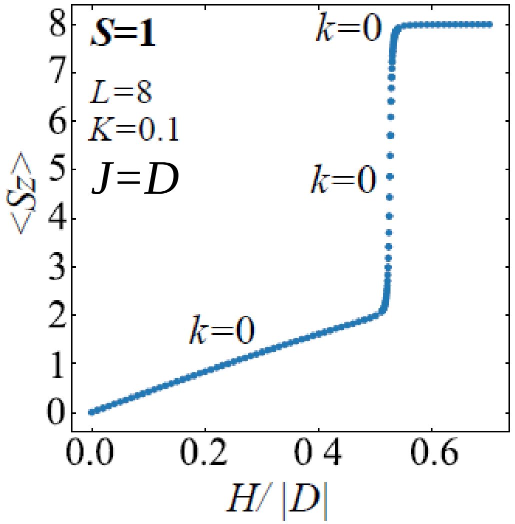
Review: Togawa *et al.*, J. Phys. Soc. Jpn. **85**, 112001 (2016)
Kishine-Ovchinnikov, Solid State Physics(2015)

Dzyaloshinskii (1965)
Chiral Soliton Lattice.

Numerical Results of Magnetization in finite size system (Exact Diagonalization under PBC)

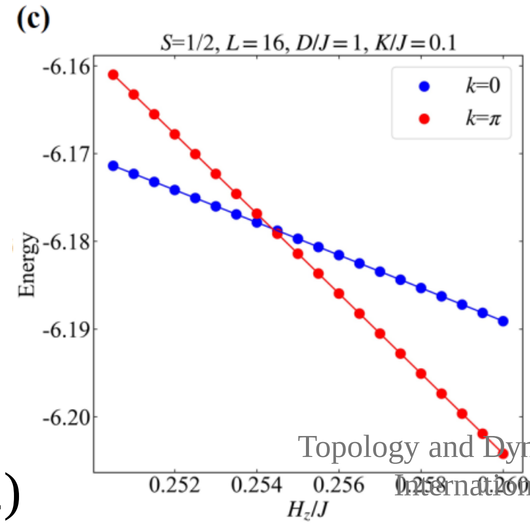


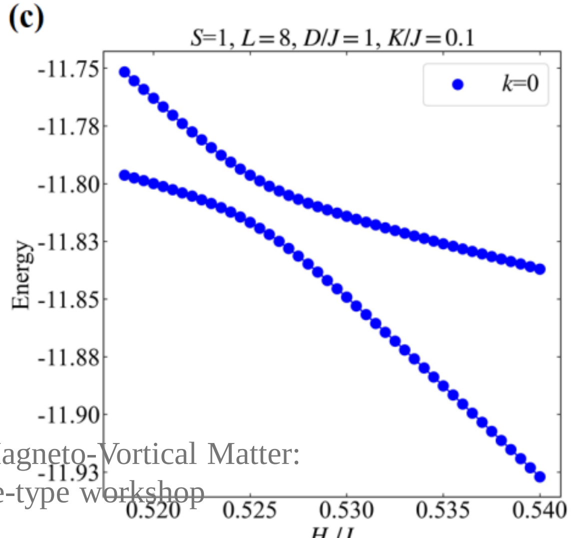
Kodama et al PRB 2023



Kodama et al PRB 2023

Discontinuous Magnetization =Level-Crossing





Continuous Magnetization =Level-Repulsion

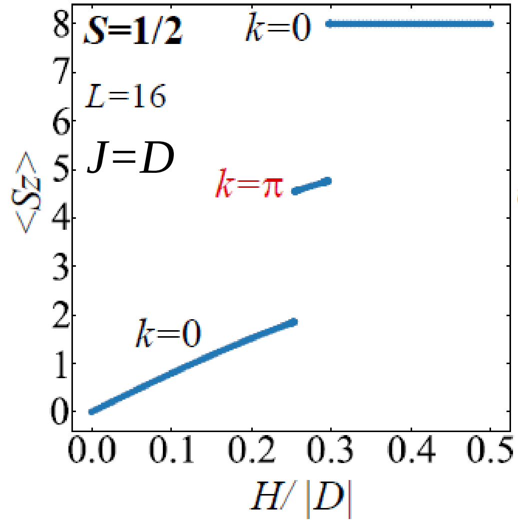
Kodama Master thesis (2022)

