©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

YHAL N21

c031-040
[N21]

c031-040-010
[N21 010]

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

H21

PHYSICS. II.

Meson Theory, Electrodynamics and related topics

Oct. 1948 ~

Princeton

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

S6

Lett. at M. I. T.

Dec. 9, 1948

Some problems of Meson Theory

The ~~fu~~ meson theory is in a curious situation today. ~~On the ham one hand we are convinced that~~ There are good reason to believe that some of fundamental ideas ~~of~~ concerning the nature of mesons are certainly correct. Nevertheless, as soon as ~~if~~ we try to work out some specific scheme of the meson theory, we ~~are~~ always meet with various kinds of difficulties which prevent us from making further development. Some of the difficulties are due to the inconsistency in the theory itself, others are the ~~inability~~ inability of ~~explaining the experiment~~ obtaining results which are in accord with experiments. ~~The situation~~ ~~Although~~ Thus everything is still problematic about mesons.

Under these circumstances, our task will be twofold:

i) First to ~~ignore the detailed~~ take out those ~~phenomena~~ problems which ~~are~~ can be ~~solved~~ ~~explained by some speci~~ understood at least qualitatively by some specific assumptions on the ~~pro~~ nature of mesons and their interaction with other kinds of elementary particles, ~~ii)~~ ~~independent~~ respective of the intrinsic defect of the present formalism,

ii) Second to investigate more fundamental problems of constructing the ~~general consistent~~ field theory, which will be devoid of divergence difficulties

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

~~the~~ necessary background for ~~the~~ future development of any specific meson theory (as well as for q.e.d.).

Part I.

Although both of these problems are not yet solved, we are now ~~at~~ the stage to be able to give definite answer at least to a part of the first question, owing to the remarkable progress in ~~cosmic ray researches~~ the fields of cosmic ray~~s~~ and high energy nuclear reaction experiments in these two years. The fundamental facts which is now firmly established are

i) the existence of ~~two~~ at least two kinds of ~~mesons~~ charged mesons

$\pi^{\pm}$

$\mu^{\pm}$

with the masses about

300 m_e 200 m_e

respectively. There ~~is are~~ is good ~~considerable~~ reason to believe that another π meson τ with the mass about 1000 ~~or 1200~~ m_e exists, but the fact is not so firmly established as the existence of π and μ-mesons.

We know various kinds of fundamental processes related to ~~the~~ one or both of these two mesons, which ~~seem now enable us to~~ seem to be sufficient to

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

determine the nature of mesons.
They can be divided into two parts.

I. First kind (Three particle processes)
i) μ-β-decay 10^{-49} erg·cm³.

~~10^{-49} erg/cm³~~

ii) μ-nucleon reaction

iii) nuclear-β-decay

II. Second kind (Two particle processes)
iv) π-μ-decay

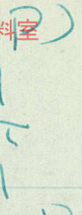
v) π-nucleon reaction

vi) absence of π-β-decay

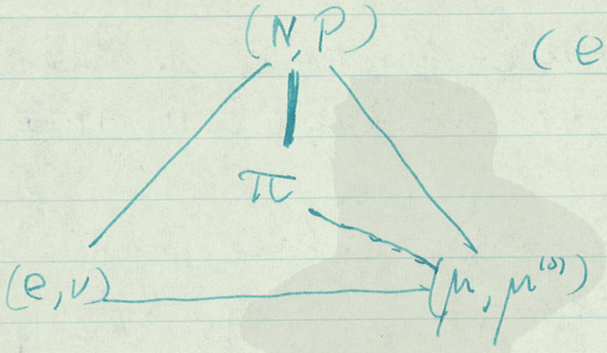
III. Third kind (processes related with neutral mesons)
vii) $\pi^0 \to \gamma + \gamma$ vector scalar
$\to \gamma + \gamma + \gamma$ ps. scalar ps. vector
through proton pair creation.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

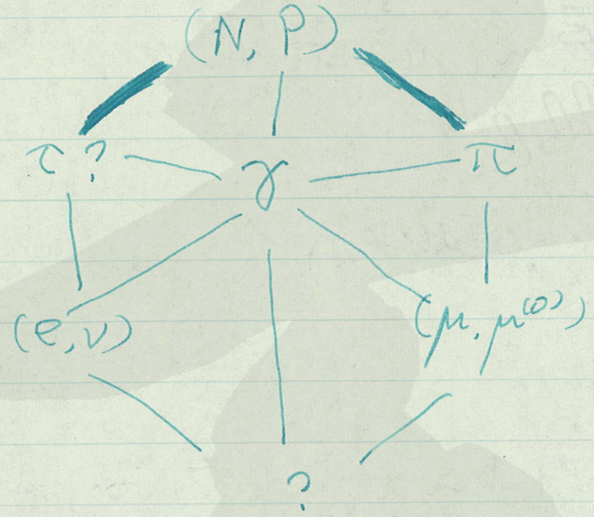
First stage



Second stage

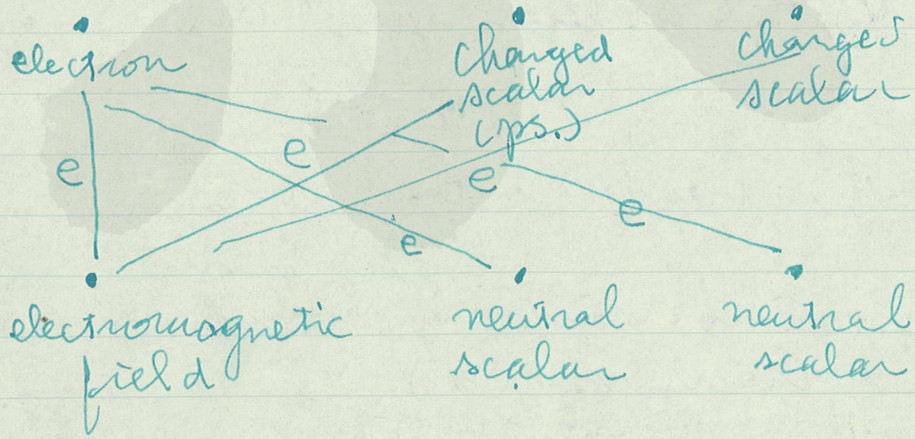


Third stage?



nucleon

μ-meson



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Part II.

Now we want to go into ~~the~~ more difficult question related with intrinsic defect of the field theory. In this connection also, mesons can play a very important role, because, as proposed by Pais and Sakata, certain kinds of mesons may ~~The existence of~~ serve to remove the undesirable divergence of in the field theory as follow ~~present~~

i) Self-energy of the electron

$f^2 = 2e^2$	$f^2 = \frac{3}{5}e^2$
neutral scalar field	ps. vector field

or 2 neutral scalar fields with $|f| = e$

$$W_{e,S} = \lim_{L\to\infty} -\frac{f^2}{\hbar c}\frac{1}{4\pi}\frac{3\mu^2}{E}\log\frac{L\hbar c}{\mu} + f.t. \quad *$$

ii) Electromagnetic self-energy of the nucleon. neutron-proton mass-difference

W_F

iii) Photon self-energy

2 charged ps scalar fields

$$W_{P,F} = -\frac{4e^2}{3\pi L_0}\int p\,dp + f.t.$$

$$W_{P,S} = \frac{2e^2}{3\pi L_0}\int p\,dp + f.t.$$

iv) Charge renormalization

?

v) F-field of nuclear force

---

* $W_{e,p} = W_{dyn} + W_{st} = \lim_{L\to\infty}\frac{e^2}{\hbar c}\frac{1}{2\pi}\frac{3\mu^2}{E}\log\frac{L\hbar c}{\mu}$

μ : rest energy of source particle.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

The above mentioned procedure of so-called "mutual compensation" will serve to remove some of the difficulties, but not all of them, because if we introduce probably one kind of new field in order to remove a divergence in some kind of self-energy, then it result in a new kind of divergence, so that we can hardly expect the consistency of the whole system of elementary particles.

Thus we cannot obviate to face with the fundamental question of constructing a consistent field theory. Recent development of the q.e.d. by Schwinger, Feynman, Tomonaga revealed the new feature of the problem:

Schrödinger eq. for the whole system has, of course, no finite solution, but nevertheless, infinite solution from the can we can take out a finite part corresponding to physically observable quantities and indeed in relativistically invariant way.

But in the case of the meson theory, it is doubtful whether we can get a finite answer by the always using the same procedure as in the case of el. dyn.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

The situation is especially serious, if we have to take into account the direct interaction between Fermi particles in addition to the interaction between Fermi and Bose particles as in el.dyn and ~~ordinary~~ original the meson theory in its original form.

There have been many attempts to solve the difficult question of consistency of the field theory. I want to suggest a possible way of approaching the problem. Namely, the introduction of "nonlocalizable field" in connection with the idea of "reciprocity" by Born.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Part I. continued

Two Problems i) nuclear Forces
ii) β-decay

were the starting points of the meson theory. There is still no satisfactory solution of these old problems.

~~In the co~~ As for β-decay, Fermi's theory ~~of β-~~ seems to be essentially correct irrespective of ~~the future~~ whether in the final analysis it can be reduced to an indirect process due to virtual emission and absorption of mesons or it is an elementary fact.

Most curious curious situation that all~~both~~ allowed and ~~most of~~ almost all of forbidden β-spectra are Fermi type suggest us that there are two distinct mechanism for the β-decay, which ~~can be an~~ are due to intermediary action of two kinds of mesons.*

As for nuclear forces, it has been always a difficult task to explain deuteron problems, as well as low and high energy nucleon scattering quantitatively.** ~~Nowadays modified symmetrical theory seems to be acceptable rather than the usual symmetrical theory, although there are still many problems remaining unsolved.~~

* Nakamura

** ~~It should be noticed that the "potential" is considerably different according to the relative energy of two nucleon, so that the comparison with experiment~~ vagueness uncleared.**

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

** It should be noticed that the phenomenological potentials ~~are different~~ should be changed according as the relative energy of two nucleon in consideration. ~~It can eas~~ Roughly speaking the range of forces will increase apparently as the relative energy increases. Strictly speaking it ~~will~~ ~~even~~ ~~be~~ not be justified even to use the potential convention forms for for relative energy comparable with the meson rest energy, where relativistic effects ~~will~~ can not be considered small.

In spite of these uncertainties, it seems very probable *** that ~~I~~ we have to adopt a combination of equal amount of exchange forces and ordinary forces in place of the mixing ratio 2:1 in the symmetrical theory. This can be regarded in terms of the meson theory to replace $\frac{1}{2}\sum_i \tau_i^{(1)}\tau_i^{(2)}$ by $\frac{1}{2}\{1+\sum_i \tau_i^{(1)}\tau_i^{(2)}\}$, which means in turn to assume the existence of two kind of neutral mesons which interact with the nucleon with the same strength as the charged mesons.

*** Serber,

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

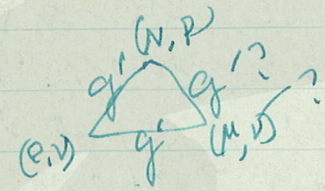
S 8

Some Problems of Meson Theory
Carnegie Institute of Technology
Pittsburgh
Jan 13, Thursday, 1949

Addition to previous lectures:

1) Nuclear Forces:
neutron-proton scattering
exchange character
a) $\tau^{(1)}\tau^{(2)}$ (symmetric theory)
b) $(1+\tau^{(1)}\tau^{(2)})$

2) β-decay



3) Fifth dimension and mass change operator.

$$U=\sum_{k_5}\left\{\sum_{k_1\cdots k_4} u(k_\mu)\, e^{ik_\mu x_\mu/\lambda}\, e^{i\lambda k_\mu p_\mu/\hbar}\right\}$$

different values of k_5 corresponds to different rest masses

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

576

Some Problems of Meson Theory

Yale University, Colloquium.
Jan 5, ~~Thursday~~, 1949
Wednesday

Lectures on meson theory and field theory

Phys. I.
1. Dep. Phys., Univ. Calif., Sept. 13, 1948.
2. Dep. Phys., Univ. Pennsyl., ~~So~~ Oct. 6, 1948.
3. Institute for Adv. Study, Princeton, Seminary at Fuld Hall. Oct. 12, 1948.

Phys. II
4. Some Works on field theory in Japan, Princeton Univ., Theor. Sem. Nov. 12, 1948
5. ~~1948~~ Fuld Hall seminar, Remarks on Non-Loc. (Symposium, Nov. 30, 1948)

Phys. ~~III~~ II
~~Phys. IV.~~
5. Dep. Phys. M.I.T., Dec. 9, 1948.
6. 7 Dep. Phys. Yale Univ., Jan. 5, 1949.
8. Dep. Phys. Carnegie Institute, Jan. 13, 1949

Part I. Meson theory in its conventional form based on perturbation theory.
Connection between π, μ-mesons with other kinds of elementary particles, which can be inferred from
i) ~~Ana~~ Analysis of fundamental collision and decay-processes.
ii) ~~Analo-Compar~~ Analysis of nuclear forces. (and β-decay)
Possible role of τ-meson.

Part II. Role of mesons in connection with the stability of whole system of elementary particles

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Self-energy of electron — C-meson or f-field.
Self-energy of photon — scalar or pseudoscalar charged ~~particle~~, meson.
Difficulty of Self-energy of C-meson
Necessity of charge Renormalization
Part III. Attempts at Self-consistent Field Theory.
Nonlocalizable Field Theory

Reference: Yukawa, On the Nature of the Meson. (Kagaku-Kikan, Vol 1, (1949), 59 in Japanese)

Certainly our knowledge about mesons has been ~~sta~~ increasing rapidly, especially in these two years. Nevertheless the whole problem is, ~~not yet settled~~ far from being. On the contrary we are more and more embarrassed with the discovery of unexpected facts which cannot easily understood in the framework of the existing theory. ~~We hope~~ Today ~~we~~ I want to discuss some fundamental problems about mesons and ~~the~~ possibilities of possible feature of the future theory.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Models and Methods in the Meson Theory

by H. Yukawa

(9) New York Meeting, American Phys. Soc. Jan. 1949
(10) Syracuse (11) Rochester Feb? 1949
(12) Cornell 18 Monday 1949.

It is indeed a great honor as well as a ~~great pleasure~~ ~~happy~~ pleasant duty for me to have the opportunity of talking about the present situation of the meson theory. However, I feel it very difficult to give a definite exposition to this summary report, because ~~because the meson theory is connected with so many phenomena so many~~ the meson theory is confronted with ~~so~~ many problems, which are not settled down as yet, and because ~~that~~ my ~~opinion~~ attitude to these meson problems has been changing so often ~~from time to time~~. Moreover I do not pretend that this report contains something very original. On the contrary it is rather a natural ~~simply rather an~~ outcome ~~common place summary~~ of frequent talks ~~with~~ ~~in~~ with many physicists in Princeton, especially with Prof. Oppenheimer, Dr. Pais, Prof. Wheeler, ~~and Mr. Tiomno~~

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

culminating in the experiments by Rome groups a

As well known the situation of meson theory changed a great deal in these two years mainly due to the discovery by Bristol physicists of two kinds of mesons in cosmic rays with characters entirely different from each other. At first sight this might have been looked upon as adding further confusion and complexity to the increasing difficulties of meson theory. However, it turned out soon that various phenomena connected with cosmic ray mesons could be understood much easier than ever by starting from the new fundamental facts:

i) π-mesons with a mass (m_π) about 300 m_e [1] interact strongly with nucleons and can decay into μ-mesons with the mean life-time (τ_π) of the order of 10^{-8} sec. [1]

ii) μ-mesons with a mass (m_μ) about 200 m_e [1] interact only weakly with the nucleon and can decay into electrons with the longer mean life-time (τ_μ) about 2×10^{-6} sec.

(1) Latest values obtained in Berkeley are $m_\pi =$ 286 m_e, $\tau_\pi = 0.8 \times 10^{-8}$ sec, $m_\mu =$ 217 m_e respectively.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

These facts were confirmed soon afterwards by experiments concerning mesons artificially produced by Berkeley cyclotron.

Owing to subsequent developments in experiment and theory, we are now almost convinced that the spins of π- and μ-mesons are an integer (either 0 or 1) and 1/2 respectively. Arguments in favour of this choice among four possibilities, which can claim equally their rights a priori as taken up by Marshak and Bethe[2], were made by Oppenheimer, Serber, ~~and~~ Wheeler and others[3] on various occasions. Similar arguments were also made independently by Taketani and others in Japan.[4] Here I want to discuss very briefly ~~the~~ essential points of these arguments. The most decisive point is the striking difference ~~of~~ in the results of capture of π- and μ-mesons by nuclei. Whereas ~~when~~ when a π-meson is captured by a nucleus, ~~very often~~ considerable part of its ~~mass~~ proper energy $m_\pi c^2 \sim 150$ mev ~~is~~ transforms into (kinetic energies

(2). Marshak and Bethe, Phys. Rev. 72 (1947),

(3)

(4)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

of nuclear particles ~~emitted~~ accompanying the disruption of the whole nucleus. This can ~~only~~ be possible only if no other neutral ~~light~~ particle such a neutral meson or a neutrino is emitted simultaneously as discussed in detail by Wheeler and Tiomno[5], which excludes the possibility of assigning spin 1/2 to π-mesons. On the contrary μ-mesons give rise very seldom, if any, to nuclear disruptions, which ~~can~~ in turn ~~in~~ in good accord with the assumption of a spin 1/2 for μ-mesons and contradict the assumption of a ~~spin~~ integer spin for them, although the possibility of ~~a spin~~ an integer spin μ-mesons coupled with nucleons only in pairs[6], cannot be excluded altogether[6].

~~Now if we develope want to develope want to construct a simple model based on these fundamental assumptions~~

(5) Wheeler and Tiomno, Bul. Amer. Phys. Soc. 24(1949), 36.

(6)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

If we accept these fundamental assumptions, interpretation of ~~other~~ fundamental processes connected with the ~~emi~~ creation or annihilation of mesons follows at once.

i) π-μ-decay:

$$\pi(0 \text{ or } 1) \rightarrow \mu(1/2) + \nu(1/2) \quad [8]$$

(or $\pi(0 \text{ or } 1) \rightarrow \mu(1/2) + \mu_0(1/2)$, where μ_0 denotes a neutral meson with a small mass of the order of me)

ii) μ-β-decay:

$$\mu(1/2) \rightarrow e(1/2) + 2\nu(1/2) \left\{ + \nu(1/2) \right)$$

(or $\mu(1/2) \rightarrow e(1/2) + \mu_0(1/2)$)

iii) π-nuclear capture

$$\pi^-(0 \text{ or } 1) + \{P(1/2) + \cdots\} \rightarrow N(1/2) + \cdots$$

iv) μ-nuclear capture

$$\mu^-(1/2) + \{P(1/2) + \cdots\} \rightarrow \nu(1/2) + \{N(1/2) + \cdots\}$$

(or $\mu^-(1/2) + \{P(1/2) + \cdots\} \rightarrow \mu_0(1/2) + N(1/2) + \cdots$

(8) Recent observations by Gardner and Lattes are in good accord with this assumption, although the existence of μ_0 with small mass cannot be excluded.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

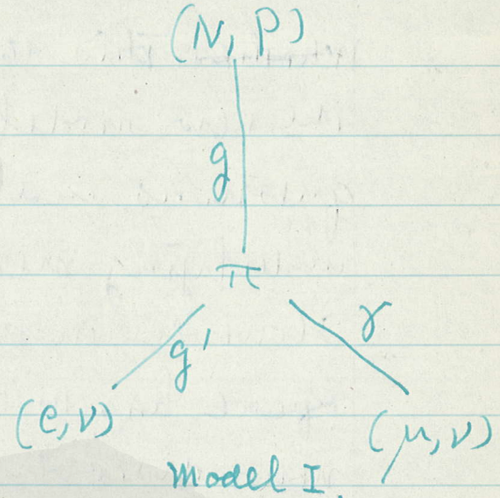
Now the simplet model, which is in conformity with the general idea of meson theory, is obviously the one first proposed by Sakata[7]. ~~in order to remove the well known~~ a few years before the discovery of π- and μ-mesons discrepancy between theoretical and observed life-times of cosmic ray mesons on the one hand and that between theoretical and observed cross-sections of scattering of mesons by nucleons. Let us call this "model I". An obvious drawback of this model is that the life-time of μ-β-decay is far too long compared with the observed value $\tau_\mu \cong 2\times10^{-6}$ sec. due to the circumstance that the coupling constant γ should be very small in order to account for the life-time $\tau_\pi \cong 10^{-8}$ sec.[8] Another difficulty is that the probability of nuclear β-decay, ~~for the of which the most typical one is the neutron-β-decay with~~

(7) Sakata

(8)

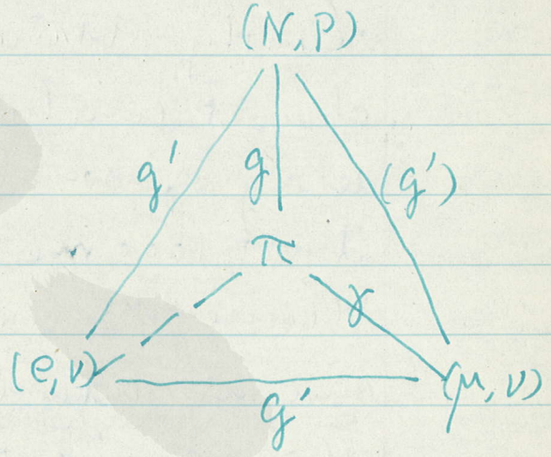
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

calculated on this basis, is 10 to 100 times smaller than the observed value,[9] unless we take advantage of peculiarity of pseudo-scalar meson theory.[10]



Model I.

Thus the modification of model I by adding direct couplings between any two of three kinds of spin ½ particles was proposed by[11] (model II). According to detailed calculation by Wheeler and Tiomno[12], these three coupling constants can be of the same order of magnitude. They may even be take the same value.



(9)

(10) Sakata,

(11)

(12) Wheeler and Tiomno,

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

~~Whether~~ This rather unexpected feature of the new model gives rise in turn to new questions and one can guess ~~no~~ possible underlying meaning, as already done by Klein[13] ~~in his own fashion.~~ However, in order to draw a complete picture, ~~of~~ one has to take into account another important new discovery. Some evidence for the existence of ~~the~~ meson with the mass of the order of 1000 me was already reported ~~by~~ more than once[14], but was not so convincing. A beautiful photograph ~~picture~~ recently obtained by Powell[15] seems to show almost conclusively the occurrence of the decay of so-called τ-meson with the mass about 1000 me into ~~3π~~ three mesons, one of which is π-mesons, very probably a negative π-meson, because the conservation laws for energy and momentum hold among these 4 mesons ~~particles~~. This ~~new~~ remarkable fact opens way to ~~so many~~ ~~various~~ new possibilities, as well as new difficulties, so that it is ~~almost~~ hard to take τ-meson into the model in a definite way. ~~One of the~~ ~~somewhere~~

(13) Klein, Nature

(14)

(15)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

where (N) denotes an antineutron

9

If we assume τ-mesons have integer spin either 0 or 1 and coupled with nucleons in the same way as π-mesons, it follows immediately that τ-meson can decay into three π-mesons through following intermediate stages for example;

$$\tau^+ \to (N) + P \to (N) + N' + \pi_1^+$$
$$\to (N) + P' + \pi_2^- + \pi_1^+ \to \pi_3^+ + \pi_2^- + \pi_1^+,$$

which can be considered as an extension of γ-decay of the neutral meson with integer spin.[16][17] In order that this is one of the most probable modes of decay of τ-meson, there should be some selection rule which prohibits the competition of lower order processes such as

$$\tau^+ \to (N) + P \to (N) + P' + \pi^{(0)}$$
$$\to \pi^+ + \pi^{(0)}$$

and

$$\tau^+ \to (N) + P \to (N) + P' + \gamma$$
$$\to \pi^+ + \gamma.$$

---

(16) Sakata and Tanikawa,

(17) Finkelstein,

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

~~This~~ Thus we can assume that

i) both the τ-nucleon coupling and π-nucleon coupling are ~~odd~~ even (vector or tensor type), which amounts to assume that τ and π ~~mesons~~ are ~~both~~ both vector mesons.

or ii) both couplings ~~is~~ are odd (scalar, pseudoscalar or pseudovector), which amounts to assume that τ and π are scalar or pseudoscular or pseudo~~scalar~~ vector mesons, and further that the condition

$$m_\alpha + m_\pi^{(0)} > m_\tau$$

is also satisfied.

Of course, ~~if~~ these conditions are not sufficient to reduce the probabilities of occurrence of τ-3π decay and other competent processes to values to be compatible with the actual life-time of τ-mesons, ~~which is certainly longer than 10^{-11} sec.~~

In order to get rid of this inconsistency, one can imagine either

i) there are unknown selection rules, by which ~~forbid~~ τ-3π and other similar

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

processes are forbidden*. It is well possible that these rules are in some way connected with the problem of quantization of the masses of elementary particles, which in turn may find its solution in five dimensional ~~description of field th~~ formulation of the field theory.**

or

ii) All ~~kinds~~ sort of processes, which are associated with the creation and annihilation of nucleon pairs in the intermediate states, will be suppressed so drastically in the future ~~that~~ theory, that it will be compatible with experiment, however the relative probabilities of occurrence competent processes ~~remain being~~ remaining roughly the same. ~~This situation~~ It this may be ~~may be connected with~~ connected with the introduction of cut-off length or universal length of the magnitude between $\frac{\hbar}{m_\pi c}$ and $\frac{\hbar}{Mc}$ in the future theory.

* In this case, τ-3π should be replaced by
$\tau \to \pi + 2\mu$ for τ: integer spin
or $\tau \to 2\pi + \mu$ for τ: half integer spin.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

12

Due to ~~At any rate, apart from~~ the uncertainty as to the nature of τ-mesons, and to our ignorance as to the existence or non-existence of mesons other than μ, π and τ-mesons, it is ~~still~~ very hard to give final ~~decision~~ ~~say~~ something definite still about the central problem of the meson theory, solution to problem of ~~that is~~ namely the deduction of nuclear forces on the basis of the interaction scheme between nucleons and mesons. ~~There are, however, a few important indications, showing that which a experimental facts~~ However, we have now a number of recently established ~~a few~~ important facts, which are strongly in favor of the basic idea of the meson theory. I want to mention only a few important facts. One is the well known ~~ex~~ high energy neutron-proton scattering ~~experiment phenome~~ observed in Berkeley.(18) The fact that the neutrons with the energy as large as 90 Mev ~~is~~ are ~~scattering~~ scattered by protons in the backward direction with ~~the~~ about the

(18)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

same probability as in the forward direction, can only be understood if ~~we~~ the considerable portion of the ~~forces~~ force between the neutron and the proton ~~is~~ has exchange character, because the relative wave length is already smaller than meson Compton wave length ~~divided by 2π~~ and the ordinary nuclear forces will give ~~predominantly~~ predominantly forward scattering. ~~As to the ratio~~ relative intensity of ordinary and Among three types of the nuclear forces, which ~~have been~~ were familiar in the meson theory, the neutral theory ~~can thus~~ is thus certainly excluded. Further the fact that the ratio of differential cross-sections of scatting at 90° and 180°, $R = \sigma(180°)/\sigma(90°)$ is about 3, is in favor of the symmetric theory rather than the charged theory. However, according to ~~calculated details~~ calculating (19) the fact that the scattering is ~~approx~~ nearly symmetric with respect to forward and backward directions, seems to indicate that the contributions of ~~odd~~ P-state and other odd states

(19) Christian

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

to ~~read~~ the scattering is very small, which in turn ~~implies~~ imply that the relative contribution of non-exchange and exchange forces is about the same in contrast to 1:2 in the case of the symmetric theory. ~~If this so~~, It is very difficult, however, to deduce this kind of interaction from ~~any~~ any possible simple model of the meson theory. There is another difficulty. ~~The calculated~~ Total cross-sections so far calculated by several authors with the same interaction constants ~~determined compatible with the~~ as in the case of deuteron and low energy scattering problems, give always ~~the values~~ numerical values ~~a little more or less~~ a little larger than the observed ~~values~~ value. ~~Thus, In spite of these discrepancies between theory and experiment, one may be the essential the situation~~ the whole situation is ~~encouraging~~ apparent

However, due to the uncertainties in the theory as well as in experiments, ~~these~~ it seems very difficult to say ~~anything definite about~~

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

it ~~seems~~ very difficult to decide how large is the true discrepancy, if any, between theoretical and observed values. ~~The whole situation~~ seems to ~~show that~~ indicate rather strongly that the meson theory is ~~essentially~~. Anyhow, it seems very likely that the ~~essential~~ present meson theory is correct in essential points and ~~that future modifications~~ we hope that ~~future modifications and refinements~~ ~~the quo~~ better agreement with experiment will be obtained by modifications and refinements of the method rather than by changing the fundamental ideas of the meson theory. ~~and nonthe less~~

~~Another important fact informations can be obtained~~ fact is the

~~Informations, which are much more directly connected with the nature of mesons, were obtained in relation with~~ has been accumulating in connection with ~~the experiment~~ mesons artificially produced by cyclotrons.

~~Informations, which~~

~~Another and non the less important fact~~

~~More direct~~

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Another and more direct criterion of the meson theory is the comparison of the observed ~~probability of~~ and theoretical probabilities of meson production by nuclear collision. ~~As pointed~~ According to calculations by Heitler[20] and others and by Oppenheimer and others[21], ~~the meson π-mesons in cosmic rays produced~~ the probability of creation of π-mesons by collision of high energy protons with nuclei due to the shaking off of virtual meson around ~~the neutrons~~ nucleons ~~seem~~ is large enough to account for the intensity of ~~mesons in~~ cosmic ray mesons, if we assume that ~~the coupling between~~ the ~~meson~~ same virtual mesons are responsible also for ~~the~~ nuclear forces.

More direct evidence for the intimate connection of ~~nuclear~~ π-mesons ~~produce creation~~ and ~~the~~ nuclear forces ~~was obtained~~ is the well known success of ~~producing~~ creating

(20) Heitler,

(21) Lewis, Oppenheimer

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

π-mesons by bombarding caibon nuclei by α-particles accelerated up to 380 Mev by Berkeley cyclotron. The threshold and yield of π-mesons are both consistent with the assumption that π-mesons have integer spin, ~~although~~ and coupled with ~~the~~ nucleons ~~with the strength~~ with the strength, which is expected from the nuclear forces[23], although it is not yet settled which one of the specific assumptions as to the type of meson fields and of ~~the~~ their coupling with nucleons is best fitted. Very recently it is reported that both ~~the~~ Berkeley and Rochester cyclotrons succeeded in ~~produc~~ creating mesons by using proton of energies about 250 Mev and 350 Mev. We can thus expect that the question will be settled in near future.

In low energy regions, where ~~we have had~~ the original results of ~~the measurements~~ values of important quantities such as deuteron binding energy, quadrupole moment and neutron-proton scattering cross-sections ~~were known with~~ measured with high precision,

(23)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

precision, ~~detailed~~ extensive calculations has been performed in these ten years on the basis of the meson theory. However, so far no one succeeded to reach satisfactory quantitative agreement with experiment by pure deductions from the meson theory. ~~On the other hand~~, This may be ~~is~~ partly due to intrinsic difficulties of the meson theory and partly due to inadequacy of specific assumptions concerning type of mesons fields and their coupling with nucleons.

On the other hand, ~~a little more or less~~ a little more phenomenological ways of attack ~~were su~~ contributed ~~very much~~ ~~to~~ give us valuable informations as to the properties of the nuclear ~~intera~~ potential. Especially, phenomenological potential ~~used by~~ of the form

$$V = -\{1 - \tfrac{1}{2} g + \tfrac{1}{2} g\, \sigma_1 \sigma_2 + \gamma S_{12}\} J(r)$$

~~int~~ proposed by Rarita and Schwinger[24], has been ~~used~~ investigated intensively. Originally they used the square ~~rectangular~~ well of depth V_0 and radius ~~range~~ r_0.

(24) Rarita and Schwinger, P.R. 59 (1941), 436.

$$S_{12} = \frac{3(\sigma_1 r_1)(\sigma_2 r_2)}{r_{12}^2} - (\sigma_1 \sigma_2)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

If we take $r_0 = 2.80 \times 10^{-13}$ cm, the best fit to deuteron binding energy E_0 and quadrupole moment is obtained by adopting the constants

$$V_0 = 6.40\ E_0 \qquad \gamma = 0.775.$$

One important criterion of this argument is to compare the percentage of admixture of D-state in deuteron ground state by comparing with the observed value $\frac{2}{3}\frac{\mu_n+\mu_p-\mu_D}{\mu_n+\mu_p-\frac{1}{2}}$. According to recent measurements (25), it is 4% in excellent agreement with theoretical prediction 3.9%, although this cannot be considered as decisive because the relativistic effects, the inclusion of which may change the situation. Recent calculations using Schwinger's variational method, meson potential of the form

$$J(r) = e^{-\kappa r}/r$$

with $\kappa = \frac{\mu c}{\hbar}$ $\mu = 320\ m_e$

can seems to fit singlet and triplet neutron-proton scattering as well as proton-proton scattering, whereas if we use square well,

(25) Arnold and Roberts, P.R. 71 (1947), 886,

(26) Blatt,
Bethe, Seminar at Princeton

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

20

for singlet
the ~~radii~~ range should be different from that for triplet. This again is in favor of meson theory, although little can be said as to the exchange character in the low energy region.

~~In spite of these~~
Now, if we want to reconcile these arguments with the formalism of the meson theory, we are at once confronted with serious difficulties. One of them is ~~the very well known~~ the appearance in nuclear forces of inadmissible singularity proportional to r^{-3} well known since the earlier stage ~~of the~~ of the meson theory. Møller-Rosenfeld[27]-Schwinger's[28] combination of pseudoscalar and vector meson fields ~~was regarded as~~ has long been regarded as a good remedy of this defect. ~~However, according to Dyson,[29] van Hove[30] Pauli and others, have shown that this this is not~~

(27)
(28)
~~(29) Dyson,~~

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

~~the case, because~~ However, as pointed out by Dyson[29] and earlier by Nelson[30], pseudoscalar theory has such a peculiar character ~~so~~ that ~~the~~ two ~~alternat~~ types of coupling, ~~name~~ i.e. the pseudoscalar and pseudovector couplings, are almost equivalent to each other. According to detailed calculation by van Hove[31] the static interaction ~~energy~~ between two nucleons due to pseudoscalar meson, which is obtained by applying ~~usual method~~ second pertubation theory as usual, but taking the recoil energy of ~~the~~ ~~no~~ nucleons into account in contrast to previous calculations, is proportional to r^{-1} for small distance r between two nucleons.* On the contrary, the interaction between two nucleons due to vector mesons, has the singularity proportional to r^{-3} as long as the tensor coupling g_2 ~~is not zero~~ exists.

Thus ~~we can no more~~ on the one hand it is of no use to mix two fields in order to avoid inadmissible singularity. But on the other hand ~~there~~

(29) (30)

(31)

* irrespective of the coupling is pseudovector pseudoscalar or

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

~~we find another~~ a new way is opened. Namely, ~~we can say there is a hope we may (as) hope that a consistent theory of nuclear forces can be constructed by~~ we may possibly arrive at a consistent theory of nuclear forces by reexamining the pseudoscalar meson theory.)

~~In this connection interesting~~ ~~and~~ results were obtained recently by applying Schwinger-Tomonaga's ~~relativistic formalism of field theory~~ to

Now, if we assume only the pseudoscalar coupling f_1 between the nucleon and the meson, it gives the same interaction ~~as in the case of~~ at large distance between nucleons at large distance as the pseudovector coupling f_2 further to considered. ~~for large distance~~ The only difference is that the coupling constant f_1 should be larger than ~~At most~~ f_2 by a factor $\frac{2M}{\mu}$. This means that ~~the~~ we have to deal with the strong coupling already at small distance of the order of $\frac{\hbar}{\mu c}$ in the pure pseudoscalar coupling.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

In this connection, interesting investigations were made recently by applying Schwinger-Tomonaga's relativistic field theory. ~~As to fundamental idea notions of this new powerful theory method,~~ I do not want to go into this new powerful method, because we shall be able to are expecting to hear on this subject from Prof. Oppenheimer ~~this~~ in this afternoon. I only mention that

i) According to Case [32] essentially, the same result as van Hove is obtained by applying Schwinger's method and

ii) also the anomalous magnetic moments of the neutron and the proton ~~are to~~ turn out to be finite ~~and indeed with correct sign in the symmetric theory~~, although ~~our~~ it is difficult to obtain ~~good~~ numerical agreement with experiment. It is still an open question ~~not certain~~ whether the discrepancy ~~is~~

(32) Case, Luttinger, Helv. Phys.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

is due the error in the model or in the method.

iii) ~~According to Dyson.~~ Neutron-electron interaction[33] turns out to be finite for pseudoscalar coupling and gives a value at least qualitatively in agreement with experiment[34]

iv) According to Dyson,[35] vacuum polarization effect in symmetric ps.sc. theory serves to reduce the singularity for $\lambda = 0$.

These arguments, all on the other hand indicate that ~~the~~ it is worth while to investigate further the pseudoscalar theory from the new point of view, ~~On the other hand it is also obvious that we need~~ (It is still an open question, whether this improved theory is a good approximation to the future

33) Slotnick, Bull. A.P.S. 24 (1949), Nr. 1, 23.

34) Rainwater, Rabi, Havens, ibid. 23.

35) Dysons Seminar

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

correct theory ~~for~~ which is free from divergence from the beginning.

Another very important problem of the meson theory is the interpretation of the β-decay. As already mentioned, the assumption that the meson, which is responsible for the nuclear force, is at same time responsible for the β-decay can not be maintained in the original form, because the probability of π-μ -decay is too small. However, the pseudoscalar theory is exceptional in this again in this ~~case~~ respect.[36] Namely, the ~~life~~ proper life-time τ_0 of pseudoscalar mesons with mass μ is given by

$$\frac{1}{\tau_0} = \frac{\mu c^2}{2\hbar}\left(\frac{f_1'}{\sqrt{\hbar c}} + \frac{f_2'}{\sqrt{\hbar c}}\frac{m}{\mu}\right)^2,$$

so that, if we take, for example, $f_1' = 0$, τ_0 becomes of the order of 10^{-6} sec., but it means ~~actually~~ again to go back ~~to~~

(36) Sakata, Prc.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

halfway to Fermi's theory, because the β-decay is due to direct coupling between ~~the~~ neutrons and the light particles in this theory. Only difference is that the interaction constant for β-decay is connected with the π-meson decay.

~~There is a~~ I want to say a few words about ~~the strange~~ curious fact ~~in connecting~~ the energy distribution of ~~β-decay~~ β-rays. ~~According to Fermi's theory as well as~~ in any kind of meson theory, the allowed ~~spectrum~~ shape should be well known Fermi ~~The fact that~~ Whereas the allowed β-ray spectrum agrees with the prediction of the original Fermi's theory as well as ~~its~~ any modification by meson theory, the fact that the forbidden spectrum except RaE seem to be also Fermi type cannot be understood easily. One possible ~~solution~~ interpretation of this curious fact is to assume ~~more than one~~ two or more ~~different~~ processes resulting in β-decay with differential selection rule and different probabilities.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

For example, ~~we can imagine~~ there can be processes leading to nuclear β-decay by

i) direct coupling

ii) through virtual creation and decay of the π-meson

iii) through virtual creation and decay of other kind of integer spin meson such as τ-meson.

But so far no detailed investigations were ~~It~~ made in this direction, except ~~low~~ The existence of mass meson was discussed by Japanese physicists (37).

~~Hitherto we~~ In the earlier stage of the meson theory, the meson has been considered only in connection with the specific nuclear phenomena. Later ~~it was pointed out by~~ a new type of a kind of the neutral meson was introduced by Bopp (39) in connection with the ~~elec~~ quantization of the electromagnetic field.

---

(37) Nakamura
(38) Nagakawa
(39) Bopp

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Theoretical Physics Seminar

Joint Meeting. Princeton University, Dep. Phys.
and Inst. Adv. Study.
Fine Hall
Friday 12, Nov, 1948

Some works on field theory in Japan.

Today I want to speak some of theoretical works done in these five or six years in Japan, because I think they have not been so easy accessible to physicists in this country. Before entering into the physical topics let me allow to express my feeling of gratitude to you for giving me the opportunity of visiting and staying here†. It is certainly not only my personal feeling, but all the physicists in Japan are very grateful for friendships of physicists in this country shown in various occasions. Most of

† in Princeton, which is a centre of theoretical study in the world.

Now the things which I shall talk today, may be new to you, because essentially the same investigations have been made also in this country and in Europe quite independently and almost simultaneously. We notice however, in spite of the equality of physical or mathematical contents, standpoint, there is a noticable difference in the genetical side. Namely stand point. The theoretical investigations have been developed always

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

in close contact with the experiment in this country. and it is certainly ~~the most~~ natural ~~and~~ sound ~~aspect~~. But in Japan the situations (as well as) are not so good., ~~the~~ theoretical investigations are apt to be too formal and speculative, because of the lack of corresponding development of experimental researches. In this connection, I feel it ~~myself~~ very fortunate, to be able to study in this country, where the theory and experiment ~~are~~ well balanced. †

Now let us go back to a few years ago, when things were much more obscure than today, but even then it was clear that there were two kinds of difficulties:

i) ~~There are~~ Divergence difficulties common to all kinds of fields interacting with each other. (At least a part since) of the

ii) ~~Difficulties~~ in the meson theory ~~which originate~~ can be ascribed to some ~~bad faults~~ false assumptions as to the ~~spins~~, properties ~~and~~ of the mesons and the modes of interaction with other particles.

Of course, ~~although~~ these two kinds of difficulties were intimately connected with each other ~~especially and even today~~ especially in the

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

and subsequent rapid development of q.e.d.

meson theory, ~~it was fairly certain we have only meet only with the first kind of difficulties in elect~~ q.e.d.

It is ~~very diffi~~ not easy even today to ~~discrimi~~ ~~which~~ discriminate between these two things, but we want first ~~assume~~ confine our attention to the first kind of difficulties.

