Chiral phase transition under acceleration and rotation

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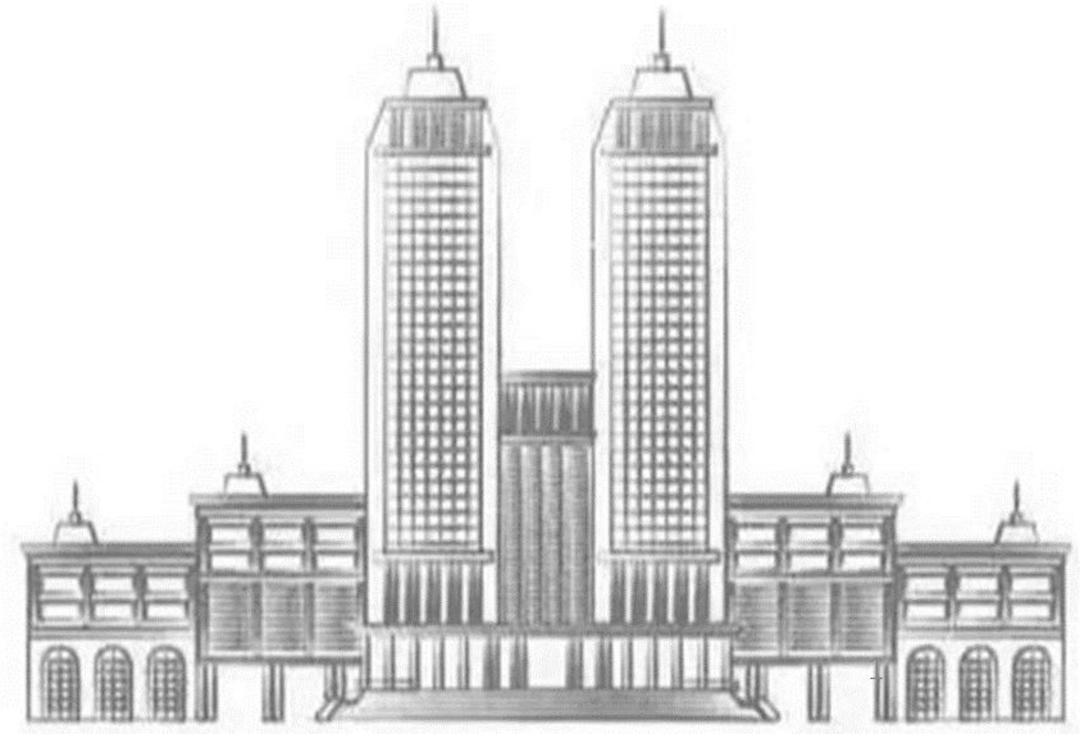
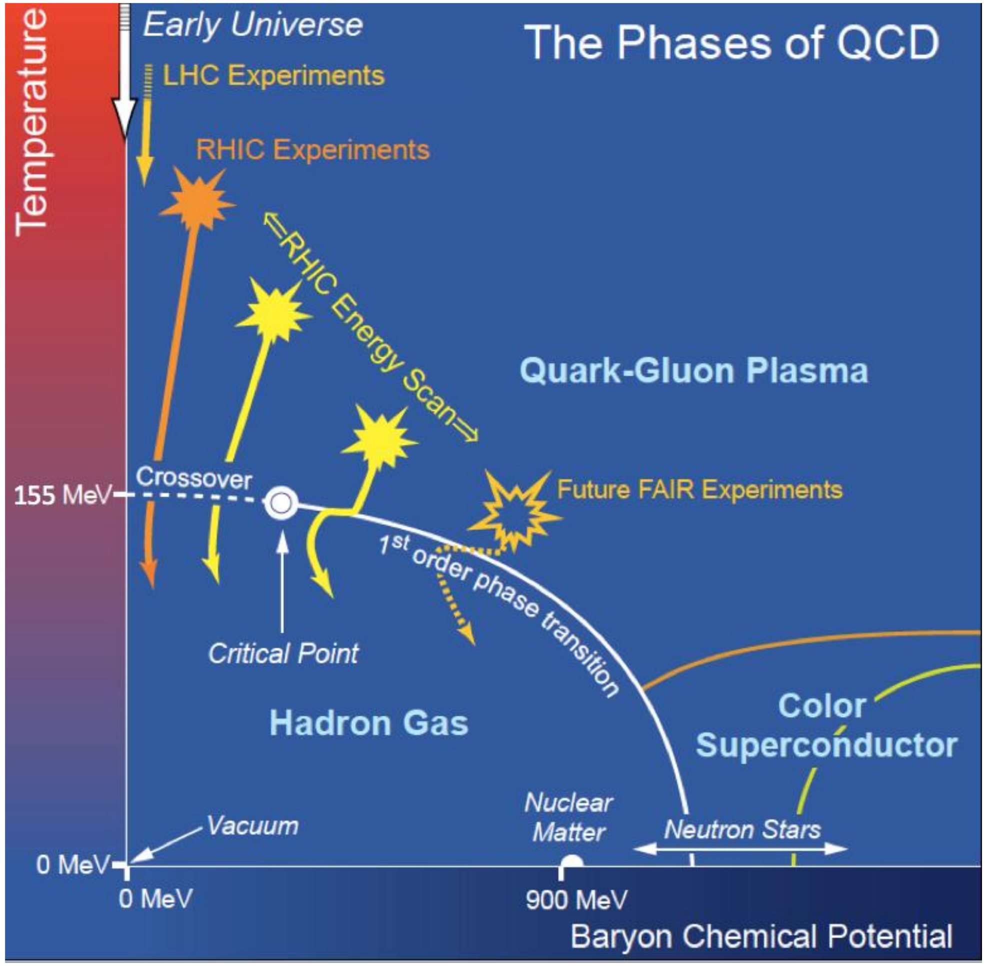
Collaborator: Hao-lei Chen, Xu-guang Huang



Outline

- Introduction： QCD phase transition under rotation
- Review: Uniform acceleration, Unruh effect, and Rindler coordinates
- Chiral symmetry under acceleration and rotation
- Summary

QCD phase transition



The Hot QCD White Paper (2015)

Order parameter for chiral phase transition:

