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§1、巨視電磁場の基礎

$$\mathbb{E} = \overline{\mathbf{e}} = \frac{1}{a^3\sqrt{\pi}^3}\iiint e(\xi\eta\zeta)\, e^{-\left(\frac{R}{a}\right)^2} d\xi\, d\eta\, d\zeta$$

$$R = \sqrt{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2}$$

$$\int_0^\infty e^{-\frac{x^2}{2}} dx = \sqrt{\frac{\pi}{2}} \qquad x = \frac{\sqrt{2}\cdot y}{a}$$

$$\frac{\sqrt{2}}{a}\int_{-\infty}^{\infty} e^{-\left(\frac{y}{a}\right)^2} dy = \sqrt{2\pi} \qquad \int_{-\infty}^{\infty} e^{-\left(\frac{y}{a}\right)^2} dy = a\sqrt{\pi}$$

$$\frac{\partial E_x(xyz)}{\partial x} = \frac{1}{a^3\sqrt{\pi}^3}\iiint e_x(\xi)\frac{\partial}{\partial x}\left\{e^{-\left(\frac{R}{a}\right)^2}\right\} d\xi\, d\eta\, d\zeta$$

$$= -\frac{1}{a^3\sqrt{\pi}^3}\iiint \underline{e_x(\xi)}\ \frac{\partial}{\partial \xi}\left\{e^{-\left(\frac{R}{a}\right)^2}\right\} d\xi\, d\eta\, d\zeta$$

$$= -\iint e(\xi)\left\{e^{-\left(\frac{R}{a}\right)^2}\right\}\Bigg|_{\xi=-\infty}^{\infty} d\eta\, d\zeta$$

$$+ \frac{1}{a^3\sqrt{\pi}^3}\iiint \frac{\partial e_x(\xi)}{\partial \xi} e^{-\left(\frac{R}{a}\right)^2} d\xi\, d\eta\, d\zeta$$

$$\frac{\partial E_x}{\partial x} = \overline{\frac{\partial e_x}{\partial x}}$$

$$-4\pi\iiint \delta(\xi-x')\,\delta(\eta-y')\,\delta'(\zeta-z')\, e^{-\frac{R^2}{a^2}}\, d\xi\, d\eta\, d\zeta$$

$$R^2=(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2$$

$$= 4\pi\, \frac{\partial}{\partial z'} e^{-\frac{(x'-x)^2+(y'-y)^2+(z'-z)^2}{a^2}}$$

$$\frac{1}{a^6\pi^3}\times$$

$$\iiint e^{-\left(\frac{R'}{a}\right)^2}\frac{\partial}{\partial\zeta} e^{-\left(\frac{R}{a}\right)^2}\, d\xi\, d\eta\, d\zeta \qquad (x\ y\ z)$$

$$=\frac{4}{a^6\pi^2}\Bigg[\int e^{-\frac{(x'-\xi)^2+(x-\xi)^2}{a^2}}\, d\xi$$

$$\times\int e^{-\frac{(y'-\eta)^2+(y-\eta)^2}{a^2}}\, d\eta$$

$$\times\int e^{-\frac{(z'-\zeta)^2+(z-\zeta)^2}{a^2}}\,\frac{2(z-\zeta)}{a^2}\, d\zeta\Bigg]$$

$$(x'-\xi)^2+(x-\xi)^2 = 2\xi^2-2(x+x')\xi+x^2+x'^2$$

$$=2\left\{\xi-\frac{(x+x')}{2}\right\}^2+\frac{1}{2}(x-x')^2$$

$$=\frac{4}{a^6\pi^2}\, e^{-\frac{1}{2a^2}(x-x')^2}\int e^{-2\frac{\left\{\xi-\frac{(x+x')}{2}\right\}^2}{a^2}}\, d\xi\ \times$$

$$\frac{4}{a^6\pi^2} e^{-\frac{(x-x')^2}{2a^2}} \int e^{-\frac{2(\xi-\cdots)^2}{a^2}} d\xi \cdot e^{-\frac{(y-y')^2}{2a^2}} \int e^{-\frac{2(\eta-\cdots)^2}{a^2}} d\eta$$

$$\times e^{-\frac{(z-z')^2}{2a^2}} \int e^{-\frac{2\zeta'^2}{a^2}} \frac{\cancel{2}(z-z')-2\zeta'}{a^2} d\zeta'$$