~~For the ~~ sake of convenience, we start from ~~more se~~ concrete but ~~more recent later~~ more concrete problem, ~~which is~~ although it comes later in ~~the~~ order ~~of time~~. As shown the famous example of

† ~~Since~~ the discovery of by Lamb and Retherford of the level shift in hydrogen ~~energy~~ atom in last year, thus the nature of the divergence difficulties in electrodynamics ~~became~~ has become very clear by theoretical analysis by various authors and especially by elaborate works by Schwinger. It is now well known ~~clear~~ that the divergence, with which we meet in quantum electrodynamics, can be separated into three types:

i) mass or ~~se~~ electron-self energy-type
ii) charge-renormalization type
iii) vacuum polarization ~~or~~ type

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

a) The nature of these three types of divergences ~~contribution to of the~~ can best illustrated ~~in the case of radiat~~ by the effect of radiative correction on the Compton scattering. ~~This problem has been investigated already in this country~~ It will not be useless to ~~introduce~~ consider in this connection a convenient and intuitive picture ~~according to~~ according to the "transition diagram method" by Koba and Takeda, which seems to be similar to a method developed recently by Dyson, although the latter is more logical and systematic.

b) Method of Self consistent subtraction

c) ~~Now~~ The question arises: where is the ~~origin of these~~ divergence difficulties ~~most simplest way~~ immediate way, most primitive ways of obviating these divergences was to subtract simply these terms from the Hamiltonian. ~~A little much more reasonable way is~~ Now that we know the nature of these ~~difficult~~ divergences more ~~origin~~ precisely, we have a more reasonable way of amalgamating them in other terms in the Hamiltonian as proposed by Lewis, Tomonaga,

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

c') Method of Mutual Compensation.
c'') Two meson theory by Tomonaga and Sakata.

c) Method of Canonical Transformation
Still more convincing way of solving the problem is the method of C.T. developed chiefly by Schwinger and also by Tomonaga.
Supermany time Theory

d) This method becomes more satisfactory, if we the whole procedure was ~~made~~ performed in relativistically and gauge invariant way as shown by Schwinger's elaborate works and Feynman's recent investigations. They starts from a relativistically invariant formulation of q.e.d.

Tomonaga's ~~new~~ exclusion of Dirac's many-time-theory.
~~We~~ I think it ~~hardly~~ almost unnecessary to talk about in detail, because it ~~very~~ is well known to most of you. I should like to only to recollect the origin of theory. It goes back to about 1942-1943, I was worrying about to construct a field theory which is completely relativistic. In this connection, I was very much interested in Dirac's paper in Physik. Zs. Sowj. 1933, in which an idea of generalized transformation function was first expressed.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

e) Tanikawa's Formulation

f) Non-localizable Field Theory

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

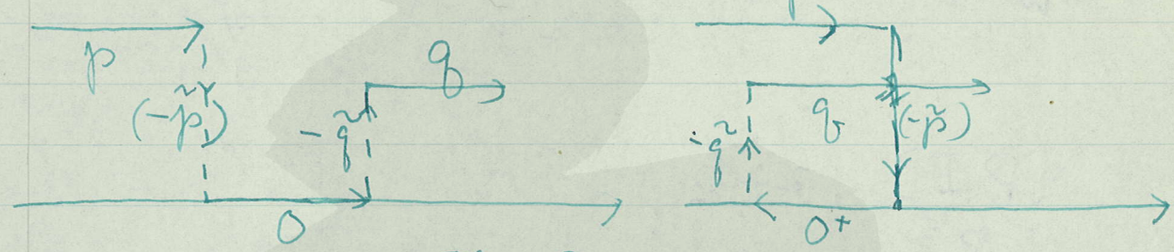
Letter: Prog. Theor. Phys. 3 (1948), 98.

Transition Diagram Method[1] I.

Ziro Koba and Gyo Takeda, Radiation Reaction in Collision Process II

— Radiative Correction for Compton Scattering — — (Prog. Theor. Phys. 3 (in Press)

○ e^4 ~~order~~ (second order perturbation)

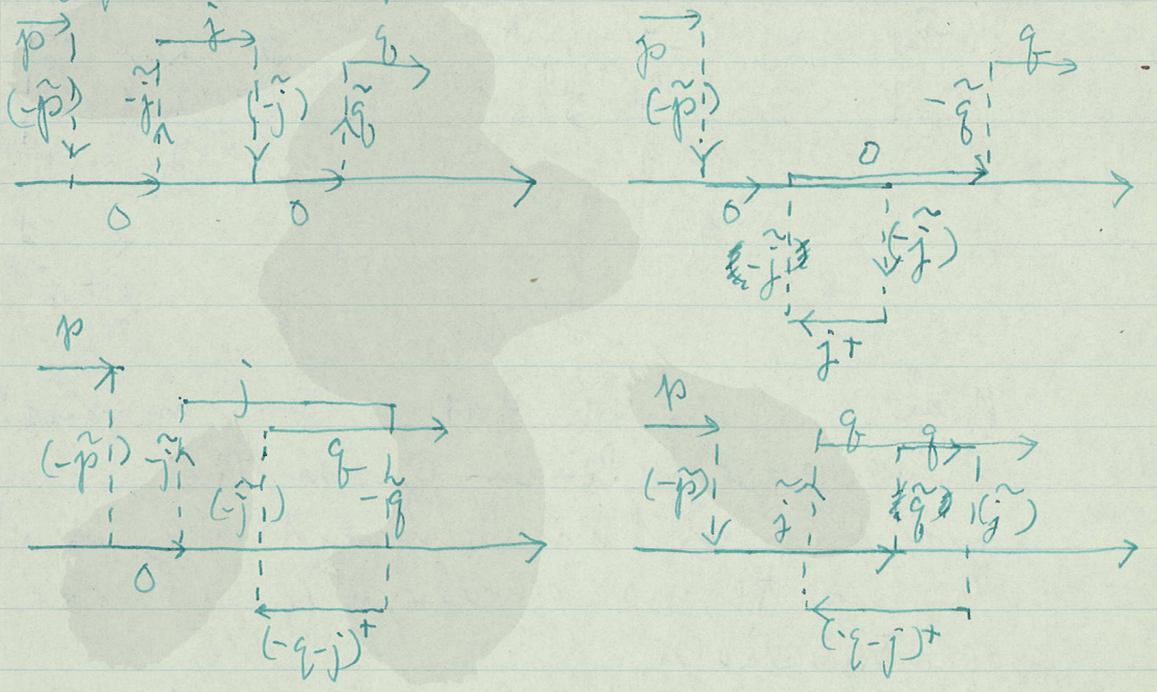


○ e^6 (fourth order (~~perturbation~~ term) × second order term)

[M]

720

number of different path



---

1) This method seems to be closely related to Feyman-Dyson's method of ~~calcu~~ integration of generalized Schrödinger equation, except that the latter is more systematic and logically rigorous.

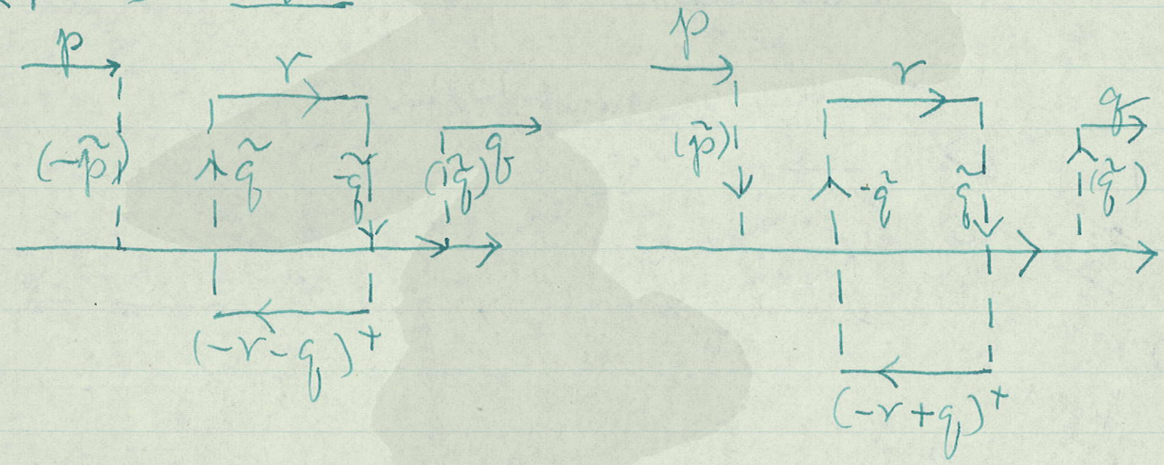
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

These Mass type processes contain divergence which can be compensated by adding the term in the Hamiltonian

$$-\delta m \int u^*(x)\beta u(x)\,dv\,, \qquad (3.26)$$

where $\delta m = \left(\frac{3me^2}{2\pi}\right)\int^{\infty}\frac{dj}{j}$ + finite terms

[PI] 240



These vacuum polarization type processes contain divergences ~~linear~~ quadratic, linear and logarithmic respectively, of which linear terms cancel when we add all contributions.

Logarithmic ~~to~~ divergence can be compensated by adding the term corresponding to charge renormalization:

$$-\delta e \int u^*(x)\,(\alpha, A(x))\,u(x)\,dv$$

where $\delta e = -\left(\frac{e^3}{3\pi^2}\right)\int^{\infty}\frac{dr}{r}$.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Quadratic divergence can be eliminate by adding

$$\frac{e^2}{3\pi^2}\int r\,dr \int A^2(x)\,dv,$$

which can be interpreted as either as changing the vacuum Maxwell equations to

$$\text{rot } H - \frac{\partial E}{\partial t} = \left(\frac{4e^2}{3\pi}\right)\int r\,dr\, A \qquad \text{div } E = 0 \quad (S.S)$$

$$\text{rot } E + \frac{\partial H}{\partial t} = 0 \qquad \text{div } H = 0$$

or as change the density current to

$$i = e u^* \alpha u - \left(\frac{e^2}{3\pi^2}\right)\int r\,dr \cdot A$$

$$\rho = u^* u,$$

formal assymmetry being due to our special choice of gauge.
Thus the total Hamiltonian can be written in the as follows:

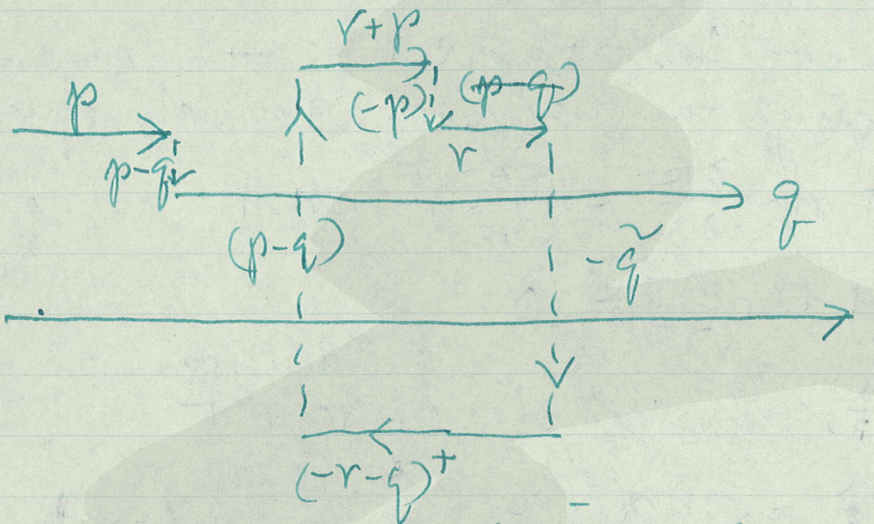
$$H = \int u^*(x)\{\alpha p + \bar{m}\beta\} u\,dv + \bar{H}_{emf}$$
$$+ \bar{e}\int (iA)\,dv + \frac{\bar{e}^2}{2}\iint \rho(x)\rho(x')/|x-x'|\,dv\,dv'$$
$$- \delta m \int u^* \beta u\,dv - \delta e \int (iA)\,dv + \left(\frac{e^2}{3\pi^2}\right)\int r\,dr \int A^2 dv$$
$$+ O(e^4)$$

in which either $\bar{H}_{emf}$ or i should be

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

modified as stated above.

[P II] 120 pathes *



These processes do not give rise to any divergencies in integrals when computed in suitable pairs, because they contain a process equivalent to breaking up of a single photon into two. The circumstance that such a process is almost forbidden will be easily understood by referring to Furry's symmetry theorem.†

* Total number of pathes: [N] + [P I] + [P II] = 1088.

† Furry, P.R. 51 (1937), 126.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Transition Diagram Method. II.

2, Koba and G. Takeda: Radiative Corrections in e^2 for an Arbitrary Process Involving Electrons, Positrons and Light Quanta*

using the modified Hamiltonian

$$H = \int u^* \{ \alpha p + m\beta \} u \, dv + \frac{1}{8\pi} \int (E^2 + H^2) \, dv$$
$$+ e \int (u^* \alpha u, A) \, dv + \frac{e^2}{2} \iint u^* u \, u^{*\prime} u' \, |x - x'|^{-1} dv dv'$$
$$- \delta m \int u^* \beta u \, dv - \delta e \int (u^* \alpha u, A) \, dv$$
$$- e \, \delta e \iint u^* u \, u^{*\prime} u' \, |x - x'|^{-1} dv dv'$$
$$+ \frac{e^2}{3\pi} \int_0^\infty r \, dr \int A^2 dv \qquad (1)$$

with

$$\left. \begin{aligned} \delta m &= \frac{3me^2}{2\pi} \int_0^\infty \frac{dk}{k} \\ \delta e &= -\frac{e^3}{2\pi} \int_0^\infty \frac{dk}{k} \end{aligned} \right\},$$

(A : transverse part of elmg. potential), we ~~obtain~~ can eliminate all the divergence difficulties as long as we are concerned with the first radiative correction $\delta\sigma \sim e^{2n+2}$ to the zeroth cross-section $\sigma_0 \sim e^{2n}$, n being the order of perturbation required to obtain a non-vanishing matrix element for the process considered.

---

* Prog. Theor. Phys. 3 (1948), 205.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

In order to confirm this general result, it is convenient to introduce a "transition diagram method". It expresses electrons and positrons as world lines in the momentum space, while emission and absorption of photons are described as "leaps" of these world lines. Any process can be characterized by the number of world lines N, ~~leaps~~ L and number of leaps L and number of real photons Q. There is a general relation

$$L = Q + 2(N-1)$$

between them.

When we take radiative ~~reaction~~ corrections into account, our diagram acquires two additional leaps. There are two distinct ways of attaching them

i) existing world lines gain new leaps, or
ii) an additional closed world line is introduced and coupled with existing ones through the virtual photons.

i) implies mass or self-energy-type divergences
ii) can be divided into $[P.I], \cdots [P.Q]$ (Q-groups).

Among them the latter $[P.I]$ afford quadratic and logarithmic divergence, which can be cancelled by vacuum ~~compensation~~ polarization terms. $[P.II]$ etc. are found to contribute nothing to divergence.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Theory of the Interaction of Elementary Particles.

IV. Problem of Vacuum Polarization.

H. Umezawa, J. Yukawa and E. Yamada, Prog. Theor. Phys. 3 (1948) 317 and in press. (Paper and Letter)

According to Sakata (Pais), self-energy type divergences can be eliminated by introducing a neutral scalar meson field or e-meson field (τ-field) as shown in the cases of elastic scattering of electrons, energy levels of hydrogen atoms.

Another type of divergence due to self-energy of a photon, which arises from the mutual interaction between a vacuum electron and a photon can be eliminated by ~~introducing~~ assuming the existence of two kinds ~~so~~charged scalar fields, because the self-energies of the photon (with momentum l_0) were found to be

$$W_{P,F} = -\frac{4e^2}{3\pi l_0}\int p\,dp + \text{finite terms}$$

$$W_{P,S} = \frac{2e^2}{3\pi l_0}\int p\,dp + \text{finite terms}$$

If we assume further the existence of a ~~v~~charged vector (or pseudovector) field, there appears

$$W_{PV} = -\frac{2e^2}{3\pi l_0 \varkappa^2}\int p^3\,dp - (\;)\int p\,dp - (\;)\int \frac{dp}{p}$$

Thus in nature there should exist only Fermi particles and scalar (or pseudoscalar)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

particles with the relative abundance of 1:2. The vector charged particles should be excluded from this point of view.

In this connection, it should be noticed that the self-energy of the neutral scalar particle (-meson (f-particle) cannot be made finite by the above assumption, because

$$W_{c,F} = -\frac{2f^2}{\pi M}\int p\,dp - \cancel{f^2}\,\text{log. div.} + \text{f.t.}$$

$$W_{c,S} = -\frac{g^2}{\pi M}\int p\,dp + \text{log. div.} + \text{f.t.}$$

$$W_{c,U} = -\frac{1}{\pi M \kappa^2}\left(\frac{g^2}{4}+\frac{f^2}{16}\right)p(p^2+\kappa^2)^{3/2} - \cdots$$

Correction to Compton Scattering

As shown by Koba, Takeda and present authors independently, the diverging terms ~~can be~~ are equivalent to the presence of extra interaction

$$-\frac{e^2}{3\pi^2}\int^\infty p\,dp\,A^2 + \frac{3e^2}{2\pi}\int^\infty \frac{dl}{l}\psi^*\beta^\mu\psi - \frac{e^3}{3\pi^2}\int^\infty \frac{dp}{p^3}\psi^*\alpha A\psi$$

~~The Among them~~ Among three terms in this expression ~~first~~ the second term can be compensated by neutral scalar meson field and the first term by charged

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

scalar meson field, whereas the third term can only be ~~eliminated~~ reduced by charge renormalization. The latter point is not satisfactory from the standpoint of Sakata theory.

Reactive Correction for the Elastic Scattering of an Electron

S. Endo, T. Kinoshita and Z. Koba

velocity of electron $\ll c$.

S.C.S:

$$\left(\frac{d\sigma}{d\sigma_0}\right) = \frac{e^2}{\pi m^2} p^2(1-\cos\theta)\left[-\frac{10}{3} + \frac{8}{3}\log\frac{2p}{m}\right]$$

which is always < 0 for $p/m \ll c$.

C-meson:

$$\left(\frac{d\sigma}{d\sigma_0}\right) = \frac{e^2}{\pi m^2} p^2(1-\cos\theta)\left\{-\frac{23}{9} + \frac{8}{3}\log\frac{2p}{m} + \frac{3}{2}\log\frac{M}{m}\right]$$

which becomes > 0 for $p/m \gtrsim 1/10$.

The divergences originating from the vacuum polarization effect are entirely taken off in the above treatment. It is not yet obvious how we should treat them.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

The problem of vacuum polarization (continued) (letter to Prog.)

Hiroomi Umezawa and Eiji Yamada

Scattering of a photon by charged scalar mesons. We assume n charged Fermi particles and $2n$ charged scalar or pseudo-scalar particles and introduce also the C-meson. Divergence of the self-energy type of charged scalar meson and photon vanishes and there remain as in the case of ordinary Compton scattering

$$-\frac{ne^3}{2\pi}\int^{\infty}\frac{dk}{k}H_1-\frac{ne^4}{\pi}\int\frac{dk}{k}H_2,$$

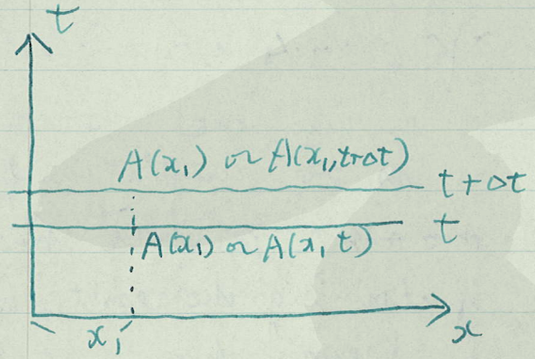
which can be removed by the same charge renormalization as in the case of Compton scattering.

More generally if n_1-Fermi charged particles and n_2-Bose charged particles coexist, the charge renormalization remains ~~is always~~ the same irrespective of the type of incident charged particle. As for the renormalization of interaction constant f ; $f \to f\left(1-\frac{f^2}{2\pi}\int\frac{dk}{k}\right)$ hold always.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

On the Relativistically Invariant Formulation of the Quantum Theory of Wave Fields.

I. Tomonaga, Prog. Theor. Phys. 1 (1946), 27
II. Kobe, Tati and Tomonaga, ibid. 2 (1947), 101 (~~Nucl~~. El. and El. Fields)
III. K. T. & T. ibid 2 (1947), 198 (Electromagnetic and Electron Fields)
IV. Kanesawa and Tomonaga, 3 (1948), 1, (Interacting Electromagnetic and Meson Fields)
V. K. & T. 3 (1948), 101, (")



Transformation for the quantized fields

1) One time theory (Heisenberg - Pauli)

Schrödinger picture

$$\lim_{\Delta t \to 0} \psi(\cdots A(x_1) \cdots t+\Delta t / \cdots A(x_1) \cdots t) = 1$$

$$i\hbar \frac{\partial \psi}{\partial t} = (H_1 + H_2 + H_{12})\psi = H\psi$$

Heisenberg picture (free fields representation)

$$\lim_{\Delta t \to 0} \psi'(\cdots A(x, t+\Delta t) \cdots / \cdots A(x,t) \cdots) = 1$$

$$i\hbar \frac{\partial \psi'}{\partial t} = H'_{12}\psi'$$

$\therefore$ $A' = A(x,t) = SAS^{-1}$ $\qquad \psi' = S\psi$.

$S = e^{\frac{i}{\hbar}(H_1+H_2)t}$

$$i\hbar \frac{\partial \psi'}{\partial t} = i\hbar \frac{\partial S}{\partial t}\psi + i\hbar S \frac{\partial \psi}{\partial t}$$

$$= -(H_1+H_2)S\psi + (H_1+H_2)S\psi + SH_{12}S^{-1}S\psi$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

(interacting fields representation)

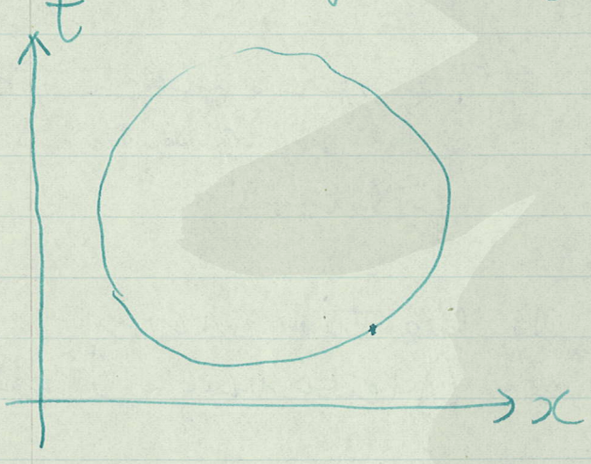
$$\psi'' = S\psi$$

$$i\hbar \frac{\partial \psi''}{\partial t} = i\hbar \frac{\partial S}{\partial t}\psi + SH\psi$$

$$i\hbar \frac{\partial S}{\partial t} = -HS :$$

$$i\hbar \frac{\partial \psi''}{\partial t} = (H'' - H)\psi''$$

2) Dirac's generalized transformation fn



$\psi(\cdots A(x,t)\cdots)$

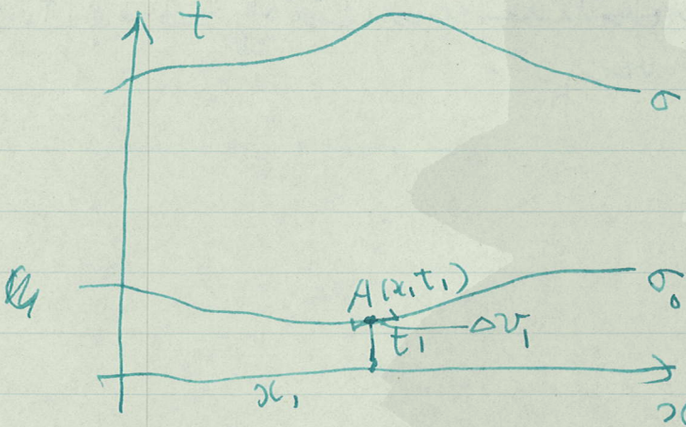
In this case, one can consider ψ as the amplitude for the ~~pro~~ a priori probability of obtaining the results $A(x,t)$ by observing them, the observations ~~interfering~~ with although one another.

(Dirac, Phys. Zs. d. Sowj. 3 (1933), 64.)

Tanikawa, Prog. Theor. Phys. 1 (1946), 12.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

3) Tomonaga's theory*



$\Psi(\cdots A(x,t_1)\cdots A(x,\psi)\cdots)$

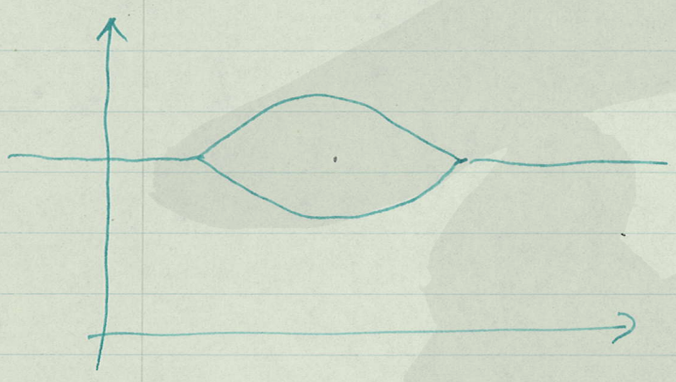
Hyperfunctional

$$i\hbar\frac{\partial\Psi}{\partial t_1} = H_{12}^{(x,t_1)}\Delta v_1\cdot\Psi$$

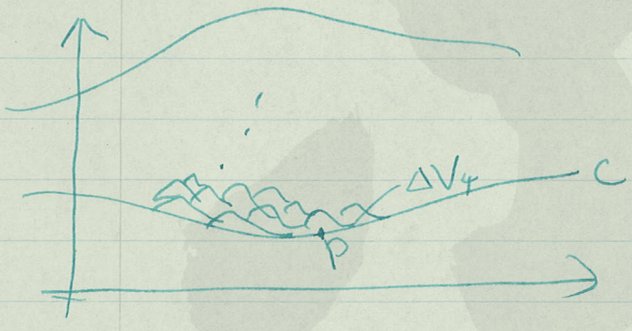
$$t_1 = t_1(x_1)$$

$$i\hbar\frac{\partial\Psi}{\partial\sigma} = H_{12}\Psi$$

$$\left(i\hbar\frac{\delta\Psi[C]}{\delta C_P} = H_{12}\Psi[C]\right)$$



$$\frac{\delta\Psi[C]}{\delta C_P} = \lim_{\substack{\Delta V_4\to 0\\ C'\to C}}\frac{\Psi[C']-\Psi[C]}{\Delta V_4}$$



---

* Dirac, Many-time theory
Bloch, Integrability

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Application of Tomonaga's theory
Case of Electrodynamics
Elimination of ~~relativ~~ auxiliary condition
in relativistic way

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Photon self-energy and related problems

i) Schwinger's covariant theory differs in canceling the self energy of the photon from Tomonaga's covariant theory, in which it remains indeterminate so far as I know (August).

On the other hand, the ordinary perturbation calculations give rise to divergence ~~corresponding to~~ equivalent to the existence of in Hamiltonian of the term

$$\dot{\varepsilon}'A = -\frac{e^2}{3\pi^2}\int^{\infty} r\,dr \int A^2(x)\,dv.$$

The covariant theory should ~~coincide~~ in all reduce to the ordinary H.P. theory, if any particular Lorentz frame and any particular gauge are ~~used~~ adopted, so that either the ~~charge~~ current density should be redefined as

$$\dot{\varepsilon} = -e u^* \alpha u - \left(\frac{e^2}{3\pi^2}\right)\int^{\infty} r\,dr\, A$$

$$= -e u^* \alpha u + \dot{\varepsilon}'$$

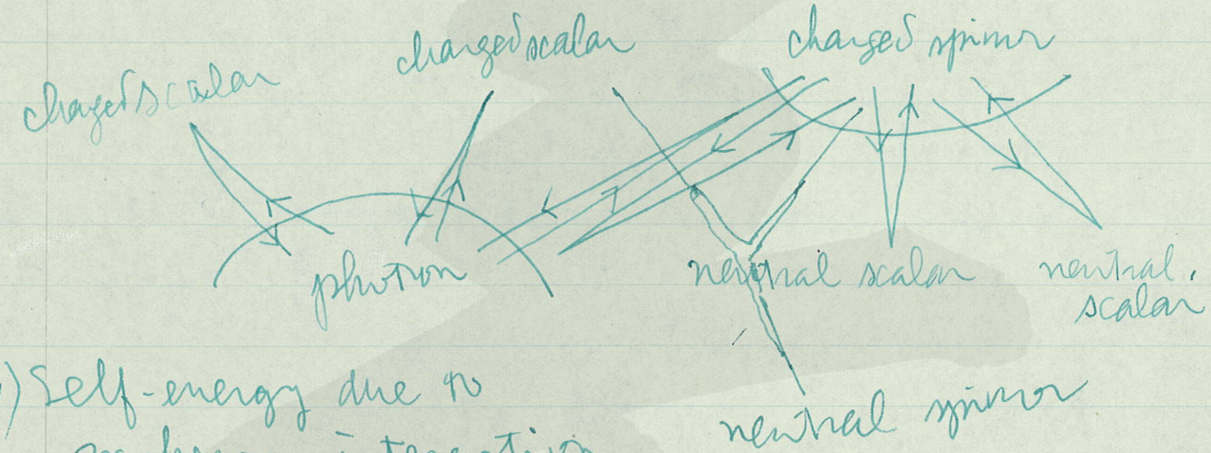
(This ~~con~~ redefinition may be generalized so as

$u^*\gamma u$, $v^*\gamma u$ etc

should all be changed in a similar manner, corresponding to the ζ-meson etc.)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

or two kinds of scalar ~~mesons~~ charged field should be introduced, in order to get convergent result for photon self-energy



iv) Self-energy due to exchange interaction

iii) 4-particle Type self-energy

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

R. Serber, A Note on Positron Theory and Proper Energies
(Phys. Rev. 49 (1936), 545)

Dirac の subtraction theory[1] は density matrix

$$(x''k''|R|x'k') = \sum_{occ} \psi_n(x''k'')\psi_n^+(x'k')$$

に対する equation of motion を積分して R が $X = X' - X''$ の length が finite の所では finite なことを示した。$x_\lambda x^\lambda = 0$ における worst singularities は

$$(x''k''|R|x'k') = \frac{1}{2}\Big[\sum_{occ} \psi_n(x''k'')\psi_n^+(x'k') - \sum_{unocc} \psi_n(x''k'')\psi_n^+(x'k')\Big]$$

なる symmetrization によって除かれる。
Singular terms S の expression は Heisenberg[2] によって与えられた。Heisenberg は更に slowly varying weak field に ~~[illegible]~~ よる vacuum polarization を 0 に renormalize に対する ~~finite~~ term をも S に入れ、R を

$$r = R - S$$

で置きかえた。F なる operator の $X = \frac{1}{2}(x' + x'')$ における expectation value は

$$\bar{F}(X) = \lim_{x \to 0} \sum_k (X - \tfrac{1}{2}x, k|F|X + \tfrac{1}{2}x, k)$$

で finite determinate である。

1) P. A. M. Dirac, Proc. Camb. Phil. Soc. 30 (1934), 150.
2) W. Heisenberg, Z. S. f. Phys. 90 (1934), 209.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$j_\lambda(X) = -e \lim_{x\to 0} \sum_{kk'} \alpha_{\lambda kk'} (X - \tfrac{1}{2}x, k' | r | X + \tfrac{1}{2}x, k)$$

Dirac's Heisenberg's

In this subtractive formalism the q. m. equations of motion and the commutation relations are not comptible. Thus, with

$$\dot{A}_i = -E_i \qquad \dot{E} = \mathrm{curl}\, H + 4\pi e \alpha (R - S)$$

$$\frac{d}{dt}[E_i(X), A_i(X')] = -4\pi e [\alpha_i S(X), A_i(X')]$$

$$\neq 0.$$

Heisenberg's second scheme[2] では Hamiltonian fn として density-matrix scheme における energy expression を採用する。

$A_0 = 0$, $x_0 = 0$ とすると

$$H = \frac{1}{2}\int dX \sum_{k'k''} \left[\alpha_{jkk''}\left\{i\frac{\partial}{\partial x_j} + \frac{1}{2}e[A_j(X+\tfrac{1}{2}x) + A_j(X-\tfrac{1}{2}x)]\right\}\right.$$

$$\left. + \beta_{k'k''}\right] [\psi^+(X+\tfrac{1}{2}x, k')\psi(X-\tfrac{1}{2}x, k'')$$

$$- \psi(X-\tfrac{1}{2}x, k'')\psi^+(X+\tfrac{1}{2}x, k')]$$

$$+ \int dX \sum_{k'} i\frac{\partial}{\partial x_0}(X - \tfrac{1}{2}x, k' | S | X + \tfrac{1}{2}x, k') + H_{EM}$$

これからでてくる Hamiltonian eq. for ψ は

$$i\dot{\psi}(X) = \left[\alpha_j\left(-i\frac{\partial}{\partial X_j} + \frac{1}{2}e\{A_j(X) + A_j(X-x)\} + \beta\right]\right.$$
$$\times \psi(X-x)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

A_i の E_i に対する eq. は

$$-\dot{A}_i = E_i - \frac{e^2}{3\pi}\left[x_i x_j E_j / x_1 x^1 - E_i \log(|x_1 x^1|/c_1)\right]$$

$$-\dot{E}_i + \mathrm{curl}\, H_i \quad \mathbf{H} = 4\pi\left[j_i(x) - e^2 S_i(j)\right]$$

$$+ e^2(\text{singular term})$$

j_i : terms linear in e.

Thus E_i, A_i will be singular as $X \to 0$, which is fatal to Heisenberg theory.

Photon self-energy can be obtained from

$$\bar{H}(x) = S^{-1}(x) H(x) S(x)$$

a transformation

by using S which diagonilizes H.

$$W = -\left(\frac{\alpha h\nu}{3\pi}\right)\left(\frac{2}{R^2} - 2\log\frac{1}{2}CR + O(R)\right) \quad (5')$$

$$\log C = 0.577\cdots$$

where z is the component of X in the direction of the electric vector of the photon, and R is the length of X. (5) is just the expectation value of $H_{EM} - h\nu$.

Serber's proposal; Commutation Rule は

$$[\psi^+(X), \psi(X')]^+ = \delta(X - X' + z)$$

にかえる.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

この方法は covariant でないが、photon self-energy が

$$W = -\frac{\alpha h\nu}{2\pi}\left\{\left(\frac{mc^2}{h\nu}\right)^2\left[8\frac{z^2}{R^4}+\left(\frac{4}{3}\right)\frac{x^2z^2}{R^4}-2\frac{z^2}{R^2}\right]\right.$$

$$\left.+\frac{1}{3}\frac{x^2}{R^2}+O(R)\right]$$

となり、e^2 の order で null photon energy を indicate している。

electrostatic self energy は

$$W_{ES} = -\left(\frac{2e^2}{\pi}\right)\left[\log\frac{1}{2}CR-1+O(R)\right]$$

(non-positron theoretical calculation では

$$W_{ES} = \frac{e^2}{2R}$$

)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

W. Heisenberg und H. Euler

Folgerungen aus der Diracschen Theorie des Positrons

(Z. f. Phys. 98 (1936), 714)

§ 1. Das mathematische Schema der Theorie des Positrons

$$\begin{cases} (x'\,t'\,k' \,|\, R \,|\, x''\,t''\,k'') = \sum_{\text{bes. Zust.}}^{(n)} \psi_n^*(x''\,t''\,k'')\,\psi_n(x'\,t'\,k') \\ (x'\,t'\,k' \,|\, R \,|\, x''\,t''\,k'') = \psi^*(x''\,t''\,k'')\,\psi(x'\,t'\,k') \end{cases}$$

$$\begin{cases} (x'\,t'\,k' \,|\, R_s \,|\, x''\,t''\,k'') = \frac{1}{2}\left(\sum_{\text{bes.}}^{(n)} - \sum_{\text{unbes.}}^{(n)}\right) \psi_n^*(x''\,t''\,k'')\,\psi_n(x'\,t'\,k') \\ (x'\,t'\,k' \,|\, R_s \,|\, x''\,t''\,k'') = \frac{1}{2}\left[\psi^*(x''\,t''\,k')\,\psi(x'\,t'\,k') - \psi(x'\,t'\,k')\,\psi^*(x''\,t''\,k'')\right] \end{cases}$$

$x_\lambda' - x_\lambda'' = x_\lambda$ $\quad t' - t'' = t$

$ct = x_0 = -x^0$ $\quad x_i = x^i$ $\quad \xi_\lambda = \frac{x_\lambda' + x_\lambda''}{2}$

$A_0 = -A^0$ $\quad A_i = A^i$

$\alpha^0 = -\alpha_0 = 1$ $\quad \alpha^i = \alpha_i$

$$(x'k' \,|\, R_s \,|\, x''k'') = u\,\frac{\alpha_\rho x_\rho}{(x^\lambda x_\lambda)^2} - \frac{v}{x^\lambda x_\lambda} + w \log(x^\lambda x_\lambda)$$

$$u = -\frac{i}{2\pi^2}\, e^{\frac{ei}{\hbar c}\int_{P'}^{P''} A^\lambda dx_\lambda}$$

Das Integral ist auf der geraden Linie von P' nach P'' zu nehmen [1]

1) Heisenberg, Z. f. Phys. 90 (1934), 209; Uehling, Phys. Rev. 48 (1935), 55; Serber, ebenda, 49.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$r = R_S - S$$

$$S = e^{\frac{ei}{\hbar c}\int_{p'}^{p''} A^\lambda dx_\lambda} \cdot S_0 + \frac{\bar{a}}{x_\lambda x^\lambda} + \bar{b}\log\left|\frac{x_\lambda x^\lambda}{C}\right|$$

S_0 bedeutet die Matrix R_S im feld- und materiefreien Raum.

$$\bar{a} = \kappa\left\{\frac{ei}{24\pi c} x_\rho x^\sigma \alpha^\lambda \left(\frac{\partial F_{\lambda\sigma}}{\partial \xi_\rho} - \delta^\rho_\lambda \frac{\partial F_{\tau\sigma}}{\partial \xi_\tau}\right)\right.$$

$$\left. - \frac{e^2}{48\pi^2 c^2} x_\rho x_\sigma x^\tau \alpha^\rho F^{\mu\sigma} F_{\mu\tau}\right\}$$

$$\bar{b} = \kappa\left\{\frac{ei}{24\pi c} \alpha^\lambda \frac{\partial F_{\tau\lambda}}{\partial \xi_\tau} + \frac{e^2}{24\pi^2 c^2} x_\lambda \alpha^\mu \right.$$

$$\left. \times \left(F_{\mu\nu} F^{\tau\lambda} - \frac{1}{4}\delta^\lambda_\mu F_{\tau\sigma} F^{\tau\sigma}\right)\right\}$$

$$C = 4\left(\frac{\hbar}{\gamma m c}\right)^2 \qquad \gamma = \text{Eulerkonst} = 1.781\ldots$$

Stromdichte

$$s_\lambda(\xi) = -e\sum_{k'k''} \alpha^\lambda_{k''k'} (\xi k' | r | \xi k'')$$

Energie- und Impulstensor

$$U^\mu_\nu(\xi) = \lim_{x\to 0}\left\{ic\hbar\frac{\partial}{\partial x_\mu} - \frac{e}{2}\left[A^\mu\left(\xi + \frac{x}{2}\right)\right.\right.$$

$$\left.\left. + A^\mu\left(\xi - \frac{x}{2}\right)\right\}\sum_{k'k''} \alpha^\nu_{k''k'} \left(\xi + \frac{x}{2}, k' | r | \xi - \frac{x}{2}, k''\right)\right.$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\psi(x,k)=\sum_n a_n u_n(x,k)$$

$$a_n^*=N_n\Delta_n V_n \;;\quad a_n=V_n\Delta_n N_n$$

$$V_n=\prod_{t\le n}(1-2N_t)$$

$$a_{n'}=a_n^*=-V_n\Delta_n N_{n'}$$

$$a_{n'}^*=-N_{n'}\Delta_n V_n \;;\quad N_{n'}=1-N_n$$

$$H=\lim_{x\to 0}\int d\xi\Big\{-\Big(c\hbar\frac{\partial}{\partial x_\ell}-\frac{e}{2}\big[A^\ell(\xi+\tfrac{x}{2})+A^\ell(\xi-\tfrac{x}{2})\big]\Big)$$

$$\times\sum_{k'k''}\alpha^\ell_{k''k'}\sum_{mn}\frac{1}{2}(a_n^*a_m-a_ma_n^*)u_n^*(\xi-\tfrac{x}{2},k'')$$

$$\times u_m(\xi+\tfrac{x}{2},k')$$

$$+\sum_{k'k''}\beta_{k''k'}mc^2\sum_{mn}\frac{1}{2}(a_n^*a_m-a_ma_n^*)u_n^*(\xi-\tfrac{x}{2},k'')$$

$$\times u_m(\xi+\tfrac{x}{2},k')$$

$$-\sum_{k'}\Big(c\hbar\frac{\partial}{\partial x_0}-\frac{e}{2}\big[A^0(\xi+\tfrac{x}{2})+A^0(\xi-\tfrac{x}{2})\big]\Big)$$

$$\times(\xi+\tfrac{x}{2},k'|S|\xi-\tfrac{x}{2},k')$$

$$+\frac{1}{8\pi}(E^2+H^2)$$

$$H_0=\sum_{E_n>0}N_nE_n-\sum_{E_n<0}N_{n'}E_n+\sum_{ge}n_{ge}\nu_{ge}$$

$H_1=$ . .

$H_2=$ . .