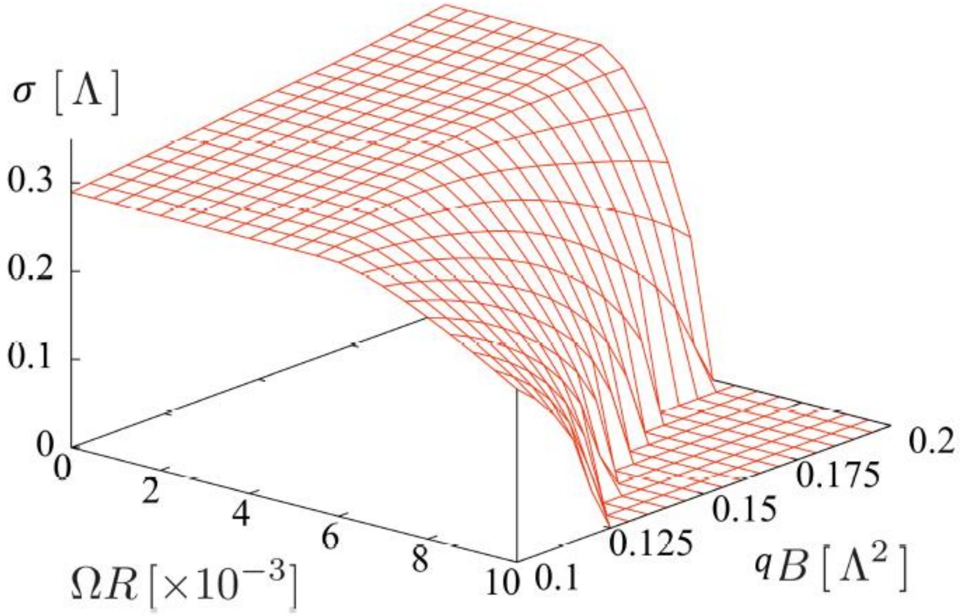
Chiral condensate $<\bar{\psi}\psi>$

QGP phase $<\bar{\psi}\psi>=0$ Chiral symmetry restoration

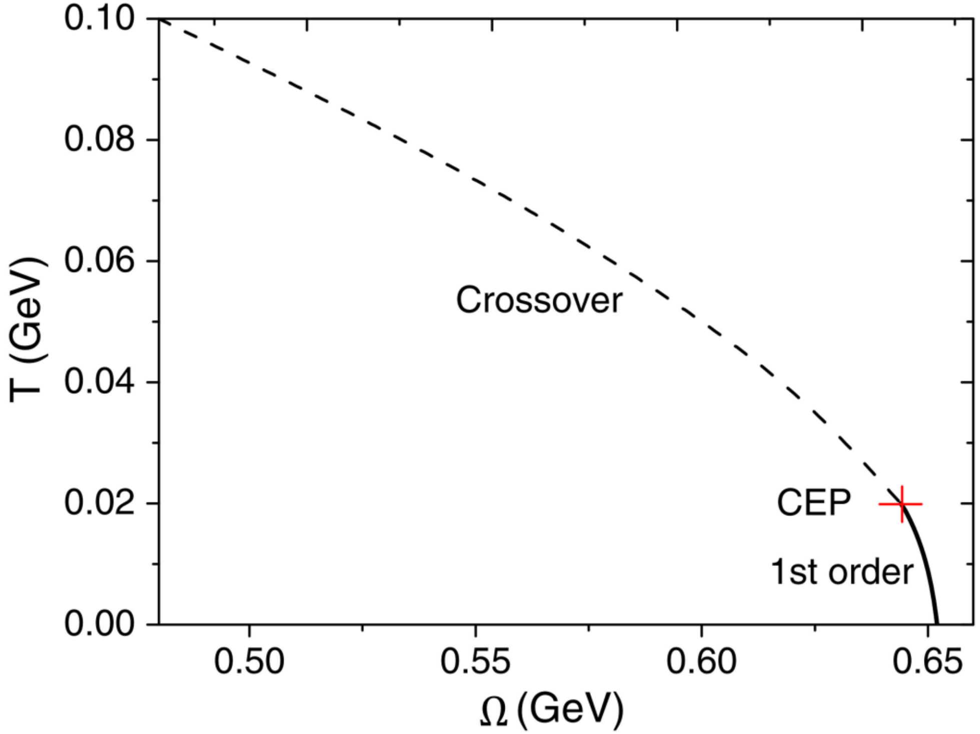
Hardronic phase $<\bar{\psi}\psi>\neq 0$ Chiral symmetry breaking

In recent years, we have focused not only on the phase transition in the T-μ plane, but also paid close attention to the phase structure under various conditions, such as magnetic fields, rotation, and acceleration.

Model calculations for rotation



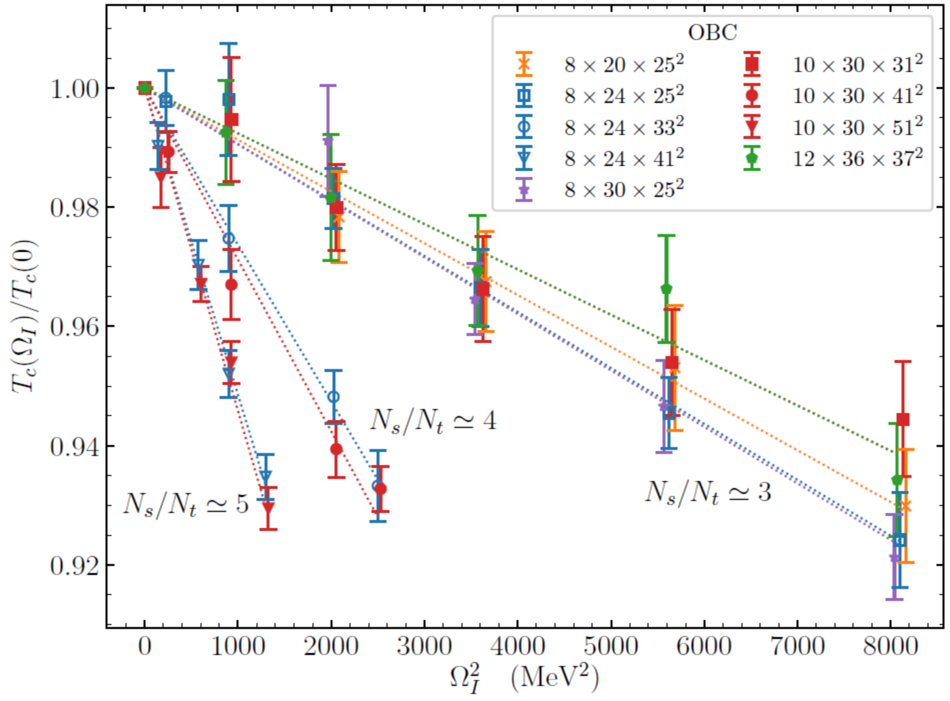
H. L. Chen, K. Fukushima, X. G. Huang and K. Mameda, Phys. Rev. D 93, 104052 (2016)