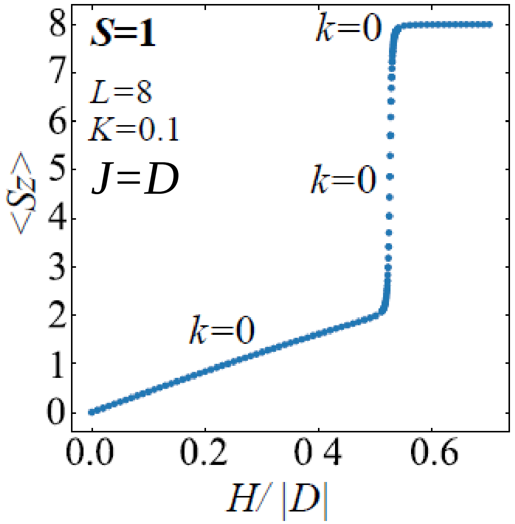
Kodama Master thesis (2022)

Issue:
Understanding of Different behavior in magnetization process between half-Integer and Integer Spins (a “Spin Parity Effect”)

Numerical Diagonalization

Under the periodic boundary condition



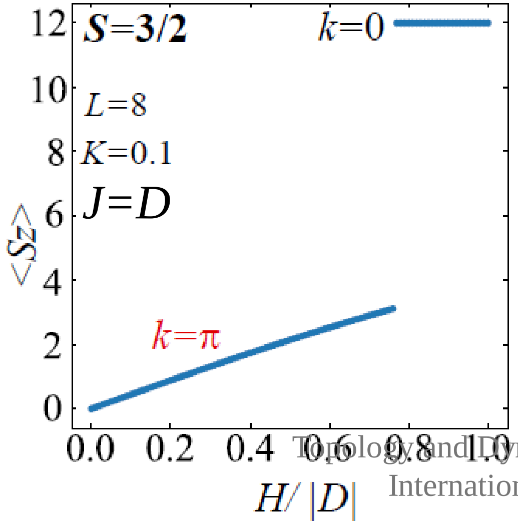


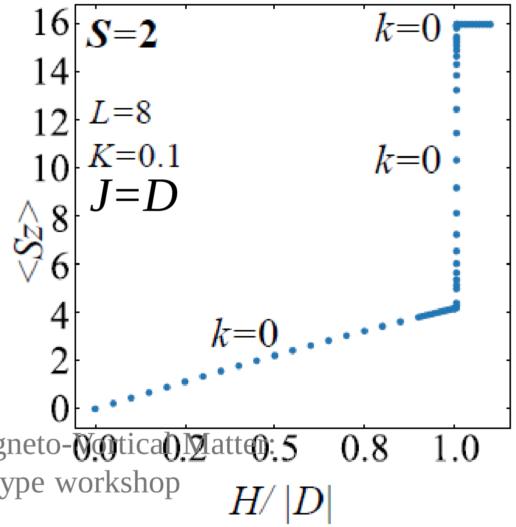
Numerical Diagonalization

Under the periodic boundary condition

Kodama et al PRB 2023

Discontinuous Magnetization =Level-Crossing





Continuous Magnetization =Level-Repulsion

Earlier Studies= Large S approach

Semiclassical Approach (Spin coherent state and Berry phase argument) for

- Nonchiral nanomagnets (Braun-Loss 1996 PRB) :
  solitons (domain wall) are generated by Ising anisotropy

- 2D chiral magnets (Takashima-Ishizuka-Balents 2016 PRB):
  Quantum skyrmion; Review; Ochoa-Tserkovnyak 2019 IJMP

We seek for a theory valid for **small S**.

Cf：Haldane Gap problem

O(3)Nonlinear Sigma model (large S + Berry phase) ⇔ AKLT model (S=1)

S=1/2 case

Clue:

Model in the J/D ⇒ 0 limit is a **canonical model** to understand spin parity effect in chiral magnet

$$\mathcal{H}_{DH} = \sum_i \left[D \left(\hat{S}_i \times \hat{S}_{i+1} \right)^y - H \hat{S}_i^z \right]$$

Number of Solitons becomes a **conserved quantity**. (Next page)

$$\hat{N} = \sum_{i=1}^{L} \left(\frac{1}{4} - \hat{S}_i^z \hat{S}_{i+1}^z \right)$$

Remark: This is opposite limit to the “solid-state limit” (J/D >> 1).
However, this limit can be realized in Rydberg Atoms(Kunimi’s talk).