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

§ 2. Berechnung der Energiedichte in der anschauliche Wellen-Theorie

§ 3. Bedeutung des Resultats in der Q.T. der Wellenfelder

§ 4. Die physikalischen Konsequenzen des Resultats

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Elementary Particles and the Principle of Reciprocity (Nature 163 (1949), 207)

by M. Born

Reciprocity applied to ψ-functions has not proved to be fertile. One has to go one step deep and apply the postulate to the L-fns for the density of wave fields.

Green started from the statistical operator ρ instead of ψ. ρ reduces to $\psi\psi^*$ for pure state but represents also "mixture".

Most general Lagrangian of a field is

$$\mathfrak{L} = F(p_k, p'_k)\,\rho$$

where F is a fn of two sets of four variables; both of them represent the momentum, but p_k operates on ρ from the left, p'_k from the right. Thus extremely simple result was obtained: the field equations can be written in the form

$$F(p_k, p_k)\,\rho = 0$$

The principle of reciprocity is applied to this fn F, namely we postulate that only self-reciprocal F have to be used for describing particles. In the case of scalar fields, a simple way of finding all self-reciprocal scalars is to determine the eigenfns of the operator $x^k x_k + p^k p_k$, where coordinates and momenta are measured in the units a and b respectively ($ab = h$). It is easy to see that the relativistic invariant eigenfunctions have the form

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\overline{F}(p) = f(P)\, e^{-\frac{1}{2}(P)}$$

where $f(P)$ are Laguerre polynomials of the arguments $P = p_K p^K$, with

$$f_0 = 1, \quad f_1 = P - 2, \quad f_2 = P^2 - 6P + 6, \cdots$$

Thus we have

$$f(P)\,\rho = 0,$$

which ~~shows that~~ can be satisfied by plane waves with momentum-energy p_K^0, if $P^0 = p_K^0 p^{0K}$ is one of the roots of $f(x) = 0$. The roots are

f_1 : $x_1 = 2$ $\qquad$ f_2 : $x_2^{(1)} = 3 + \sqrt{3}$, $\; x_2^{(2)} = 3 - \sqrt{3}$

If χ_r is one of these, one have a wave which satisfies

$$(p_K p^K - \chi_r)\,\psi = 0$$

so that $\sqrt{\chi_r}$ is a possible restmass. ~~To show this rigorously~~. Green proved this rigorously by showing that the total energy and momentum of the field appear in the form

$$P_K = \sum n_r\, p_K^{0(r)} p^{0rK} \qquad p_K^{0(r)} p^{0rK} = \chi_r$$

where n_r are operators with integral eigenvalues. One has already for scalar fields an infinite number of diff. particles, the lowest masses being

$$\sqrt{2} = 1.414 \qquad \sqrt{3+\sqrt{3}} = 2.175$$
$$\sqrt{3-\sqrt{3}} = 1.126 \ldots$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

their ratios correspond roughly to some experimental results on mesons.

Lagrangian for particles with spin can be found in a similar way.

This can be regarded as a rational generalization of relativity. The theory proposed here for scalar fields is equivalent to demanding as fundamental invariant the sum $R + P = x^k x_k + p^k p_k$. For spin ½ one has to define the distance by taking the root of R and P with the help of properly chosen Dirac matrices.

It remains to consider the problem of interaction. Whatever the success or failure of the special hypothesis suggested here, I am convinced that in the future quantum theory of particles has to solve the problem, to determine Green's F-fns from some general principle.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Quantised Field Theories and the Principle of Reciprocity

H.S. Green, Nature 163 (1949), 208.

Chang, T.S., Proc. Camb. Phil. Soc. 42 (1946), 132
44 (1948), 76.

de Wet, 44 (1948), 546.

There is now little doubt that the existence of mesons with several different masses requires a wave equation of the type

$$f(P)\psi = 0 \qquad P = p_k p^k = E^2 - \vec{p}^2,$$

the masses of the different particles being obtained as the roots of the equation $f(m^2) = 0$.

$$\mathcal{L} = F(p_k, p_{\ell'})\rho$$

$$\begin{cases} F(p_{k'}, p_{\ell'})\rho = (F + \pi_k G^k)\rho = 0 \\ \qquad\qquad \pi_k = p'_k - p_k \\ F(p_k, p_{k'})\rho = (F + \pi_{k'} G^{k'})\rho = 0 \\ \qquad\qquad \pi_{k'} = p_k - p_{k'} \end{cases}$$

$$\pi_k (G^k + G^{k'})\rho = 0$$

$$\pi_k (F\delta^k_\ell - G^k p_\ell - G^{k'} p_{\ell'})\rho = 0$$

$R^k = \frac{1}{2}(G^k + G^{k'})$: density-flux,

$\mathcal{L}^k_\ell = (F\delta^k_\ell - G^k p_\ell - G^{k'} p_{\ell'})$: canonical energy flux tensor.

Thus R^4 is the true density operator and

$$p_k = \int \tfrac{1}{2}(p_k + p_{k'}) R^4 \, d\sigma$$

is the Hamiltonian energy-momentum vector

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

differing (in the integrand) from L_K^+ only by a space divergence.

Commutation Rules are simply

$$\zeta_i R^+ \pm R^+ \zeta_i = \zeta_i$$

where ζ_i is a creation operator for a particle with mass m_i. If ρ is expressed

$$\rho = \sum_{ij} \psi_i(x_k)\, \psi_j^*(x_\ell')$$

the commutation rules reduce to

$$\tfrac{1}{2}(G_4 + G_4')\,[\psi_i(x_k),\, \psi_j^*(x_\ell')] = \delta(\underline{x} - \underline{x}'), \qquad t' = t$$

δ_{ij}

From this one can easily show that the field consists of particles with different but definite masses, to each of which can be assigned a definite energy and momentum. One can show further that the energy–momentum operator P_K has the property of the differential operator in commutation with ψ_i, so that the transition to q.m. can now be completed by expressing the field equations in operational form. When applied to Born's self-reciprocal Lagrangians, one obtains a quantised wave field in which particles with different

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

waves co-exist without interaction.
Particles belonging to different eigenfunctions may however interact directly, and it appears to be possible to determine the form of interaction uniquely from the principle of reciprocity, restricted only by the requirements of the conservation of charge, momentum and spin (ang. mom.) which are consequences of the theory.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Meson Masses and the Principle of Reciprocity

Max Born
Antonio E. Rodrigues

$L_n^{(1)}(x) = 0$ Laguerre polynomials of the first kind

n					
1	2				
2	1.26	4.73			
3	0.93	3.31	7.75		
4	0.74	2.57	5.73	10.9	
5	0.53	2.11	4.61	8.40	14.26

$$(p^2 - E^2 - \varkappa b^2)\,\psi = 0$$

$n = 1$: $\mu = \dfrac{b\sqrt{\varkappa}}{c} = \dfrac{\hbar\sqrt{2}}{ac}$

$a = \dfrac{e^2}{\mu c^2}$: $\dfrac{\mu}{m} = 137\cdot\sqrt{2} \cong 194.$

In the case of particles with spin, there is one with 0 rest-mass, so that the electron's mass should be due to self-energy of surrounding electromagnetic field.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Particles with spin and the Principle of Reciprocity.
Kai-Chia Cheng
Antonio E. Rodriguez

spin 0 or 1: metric operator

$$S = x^r x_r + p^r p_r$$

spin 1/2: $S = (p^r + i\gamma x^r)\alpha_r$ $\quad \gamma = \alpha_1\alpha_2\alpha_3\alpha_4$

reciprocity: $S(x_r p_r) = T^{-1} S(p_r, -x_r) T.$

with $T = \dfrac{1+i\gamma}{1+i}$

$$S\mathcal{F} = \mathcal{F}_S$$

S: eigensymbol depending only on γ.

± 1

$\gamma = \pm 1$

$$\mathcal{F}_1 = 2\{(\alpha_r p^r) L_n^{k+2}(P) \pm \sqrt{n-k-1}\, L_n^{k+1}(P)\} \times (\alpha_r p^r)^k e^{-P/2} g$$

$$\mathcal{F}_2 = 2\{(\alpha_r p^r)[L_n^{k+1} - L_n^{k+2}] \pm \sqrt{n+1}\, L_n^{k+1}\} (\alpha_r p^r)^k e^{-P/2} g$$

g: 3-dim. sph. fn

$n = 1, 2, 3, \cdots \quad k = 1, 2, \cdots n-1.$

Roots of $\mathcal{F}_1 = 0$ $\mathcal{F}_2 = 0$: $\frac{\mu}{m} = 137\kappa$ due to $(\alpha_r p^r)^k$, for $k > 1$. theory predicts the existence of neutrino satisfying $\alpha_r p^r = 0$

There are great many mesons with spin 1/2.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

Lagrangian Formalism in Field Theory

The Lagrangian in Quantum Mech.

P.A.M. Dirac, (P.Z. Sowj. 3 [1933), 64]

Contact Transformations

p_r, q_r; $P_r Q_r$ $\quad q_r, Q_r$: indep variables

$$p_r = \frac{\partial S}{\partial q_r} \qquad P_r = \frac{-\partial S}{\partial Q_r}$$

(q'/Q') is the quantum analogue of $e^{iS/\hbar}$

$$(q'|\alpha|Q') = \int (q'|\alpha|q'')\, dq'' (q''/Q')$$

$$= \int (q'/Q'')\, dQ'' (Q''|\alpha|Q')$$

$$(q'|q_r|Q') = q_r' (q'/Q')$$

$$(q'|p_r|Q') = -i\hbar \frac{\partial}{\partial q_r'} (q'/Q')$$

$$(q'|Q_r|Q') = Q_r' (q'/Q')$$

$$(q'|P_r|Q') = i\hbar \frac{\partial}{\partial Q_r'} (q'/Q')$$

$$(q'|f(q)g(Q)|Q') = \int\int (q'|f(q)|q'')\, dq'' (q''/Q'')$$

$$\times dQ'' (Q''|g(Q)|Q') = f(q')\, g(Q')\, (q'/Q')$$

$$(q'|\Sigma_k f_k(q)\, g_k(Q)|Q') = \Sigma_k f_k(q')\, g_k(Q')\, (q'/Q')$$

$$(q'|\alpha(qQ)|Q') = \alpha(q'Q')\, (q'/Q')$$

$\alpha(qQ)$: well ordered.

$\alpha = p_r$: $\qquad (q'/Q') = e^{iU/\hbar}$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所・湯川記念館史料室

$$(q'|p_r|Q') = \frac{\partial U(q'Q')}{\partial q'_r}(q'|Q')$$

$$p_r = \frac{\partial U(qQ)}{\partial q_r}$$

$$P_r = -\frac{\partial U(qQ)}{\partial Q_r}$$

} U: well ordered

The Lagrangian and the Action Principle

$q, p = q_t, p_t \;;\; Q, P = q_T, p_T$

$$S = \exp\left[i\int_t^T L\,dt/\hbar\right]$$

$$\downarrow$$

$$(q_t/q_T)$$

$$\underline{(q_{t+dt}/q_t) \to \exp(iL\,dt/\hbar)}$$

We ought to consider the classical Lag. not as a fn of the coordinates and velocities, but rather as a fn of the coordinates at time t and coordinates at time t+dt.

$$A(tT) = \exp\left(i\int_T^t L\,dt/\hbar\right)$$

$$A(tT) = A(t\,t_m)A(t_m\,t_{m-1})\cdots A(t_1,T)$$

$$(q_t/q_T) = \int (q_t/q_m)\,dq_m(\;)\cdots dq_1(q_1/q_T)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

The integrand has the form $e^{iF/\hbar}$, which changes very rapidly with the small change of q's, if we consider $\hbar$ very small, so that the integration with respect to q's contribute very little to (q_{τ}/q_{τ}) except where the $e^{iF/\hbar}$ is stationary with respect to small variation of q's.

Application to Field Dynamics

g.t.f for the whole region will equal the product of the g.t.f. for the two parts, integrated over all values for the coord. on the common boundary of two parts.

$|(q_{\tau}/q_{\tau})|^2$: a priori prob. (for any closed surface)

↓ more general

restricted def.: transformation fn.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Lagrangian Formalism in supermany time theory

$$(q_{t+dt}/q_t) = \exp(iL\,dt/\hbar)$$

$$(q(\sigma+d\sigma)/q(\sigma)) \to \exp\left(\int_\sigma iL\,d\omega/\hbar\right)$$

$$\to 1 + \frac{i}{\hbar}\int_\sigma L\,d\omega$$

What is q? What are the ~~independent~~ arguments in L? In particle theory, arguments should be coordinates. not velocity, so in field theory, they should be field quantities and their derivatives in space-like direction but not in time-like direction. In actual problems, Lagrangian ~~should not~~ contains time derivative ~~for the~~ of the field quantities, so that, in quantum theory, we have to get the commutation relations ~~for~~ only by ~~going~~ ~~virtual~~ going around through the Hamiltonian formalism* and ~~at the same time~~ afterwards going back to Schrödinger representation

$$(q', s_0'/q'', s_0'') \to 1 + \frac{i}{\hbar}\int_\sigma L\left(q, \frac{\partial q}{\partial s_0}, \cdots\right) ds_0 (ds)^3$$

* We assume an infinitesimal comm. rel. ~~for~~ between quantities space-like surface, and ~~do not~~ assume nothing in advance between two quantities ~~on~~ ~~like-like wi on diff.~~ at points ~~like~~ time-like with each other.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Tomonaga, On the Generalized Transformation Functions (Prog. 1 (1946), 12)
Supplementary Note (2 (1947), 219)

$T(q_T' q_t') = M_{q_T'} D(T-t) M_{q_t'}$

$D_2(T-t) = \exp\left(-\frac{i}{\hbar} H (T-t)\right)$

$M_{q_t'} = M_{q_t'} \qquad M_{q_t'}^2 = M_{q_t'}$

(observation operator)

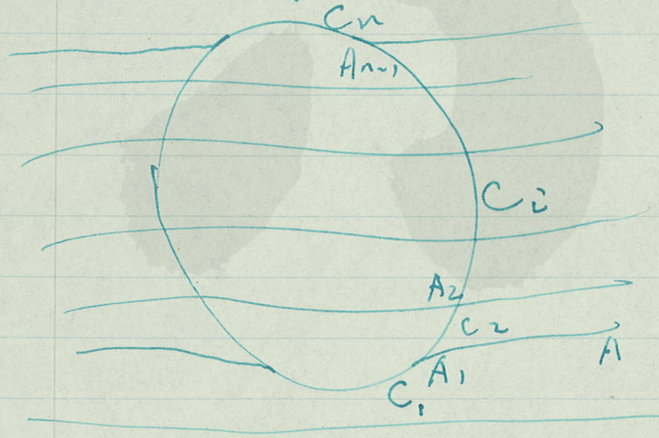
(M_q represent a state-vector in the complete normalized orthogonal system belonging to the operators q and take out only the part depending on the proper values q' of the q's

$T(q_T'' q_t') = M_{q_T''} D(T-t) M_{q_t'}$

(transf. operator)

Trace $\tilde{T}(\,,\,) T(\,,\,)$

$= |(q_T'' | D(T-t) | q_t')|^2$



† Another way of formulation
see next page →

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Watanabe, On Dirac's G.T.F.
I. Prog. 2 (1947), 71
II. Prog. 3 (1948), in press

curvilinear coordinates
again to Hamiltonian formalism

Feynman, Rev. Mod. Phys.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

On the relativistic invariance of quantized field theories

By J. S. de Wet, Balliol College, Oxford.

(Proc. Roy. Soc. A195 (1948), 365)

P. Weiss, P. R. S. A 169 (1938), 102.
T. S. Chang, P. C. P. S. 42 (1946), 132; 43 (1947) 196.
de Wet, P. C. P. S. (1948), 546
(On the Quantization of Field Theories Derived From Higher Order Lagrangians)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Oct. 15, 1948

Electron Theory
by Oppenheimer
(Solvay Report, 1948)

First Step of Schwinger's Theory

先ず $V = \exp \frac{e}{mc}[\vec{Z}\cdot\nabla]$

[illegible]の如き contact transformation を行う. 但し $\vec{Z}$ は Hertzvector から electron の quasi-static field を減じたもの.

これではしかし $\nu \lesssim \frac{mc^2}{\hbar}$ に対して reactive correction が converge しないから, 更に pair creation, annihilation, light quantum 相互の interaction を考慮した第二の Transf. が必要だが この場合には closed solution は得られていない, $\frac{e^2}{\hbar c}$ を小さいとして expand した結果 first order の correction だけ考える範囲で

a) an infinite term in the electron's electromagnetic inertia

b) an ambiguous light quantum self energy

c) no other effects for a single electrons or photon

d) interaction of order e^2 between electrons, positrons, and photons ~~with~~ which in this order, correspond to the familiar Møller interactions and Compton effect and pair production probabilities.

e) an infinite vacuum polarizability

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

f) the familiar frequency dependent finite polarizability for external electromagnetic fields

g) Emission and absorption probabilities equivalent to those of the Dirac theory for an electron in an external e.m. field

h) New reactive corrections of order e^2 to the effective charge and current distribution of an electron, which corresponding to vanishing total supplementary charge, and to currents of the order $e^3/\hbar c$ distributed over dimensions of the order $\hbar/mc$, and which include the supplementary potential I, and the supplementary magnetic moment $\frac{e^2}{2\pi\hbar c}\frac{e\hbar}{2mc}\vec{\sigma}$ as special (non-relativistic) limiting cases.

この計算を higher order に拡張しようとすると、h) が gauge 及び Lorentz transf. に対して invariant でないという問題を満足せねばならぬ。$e^2/\hbar c$ までの order ならば Stueckelberg の 4-dimensional perturbation theory, Feyman の theory 等色々あるが Tomonaga-Schwinger が最も general で、Schwinger に従って

$$\frac{\delta\Psi}{\delta\sigma} = -\frac{1}{i} J^{\mu}(P) A_{\mu}(P') \Psi$$

なる基礎方程式から出発して $e, e^2, \ldots$ 等の term

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

の中で virtual transition を contact transf によって
を eliminate して行くのがよい。ここでできる
light quantum self-energy は gauge 及び Lorentz
invariant であるから、これを 0 と取り得ることになる。(直接計算すれば $0\times\infty$, 或 Pais が f-theory で
stress を計算して $\frac{e^2}{2\pi\hbar c}mc^2$ (electron rest system)
を得たのと同様)

さて以上の結果~~から~~に対して残る疑問は

1) e の higher power に extend できるか、その場合
finite, unique でしかも experiment と一致する結果が得られるか
2) e の expansion によらぬ rigorous な procedure であるか。
3) finite でないのは el. mg. inertia of electron,
charge の polarization effect 等の直接観測できない
量に限られるという事実はどの位 general か?
other spin の charged particle に拡張できるか?
4) Yukawa-field ではどうか?
5) electrodynamics はいかなる意味で close
しているか?

Case の計算によると pseudoscalar theory で magnetic
moment は finite になるが、neutron-proton
moment の差が neutron-moment 自身と同程度
となり実験とあわない。proton-neutron mass
difference が ∞ となり、sign も逆になる。
一般に meson-field, strong coupling の場合は
全く新しい分野へ入って行くことになると思われる。

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar　　Saturday　Nov. 20, 1948

Prof. Placzek:

Neutron - Electron Interaction

$$(\sigma_{free})_{rest} = \left(\frac{M}{1+\mu}\right)^2 \sigma_{bound\,fixed}$$

$$\sigma_{free} = \left(\frac{v_{rel}}{v_n}\right)_{Av} (\sigma_f)_{rest}$$

$$\sigma_{free} = \left(\frac{v_{rel}}{v_n}\right)_{Av} \left(\frac{M}{1+\mu}\right)^2 \sigma_{fixed}$$

$$\left(\frac{v_{rel}}{v_n}\right)_{Av} = \begin{cases} u^2/3 & u<1 \\ \frac{1}{3u} & u>1 \end{cases} \qquad \frac{v}{v_n} = u$$

$$= 1\tau \frac{1}{3} \sum_s \frac{(E_{s\,kin})_{Av}}{\mu_s E}$$

Fermi: Angular distribution: negative

Rabi: Energy dist dependence:

$$\tau = \frac{\hbar}{\Delta E} \ll \frac{1}{\hbar\omega}$$

$$\Delta E = \frac{2}{\mu} E : \qquad \frac{2}{\mu} E \gg \text{level spacing } \overline{E}$$

$$E \gg 1\,eV$$

$$F = \sum_s A_s e^{i\vec{K}\vec{r}_s} \qquad \sigma_{fix} = |A_s|^2$$

$$d\sigma_b^a = \frac{1}{4\pi} \frac{k'_{ab}}{k} |(F)_n^a|^2 d\Omega \qquad \vec{K} = \vec{k} - \vec{k}'_{ab}$$

$$\sigma_b^a = \frac{1}{k} \frac{k'_{ab}}{b} \int_{(k-k'_{ab})^2}^{(k+k'_{ab})^2} |F_{ab}^a|^2 d\frac{\Omega}{4\pi}$$

$$= \frac{1}{4k^2} \int_{(K-k'_{ab})^2}^{} |F_{ab}^a|^2 dK^2$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\sigma^a = \frac{1}{4k^2} \sum_b \int |F_{\not b}^a|^2 d\tilde{k}$$

Seminar Saturday Dec. 4, 1988

Płaczek, continued

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar Nov. 22, 1948

Dyson: Continuation

Semi-Heisenberg, Not Schwinger

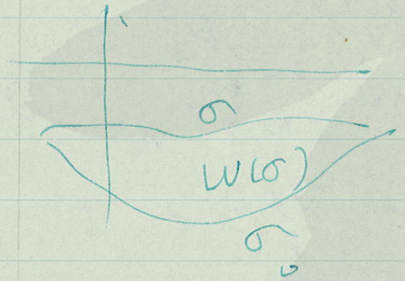
$$i\hbar c \frac{\partial \Psi'}{\partial \sigma} = H_I \Psi' \qquad \Psi = S(\sigma) Q \Psi'$$

$$H_I = Q^{-1} S(\sigma)^{-1} H_e S(\sigma) Q$$

$$\log S(\infty) = i\eta_\sigma = \left(\frac{-i}{\hbar c}\right) \delta_4(K-K') V$$

$$S'(\sigma) = \prod_{\sigma_0}^{\sigma} \left(1 - \frac{i}{\hbar c} \int_B H_1(x)\, dx\right)$$

$$T'(\sigma) = \exp\left(\frac{i}{\hbar c} W(\sigma) V\right)$$



$$i\hbar c \frac{\partial S'}{\partial \sigma} = H_1 S'$$

$$i\hbar c \frac{\partial T'}{\partial \sigma} = -T' V$$

$$\Psi(\sigma) = S'(\sigma) T'(\sigma) \Psi'(\sigma) = Y' \Psi'$$

$$i\hbar c \frac{\partial \Psi'}{\partial \sigma} = (V + Y'^{-1} H_e Y') \Psi'$$

$\sigma_0 \to -\infty$: $T'(\sigma) \to (S(\infty))^{-\frac{1}{2}} = Q$

$$i\hbar c \frac{\partial \Psi'}{\partial \sigma} = (V + H_I) \Psi'$$

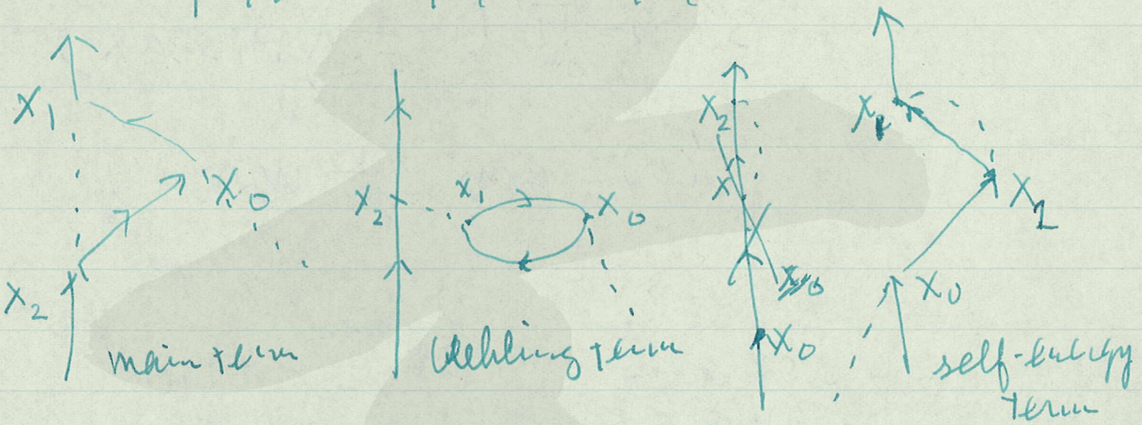
Schwinger

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$S_F(x) = \sum_{n=0}^{\infty} \left(\frac{i}{\hbar c}\right)^n \frac{1}{n!} \int_{-\infty}^{+\infty} P(H_e(x_0), H_1(x_1), \cdots H_1(x_n))\, dx_1 \cdots dx_n$$

$$\iiint_{-\infty}^{+\infty} P(H_e(x_0), H_1(x_1), H_1(x_2))\, dx_1\, dx_2$$

$\bar{\psi}\psi A^2$　$\bar{\psi}\psi A$　$\bar{\psi}\psi A$



$$S_F(-x) = i|S|(x) + S_1(x)$$

$$D_F(-x) = i|D|(x) + D_1(x)$$

$$D_1 \approx \delta(k^2)\frac{1}{2\pi}\,;$$

$$|D| \approx \frac{1}{4\pi^2 k^2}\,;$$

$$S_1 \propto \delta(k^2+\kappa_0^2)(\gamma_\mu k_\mu + i\kappa_0)$$

$$|S| \approx \frac{\gamma_\mu k_\mu + i\kappa_0}{k^2+\kappa_0^2}$$

$$D_F \approx \frac{1}{2\pi}\delta_+(k^2) \qquad S_F = \delta_+(k^2+\kappa^2)(\gamma_\mu k_\mu + i\kappa_0)$$

$$\delta_+ = \delta(x) + \frac{i}{\pi x}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Julian Schwinger, Recent Developments in Quantum Electrodyn.
(Summer Symposium, Univ. Mich.)
~~I. A Cov~~ 1948

I, A Covariant Formulation

Introduction

1. Covariance in the Heisenberg Representation
2. Interaction Representation
3. Covariant Elimination of the longitudinal Field
4. Invariant Collision Operator

II. Application of the Theory

Polarization of the Vacuum

$$\Box^2 A_\mu(x) - \frac{\partial}{\partial x_\mu}\left(\frac{\partial A_\nu(x)}{\partial x_\nu}\right) = -\frac{1}{c} J_\mu(x)$$

$$i\hbar c \frac{\delta \Psi[\sigma]}{\delta \sigma(x)} = -\frac{1}{c} j_\mu(x) A_\mu(x) \Psi[\sigma]$$

zeroth order $\Psi[\sigma] = \Psi_0$

first order

$$i\hbar c \frac{\delta \Psi[\sigma]}{\delta \sigma(x)} = -\frac{1}{c} j_\mu(x) A_\mu(x) \Psi_0$$

$$\Psi[\sigma] = \left\{1 + \frac{i}{\hbar c}\int_{-\infty}^{\sigma} j_\mu(x') A_\mu(x')\, d\sigma'\right\}\Psi_0$$

provided that $A^{-\infty}_\mu(x) = 0$ in the remote past.

The expectation value in the ~~value~~ vacuum of the perturbed current vector $j_\mu(x)$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

is

$$\langle j_\mu(x)\rangle = (\Psi[\sigma], j_\mu(x)\Psi[\sigma])$$
$$= (\Psi_0, e^{iS} j_\mu(x) e^{-iS}\Psi_0)$$

$$\langle j_\mu(x)\rangle = -i\langle [j_\mu(x), S(\sigma)]\rangle$$
$$= \frac{i}{2\pi c^2}\int_{-\omega}^{+\omega} \langle j_\mu(x), j_\nu(x')\rangle_0 A_\nu(x') \times \xi[\sigma,\sigma']\, d\omega'$$

$$= -4ie^2c\left\{\frac{\partial\Delta}{\partial x_\mu}\frac{\partial\Delta^{(1)}}{\partial x_\nu} + \frac{\partial\Delta}{\partial x_\nu}\frac{\partial\Delta^{(1)}}{\partial x_\mu} - \delta_{\mu\nu}\left[\frac{\partial\Delta}{\partial x_\lambda}\frac{\partial\Delta^{(1)}}{\partial x_\lambda} + \kappa_0^2\Delta\Delta^{(1)}\right]\right\}$$

$$= -\frac{8e^2}{\pi c}\int G(\lambda) J_\nu(x')\, d\omega'$$

$J_\nu = 0$;

$$\therefore \langle j_\mu(x)\rangle = 0 ;$$

which

so that there is no current, interact with the light wave and change its energy. Thus photon mass is zero. Only a theory which is properly gauge-invariant will yield this result.

The Electron Self-Energy

Lamb shift $\frac{8\alpha^3}{3\pi}\frac{Z^4}{n^3}Ry\left(\ln\frac{mc^2}{\Delta E} - \ln 2 + \frac{3}{8} - \left(\frac{1}{5}+\frac{1}{2}\right)\right)$

$\simeq 1040$ Mc. (Lamb exp. 1046 ∓ 20)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\frac{2e^3}{\hbar c}\iint dx_1 dx_2 \Big[\bar{\psi}(x_1)\gamma_\lambda S_F(x_0-x_1)\gamma_\mu A^{e}_{\mu}(x_\mu) S_F(x_2-x_0)$$
$$\times \gamma_\lambda \psi(x_2) D_F(x_2-x_1)\Big]$$

$$\text{''} \iint dx_1 dx_2 \Big[\bar{\psi}(x_1)\gamma_\lambda S_F(x_2-x_1)\gamma_\lambda S_F(\underline{\underline{x_0}}-x_2)$$
$$\times \gamma_\mu A^{(e)}_{\mu}(x_0)\psi(\underline{\underline{x_0}}) D_F(x_2-x_1)\Big]$$

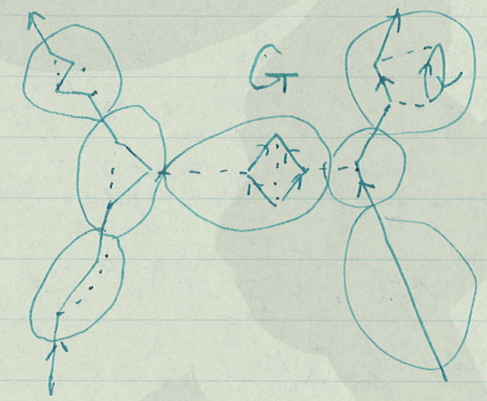
Seminar Nov. 23, 1948

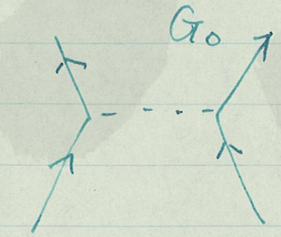
Dyson, Continued.

$N = E_{in} + 1 - V$ $\qquad$ $D = E_{inel} + 2E_{inth}$

$$D - 4N = \frac{3}{2}E + P - 4$$
$$\geq 1$$

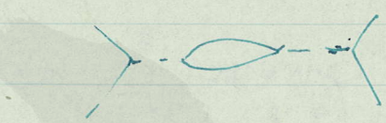
for convergence



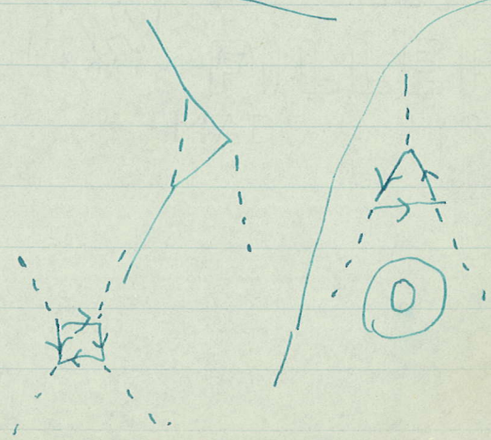


Møller scattering

Divergence:







©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$D_F'(x-y)\delta_{\mu\nu} = \langle P(A_\mu(x), A_\nu(y), S(\infty)\rangle$

2nd order

$D_F' = (1-R_1)D_F + e^2 F(k^2) D_F + \cdots$

$\varepsilon(x-y) S'_{F\alpha\beta}(x-y) = \langle P(\psi_\beta(x), \bar\psi_\alpha(y), S(\infty))\rangle$

$S_F' = (1-R_2) S_F + e^2 [F_1(k^2+k_0^2) + (\not k + i k_0) F_2(k^2+k_0^2)] S_F$

$\Gamma_\mu' = \gamma_\mu + \Lambda_\mu + \cdots$

$\Gamma_\mu = (1+R_2)\gamma_\mu + e^2 [G_1(k^2)\gamma_\mu + G_2(k^2) \not k \sigma_{\lambda\mu}]$

$\Lambda_\mu = \int d\alpha p \, \gamma_\lambda S_F(p+k_4) \gamma_\mu S_F(p+k_5) * \gamma_\lambda D_F(p+k_6)$

(wave fn renormalization
charge renormalization

$H_1 = -\frac{1}{c} j_\mu A_\mu - \delta m \bar\psi\psi$
$H_0 = H_0' + \delta m \bar\psi\psi$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Oct, 22, 1948

Neutron-Proton and Neutron-Neutron Scattering at High Energies

F. Rohrlich and J. Eisenstein
Lyman Laboratory of Physics, Harvard University
Cambridge, Massachusetts

Well Shape:
Bohm, Richman, 71 (1947), 567
Total cross-section: n-p, 25 Mev. ~ 6 Mev
Sleator, 72, 207
charged or symmetrical ~~tensor~~ tensor theory
Yukawa fits.
P-P, 14.5 Mev
Wilson, etc 71, 560; 72, 1132
negative p-wave contribution → exchange force (tensor?, Rarita-Schwinger type)

Present method:
cross-sections for high energy scattering
(rectangle, Yukawa well)
(binding energy of the deuteron
epithelmal n-p cross-section
low energy p-p cross-section
(assumed equality of p-p and n-p forces)

---

slow neutron scattering by ortho- para-hydrogen and by hydrogen in crystals indicate that the ranges of singlet and triplet forces may be much shorter than was commonly accepted (Blatt, 74, 92)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Approx. Calculation:

Eisenstein and Rohrlich, 23, 691, 1411.

Camac and Bethe, 23, 191

T. Y. Wu, 23, 934, 1132L

Asakin and Wu, 23 972.

Massey, Burshop. and Hu, 23, 1403.

Variation method

Schwinger, 72, 742.

(Nuclear Physics 295-296, New York University, 1947)

scattering: $u'' + k^2 u + \lambda f(x) u = 0$ *

ground state:

$$u'' - \eta^2 u + \lambda f(x) u = 0$$

* を integral eq. に直す

$$\left(\frac{d^2}{dx^2} + \kappa^2\right) G(x, x') = -\delta(x - x')$$

$$G(0, x') = 0$$

$$G(x, x') = \frac{1}{k} \sin k x_< \cos k x_>$$

$$u(x) = \sin kx + \int_0^\infty G(x, x') \lambda f(x') u(x') dx' \quad \dagger$$

$$u \sim \sin kx + \frac{\lambda}{k} \cos kx \int_0^\infty \sin kx' f(x') u(x') dx'$$

$$\propto \sin(kx + \delta)$$

$$\tan\delta = \frac{\lambda}{\kappa} \int_0^\infty \sin kx f(x) u(x) dx$$

† に, $f(x) u(x)$ を乗じて積分すれば

これで

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$I[u] = \frac{\int_0^\infty dx\, f(x)u(x)\left[u(x) - \int_0^\infty dx'\, G(x,x')\lambda f(x')u(x')\right]}{\left[\int_0^\infty dx\, u(x) f(x) \sin kx\right]^2}$$

なる functional を考えて見ると，u が T を満足するなら

$$I[u] \to \left[\int u f \sin kx\, dx\right]^{-1} = \frac{\lambda}{\kappa} \cot\delta$$

となる．しかるに $I[u]$ は T を満す u の所で extremum になっている．
従って近似函数を使って $I[u]$ を計算すれば phase shift δ に対するよい近似値が得られることになる．

Phenomenological theory

Tensor Forces $\quad S_{12} = 3\frac{(\sigma_1 r)(\sigma_2 r)}{r^2} - \sigma_1\sigma_2$

~~Vth~~

Triplet Potential

$$V(r) = f(\sigma_1\sigma_2, \tau_1\tau_2)\left[J(r) + \gamma S_{12} K(r)\right]$$

neutral theory $\quad f = -1$

charged theory $\quad f = \frac{1+\sigma_1\sigma_2}{2} \cdot \frac{1+\tau_1\tau_2}{2}$

symmetrical theory $\quad f = \frac{1}{3}\sigma_1\sigma_2 \cdot \tau_1\tau_2$

neutron-neutron $\quad \frac{1+\tau_1\tau_2}{2} = -1$,

Conclusions

90 MeV $\quad$ n-p cross-section

$\sigma_{np} = 0.083 \pm 0.004$ barns

angular distribution

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$R = \sigma_{np}(\pi) / \sigma_{np}(\pi/2 \cong 3$$

と最もよく合うのは

i) symmetrical tensor Yukawa well に relativistic correction (5〜10%) を施したもの~~であろうか~~.

ii) 或は neutral と charged の contribution は同等にしたもの 即ち

$$f(\sigma_1, \sigma_2, \tau_1, \tau_2) = -\frac{1}{2}\left(1 - \frac{1+\sigma_1\sigma_2}{2}\,\frac{1+\tau_1\tau_2}{2}\right)$$

$$= -\frac{1}{2}\left(1 + (-1)^{-L}\right)$$

で, tensor force は even state でだけ働く場合*.

---

† N.P. Triplet Cross-section は symm. tensor, Yukawa well で

		$\times 10^{-13}$ cm	Depth MeV	Meson Mass
I)	Tensor Range	1.400	44.969	280
	Ordinary Range	1.202	30.817	326
II)	Tensor Range	1.556	27.988	250
	Ordinary Range	1.202	45.286	326

のどちらがもよい. この combination は最近の Harvard の Felbach? によると, H_3 の binding をよく説明する.

---

* Serber は central force だけがよいといっている? しかし tensor force が全然なければ 90° scattering の cross-section は殆んど 0 になってしまう. (粒々 weak long range Tensor force があればよい.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Reprinted from THE PHYSICAL REVIEW, Vol. 73, No. 6, 641–642, March 15, 1948
Printed in U. S. A.

Neutron-Proton Scattering at 100 Mev

JULIAN EISENSTEIN AND FRITZ ROHRLICH
Lyman Laboratory, Harvard Universly, Cambridge 38, Massachusetts
February 7, 1948

A VALUE $0.083\pm0.004\times10^{-24}$ cm² for the neutron-proton total cross section at 90 Mev has recently been reported by Cook, McMillan, Peterson, and Sewell.[1]

We have made calculations to determine whether it is possible to fit this value of the cross section if one uses existing phenomenological theories of nuclear forces. The results at 100 Mev for a rectangular well interaction potential are as follows:

1. Central, non-exchange force. Triplet range, 2.80×10^{-13} cm; depth, 21.213 Mev. Singlet range, 2.80×10^{-13} cm; depth 11.904 Mev. Exact cross section, 0.162×10^{-24} cm². Born approximation, 0.169×10^{-24} cm².

Triplet range, 1.80×10^{-13} cm; depth, 43.476 Mev. Singlet range, 2.80×10^{-13} cm; depth 11.904 Mev. Exact cross section, 0.117×10^{-24} cm². Born approximation, 0.123×10^{-24} cm².

2. Central, symmetrical exchange force. Triplet and singlet ranges, 2.80×10^{-13} cm; depths as above. Exact cross section, 0.111×10^{-24} cm². Decreasing the triplet range increases the cross section. In this case the Born approximation to the singlet cross section is too large by a factor of approximately two.

3. Tensor, non-exchange force. Triplet and singlet ranges, 2.80×10^{-13} cm. Rarita and Schwinger[2] constants. Born approximation, 0.205×10^{-24} cm².

4. Tensor, symmetrical exchange force. Ranges, 2.80×10^{-13} cm. Rarita and Schwinger constants. The Born approximation for the triplet cross section and the exact value for the singlet cross section yield a total cross section of 0.113×10^{-24} cm².

Experiments on the angular distribution at 90 Mev were reported by Hadley *et al.*[3] They found 0.3 for the ratio of scattering at 90° to backward scattering in the center of gravity system.

The exact solution at 100 Mev for a central, symmetrical exchange force using the first five phase shifts gives $\sigma(0):\sigma(\pi/2):\sigma(\pi)=2.24:0.062:6.87$. For the tensor, symmetrical theory in the above combination of exact and Born cross sections we obtain $\sigma(0):\sigma(\pi/2):\sigma(\pi)=1.22:0.51:4.03$.

These figures show that only the symmetrical tensor theory is in close enough agreement with experiments so that the remaining discrepancies might be accounted for by relativistic corrections. Comparison of the symmetrical non-tensor theory with the symmetrical tensor theory shows that the tensor force decreases appreciably the forward and backward scattering and accounts for nearly all the scattering at right angles. In the symmetrical tensor theory a reduction of the triplet range increases the tensor part, i.e., the constant[4] γ thus increasing the ratio $\sigma(\pi/2):\sigma(\pi)$ and the total cross section. The same reduction made for the tensor force range rather than the triplet range is more effective in increasing γ and the above ratio.

As an example we used constants calculated by Guindon[5] to fit the epithermal neutron-proton cross section, the binding energy of the deuteron, and the quadrupole and magnetic moments. He assumed an interaction potential which does not contain a spin exchange force ($g=0$),[4] both the non-tensor and tensor parts being rectangular wells of ranges r(non-tensor)$=2.80\times10^{-13}$ cm and r(tensor) $=2.654\times10^{-13}$ cm and depths $V_0=11.904$ Mev and γV_0 ($\gamma=1.149$), respectively. With these constants we find for the tensor, symmetrical case $\sigma(0):\sigma(\pi/2):\sigma(\pi)=1.26:0.61:3.44$.