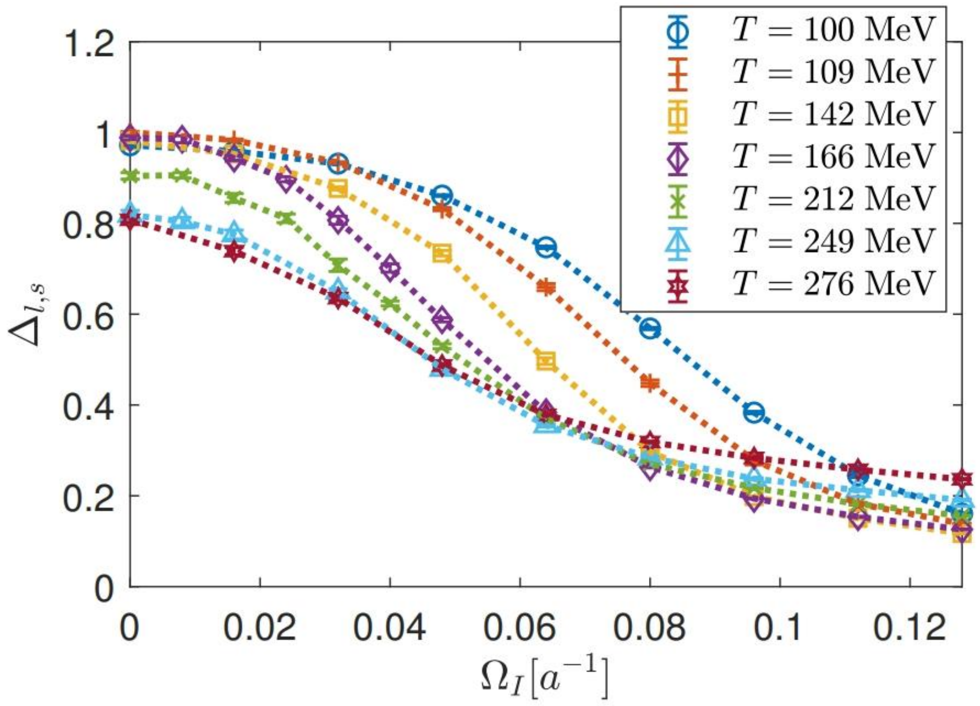
Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016) arXiv:1606.03808

Model calculations indicate that real rotation suppresses the chiral condensate and lowers the critical temperature, meaning rotation leads to chiral symmetry restoration.

Lattice result for rotation



Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.



Ji-Chong Yang and Xu-Guang Huang arxiv:2307.05755

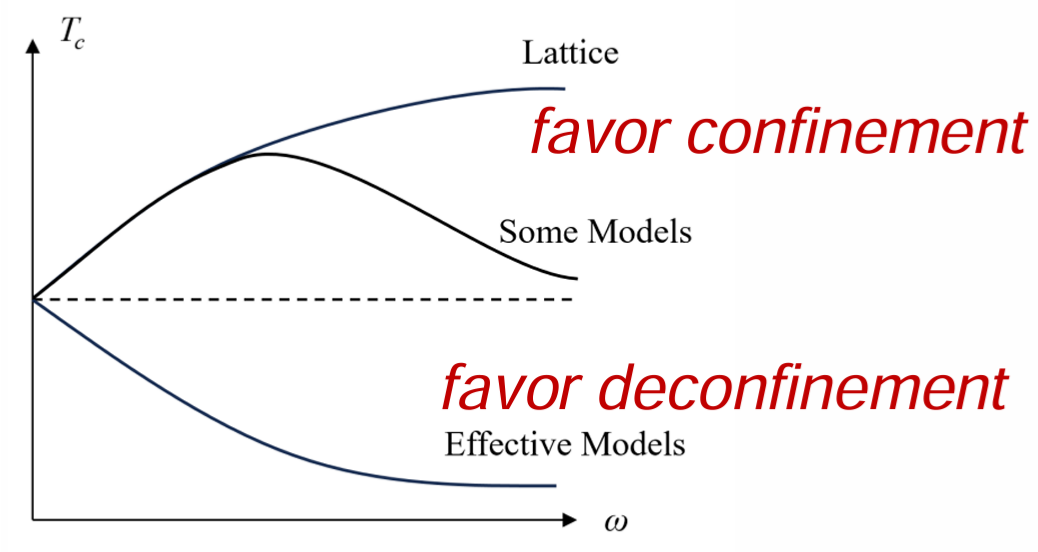
Lattice's results suggest that imaginary rotation promotes chiral symmetry restoration, which naively implies that real rotation promotes chiral symmetry breaking.

Rotation puzzle

Thus, there is a puzzle in the QCD phase transition under rotation.

The answer to this puzzle remains unknown.

The serious calculation in QCD is still required.



Form Pengfei Zhuang's talk in 2024

This puzzle is not the main stuff for today's talk, let's go to acceleration

NJL model in general spacetime

$$\mathcal{L}_{NJL} = \bar{\psi}[i\gamma^\mu\nabla_\mu - m_0]\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + (\bar{\psi}i\gamma^5\psi)^2\right]$$

$$Z = \int D[\bar{\psi},\psi] exp\left(i\int d^4x\sqrt{-g}\mathcal{L}_{\mathrm{NJL}}\right)$$

$$= \int D[\bar{\psi},\psi,\sigma,\pi] exp\left\{i\int d^4x\sqrt{-g}\left[\bar{\psi}\left(i\gamma^\mu\nabla_\mu - m - i\gamma^5\pi\right)\psi - \frac{\sigma^2+\pi^2}{2G}\right]\right\}$$

$$\Gamma = \frac{1}{i}\ln Z = -\int d^4x\sqrt{-g}\frac{\sigma^2+\pi^2}{2G} + \frac{1}{i}\ln\det\left(i\gamma^\mu\nabla_\mu - m - i\gamma^5\pi\right)$$

$$V_{eff} = -\frac{\Gamma}{\int d^4x\sqrt{g}} = \frac{\sigma^2+\pi^2}{2G} - \frac{1}{i}\mathrm{tr}\ln(i\gamma^\nu\nabla_\nu - m - i\gamma^5\pi)$$

$$\frac{\partial V_{\mathrm{eff}}}{\partial\sigma} = 0, \frac{\partial V_{\mathrm{eff}}}{\partial\pi} = 0$$

$$\frac{m - m_0}{G} = i\,\mathrm{tr}(S)$$

$$\frac{\pi}{G} = i\,\mathrm{tr}(i\gamma^5 S)$$

Where S the propagator

Unruh effect

The famous effect induced from acceleration is Unruh effect.