Proposal for realizing quantum spin models with Dzyaloshinskii-Moriya interaction using Rydberg atoms
Masaya Kunimi, Takafumi Tomita, Hosho Katsura, Yusuke Kato

Conserved Quantity=Soliton Number Operator

$$\hat{N} = \sum_{i=1}^{L} \left(\frac{1}{4} - \hat{S}_i^z \hat{S}_{i+1}^z \right)$$

1/2 + 1/2 = **1**

$$\hat{N}|0001111100\rangle = |0001111100\rangle,$$

$$\hat{N}|0011100110\rangle = 2|0011100110\rangle.$$

1/2 +1/2+1/2+1/2 = **2**

0 for parallel spins “00” or “11”

1/2 for antiparallel spins “01” or “10”

Conventional Basis

0

Sz=+1/2

1

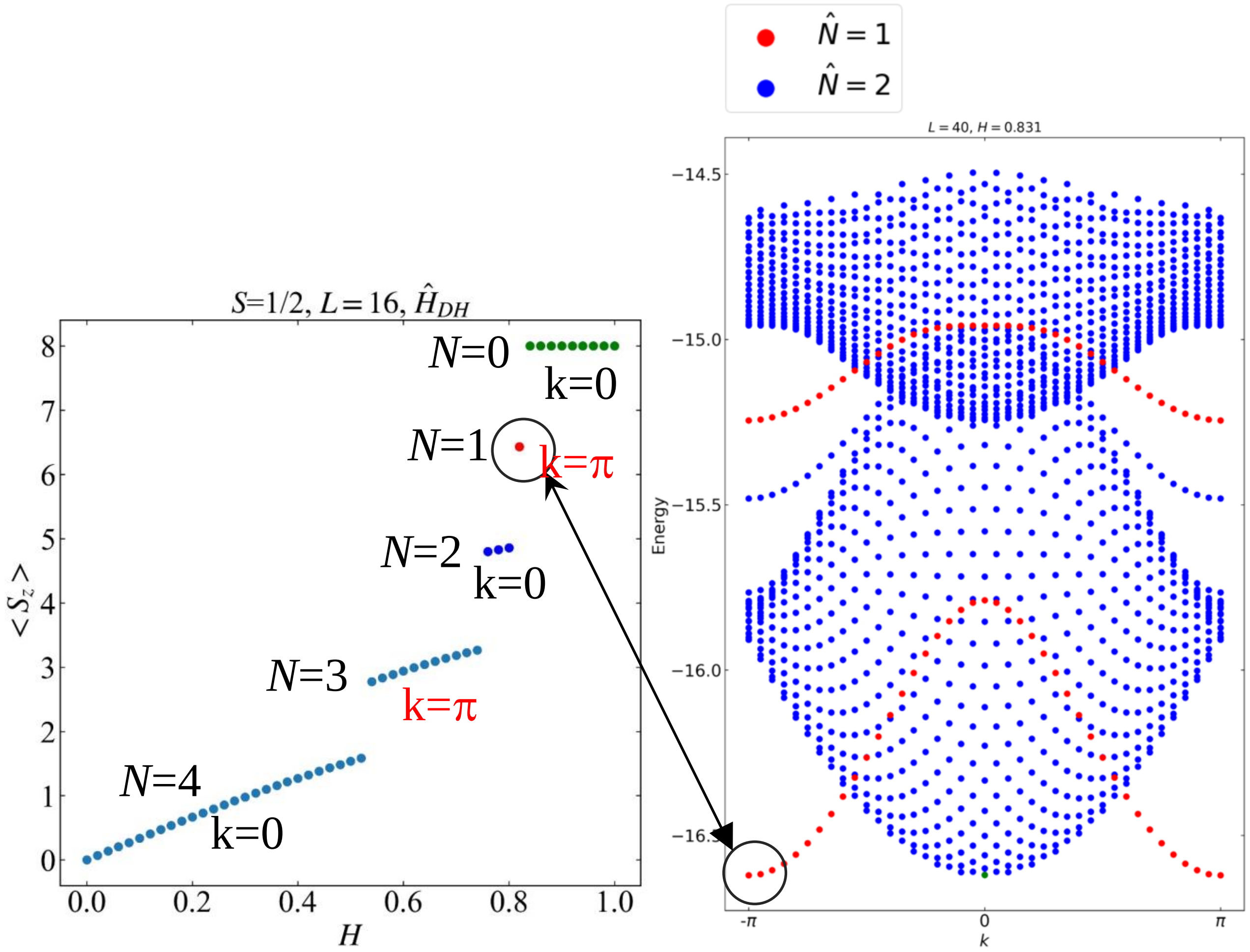
Sz=-1/2

$$|n_1, n_2, \cdots, n_L\rangle =: |\boldsymbol{n}\rangle$$

$$n_i = 0, 1, \cdots 2S \ (\ i = 1, \cdots, L)$$

$$\hat{S}_{i,z}|\boldsymbol{n}\rangle = (S - n_i)|\boldsymbol{n}\rangle$$

Numerical Results for H_{DH} model (under the PBC)



- **Magnetization/deMagnetization Processes consisting of successive escape/penetration of Solitons,**