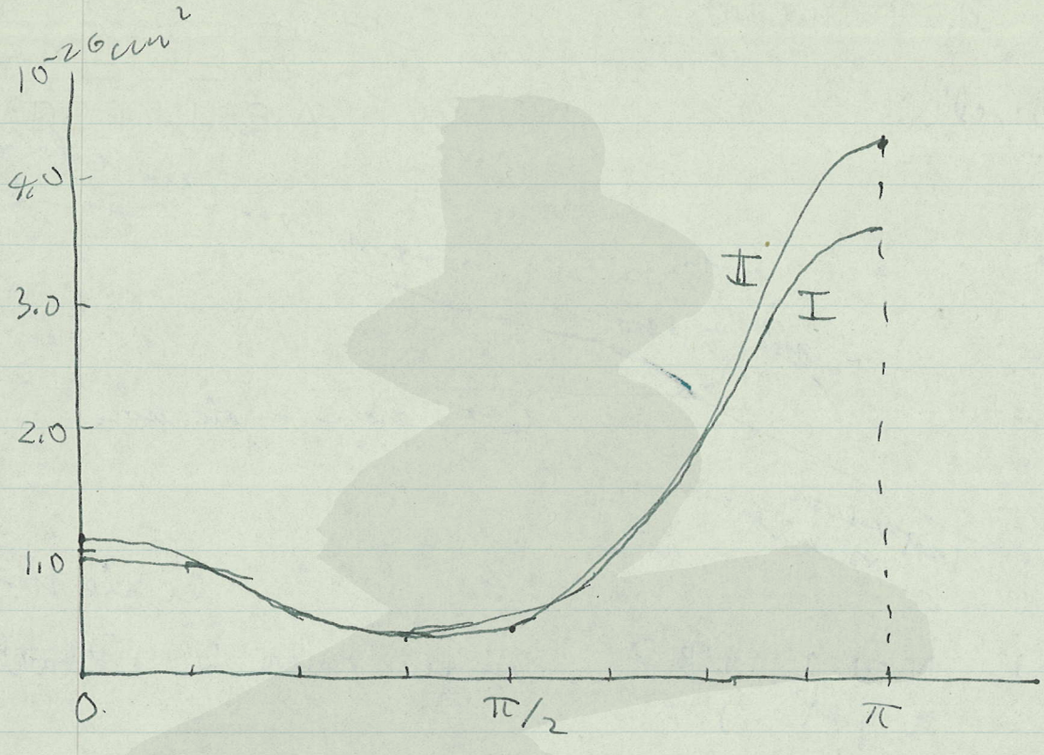
Calculations were also made with Yukawa potentials and a central non-exchange force. They led to considerably lower cross sections. The singlet range 1.8×10^{-13} cm and depth 45.80 Mev found by Hoisington, Share, and Breit[6] in an analysis of proton-proton scattering were used. The triplet range was also taken to be 1.18×10^{-13} cm and the depth interpolated from calculations of Hulthén[7] to be 67.45 Mev. The cross section, in first Born approximation, is 0.111×10^{-24} cm² at 100 Mev. This value is 35 percent below the corresponding value obtained with a rectangular well potential. It is difficult to estimate the cross section on the basis of a symmetrical theory because the Born approximation value of the singlet cross section is so far off. For the range and depth previously used the triplet cross section is 0.093×10^{-24} cm². The singlet cross section will probably be about the same as in the rectangular well case. Consequently, the total cross section will be about 0.11×10^{-24} cm². For a Yukawa potential a symmetrical exchange force therefore apparently does not lead to as strong a reduction in the total cross section as it does for a rectangular well.

Constants for the Yukawa potential in the tensor force case are being calculated.[8] The triplet cross section obtained in the symmetrical tensor case with tentative values of the constants is about 0.09×10^{-24} cm².

A detailed report on these calculations will be presented later.

[1] Leslie J. Cook, Edwin M. McMillan, Jack M. Peterson, and Duane C. Sewell, Phys. Rev. **72**, 1264 (1947).
[2] William Rarita and Julan Schwinger, Phys. Rev. **59**, 436 (1941).
[3] Hadley, Leith, York, Kelly, and Wiegand, Bull. Am. Phys. Soc. **23**, 15 (1948).
[4] For notation see reference 2.
[5] W. Guindon, Theses, Massachusetts Institute of Technolgy.
[6] L. E. Hoisington, S. S. Share, and G. Breit, Phys. Rev. **56**, 884 (1939).
[7] L. Hulthén, Arkiv. mat. astr. fys. **28A**, No. 5 (1942).
[8] Feshbach, Schwinger, and Eisenstein (to be published).

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室



Christian and Hart, Neutron-Proton Scattering

$1 - a + aP_x$ $\quad a \approx 1/2$

meson potential range $1.25 \pm 0.10 \times 10^{-13}$ cm

$a = 0.50 \sim 0.55$

high energy cross-section 20% higher

S, D contribute, because asymmetry is small. almost 90% of scattering

meson potential can fit all overall experiments from lower to higher energy range.

~~central forces~~ square well gives smaller cross-section, but if include tensor forces, the deviation from experiment becomes larger.

Experiments: Bruckner, Hartsough, Haywood and Powell

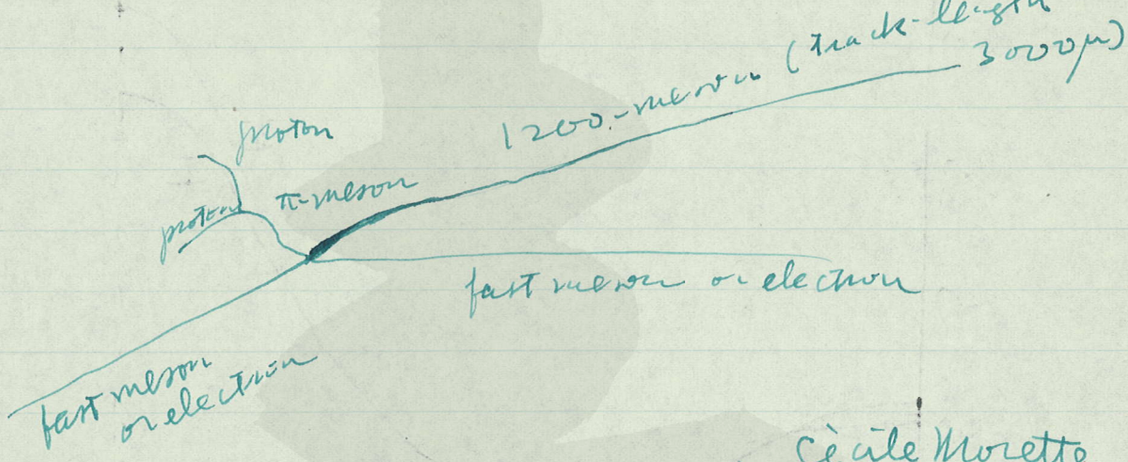
〃 : Segrè et al.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Nov. 10.

More evidence for the existence of τ-meson

Powell:



(Nov 1, 1948 の手紙)　Aage Bohr より ~~Powell~~ Cécile Morette へ

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Type of Nuclear Forces and Meson Model

N-P-scattering,
Cristian and Hart, Bul. A.P.S.
24 (1942), Nr. 2, p. 14.
small angular asymmetry in 90 MeV.
95% comes from S, D state.

$\frac{1}{2}(1 + P_x)$

Spin indep. and spin dep. Central Forces

	1	$\tau_3^{(1)}\tau_3^{(2)}$	$\frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}$	$\frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\cdot\frac{1+\sigma_1\sigma_2}{2}$	$\tau_3^{(1)}\tau_3^{(2)}\frac{1+\sigma_1\sigma_2}{2}$
^3S	1	−1	−1	−1	−1
^1S	1	−1	+1	−1	+1
$(N,P)\ ^3P$	1	−1	+1	+1	−1
^1P	1	−1	−1	+1	+1

		$\tau_3^{(1)}\tau_3^{(2)}$	$\tau_3^{(1)}\tau_3^{(2)}\frac{1+\sigma_1\sigma_2}{2}$
(N,N)	^1S	1	−1
(P,P)	^3P	1	+1

$$T = \left(a_1 + a_2\frac{1+\sigma_1\sigma_2}{2}\right) + \left(b_1 + b_2\frac{1+\sigma_1\sigma_2}{2}\right)\tau_3^{(1)}\tau_3^{(2)}$$
$$+\left(c_1 + c_2\frac{1+\sigma_1\sigma_2}{2}\right)\frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}$$

(N,P)

^3P: $(a_1 + a_2) - (b_1 + b_2) + (c_1 + c_2) = 0$

^1P: $(a_1 - a_2) - (b_1 - b_2) - (c_1 - c_2) = 0$

$$a_1 - b_1 + c_2 = 0$$
$$a_2 - b_2 + c_1 = 0,$$

$(N,P)\ ^1S$: $(a_1 - a_2) - (b_1 - b_2) + (c_1 - c_2) = 0$

$(P,P)\ ^1S$: $= (a_1 - a_2) + (b_1 - b_2)$

$$2(b_1 - b_2) = c_1 - c_2$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\therefore \quad (a_1 - a_2) = 3(b_1 - b_2)$$

$$a_1 - a_2 = 3\alpha \qquad b_1 - b_2 = \alpha \qquad c_1 - c_2 = 2\alpha$$

$$a_1 + a_2 = A \qquad b_1 + b_2 = B \qquad c_1 + c_2 = C$$

$$A - B + C = 0.$$

$$a_1 = \frac{A+3\alpha}{2} \qquad a_2 = \frac{A-3\alpha}{2}$$

$$b_1 = \frac{B+\alpha}{2} \qquad b_2 = \frac{B-\alpha}{2}$$

$$c_1 = \frac{C+2\alpha}{2} \qquad c_2 = \frac{C-2\alpha}{2}$$

$$T = \frac{A+3\alpha}{2} + \frac{A-3\alpha}{2}\frac{(1+\vec{\sigma}_1\vec{\sigma}_2)}{2}$$

$$+\left\{\frac{B+\alpha}{2} + \frac{B-\alpha}{2}\frac{(1+\vec{\sigma}_1\vec{\sigma}_2)}{2}\right\}\tau_3^{(1)}\tau_3^{(2)}$$

$$+\left\{\frac{C+2\alpha}{2} + \frac{C-2\alpha}{2}\frac{(1+\vec{\sigma}_1\vec{\sigma}_2)}{2}\right\}\frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}$$

$$= \left\{\frac{3}{4}A + \frac{A}{4}(\vec{\sigma}_1\vec{\sigma}_2) + \left(\frac{A-3\alpha}{4}\right)(\vec{\sigma}_1\vec{\sigma}_2)\right\}$$

$$+\left\{\frac{3}{4}A + \frac{3}{4}C + \frac{A+CB-\alpha}{4}(\vec{\sigma}_1\vec{\sigma}_2)\right\}\tau_3^{(1)}\tau_3^{(2)}$$

$$+\left\{\frac{3}{4}C + \frac{C-2\alpha}{4}(\vec{\sigma}_1\vec{\sigma}_2)\right\}\frac{\tau\tau + \tau\tau}{2}$$

$$= \frac{A}{4}(3 + \vec{\sigma}_1\vec{\sigma}_2)\cdot(1 + \tau_3^{(1)}\tau_3^{(2)})$$

$$+ \frac{C}{4}(3 + \vec{\sigma}_1\vec{\sigma}_2)\cdot\frac{(2\tau_3^{(1)}\tau_3^{(2)} + \tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)})}{2}$$

$$+ \frac{\alpha}{4}(\vec{\sigma}_1\vec{\sigma}_2)(3 + \tau_3^{(1)}\tau_3^{(2)} + \tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)})$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

'S: $T = +\frac{\alpha}{4}(4)(3 + \tau_3^{(1)}\tau_3^{(2)} + \cdots + \cdots)$

'P: $T = +\frac{\alpha}{4}(+4)(3 + \tau_3^{(1)}\tau_3^{(2)} + \cdots + \cdots)$

N;P: $3 - 1 - 1 - 1 = 0$,

(N,P;)

3P: $T = C \times 0 + \frac{\alpha}{4}(\cdots)\alpha \times 0 = 0$

Further if we take:

$A = -C$:

$$T = \frac{A}{4}(3 + \sigma_1\sigma_2)(1 - \tau_1\tau_2) + \frac{\alpha}{4}(1 - \sigma_1\sigma_2)(3 + \tau_1\tau_2)$$

i) $A = \alpha$:

$$T = \frac{\alpha}{4}\{6 - (\sigma_1\sigma_2)(\tau_1\tau_2) + 2(\tau_1\tau_2) - 2(\sigma_1\sigma_2)\}$$

ii) $A = -\alpha$.

$$T = -\alpha\{(\sigma_1\sigma_2) + (\tau_1\tau_2)\}$$

$$= -\alpha\{(1 + \sigma_1\sigma_2)(\sigma_1\sigma_2) + \tau_3^{(1)}\tau_3^{(2)} + \tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}\}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar

Ch. Møller, Centre of Gravity in Relativistic System

Newtonian mechanics

$\mu = \mu(x,t)$

$X = \frac{1}{M}\int \mu \cdot \underline{x}\, dv$

relativistic mechanics

$\mu = \frac{h}{c^2}$ h: energy density

$T_{ik}(x_i)$ $\quad x_i = (x, y, z, ict)$.

$g_i = \frac{T_{i4}}{ic}$ $\quad \frac{\partial T_{ik}}{\partial x_k} = 0$

$= \{g \cdot \frac{ih}{c}\} = \{P, \frac{i}{c}H\}$

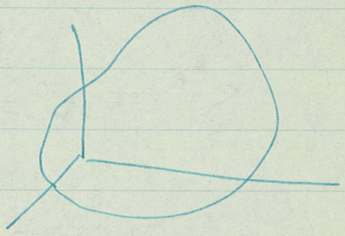
$P_i = \int g_i\, dv$ is a four vector for a (closed system with no singularity.

$M_{ik} = \int (x_i g_k - x_k g_i)\, dv = -M_{ki}$: const. of motion

$P_i P_i = -M_0^2 c^2$

$\underline{M} = \{M_{23}, M_{31}, M_{12}\}$

$= \int \{\underline{x} \times \underline{g}\}\, dv$

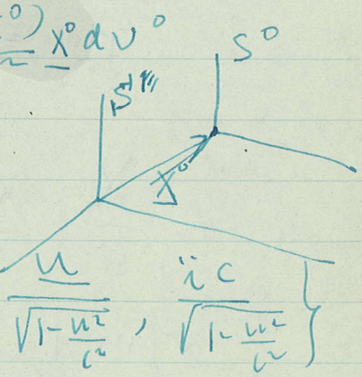


S^0 : system of inertia

$P_i^0 = \{0, 0, 0, iM_0 c\}$

centre of gravity $\quad \underline{X}^0 = \frac{c^2}{H^0}\int \frac{h^0(\underline{x}^0 t^0)}{c^2}\underline{x}^0\, dv^0$

$\underline{u} = \frac{c^2 \underline{P}}{H}$



$X_i = X_i(\tau)$ $\quad u_i = \frac{dX_i}{d\tau} = \left\{\frac{\underline{u}}{\sqrt{1-\frac{u^2}{c^2}}}, \frac{ic}{\sqrt{1-\frac{u^2}{c^2}}}\right\}$

$P_i = M_0 u_i$

$m_{ik} = \int \{(x_i - X_i) g_k - (x_k - X_k) g_i\}\, dv^0$

$= M_{ik} - (X_i P_k - X_k P_i)$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\frac{d m_{ik}}{dt} = 0$$

$$\underline{m} = (m_{23}, m_{31}, m_{12})$$

$$i\underline{n} = (m_{14}, m_{24}, m_{34})$$

$$= \int \rho_4 \underline{x}\, dv - \underline{X} P_4 - (X_4 - X_4)\underline{P}_E$$

$$\underline{X} = \frac{1}{P_4} \int \rho_4 \underline{x}\, dv \Big|_{x_4 = X_4} - \frac{i\underline{n}}{P_4}$$

centre of gravity = coord. of centre of (gravity) mass in general

$$\underline{X}(S)$$

$$\underline{X} = \underline{X}(S) - \frac{c\underline{n}}{H}$$

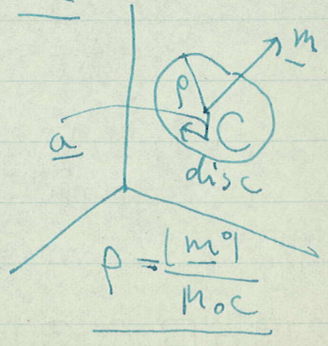
S^0 : $n^0 = 0$, $m^0_{i4} = 0$ or $m_{ik} P_k = 0$ or $m_{ik} P_k = 0$

$$m_{ik} = M_{ik} \mp \frac{1}{M_0^2 c^2} (M_{il} P_l P_k - M_{kl} P_l P_i)$$

$$\underline{n} = \frac{\underline{v} \times \underline{m}^0}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\underline{X} = \underline{X}(S) = \frac{\underline{v} \times \underline{m}^{(0)}}{M_0 c^2} = \underline{a} \quad S^{(0)}$$

$|m^0|$, M_0, $\underline{h}$



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Mathisson (1938), Weisenhof.

$\underline{X}$: Centre of gravity (~~at rest~~) / (in its own system (at rest))

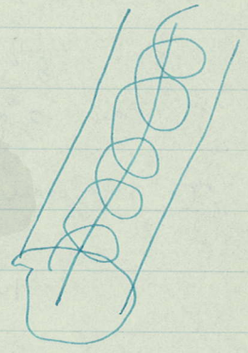
$$\omega^0 = \frac{M_0 c^2}{|m^0|}$$

$\underline{X}(s)$; at rest in its own system momentarily.

External Force;

$$\frac{\partial T_{ik}}{\partial x_k} = f_i$$

centre of gravity can ~~body~~ not be defined uniquely.



Pryce.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Oppenheimer　　Ritson,

Powell et al　　Brown, Camerini, Fowler
　　Muirhead,

Mechanism of $\pi^- + \mu^-$ capture

Mechanism of μ-decay

electron energy MeV/c

36 ± 8

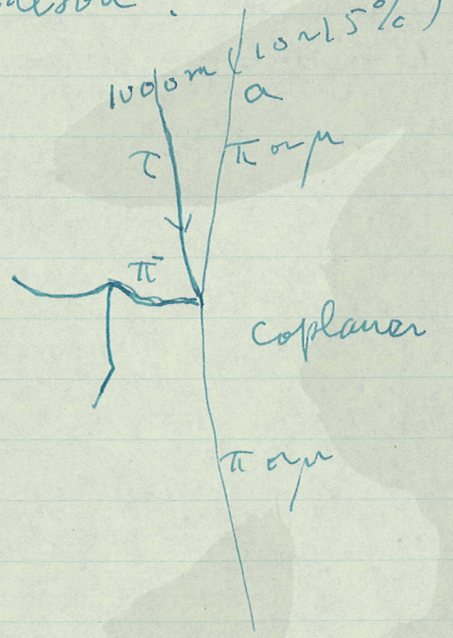
63 ± 16

25 ± 5

69 ± 12

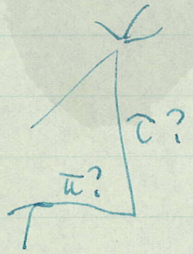
38 ± 6

τ-meson : (10 ~ 15%) → (scattering ; 1200 ± 500 m)



a :	p	π	μ	e
b	70	103	80	.4
a				>1000
Sc	245	176	100	68
mom. Bal.	98	98	98	98

Rochester.



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Møller, continued

Quantum mechanical system:

$T_{ik} = T_{ki} = T_{ik}(F(x_i))$ $\quad P_i = \int j_i \, dv = \int \frac{T_{ik}}{ic} dv$

A : $\frac{\partial T_{ik}}{\partial x_k} = 0$

B : $\bar{x}_i = x_i - q_i$ $\quad F(x)$, $\bar{F}(\bar{x})$

$$\frac{\partial \bar{F}}{\partial x_i} = \frac{i}{\hbar}[P_i, \bar{F}]$$

C $\quad \bar{x}_i = x_i + q_{ik} x_k$ $\quad q_{ik} = q_{ki}$

$$\delta^* \bar{F} = \frac{q_{kl}}{2i\hbar}[M_{kl} \bar{F}]$$

$f(\bar{F}; x_i)$, $\quad \delta^* f =$

$[P_i, P_k] = 0$

$[P_i, P_{kl}] = -\delta_{ik} P_l - \delta_{il} P_k$

$[M_{ik}, M_{lm}] = \delta_{il} M_{km} - \delta_{im} M_{kl} - \delta_{kl} M_{im} + \delta_{km} M_{il}$

$[M_0, P_i] = [M_0, M_{kl}] = 0$

$[M_x, M_y] = i\hbar M_z$

$[P_i, m_{kl}] = 0$

$[m_{ik}, m_{lm}] =$

$[m_x, m_y] = i\hbar \left(m_z + \frac{m P}{(M_0 c)^2} P_z \right)$

$[\bar{x}_i, P_k] = i\hbar \delta_{ik}$

$[P_i, P_k] = 0$

$\underline{X} \times \underline{X} = \frac{i\hbar}{(M_0 c)^2} \left(\underline{m} - c(\underline{n} \times \underline{P}) H^{-1} \right)$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$(\overset{\times}{X} \times \overset{\times}{X}) = 0$ $\qquad [\overset{\times}{x_i}, p_k] = \delta_{ik}$

$(\overset{\times}{m}_x, \overset{\times}{m}_y) = i\hbar \overset{\times}{m}_z$

Finkelstein:

Dirac electron:

$$\underline{m} = \frac{\hbar}{2}\underline{\sigma} + \frac{\hbar}{2m_0 c}\rho_2(\underline{\sigma} \times \underline{p})$$

$$\frac{1}{2}(m^2 - n^2) = \frac{3\hbar^2}{4}$$

$$(\overset{\times}{m})^2 = \frac{3\hbar^2}{4}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

R. J. Finkelstein
On the Quantization of a Unitary Field Theory

Maxwell-Yukawa and the Scalar-Scalar Field[1]:

$$L = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \psi)(D_\mu^* \psi^*) + \sigma^2 \psi \psi^* \qquad (1)$$

$$\left.\begin{aligned} D_\mu \psi &= (\partial_\mu - i\varepsilon \varphi_\mu)\psi \\ D_\mu^* \psi^* &= (\partial_\mu + i\varepsilon \varphi_\mu)\psi^* \end{aligned}\right\} \quad (1-a)$$

This case was considered by Rosen[2], as a modification of Maxwellian Lagrangian with contains the potential explicitly and is still gauge invariant. The signs are so chosen that the energy of free el. mag. field is positive, while that of the free meric field is negative.

$$(D_\lambda D_\lambda - \sigma^2)\psi = 0 \qquad (2)$$

$$F_{\mu\lambda,\lambda} = s_\mu \qquad (3)$$

with
$$s_\mu = i\varepsilon(\psi^* D_\mu \psi - \psi D_\mu^* \psi^*) \qquad (3-a)$$

Put:
$$\psi = \theta(r) e^{i\omega t} \qquad \varphi_\mu = (\underline{0}, i\varphi)$$

We have

---

(1) Bohm pointed out that one can expect to form a particle of positive energy by locking the Dirac field to a scalar Yukawa field, and that one could perhaps get a useful theory of the nucleon in this way.

(2) Rosen, Mennius, P. R. 62 (1942), 436.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$r^{-2}\frac{d}{dr}r^2\frac{d\theta}{dr}+\left(\frac{\omega}{c}+\epsilon q\right)^2\theta-\sigma^2\theta=0 \qquad (4\text{-}a)$$

$$r^{-2}\frac{d}{dr}r^2\frac{d\theta}{dr}+2\epsilon\left(\frac{\omega}{c}+\epsilon q\right)\theta^2=0 \qquad (4\text{-}b)$$

Change to dimensionless variables:

$$\left.\begin{aligned} z&=\sigma^{-1}(\omega/c+\epsilon q)\\ y&=\sqrt{2}\,\sigma^{-1}\epsilon\theta\\ x&=\sigma r\end{aligned}\right\} (5\text{-}a)$$

and take $\eta=xy$ $\zeta=xz$. (5-b)

$$\eta''+\left(\zeta^2/x^2-1\right)\eta=0 \qquad (6\text{-}a)$$

$$\zeta''+\eta^2x^{-2}\zeta=0 \qquad (6\text{-}b)$$

boundary conditions:

z, y: finite at $x=0$

$\frac{dz}{dx}=\frac{dy}{dx}=0$ at $x=0$

or

$$\left.\begin{aligned}&\zeta(0)=\eta(0)=0\\ &\zeta'(0)=z(0),\quad \eta'(0)=y(0)\end{aligned}\right\} (7\text{-}a)$$

$$\lim_{r\to\infty}\begin{cases}\varphi=0\\ \theta=0\end{cases} \quad\text{or}\quad \lim_{x\to\infty}\begin{cases}\zeta/x=\omega/c\sigma\\ \eta/x=0\end{cases} \qquad (7b)$$

Asymptotically by (6b), (7)

$$\zeta=\alpha Q+\beta x$$

$$\beta=\omega/c\sigma$$

$\varphi=Q/r$ at large distance and

$$\int\zeta_0\,dx=4\pi Q \qquad (8)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

It may be shown that (6) and (7) possess nodeless solutions[3] depending on the four physical constants (c, σ, ε, Q); the frequency ω has a corresponding eigenvalue.

Energy-momentum tensor follows uniquely from the eq.

$$\delta L = -\frac{1}{2}\int T_{\mu\nu}\,\delta g^{\mu\nu}\sqrt{-g}\,d\tau \qquad (9a)$$

$$-T_{\mu\nu} = -2\frac{\partial L}{\partial g^{\mu\nu}} - L\,g_{\mu\nu}$$

corrected canonical tensor

$$-T_{\mu\nu} = \frac{1}{4}F_{\lambda\sigma}F_{\lambda\sigma}g_{\mu\nu} - F_{\alpha\mu}F_{\alpha\nu} + (D_\mu^*\psi^*)(D_\nu\psi) + (D_\nu^*\psi^*)(D_\mu\psi) - [(D_\mu^*\psi)(D_\mu\psi) + \sigma^2\psi^*\psi]\,g_{\mu\nu}$$

rest mass: $Mc^2 = -\int T_{44}\,d\underline{r}$

For small εQ, $Mc^2 \sim -\varepsilon^{-1}Q\sigma$, which is negative.*

Scalar-scalar field

$$L = -(\partial_\lambda\psi)(\partial_\lambda\psi^*) - \sigma^2\psi\psi^* - (\partial_\lambda\varphi)(\partial_\lambda\varphi) + g\psi\psi^*\varphi^2 \qquad (1)$$

From this we obtain

(3) ~~Cf~~ Rosen, ~~loc. cit.~~, Phys. Rev. 55 (1939), 94; also Jehle, unpublished.

* This defect can be avoided by adding divergenceless tensor to $T_{\mu\nu}$, or by considering scalar-scalar coupling.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\nabla^2\theta + (\omega^2/c^2 - \sigma^2)\theta = -g\theta\varphi^2$$
$$D^2\varphi = -g\theta^2\varphi$$

Put $\psi = \theta e^{i\omega t}$, $\frac{\partial\varphi}{\partial t} = 0$

$$\eta = g^{\frac{1}{2}} r\theta \qquad \zeta = g^{\frac{1}{2}} r\varphi$$
$$x = ar \qquad a^2 = \sigma^2 - \omega^2/c^2$$

we again obtain (6) with the boundary condition $\zeta/x = \eta/x = 0$. For $\omega = 0$, we have a particle like solution with positive energy.

If the charge Q is fixed a discrete spectrum of rest masses follows uniquely. In ~~non~~-quantized theory, charge cannot be fixed. In quantized theory there is a relation

$$q = Q\min/\hbar c$$

Quantization

Commutation relations between the particle observables are already determined by the postulated commutators of the field operators.

B.E.:
$$[\pi_\sigma(x), \psi_{\sigma'}(x')] = -i\hbar\,\delta_{\sigma\sigma'}\,\delta(x-x')$$
$$[\pi^*_\sigma(x), \psi^*_{\sigma'}(x')] = -i\hbar\,\delta_{\sigma\sigma'}\,\delta(x-x') \qquad (11a)$$

F.D.:
$$[\psi_\sigma(x), \psi^*_{\sigma'}(x')]_+ = \delta_{\sigma\sigma'}\,\delta(x-x')$$
$$[\psi_\sigma(x), \psi_{\sigma'}(x')]_+ = [\psi^*_\sigma(x), \psi^*_{\sigma'}(x')]_+ = 0 \qquad (11b)$$

Momentum and Angular Momentum

$$M_{\lambda\mu\nu} = T_{\lambda\mu}x_\nu - T_{\lambda\nu}x_\mu + \sum_{\alpha,\beta}\frac{\partial L}{\partial(D_\lambda\psi^{(\alpha)})}S^{(\alpha\beta)}_{\mu\nu}\psi^{(\beta)} + \text{*} \qquad (13)$$

where
$$\delta\psi^{(\alpha)} = \sum_{\lambda<\mu}\sum_{\beta} S^{(\alpha\beta)}_{\lambda\mu}\,\delta\omega_{\lambda\mu}\,\psi^{(\beta)}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\delta x_\lambda = \sum_i \delta\omega_{\lambda\mu} x_\mu$$

$$T_{\lambda\nu,\lambda} = 0 \qquad M_{\lambda\mu\nu,\lambda} = 0$$

$$ic\,G_\nu = \int T_{4\nu}\, d\underline{x}$$

$$ic\,M_{\mu\nu} = \int M_{4\mu\nu}\, d\underline{x}$$

$$G_n = \int g_n\, d\underline{x} \qquad (14)$$

$$M_{\mu\nu} = \int (g_\mu x_\nu - g_\nu x_\mu)\, d\underline{x} + \int S^{(\alpha\rho)}_{\mu\nu}\, \pi^{(\alpha)}\, \psi(\rho)\, d\underline{x} \qquad (15)$$

$$[G_\alpha, f] = i\hbar \frac{\partial f}{\partial x_\alpha}$$

If there's no external field;

$$[G_\alpha, G_\beta] = 0 \qquad \alpha, \beta = 1 \cdots 4 \quad (18)$$

$$\vdots$$

$$[M_{ij}, M_{jk}] = i\hbar M_{ik} \qquad i\,j\,k = 1,2,3$$

$$[M_{\alpha\beta}, M_{\gamma\delta}] = (g_{\alpha\delta} M_{\beta\gamma} + g_{\beta\gamma} M_{\alpha\delta} - g_{\alpha\gamma} M_{\beta\delta} - g_{\beta\delta} M_{\alpha\gamma})\, i\hbar \qquad (20)$$

$$[M_{\alpha\beta}, G_\gamma] = (G_\alpha g_{\beta\gamma} - G_\beta g_{\alpha\gamma})\, i\hbar \qquad (21)$$

Non-relativistic definition of position

$$\lambda_i \int \rho\, d\underline{x} = \int x_i \rho\, d\underline{x} \qquad (22)$$

$$\rho = \psi^* \psi.$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Relativistic Definition of Position (classical)

$$X_i = (G_4)^{-1} \int g_4 x_i \, d\underline{x} \qquad (28)$$

(quantized field)

$$[X_\alpha, X_\beta] = (G_\alpha X_\beta - G_\beta X_\alpha) G^{-1} \cdot i\hbar \qquad (37)$$

$$[G_\alpha, X_\beta] = -i\hbar (g_{\alpha\beta} - G_\alpha G_\beta G^{-1}) \qquad (36)$$

equation of motion

$$\frac{dM_{i4}}{dx_4} = 0$$

$$\frac{dX_i}{dt} = G_i G_4^{-1}$$

Weak external field:

$$\frac{dG_i}{dt} = \frac{i}{\hbar}[H^{int}, G_i] = \frac{i}{\hbar}\int [\hat{H}', G_i] dx'$$

$$= -\int \left(\frac{d\hat{H}}{dx_i}\right)_{ext} d\underline{x}$$

mass and charge:

$$-M^2 = G_\alpha G_\alpha = G_4^2 + \underline{G}^2$$

$$[M_{\alpha\beta}, M^2] = 0$$

$$[G_\alpha, M] = 0$$

$$[M^2, X_\alpha] = 0$$

$$Q = \int s_4 \, d\underline{x}$$

$$= -\varepsilon \sum_\sigma \int (\pi^{(\sigma)} \psi^{(\sigma)} - \pi^{+(\sigma)} \psi^{+(\sigma)}) \, d\underline{x}$$

$$[Q, G_\alpha] = 0$$

$$[Q, M_{\alpha\gamma}] = 0$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$[Q, M] = 0$$

Charge current vector

$$j_\mu = Q \frac{dX_\mu}{ds}.$$

Connection between charge and coupling constant:

$$\psi = \left(\frac{\hbar}{2}\right)^{\frac{1}{2}} (\alpha + \beta^*)$$

$$\pi = i\left(\frac{\hbar}{2}\right)^{1/2} (\alpha^* - \beta)$$

$$Q = (\varepsilon\hbar/2) \int [\alpha\alpha^* + \alpha^*\alpha - \beta\beta^* - \beta^*\beta]\, d\underline{x}$$

$$[\alpha(x), \alpha^*(x')] = \delta(x - x') \text{ etc.}$$

$$Q = \hbar\varepsilon \sum (N_a - N_b)$$

$$\varepsilon = e/\hbar.$$

e: min. positive value of Q.

External Fields:

$$[F_{\alpha\beta}, M_{\gamma\delta}] = i\hbar (F_{\alpha\delta} g_{\beta\gamma} + F_{\beta\gamma} g_{\alpha\delta} - F_{\beta\delta} g_{\alpha\gamma} - F_{\alpha\gamma} g_{\beta\delta})$$

$$[F_{\alpha\beta}, G_\gamma] = -i\hbar \frac{\partial F_{\alpha\beta}}{\partial x_\gamma}$$

$$[F_{\alpha\beta}, X_\gamma] \neq 0$$

$$[F_{\alpha\beta}, \mu_{ij}] = 0 \qquad \mu_{ij} = \frac{1}{2}\int (x_i \delta_j - x_j \delta_i)\, d\underline{x}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

○ T. Shimose, On Fundamental Equations for Elementary Particles
(read before 3rd Symposium on the Theory of Elementary Particles at Tokyo University, Oct. 16, 17, 1948)

$$\left.\begin{aligned} x+iy &= \xi_1^* \eta_1 \pm \xi_2^* \eta_2 \\ x-iy &= \eta_1^* \xi_1 \pm \eta_2^* \xi_2 \\ z+ct &= \xi_1^* \xi_1 \pm \xi_2^* \xi_2 \\ -z+ct &= \eta_1^* \eta_1 \pm \eta_2^* \eta_2 \end{aligned}\right\} \begin{matrix} + \text{ time like} \\ - \text{ space like} \end{matrix}$$

$$\left(S_{\xi_1}\frac{\partial}{\partial \xi_1} + S_{\eta_1}\frac{\partial}{\partial \eta_1} + S_{\xi_2^*}\frac{\partial}{\partial \xi_2^*} + S_{\eta_2^*}\frac{\partial}{\partial \eta_2^*}\right)\psi = -\varkappa\psi$$

time like:

$$\frac{\partial}{\partial \xi_1} = \frac{1}{2}\eta_1^*\frac{\partial}{\partial x} + \frac{1}{2}i\eta_1^*\frac{\partial}{\partial y} + \frac{1}{2}\xi_1^*\frac{\partial}{\partial z} + \frac{1}{2}\xi_1^*\frac{\partial}{\partial t}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

G. C. J. Zwanikken

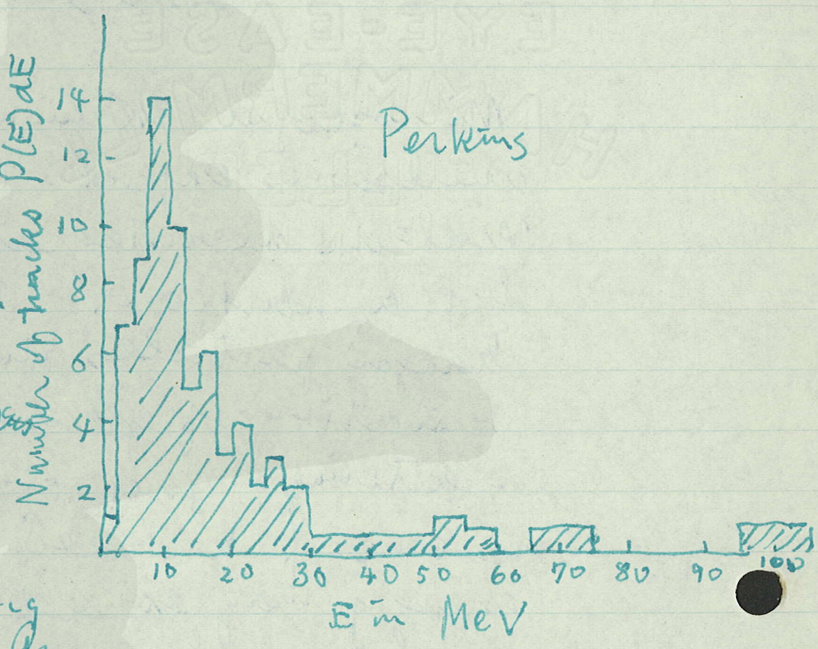
On the penetration of nucleons through heavy nuclei (Physica 14 (1948), 530)

Microscopic examination of photographic emulsions exposed to cosmic radiation. Nuclear disintegration presumably caused by a neutron. The length of path of charged particles in the emulsion, the variation of grain density and the Coulomb scattering gives conclusions about the nature of knocked out particles. These energy may exceed 100 MeV. Protons are most frequent. On the average an equal number of neutrons must be expelled. The energy transmitted is of the order of a few hundreds of MeV. Larger energies are, however, also frequent.

Heisenberg (1937, Leipz. Ber. 89 (1), 369) and Bagge (1939, Ann. Phys. 35, 118) tried to draw conclusion about the range $\varkappa^{-1}$ of nuclear potential from the energy distribution of knock-out nucleons assuming the Gaussian potential. Ortner (1940, Wien. Ber. 149, 259) compared the result with experiment and found a good agreement. However,

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

more recent data of Perkins (1947, Nature 160 279, 1947) indicate that the number of high energy particles is too high to be explained by a Gauss-potential.



Here all calculations were made using meson potential $e^{-\kappa r}/\kappa r$ and referring to Rosenfeld's book.
Fermi gas for nucleus. Born approximation.
Exchange effect.
The values for κ^{-1} from 1.8 ~ 2.2 agree equally well with the experiment.
Energy of incident neutron was supposed to be larger than 200 MeV.

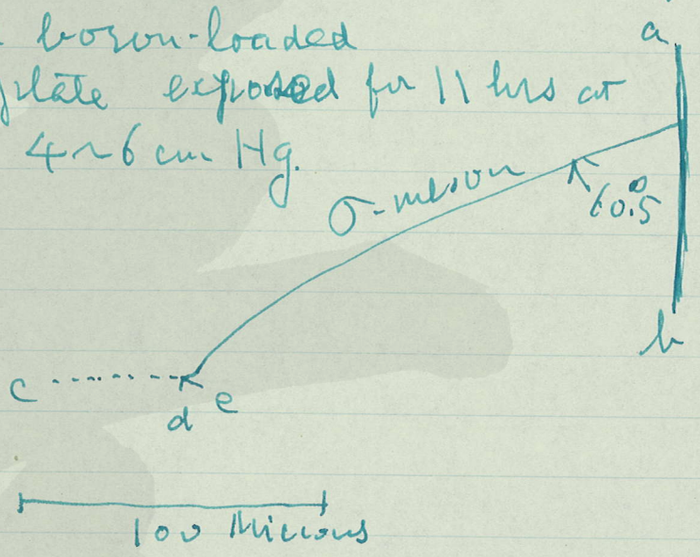
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

J. J. Lord and Marcel Schein

Creation of a "σ" meson by a highly ionizing nucleus in the Cosmic Radiations

(Freier, Lofgren, Ney, Oppenheimer, Bradt and Peters, P. R. 1948, 74, 213)

100μ-thick boron-loaded Ilford C2 plate exposed for 11 hrs at altitude 4~6 cm Hg.



estimated energy of $\overline{a,b}$: < several bev.

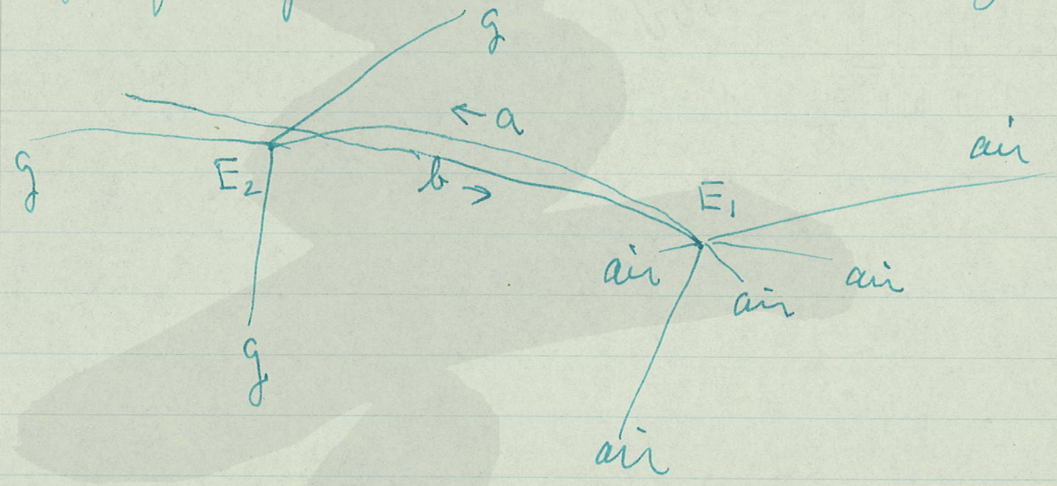
" charge " " $\eta \sim 20\,e$

The absence of a large nuclear disintegration demonstrates that the meson was created in a pure nucleon-nucleon interaction. This seems to indicate that nuclei can be greatly transparent to the bombardment of charged nuclei of energies capable of producing a meson.