The Hawking–Unruh effect predicts that the accelerated observer sees Minkowski vacuum state as a thermal bath of particles with temperature $T = a/2\pi$.

Define the creation and annihilation operator in acceleration frame: $a_R(\omega)$and $a_R^\dagger(\omega)$

We have $a_R|0\rangle_R=0$, where $|0\rangle_R$ is Rindler vacuum

According to the Unruh effect we have

$${}_M\langle 0|a_R^\dagger a_R\ |0\rangle_M \sim \left(exp\left(\frac{2\pi\omega}{a}\right)-1\right)^{-1}$$

Unruh effect in heavy ion collisions

Acceleration will provide a temperature $T = a/2\pi$. Temperature will surely affect the QCD phase transition

So is it important in heavy-ion collisions?

May be Yes

According to Dmitri's work[1], the Unruh effect under strong color fields should be observable.

It said that the strength of the color-electric field $E \sim Q_s^2/g$, where Q_s is the saturation scale and g is the strong coupling and the typical acceleration it provided is $a \sim Q_S \sim 1GeV$ ($T = \frac{a}{2\pi} \sim 200MeV > T_c \cong 150MeV$)

[1] Kharzeev D, Tuchin K. From color glass condensate to quark–gluon plasma through the event horizon[J]. Nuclear Physics A, 2005, 753(3-4): 316-334.

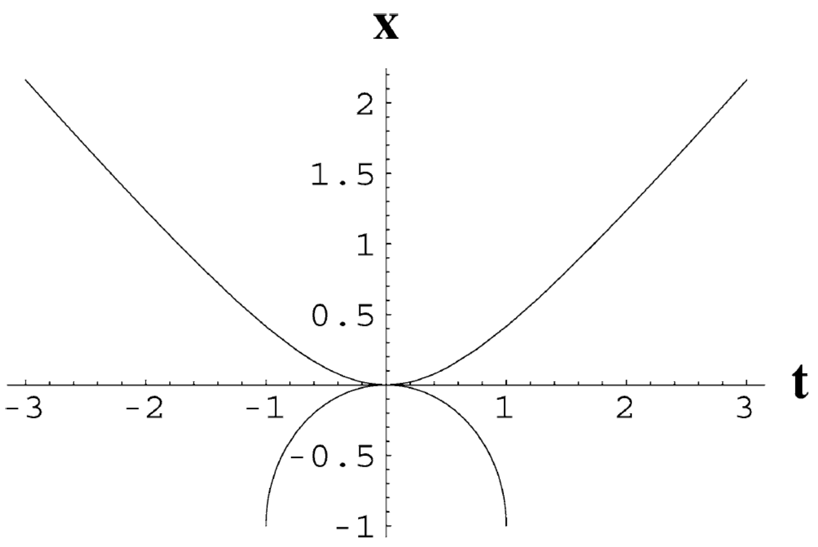
Uniform acceleration in relativity case

The equation of motion for a uniform acceleration particle:

$$v(t) = \frac{at}{\sqrt{1+a^2t^2}},$$

$$z(t) = a^{-1}\left(\sqrt{1+a^2t^2} - 1\right)$$

$$a = \frac{d}{dt}\frac{v}{\sqrt{1-v^2}}$$



Taken from Kharzeev D, Tuchin K. Nuclear Physics A, 2005, 753(3-4): 316-334.

The trajectory is hyperbola in Minkowski coordinates :

$$\left(z+\frac{1}{a}\right)^2 - t^2 = \frac{1}{a^2}$$

Rindler spacetime

Minkowski coordinates (T,X,Y,Z)

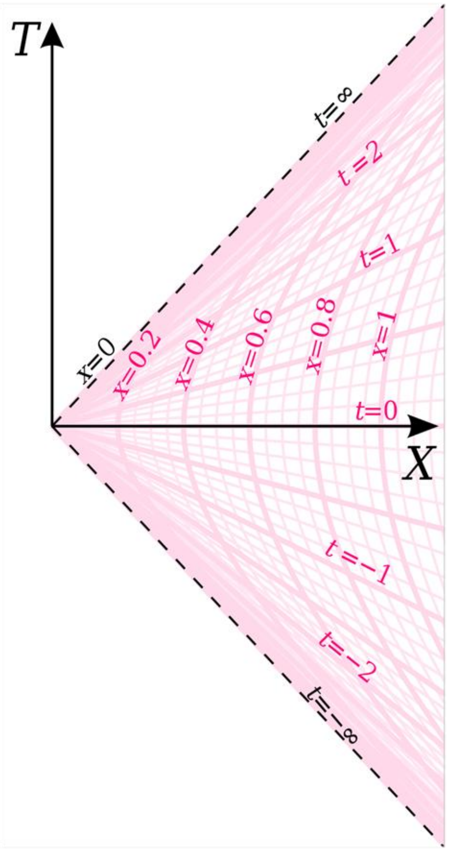
$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2$$

Coordinates transformation :

$$T = x\sinh(t)\ , X = x\,cosh(t), Y = y, Z = z$$

Rindler coordinates (t,x,y,z)

$$\boldsymbol{ds^2 = -x^2dt^2 + dx^2 + dy^2 + dz^2}$$



For a uniform acceleration particle,

the world line in Minkowski coordinates:

$$T = x\sinh(a\tau)\ , X = x cosh(a\tau)$$

the world line in Rindler coordinates:

$$x = \frac{1}{a}\ , t = a\tau$$

Euclidean Rindler coordinate

$$T = \xi\, sinh(t) \qquad T_E = \xi \sin(t_E)$$
$$Z = \xi\, cosh(t) \quad \Rightarrow \quad Z = \xi\, cos(t_E)$$

