N 17

INS (18, 19)

~~Change~~ On the Conservation of the Total Angular Momentum in the Process of β-Disintegration

On ~~the~~ Application of ~~Theorems of the Theorems~~ ~~The Angular Momentum~~ of The Rotation Group ~~Theorem~~ to the Theory of β-Disinteg.

By Hideki Yukawa

(March 6, 1937)

~~Ele~~ Electron, neutrino を二つのstateとして有する light particle が Dirac typeの式に従ふと~~する~~し、之が nucleusとの相互作用は、~~(r, exp)~~ $\vec r$, ~~...~~ Coulomb field以外に

$$g'\tau' U \exp(-2\pi i \nu T) \qquad e A_0(r)$$

なる perturbation term を有するとすると、~~spin angular momentum~~ が nucleusのtotal spinをあらはすvectorを $\vec S$ とし、light particleのtotal spinをあらはすvectorを $\vec M = \vec m + \frac{h}{4\pi}\vec\sigma$ とすると、total angular momentumのconservationの ~~成立の~~ 条件は、相互のinteractionの形如何に関せず一般に

$$U\,(\vec S + \vec M) - (\vec S + \vec M)\,U = 0.$$

で与へられる筈である。

$\vec\alpha, \beta$ ~~$U(\vec r)$, $\alpha_x \alpha_y$ = ...~~ の積を出来るもので其の中 linearly independent なものは16ある。之は

$$\overset{(1)}{1},\ \overset{(3)}{\vec\alpha},\ \overset{(1)}{\beta},\ \overset{(3)}{\alpha_x\alpha_y,\ \alpha_y\alpha_z,\ \alpha_z\alpha_x}$$

$$\overset{(3)}{\beta\vec\alpha},\ \overset{(1)}{\alpha_x\alpha_y\alpha_z},\ \overset{(3)}{\beta\alpha_x\alpha_y,\ \beta\alpha_y\alpha_z,\ \beta\alpha_z\alpha_x},\ \overset{(1)}{\beta\alpha_x\alpha_y\alpha_z}$$

$$1,\ \rho_1\vec\sigma,\ \rho_3,\ \rho_3\vec\sigma$$

$$\rho_2\vec\sigma,\ \rho_1,\ \rho_3\vec\sigma,\ \rho_2$$

$x_0 = ict,\ x_1 = x,\ x_2 = y,\ x_3 = z$ とすると

然らば $(-i\beta,\ \beta\vec\alpha)$ が four vector をなす。

従って

$$\begin{array}{c|cccc}
 & i\beta & \beta\alpha_x & \beta\alpha_y & \beta\alpha_z \\ \hline
-i\beta & -1 & -i\alpha_x & -i\alpha_y & -i\alpha_z \\
\beta\alpha_x & i\alpha_x & -1 & -i\alpha_x\alpha_y & +i\alpha_z\alpha_x \\
\beta\alpha_y & i\alpha_y & i\alpha_x\alpha_y & -1 & i\alpha_y\alpha_z \\
\beta\alpha_z & i\alpha_z & -i\alpha_z\alpha_x & -i\alpha_y\alpha_z & -1
\end{array}$$

が four tensor をなす。

$$-i\alpha_x\alpha_y = -i(\rho_1\sigma_x\cdot\rho_1\sigma_y) = -i(i\sigma_z) = \beta\alpha_z$$

これの antisymm. part (sym, abv)

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & -i\alpha_x & \cdots & \\ i\alpha_x & 0 & \cdots & \\ i\alpha_y & i\alpha_x\alpha_y\ 0 & \cdots & \\ i\alpha_z & -i\alpha_z\alpha_x & i\alpha_y\alpha_z\ 0 & \end{pmatrix}$$