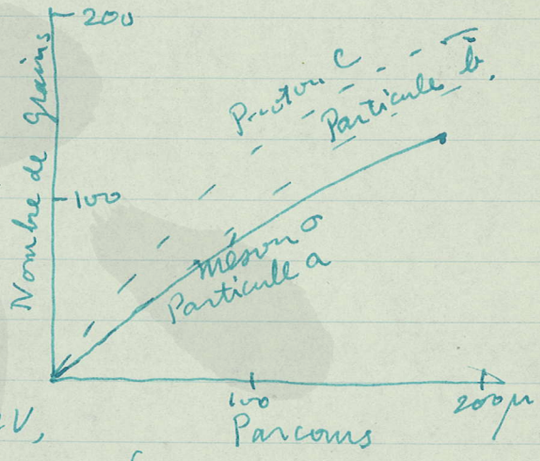
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Lepince-Ringuet, Fong, Jauneau et Morellet. Observation probable d'un méson 1000 donnant naissance à une étoile cosmique
(C.R. Paris. 226 (1948), 1897)

Illford plates. 4300 m height



If we assume the star at E1 to be caused by particle b and the mass of the created meson to be that of π-meson, then the energy liberated at E1 is at least 400 MeV, which is in good accord with the assumption that b is a meson with a mass larger than 800 me.



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Bradt and Peters, Bristol Symposium
1948 (in press)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

Observations with Electron-Sensitive Plates Exposed to Cosmic Radiation

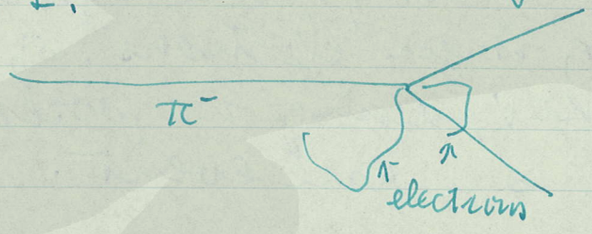
Brown, Camerini, Fowler, Muirhead and Prof. Powell
H.H. Wills Laboratory, University of Bristol,
and Rotson, Clarendon Laboratory, Oxford

Nature 163 (1949), 47, 82.

Part I. Decay of μ-Mesons

30 disintegr. produced by π⁻-particles.

Fig 1.



Jungfraujoch
11,000 ft
Kodak
type N.T. 4.
Beriman

only 3 among 30 show the tracks of associated electrons. Number of electrons with energy larger than 15 KeV must be small.

30 - $\pi^+ - \mu^+$ - decay

Miss Brown

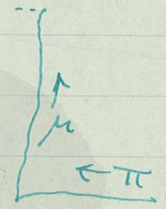


Fig. 6

μ - β - decay

momenta of decay particles

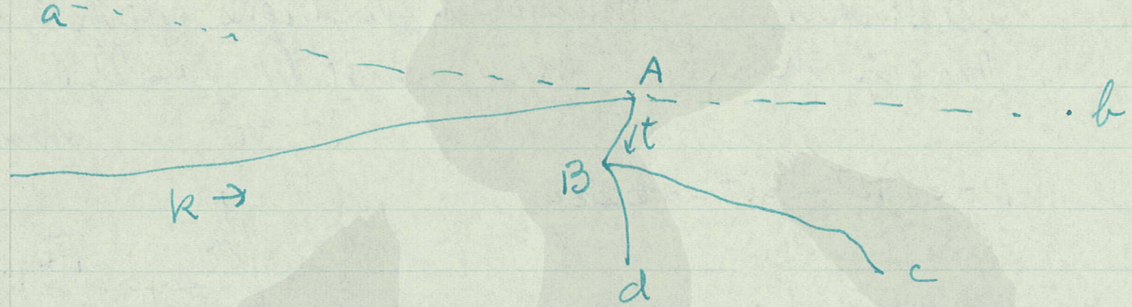
15 ± 3, 25 ± 5, 38 ± 6, 42 ± 2,
44 ± 4, 48 ± 6 MeV/c.

65 ± 10 % of μ-mesons emitted secondaries, which is consistent with the estimation of proportion of π-particles stopped in gelatine and in silver bromide. As in the case of π-mesons, of the hundred events in which

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

we can distinguish the track of the fast particle arising from the decay of a μ-meson, there is no case in which the track of an electron of short range arising from the point of disintegration can be distinguished. It is reasonable to suppose that in about a third of these events we are observing the spontaneous decay of a negative ~~proton~~ meson. It follows that for the μ-particles also, the ejection of Auger electrons, of energy greater than 15 keV, during the process of atomic capture, is a rare event.

Part II.



Observer: Mrs. W. J. van de Meiwe.

Fig. 8.

Table I. Determination of m_p/m_K by grain-counting

1.77 1.88 1.49 1.64 2.17 1.79 1.32
1.71 1.66 1.21 1.69 2.13 1.55

Mean value: 1.70 → $m_K = 1.080 \pm 160\ m_e$.

Determination from small-angle scattering

$m_K = 1,800 \pm 400\ m_e$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

B: star formation by π^-.
A: three tracks are coplanar!
~~Table 2.~~

Table 3. Estimates of the mass of K based on total release of mass and energy

(i) $k \to \pi^- + \pi + \pi$

	Track a	Track b	Track c
Particle	π	π	π^-
Rest-mass (m_e)	286	286	286
Energy in m_e	61	64	2

Total $m_K = 985\ m_e$

(ii) $k \to \pi^- + \mu + \nu$

Particle	μ	μ	π^-
Rest-mass	212	212	286
Energy	76	81	2

Total $m_K = 869\ m_e$

(iii) $k \to 2\pi + \mu$: $m_K = 925\ m_e$

Table 4. Comparison of observed and calculated values of the grain-density in track b, for various assumptions regarding the nature of a and b.

	length of track (microns)	number of grains	Grain density (per 100 μ)	Assume Particles 1	2	3	4
a	2100	1025	49 ± 1.5	π	μ	π	μ
b	116	59	51 ± 6	π	μ	μ	π
Calculated grain density in b				45	45	34	69
order of plausibility:				(1)	(2)	(3)	

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Bradt and Peters, Report to Bristol Symposium 1948 (in the press)
(Alichanian, Alichanov and Weissenberg, J. Exp. and Theor. Phys., U.S.S.R. 18, 301 (1948). and other references.)

T-mesons with 800 mc give no recorded secondary particles at the end of their range.

Note added in proof: Peters and Bradt, 90,000 ft Ilford C2 emulsions
T-mesons emits a particle of smaller mass, which produces a nuclear disintegration
The ranges of secondary particle in three events of this type: 20, 25, 45 μ.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

On the Interpretation

τ - ~~3π~~ three meson-decay process　　Jan 17, 1949

Assumption: τ: integer spin, coupled with nucleon.

$$\tau^+ \xrightarrow[g_\tau]{} (N) + P \xrightarrow[g]{} (N) + N' + \pi_1^+$$
$$\xrightarrow[g]{} (N) + P' + \pi_2^- + \pi_1^+$$
$$\xrightarrow[g]{} \pi_3^+ + \pi_2^- + \pi_1^+$$

Similar Processes calculated by Finkelstein[1]

(A) $\pi^0(0) \to \gamma_1 + \gamma_2$　　$\tau_A = 1 \times 10^{-16}$ sec.

(B) $\pi^0(1) \to \gamma_1 + \gamma_2 + \gamma_3$　　$\tau_B = 2 \times 10^{-11}$ sec.

(C) $\pi^0(1) \to \cancel{M_0 + \gamma}\ \pi^0(0) + \gamma$　　$\tau_C = 1 \times 10^{-18}$ sec.

(D) $\pi^{\pm}(1) \to \cancel{M}\ \pi^{0}(0) + \gamma$　　$\tau_D = 4 \times 10^{-18}$ sec.

Constants used[2]

$m_\pi^{(0)(1)} / m_\pi^{(0)}(0) = 1.6$　　$f_0^2/\hbar c = 0.05$

$f_0 / m_\pi^{(0)}(0) = f_1 / m_\pi^{(0)}(1)$

$m_\pi^{(0)}(0) = 177\, m_e$

Selection rule

A process is forbidden if it involves an odd number of even matrix elements

even: vector, antisym. tensor interactions

odd: scalar, pseudoscalar, pseudovector interactions.

$\tau - 3\pi$ process: even number of elements.

~~competitive processes:~~

$\cancel{\tau^+ \to (N) + P \xrightarrow[\text{odd even}]{} (N) + P' + \gamma \to \pi^+ + \gamma}$ π^0 π^0

(1) Finkelstein, Phys. Rev. 72 (1947), 415;
Tanikawa,
Nakamura,
(2) Jauch and Hu, Phys. Rev. 65 (1944), 289.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Competitive Processes of lower order

i) $\tau^+ \to (N) + P \xrightarrow{\text{odd}} (N) + P' + \pi^0 \to \pi^+ + \pi^0$

ii) $\tau^+ \to (N) + P \xrightarrow[\text{even}]{\text{odd}} (N) + P' + \gamma \to \pi^+ + \gamma$

In order that ii) is forbidden

(τ^+-(N,P) coupling
π^+-(N,P) coupling

both odd → scalar, pseudoscalar, pseudovector
both even → vector, tensor

Moreover in order that i) is also forbidden

	τ-$(N\,P)$	$\pi^\pm$-(N,P)	π^0-(N,P)
a)	even	even	odd even
b)	odd	odd	even
c)	odd	odd	odd

c) $m_\pi + m_\pi^{(0)} > m_\tau$.

Order of magnitude of $\tau - 3\pi$ decay probability

$$\frac{1}{\tau_{\tau 3\pi}} \sim \left(\frac{m_\pi c^2}{\hbar}\right)\left(\frac{g^2}{\hbar c}\right)^3 \cdot \left(\frac{g^2 \tau}{\hbar c}\right)^2 \left\{ 1 + \left(\frac{\tau}{\mu c^2}\right) \tilde{\alpha} E \times \cdots \right\}$$

$$\tau_{3\pi} \sim 10^{-23}\, 10^3 \cdot \left(\frac{g^2 \tau}{\hbar c}\right)^{-2}$$

$$\sim 10^{-20}.$$

$$\left(\frac{g^2 \tau}{\hbar c}\right) \sim 10^{-5} : \to \tau_{3\pi} \to 10^{-10}$$

In order that $\tau \to \pi + \pi + \pi$ is forbiden.

τ-$(N\,P)$ even odd
$\pi^\pm$-(N,P) odd even.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Dec. 16, 1948 From R.E. Peierls to J.R. Oppenh.

... The first is that you drew attention to the results by Finkelstein, according to which the life-time of Σ-mesons for decay into π with the emission of one or two photons was sure to be much shorter than observed. The published paper by Finkelstein covers only some special cases. We have looked into this problem more generally. It is ~~clear~~ easier, of course, to survey the case of the emission of one photon. In this case it seems possible to choose the spin and parity of the two mesons and type of coupling in such a way that this process is forbidden. We are proposing also to study a way that this process is forbidden. . . .

The second point also concerns work by Finkelstein. We have played a bit and found it very hard to decide whether they have only one solution or an enumerable set. . . .

Another point is that you mentioned the question of finding an operator representing the coordinates of the centre of this structure in the quantized equations. . . . I see no reason, however, why one should expect to have an exact coordinate operator which is defined to better accuracy than the size. . . .

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Dec. 30, 1948 From Oppenheimer to Peierls

To the best of my knowledge, no systematic search for selection rules to lengthen the life-time of τ-meson has been made. . . . Surely one must also investigate stability with regard to electron-positron pairs. We are looking for a factor which is something like 10^{-9}, since certainly the characteristic decay process is not a gamma instability or an el-pos. pair instability. . . . If one does not wish to question the materialization processes for nucleons, one may believe the production of the τ-meson is always accompanied by the production of another particle of equal or greater mass. . . . I meant really only that we should keep open-minded about the materialization process and this might be one of the ways that we might learn about them in the not too remote future. . . . As far as the non-linear theories, I profoundly agree with every word you write. . . .

The essential question here is to understand the relation between the classically coupled fields and the quantum theory. . . . The difficulties of the order of operators do not occur in a classical field theory and if one should solve the classical problem, one would have a starting point for perturbation theory based on the smallness of h and not of e. To date this is a program only

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

and I suspect that there are some deep reasons why that will not work, because a number of us have been trying without success.

. . . The Lamb shift appears to be settling down to 1052 megs., a result which was not obtained by the highbrow F. S. T. methods, but by much more pedestrian ones. There is still a real question as to the complete uniqueness of procedures; but as to the moment . . . it does not appear that this has contributed to the uncertainty of the result, nor that only rather arbitrary and odd methods of following the integrations can give a different answer.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Jan. 19, Wednesday

Columbia Univ.

Prof. Lamb:

Lamb shift exp. 1061 ± 5

F.S.T. $1072 \sim 3$

K.L.V F.W. $1051 \sim 2$

$\Delta_H - \Delta_D \leq 1$ mc

size effect $\sim 1/2$ mc

f-field effect $5 \sim 50$ mc

$\Delta_{He} \sim 13.2 \times 1061 \sim 14 \cdots$

no definite sign for specific nuclear interaction.

Dr. Bernaldini:

Berkeley proton - neutron - meson

Mr. Slotnick:

neutron-Electron Interaction

pseudoscalar coupling:

symmetric theory -14.7 Kev. $\mu = m_\pi$.

experiment $-13.$ Kev.

not so sensitive to meson mass.

nuclear magnetic moment:

$\mu_n = \cancel{1} + \alpha - \frac{M}{m_\pi}\alpha$

$\mu_p = 1 - \alpha + \frac{M}{m_\pi}\alpha$.

$\alpha \gtrsim \frac{M}{m_\pi}\alpha$ no cut off

$\alpha \ll \frac{M}{m_\pi}\alpha$ cut off above $\sim m_c$.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Theoretical Seminar

Prof. Bethe : scattering phenomena in cl. N. phys.
(Jan. 15, 1949)

N-P-scattering (moderate energy)
Schwinger-Blatt
V effective depth
effective range

$u_1'' + k_1^2 u_1 - V u_1 = 0$
$u_2'' + k_2^2 u_2 - V u_2 = 0$

u_2
u_1

$$u_2 u_1' - u_1 u_2' \Big|_0^R = (k_2^2 - k_1^2) \int_0^R u_1 u_2 \, dr$$

$$\psi = \frac{\sin(kr+\delta)}{\sin\delta}$$

$u \to \psi$ outside

$$\psi_2 \psi_1' - \psi_1 \psi_2' \Big|_0^R = \int_0^R \psi_1 \psi_2 \, dr \cdot (k_2^2 - k_1^2)$$

$$(k_2^2 - k_1^2) \int_0^R (\psi_1 \psi_2 - u_1 u_2) \, dr = (\psi_2' - \psi_1')_0$$

$$\psi'\Big|_0 = k \cot\delta$$

$k_1 = 0$: $k_1 = k$

$$(k \cot\delta)_{k=0} = -1/a$$

$\frac{1}{2}$ / $\frac{1}{2\pi^2}$ square well

$$k \cot\delta = -\frac{1}{a} + k^2 \underbrace{\int_0^\infty (\psi_0 \psi - u_0 u) \, dr}_{\frac{1}{2}\rho(E,0)}$$

$$\rho(E,0) = \rho(0,0) = r_0$$

$$k \cot\delta = -\frac{1}{a} + \frac{1}{2} k^2 r_0$$

$$\psi = \cos kr + \cot\delta \sin kr = 1 - \frac{1}{2}k^2 r^2 + \cdots + k\cot\delta \cdot r + \cdots$$

$$= 1 - \frac{r}{a} + \frac{1}{2} k^2 r (r_0 - r)$$

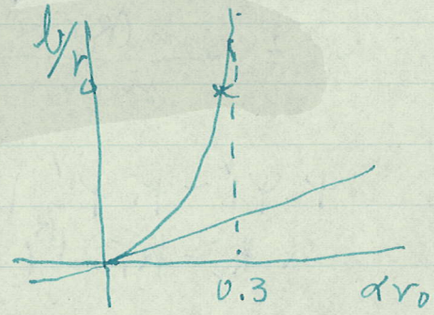
$$\rho(E,0) = r_0 + \beta k^2 r_0^3 \qquad \beta \ll 1,$$

10 Mev : $k r_0 \sim 1/2$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Singlet
$\sigma_{np} = 20.36 \pm 0.10$ barns.
Triplet parahydrogen. $3a_1 + a_0$
$-a_0 = 2.375 \times 10^{-12}$. $a_1 = 0.53 \times 10^{-12}$
^3D: 2.21 Mev ($C^{14}(p,n)N^{14}$) H_2-D
$r_{0t} = 1.56 \times 10^{-13}$ cm ± 0.13

$b_{sq.\ well} = 1.8$
Intrinsic range $b \leftrightarrows r_0$ (for 0-binding energy)
$V_0 \frac{e^{-\mu r}}{\mu r}$



$\gamma : e^{-\gamma r}$
$\gamma = -\frac{1}{a} - \frac{1}{2}\gamma^2 r_0$

p-p = n-p.
$r_{0s} \sim 2.7$
$b = 2.4$ Yukawa → 320 m.
$b = 2.5$ Square
Same shape → Yukawa, $e^{-\mu r}$ → long tail
× Square, Gauss.

Proton-proton scattering:
$v'' + k^2 v - Vv = \frac{2}{Dr} v$ $\quad D = \frac{2\hbar^2}{Me^2} = 5.176 \times 10^{-13}$ cm

$\varphi'' + k^2 \varphi = \frac{2}{Dr} v$

$v \to \varphi$ $\quad \varphi = 1$ $\quad r = 0$

$(\varphi_2' - \varphi_1')\Big|_r = (k_2^2 - k_1^2) \int_r^{\infty} (\varphi_1 \varphi_2 - v_1 v_2) dr$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

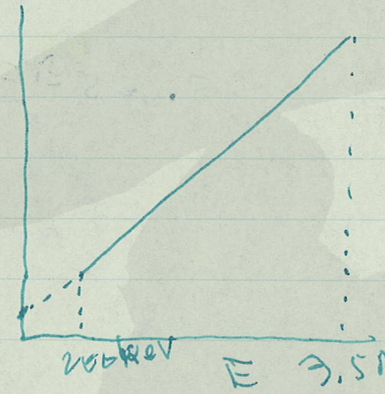
$$\varphi = 1 + \frac{2r}{D}\left(\ln\frac{2r}{D} + \cdots\right)$$

$$\varphi'\Big|_{r\to 0} \sim \ln(0) + \cdots$$

$$\psi = C_0(G + F\cot\delta)$$

$$C_0 = \sqrt{\frac{2\pi\eta}{e^{2\pi\eta}-1}} \qquad \eta = \frac{e^2}{\hbar v}$$

$$f(E) = \frac{2\pi\cot\delta}{e^{2\pi\eta}-1} + g(\eta) = \frac{1}{D}k^2 p(E,0) + f(0)$$



Williams (minnesota)

$$r_0 = 2.71 \pm 0.1$$

u, ψ $\quad r_1 : \quad \frac{\psi'}{\psi} = \left(\frac{\varphi'}{\varphi}\right)_{r_1}$

$$\int_{r_1}^{R} \varphi\psi\,dr = \int_0^R uv\,dr$$

$$\psi = 1 - \alpha_p r$$

$\alpha_p = 0.58 \times 10^{12}\ cm^{-1}$ Square
$= 0.52$ " Yukawa

$\alpha_N = 0.42$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Tensor forces:

$$\frac{1}{2} r_0 = \int (\psi^2 - u^2 - w^2)\, dr$$

w : d-function

anisotropy: $Q\sqrt{2}\, k^2$

photoel. effect:

photmg. effect: $r_{0s} - r_{0t}$

$$r_1 = \left(\frac{Q}{P_0 \alpha} \right)^{1/3} \sim 3 \times 10^{-13}$$

$$\frac{e^{-\mu r}}{\mu r} S_{12} \qquad S_{12} = 3 \frac{\underline{\sigma}_1 \underline{r} \cdot \underline{\sigma}_2 \underline{r}}{r^2} - \underline{\sigma}_1 \underline{\sigma}_2$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

D. C. Padfield:
Tensor Forces and the Neutron-Proton Interaction
(Nature 163 (1949), 22, Jan. 1)

Blatt: Phys. Rev. 74 (1948), 92
effective triplet range 1.30 ~ 1.55 × 10^{-13} cm
square well 1.5 ~ 1.7 × 10^{-13} cm

Rarita, Schwinger: Phys. Rev. 59 (1941), 436.
Gwinn: Phys. Rev. 74 (1948), 145.
r_0: range of central force
εr_0: range of tensor force
J: depth of square central well
γJ: depth of square tensor well

Conditions:
E = 2.185 MeV
$a = 2.73 \times 10^{-27}$ cm^2
4 % D state (1)

r_0	$2r_0$	J (MeV)	γJ
1.6	3.08	39.3	10.04
1.8	3.04	31.8	9.52
2.0	3.00	25.9	9.46
2.2	2.94	20.2	9.85

error: 5%.

(1) Arnold and Roberts, P. R. 71 (1947), 878, magnetic moments of neutron, proton and deuteron. But, it's dubious for smaller range, c.f. Sachs, 72, 91; Primakoff, 118; Breit and Bloch, 135.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Scattering of like Particles at 100 MeV.

By Barker and Ravenhall

Barker, Nature, 161 (1948), 726.

Ashkin and Wu, P.R. 73 (1948), 973.

$V \exp(-r/a)$ $\quad V = 191.2$ MeV, $a = 0.5 \times 10^{-13}$ cm

for even singlet

$V = 313.5$ MeV for odd triplet, exchange

$V = 104.5$ MeV for odd triplet symmetric

	Exchange	Symmetric
N-P:	9.1×10^{-26} cm^2	5.9×10^{-26}
N-P:	8.7×10^{-26}	8.8×10^{-26}

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Burhop and Yadav, Fast Neutron-Proton Scattering and the law of Interaction between Nucleons (Univ. College, London)
(Nature 162 (1948), 738)
Hu and Massey, Proc. Roy. Soc. in press
Rarita and Schwinger model:

$$^1V_{even} = -\frac{\hbar^2}{M r_0^2} g V\left(\frac{r}{r_0}\right)$$

$$^3V_{even} = -\frac{\hbar^2}{M r_0^2} a(1+\gamma S_{12}) V\left(\frac{r}{r_0}\right)$$

(1) Spherical well $V\left(\frac{r}{r_0}\right) = \begin{cases} 1 & \text{for } r < r_0 \\ 0 & r > r_0 \end{cases}$

(2) Yukawa potential $V\left(\frac{r}{r_0}\right) = \frac{r_0}{r}\exp\left(-\frac{r}{r_0}\right)$

(3) Exponential pot. $V\left(\frac{r}{r_0}\right) = \exp\left(-\frac{r}{r_0}\right)$

(4) Gaussian pot. $V\left(\frac{r}{r_0}\right) = \exp\left(-\left(\frac{r}{r_0}\right)^2\right)$

Born-approximation
Alskin and Wu, Phys. Rev. 73 (1948), 973
(validity): B.A. gives in general too large a value of the total cross-section

Table 1 (c.s.) $r_0 = 2.8 \times 10^{-13}$

Interaction	I	II	III	I	II	III ($\sigma_{180°}/\sigma_{90°}$)
Exact Theory	14.10×10^{-26}	15.77	23.38	10.2	3.67	0.58
Born App.	19.05	23.32	23.32	11.3	3.56	0.53

but fairly good for $\sigma_{180°}/\sigma_{90°}$.

---

* Chew; Goldberger, P.R. 73 (1948), 1408.

Type I. symmetrical III. neutral
II. charged

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Table 2.

Shape / Neutron Energy (MeV)	Range $\times 10^{13}$ cm	C.S. $\times 10^{26}$ cm² I 100	I 250	II, III 100	II, III 250	Ratio $\sigma_{100}/\sigma_{200}$ I 100	I 250	II 100	II 250	III 100	III 250
Spherical well	3.49	19.0	7.5	22.1	8.5	75	1240	37	957	0.12	0.68
	2.18	25.5	15.3	48.9	30.8	2.4	3.1	0.44	0.62	0.98	0.01
Yukawa-pot.	2.18	14.1	6.0	16.6	6.6	90	391	269	648	0.34	0.31
	1.74	11.9	5.5	13.8	6.4	38	152	684	18.9	0.40	0.35
	1.18*	9.1	5.9	16.5	10.4	6.2	14.3	1.29	0.61	0.59	0.43
Exp. p.	1.09	15.3	6.2	17.6	7.0	995	562	430	237	0.28	0.20
Gaussian p.	2.18	14.9	6.5	17.7	7.5	352	266	12.3	9.9	0.25	0.13

(1) Total cross-section II, III is always larger than I,

(2) ratio is greatest for I and always <1 for III,

(3) As the range of interaction decreases, the total cross-section passes through a minimum and increases again, while ratio continually diminishes.

Increase of total cross-section at very short ranges is associated with the increasing importance of the tensor force. In the extreme case of 2.18×10^{-13} cm spherical well, the non-central interaction is so large that the central interaction has to be made repulsive to give the correct deuteron binding energy.

(4) Decrease of total C.S. with increase

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

of neutron energy is more rapid with the longer range interactions.

(5) Spherical well gives a larger c.-s. and smaller ratio.

~~(6)~~

Berkeley measurements of total c.s. 8.3 $\times 10^{-26}$ (± 0.4) cm^2 for 90 ± 13 MeV. neutrons.* Only one approaching this is Yukawa potential 1.18×10^{-13}, Type I. ($9.1 \times 10^{-26} cm^2$) (Chew and Goldberger), the triplet interaction being almost wholly noncentral.

$\sigma_{180°}/\sigma_{90°} \sim 3$ ** rule cut III. Yukawa I. 1.18×10^{-13} cm gives 6.2. In this case sharp increase in differential cross-section near 180°, so that it is expected that the measurement gives a too small ratio $\sigma_{180°}/\sigma_{90°}$.

---

* Cook, McMillan, Peterson and Sewell, P.R. 72 (1947), 1264.

** Hadley, Leith, York, Kelly and Wiegand, P.R. 73 (1948), 541

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Fürth, Masses of Mesons
(Nature 162 (1948), 772, Nov. 13)

$$\frac{m_p}{m_\pi} = 2\pi \qquad m_\pi = 293$$

$$\frac{m_\pi}{m_\mu} = 1 + \frac{1}{\pi} = 1.32$$

Rochester and Butler (Proc. Phys. Soc. 61 (1948), 307) (Manchester)
The Penetrating Particles in Cosmic-Ray Showers.
I. Heavily-ionizing Particles

μ-meson creation　4 MeV よりずつと大きな μ-meson が ~~exp~~ nuclear explosion の際に発生するらしい？

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Die Mesonenausbeute beim Beschuß von leichten Kernen mit α-Teilchen

Von H. Koppe
Max-Planck-Institut für Physik, Göttingen

(Z.S.f. Naturforschung, Band 3a, April 1948. 251)

Anregungsenergie pro Nucleon:

$$U = \frac{M_2}{M^2} E \qquad M = M_1 + M_2$$

M_1: Partikelmass　　E: Partikelenergie

M_2: Kernmass

Temperatur ~~der~~ des Zwischenkerns (Bagge)

$$T_0 = 3.8\sqrt{U}. \qquad U \text{ in MeV.}$$

Elektronenpaargas (Houtermans, Jensen)

Energiedichte

$$u = 2 \cdot \frac{7}{8} \frac{\pi^2}{16 c^3 \hbar^3} T^4$$

Strahlungsverlust für Oberfläche O.

$$q = \frac{11\pi^2}{240} \frac{1}{c^2\hbar^3} T^4 O \qquad (4)$$

(Stefan-Boltzmann Gesetz unter Hinzufügung eines Faktors $1 + \frac{7}{4} = \frac{11}{4}$)

Temperaturverlauf

$$\frac{\partial}{\partial t} M \frac{T^2}{3.8^2} = -\frac{11\pi^2}{240} \frac{O T^4}{c^2\hbar^3}$$

$$\frac{1}{T} = \sqrt{\frac{1}{T_0^2} + 2\beta t}$$

$$\beta = \frac{3.8^2}{480} \cdot \frac{11\pi^2}{c^2\hbar^3} \cdot \frac{O}{M}.$$

Wahrscheinlichkeit des Vorhandenseins des Mesons

$$n(\varepsilon) = \frac{(2\mu)^{3/2}}{\pi^2\hbar^3} \varepsilon^{1/2} \{ e^{(\mu c^2 + \varepsilon)/T} + 1 \}^{-1}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Anzahl der in der Zeiteinheit herausfliegenden Mesonen:

$$\nu = \frac{\Omega_n T^2}{\pi^2 \hbar^3} e^{-\mu c^2/T}$$

Die Ausbeute an Mesonen:

$$\eta = \int_0^\infty \nu(T)\, dt = \frac{\Omega_n}{\pi^2 \hbar^3} \int_0^\infty T^2 e^{-\mu c^2 \sqrt{1/T_0^2 + 2/3\tau}}\, dt$$

$$\cong 0.031\, T_0 M e^{-\mu_1 c^2/T_0}$$

$$\cong 1.7 \times 10^{-4}$$

Critic: It seems that the above result is entirely independent of strength of coupling of mesons with nucleons, ~~It~~ which is certainly contradict with the fact that much more π-mesons are created than μ-mesons. Also it is independent of the spin of the meson, which again ~~is a~~ seems to be an inadmissible simplification over of whole situation.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

Nov Dec. 3, 1948

Uhlenbeck, Beta Decay

Successive γ-~~decay~~ emission
Hamilton γ-γ
Falkoff { gen. theory
β-γ
Ling { γ-e⁻

Angular distribution fn
L=1 $F_0^1(\theta) = (1+\lambda) - 2\lambda \cos^2\theta$

$F_{\pm1}^1(\theta) = 1 + \lambda \cos^2\theta$

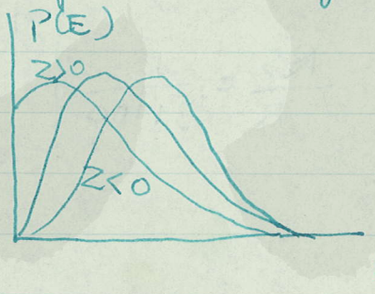
λ=1 γ-dipole
λ=-1 scalar
λ=0 P, AV · β trans. allowed

1°: $P \to N + e^+ + \nu$

$N \to P + e^- + \nu$

2°: K-capture $P + e^- \to N + \nu$
(positron capture)

3°:



4° Connection between lifetime and ΔS.

S, PV } Fermi sel. ΔJ = 0
T, AV } Gamow-Teller ΔJ = 0 ± 1.
~~yes~~ no

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Difficulties

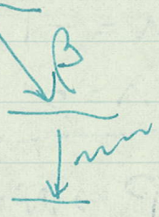
a) Be^{10} → forbidden → B^{10} ← allowed ← C^{10}

b) low energy diff.

c) . shape of forb. spectrum
RaE C^{14} : not Fermi

β-γ-decay

A] No angular correlation for allowed β-transition



B] $R + S\cos^2\theta$ for 1st forb.
S/R (We, Z, ; type of int. ; J)

C] $R + S\cos^2\theta + T\cos^4\theta$

$J-1 \to J \to J+1$: $\frac{RS}{R} = \frac{1}{3+10\phi}$

$\phi = -\frac{1}{3}$ S
$\frac{5}{3}$ PV
1 AV
T

Wheeler: Snell, neutron - β-decay
life-time 13 min
< 15 min, > 10 min.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar

Dec. 6, 1948

Bargmann : Scattering Potential and Phase shift

(1) $u = e^{\frac{\Delta}{2}r} - \gamma e^{-\frac{\Delta}{2}r}$ $[0<\gamma<1]$

(2) $u = (r-a)^2 + \beta^2$ $\beta > a \geq 0$

Past $\int_0^\infty |V| dr < \infty$

$S(k) = \frac{f(k)}{f(-k)} = e^{2i\eta}$

bound state $f(-ix) = 0$ $E = -x^2$.

Mor, Redundant Zero in S-matrix scheme

Moller, $\frac{dS}{dx} < 0$ ×

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

W.H. Ramsey - On Schwinger's Theory of Nuclear Forces

(Proc. Phys. Soc. 61 (1948), 297)

Ramsey (Proc. Roy. Soc. ~~1944~~ 191 (1948), 125; in press) has shown that the proton-proton scattering set a lower limit of 275 me in Møller-Rosenfeld theory.

Schwinger theory (P.R. 61 (1942), 387)

$$(g_2/\lambda)^2 = (f_2/\varkappa)^2$$

g_2 : vector ~~vector~~ tensor coupling

f_2 : pseudoscalar - pseudovector coupling

$$\alpha = f_2^2/(f_2^2 + 2g_2^2 - g_1^2)$$

Case 1. : $0 < \alpha < 1$ or $2g_2^2 > g_1^2$

(Gauch-Hu (1944) : $g_1^2 = 0$, $\hbar\varkappa/c = 1177$ me)

This can not account for the p'-p scattering.

Case 2. $\alpha > 1$ $\quad f_2^2 > g_1^2 - 2g_2^2 > 0$

$$g_1^2 > 0.5\,(\lambda/\varkappa)^2 \hbar c \;\}$$

$$\lambda > \varkappa$$

these values cannot give correct binding energy of the deuteron

Thus in any case, Schwinger theory cannot account for correctly the quadrupole moment of the deuteron, and it offers no hope of a detailed agreement with p-p scattering.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Tiomno[2] and Wheeler: Charge-Exchange Reaction of the μ-Meson with the Nucleus (Rev. Mod. Phys. (1949)

$\mu^- + P \rightarrow \mu_0 + N$

Zero-meson brought to rest in matter very rarely results in nuclear disruption as shown by Bristol groups, by Hattes and Gardner and by Chang[4]. Chang's observations emphasize the two conclusions (i) that the whole rest energy of the meson is certainly not imparted to the nucleus, (ii) and considerable part of it should go off in the form of a neutral particle. The process should thus be called a charge-transfer reaction rather than a capture reaction. Some at least of the rest energy of the μ-meson is given to the nucleus, as roughly one or two neutrons come off on the average per meson stopped in lead according to Sands and collaborators.[8] This cannot be understood if nucleons alone left at the end of the meson reaction, because the energy left over for the nucleons would release protons as well as neutrons, contrary to the findings of Chang.

(97 (1948); Groetzinger and McClure, 341)

(8) Sands and Stoner, Comfort and Crouch, Phys. Rev. 74,
(2) Brenner Aires Convention - U.S. State Department Fellow from the University of São Paulo
(4) Chang, Rev. Mod. Phys. (1949)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Consequently it is reasonable to believe that some neutral, non-nucleonic entity carries away a large part of the rest energy of the μ-meson. Since the experiments of Sard and ~~Att~~ Althaus and Piccioni [9] rule out a photonic character for this radiation, we are led to describe the entity as a neutral meson - without thereby ruling out the probability that it is a ~~neutrino~~ photon [10].
Thus the process is pictured as

$$\mu^- + P \rightarrow N + \mu_0 \quad (1)$$

to be compared with the π-μ-decay:

$$\pi \rightarrow \mu + \mu_0 \quad (2)$$

and π-meson production:

$$N \rightarrow P + \pi \quad (3)$$

The interaction Hamiltonian takes the form

$$H = g O_{HL} (\tau_H \tau_L + \tau_H^* \tau_L^*) \delta(x_H - x_L) \quad (4)$$

in analogy with the cases

$$\mu^- \rightarrow \mu^0 + e^- + \nu$$
$$N \rightarrow P + e^- + \nu .$$

The coupling constant g has the dimension gm/(cm)³. The decay rate of reaction between a meson and a proton is

$$A_H (sec^{-1}) = 190 \, (g/10^{-49} \, gm \, cm^3)^2$$

(9) Sard and Althaus, P.R. 73 (1948), 1251; Piccioni, P.R.
(10) Pontecorvo, P.R. 72 (1947), 246 ✓

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

for $\mu = 210\,m$, $\mu_0 = 0$

$$A = \left(\frac{2\pi}{\hbar}\right)\frac{1}{\Delta E}\sum_F |H_{FI}|^2 \left(\frac{L}{h}\right)^3 4\pi p_0^2 \Delta p_0$$

$$= (2\pi^2)^{-1}\left(\frac{g^2}{\hbar mc^2}\right)\cdot\left(\frac{\mu c}{\hbar}\right)^6\left(\frac{e^2}{\hbar c}\right)^3\left(\frac{\mu}{m}\right)^3$$

$$\times\left(\frac{p_0}{\mu c}\right)^2\left\{\frac{p_0}{\mu c}+\left[\left(\frac{\mu_0 c}{p_0}\right)^2+1\right]^{-\frac{1}{2}}\right\}^{-1} \qquad (1)$$

for a meson bound in the K-orbit in the field of a proton.

On the other hand, the Z^4 law of meson reaction probability, together with the observation that A has the

* Partition of Energy-Momentum for the case of free-nucleon–μ-meson collision

$$\mu c^2 = \frac{1}{2}MV^2 + \mu_0 c^2\sqrt{1-\frac{v^2}{c^2}}$$

$$MV = \frac{\mu_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow v \cong c, \quad MV \cong \mu c$$

$$\mu c^2 = \frac{\mu_0}{2M}\frac{\mu_0 v^2}{1-\frac{v^2}{c^2}} + \frac{\mu_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{\mu}{\mu_0} = \frac{\mu_0}{2M}\cdot\frac{\beta^2}{1-\beta^2} + \frac{1}{\sqrt{1-\beta^2}}$$

$$\frac{\mu_0}{\mu} = x, \quad \frac{\mu}{2M} = \delta \quad \frac{1}{\sqrt{1-\beta^2}} = x$$

$$\frac{1}{x} = x\delta(x^2-1) + x$$

$$x\delta x^2 + x - \frac{1}{x} - x\delta = 0$$

$$x = \frac{-1 \pm \sqrt{1+4\delta+4(x\delta)^2}}{2x\delta}$$

$$\cong \frac{1}{2x\delta}\left(2\delta + \frac{2}{1}\delta^2 + 2(x\delta)^2\right)$$

$$\frac{\mu_0 c^2}{\sqrt{1-\beta^2}} \cong \mu c^2 - \mu c^2\cdot\delta = \mu c^2 - 5.6\,MeV \qquad \frac{1}{2}MV^2 \cong 5.6\,MeV$$

$$\frac{(\mu c^2)^2}{2Mc^2} = \frac{1}{2}MV^2$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

value $1/2.15\times10^{-6}$ for a nucleus of charge number $Z\sim10^{(11)}$, allows one to obtain by extrapolation to hydrogen the rough estimate of charge-exchange reaction rate

$$A_H \sim \frac{1}{(10)^4}/2.15\times10^{-6} = 47/\text{sec}, \quad (6)$$

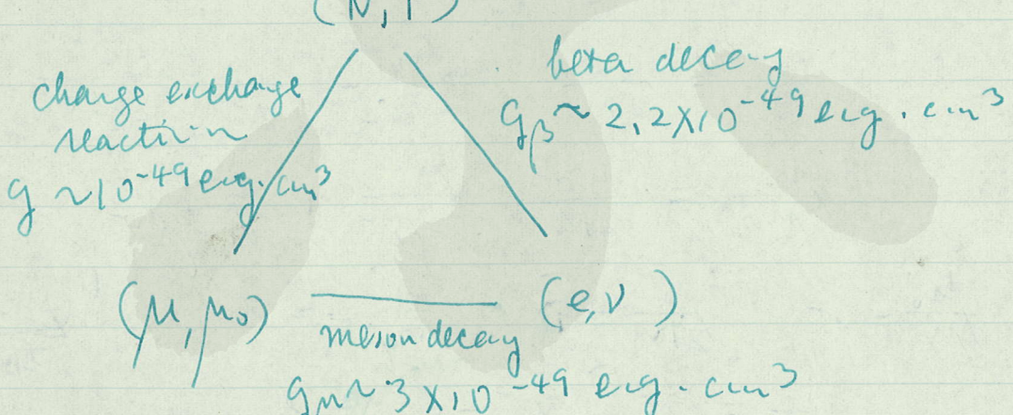
which gives

$$g \sim 0.5\times10^{-49}\ \text{gm}\cdot\text{cm}^3$$
$$\sim 1\ eV\cdot\left(\frac{e^2}{mc^2}\right)^3$$

More accurate value is obtained by considering the effects of motion and binding of the protons in the nucleus:

$$g \sim 0.9\times10^{-49}\ \text{gm}\cdot\text{cm}^3$$

Thus we have a remarkable result:



(11) J. Sigurgeirsson and Yamakawa, Rev. Mod. Phys. (1949); Wheeler, ibid. (1949)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Of course, the possibility that, for instance, the coupling of μ-meson field and the nucleon field could be an indirect consequence of the direct coupling between π-mesons and nucleons and between π-mesons and μ-mesons (16)

Practically independent of the magnitude of the coupling constant is the nuclear excitation following the meson charge exchange reaction. The free particle model predicts the highest average energy and all three models (free, dipole, and Hartree) predicts expectations very much smaller than the rest mass of the lost μ-meson, of the order of 10MeV.* What will happen to a nucleus with 10MeV of excitation? If it is a heavy nucleus, where the Coulomb potential barrier against protons is of the order of 10MeV to 15MeV, so that it is very much less likely for a proton to emerge than for a neutron. For low atomic number, the binding energies are larger, so that there will be many cases where

(16) Marty and Prentki, J. de Phys. No. 4 1948; Lopes, P.R. in press.

* Generally smaller than 20 MeV.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

the nucleus gets too little excitation to emit any kind of nucleon. Furthermore there is a slight preference for neutron emission over proton emission, due to the fact that the meson collide originally with a proton, which change into a neutron, ~~with the neutron~~, so that the latter may emitted with consider part of recoil energy.