If we define a field in Euclidean coordinate: $\phi \equiv \phi(T_E, Z)$

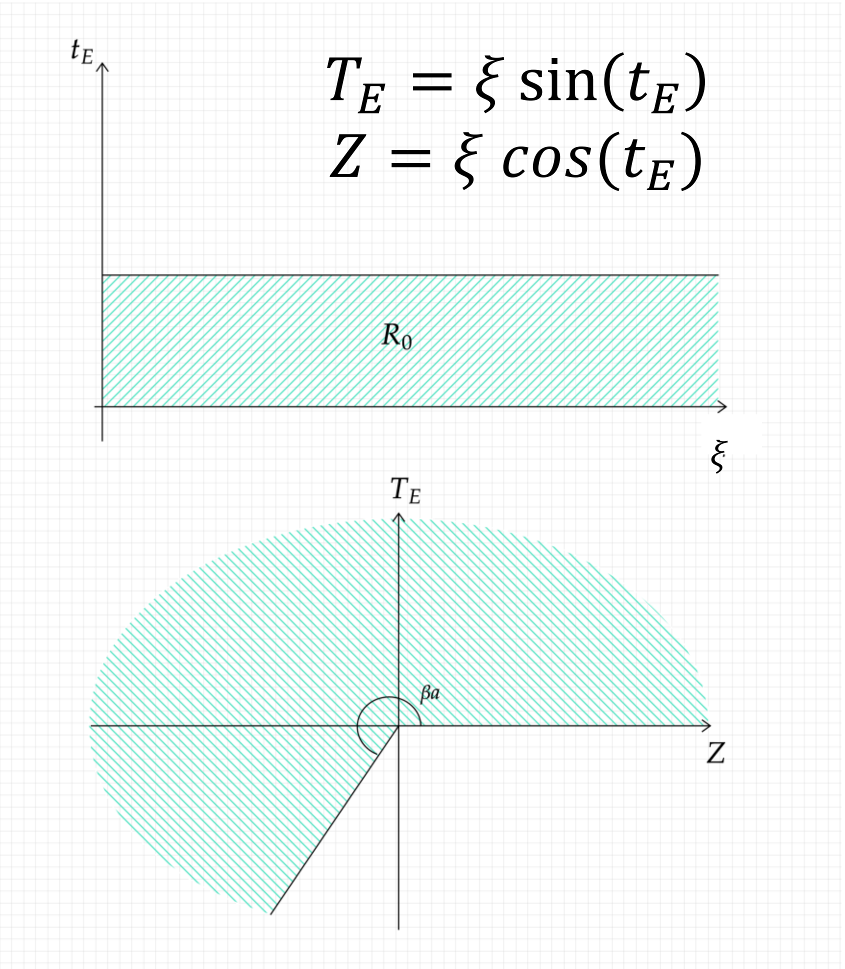
Because we did not apply any period condition to this field, it can be regarded as a field in a zero-temperature background.

It is readily that ϕ gets a period in Euclidean Rindler coordinate which is $\phi(t_E, \xi) = \phi(t_E + 2\pi, \xi)$

Remember the relation between Rindler time and uniform acceleration particle's proper time is $t = a\,\tau$, which means $\phi(\tau, \xi) = \phi(\tau + i\, 2\pi/a, \xi)$

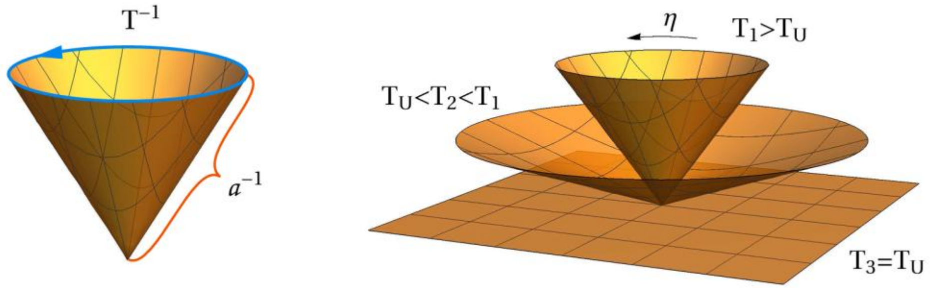
If we naively regard this period of proper time as the temperature of an accelerated observer feels, we will have $T = \frac{a}{2\,\pi}$

Euclidean Rindler coordinate



If we identify the hypersurface $t_E = 0$ and $t_E = a\beta$

The manifold it represented will become a cone with an angle deficit $2\pi(1-\nu^{-1})$ Where $\nu = \frac{2\pi T}{a} = T/T_U$



In previous studies, this temperature was always set to T_U which is the Unruh effect suggests.

If a particle moves in a finite temperature background, it may feel a temperature large than T_U. Thus, we are interested in a phase diagram in $T - a$ plane.

Scalar under acceleration

The explicit form of the KG equation in Rindler coordinate is

$$(-\frac{1}{\xi^2}\frac{\partial^2}{\partial\eta^2}+\frac{\partial^2}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial}{\partial\xi}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-m^2)\phi=0$$

The eigenfunctions

$$\phi=\frac{1}{\sqrt{2\omega}}\frac{1}{2\pi}\sqrt{\frac{2\omega\sinh\pi\omega}{\pi^2}}K_{i\omega}(m_\perp\xi)\mathrm{e}^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}}$$

K_μ is the modified Bessel function satisfying the equation:

$$z^2\frac{d^2K_\nu(z)}{dz^2}+z\frac{dK_\nu(z)}{dz}=(z^2+\nu^2)\,K_\nu(z)$$

The Green function:

$$G_\beta(x_E,x'_E)=\sum_n\int_0^\infty\frac{\mathrm{d}\omega}{\beta}\frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\frac{\mathrm{e}^{i\omega_n(\tau-\tau')+i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}}{\omega_n^2+\omega^2}\frac{2\omega\sinh\pi\omega}{\pi^2}K_{i\omega}(m_\perp\xi)K_{i\omega}(m_\perp\xi').$$

Scalar under acceleration

The Green function:

$$G_\beta(x_E, x'_E) = \sum_n \int_0^\infty \frac{\mathrm{d}\omega}{\beta} \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\mathrm{e}^{i\omega_n(\tau-\tau')+i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}}{\omega_n^2+\omega^2} \frac{2\omega\sinh\pi\omega}{\pi^2} K_{i\omega}(m_\perp\xi)K_{i\omega}(m_\perp\xi'),$$

The trace of Green function in massless limit:

$$\mathrm{tr}G(x, x'|m=0) = \frac{1}{2\pi^3}\int_0^\infty \mathrm{d}\omega \frac{(2\pi T)^2\pi\omega}{2\tanh\omega\pi}$$