- **One-Soliton states (N=1) have minimum energy at k=π**
  Two-Soliton states (N=2) have minimum energy at k=0,

We prove that the ground state has k= 0 (π), when N=even (odd)

Four Important Operators in the proof

- N : Soliton Number Operator
- T :One-site Translation Operator

$$\hat{T}|n_1,\cdots,n_L\rangle = |n_L, n_1, \cdots, n_{L-1}\rangle$$

- H_{DH}: Hamiltonian
- U : Sign-Changing Operator (Unitary)

Commuting Set

Definition of *U*

$$\hat{U}|\boldsymbol{n}\rangle = \underbrace{(-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle}_{\textbf{Signed Basis}}$$

$$|n_1, n_2, \cdots, n_L\rangle =: |\boldsymbol{n}\rangle$$

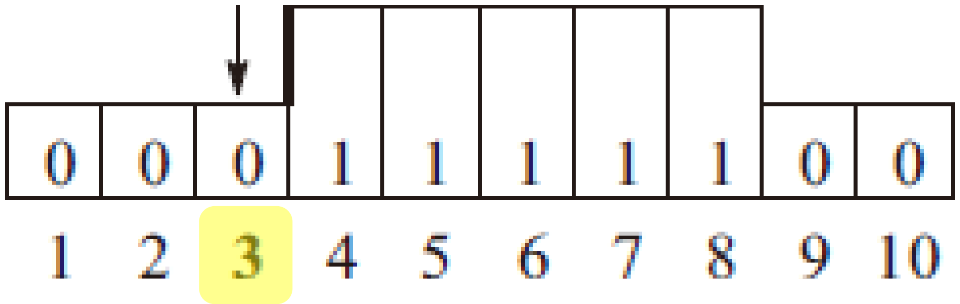
$$\delta(\boldsymbol{n}) = \sum_{i=1}^{L} \underbrace{i\left(n_{i+1} - n_i + |n_{i+1} - n_i|\right)/2}.$$

1 for $n_i\ n_{i+1}$ = “ 01 ”
0 otherwise

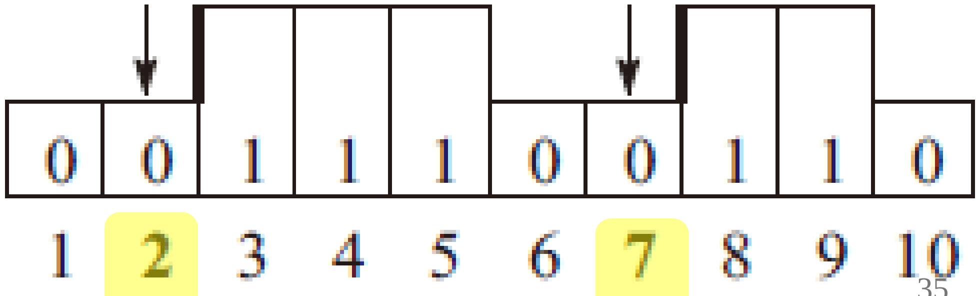
δ(**n**): sum of the coordinates of the left edge of each soliton