The calculated low excitation of the nucleus in the p-charge exchange reaction, and the observed high excitation in the reaction of π-meson with the nucleus (≈100 MeV), argue that the π-capture process is not a charge exchange reaction. Thus if the elementary transition were $\pi^- + p \Rightarrow N + \pi^0$ (say a neutrino), the free particle model permit a transfer of energy to the nucleus at most about 30 MeV. This argument excludes a value of ½ for the spin of the π-meson.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Energy Spectrum of Electrons from meson decay

By Tiomno and Wheeler (R. M. P. (1949) (3))

$$\mu \to \mu_0 + \nu + e$$

or $$\mu \to 2\nu + e$$

Emission of even a single photon is inconsistent with experiment (4). We assume μ, μ_0 have spin 1/2, although the possibility that they both have spin 0 and couples with the nucleon always in pair. Klein (1) suggested that three particle decay hypothesis is closely related to nuclear β-decay ($n \to p$, $H^3 \to He^3 \ldots$), so that μ, μ_0 have naturally spin 1/2.

The choice of form of coupling makes a difference in the shape of the spectrum of decay electrons, unlike the case of β-decay. It is due to relativistic effect.

$$H_S = g_S (\beta_H \beta_L)(\tau_H \tau_L + \tau_H^* \tau_L^*)\,\delta(\vec{x}_H - \vec{x}_L)$$

$$H_V = g_V (1 - \vec{\alpha}_H \vec{\alpha}_L)(\tau_H \tau_L + \tau_H^* \tau_L^*)\,\delta(\vec{x}_H - \vec{x}_L)$$

$$H_T = g_T \left(\beta_H \vec{\sigma}_H \cdot \beta_L \vec{\sigma}_L + \beta_H \vec{\alpha}_H \cdot \beta \vec{\alpha}_L\right)(\tau \ldots)\,\delta(\ldots)$$

$$H_{PV} = g_{PV} (\vec{\sigma}_H \vec{\sigma}_L - \gamma_H^5 \gamma_L^5) \quad (\ldots)(\ldots)$$

$$H_{PS} = g_{PS} (\beta_H \gamma_H^5 \beta_L \gamma_L^5) \quad (\ldots)(\ldots)$$

---

(3). Møller in Marty and Prentki, J. de Phys. 4 (1948), 147.
Marshak, P. R. 73 (1948), 1226.

(4) Hinches, Pontecorvo, ibid. 73 (1948), 257
Sens and Athens, 125 1; Piccioni, in press.

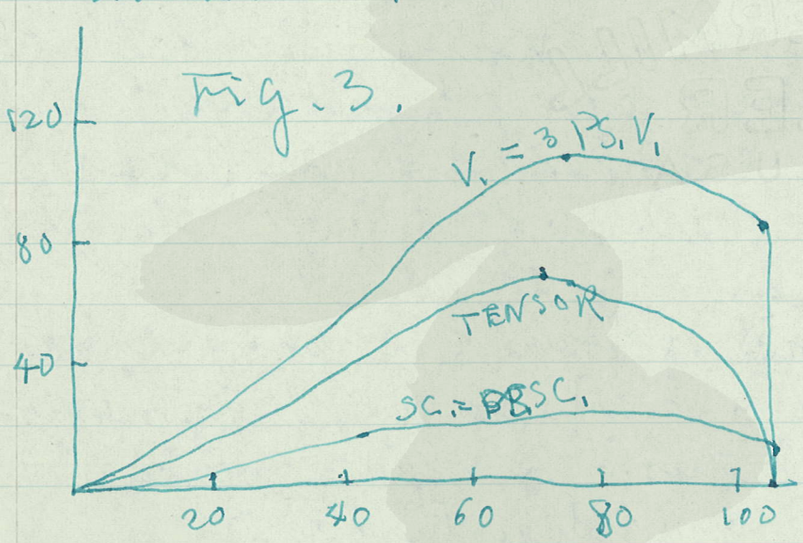
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$\begin{pmatrix}\mu^+\\ \bar{e}\end{pmatrix} \to \begin{pmatrix}\nu\\ \nu\end{pmatrix}$ $\mu = 210\,m$

Theory	g_μ/g_β
Scalar	2.9
Vector	1.0
Tensor	1.4
PS. Vector	1.7
PS. SCALAR	.025

for neutron half life 30 min.

Fig. 3.



○ Transition prob. for μ_0 ~~with no~~ not a neutrino

$$dW = \left(\frac{2\pi}{\hbar}\right) \frac{4\pi p_e^2\, dp_e}{h^6} \frac{1}{dE} \int I\, dp_{\nu_x}\, dp_{\nu_y}\, dp_{\nu_z}$$

with the condition

$$\mu c^2 \le E_\nu + E_e + E_{\mu_0} \le \mu c^2 + dE$$

○ Transition prob. for μ_0 a neutrino decay mechanism is not unique. $\begin{matrix}\mu^+ \to \nu\\ \bar{e} \to \nu\end{matrix}$

i) Two neutrino creation (Antisym. charge ex.)

ii) neutrino and antineutrino creation

iii) $\mu^- \to \nu$ $\quad \bar{\nu} \to e$ (Simple charge exchange)

iii) $\mu^- \to e$ $\quad \bar{\nu} \to \nu$ (Charge retention)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

○ Energy spectrum for $\mu_0 = \nu$

i) → Fig. 3,
ii) → Fig. 4,
iii) → Fig. 5.

○ Energy spectrum for $\mu_0 \neq \nu$

$\mu_0 = 0, 20, 40, 60, 80$ → Fig. 1,

○ Determination of coupling constant

half life-time of the free neutron (14)(15)

$$\frac{0.693}{(\text{half life})} = \frac{1}{\tau_\beta} = \left(\frac{32\pi^3 g^2}{h m c^2}\right)\left(\frac{mc}{h}\right)^6 J$$

J : dimensionless quantity" different between (sc. vect. psv. tensor)
and (spsc), where the rate of decay depends upon the speed of recoil of the proton,
for (half life) = 30 min, we have

$g_\beta = 2.24 \times 10^{-49}$ erg. cm^3 (except ps.sc.)

(for 13 min (2.59 for ps.sc.)

$g_\beta = 4 \times 10^{-49}$ erg. cm^3
$(g_\beta)_{PS} \cong 5 \times 10^{-49}$ erg. cm^3

(15) Wigner, Phys. Rev. 56 (1939), 519
(14) Snell, Science (1948), 169, His new experimental value is about 13 min, Old value is about 30 min

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

○ Comparison with Experiment:

i) Decrease of μ_0 raises both the upper limit and the maximum, but not a large influence on coupling constant ~~bet~~ for μ_0 between 0 and 20 m.

ii) Charge retention process can be objected, because the process

$$\mu^- \to e^+ + 2e^-$$

can equal claim to occur, in contradiction with experiment.

iii) $\mu \to e + 2\nu$ ~~fa~~ is in favore compared with $\mu_0 \neq \nu$, because the coupling constants ~~β~~ g_μ, g_β are near to each other. Especially vector or tensor antisym. charge exchange is promising, because ~~it~~ they are the cases, which can be adopted in the β-decay theory[17] (or vector charge retention theory)

(17) Konopinski, R.M.P. 15
Hamilton 71, 545
Kofoed-Hansen, 73, 675.

Oct. 22, 1948

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

On the Range of the Electrons in Meson Decay

~~J. Schwinger~~
J. Steinberger

13 cloud chamber pictures, which permit a determination of the energy of the electron produced by meson decay.

15 ± 3 (5)	42 ± 8 (4)
24 (3)	42 ± 12 (4)
20 ~ 50 (4)	43 ± 9 (4)
20 ~ 50 (4)	48 ± 10 (4)
40 ± 12 (4)	53 ± 15 (4)
40 ± 8 (4)	70 ± 35 (2)
	70 (with large error) (4)

(3) Anderson, Adams et al, P.R. 72 (1947), 724
(4) Thompson, P.R. 74 (1948), 490
(5) Fowler, Cool, Street, 74, 101
(2) Wil~~son~~iams, Robert, ~~145~~ Nature (1940), 102

(6) Coversci, Piccioni, Piccioni, 71 (1947), 209 : iron での decay electron の absorption を測定し, average energy を 45 ~~を~~ Mev ± 20% と推定

Steinberger は polystyrene, (hydrocarbon) での absorption を Chicago (600 ft), Mt. Evans (14200 ft) でやり 8000 counts を得た。その結果は (8) Hinks, Pontecorvo, P.R. 74, 697 と一致している。それより statistics はずっとよい。

結果は次の図の如く, E = 50 Mev or 25 Mev の unique value としては説明が出来

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

ない、従って

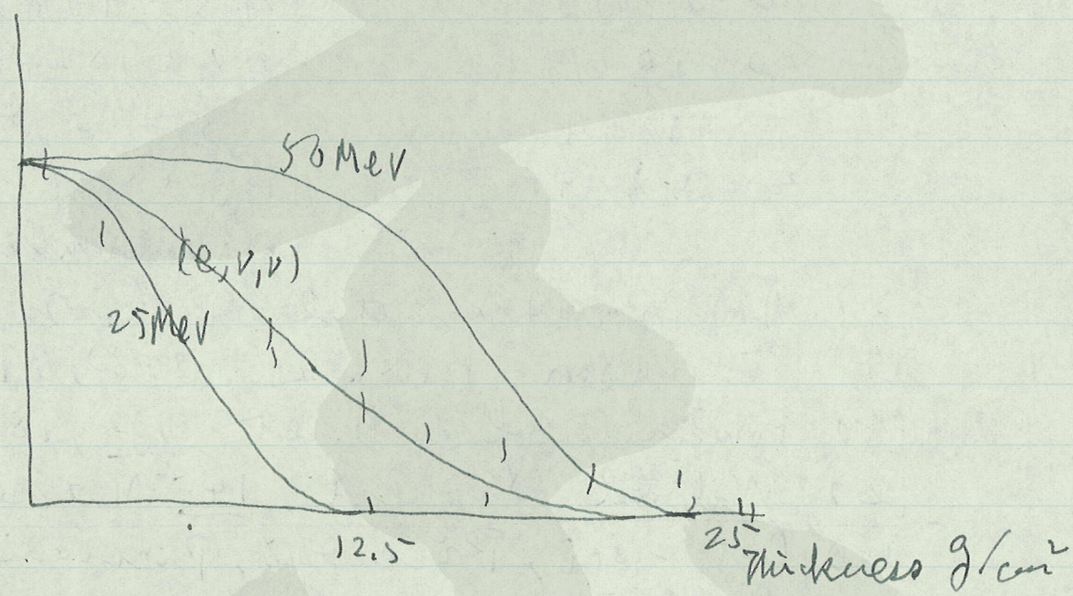
i) electron が幾つかの line spectrum から成る。
即ち 幾種類 ものmass の違った neutral
partner を仮定するか

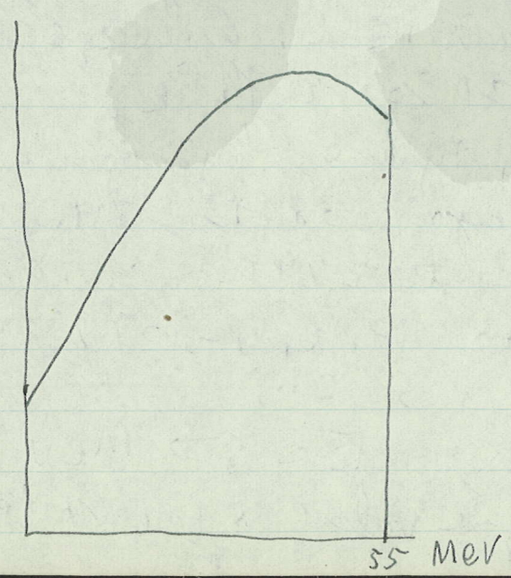
ii) three particle decay と考えるか

である。後者を採って 特に

$$\mu^{\pm} \to e^{\pm} + \nu' + \nu$$

を仮定すると 図の如く実際とよくあう





各実験とよくあうような energy distrib を推定すると 左の図のようになる

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

○ Dec. 9, 1948

Cosmic Ray Colloquium, M. I. T.

Rossi. Bob. T.

μ-meson decay cloud chamber
photographs 12 ~~pho~~ tracks all
between 35 ~ 53 MeV/c.
only one at 61
and one 18
average 42 ± 5 MeV.
Prob. that $\mu \to e + 2\nu$: $< 1/30$.
electrons lie all close to 42 MeV is

2-particle — 3-particle
↓
two stage decay

○ Chang, A Cloud Chamber Study of Meson Absorption by Thin Pb, Fe and Al Foils (Rev. Mod. Phys. 1949 Jan.); Washington meeting of A. P. S. May 1, 1948)

Meson stopped and giving rise to ~~no~~ neither heavy charged particle nor decay electron, ~~identi~~ supposed to be negative meson: Number of them: Pb-27, Fe-14; Al-12; No protons are emitted when a negative μ-meson interacts with a Pb nucleus. (probably with Fe, Al); high energy ~~electrons~~ photons were not emitted:

$$\mu^- + {}^A_Z N \to {}^A_{Z-1} N^* + \mu_0$$

Photoelectrons (1~5 MeV) 6 for Pb, 1 for Fe, 0 for Al.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar　Dec. 14, 1948

Q Watson, Hyperfine Structure

○ One Body Problem: of Hydrogen

$H = H_e + V + e\alpha_1 \underline{A}(x_1)$

$\underline{A} = -\mu_p \frac{\underline{\sigma_2} \times \underline{r}}{r^3}$

$H'' = -e\mu_p \frac{\alpha_1 \underline{\sigma_2} \times \underline{r}}{r^3} \to e\mu_p \frac{1}{2\pi^2} \frac{i\underline{\alpha_1} \underline{\sigma_2} \wedge \underline{k}}{k^2}$

$n\left(\frac{m}{M}\right)\alpha^4$

Field Theory:

$H_p' = \mu_p \underline{\sigma_2}\, \underline{H}(x_2)$　　$H_e' = e\alpha_1 A(x_1)$

$\underline{H} = \nabla \times \underline{A}$

$W = \left\{ \frac{H_p' H_e'}{\varepsilon_{p'} - \varepsilon_p} + \frac{H_e' H_p'}{-k} \right\}_{\text{sym.}}$

○ Two body problem: *

$H = \underbrace{H_p + H_e}_{H_0} + H_p' + V + H' + H_\gamma$

$\psi = \begin{pmatrix} \psi_{++} \\ \psi_{-+} \\ \psi_{+-} \\ \psi_{--} \end{pmatrix}$

Field Theory:

$e^{iS} H e^{-iS}$　　$S = S_e + S_\mu$　　$H' + [iS, H_0] = 0$

correction $\alpha^2 \{\ln \tfrac{1}{2}\}^2 \sim 20\alpha^2$!!!

---

* Breit, Phys. Rev. 74 (1948)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$[(\alpha_1 - \alpha_2)\hat{p} + \rho_1 m + (\beta_2 M + V)]\psi = \varepsilon\psi$$

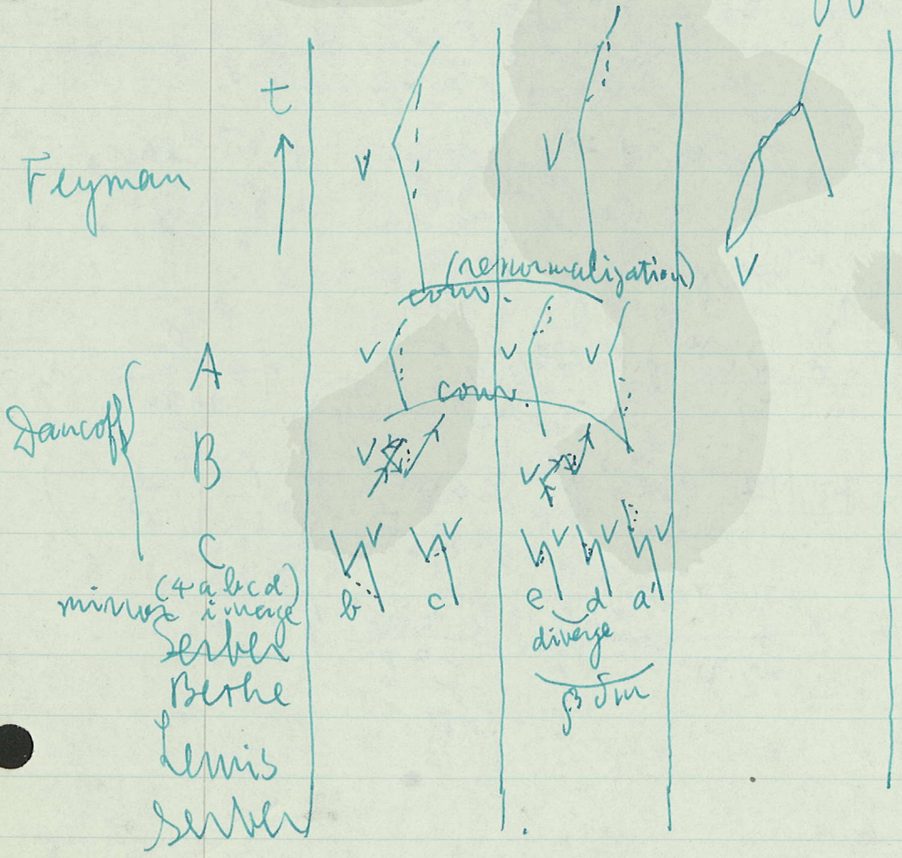
$$\therefore \alpha_1 p_1 + \alpha_2 p_2 = \frac{1}{2}(\alpha_1 + \alpha_2)(p_1 + p_2) + \frac{1}{2}(\alpha_1 - \alpha_2)(p_1 - p_2)$$

$$(p_1 + p_2)\psi = K\psi$$

$K = 0$!

$$\psi = \begin{pmatrix} \psi_+ \ (8\text{ comp}) \\ \psi_- \ (8\text{-comp}) \end{pmatrix}$$

○ Morette, Feynman Diagram and Dancoff ?



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Morette: Meson - Nucleon Interaction

Heisenberg Van Hove: $H_0 + H_1 + H_\delta$

$\downarrow$

$H_0 + H_\delta + H_2$

1) pseudoscalar pseudoscalar pseudovect (Dyson)

$$I_\mu \frac{\partial \rho}{\partial x_\mu} = \frac{\partial (\Sigma_n \phi)}{\partial x_n} - \phi \frac{\partial \Sigma_n}{\partial x_\mu}$$

e^{iG} $G = \int \Sigma \phi \, dc$

$$H_2 = W_1 + W_2$$

$$W_1 = \frac{\rho_2^{(1)} \rho_2^{(2)}}{\varepsilon^2 - (E_m - E_n)^2} \qquad \nu^{-1}$$

$$W_2 = \frac{E_m - E_n}{\cdots} \qquad \nu^{-2}$$

Theoretical Seminar, Dec. 17, Friday.

Kirong, Hydrodynamical Theory of Stopping Power

classical free

$$K = -\frac{4\pi N z (Qe)^2}{m v^2} \ln \frac{\varepsilon^{3/2} v^2}{4\pi^{1/2} \rho \eta}$$

quantum bound

$$K_{Bethe} = -\frac{4\pi N z (Qe)^2}{m v^2} \ln \frac{2 m v^2}{E}$$

$$K_{Kramers} = -\frac{4\pi N z (Qe)^2}{m v^2} \ln \frac{a^3 m^2 v^2}{e^2 h}$$

$$\eta = \frac{h}{16 \pi a^3} \qquad \left(\sqrt{\frac{e^2 m}{a^5}} \right)$$

$\sim 10^{15}$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Relativistic Terms in the Interaction between Nucleons in Pseudoscalar and Vector Meson Theory

L. Van Hove
Institute for Theoretical Physics, Copenhagen, Denmark

1. Møller-Rosenfeld neglected all velocity dependent variables, whereas Kemmer neglected only recoil energies of nucleons. Our purpose is the investigation of the second order interaction, all terms being taken into account. Our calculation revealed that the singularity proportional to r^{-3} was reduced for pseudoscalar, but not for vector, so that compensation device by Møller-Rosenfeld did not work any more.

2. Pseudoscalar meson fields

$$L = L_{\text{free nucl.}} + L_{\text{free mes}} + L_1 \qquad (1)$$

$$L_{\text{free nucl}} = \frac{-1}{2i} \int [\psi^* (\vec{\alpha} \operatorname{grad} \psi) - (\operatorname{grad} \psi^*, \vec{\alpha}) \psi + \psi^* \frac{\partial \psi}{\partial t} - \frac{\partial \psi^*}{\partial t} \psi + 2iM \psi^* \beta \psi] \, d_3 \vec{x} \qquad (2)$$

$$L_{\text{free mes.}} = -\frac{1}{2} \int [(\operatorname{grad} U \cdot \operatorname{grad} U) - \left(\frac{\partial U}{\partial t}\right)^2 + \varkappa^2 U^2] \, d_3 \vec{x} \qquad (3)$$

$$L_1 = -\int \psi^* \{ f_1 \rho_2 U + \frac{f_2}{\varkappa} ((\vec{\sigma} \operatorname{grad} U) + \rho_1 \frac{\partial U}{\partial t}) \} \times \psi \, d_3 \vec{x} \qquad (4)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$H = H_{\text{free nucl}} + H_{\text{free mes}} + H_1 + H_{\text{int}} \qquad (5)$$

$$H_{\text{free nucl}} = \frac{1}{2i}\int\left[\psi^*(\vec{\alpha}\,\text{grad}\,\psi) - (\text{grad}\,\psi^*,\vec{\alpha})\psi \right.$$

$$\left. + 2iM\psi^*\beta\psi\right] d_3\vec{x} \qquad (6)$$

$$H_{\text{free mes.}} = \frac{1}{2}\int\left[(\text{grad}\,U\cdot\text{grad}\,U) + \Pi^2 + \pi^2 U^2\right] d_3\vec{x} \qquad (7)$$

$$H_1 = \int \psi^*\left\{f_1\rho_2 U + \frac{f_2}{\pi}\left[(\vec{\sigma}\,\text{grad}\,U) + \rho_1\Pi\right]\right\}\psi\, d_3\vec{x} \qquad (8)$$

$$H_{\text{int}} = \frac{1}{2}\left(\frac{f_2}{\pi}\right)^2\int(\psi^*\rho_1\psi)^2 d_3\vec{x} \qquad (9)$$

$$\Pi = \frac{\delta L}{\delta\left(\frac{\partial U}{\partial t}\right)}$$

Dirac eq. for free nucleon

$$\left[(\vec{\alpha}\,\vec{p}_m + \rho M)\chi_m = E_m\chi_m\right] \qquad (11)$$

$$\psi_m = \chi_m \exp(i\vec{p}_m\vec{x}) \qquad (10)$$

$$(H_1)_{n+k\,E\,m} = (H_1)^*_{m\,E\,n+k}$$

$$= \frac{1}{\sqrt{2\varepsilon_k}}\chi_n^*\left\{f_1\rho_2 - i\frac{f_2}{\pi}(\vec{\sigma}\vec{k} - \rho_1\varepsilon_k)\right\}\chi_m$$

with $\varepsilon_k = \sqrt{\pi^2 + k^2}$, $\quad \vec{k} = \vec{p}_m - \vec{p}_n$

$$i\chi_n^*(\vec{\sigma}\vec{k} - \rho_1\varepsilon_k)\chi_m = i(E_m - E_n - \varepsilon_k)\chi_n^*\rho_1\chi_m$$

$$-2M\chi_n^*\rho_2\chi_m$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$(H_1)_{n+k\leftarrow m} = (H_1)^*_{m\leftarrow n+k}$$

$$= \frac{1}{\sqrt{2\varepsilon_R}} \chi_n^* \left[f_3 \rho_2 - i \frac{f_2}{\pi} (E_m - E_n - \varepsilon_k) \rho_1 \right] \chi_m \qquad (12)$$

with $f_3 = f_1 + (2M/\pi) f_2$

3. Second order interaction in pseudoscalar theory.

In Hamiltonian (5), H_1 is carried over to higher order by canonical transf.

$$\exp iS$$

with

$$S_{n'\leftarrow n} = i (H_1)_{n'\leftarrow n} / E_n - E_{n'} \qquad (13)$$

in momentum representation, where E_n, $E_{n'}$ are the eigenvalue of the (diagonal) matrix $H_{free\ nucl} + H_{free\ mes.}$

$$\exp(iS) H \exp(-iS) = H_{free\ nucl} + H_{free\ mes} + H_2 + H_{dir,1} + \text{higher order} \qquad (14)$$

$$(H_2)_{n'\leftarrow n} = \frac{1}{2} \sum_{n''} \left(\frac{1}{E_n - E_{n''}} + \frac{1}{E_{n'} - E_{n''}} \right) (H_1)_{n'\leftarrow n''} (H_1)_{n''\leftarrow n} \qquad (15)$$

Consider two-nucleon system with centre of gravity at rest:

$$\Phi = \frac{1}{\sqrt{2}} \sum_{m_1 m_2} \bar{\Phi}_{m_1 m_2} \overset{(1)}{\psi}_{m_1} \overset{(2)}{\psi}_{m_2} \rightarrow \Psi = \frac{1}{\sqrt{2}} \sum_{n_1 n_2} \Psi_{n_1 n_2} \overset{(1)}{\psi}_{n_1} \overset{(2)}{\psi}_{n_2} \qquad (16)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Non-vanishing terms satisfy

$$\Phi_{m_1 m_2} = -\Phi_{m_2 m_1} \qquad \Psi_{n_1 n_2} = -\Psi_{n_2 n_1}$$

$$\vec{p}_{m_1} + \vec{p}_{m_2} = \vec{p}_{n_1} + \vec{p}_{n_2} = E_{m_1} - E_{m_2} = E_{n_1} - E_{n_2} = 0$$

$$(H_2)_{\Psi \leftarrow \Phi} = -\left(\frac{f^2}{\pi}\right)^2 \sum \Psi^*_{n_1 n_2} \chi^{(1)*}_{n_1} \chi^{(2)*}_{n_2} \Phi_{m_1 m_2} + (W_1 + W_2)_{\Psi \leftarrow \Phi} \tag{18}$$

$$(W_1)_{\Psi \leftarrow \Phi} = -f_3^2 \sum \Psi^*_{n_1 n_2} \chi^{(1)*}_{n_1} \chi^{(2)*}_{n_2} \frac{\rho_2^{(1)} \rho_2^{(2)}}{\xi_k^2 - (E_{m_1} - E_{n_1})^2} \times \chi^{(1)}_{m_1} \chi^{(2)}_{m_2} \Phi_{m_1 m_2} \tag{19}$$

$$(W_2)_{\Psi \leftarrow \Phi} = i \frac{f_2}{\pi} f_3 \sum \Psi^*_{n_1 n_2} \chi^{(1)*}_{n_1} \chi^{(2)*}_{n_2} \frac{E_{m_1} - E_{n_1}}{\xi_k^2 - (E_{m_1} - E_{n_1})^2} (\rho_1^{(1)} \rho_2^{(2)} + \rho_1^{(2)} \rho_2^{(1)}) \chi^{(1)}_{m_1} \chi^{(2)}_{m_2} \Phi_{m_1 m_2} \tag{20}$$

with $\vec{k} = \vec{p}_{m_1} - \vec{p}_{n_1}$. The first term in (18) gives $\delta(\vec{r})$-type interaction and equal to $(-H_{dir})_{\Psi \leftarrow \Phi}$ so that complete second order interaction is

$$W = H_2 + H_{dir} = W_1 + W_2 \tag{21}$$

For large relative momentum p or small relative distance r, $\langle W_1 \rangle$ becomes proportional to p or r^{-1} and $\langle W_2 \rangle$ to p^2 or r^{-2}. For any real transition of two nucleon system, occurring with conservation of energy, the matrix element of W_2 vanishes. It has no diagonal ~~momentum~~ (can be) elements in momentum representation and carried over to fourth order by canonical transformation, the second order interaction vanishes for

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$f_3 = f_1 + \frac{2M}{\varkappa} f_2 = 0$$

It is true for arbitrary system of reference, where the centre of gravity is not at rest.

4. An alternative method.

Dyson's transformation[3] $\exp(iS')$ with

$$S'_{r' \leftarrow r} = i(H'_1)_{r' \leftarrow r} / (E_r - E_{r'}) \qquad (22)$$

$$H'_1 = \int \psi^* \{ -\frac{2M}{\varkappa} f_2 \rho_2 U + \frac{f_2}{\varkappa} (\vec{\sigma}, \operatorname{grad} U) + \rho_1 \Pi \} \psi \, d_3 \vec{x}$$

Due to (12):

$$(S')_{n+k \leftarrow \dots} = (S')^*_{m \leftarrow m+k} = \frac{1}{\sqrt{2 z_k}} \frac{f_2}{\varkappa} \chi^*_m \rho_1 \chi_m$$

$$\exp(iS') \cdot H \cdot \exp(-iS') = H_{\text{free nucl}} + H_{\text{free mes.}} + H''_1 + H'_2 + H_{\text{dir}} \qquad (23)$$

$$H''_1 = f_3 \int (\psi^* \rho_2 \psi) U \, d_3 \vec{x} \qquad f_3 = f_1 + \frac{2M}{\varkappa} f_2$$

$H'_2 + H_{\text{dir}}$ giving a vanishing contribution to second order interaction.

The procedure of §3. corresponds to $\exp(iS'')$ with $S'' = S - S'$, S, S' being defined by (13) and (22). Instead of (21), one gets

$$W' = W_1$$

---

(3) Dyson, P.R. 73 (1948), 929 ; he investigated the case $f_1 = 0$; He gave W_1 instead of $W_1 + W_2$.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

the transform ~~exp iS~~ U defined by

$$\exp(iS'')\exp(iS') = V\exp(iS)$$

removes W_2-term from W and carries it over to fourth order interaction.[5]

5. Vector meson field

In (1), $L_{free\,mes}^{v}$ and L_1 is replaced by

$$L_{free\,mes}^{v} = -\frac{1}{2}\int\{(\mathrm{curl}\,\vec{U})(\mathrm{curl}\,\vec{U}) - \left\{\left(\frac{\partial\vec{U}}{\partial t} - \mathrm{grad}\,U_0\right)\left\{\frac{\partial\vec{U}}{\partial t} - \mathrm{grad}\,U_0\right\} + \varkappa^2(\vec{U}\vec{U} - U_0^2)\right\} \times d_3\vec{x}$$

$$L_1^{v} = -\int\psi^*\{g_1(\vec{\alpha}\vec{U} + U_0) + \frac{g_2}{\varkappa}[(\beta\vec{\sigma}\,\mathrm{curl}\,\vec{U}) - (\vec{\gamma}\{\frac{\partial\vec{U}}{\partial t} - \mathrm{grad}\,U_0\})]\}\psi\, d_3\vec{x}$$

$$\vec{\gamma} = -i\beta\vec{\alpha} = i\vec{\alpha}\beta$$

and in (5)

$$H_{free\,mes}^{v} = \frac{1}{2}\int[(\mathrm{curl}\,\vec{U}\cdot\mathrm{curl}\,\vec{U}) + \varkappa^2(\vec{U}\cdot\vec{U}) + \frac{1}{\varkappa^2}(\mathrm{div}\,\vec{\Pi})^2 + (\vec{\Pi}\cdot\vec{\Pi})]\, d_3\vec{x}$$

$$H_1^{v} = \int\psi^*\{g_1(\vec{\alpha}\cdot\vec{U} - \frac{1}{\varkappa^2}\mathrm{div}\,\vec{\Pi}) + \frac{g_2}{\varkappa}[(\beta\vec{\sigma}\cdot\mathrm{curl}\,\vec{U}) - (\vec{\gamma}\cdot\vec{\Pi})]\}\psi\, d_3\vec{x}$$

$$H_{dis}^{v} = \frac{1}{2}\left(\frac{g_1}{\varkappa^2}\right)^2\int(\psi^*\psi)^2 d_3\vec{x} + \frac{1}{2}\left(\frac{g_2}{\varkappa}\right)^2 \times\int(\psi^*\vec{\gamma}\psi)(\psi^*\vec{\sigma}\psi)\, d_3\vec{x}$$

---

5) Nelson (P.R. 60 (1941) 830)'s procedure seems not convincing, because the eq. of motion deduced from the new Lagrangian is not equivalent to the previous ones.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

A canonical transformation defined by the formula analogous to (13) gives to the Hamiltonian the form (14), $H_{free\ mes.}$, H_{dir} and H_2 being replaced by $H^{\nu}_{free\ mes.}$, H^{ν}_{dir} and a matrix H_2^{ν} which is easily deduced.[6] For an arbitrary transition $\Phi \to \Psi$ of the two-nucleon system, one gets

$$(H_2^{\nu}) = -\sum \psi^*_{n_1 n_2} \chi^{(1)*}_{n_1} \chi^{(2)*}_{n_2} \frac{1}{\varepsilon_k^2 - (E_{m_1} - E_{n_1})^2} \times$$
$$\Big\{ g_1^2 \Big[(\vec{\alpha}^{(1)} \vec{\alpha}^{(2)}) + \frac{1}{\pi^2} \Big\{ k^2 + (\vec{\alpha}^{(1)} \vec{k})(\vec{\alpha}^{(2)} \vec{k}) \Big\}$$
$$+ \Big(\frac{q_1^2}{\pi}\Big)^2 \Big[\varepsilon_k^2 (\vec{\gamma}^{(1)} \vec{\gamma}^{(2)}) - (\vec{\gamma}^{(1)} \vec{k})(\vec{\gamma}^{(2)} \vec{k}) + (\vec{\gamma}^{(1)} \vec{\gamma}^{(2)})(\vec{\alpha}^{(1)} \vec{k})(\vec{\alpha}^{(2)} \vec{k}) + k^2 \beta^{(1)} \beta^{(2)} \Big]$$
$$+ i g_1 \frac{q_1^2}{\pi} \Big[((\vec{\gamma}^{(1)} - \vec{\gamma}^{(2)}) \vec{k}) (1 - (\vec{\alpha}^{(1)} \vec{\alpha}^{(2)})) - i \beta^{(1)} (\vec{\alpha}^{(2)} \vec{k}) + i \beta^{(2)} (\vec{\alpha}^{(1)} \vec{k}) \Big] \Big\} \times \chi^{(1)}_{m_1} \chi^{(2)}_{m_2} \Phi_{m_1 m_2} \qquad (24)$$

with $\vec{k} = \vec{p}_{m_1} - \vec{p}_{n_1}$. Using the properties of Dirac matrix, we obtain

$$(H_2^{\nu})_{\Psi \leftarrow \Phi} = (-H^{\nu}_{dir} + W_1^{\nu} + W_2^{\nu} + W_3^{\nu})_{\Psi \leftarrow \Phi} \qquad (25)$$

$$(W_1^{\nu})_{\Psi \leftarrow \Phi} = -\sum \psi^*_{n_1 n_2} \chi^{(1)*}_{n_1} \chi^{(2)*}_{n_2} \frac{g_3^2 [(\vec{\alpha}^{(1)} \vec{\alpha}^{(2)}) - 1 - \beta^{(1)} \beta^{(2)}] + (g_1^2 - g_2^2)(\vec{\gamma}^{(1)} \vec{\gamma}^{(2)})}{\varepsilon_k^2 - (E_{m_1} - E_{n_1})^2} \chi^{(1)}_{m_1} \chi^{(2)}_{m_2} \Phi_{m_1 m_2} \qquad (26)$$

6. Bhabha, P.R.S. A 166 (1939), 501.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$(W_2^{(v)})_{\Psi\leftarrow\Phi} = \frac{g_1 g_2}{\kappa}\sum \Psi^*_{n_1 n_2}\chi^{(1)*}_{n_1}\chi^{(2)*}_{n_2}\times$$

$$\frac{(E_{m_1}-E_{n_1})(\vec{\alpha}^{(1)}\vec{q}^{(2)})-2(E_{m_1}+E_{n_1})}{\zeta k^2-(E_{m_1}-E_{n_1})^2}(\beta^{(1)}+\beta^{(2)})\chi^{(1)}_{m_1}\chi^{(2)}_{m_2}\Phi_{m_1 m_2} \quad (27)$$

$$(W_3^{(v)})_{\Psi\leftarrow\Phi} = -\left(\frac{g_2}{\kappa}\right)^2\sum \Psi^*_{n_1 n_2}\chi^{(1)*}_{n_1}\chi^{(2)*}_{n_2}$$

$$\left[1-\frac{8E_{m_1}E_{n_1}}{\zeta k^2-(E_{m_1}-E_{n_1})^2}\right]\beta^{(1)}\beta^{(2)}\chi^{(1)}_{m_1}\chi^{(2)}_{m_2}\Phi_{m_1 m_2} \quad (28)$$

with $g_3 = g_1 + \frac{2M}{\kappa}g_2$, and the total interaction is given by

$$W^v = W_1^v + W_2^v + W_3^v.$$

For large p or small r, $\langle W_1^v\rangle, \langle W_2^v\rangle, \langle W_3^v\rangle$ behave as $p \sim r^{-1}$, $p^2 \sim r^{-2}$, $p^3 \sim r^{-3}$ respectively, all having nonvanishing diagonal elements.

6. Discussions of the results. (29)

i) For $E_{m_1} \sim E_{n_1} \sim M$, $\rho_1 \sim 0$; $\rho_2 \sim 0$, $\vec{\alpha} \sim 0$, (sw), Pssc. field gives vanishing static interaction. Vector field gives $e^{-\kappa r}/r$ and contact interaction.

ii) Kemmer cal. with the assumption $M \gg \kappa$, whereas finite value for M/κ is used in this case (this ratio comes in f_3, g_3)

iii) Deuteron problem: non-static interaction cannot be treated as perturbation, because static interaction may happen to vanish.

iv) It is hopeless to discard inadmissible singularities by mixing two fields.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Five Dimensional Formalism

1. Møller, On the Theory of Mesons, 1941 (Math. Phys. Communic., Academy of Denmark)

2. Rosenfeld, Meson Theories in five dimensions. 1942 (Proceedings of Nederl. Akademie van Wetenschappen, Vol XLV, No. 2, p. 3.)

3. Lubanski and Rosenfeld, Physica IX. (1942), 117.

4. A. Pais, Energy Momentum Tensor in Projective Relativity Theory, Physica VIII (1941), 1137.

5. A. Pais, Meson Fields in Projective Space (ibid. IX (1942), 267)

6. Watanabe, Sci. Pap. I.P.C.R. 39 (1941), 157 42 (1944), 1.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

L. Rosenfeld: Non-central coupling in the Mixed meson theory of nuclear forces
(Kgl. Danske Videns. Selsk. Math.-Fys. Med. XXIII. Nr. 13 (1945))

Non-central interaction introduced through the operator

$$S^{(12)} \equiv -\frac{1}{\pi^2}(\vec{\sigma}^{(1)}\,\mathrm{grad}^{(1)})(\vec{\sigma}^{(2)}\,\mathrm{grad}^{(2)})\varphi(r)$$

can be decomposed into

$$S^{(12)} \equiv \frac{1}{3}V_\sigma^{(12)} + V_D^{(12)} - \frac{1}{3}C_\sigma^{(12)}$$

$$\begin{cases} V_\sigma^{(12)} \equiv \vec{\sigma}^{(1)}\vec{\sigma}^{(2)}\varphi(r) \\ V_D^{(12)} \equiv D^{(12)}F(r) \\ \quad \text{with } D^{(12)} \equiv (\vec{\sigma}^{(1)}\vec{x}_0)(\vec{\sigma}^{(2)}\vec{x}_0) - \frac{1}{3}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) \\ \quad F(r) \equiv \left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right)\varphi(r) \\ C_\sigma^{(12)} \equiv \frac{3}{4\pi\varkappa^2}\lim_{\zeta\to 0}\int \delta(\vec{x}-\vec{\zeta})(\vec{\sigma}^{(1)}\vec{\zeta}_0)(\vec{\sigma}^{(2)}\vec{\zeta}_0)\,d\Omega \\ \quad \varphi(r) \equiv e^{-\varkappa r}/4\pi r. \\ \quad \varkappa \equiv \mu c/\hbar \end{cases}$$

If we assume the main part of nuclear forces to be represented by a central, charge-independent potential, ~~the saturation requirements fix~~ we obtain a restriction for the factor

$$A + B\,\vec{\tau}^{(1)}\vec{\tau}^{(2)}$$

$$A \ll B$$

from saturation requirements, which is satisfied by symmetric theory with the factor

$$T^{(12)} = \vec{\tau}^{(1)}\vec{\tau}^{(2)}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Ferretti, Nuovo Cimento 1 (1943), 25, however, showed that the M-R. theory gives rise to divergence due to velocity dependent interactions, which is necessary for accounting for deuteron quadruple moment. It can be shown, however, this velocity dependent term reduces to axial dipole interaction of the form $D^{(12)}$, in agreement with expectation on general invariance grounds discussed by Wigner, Phys. Rev. 51 (1937), 106; Wigner and Eisenbud, Phys. Rev. 56 (1939), 214.

§1. Nuclear Forces on Symmetrical Meson Theory.

$$V = g_1^2 V_{g_1} + g_2^2 V_{g_2} + f_2^2 V_{f_2}$$

$$(5)\quad \left\{ \begin{aligned} V_{g_1} &\equiv \frac{1}{2}\int n(\vec{x})\, n(\vec{x}')\, \varphi(|\vec{x}-\vec{x}'|)\, dv dv' \\ V_{g_2} &\equiv -\frac{1}{2\pi^2}\int [\vec{s}(\vec{x}') \wedge \mathrm{grad}'\varphi]\, \mathrm{rot}\, \vec{s}(\vec{x})\, dv dv' \\ &\quad + \frac{1}{2\pi^2}\int \vec{s}^{\,2}\, dv \\ V_{f_2} &\equiv \frac{1}{2\pi^2}\int \{\vec{s}(\vec{x}')\cdot \mathrm{grad}'\varphi]\, \mathrm{div}\, \vec{s}(\vec{x})\, dv dv' \\ n &= \sum \tau^{(i)} \delta(\vec{x}-\vec{x}^{(i)}) \\ \vec{s} &= \sum_i \tau^{(i)} \vec{\sigma}^{(i)} \delta(\vec{x}-\vec{x}^{(i)}) \end{aligned} \right.$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

or $$V_{g_1} = \frac{1}{2}\sum_{i,k} T^{(ik)} \varphi(r^{ik})$$

$$V_{g_2} = \frac{1}{2}\sum_{i,k} T^{(ik)} \left[V_\sigma^{(ik)} - S^{(ik)}\right]$$

$$V_{f_2} = \frac{1}{2}\sum_{i,k} T^{(ik)} S^{(ik)}$$

If we take $g_2^2 = f_2^2$, we are left with a purely central potential; in particular, the spin-spin coupling V_σ is entirely responsible for the separation of ^3S, ^1S.