In NLσ model $\sigma^2 = f_\pi^2 - \frac{\nu^2}{\xi^2}\frac{N}{48\pi^2}$

Critical temperature $T_c = a\xi\sqrt{\frac{12 f_\pi^2}{N}}$

The accelerated observer is on $\xi = 1/a$

Spinor under acceleration

The explicit form of the Dirac equation in Rindler coordinate is

$$\left[\frac{\gamma^{\hat{0}}}{\xi}\left(i\partial_0 + \frac{i}{2}\gamma^{\hat{0}}\gamma^{\hat{3}}\right) + i\gamma^{\hat{i}}\partial_i - m\right]\psi = 0$$

Directly solving the Dirac equation might be challenging. Thus, we define the Green's function:

$$\left(\hat{D} - s\right) S(x, y; s) = \frac{1}{\sqrt{-g}}\delta(x, y),$$
$$\left(\hat{D} + s^{\dagger}\right) G(x, y; s) = S(x, y; s),$$
$$\left(\hat{D}^2 - s^{\dagger} s\right) G(x, y; s) = \frac{1}{\sqrt{-g}}\delta(x, y)$$

Spinor under acceleration

The explicit form of the Dirac equation's square is

$$-\frac{1}{\xi^2}\left(i(i\partial_0)-\frac{\gamma^{\hat{0}}\gamma^{\hat{3}}}{2}\right)^2+\partial_3^2+\frac{1}{\xi}\partial_3+\partial_1^2+\partial_2^2-m^2$$

Compared to the case of a scalar field, there is only an additional shift in the energy part. The eigenfunctions is

$$e^{-i\omega t}e^{i\mathbf{k}\cdot\mathbf{x}}K_{i\omega-\frac{\gamma^{\hat{0}}\gamma^{\hat{3}}}{2}}(m_\perp\xi)$$

The function contains $\boldsymbol{\gamma}^0\boldsymbol{\gamma}^3$ obeyed $\quad f(\gamma^0\gamma^3)=P^+f(1)+P^-f(-1)$

Where, $\quad P^\pm=\frac{1\pm\gamma^0\gamma^3}{2}$

Spinor under acceleration

The Green's function $\boldsymbol{G}$ is

$$G_\beta(x,x') = \int_0^\infty \mathrm{d}\omega \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\sinh(\beta\omega/2 - \omega|\tau-\tau'|)}{\cosh(\beta\omega/2)} \frac{\sinh\pi(\omega + i\gamma^0\gamma^3/2)}{\pi^2} K_{i\omega-\gamma^0\gamma^3/2}(m_\perp\xi) K_{i\omega-\gamma^0\gamma^3/2}(m_\perp\xi') \mathrm{e}^{i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}.$$

If we apply the NJL model calculation with the massless limit

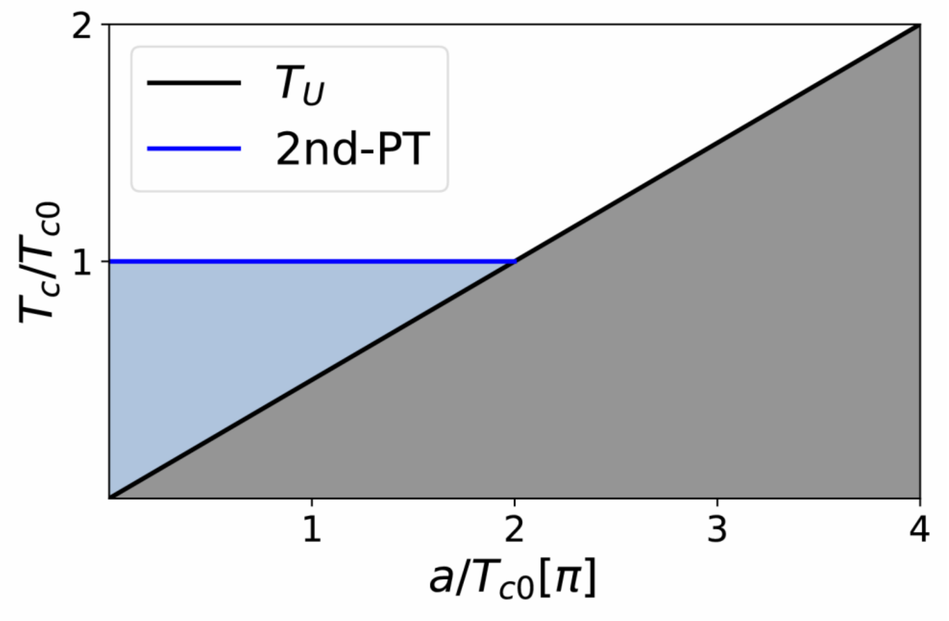
$$\begin{aligned} Tr\, G_\beta(m=0) &= \int_0^\infty d\omega \frac{\omega}{\pi^2\xi^2} \tanh(\beta\omega/2) \\ &= \frac{1}{\pi^2\xi^2}\int_0^\infty \omega(1 - \frac{2}{e^{\beta\omega}+1}) \end{aligned}$$

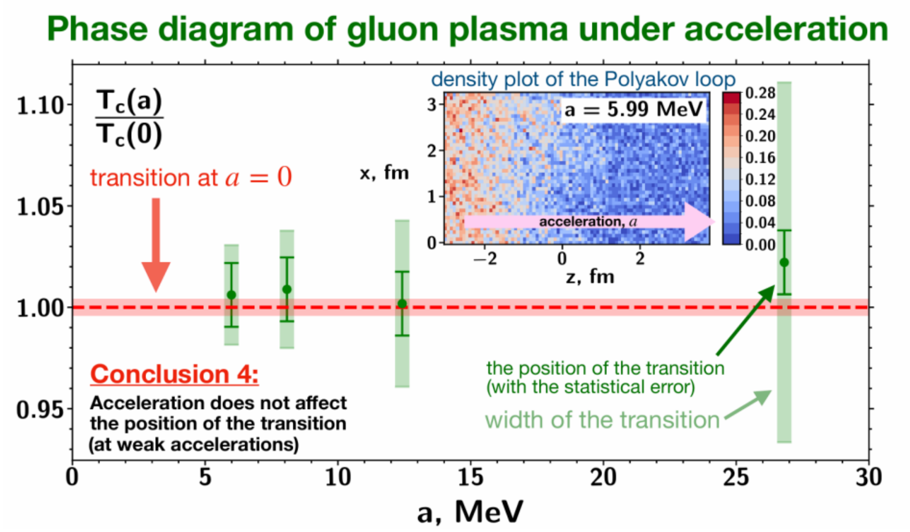
We can obtain the critical temperature is

$$T_c = a\xi\sqrt{\frac{3\Lambda^2}{\pi^2} - \frac{6}{G}}.$$

It is trivial in $\boldsymbol{\xi = \frac{1}{a}}$ again.