これは form tensor を作る。

又は

1つとれ。 $\int \tilde{v}(U\vec{S} - \vec{S}U)u$

$(\vec{S})^2 u = I(I+1)u$ $\quad u = \begin{Bmatrix} u_I \\ u_{I-1} \\ \vdots \\ u_{-I} \end{Bmatrix}$ $\quad (2I+1)$

$$\left.\begin{aligned} (S_x + iS_y)u_M &= \sqrt{I(I+1) - M(M+1)}\, u_{M+1} \\ (S_x - iS_y)u_M &= \sqrt{I(I+1) - M(M-1)}\, u_{M-1} \end{aligned}\right\} \quad M = -I, \cdots +I$$

$$S_z u_M = M u_M.$$

$$\int \tilde{v}_N U u_M = U_{NM}.$$

$\cancel{\tilde{v}(\vec{S})^2 = \tilde{v}\, J(J+1)}$

$\cancel{\tilde{v}(S_x + iS_y) = (S_x - iS_y)\tilde{v}}$

$$\int \tilde{u}_n \vec{S} U u_m = \cancel{U\vec{S}v} \quad \cancel{\int \tilde{u} U \vec{S} u_m}$$

$$= \int \overline{\tilde{u}_m U \vec{S} u_n}$$

$$\int \tilde{v}\vec{S}Uu = \int \overline{\tilde{u}U\vec{S}v}$$

$$\int \tilde{v}_N U \cancel{u_M} (S_x + iS_y) u_M = \sqrt{I(I+1) - M(M+1)} \quad U_{NM+1}$$

$$\int \tilde{v}_N U (S_x - iS_y) u_M = \sqrt{I(I+1) - M(M-1)} \; U_{NM-1}$$

$$\int \tilde{v}_N U S_z u_M = M \, U_{NM}.$$

$$\int \tilde{v}_N (S_x + iS_y) U u_M = \int \tilde{u}_M U (S_x + iS_y) v_N$$

$$= \sqrt{J(J+1) - N(N+1)} \quad U_{N+1,M}$$

$$\int \tilde{v}_N (S_x - iS_y) U u_M = \sqrt{J(J+1) - N(N-1)} \quad U_{N-1,M}$$

$$\int \tilde{v}_N S_z U u_M = N \, U_{N,M}$$

$$\int \tilde{v}_N \left(U(S_x + iS_y) - (S_x + iS_y) U \right) u_M$$
$$= \sqrt{I(I+1) - M(M+1)} \, U_{N,M+1} - \sqrt{J(J+1) - N(N+1)} \, U_{N+1,M}$$

$$\int \tilde{v}_N \left(U(S_x - iS_y) - (S_x - iS_y) U \right) u_M$$
$$= \sqrt{I(I+1) - M(M-1)} \, U_{N,M-1} - \sqrt{J(J+1) - N(N+1)} \, U_{N-1,M}$$

$$\int \tilde{v}_N \left(U S_z - S_z U \right) u_M = \cancel{\sqrt{I(I+1) - MA}} \, (M - N) U_{N,M}.$$

$$M_x + iM_y = M_p \qquad M_x - iM_y = M_q$$

とおいて、

$$U_{NM} M_p - M_p U_{NM} + \sqrt{I(I+1) - M(M+1)} \, U_{N,M+1}$$
$$- \sqrt{J(J+1) - N(N+1)} \, U_{N+1,M} = 0$$

$$U_{NM} M_q - M_q U_{NM} + \sqrt{I(I+1) - M(M-1)} \, U_{N,M-1}$$
$$- \sqrt{J(J+1) - N(N-1)} \, U_{N-1,M} = 0.$$

$$U_{NM} M_z - M_z U_{NM} + (M-N) U_{NM} = 0.$$

$$\cancel{U_{NM}}\ M_z \cdot \Psi_{j,\cancel{m}u} = u \Psi_{j,u}.$$

$$U_{NM}^{u'u}\, u - u' U_{NM}^{u'u} \cancel{=} + (M-N) U_{NM}^{u'u} = 0$$

$U_{NM}^{u'u} \neq 0$ for $\underline{u - u' + M - N = 0}$

$(u, M) \to (u', N)$

$N = -1, 0, +1.$

$I = 0 \to \cancel{J}\ J = 1.$

$M = 0.$

$$\left\{\begin{array}{l} U_{N\cancel{M}} M_p - \cancel{\quad} M_p U_{N\cancel{M}} - \sqrt{2 - N(N+1)}\, U_{N+1} = 0 \\ U_N M_q - M_q U_N - \sqrt{2 - N(N-1)}\, U_{N-1} = 0 \end{array}\right.$$