Ex.

$$(-1)^3|0001111100\rangle \qquad (-1)^3$$



$$(-1)^{2+7}|0011100110\rangle \qquad (-1)^{2+7}$$



$\hat{U}$: Properties of Sign-Changing Operator (Unitary)

$$[\hat{U}, \hat{N}] = 0$$

$$\langle \boldsymbol{n} | \hat{U} \hat{\mathcal{H}}_{DH} \hat{U} | \boldsymbol{n} \rangle \leq 0, \quad \text{for} \quad \boldsymbol{n} \neq \boldsymbol{n}'$$

Off diagonal matrix element is non-positive

$$\hat{U}\hat{T} = \hat{T}\hat{U} \exp\left(i\pi\hat{N}\right)$$

In the eigenspace with N even (odd), U commutes with T
anticommutes with T

$$\langle \boldsymbol{n}|\hat{U}\hat{\mathcal{H}}_{DH}\hat{U}|\boldsymbol{n}\rangle \leq 0, \quad \text{for} \quad \boldsymbol{n} \neq \boldsymbol{n}'$$

→ The ground state of $\hat{U}\hat{\mathcal{H}}_{DH}\hat{U}$ has the crystal momentum k=0 (Theoem.Perron-Frobenius)

$$\hat{U}\hat{T} = \hat{T}\hat{U}\exp\left(i\pi\hat{N}\right)$$

→ In the eigenspace with N even (odd),
U conserves the crystal momentum
U changes the crystal momentum by π

➡ In the eigenspace with N even (odd),
the ground state of $\hat{\mathcal{H}}_{DH}$ has the crystal momentum 0 (π)

Higher S cases

Higher S: Several Soliton Number Operators $\hat{N}_f$

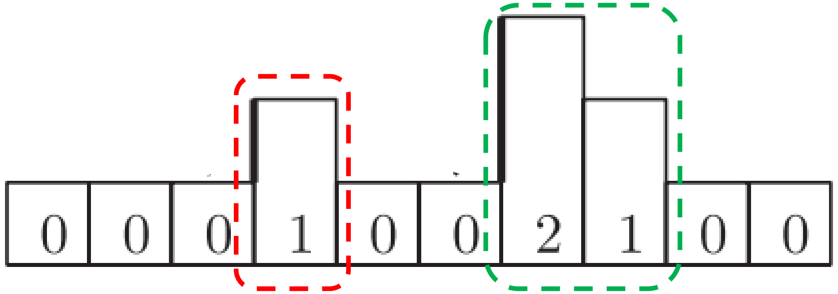
f (=1,2, ・・・2S) with amplitude/height f

$$|n_1, n_2, \cdots, n_L\rangle =: |\boldsymbol{n}\rangle$$

$$n_i = 0, 1, \cdots 2S \ (\ i = 1, \cdots, L)$$

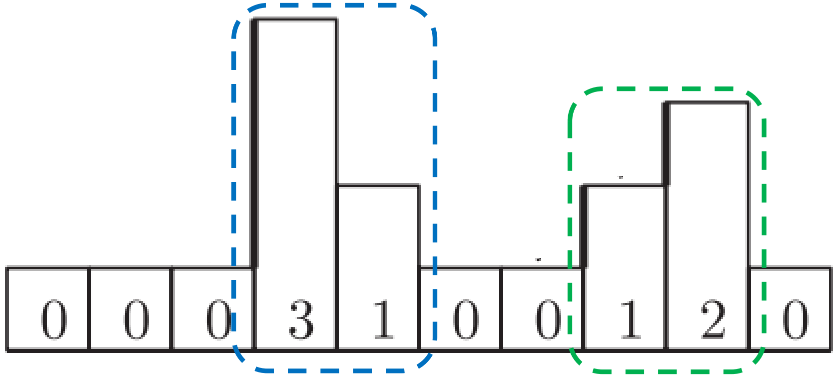
$$\hat{S}_{i,z}|\boldsymbol{n}\rangle = (S - n_i)|\boldsymbol{n}\rangle$$