Spinor under acceleration





Form Maxim Chernodub's poster in Chirality2024

It's frustrating that acceleration does not seem to affect the critical temperature.

This temperature is defined by the accelerated observer's proper time. This result means the phase transition only depends on the temperature the observer feels.

The relationship between temperature in an accelerated frame(T) and inertial frame(T_I) remains unknown. The diagram in $T_I - a$ plane may be non-trivial.

For now, we only know $T(a=0) = T_I$ and $T = \frac{a}{2\pi} \rightarrow T_I = 0$

Under rotation and acceleration

$$\mathcal{L}_{NJL} = \bar{\psi}[i\gamma^{\mu}\nabla_{\mu} - m_0]\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right]$$

$$g_{\mu\nu} = \begin{pmatrix} (1+az)^2 - \omega^2 r^2 & \omega y & -\omega x & 0 \\ \omega y & -1 & 0 & 0 \\ -\omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\{\gamma_{\mu}(x), \gamma_{\nu}(x)\} = 2g_{\mu\nu}(x), \{\gamma_{\hat{m}}, \gamma_{\hat{n}}\} = 2\eta_{\hat{m}\hat{n}}$$

$$g_{\mu\nu}g^{\nu\rho} = \delta_{\mu}^{\rho}, g^{\mu\nu}(x) = e_{\hat{m}}^{\mu}(x)e^{\nu\hat{m}}(x), \gamma_{\mu}(x) = e_{\mu}^{\hat{m}}(x)\gamma_{\hat{m}}.$$

$$\gamma^0(x) = \frac{1}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\gamma^{\hat{0}}, \gamma^i(x) = \frac{(\boldsymbol{\omega}\times\boldsymbol{x})^i}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\gamma^{\hat{0}} + \gamma^{\hat{\imath}}$$

Covariant derivative :

$$\nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu}, \Gamma_{\mu} = -\frac{i}{4}\omega_{\mu\hat{\imath}\hat{\jmath}}\sigma^{\hat{\imath}\hat{\jmath}}, \sigma^{ij} = \frac{i}{2}[\gamma^{\hat{\imath}}, \gamma^{\hat{\jmath}}], \omega_{\mu\hat{\imath}\hat{\jmath}} = g_{ab}e_{\hat{\imath}}^{a}\nabla_{\mu}e_{\hat{\jmath}}^{b}$$

$$\Gamma_0 = -\frac{i}{2}\boldsymbol{\omega}\cdot\sigma + \frac{1}{2}\boldsymbol{a}\cdot\boldsymbol{\alpha},$$

NJL model action in rotation and acceleration frame:

$$S = \int d^4x \left\{\bar{\psi}\left[i\gamma^{\hat{\mu}}\partial_{\mu} + i a\cdot x\gamma^{\hat{\imath}}\partial_i + \frac{i}{2}a\cdot\gamma + \gamma^{\hat{0}}\omega\cdot J - m_0\phi\right]\psi + \frac{G}{2}\phi\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\psi\right)^2\right]\right\}$$

Where $\phi = 1 + \boldsymbol{a}\cdot\boldsymbol{x}$

Dirac equation

The Dirac equation:

$$\left(\frac{i\gamma^{\hat{\mu}}\partial_\mu + i\boldsymbol{a}\cdot\boldsymbol{x}\gamma^{\hat{\imath}}\partial_i + \frac{i}{2}\boldsymbol{a}\cdot\boldsymbol{\gamma} + \gamma^{\hat{0}}\boldsymbol{\omega}\cdot\boldsymbol{J}}{\phi} - m\right)\psi = 0$$

To simplify the formalism, acceleration and angular velocity are chosen along the z-direction.

Thus, the Dirac equation's square is

$$D^2 = \frac{1}{\phi^2}\left\{\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right]^2 - \frac{1}{4}a^2\right\} + \partial_3^2 + \frac{1}{\phi}a\partial_3 + \hat{\gamma}^0\hat{\gamma}^3\frac{a}{\phi^2}i\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right] + \partial_1^2 + \partial_2^2$$

Similarly, it can be seen that compared to the non-rotating case, there is only an additional shift in the energy part due to rotation.

Gap equation

When $\boldsymbol{m_0 = 0}$, we can set $\boldsymbol{\pi}$ condensate $\boldsymbol{\pi = 0}$

$$\frac{m}{G} = i\,\mathrm{tr}(S)$$

When $\boldsymbol{T = \frac{a}{2\pi}}$, , $\boldsymbol{m_0 = 0}$, the gap equation become

$$\frac{1}{G} = \sum_{l,k,s_1} \int d\,\Omega \frac{1}{2\pi} \frac{1}{N_{l,k}^2} \frac{-is_1}{2a} \frac{cosh(\pi\,\Omega/a)}{\pi^2} \left\{\tanh\left(\frac{\Omega-\omega j}{2T}\right) + \tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r) + J_{l+1}^2(p_{l,k}r)\right]$$