$$U_N M_{\cancel{z}z} - M_z U_N - N U_N = 0.$$

$$\left.\begin{array}{l} U_{-1} M_p - M_p U_{-1} \cancel{-\sqrt{2}} = \cancel{0} \sqrt{2}\, U_{\cancel{1}0} \\ U_{-1} M_q - M_q U_{-1} = \cancel{\sqrt{2}}\ 0 \\ U_{-1} M_z - M_z U_{-1} = \cancel{0} - U_{-1} \end{array}\right\}$$

$$\left.\begin{array}{l} U_0 M_p - M_p U_0 = \cancel{0} \sqrt{2}\, U_1 \\ U_0 M_q - M_q U_0 = \sqrt{2}\, U_{-1} \\ U_0 M_z - M_z U_0 = \cancel{0} \cdot 0 \end{array}\right\} \quad -2i$$

$$\left.\begin{array}{l} U_{+1} M_p - M_p U_{+1} = 0 \\ U_{+1} M_q - M_q U_{+1} = \cancel{m} \sqrt{2}\, U_0 \\ U_{+1} M_z - M_z U_{+1} = \cancel{0} + U_{+1} \end{array}\right\}$$

$$U_{-1}\underset{M_q}{\cancel{M_p}} - \cancel{M_p}\underset{M_q}{\cancel{U_{-1}}} = \cancel{\sqrt{2}\,U_0}\, M_q$$

$$\cancel{M_p U_{-1} M_q - M_p M_q U_{-1} = 0}$$

$$\left.\begin{aligned} U_{-1}M_x - M_x U_{-1} &= \frac{U_0}{\sqrt{2}} \\ U_{-1}M_y - M_y U_{-1} &= \frac{U_0}{\sqrt{2}\,i} \\ U_{-1}M_z - M_z U_{-1} &= \cancel{0}\, U_{-1} \end{aligned}\right\} \text{etc.}$$

$$\left.\begin{aligned} U_{+1}M_x - M_x U_{+1} &= \frac{U_0}{\sqrt{2}} \\ U_{+1}M_y - M_y U_{+1} &= -\frac{U_0}{\sqrt{2}\,i} \\ U_{+1}M_z - M_z U_{+1} &= \cancel{0}\, U_{+1} \end{aligned}\right\} \text{etc.}$$

$$\left.\begin{aligned} \frac{1}{\sqrt{2}}(U_{-1} - U_{+1}) &= U_x \\ \frac{i}{\sqrt{2}}(U_{-1} + U_{+1}) &= U_y \end{aligned}\right\}$$

$$\cancel{U_p M_x - M_x U_p = U_0}$$
$$\cancel{U_q M_x - M_x U_q = U_0}$$

$$\left.\begin{aligned} U_x M_x - M_x U_x &= 0 \\ U_x M_y - M_y U_x &= -i\,U_0 \\ U_x M_z - M_z U_x &= U_y \end{aligned}\right\}$$

$$\left.\begin{aligned} U_y M_x - M_x U_y &= U_0 \\ U_y M_y - M_y U_y &= 0 \\ U_y M_z - M_z U_y &= -U_x \end{aligned}\right\}$$

$$\left.\begin{aligned} U_0 M_x - M_x U_0 &= U_y \\ U_0 M_y - M_y U_0 &= i\,U_x \\ U_0 M_z - M_z U_0 &= 0 \end{aligned}\right\}$$