Ex. S=3/2,



(N_1, N_2, N_3)=(1,1,0)

$$|00100000\rangle, \quad (N_1, N_2, N_3) = (1, 0, 0)$$
$$|01211000\rangle, \quad (N_1, N_2, N_3) = (0, 1, 0)$$
$$|01123100\rangle, \quad (N_1, N_2, N_3) = (0, 0, 1)$$
$$|01310230\rangle, \quad (N_1, N_2, N_3) = (0, 0, 2)$$



(N_1, N_2, N_3)=(0,1,1)

$\hat{U}$: Sign-Changing Operator (Unitary)

- $[\hat{U}, \hat{N}_f] = 0$

- $\langle \boldsymbol{n} | \hat{U} \hat{\mathcal{H}}_{\rm p} \hat{U} | \boldsymbol{n}' \rangle \leq 0, \quad \text{for } \boldsymbol{n} \neq \boldsymbol{n}'$; off diagonal matrix element is non-positive

→ the ground state of $\hat{U} \hat{\mathcal{H}}_{\rm p} \hat{U}$ has crystal momentum k=0 Cf. Perron-Frobenius Threorem

"Total Height of Solitons"

- $\hat{U} \hat{T} \hat{U} = \pm \hat{T}$ When $\sum_{f=1}^{2S} f N_f$ is even (odd), U conserves the crystal momentum (changes the crystal momentum by π)

→When $\sum_{f=1}^{2S} f N_f$ is even (odd) , the ground state of $\hat{\mathcal{H}}_{\rm p}$ has the crystal momentum 0 (π)

Height Parity Effect⇒Spin Parity Effect

→When $\sum_{f=1}^{2S} f N_f$ is even (odd) , the ground state of $\hat{\mathcal{H}}_{\mathrm{p}}$ has the crystal momentum 0 (π)

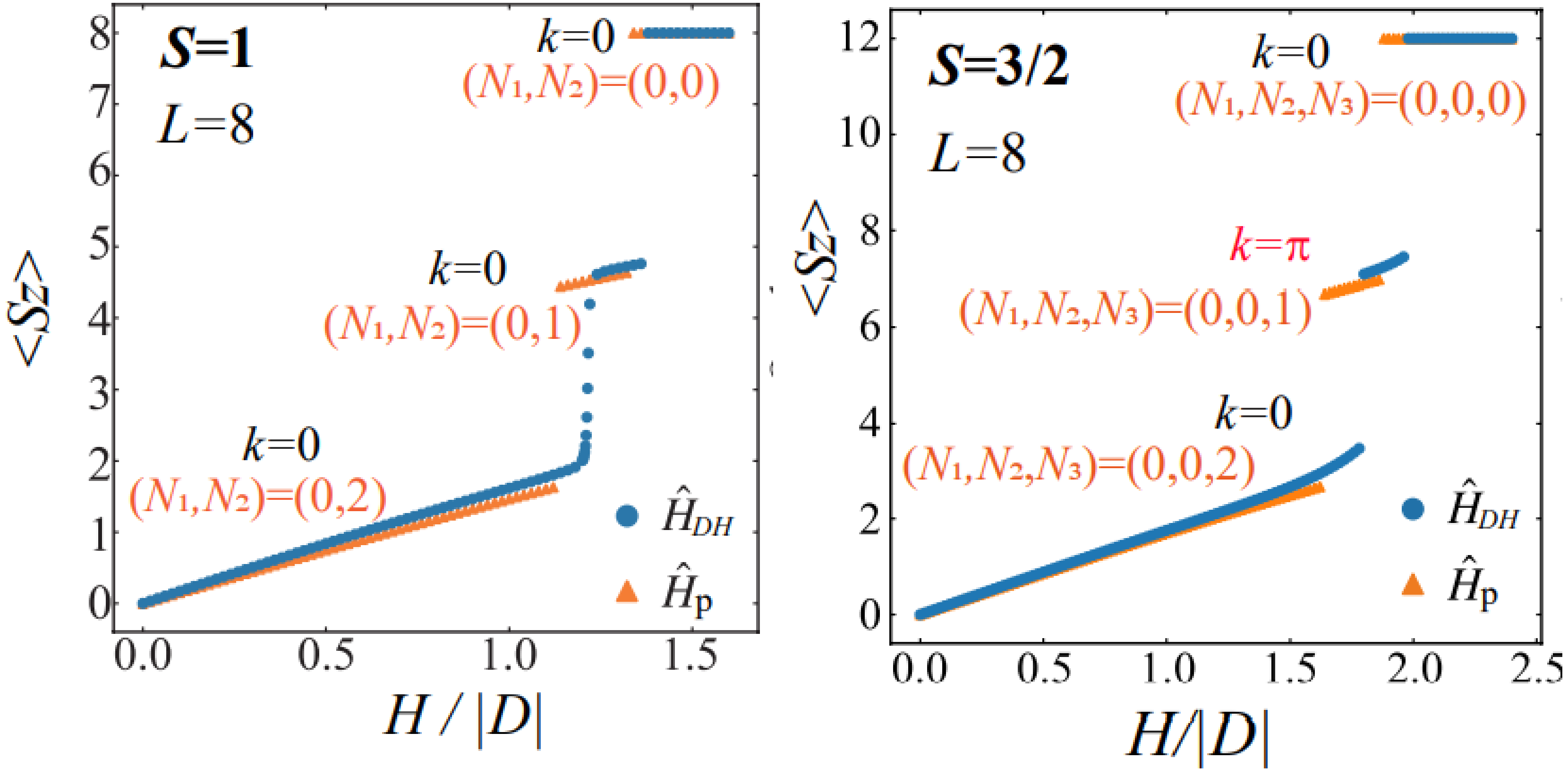
The crystal momentum in the ground state is determined by height of soliton as well as soliton numbers (“Height Parity Effect”)。