Besides the static potential (5), a non-static coupling

$$W = \frac{g_1 g_2}{\pi}\int \{\vec{m}(\vec{x}')\vec{\tau}(\vec{x}) \operatorname{grad}\varphi + [\vec{s}(\vec{x}') \wedge \operatorname{grad}'\varphi]\vec{m}(\vec{x})\} dv dv'$$
$$+ \frac{f_1 f_2}{\pi}\int \vec{s}(\vec{x}')\vec{\tau}(\vec{x}) \operatorname{grad}\varphi \, dv dv' \quad (8)$$

to the first order of approx. in the nucleon velocities, with

$$\left.\begin{aligned} \vec{I} &= -\sum_i \tau^{(i)} \rho_2^{(i)} \vec{\sigma}^{(i)} \delta(\vec{x} - \vec{x}^{(i)}) \\ \vec{m}\vec{\tau} &= \sum_i \tau^{(i)} \rho_1^{(i)} \vec{\sigma}^{(i)} \delta(\vec{x} - \vec{x}^{(i)}) \\ \vec{\tau} &= \sum_i \tau^{(i)} \rho_2^{(i)} \delta(\vec{x} - \vec{x}^{(i)}) \end{aligned}\right\}$$

Can approximate expressions for the source densities can be obtained by a procedure similar to Gordon's decomposition in Dirac's

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

theory, — denoting symmetrization ;

$$\vec{m} \cong \sum_i x^{(i)} \frac{\vec{p}^{(i)}}{Mc^2} \delta(\vec{x}-\vec{x}^{(i)}) + \frac{\hbar}{2Mc} \operatorname{rot} \vec{S}$$

$$- \frac{\hbar}{2Mc} \frac{d\vec{t}}{c\,dt}$$

$$\vec{t} \cong \sum_i \tau^{(i)} \vec{\sigma}^{(i)} \wedge \frac{\vec{p}^{(i)}}{Mc^2} \delta(\vec{x}-\vec{x}^{(i)})$$

$$+ \frac{\hbar}{2Mc} \operatorname{grad} m + \frac{\hbar}{2Mc} \frac{d\vec{m}}{dt} \qquad (9)$$

$$x \cong - \frac{\hbar}{2Mc} \operatorname{div} \vec{S} - \frac{\hbar}{2Mc} \frac{dq}{dt}$$

$$q \cong \sum_i \tau^{(i)} \vec{\sigma}^{(i)} \frac{\vec{p}^{(i)}}{Mc} \delta(\vec{x}-\vec{x}^{(i)}) + \frac{\hbar}{2Mc} \frac{dr}{dt},$$

the terms containing time derivatives are of higher order in velocities and should be neglected. Thus we obtain from (9)

$$W_{sp.orb.} \cong - \frac{2g_1 g_2}{r} \frac{1}{Mc^2} \frac{1}{2} \sum_{i,k} \tau^{ik} \varphi(r^{(ik)}) \times (\vec{\sigma}^{(i)} + \vec{\sigma}^{(k)}) (\vec{r}^{(ik)} \wedge \vec{p}^{(ik)})$$

$$= \frac{2g_1 g_2}{r} \frac{Mm}{M} \frac{1}{2} \sum_{ik} \tau^{(ik)} \times \left(\frac{1}{r^{(ik)}} + \frac{1}{r_1^{2(ik)} r} \right) \varphi(r^{(ik)}) (\vec{\sigma}^{(i)} + \vec{\sigma}^{(k)}) (\vec{r}^{(ik)} \wedge \vec{p}^{(ik)})$$

and other terms includes

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$C_0 = \frac{1}{2\pi^2}\int \pi^2 dv = \frac{1}{2\pi^2}\sum_{i,k} T^{(ik)} f(\vec{x}^{(ik)})$$

with

$$C_0 = \frac{1}{2\pi^2}\int \vec{\sigma}^2 dv = \frac{1}{2}\sum_{i,k} T^{(ik)} C_\sigma^{(ik)}$$

$$C_\sigma^{(ik)} = \frac{1}{\pi^2}\vec{\sigma}^{(i)}\vec{\sigma}^{(k)} f(\vec{x}^{ik}).$$

~~C_0 reduces to~~ In all, we have

$$W = -g_1 g_2 \frac{M_m}{M}(V_{g_1} - C_0)$$

$$- \frac{2 g_1 g_2 \cdot f_1 f_2}{3}\frac{M_m}{M}\frac{1}{2}\sum_{i,k} T^{(ik)} V_\sigma^{(ik)}$$

$$+ \frac{M_m}{M}\frac{1}{2}\sum_{i,k} T^{(ik)}\left\{ g_1 g_2 C_\sigma^{(ik)} - \frac{g_1 g_2 - f_1 f_2}{3} C_{\tilde{\sigma}}^{(ik)}\right\}$$

$$+ (g_1 g_2 + f_1 f_2)\frac{M_m}{M}\frac{1}{2}\sum_{i,k} T^{(ik)} V_D^{(ik)} + W_{sp\text{-}orb.},$$

first three lines represent central and contact interactions, the ~~lines one~~ last line non-central couplings of axial dipole and spin-orbit types.

§2. The Deuteron Problem

all matrixelements of the contact transform. between stationary states (for central field.) vanish on account of radial dependence except §-5. In this calculation

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$C_\sigma^{(12)}$ is equivalent to $C_\sigma^{(12)}$, so that

$$W = \frac{M_m}{M} T^{(12)} q - g_1 g_2 \varphi(r) - \frac{2g_1g_2 - f_1f_2}{3} \times V_\sigma^{(12)} + g_1 g_2 \frac{\delta(\vec{x})}{\kappa^2} + \frac{2g_1g_2 - f_1 - f_2}{3} C_\sigma^{(12)}$$

$$+ (g_1 g_2 + f_1 f_2) V_D^{(12)} + W_{sp.orb.} \qquad (15)$$

the only term bringing about a mixing of states with different orbital momenta is the axial dipole coupling, the spin-orbit coupling being diagonal in this case. As shown by Feretti (15) ~~gives~~ contains the same kind of singularity as $S^{(12)}$. But it is smaller by a factor M_m/M, so that it can be considered as perturbation, just as in the theory of fine structure and hyperfine structure of atomic spectra. Namely, the apparent divergence of r^{-3} in S-state is due to the neglect of relativistic correction $\left(1 - \frac{\varepsilon - V}{M c^2}\right)^{-2}$. (Bethe, Handb. XXIV/1, 385~386; Pauli, ibid., eq. (89)) Thus both the spin-orbit coupling and the diagonal element of $V_D^{(12)}$ for S states vanish. Furthermore, contact interaction can be eliminated by adding invariants to Hamiltonian, so that the term

$$g_1 g_2 \frac{M_m}{M} C_\sigma = \frac{g_1 g_2}{2\kappa^2} \frac{M_m}{M} \int \rho^2 dv$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

disappears by adding

$$\frac{g_1 g_2}{2\pi^2}\frac{Mm}{M}\int(\vec{\nabla}^2 - \nabla^2)\,d\omega$$

Similarly C_0 can be reduced to high order in velocities by adding invariant proportional to $\int(\vec{S}^2 - t^2)\,d\omega$

Thus the whole interaction can be divided into central interaction

$$V_0 = \left(g_1^2 - \frac{Mm}{M} g_1 g_2\right) V_{g,\tau}\left(g_2^2 - \frac{Mm}{M}\frac{2g_1 g_2 - f_1 f_2}{3}\right) \times \frac{1}{2}\sum_{i,k} T^{ik} V_0^{(ik)}$$

and non-static interaction

$$W_{eff} = \frac{Mm}{M}(g_1 g_2 + f_1 f_2)\frac{1}{2}\sum_{i,k} T^{(ik)} V_D^{(ik)} + W_{sp.orb.}$$

In the cut-off theory, the non-central interaction appears with a coef. $\frac{1}{3}$ as large as that of the central interaction, giving rise to a large contribution to the energy level of ground state. Nevertheless the admixture of D-state is small. In M.K. theory, the non-central potential ~~only~~ changes everything only by a small amount. In particular, the sign and magnitude of quad. moment, which is proportional to $g_1 g_2 + f_1 f_2$,

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

fixes the signs of g_1, f_1, g_2 / where $|g_2| = |f_2|$ f_2. Namely, from

$$\left.\begin{aligned} & g_1^2 - \frac{M_m}{M} g_1 g_2 = \alpha > 0 \\ & g_2^2 - \frac{1}{3}\frac{M_m}{M}(2 g_1 g_2 - f_1 f_2) = \beta > 0 \\ & g_1 g_2 + f_1 f_2 = \gamma > 0 \\ & g_2^2 = f_2^2, \end{aligned}\right\}$$

where α, β, γ are comparable numerical quantities, we obtain two kind of solutions essentially distinct according as the sign of $g_1 g_2$.

If we further assume that

$$2 g_1 g_2 - f_1 f_2 = 0,$$

C_0 and the additional spin-spin interactions eliminated, so that g_2^2 is fixed from β. The absolute value of quad. moment is predicted by the theory. The only question is whether the quad. mom thus obtained is large enough.

An estimate by Multhén (Arkiv. för mat., Astron. och Fys. 29 A. No. 33, 1943) suggests rather a relation

$$g_1 g_2 - f_1 f_2 = 0.$$

§3. Velocity-dependent Coupling on Neutral Mixed Theory.

In this case, the source densities are

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Scalar field:
scalar density $\approx f_1' n$
vector density $\approx \frac{1}{\varkappa} f_2' \vec{m}$, $f_2' n$
Pseudovector field:
pseudovector density $\approx g_1' \vec{s}$, $g_1' q$
pseudotensor density $\approx \frac{1}{\varkappa} g_2' \vec{t} \approx \frac{1}{\varkappa} g_2' \vec{s}$

$(q \rightleftharpoons \rho_1 \delta(\cdot\cdot))$

Static Interaction:

$$V = -\frac{1}{2} \sum_{i,k} \{ f_1'^2 \varphi(r^{(ik)}) + g_1'^2 V_\sigma^{(ik)} - (g_1'^2 - g_2'^2) S^{(ik)} \},$$

the last term drops, if one puts $g_1'^2 = g_2'^2$

Non-static coupling:

$$W = -\frac{f_1' f_2'}{\varkappa} \int n(\vec{x}') \vec{m}(\vec{x}) \operatorname{grad} \varphi \, dv dv'$$
$$+ \frac{g_1' g_2'}{\varkappa} \int \{ q(\vec{x}') \vec{s}(\vec{x}') \operatorname{grad} \varphi + \vec{t}(x') [\vec{s}(x) \wedge \operatorname{grad} \varphi] \} dv dv',$$

from which we know that there's no mixing of states with different orbital momenta. Thus in the neutral theory, no alternative to the cut-off procedure to explain the quadrupole moment of the deuteron.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

J. M. Luttinger, On the Magnetic Moment of the Neutron and Proton.
(Helv. Phys. Acta, XXI (1948), 483)
(Letter: Phys. Rev. 74 (1948), 893.
for electron g-factor)

Pseudoscalar theory:
equivalence of pseudoscalar and pseudovector coupling ~~is~~ can be proved only, if we take the interaction terms quadratic in the coupling constant, which removes the $1/r^3$ singularity, in accord with L. van Hove, ~~in~~ whereas Dyson dropped this term.

II. Neutral Theory

$$L = L_p + L_M + L' + L''$$

$$L_p = -\psi_p^* \left(\frac{1}{i}\frac{\partial}{\partial t} + \frac{1}{i}\alpha(\nabla - ieA_0) + M\beta \right)\psi_p$$

$$L_M = \frac{1}{2}\left(\left(\frac{\partial \Phi}{\partial t}\right)^2 - (\nabla\Phi)^2 - \mu^2\Phi^2 \right)$$

$$L' = -\sqrt{4\pi}\, i g \left(\psi_p^* \beta\gamma_5 \Phi \psi_p\right)$$

$$L'' = -2\pi g^2 \lambda \left\{ (\psi_p^* \sigma \psi_p)(\psi_p^* \sigma \psi_p) - (\psi_p^* \gamma_5 \psi_p)(\psi_p^* \gamma_5 \psi_p) \right\}$$

$$\gamma_5 = i\alpha_x \alpha_y \alpha_z \quad *$$

(L'' gives no contribution to the magnetic moment, so that λ can take arbitrary values in our problem)

* Here α_x, α_y is interchanged for convenience.
$\hbar = c = 1$. A_0: potential of homogeneous mag. field.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$H = H_p + H_M + H' + H''$$

$$H_p = \psi_p^* \left(\frac{1}{i} \alpha (D - ieA_0) + M(\beta) \right) \psi_p$$

$$H_M = \frac{1}{2} \left(\dot{\pi}^2 + (\nabla \Phi)^2 + \mu^2 \Phi^2 \right)$$

$$H' = \sqrt{4\pi}\, i g \left(\psi_p \beta \gamma_5 \Phi \psi_p \right)$$

$$H'' = 2\pi g^2 \lambda \left[(\psi_p^* \sigma \psi_p)\cdot(\psi_p^* \sigma \psi_p) - (\psi_p^* \gamma_5 \psi_p)\cdot(\psi_p^* \gamma_5 \psi_p) \right]$$

$$\psi_p = \sum_n a_n \psi_n \quad \cdots$$

$$\Phi = \sum_p \frac{\alpha_p + \alpha_{-p}^*}{\sqrt{2\omega_p}} e^{ipx}$$

$$\pi = i \sum_p \sqrt{\frac{\omega_p}{2}} \left(\alpha_{-p}^* - \alpha_p \right) e^{ipx}$$

$$a_n^* a_{n'} + a_{n'} a_n^* = \delta_{nn'}$$

$$\alpha_p \alpha_{p'}^* - \alpha_{p'}^* \alpha_p = \delta_{pp'}$$

$$\omega_p = \sqrt{\mu^2 + p^2}$$

$$H_p = \sum_n a_n^* a_n E_n$$

$$H_M = \sum_p \alpha_p^* \alpha_p \omega_p$$

$$H' = \sqrt{4\pi}\, i g \sum_{n,n'} \sum_p a_n^* a_{n'} \alpha_p^* \frac{Q_{nn'}^{(p)}}{\sqrt{2\omega_p}} + \text{c.c.}$$

$$H'' = 2\pi g^2 \lambda \sum_{\substack{n,n' \\ m,m'}} a_n^* a_{n'} a_m^* a_{m'} R_{nn'mm'}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$Q^{(p)}_{nn'} \equiv \int d^3x\, (\psi_n^* \beta \gamma_5 \psi_{n'}) e^{-ipx}$$

$$R_{nn'mm'} \equiv \int d^3x \left[(\psi_n \sigma \psi_{n'})(\psi_m^* \sigma \psi_{m'}) - (\psi_n^* \gamma_5 \psi_{n'})(\psi_m^* \gamma_5 \psi_{m'}) \right]$$

As in the case of the electron, we calculate the energy of one proton in a state "m" to terms proportional to the square of the coupling constant. This means to calculate the mean value of H'' and second-order perturbation terms from H'. The result is

$$E = E' + E''$$

$$E' = -2\pi g^2 \left[\sum_p \frac{1}{\omega_p} \left\{ \sum_{E_n > 0} \frac{|Q^{(p)}_{mn}|^2}{\omega_p + E_n - E_m} - \sum_{E_n < 0} \frac{|Q^{(p)}_{mn}|^2}{\omega_p + |E_n| + E_m} \right\} + \sum_p \frac{Q^{(p)*}_{mm} Q^{(p)}_{mm} + Q^{(p)*}_{mm}}{\omega_p^2} \right]$$

$$E'' = 2\pi g^2 \lambda \left(\sum_{E_n > 0} - \sum_{E_n < 0} \right) R_{mnnm} + 2\pi g^2 \lambda \sum_{E_n < 0} R_{mnnm}$$

As shown in the case of the electron, if we take as state "m" the solution of Dirac equation with energy M, any infinities in the mass or charge of electron will not effect the magnetic moment. This means

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

that simply by developing the quantity E in a power series in H_0 (external field) the linear term in H_0 will converge and will give us directly the change in the magnetic moment of the nucleon. In fact in the pseudoscalar theory[1] all the infinities are simply mass and charge infinities just as in electrodynamics.

$$E' = -\pi g^2 \sum_p \frac{1}{\omega_p} \left\{ \sum_{n=0}^{\infty} \cdots \right) \rightarrow$$

$$E'' = \pi g^2 \lambda \sum_{p\mu} \sum_{n=0}^{\infty} \left(\cdots = 0 \right.$$

$$\rightarrow E_p' = \frac{g^2 (e H_0)}{8\pi M} \left(1 + 2\delta^2 - \frac{2\delta^3 (3-\delta^2)}{\sqrt{4-\delta^2}} \cos^{-1} \frac{\delta}{2} \right.$$

$$\left. - 2(1-\delta^2)\delta^2 \log \frac{1}{\delta} \right)$$

$$\delta = \mu / M.$$

$$\mu_p = -\frac{\partial E_p'}{\partial H_0} = -\frac{g^2}{\pi} \left(\frac{1}{4} + \frac{\delta^2}{2} - \frac{\delta^3 (3-\delta^2) \cos^{-1} \delta/2}{2\sqrt{4-\delta^2}} \right.$$

---

1) Villars has shown that the magnetic moment of the nucleon diverges for the vector theory with tensor coupling, indicating the presence of infinity other than mass and charge types, and preventing the calculation on the basis of M-R-mixture.
Vector theory with vector coupling gives convergent result.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$-\frac{(1-\delta^2)\delta^2}{2}\log\left(\frac{1}{\delta}\right)$$

III. Charged Theory

$$E_N^c = \frac{(\epsilon+1_0)g^2}{2\pi}\frac{1}{\mu}\left(1-\frac{\delta(2-\delta^2)}{\sqrt{4-\delta^2}}\cos^{-1}\frac{\delta}{2}+\delta^2\log\frac{1}{\delta}\right)$$

$$\mu_N^c = -\frac{g^2}{\pi}\left(1-\frac{\delta(2-\delta^2)}{\sqrt{4-\delta^2}}\cdot\cos^{-1}\frac{\delta}{2}+\delta^2\log\frac{1}{\delta}\right)$$

$$E_P^c = -\frac{g^2(\epsilon+1_0)}{2\pi}\left(\frac{1}{2}-\delta^2-\frac{\delta(2-4\delta^2+\delta^4)}{\sqrt{4-\delta^2}}\cos^{-1}\frac{\delta}{2}\right.$$
$$\left.+\delta^2(2-\delta^2)\log\frac{1}{\delta}\right)$$

$$\mu_P^c = \frac{g^2}{\pi}\left(\frac{1}{2}-\delta^2-\frac{\delta(2-4\delta^2+\delta^4)}{\sqrt{4-\delta^2}}\cos^{-1}\frac{\delta}{2}\right.$$
$$\left.+\delta^2(2-\delta^2)\log\frac{1}{\delta}\right)$$

IV. The Pseudovector Coupling

$$L' = -\sqrt{4\pi}f\left(\psi_p^*(\sigma\cdot\nabla\tau - \gamma_5\frac{\partial\tau}{\partial t})\psi_p\right)$$

$$L'_c = -\sqrt{8\pi}f\left[\left(\psi_p^*\left[\sigma\cdot(\nabla-ieA_0)\tau\right.\right.\right.$$
$$\left.\left.\left.-\gamma_5\cdot\frac{\partial\tau}{\partial t}\right]\psi_N\right)\right.$$
$$\left.+\left(\psi_N^*\left[\sigma\cdot\nabla+(ieA_0)\tau^*-\gamma_5\frac{\partial\tau^*}{\partial t}\right]\psi_p\right)\right]$$

If we assume $g^2 = \left(\frac{2M}{f}\right)^2$, this gives identical results. ($\frac{f}{M})^2 \sim g$)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

V. Summary of Results [2]

A) $\mu \sim 325$ m

$\mu_{IV} = 0$ $\quad \mu_N^c \sim -8$ $\quad \mu_N^s \sim -8$

$\mu_p \sim -2.8$ $\quad \mu_p^c \sim 2.5$ $\quad \mu_p^s \sim -.3$

B) $\mu \sim 200$ m

$\mu_N = 0$ $\quad \mu_N^c \sim -9.3$ $\quad \mu_N^s \sim -9.3$

$\mu_p \sim -2.8$ $\quad \mu_p^c \sim 3.6$ $\quad \mu_p^s \sim .8$

Exp.

$\mu_N = -1.9103$

$\mu_p = 1.7896$

Ratio of μ_N/μ_p (theor.) is indep. of coupling const. and ~~is~~ does not agree with experiment.
Whether the error lies in the model (pss. ms) or method (pert. theory) remains open.

2) The relationship:

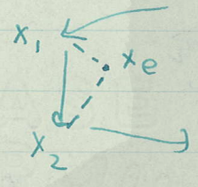
$$\mu_N^c + \mu_p^c = 2\mu_N^N$$

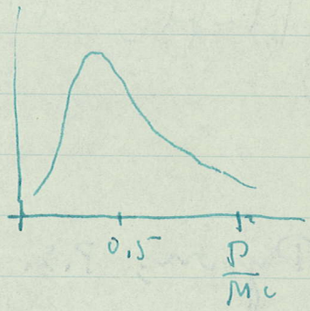
is generally satisfied.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar Jan. 22, 1949

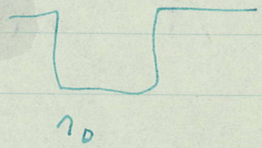
Case : P.S. theory





$$\frac{1}{\chi} = \int_0^1 \frac{y^3\,dy}{\chi_0^2 q^2 + (1-y)\chi^2}$$

$$V_{ne} = -\left(\frac{f^2}{4\pi\hbar c}\right)(300\ eV)$$



Watson-Feenberg hyperspherical Correction

$$\psi^+ \frac{\delta\epsilon^2}{\kappa^2(k^2-\delta\epsilon^2)}\psi$$

$$R = \alpha^2\left[\frac{1}{2}\ln\frac{a_0}{r_c} - 4.13\right]$$

$R = 1.33\alpha^2$ — 0.1% Mc

$R = 2.10\alpha^2$ — 9.6% Mc

$R = 1.61\alpha^2$ — 7.4%

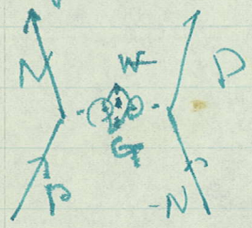
$\alpha \sim 137.039$ $\left(\frac{1}{40.000}\right)$

$.035 \pm .01$ e, k, c

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Bethe, Lamb shift
H-He $2^4 \ln \frac{K}{2^2}$

Dyson, P.S. meson theory.



$g\bar{\psi}_i j_{5i}$, $j_{5i} = \bar{\psi}\gamma_5\tau_i\psi$

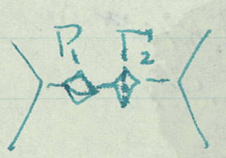
$j_5(x)\, j_5(x')\, \Delta(x-x')$

$$V_G(\Delta p) = \tau_1\cdot\tau_2\; j_5(p)\, j_5(q) \frac{1}{(\Delta p)^2+\mu^2} \times F_G(\Delta p) \frac{1}{(\Delta p)^2+\mu^2}$$

$$\Gamma_G = \Gamma_0 + \Gamma_1 \times (\Delta p^2+\mu^2) + \Gamma_2 (\Delta p^2+\mu^2)\times(\Delta p^2+\mu^2)$$

↓ $\delta\mu^2\bar{\psi}\psi$　　↓ g-renormalization

$$V_G(\Delta p) = \tau_1\tau_2\; j_5(p)\, j_5(q) \frac{\Gamma_2(\Delta p)}{(\Delta p)^2+\mu^2}$$



$\Gamma_2 \to F_{G_1}(\Delta p) F_{G_2}(\Delta p)$

$$V_{\Gamma_1\Gamma_2} = \sum_G V_G = \tau_1\tau_2\cdots \frac{1}{(\Delta p)^2+\mu^2}\left(\frac{1}{1-F_{\Gamma_1}}\right) \times \left(\frac{1}{1-F_{\Gamma_2}}\right)\cdots(\cdots)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$F_{P_1} = -\frac{g^2}{\pi}\int_0^1 \log\left(1 + x(1-x)\frac{\Delta p^2+\mu^2}{M^2}\right)dx$$

$$\simeq -\frac{g^2}{6\pi}\frac{\Delta p^2+\mu^2}{M^2}$$

$$\sum_q V_G(\Delta p) = \cdots \frac{1}{\Delta p^2+\mu^2}\left(\frac{1}{1+\frac{g^2}{6\pi}\frac{\Delta p^2+\mu^2}{M^2}}\right)$$

$$\downarrow \quad \frac{2\alpha}{3\pi}\frac{\Delta p^2+\mu^2}{\mu^2}$$

$$g^2 = \frac{M^2}{\mu^2}4\alpha.$$

~~$\frac{g^2}{6\pi}$~~ $\alpha \sim 1/2$.

$$V_0 = B\tau_1\tau_2\left[\frac{1}{3}\sigma_1\sigma_2\left(\frac{e^{-\rho\mu}}{\rho\mu} - \frac{1}{4\pi\mu^2}\delta(\vec{\rho})\right)\right.$$
$$\left. + \frac{1}{3}S_{12}\,e^{-\rho\mu}\left(\frac{1}{\rho\mu} + \frac{3}{(\rho\mu)^2} + \frac{3}{(\rho\mu)^3}\right)\right]$$

$$V_1 = B'\tau_1\tau_2\left[\frac{1}{3}\sigma_1\sigma_2\left(e^{-\rho\mu} - e^{-3\rho\mu}\right)\frac{1}{\rho\mu}\right.$$
$$\left. + \frac{1}{3}S_{12}\left[\underbrace{(\quad)e^{-\rho\mu} - (\quad)e^{-3\rho\mu}}_{\rho^{-1}}\right]\right]$$

Yukawa,

$\tau \to \pi + \pi + \pi$?

prohibited by selection rule for if

i) $\tau^{**} \to \pi + \pi + \pi$

ii) $\tau \to \pi + \pi^{(0)}$

iii) $\tau \to \pi + \gamma$ $\longrightarrow$

	τ–N	π–N	$\pi^{(0)}$–N	γ–N
	~~even~~	~~odd~~	~~even~~	~~odd~~
	odd	even		
	even	odd	odd	
	~~even~~	~~even~~	~~odd~~	even
	~~odd~~	odd	even	
	odd	even	odd	even
	even	even		
	odd	odd		

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

Case, Electromagnetic Properties of Nucleons. Feb-8, 1949.

Pseudoscalar Mesons - Pseudoscalar Coupling

$$(\Box^2 - \varkappa^2)\phi_\nu = 0$$

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} - \varkappa_0\right)\psi = 0$$

$$\left(\tilde{\gamma}_\mu \frac{\partial}{\partial x_\mu} + \varkappa_0\right)\tilde{\psi} = 0$$

$$i\hbar c \frac{\delta \Psi}{\delta \sigma} = \left(R_\nu \phi_\nu + H_{ext}\right)\Psi$$

$$R_\nu = i f_\nu \tilde{\psi} \Gamma_\nu \psi \qquad \Gamma_\nu = \tau_\nu \gamma_5$$

ϕ_ν : real field $\qquad \tau_4 = 1.$

$$H_{ext}^{(1)} = -\frac{iec}{\hbar} \bar{\psi} \gamma'_\mu \psi A_\mu \qquad \gamma'_\mu = \frac{1-\tau_3}{2}\gamma_\mu$$

$$H_{ext}^{(2)} = -\frac{e}{\hbar c}\left(\phi_1 \frac{\partial \phi_2}{\partial x_\mu} - \phi_2 \frac{\partial \phi_1}{\partial x_\mu}\right) A_\mu$$

$$+ \frac{e^2}{2\hbar^2 c^2}\left(A_\mu^2 + (n_\mu A_\mu)^2\right)\left(\phi_1^2 + \phi_2^2\right)$$

$$S(\sigma_0) = 1 + \left(-\frac{i}{\hbar c}\right)\int_{-\infty}^{\tau_{\sigma_0}} \mathcal{H}_i(x_1)\, dx_1$$

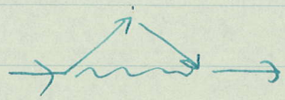
$$+ \left(\frac{-i}{\hbar c}\right)^2 \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2\, \mathcal{H}_i(x_1)\mathcal{H}_i(x_2) \cdots$$

$$\Psi = S(\sigma)\, S^{-1/2}(\infty)\, \Psi'$$

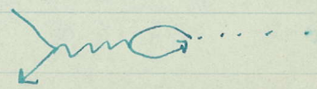
$$i\hbar c \frac{\delta \Psi'}{\delta \sigma} = H_E(x_0)\, \Psi'$$

H_E to order f^2.

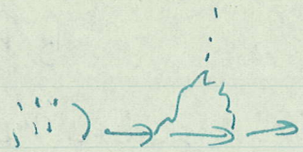
i)



ii)



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室



neutron-electron:

$$H_{eff} = \psi^* \psi V(r).$$

$$V(r) = (2\pi)^{\frac{3}{2}} V_p(p=0)$$

$$V_0 = -\left(\frac{\hbar^2}{4\pi\pi c}\right) \frac{1}{\alpha^2} \left(\frac{m}{M}\right)^2 mc^2 \frac{3}{2\pi}$$

$$\left\{ -\frac{13}{12} + \frac{1}{2} \log \frac{1}{\lambda^{1/2}} + \frac{35\pi}{48} \lambda^{1/2} \right\}$$

$$\lambda = \left(\frac{m}{M}\right)^2.$$

$\lambda^{1/2} \approx 1/6$: $V_0 \simeq -270 \left(\frac{\hbar^2}{4\pi\pi c}\right)$ eV.

$\approx 1/10$: $V_0 \simeq -410$ (〃) eV.

anomalous mag. moment:

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Van Hove — with recoil

Araki — without recoil

Matrix Elements in Meson Theory

i) Pseudoscalar Meson Theory ~~negle~~

	momentum	energy	wave fn
free proton	$\hbar \vec{K}_p$	$\hbar c K_p^{(0)}$	ψ_p
free neutron	$\hbar \vec{K}_n$	$\hbar c K_n^{(0)}$	φ_n
positive meson	$\hbar \vec{k}$	$\hbar c k^{(0)}$	U

$$\vec{K}_p = \vec{K}_n + \vec{k}$$

Interaction Term;

$$H' = \frac{4\pi c}{\varkappa} f_2 \tilde{\varphi}_n \rho_1 U^+ \psi_p + \frac{f_2}{\varkappa} \tilde{\varphi}_n \vec{\sigma}\, \mathrm{grad}\, \tilde{U} \psi_p$$

$$U^+ = + \frac{1}{4\pi c^2} \frac{\partial \tilde{U}}{\partial t}$$

$$H' = + \frac{f_2}{\varkappa c} \tilde{\varphi}_n \cdot i k^{(0)} \rho_1 \tilde{U} \psi_p - \frac{f_2}{\varkappa} \tilde{\varphi}_n (\vec{\sigma} \cdot i \vec{k}) \psi_p$$

$$= + \frac{i f_2}{\varkappa c} \tilde{\varphi}_n \rho_1 k^{(0)} \tilde{U} \psi_p - \frac{i f_2}{\hbar \varkappa} \tilde{\varphi}_n (\vec{\sigma} \vec{p}) \psi_p \cdot \tilde{U}$$

$$+ \frac{i f_2}{\hbar \varkappa} \{\tilde{\varphi}_n (\vec{\sigma} \vec{p})\} \psi_p \cdot \tilde{U}$$

$$(c \rho_1 \vec{\sigma} \vec{p} + \rho_3 M c^2) \psi_p = W_p \psi_p = \hbar c K_p^{(0)} \psi_p$$

$$(\vec{\sigma} \vec{p}) \psi_p - i \rho_2 M c \psi_p = \hbar K_p^{(0)} \rho_1 \psi_p$$

$$\tilde{\varphi}_n (c \rho_1 \vec{\sigma} \vec{p} + \rho_3 M c^2) = \hbar c K_n^{(0)} \tilde{\varphi}_n$$

$$\tilde{\varphi}_n (\vec{\sigma} \vec{p}) + i \cancel{\rho_2} M c \tilde{\varphi}_n \rho_2 = \hbar K_n^{(0)} \tilde{\varphi}_n \rho_1$$

$$\therefore -\frac{i f_2}{\varkappa \hbar} \tilde{\varphi}_n (\vec{\sigma} \vec{p}) \psi_p \tilde{U} + \frac{i f_2}{\varkappa \hbar} \{\tilde{\varphi}_n (\vec{\sigma} \vec{p})\} \psi_p \tilde{U}$$

$$= + \frac{2 f_2 M c}{\varkappa \hbar} \tilde{\varphi}_n \rho_2 \psi_p \cdot \tilde{U} - \frac{i f_2}{\varkappa} (K_p^{(0)} - K_n^{(0)}) \tilde{\varphi}_n \rho_1 \psi_p \cdot \tilde{U}$$

$$H' = \underbrace{+ \frac{i f_2}{\varkappa c} \tilde{\varphi}_n \rho_1 k^{(0)} \tilde{U} \psi_p}_{(k^{(0)} + K_n^{(0)} - K_p^{(0)})} + \underline{\frac{2 f_2 M}{\mu} \tilde{\varphi}_n \rho_2 \psi_p \tilde{U}}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

ii) Vector Meson Theory

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Dyson. Seminar Feb. 16, 1949

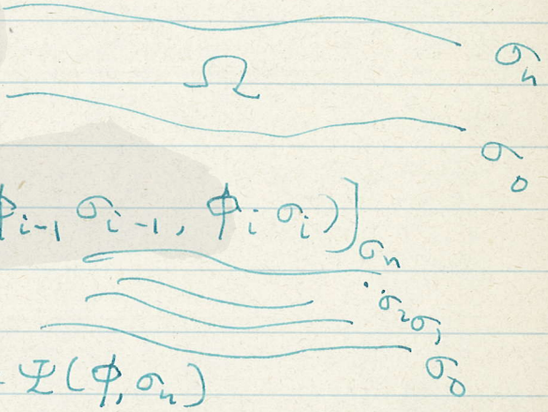
Feynman's Lagrangian Formalism

Rev. Mod. Phys. 20 (1948), 367.

Quantum Mechanics

Probability amplitude for any given ψ in Ω is $A \exp[iW/\hbar c]$. Amplitude for given ψ_0 on σ_0 and ψ_n on σ_n is $A \Sigma \exp[iW/\hbar c]$ summed over all ψ in Ω with these values.

(classical theory $W = \int_\Omega L \, d\omega$)



$$A \sum_{\phi_1 \cdots \phi_{n-1}} \exp\left[\frac{i}{\hbar c} \sum_1^n W_{clas}(\phi_{i-1}\sigma_{i-1}, \phi_i \sigma_i)\right]$$

This is wave-fn $\Psi(\phi_n, \sigma_n) = \Psi(\phi, \sigma_n)$

$$\frac{\partial \Psi}{\partial \phi} = \frac{i}{\hbar c}\left(\frac{\partial W_{clas}}{\partial \phi}\right)\Psi = \frac{i}{\hbar c}\pi\Psi.$$

$$\frac{\partial \Psi}{\partial \sigma_n} = \frac{i}{\hbar c}\left(\frac{\partial W_{clas}}{\partial \sigma_n}\right)\Psi = -\frac{i}{\hbar c} H\Psi$$

Weiss, P.R.S. 1938

Classical Dynamics

Lagrangian $L = L(\phi, \phi_\mu)$ $\quad \phi_\mu = \frac{\partial \phi}{\partial x_\mu}$

action $W = \int_\Omega L \, d\omega$



Vary ϕ by $\delta\phi$

Boundary σ of Ω by δn along normal

Then $\delta W = \int d\sigma [\pi \delta\phi - H \delta n]$

$\pi = n_\mu \frac{\partial L}{\partial \phi_\mu}$, $H = \pi \frac{\partial \phi}{\partial n} - L$.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Perturbation Theory

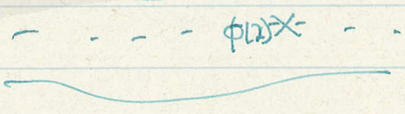
$L = L_0 + L_1$, $\quad W = W_0 + W_1$

$$A \sum_{\phi_{in}} \exp\left[i W_0 / \hbar c \right] \to \Psi_0(\phi_0(\sigma_0), \phi_{out}(\sigma_{out}))$$

$$+ A \frac{i}{\hbar c} \int_\Omega dx \sum_{\phi(x)} L_1(x) \sum_{\phi_{in},\, \phi(x)\ \text{fixed}} \exp\left[i W_0 / \hbar c \right]$$

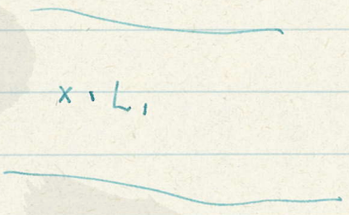
$$\Psi_0(\phi_0(\sigma_0), \phi(x)) \Psi_0(\phi(x), \phi_{out}(\sigma_{out}))$$

$(D_F \quad , \quad S_F)$



$+ \cdots\cdots$

Discrete particle case



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

Seminar Feb. 24, 1949

Feynman:

$$i\frac{\partial\psi}{\partial t} = H\psi \qquad \psi(x_2 t_2) = \int K(x_2, t_2; x_1, t_1)\,\psi(x_1, t_1)\,dx_1\,dt_1$$

$$K(2,1) = \sum_n \varphi_n^*(x_1)\varphi_n(x_2)\,e^{-iE_n(t_2-t_1)} \qquad t_2 > t_1$$

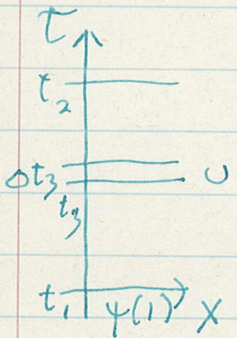
$$= 0 \qquad t_2 < t_1$$

$$\left(i\frac{\partial}{\partial t_2} - H_2\right)K(2,1) = i\delta(2,1)$$

and for $t_2 < t_1$ $\quad K(2,1) = 0$]

$H = H_0 + U$. $\quad K_0$: K for free particle

$$K(2,1) = K_0(2,1) + K^{(1)}(2,1) + K^{(2)}(2,1) + \cdots$$



$$\psi(x_3 t_3) = \int K_0(3,1)\psi(1)\,d^3x_1$$

$$\psi(x_3, t_3+\Delta t_3) = e^{-iH\Delta t_3}\psi(x_3, t_3) \quad \text{or } (1 - iH_0\Delta t_3 - iU\Delta t_3)$$

$$\Delta\psi(x_3, t_3+\Delta t_3) = -iU(x_3, t_3)\Delta t_3\,\psi(x_3, t_3)$$

$$\Delta\psi(2) = -\int K_0(2,3)\,iU(3)\Delta t_3\,d^3x_3\,\psi(x_3 t_3)$$

$$K^{(1)}(2,1) = -i\int K_0(2,3)\,U(3)\,K_0(3,1)\,\Delta t_3\,d^3x_3$$

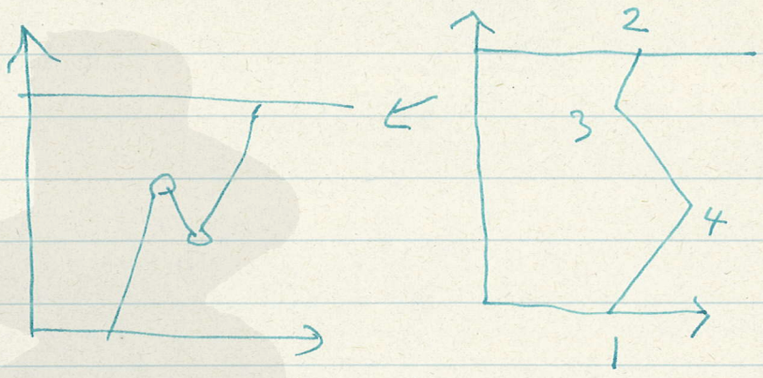
U all time:

$$K^{(1)}(2,1) = -i\int K_0(2,3)\,U(3)\,K_0(3,1)\,d^4\tau_3.$$

$$K^{(2)}(2,1) = -\int\int K_0(2,3)\,U(3)\,K_0(3,4)\,U(4)\,K_0(4,1)\,d^4\tau_3\,d^4\tau_4$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

hole theory?



$$K_+(2,1) = \sum_n \tilde{\varphi}_n(x_1)\,\varphi_n(x_2)\,e^{-iE_n(t_2-t_1)}$$

$$\tilde{\varphi} = \beta\varphi^*$$

only for positive energy states for $t_2 > t_1$

$$= -\sum_n \tilde{\varphi}_n(x_1)\,\varphi_n(x_2)\,e^{-iE_n(t_2-t_1)}$$

neg. $\qquad t_2 < t_1$

$a_\mu \gamma_\mu = \not{a} \qquad \not{a}\not{b} + \not{b}\not{a} = 2a\cdot b \qquad \not{a} = a^2$

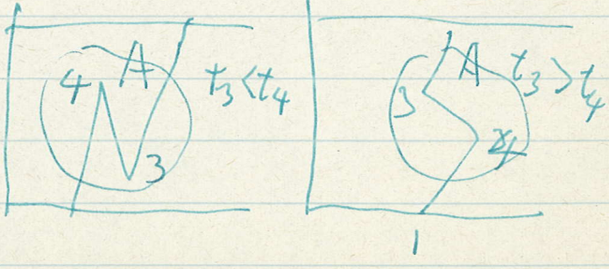
$\gamma_\mu \frac{\partial}{\partial x_\mu} = \not{\partial}$

$(i\not{\partial} - \mu)\,\psi = \not{A}\psi.$

$(i\not{\partial}_2 - \mu)\,K_+(2,1) = i\delta(2,1)$

$$K_+^{(1)}(2,1) = -i\int K_+(2,3)\,\not{A}(3)\,K_+(3,1)\,d\tau_3$$

$$K_+^{(2)}(2,1) = -\int K_+(2,3)\,\not{A}(3)\,K_+(3,4)\,\not{A}(4)\,K_+(4,1)\,d\tau_3\,d\tau_4$$



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

More Electrons.