$$\zeta' = \zeta - \frac{z+z'}{2}$$

$$z-\zeta = z - \zeta' \cancel{+} - \frac{z+z'}{2}$$

$$= \frac{z-z'}{2} - \zeta'$$

$$\zeta = \frac{a}{\sqrt{2}} \qquad a\sqrt{\frac{\pi}{2}}$$

$$\frac{2}{a^{\cancel{6}}\pi} e^{\frac{(x-x')^2+(y-y')^2+(z-z')^2}{2a^2}}$$

$$\times \left\{ \cancel{-} a\sqrt{\frac{\pi}{2}} \cancel{+} (z-z') \cancel{- \int \frac{\partial}{\partial\zeta'} e^{-\frac{2\zeta'}{a^2}} d\zeta'} \right\}$$

$$\frac{1}{a^6\pi^3}\iiint_{-\infty}^{+\infty} e^{-\frac{R'^2}{a^2}} \frac{\partial}{\partial \xi} e^{-\frac{R^2}{a^2}} d\xi\, d\eta\, d\zeta$$

$$R^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2$$

$$R'^2 = (x'-\xi)^2 + (y'-\eta)^2 + (z'-\zeta)^2$$

$$= \frac{-1}{a^6\pi^3}\iiint \cancel{\frac{\partial}{\partial \zeta}} e^{-\frac{R'^2}{a}} \frac{\partial}{\partial z} e^{-\frac{R^2}{a^2}} d\xi\, d\eta\, d\zeta$$

$$= -\frac{1}{a^6\pi^3}\frac{\partial}{\partial z}\iiint e^{-\frac{R'^2+R^2}{a^2}} d\xi\, d\eta\, d\zeta$$

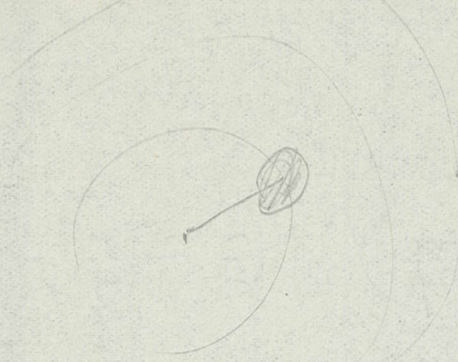
$$R^2 + R'^2 = (x^2 - 2x\xi + \xi^2 + x'^2 - 2x'\xi + \xi^2) + \cdots$$

$$= 2\left(\xi - \frac{x+x'}{2}\right)^2 + \frac{(x-x')^2}{2} + \cdots$$

$$= -\frac{1}{a^6\pi^3}\frac{\partial}{\partial z} e^{-\Sigma\frac{(x-x')^2}{2a^2}} \int e^{-\frac{2\xi^2}{a^2}} d\xi \int e^{\cdots} d\eta \int e^{\cdots} d\zeta$$

$$= \frac{2^{\frac{3}{2}}}{a^3(\pi)^{\frac{3}{2}}} \frac{z-z'}{\cancel{2}a^2} e^{\cdots} a^3\left(\frac{\pi}{2}\right)^{\frac{3}{2}}$$

$$= \frac{2^{\frac{3}{2}}}{a^5\pi^{\frac{3}{2}}} (z-z') e^{\cdots}$$



$$\sum_j \tilde{a}_j \tilde{u}_j \times \sum_i a_i u_i = \sum \tilde{a}_i a_i \int \tilde{u}_j u_i$$

$$= \sum_i n_i \ldots$$

$$+ \sum_{j \neq i} \tilde{a}_j a_i \tilde{u}_j u_i$$

$$\sum_i \int \rho_i(r - R_i)\, dv \; e^{-\frac{r^2}{a^2}}$$

$$= \sum \int \rho_i(x_i + X_i, \ldots)\, dx\, dy\, dz \; e^{-\frac{x^2}{a^2} - \frac{y^2}{a^2} - \frac{z^2}{a^2}}$$

$$e^{-\frac{\{(x-X_i)^2 + 2X_i(x-X_i) + X_i^2\}}{a^2}}$$

$$= e^{-\frac{X_i^2}{a^2}} \left(1 - \frac{2X_i(x-X_i)}{a^2}\right)$$

$$\int p \,\mathrm{grad}\, \phi$$

$$= \int \mathrm{div}\, p\, \phi \ldots \rho_i - e \sum_i e^{-\frac{X_i^2}{a^2}}$$

$$= \int \mathrm{div}\, p \cdot \phi \quad 0$$

$$- \sum_i 2 \int \rho_i \frac{R_i(r - R_i)}{a^2}\, dv \; e^{-\frac{X_i^2}{a^2}}$$

$$+ \sum_i (\mathrm{grad}\, p)\, e^{-\frac{X_i^2}{a^2}}$$

$$+ \sum_i \mathrm{div} \left(\sum_i p(R_i)\, e \right) - \mathrm{grad}\, p \cdot e \; \sum_i \mathrm{div}\, p \cdot e^{-\frac{X_i^2}{a^2}}$$