野田氏の計算 I.　　　　N18

19. 1. 19.

①

$$e_x^{(1)} \quad h_y^{(1)}$$

$$e_x(x')\,h_y(x'') - h_y(x'')\,e_x(x') = i\hbar\,[e_x(x'), h_y(x'')]$$

$$[e_x(x'), h_y(x'')] = -4\pi\,\delta(x'-x'')\,\delta(y'-y'')\,\delta'(z'-z'') \quad ; \quad \delta'(z'-z'') = \frac{\partial}{\partial z'}\delta(z'-z'').$$

$$E_x^{(1)} = \overline{\overline{e}}_x^{(1)} = \frac{1}{N'}\iiint e_x(x)\, e^{-\frac{1}{a^2}\{(x-x')^2+(y-y')^2+(z-z')^2\}}\,dv.$$

$$N' = \iiint e^{-\frac{1}{a^2}\{(x-x')^2+(y-y')^2+(z-z')^2\}}\,dv.$$

$$H_y^{(1)} = \overline{\overline{h}}_y^{(1)} = \frac{1}{N''}\iiint h_y(x).\, e^{-\frac{1}{a^2}\{(x-x'')^2+(y-y'')^2+(z-z'')^2\}}\,dv$$

$$N'' = \iiint e^{-\frac{1}{a^2}\{(x-x'')^2+(y-y'')^2+(z-z'')^2\}}\,dv.$$

今,　$f(\vec{x}-\vec{x}') = e^{-\frac{1}{a^2}\{(x-x')^2+(y-y')^2+(z-z')^2\}}$ トオケバ,

$$[E_x^{(1)}, H_y^{(1)}] = \left[\frac{1}{N'}\int e_x^{(\prime\prime\prime)} f(\vec{x}'''-\vec{x}')\,dv''',\ \frac{1}{N''}\int h_y^{(IV)} f(\vec{x}^{IV}-\vec{x}'')\,dv^{IV}\right]$$

$$= \frac{1}{N'N''}\iint f(\vec{x}'''-\vec{x}')\,f(\vec{x}^{IV}-\vec{x}'')\,[e_x^{(\prime\prime\prime)}, h_y^{(IV)}]\,dv'''\,dv^{IV}.$$

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$$= -\frac{4\pi}{N'N''}\iint f(\vec{x}'''-\vec{x}')\,f(\vec{x}^{IV}-\vec{x}'')\,\delta(x'''-x^{IV})\,\delta(y'''-y^{IV})\,\delta'(z'''-z^{IV})\,dv'''\,dv^{IV}.$$

$$= -\frac{4\pi}{N'N''}\int\cdots\int e^{-\frac{1}{a^2}\{(x'''-x')^2+(y'''-y')^2+(z'''-z')^2+(x^{IV}-x'')^2+(y^{IV}-y'')^2+(z^{IV}-z'')^2\}}\,\delta(x'''-x^{IV})\,\delta(y'''-y^{IV})\,\delta'(z'''-z^{IV})\times$$

$$dx'''\,dy'''\,dz'''\,dx^{IV}\,dy^{IV}\,dz^{IV}$$

$$= -\frac{4\pi}{N'N''}\int\int e^{-\frac{1}{a^2}\{(x^{IV}-x')^2+(y^{IV}-y')^2+(z'''-z')^2+(x^{IV}-x'')^2+(y^{IV}-y'')^2+(z^{IV}-z'')^2\}}\,\delta'(z'''-z^{IV})\,dz'''\,dx^{IV}\,dy^{IV}\,dz^{IV}.$$

$$= -\frac{8\pi}{a^2N'N''}\int\int e^{-\frac{1}{a^2}\{(x-x')^2+(y-y')^2+(z-z')^2+(x-x'')^2+(y-y'')^2+(z-z'')^2\}}\,(z-z')\,dx\,dy\,dz.$$