$\frac{x}{i\hbar} = X$　　$\underline{x}$

$M_x =$

ψ, ψ^* であらわすと

ρ scalar　　$(1, i\vec{\alpha})$ four vector

$\beta\vec{\sigma}\begin{pmatrix} 0 & & 1 \\ & 0 & \\ & & 0 \end{pmatrix} = (\beta\vec{\sigma}, \beta\vec{\alpha})$ six vector

$(\alpha_x \alpha_y \alpha_z, \vec{\sigma})$ pseudovector　　$\beta\gamma_5$ pseudoscalar

$(p_0, icp_{x,y,z})$ vector　　(ict, x, y, z) vector

$$\vec{M}(eA_0 + e\vec{\alpha}\vec{A}) - (eA_0 + e\vec{\alpha}\vec{A})\vec{M} = 0,$$

$$(\vec{m} + \tfrac{\hbar}{2}\vec{\sigma})(eA_0 + e\vec{\alpha}\vec{A}) - (eA_0 + e\vec{\alpha}\vec{A})(\vec{m} + \tfrac{\hbar}{2}\vec{\sigma}) = 0$$

$$(\vec{m}\, eA_0 - eA_0\vec{m}) + \cancel{\tfrac{\hbar}{2}\vec{\sigma}A_0 - A_0\tfrac{\hbar}{2}\vec{\sigma}}$$

$$+ \frac{e\hbar}{2}\{\vec{\sigma}(\vec{\alpha}\vec{A}) - (\vec{\alpha}\vec{A})\vec{\sigma}\} + \{e\vec{m}(\vec{\alpha}\vec{A}) - (\vec{\alpha}\vec{A})\vec{m}\} = 0$$

$$\|$$

$$e\rho_1\{\tfrac{\hbar}{2}(\vec{\sigma}\sigma_x - \sigma_x\vec{\sigma}) + (\vec{m}A_x - A_x\vec{m})\}\sigma_x + \cdots$$

$$+\{[illegible] + (\vec{m}_x A_x - A_x\vec{m}_x)\}\sigma_x$$

$$+\{+i\hbar\sigma_z + (\vec{m}_x A_y - A_y\vec{m}_x)\}\sigma_y$$

$$+\{-i\hbar\sigma_y + (\vec{m}_x A_z - A_z\vec{m}_x)\}\sigma_z = 0.$$

ρ_3,　$(1, \boxed{\rho_1\vec{\sigma}})$　$(\boxed{\rho_3\vec{\sigma}}, \boxed{\rho_2\vec{\sigma}})$,　$(\rho_1, \boxed{\vec{\sigma}})$,　ρ_2

$\downarrow$　　$\downarrow$　　$\downarrow$　　$\downarrow$　　$\downarrow$

0　　$(\boxed{\rho_1\vec{\sigma}})$　　$(\boxed{\rho_3\vec{\sigma}}, \boxed{\rho_2\vec{\sigma}})$　　$(\boxed{\vec{\sigma}})$　　0.

思ひ出[illegible]

$$M_p M_q = (M_x + iM_y)(M_x - iM_y)$$
$$= M_x^2 + M_y^2 \mp i(M_x M_y - M_y M_x)$$
$$M_q M_p = M_x^2 + M_y^2 + i(\quad - \quad)$$

$$U_{-1} M_p M_q - M_p M_q U_{-1} = \sqrt{2}\, U_0 M_q$$
$$\cancel{U_{-1} M_p M_q}$$
$$U_{-1} M_q M_p - M_q M_p U_{-1} = +\sqrt{2}\, M_q U_0$$
$$U_{-1}(\ +\ ) \pm (\ +\ ) U_{-1} = \sqrt{2}(U_0 M_q + M_q U_0)$$

$$U_{-1} M_p M_q - M_p U_{-1} M_q - \cancel{M_p U_{-1} M_q + M_p M_q}$$
$$\cancel{-U_{-1} M_q} - M_q U_{-1} M_p + M_q M_p U_{-1} = 2 U_{-1}.$$