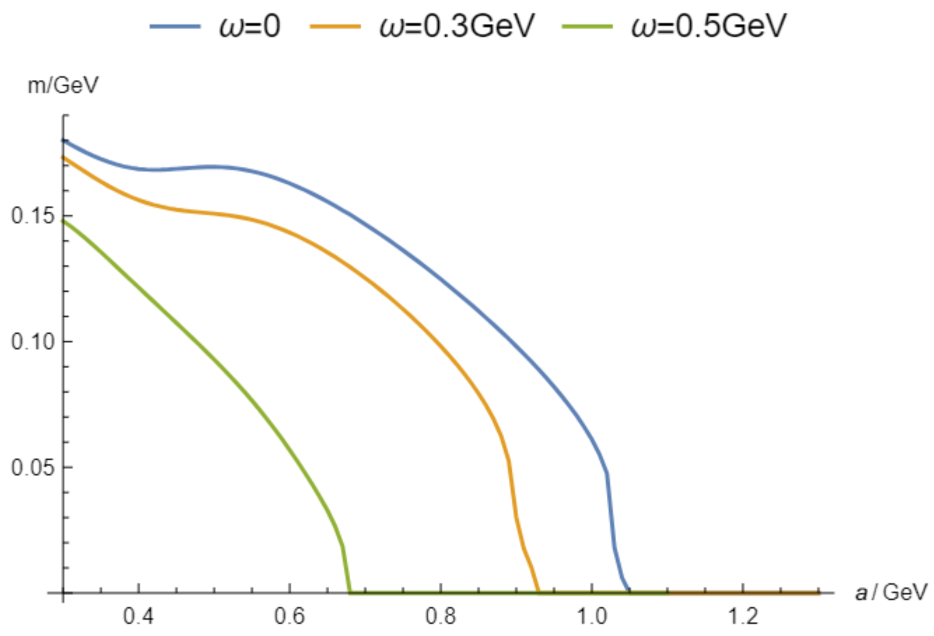
If we ignore the boundary, set $\phi = 1$, and take the non-rotation limit, we have

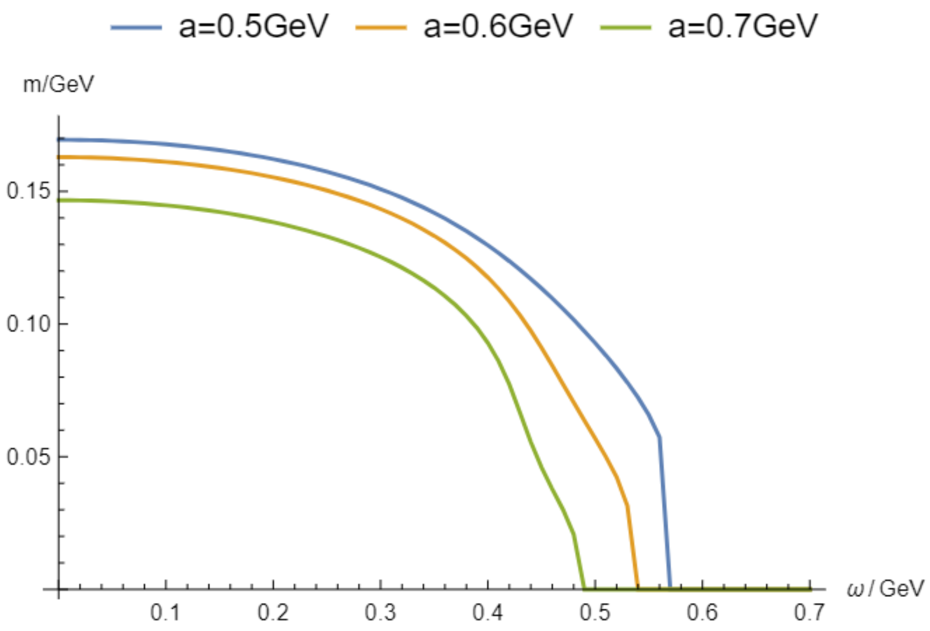
$$1 = G\int d\Omega \int \frac{d^2p_t}{(2\pi)^2} \frac{-i}{a} \frac{sinh(\pi\Omega/a)}{\pi^2} \left\{K^2_{\frac{i\Omega}{a}+\frac{1}{2}}(\alpha) - K^2_{\frac{i\Omega}{a}-\frac{1}{2}}(\alpha)\right\}$$

Where, $\boldsymbol{\alpha = \left[\frac{p_{l,k}^2+m^2}{a^2}\right]^{\frac{1}{2}}}$

which is consistent with previous work.

Result



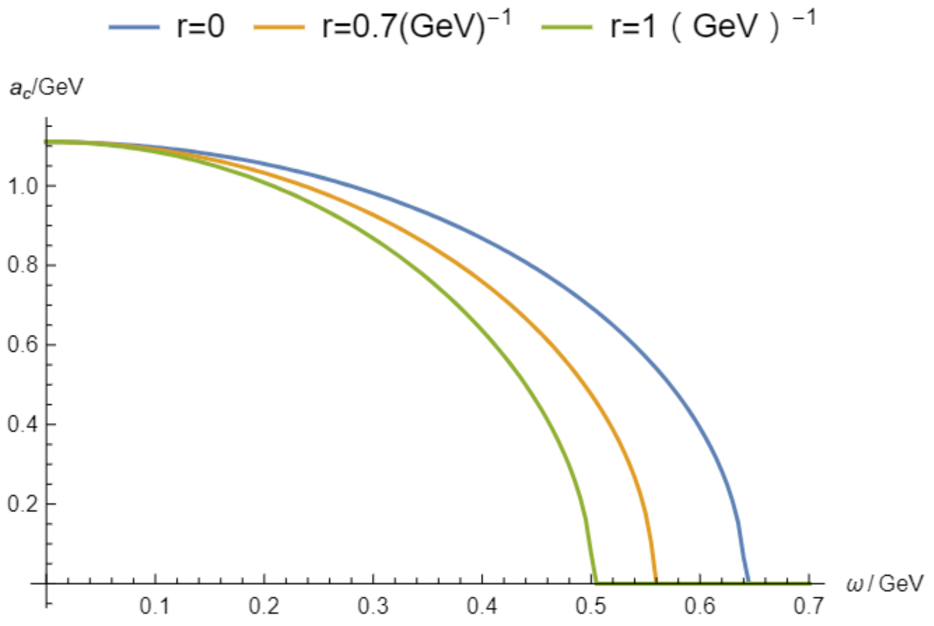


- $T = T_U$
- **both acceleration and rotation restore the chiral symmetry**

Massless limit

When $m \to 0$, the gap equation become:

$$G\left(\frac{\Lambda^2}{2\pi^2} - \frac{a^2(r^4\omega^4+1) - r^2\omega^4 + 3\omega^2}{24\pi^2(r^2\omega^2-1)^2}\right) = 1$$



The critical acceleration (a_c) decreases as the angular velocity increases.
The effects of rotation become increasingly significant with increasing radius.

Summary

- We develop the formulism under rotation and acceleration.
- The critical temperature is independent of acceleration
- Both the acceleration and rotation restore the chiral symmetry.
- A phase diagram in $a-\omega$ plane are obtained.

Outlook

- The relationship between temperature in acceleration frame(T) and inertial frame(T_I) remains unclear. A T_I-a diagram may non-trivial.

Thanks!