$$= -\frac{8\pi}{a^2N'N''}\int dx\, e^{-\frac{1}{a^2}\{(x-x')^2+(x-x'')^2\}}\int dy\, e^{-\frac{1}{a^2}\{(y-y')^2+(y-y'')^2\}}\int dz\, e^{-\frac{1}{a^2}\{(z-z')^2+(z-z'')^2\}}\,(z-z')\,dz.$$

$$= +\frac{8\pi}{a^2(a^3\pi^{\frac{3}{2}})^2}\cdot e^{-\frac{(x'-x'')^2}{2a^2}}\,a\sqrt{\frac{\pi}{2}}\cdot e^{-\frac{(y'-y'')^2}{2a^2}}\,a\cdot\sqrt{\frac{\pi}{2}}\;e^{-\frac{(z'-z'')^2}{2a^2}}\cdot\frac{a(z'-z'')}{2}\sqrt{\frac{\pi}{2}}$$

$$= \frac{1}{a^5}\sqrt{\frac{2}{\pi}}\cdot(z'-z'')\,e^{-\frac{1}{2a^2}\{(x'-x'')^2+(y'-y'')^2+(z'-z'')^2\}}.$$

③

1. $N' = N'' = \iiint e^{-\frac{r^2}{a^2}} r^2 dr \sin\theta\, d\theta\, d\varphi = 4\pi \int_0^\infty e^{-\frac{r^2}{a^2}} r^2 dr$

$= 4\pi \frac{1}{(2\cdot\frac{1}{a^2})} \frac{1}{2} \sqrt{\frac{\pi}{\frac{1}{a^2}}} = 4\pi \frac{a^2}{4} a\sqrt{\pi} = \pi^{\frac{3}{2}} a^3.$

2. $\int_{-\infty}^{\infty} e^{-\frac{1}{a^2}\{(x-x')^2+(x-x'')^2\}} dx.$

$x-x' = X$

$x-x'' = x-x'+x'-x'' = X+(x'-x'')$

$= \int_{-\infty}^{\infty} e^{-\frac{1}{a^2}[X^2+\{X+(x'-x'')\}^2]} dX$

$= e^{-\frac{(x'-x'')^2}{a^2}} \int_{-\infty}^{\infty} e^{-\frac{2}{a^2}X^2 - \frac{2}{a^2}(x'-x'')X} dX$

$= e^{-\frac{(x'-x'')^2}{a^2}} \cdot e^{\frac{\{\frac{2}{a^2}(x'-x'')\}^2}{4\cdot\frac{2}{a^2}}} \cdot \sqrt{\frac{\pi}{\frac{2}{a^2}}}$

$= a\sqrt{\frac{\pi}{2}}\, e^{-\frac{(x'-x'')^2}{a^2} + \frac{(x'-x'')^2}{2a^2}}$

$= a\sqrt{\frac{\pi}{2}}\, e^{-\frac{(x'-x'')^2}{2a^2}}$

(4)

3 $\displaystyle\int_{-\infty}^{\infty} e^{-\frac{1}{a^2}\left\{(z-z')^2+(z-z'')^2\right\}}(z-z')\,dz$ $\qquad z-z'=Z$

$$= e^{-\frac{(z'-z'')^2}{a^2}}\int_{-\infty}^{\infty} e^{-\frac{2}{a^2}Z^2-2\cdot\frac{(z'-z'')}{a^2}Z}\cdot Z\,dZ$$

$$= -e^{-\frac{(z'-z'')^2}{a^2}}\cdot\frac{\frac{1}{a^2}(z'-z'')}{\frac{2}{a^2}}\sqrt{\frac{\pi}{\frac{2}{a^2}}}\cdot e^{\frac{\left(\frac{z'-z''}{a^2}\right)^2}{\frac{2}{a^2}}}$$

$$= -\frac{a\cdot(z'-z'')}{2}\sqrt{\frac{\pi}{2}}\;e^{-\frac{(z'-z'')^2}{2a^2}}$$

4.