$$U_{-1} M_p M_q - M_p M_q U_{-1} - U_{-1} M_q M_p + M_q M_p U_{-1} = 2 U_{-1},$$

$$\overset{\frac{y+ix}{\sqrt{2}}}{U_{-1}} M_x - M_x U_{-1} = \frac{1}{\sqrt{2}} U_0$$
$$U_{-1} M_y - M_y U_{-1} = \frac{1}{\sqrt{2}\,i} U_0$$
$$U_{-1} M_z - M_z U_{-1} = -U_{-1}$$

$$\frac{1}{\sqrt{2}}(U_{-1} + U_{+1}) M_x - M_x (U_{-1} + U_{+1}) = U_0 \quad \frac{1}{\sqrt{2}}$$
$$\frac{1}{\sqrt{2}}(U_{-1} + U_{+1}) M_y - M_y (U_{-1} + U_{+1}) = 0$$
$$\frac{1}{\sqrt{2}}(U_{-1} + U_1) M_z - M_z (U_{-1} + U_{+1}) = \frac{1}{\sqrt{2}}(-U_{-1} + U_1)$$

$$\overset{\sqrt{2}\,y}{U_0} M_x - M_x U_0 = \frac{1}{\sqrt{2}}(U_1 + U_{-1})$$
$$U_0 M_y - M_y U_0 = \frac{1}{\sqrt{2}\,i}(U_1 - U_{-1})$$
$$-i\sqrt{2}\,x$$
$$U_0 M_z - M_z U_0 = 0$$

$$\frac{1}{\sqrt{2}}(U_{-1} - U_{+1}) M_z -$$

$$\frac{1}{\sqrt{2}}(y - ix) = U_{+1} M_x - M_x U_1 = \frac{1}{\sqrt{2}} U_0$$
$$y - ix = U_1 M_y - M_y U_1 = \frac{-1}{\sqrt{2}\,i} U_0$$
$$U_1 M_z - M_z U_1 = U_1$$

迅 [illegible] $\frac{1}{i} U_0$

$$M_x = y p_z - z p_y$$

$$x(y p_z - z p_y) - (y p_z - z p_y) x = 0$$

$$y(\quad '' \quad) - (\quad '' \quad) y = z(-i\hbar)$$

$$z(\quad '' \quad) - (\quad '' \quad) z = y(+i\hbar)$$

$$x M_x - M_x x = 0$$

$$y M_x - M_x y = (-i\hbar) z$$

$$z M_x - M_x z = (i\hbar) y$$

$$\frac{1}{\sqrt{2}i}(U_{+1} + U_{-1}) = U_y \qquad \frac{1}{\sqrt{2}}(+U_{-1} \mp U_1) = U_x$$

$$U_y M_x - M_x U_y = -i U_z$$

$$U_y M_y - M_y U_y = 0$$

$$U_y M_z - M_z U_y = i U_x$$

$$U_z M_x - M_x U_z = +i U_y$$

$$U_z M_y - M_y U_z = +i U_x$$

$$U_z M_z - M_z U_z = 0$$

$$\frac{1}{\sqrt{2}}(U_1 - U_{-1}) = U_x$$

$$\frac{1}{\sqrt{2}i}(U_1 - U_{-1}) = i U_x$$

$$\left.\begin{aligned} U_x M_x - M_x U_x &= 0 \\ U_x M_y - M_y U_x &= -i U_z \\ U_x M_z - M_z U_x &= -i U_y \end{aligned}\right\}$$

$$\cancel{\frac{1}{\sqrt{2}i}(U_{+1} + U_{-1})}$$

$$\vec{u}_0 + (\vec{\sigma}\ [illegible]\ \vec{u}). =$$

Fermi $\vec{u}_0 = \frac{\vec{r}}{r} G(r). \quad \vec{u} = \frac{\vec{r} \times \vec{\sigma}}{r^2}\ [illegible] \times \vec{F}(r).$