Ex. ：Lowest energy state in the one-soliton state

	S=1/2	**S=1**	**S=3/2**	**S=2**	**S=5/2**
f=1	$k_{min}=\pi$	$k_{min}=\pi$	$k_{min}=\pi$	$k_{min}=\pi$	$k_{min}=\pi$
f=2		$k_{min}=0$	$k_{min}=0$	$k_{min}=0$	$k_{min}=0$
f=3			$k_{min}=\pi$	$k_{min}=\pi$	$k_{min}=\pi$
f=4				$k_{min}=0$	$k_{min}=0$
f=5					$k_{min}=\pi$

Only solitons with f=2S contribute to the ground state⇒Spin Parity Effect

Height Parity Effect ⇒Spin Parity effect



In the ground states, **only solitons with maximum height f=2S contribute**.

"Height parity effect"
Soliton with maximum height contribute ⟹ "Spin parity effect".

Summary of the second part:

- Different behavior in magnetization process between half-Integer and Integer Spins (a "Spin Parity Effect") in chiral ferromagnetic spin chain

  half-Integer Spin: Level crossing ⇔ Integer Spin: Level Repulsion

- **Models in the limit D/J ⇒ ∞ are canonical models** to understand Spin parity effect in monoaxial chiral ferromagnetic chain. This limit can be realized in Rydberg atom quantum simulators

  (Kunimi-Tomita-Katsura-Kato: Phys Rev. A).

- Essential is "height parity effect", a soliton with odd (even) height f has the k=π (k=0) in the lowest energy state.

- In the low energy sector, only solitons with maximum height f=2S contribute. It results in the spin parity effect in the magnetization process.

Summary

I: Introduction to Classical Chiral magnet

- Chiral magnet, Dzyaloshinskii-Moriya Interaction, Chiral Soliton Lattice
- Sine-Gordon Theory of Chiral Soliton Lattice
- Example of Real Material: CrNbS
- Similarity with Chiral Liquid Crystal, Vortex in type II superconductors,
- Hysteresis in Continuous Phase Transition: Common Property between Chiral magnets and Type II Superconductors Shinozaki et al, 2018

II: Quantum Spin Chain of monoaxial Chiral magnet Kodama et al , 2023

- Numerical Result: Difference between Half integer spin and Integer spin in chiral magnet
- Theory for S=1/2
- Theory for higher Spin ”height Parity effect”→”Spin Parity effect”