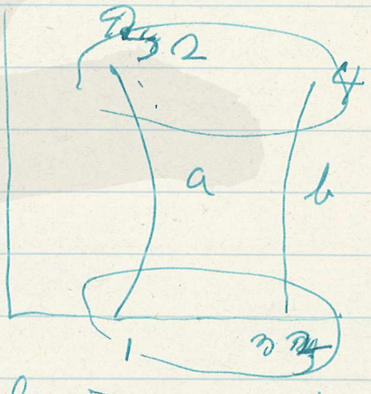
$\frac{\hbar}{i}\frac{\partial\psi}{\partial t} = H\psi$ $\qquad \psi(x_a, x_b, t)$

$K(x_{a2}, x_{b2}, t_2; x_{a1}, x_{b1}, t_1)$

free (No interaction)

$K^{(a)}(x_{a2}, t_2; x_{a1}, t_1) K^{(b)}(x_{b2}, t_2; x_{b1}, t_1)$

$K(2,4;1,3)$



Interaction e^2/r_{12} on only interval Δt_0 at t_0

normal. Schrödinger

$K^{(1)}(2,4;1,3) = -ie^2 \int K^{(0)}(2,4;5,6) \frac{1}{r_{56}}$

$K^0(5,6;1,3)\, d^3x_5 d^3x_6 \Delta t_0$

$(t_5 = t_6 = t_0)$

all time

$= -ie^2 \iint K^{(a)}(2,5) K^{(b)}(4,6) \frac{1}{r_{56}} K^{(a)}(5,1) K^{(b)}(6,3)$

$\delta(t_5 - t_6)\, d^3x_5 d^3x_6\, dt_5 dt_6$

$d\tau_5\, d\tau_6$

$\delta(t_5 - t_6 - r_{56})\, X$

$\delta_+(t_5 - t_6 - r_{56})$

$\delta_+(x) = \int_0 e^{-i\omega x} d\omega/\pi = \lim_{\varepsilon\to 0} \frac{(\hbar i)^{-1}}{x - i\varepsilon} = \delta(x) + \frac{1}{\pi i x}$

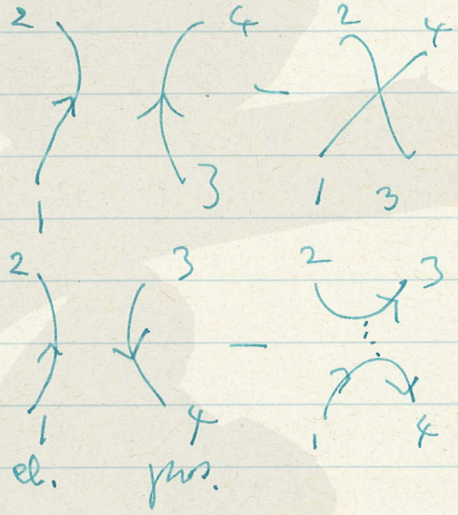
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

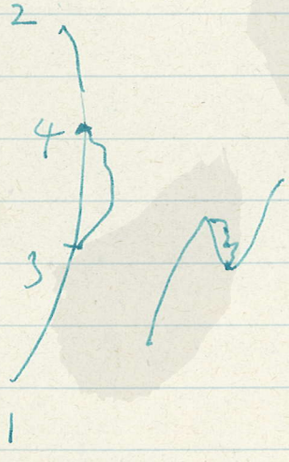
$$\frac{1}{2\lambda_{5\delta}}\left[\delta_+(\tau_{5\delta}-\lambda_{56})+\delta_+(-\tau_{56}-\lambda_{56})\right]$$

$$=\frac{1}{2}\,\delta_+(t_{5\delta}^2-\lambda_{5\delta}^2)$$

$$K^{(1)}(2,4;1,3)=-ie^2\iint K_+^{(a)}(2,5)\,K_+^{(b)}(4,6)$$

$$\gamma_\mu^{(a)}\gamma_\mu^{(b)}\,\delta_+(D_{56}^2)\;K_+^{(a)}(5;1)\,K_+^{(b)}(6,3)\,d\tau_5\,d\tau_6$$





$$K^{(1)}(2,1)=\int K_4(34)\gamma_\mu K_+(4,3)\gamma_\mu K_4(3,1)$$

$$\delta_+(\Delta_{43}^2)\,d\tau_3\,d\tau_4$$

$$\downarrow$$

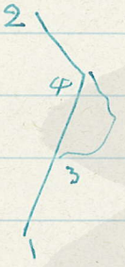
$$J_+$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Feb. 25

Statistics

Self-energy



$$\int \psi(4)\gamma_\mu K_+(4,3)\gamma_\nu \psi(3)$$
$$\delta_+(s_{43}^2)\,d\sigma_3\,d\sigma_4$$

$$K_+(2,1) = \frac{1}{4\pi^2}\int (\not p - \mu)^{-1} e^{-ipx}\,d^4p.$$

$$\delta_+(s_{12}^2) = \pi^{-1}\int \exp(-ikx)\frac{d^4k}{k^2}.$$

$\delta_+ \to f$

$\frac{1}{k^2} \to g(k^2) \to \frac{1}{k^2}$ for $k \to 0$

$k^2 g(k^2) \to 0$ for $k \to \infty$

$k^4 g \to$ finite, 0

$$g(k^2) = \int_0^\infty \left(\frac{1}{k^2} - \frac{1}{k^2-\lambda^2}\right) G(\lambda^2)\,d\lambda^2$$

$\ln \frac{\Lambda}{\mu}$

Bethe & Stueckelberg $\sum_i c_i = 0$ $\sum_i c_i m_i^2 = 0$

Pauli: Continuous spectrum of masses.

Meson Theory $\frac{1}{k^2} \to \frac{1}{k^2-m^2}$

Vector! Vector

scalar vector $\to 0$.

ps. ps.v. $\cong$ ps. ps.s.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

spin 0 charged ~~neu~~ particle

self-energy

$$\int \frac{1}{(p+k)^2-\mu^2} \frac{d^4k}{k^2} C(k^2)$$

$$\int \frac{k_\mu}{(p+k)^2-\mu^2} \frac{d^4k}{k^2} C(k^2)$$

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[ax+b(1-x)]^2}$$

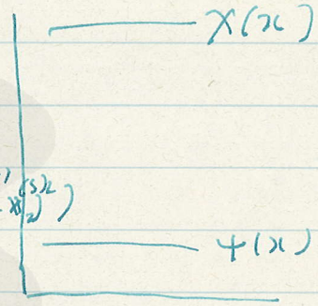
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Feb. 26.

Feynman

Rev. Mod. Phys. 1948.

$e^{iS/\hbar}$ $\quad S=\int L\,dt$



$e^{i(S_1+S_2+R)}$

$$R=\iint \left(1-\dot{x}_1^{(t)}\dot{x}_2^{(s)}\right)\delta\left((t-s)^2-(x_1^{(t)}-x_2^{(s)})^2\right)dt\,ds$$

S_p

$$S_{osc}=\int \frac{1}{2}\left[\dot{q}^2-\omega^2 q^2\right]dt+\int \gamma(t)q(t)\,dt$$

$$\gamma(t)=\sum_n e_n(e\cdot\dot{x}_n)\cos(k x_n)$$

$$\text{Ground}_{\chi}\text{-ground}_{\psi}=\int \tilde{\chi}(x)\tilde{\varphi}_0(q)e^{iS_p}e^{iS_{osc}}\psi(x)\varphi_0(q)\,dx\,dx'\,d\,\text{Path}_{osc}\,dq\,dq'$$

part.

$$G_{00}=\int \varphi_0^*\,e^{iS_{osc}}\varphi_0\,d\,\text{Path}_{osc}\,dq\,dq'$$

$$G_{00}=e^{-i\omega\frac{T_2}{t_2-t_1}}\exp\left[\frac{i}{\hbar}\frac{\hbar}{2\omega}\int_{t_1}^{t_2}\int_{t_1}^{t}e^{-i\omega(t-s)}\gamma(t)\gamma(s)\,dt\,ds\right]$$

elimination of oscillator.

$$G_{00}\quad\left(1-\dot{x}_n^{(t)}\cdot\dot{x}_m^{(s)}\right)\Big|\left[\delta_+(|t|-r)-\delta_+(|t|+r)\right]$$

$$\delta_+(t^2-r^2)$$

δ in $R\to\delta_+$

$$\cdots\langle x_{i+2}|x_{i+1}\rangle\langle x_{i+1}|x_i\rangle\cdots$$

x_{i+1}

x_i

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$AB \to A_1 B_2$

e^{iHt} $\qquad$ $e^{i\int H_t dt}$

$$e^{i\int_0^T T_t dt} \int_0^T V_t dt = \int_0^T dt' e^{-i(T(t-t'))} V e^{iT(t')}$$

Equivalence of hole theory of Dirac & back track theory.

$\chi_g \quad (\bar{\chi}_g e^{iHT} \chi_f)$

A

χ_f

$\chi_0 = vac$

$F^* = \int \psi^*(x) f_{pos}(x) d^3x$

one of electron $f_{pos}(x)$.

$\chi_f = F^* \chi_0$

pos. in φ_1

$\bar{F} = \int \psi(x) \varphi_1^*(x) d^3x$

$\downarrow$ neg.

$\chi_f = F_1^* F_2^* \cdots \bar{F}_1 \bar{F}_2 \cdots \chi_0$

$F^* G + G F^* = \int f(x) g(x) d^3x$

$(\bar{\chi}_0 G G_2 \cdots \Psi_1^* \Psi_2^* \cdots e^{iHT} F_1^* F_2^* \cdots \bar{F}_1 \bar{F}_2 \cdots \chi_0)$

$= \text{Tr}(\bar{\chi}_0 e^{iHT} \chi_0)$

$(\because \; e^{iHT} F_{pos}^* = F_{pos}'^* e^{iHT} - e^{iHT} F_{neg}''^*$

f $\qquad$ f' $\qquad$ f''

$f'(2) = \int K_+(2,1) f(1) \, d^3V_1$

$f''(2) = \int K_-(2,1) f(1) \, d^3V_1$ $\qquad t_1 = t_2$)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

spin 0 particle

$$S = \int \frac{1}{2}(\dot{x}_\mu \dot{x}_\mu)\, dw + \int \dot{x}_\mu A_\mu\, dw$$

$\psi(x_\mu, w)$

$$\chi \quad e^{iS} \quad \psi$$

$$-\frac{\hbar}{i}\frac{\partial \psi}{\partial w} = \frac{1}{2}\left(\frac{\hbar}{i}\frac{\partial}{\partial x_\mu} - A_\mu\right)^2 \psi$$

$$\psi = e^{i\mu^2 w/2\hbar}\, \varphi(x_\mu)$$

$$\int \left(\chi\, e^{i\left(S - \frac{\mu^2}{2}w\right)} \psi\right) dw$$

$$\int \chi\, e^{i(p_\mu^2 - \mu^2)w}\, \varphi\, dw$$

$$\downarrow$$

$$\frac{1}{p_\mu^2 - \mu^2}$$

spin 1/2 particle

$$\iint \gamma_\mu \gamma_\nu F_{\mu\nu}\, dw$$

second order Dirac equation.

$$\int_0 \bar{\varphi}_f\, e^{i\int (\not{p}_w - \not{A}_w - \mu) dw}\, \varphi_i\, dw$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

1. Max Born, Elementary Particles and the Principle of Reciprocity
Nature 163, 207 (1949)

Green's method of quantization:
Statistical matrix $\rho \to \psi\psi^*$ for pure state
Lagrangian

$$L = F(p_k, p'_k)\rho$$

p_k operate on ρ from the left, p'_k from the right.

Field eq. $F(p_k, p_k)\rho = 0$ (1)

F: self-reciprocal:
Scalar Field
A simple way of finding all self-reciprocal scalars is to determine the eigenfns of the operator $x^k x_k + p^k p_k$, where the coordinates and momenta are measured in the units a and b ($ab = \hbar$). Relativistic invariant eigenfns have the form

$$F(p) = f(P)\, e^{-P/2} \qquad (2)$$

$f(P)$: Laguerre polynomial

$$P = p_k p^k$$

$$f_0 = 1, \quad f_1 = P - 2, \quad f_2 = P^2 - 6P + 6 \qquad (3)$$

(1) reduces to

$$f(p)\rho = 0 \qquad (4)$$

(4) can be satisfied by plane waves with momentum-energy p^0_k, if $P^0 = p^0_k p^{0k}$ is one of the roots of $f(x) = 0$. These roots are positive numbers:

$f_1 = 0$: $x_1 = 2$, $\quad f_2 = 0$: $x_2^{(1)} = 3 + \sqrt{3}$ $\quad x_2^{(2)} = 3 - \sqrt{3}$ etc

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

If x_r is one of these,

$$(p_\kappa p^\kappa - x_r)\psi = 0 \qquad (5')$$

thus $\sqrt{x_r}$ is a possible rest mass.

Quantized field theory:

Total energy momentum $P_\kappa = \sum_r n_r p_\kappa^{(r)}$ $\qquad p_\kappa^{(r)} p_\kappa^{(r)k} = x_r \qquad (6)$

n_r: operators with integral eigenvalues.

Infinite number of masses for scalar particles;

lowest mass: $\sqrt{2} = 1.414$

$\sqrt{3+\sqrt{3}} = 2.175$ $\qquad \sqrt{3-\sqrt{3}} = 1.126\cdots$

Lagrangian for particles with spin: ...

The new theory demands the invariants of $R+P = x^\kappa x_\kappa + p^\kappa p_\kappa$ instead of R in relativity.

Eigenvalues of $R+P$ are integers, indicating a kind of discontinuous structure of x- and p- space, while F determines the dynamical laws. For spin ½ one has to define the distance by taking the root of R and P with the help of properly chosen Dirac matrices.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

2. H.S. Green, Quantized Field Theories and the Principle of Reciprocity

(Nature 163, 208 (1949))

Pauli, R.M.P., 76 (1948).
Chang, Proc. Camb. Phil. Soc. 42, 132 (1946); 44,
de Wet, ibid. 44, 546 (1948).

$$F(P)\psi = 0 \qquad P = p_k p^k = E^2 - p^2, \quad (1)$$

masses of the particles being obtained as the roots of $F(m^2) = 0$

Lagrangian:

$$L = F(p_k, p_{k'})\rho \qquad p_k = i\hbar \frac{\partial}{\partial x^k}, \quad p_{k'} = -i\hbar \frac{\partial}{\partial x^{k'}} \quad (2)$$

Field Equations:

$$F(p_k, p_{k'})\rho = (F + \pi_k G^k)\rho = 0, \qquad \pi_k = p'_k - p_k,$$
$$F(p_k, p_{k'})\rho = (F + \pi_{k'} G^{k'})\rho = 0, \qquad \pi_{k'} = p_k - p'_k \quad (3)$$

multiply first by 1, p_k, $p_k p^{k'}$ ... and second by 1, $p'^{k'}$, $p'_k p^{k'}$ ... and subtracting

$$\pi_k (G^k + G^{k'})\rho = 0$$
$$\pi_k (F\delta_l{}^k - G^k p_l - G^{k'} p'_l)\rho = 0 \quad (4)$$

with

$$R^k = \tfrac{1}{2}(G^k + G^{k'})\rho$$

interpreted as the density-flux vector.

$$L_l{}^k = (F\delta_l{}^k - G^k p_l - G^k p'_l)\rho$$

as canonical energy flux tensor.

R^4 is the time density operator

$$P_k = \int \tfrac{1}{2}(p_k + p'_k) R^4 \, d\Omega \quad (5)$$

is Hamiltonian energy-momentum vector, which

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

differs from L_k^4 (integrated) by a space divergence.

Commutation rule

$$\zeta_i R^4 \pm R^4 \zeta_i = \zeta_i \qquad (6)$$

ζ_i : creation operator for a particle with mass m_i

For $\rho = \sum_{ij} \psi_i(x_k)\, \psi_j^*(x_\ell')$, (7)

(6) reduces to

$$\tfrac{1}{2}(G_{T_4} + G_{T_4'})\,[\psi_i(x_k), \psi_j^*(x_\ell')] = \delta(x - x')\,\delta_{ij}, \qquad (T' = t) \qquad (8)$$

Self-reciprocal Lagrangian:

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

3. Max Born and Antonio E. Rodriguez
Meson Masses and the Principle of Reciprocity
(Nature 163, 320 (1949)).

$$L_n^{(1)}(x) = 0 \qquad (1)$$

$L_n^{(1)}$: Lagnerre polynomials of first kind.

n	x				
1	2				
2	1.26	4.73			
3	0.93	3.31	7.175		
4	0.74	2.57	5.73	10.9	
5	0.53	2.11	4.61	8.40	14.26

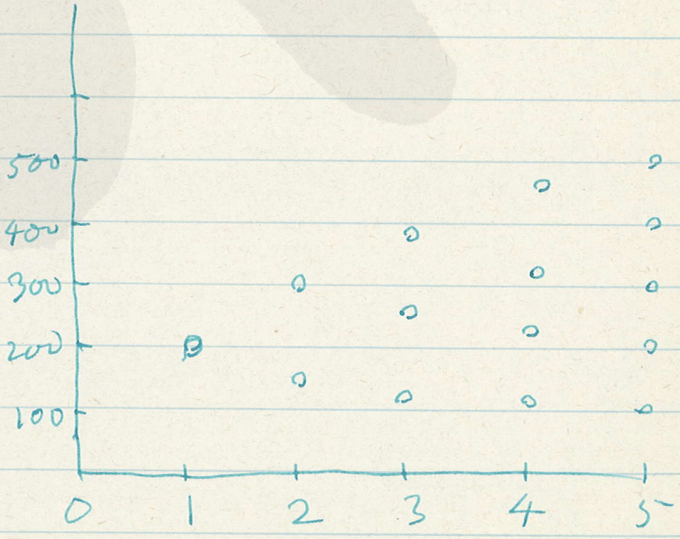
$$(p^2 - E^2 - x b^2)\psi = 0$$

$$m = b\sqrt{x}/c$$

$$m = \hbar\sqrt{2}/ac$$

$$a = e^2/\mu c^2 \qquad \mu = \text{electron mass}$$

$$\frac{m}{\mu} = \frac{\hbar c}{e^2}\sqrt{2} = 137\cdot\sqrt{2} = 194$$



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

4, Kai-Chia Cheng and A. E. Rodriguez
Particles with Spin and the Principle of Reciprocity
(Nature 163, 367 (1949)

spin 0 or 1,

metric operator: $S = x^k x_k + p^k p_k$

spin 1/2

$$S = (p^k + i\gamma \not{x} x^k)\alpha_k$$

$$\gamma = a_1 a_2 a_3 a_4$$

reciprocity equation:

$$S(x_k, p_k) = T^{-1} S(p_k, -x_k) T$$

$$T = \frac{1+i\gamma}{1+i}$$

self-reciprocital functions are solutions of

$$S\mathbb{F} = \mathbb{F} S$$

s: eigen-symbol depending only on γ.

These solutions can be expressed in terms of Laguerre polynomials $L_n^k(P)$, with $P = p_k p^k$

$$F_1 = 2\{(\alpha_r p^r) L_n^{k+2}(P) \pm \sqrt{n-k-1}\, L_n^{k+1}(P)\}$$
$$(\alpha_r p^r)^k e^{-P/2} g, \qquad (4)$$

$$F_2 = 2\{(\alpha_r p^r)[L_n^{k+1}(P) - L_n^{k+2}(P)] \pm \sqrt{n+1}$$
$$L_n^{k+1}(P)\}(\alpha_r p^r) e^{-P/2} g. \qquad (5)$$

corresponding to $\gamma = \pm 1$, where g is a three-dimensional spherical fn, $n = 1, 2, 3, \cdots$, $k = 0, 1, 2, \cdots n-1$.

The root of the equations $F_1 = 0$, $F_2 = 0$ are

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

the mass factors κ^2.

$$\mu = \frac{\hbar\kappa}{ca} \qquad a = \frac{e^2}{mc^2}$$

$$\frac{\mu}{m} = \frac{\hbar c}{e^2}\kappa = 137\kappa \qquad m: \text{electronic mass}$$

As (4) and (5) both have the factor $(\alpha_r p^r)^K$, for odd $K > 1$, the theory predict neutrinos

$$\alpha_r p^r = 0 \qquad (6)$$

It is very likely that the same factor represents electrons as charged particle neutrinos.
For even K, one has particle satisfying

$$(\alpha_r p^r)^2 = \gamma_r p^r = P = 0$$

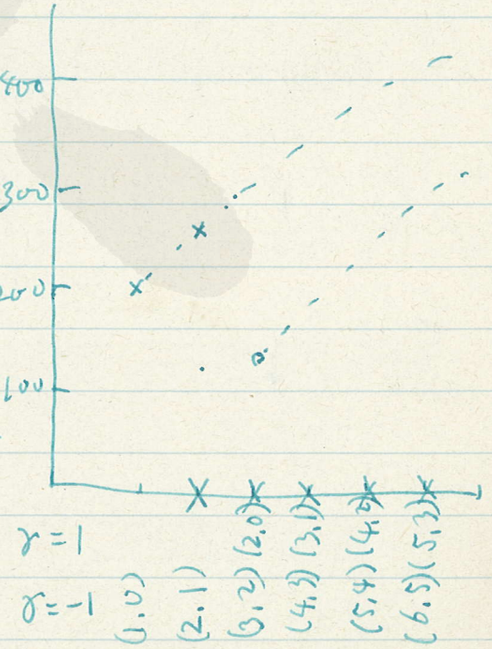
but with different spin properties from the photon.

For $\gamma = 1$ $\quad k = n-2$

m	n	κ
	2	1
		2
	3	0
		1.30
		2.30
	4	0
		1.56
		2.56
	5	0
		1.79
		2.79
	6	0
		2
		3

$\gamma = -1$ $\quad k = n-1$

n	κ
1	1.41
2	0
	1.73
3	0
	2
4	2.23
	2.33
5	0
	2.45
6	0
	2.64



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

April 23, 1949

Pais, Mass Spectrum for Elementary Particles

$$\{\gamma_\mu \partial_\mu + \varkappa e^{\lambda^2 \Box}\}\psi = 0$$

$$\left(e^{\lambda^2 \frac{\partial^2}{\partial x^2}} f(x) = \frac{1}{\sqrt{4\pi\lambda}} \int_{-\infty}^{+\infty} d\xi\, f(\xi)\, e^{-(x-\xi)^2/4\lambda}\right)$$

$$\{\Box \overline{\text{or}}\ \varkappa^2 e^{2\lambda^2 \Box}\}\psi = 0$$

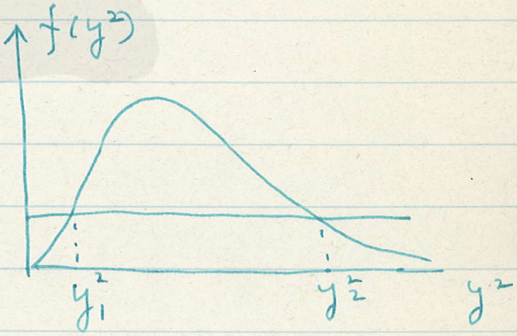
$$\{y^2 - \varkappa^2 e^{+2\lambda^2 y^2}\} e^{ikx} = 0$$

$$f(y) = y^2 e^{-2\lambda^2 y^2} = \varkappa^2$$

$$y_1 \ll \frac{1}{\lambda}.$$

$$y_1 \cong \varkappa \ll \frac{1}{\lambda}$$

$$y_2 \gg \varkappa$$



$$\underbrace{y^2\left(1 - 2\lambda^2 y^2 + \frac{(2\lambda^2 y^2)^2}{2} - \dots\right)}_{n+1} = \varkappa^2$$

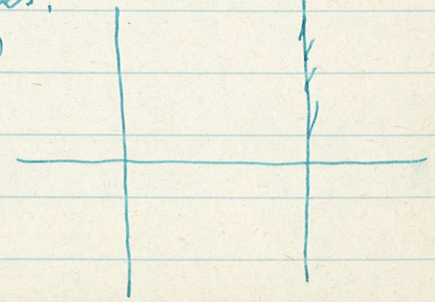
$n+1$ masses are there

What kinds of solutions, other than the above two solutions connected with the ordinary particles with real masses?

For $y^2 < 0$ $\quad -y^2 = z > 0$

$$\frac{\partial^2}{\partial z} + \varkappa^2 e^{-2\lambda^2 z} = 0$$

no $y^2 < 0$



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Spinor Particle:

$$\sum_n c_n (\gamma_\mu \partial_\mu)^n \psi = 0$$

$$\left\{ \sum_{n=0}^{\infty} c_{2n} \Box^n + \sum_{n=0}^{\infty} c_{2n+1} (\gamma_\mu \partial_\mu) \Box^n \right\} \psi = 0$$

$$\{ f_0(\Box) + f_1(\Box) \gamma_\mu \partial_\mu \} \psi = 0$$

Pais' case $\quad f_0(\Box) = \varkappa e^{\lambda^2 \Box}$
$\qquad\qquad\qquad\quad f_1(\Box) = 1$

Electron — two masses
Neutrino — one mass

Scalar (Pseudoscalar) Particle:

$$\left\{ \sum_{n=0}^{\infty} c_n \Box^n \right\} U = 0$$

$$f(\Box) U = 0.$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

P. A. M. Dirac

Unitary representation of the Lorentz group

(Proc. Roy. Soc. A. 183, (~~1944~~, 1945), 284)

definition of expansion:

$$Q = \sum_0^\infty A_{nrst}\, \xi_0^{n-u-1}\, \xi_1^r\, \xi_2^s\, \xi_3^t$$

scalar product:

$$\sum_0^\infty n!^{-1}\, r!\, s!\, t!\, A_{nrst}\, B_{nrst}$$

within the light cone

$$\xi_0^2 - \xi_1^2 - \xi_2^2 - \xi_3^2 > 0,$$

the series is absolutely convergent. For homog. f. of degree $u-1$,

$$Q = \sum_n \xi_0^{n-u-1} \sum_{rst} A_{nrst}\, \xi_1^r\, \xi_2^s\, \xi_3^t$$

~~is absolutely convergent~~

where

$$r+s+t = n+u$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

H. T. Flint, Professor of Physics,
University of London, Bedford College

A Study of the Nature of the Field Theories of the electron and positron and of the meson

(Proc. Roy. Soc. A185 (1946), 14 ~ 34)

Kaluza: S. B. preuss. Akad. Wiss. 1921, p. 966,

Klein: Z. f. Phys. 46 (1927), 188,

$$d\sigma^2 = \gamma_{\mu\nu}\, dx^\mu dx^\nu$$

$$\left.\begin{aligned} \gamma_{mn} &= g_{mn} + \gamma_{55}\alpha^2\phi_m\phi_n \\ \gamma_{m5} &= \gamma_{5m} = \gamma_{55}\alpha\phi_m \end{aligned}\right\}$$

$$\gamma^{mn} = g^{mn}$$

$$\gamma^{m5} = -\alpha\phi^m \qquad \gamma^{55} = \frac{1}{\gamma_{55}} + \alpha^2\phi_m\phi^m$$

Fisher: P. R. S. A 123 (1929), 489

Fisher and Flint: ibid. 126, 644,

The particle trajectories are null geodesics

Flint: Phil. Mag. (7) 29 (1940), 330.

$\alpha = e/m_0c^2$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Coordinate Operators in Quantum Mechanics,

H. T. Flint (Bedford College, University of London)

(Nature 163 (1949), 132)

Snyder, P. R. 71 (1947), (7), 38.

$$x \Rightarrow a\left(x\frac{\partial}{\partial x}\Big|_0 - \eta_0 \frac{\partial}{\partial x}\right)$$

$$\frac{\partial}{\partial x^5} = 2\pi i m_0 c/h \qquad x^5 = h/m_0 c$$

$$X = x - \frac{h}{m_0^2 c^2} u_x$$

$$u_x = \left(\frac{h}{i}\frac{\partial}{\partial x} - \frac{e}{c}\varphi_x\right)$$

Flint, P. R. 74 (1948), ~~20~~ (2), 209.

The groups of angular momentum and co-ordinate operators can be regarded as a single tensor, (Flint, P. R. S. A. 185 (1945), 22) so that coordinate should be extended to take the form

$$X = x - \frac{h}{m_0^2 c^2} u_x + \frac{ih}{2m_0 c}\beta\alpha_x .$$

and $$X_4 = x_4 - \frac{h}{m_0^2 c^2} u_4 - \frac{h}{2m_0 c}\beta .$$

where

$$x_4 = ict \qquad u_4 = \left(\frac{h}{i}\frac{\partial}{\partial x^4} - \frac{ei}{c}\varphi\right)$$

Neglecting the second term, the proper values of X are $x \pm h/2m_0 c$. Thus if the coordinate x is chosen in the usual way, the particle is described as be located at

$x + h/2m_0 c$ or $x - h/2m_0 c$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Further

$$m_0 \dot{X} = -\beta u_x - he(\vec{E} + \vec{H} \times \vec{a})_x / m_0 c^2$$

mass × velocity = momentum

$$\alpha_x \dot{X} + \alpha_y \dot{Y} + \gamma_z \dot{Z} - i \dot{X}_4 = 0$$

except for a small imaginary value.

These expressions suggest that a solution of certain difficulties is to be found not in an intrinsic property of the space of physics, but in an intrinsic property of matter.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

E.C.G. Stückelberg et D. Rivier

Opérateurs non linéaires en théorie des quanta

(Helv. Phys. Acta. 19 (1946), 240)

$$\overline{(\Delta P)^2}\cdot\overline{(\Delta X)^2} = \frac{\hbar^2}{4}\left(1-\frac{\lambda^2}{\overline{(\Delta X)^2}}\right)^{-1}$$

$$i[P,X] = \left(1-\frac{\lambda_0^2}{\overline{(\Delta X)^2}}\right)^{-\frac{1}{2}}\mathbf{1}$$

$$\Psi^*(P) = S(P/X)\,\Psi(X)$$

$$S(P/X) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{i\left(1-\frac{\lambda_0^2}{\overline{(\Delta X)^2}}\right)^{\frac{1}{2}}}{\hbar}PX\right]$$

$$S(P/X)[X] = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{i\left(1-\frac{\lambda_0^2}{(X,(\Delta X)^2X)}\right)^{\frac{1}{2}}}{\hbar}PX\right]$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Canonical Transformation of the Hamiltonian in Meson Field Theory.

Le Couteur, L. Rosenfeld
(Phil. Mag. 40 (1949), 151)

Pseudoscalar Meson Theory

nuclear forces:

$$\left(f_1 + 2f_2 \frac{M}{\mu}\right)^2 = \left(\frac{2M}{\mu}\right)^2 \left\{f_2^2 + f_2 f_1 \frac{\mu}{M} + \left(f_1 \frac{\mu}{2M}\right)^2\right\}$$

β-decay: decay constant is proportional

$$\left(f_1 + f_2 \frac{M_p + M_n}{\mu}\right)^2 \left(\bar{f}_1 + \bar{f}_2 \frac{m}{\mu}\right)^2$$

in agreement with Nelson, (P. R. 60 (1941), 830)

life-time t_0 of meson is

$$\frac{1}{t_0} = c.\kappa \left\{\frac{\bar{f}_1}{\sqrt{t_0}} + \frac{\bar{f}_2}{\sqrt{t_0}} \frac{m}{\mu}\right\}^2$$

(~~Necessity~~ Possibility of direct coupling)

Scalar

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Jánossy and McCusker
Nature of Penetrating Particles in Air Showers

(Dublin Institute for Advanced Studies)
(Nature 163 (1949), 181)

Two groups of investigators (~~one~~ Auger, Daudin, Freon and Maze C.R. 226 (1948), 169 and Broadbent and Jánossy, Proc. Roy. Soc. A, 192 (1948), 364) agree that penetrating particles which constitute a small percentage of extensive air showers mainly consisting of electrons and photons. However, Auger's group suggested that they are "λ-mesons" with charge ±e and mass about 3 mₑ whereas Jánossy's group suggested that they are heavy electrons of mass about 10 mₑ.

Recent experiments by Chowhuri, Jánossy and McCusker lead to the conclusions:

(1) Penetrating particles in air showers are produced locally in the first few centimeter of lead.

(2) They are highly unstable and have a considerable chance of decay along the 50-cm gap between roof and absorber. An apparent ($\frac{1}{3}$) life of the order of 10^{-9} sec. is to be expected.

(3) While the thin lead roof shows a considerable effect, no noticeable effect is shown by a mass-equivalent brick roof. It must be concluded that the primaries of the penetrating particles are strongly absorbed in lead but

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

not in the mass-equivalent of brick. This makes it virtually certain that the primaries of the penetrating particles are either electrons or photons or both.

(4) Experiment evidence is incompatible with the view that the penetrating particles are produced by nucleons; nucleons would show similar effect in lead and brick roofs. Thus the present experiment further supports the view that the local penetrating showers which are produced by nucleons are qualitatively different from the extensive penetrating showers.

The number of penetrating events should be proportional to

$$K = \Sigma(\text{prod}) / \Sigma(\text{abs.})$$

As $\Sigma(\text{abs}) \sim Z^2$ per atom, and the comparison of lead and iron absorbers has shown experimentally that $\Sigma(\text{prod}) / \Sigma(\text{abs.})$ is indep. of Z,

$$\Sigma(\text{prod}) \sim Z^2$$

$K \sim 1/50$. $\quad \Sigma(\text{prod}) \sim \dfrac{\Sigma(\text{rad})}{50}$

∴ (a) particles somewhat heavier than electrons (heavy electron or λ-meson)
(b) neutral particles emitted by electrons as a kind of collision radiation.

The latter hypothesis seems unlikely, because penetrating agents are found to discharge

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Jánossy and McCusker (continued)

frequently two covered trays (m) and (d) on top of each other, Heavy electrons observed must have energies exceeding 200 MeV. as they penetrate 15 cm of Pb. They are presumably produced by photons. If their rest energy is of the order of a few MeV, then the proper life is estimated to be of the order of $\tau \sim 10^{-11}$ sec. This short life fully accounts for the absence of cloud chamber evidence (see, for example, Fretter, P.R. 73 (1941), 41) Jánossy, Rochester and Broadbent (Nature 155 (1945), 142) obtained a picture which may possibly show a decaying heavy electron in the left-hand bottom corner. The particles penet. a lead plate see on the same photograph might very well be examples of heavy electrons.

The probability that a heavy electron $E' > 200$ MeV traversing the * 30 cascade units is about

$\exp(-30 K \alpha(E'))$ $K \sim (m_e/m_h)^2$,

~~30K~~ $\alpha(E')$ is a fn derived from cascade theory. Now ~~from~~ $K = 1/50$, $K = 1/30$, $\alpha = 1$ are reasonable values, so that $m_h \sim 6 m_e$.

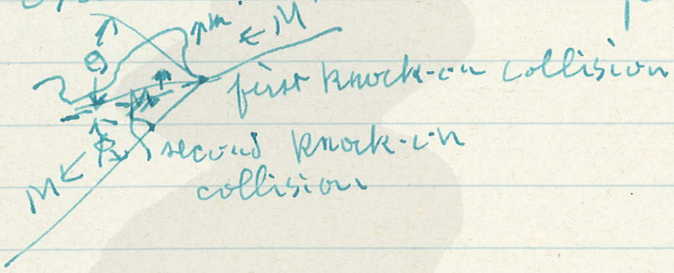
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Evidence for the Existence of a Low-Mass Meson

F. W. Cowan, (Cal. Tech.)

(Science Nov. 12, 1948 108, 534)

Altitude 8,850 m. random expansion



no magnetic field

Energy transferred to the electron is 75,000 eV

$$\frac{M}{m} = \frac{\sin(2\theta+\phi)}{\sin\phi}$$

$\theta = 39°$ $\phi = 5.0°$: $\frac{M}{m} = 11.4$

maximum value $\left(\frac{M}{m}\right)_{max} = \frac{1}{\sin\phi} = 11.5$

The possibility of M=m can be ruled out on the basis of the ionization and degree of multiple scattering.

second collision gives an upper limit 30 to M/m.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

R. Wilson, Collie, Halban (Oxford)
Cross-sections for the Disintegration of the Deuteron by 2.76 MeV γ-Rays.

(Nature 163 (1949), 245)

(γ-rays from forbidden lines RaTh 3.2 MeV.
Na^{24} 4.14 MeV)
γ-rays from Radiosodium 2.76 MeV.

RaTh cross-section	14.8 ± 1.0	
Ra Na "	15.6 ± 1.0	
Ratio	1.05 ± 0.08	
Theoretical Ratio		meson mass
$\frac{\sigma_{Na}}{\sigma_{RaTh}}$	$= 1.21$	296
	1.22	197

(Hulthén)

Primary Soft Component of Cosmic Radiation

Jánossy (Nature ibid. 246)
Rossi-Hulsizer's experiment does not lead to the conclusion that soft primary is only a few percent of total primary intensity.

Rossi and Hulsizer (")
Upper limit is about 11 % for the number of electrons in the primary radiation.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Knoek, Continuous γ-Emission in N-P collisions
(Proc. Phys. Soc. A 62 (1949), 19

exchange-force model for energies up to 20 MeV. cross-section is about 10^{-28}–10^{-29} cm² too small to be observed,

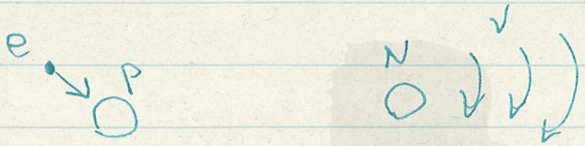
a) electric dipole $\frac{1}{2} e r$

b) magnetic dipole $\mu_p \sigma_p + \mu_n \sigma_n$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar Feb. 11, 1949

Wheeler, Problem of Nuclear Electrons.



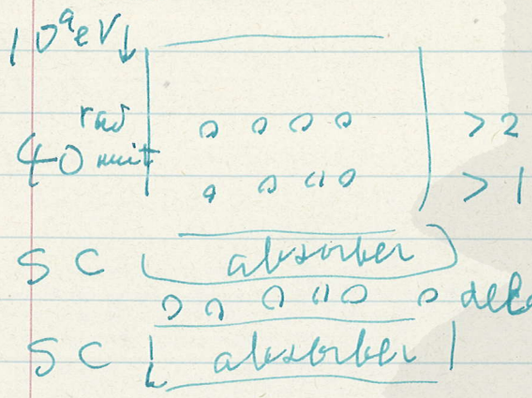
I. Purely bipartite
II. Highly polymorphic
III. Not a multi-field pict.
IV. Relation to meson theory.
V. Neutrino.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar　　Feb. 12, Saturday.

Morette : Colorado Experiments

Piccioni : Hard Shower Producing Particle



C : S = 1.9　　μ

C : S = 1.1　　π

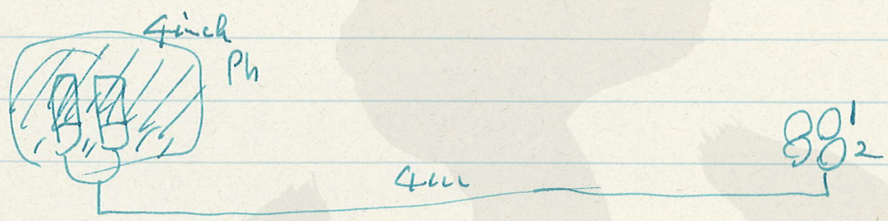
exp. 1.1

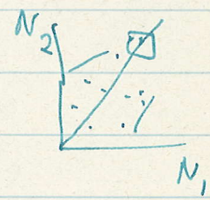
Altitude dependence

10^9　$e^{-x/120}$

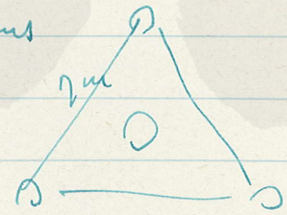
10^{11}　$e^{-x/60}$

Air Shower　　Lewis





Williams



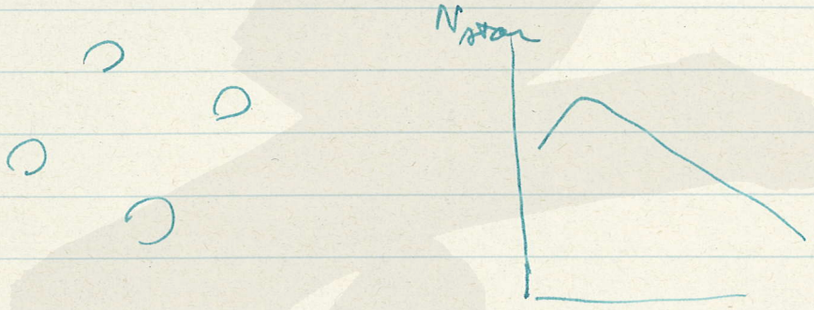
600 particles

Core structure

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Christy, Mill :

Lewis, Absorption of Star Producing Particle



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Colloquium, Feb. 17, 1949

Bernaldini, Absorption of Nucleonic Component of Cosmic Rays.

	number	energy
{ hard shower	10	BeV

star

locally generated hard shower → π-mesons (Piccioni)

absorption of hard shower producing particle.

$118 \leq \lambda_a \leq 135$ grm cm^{-2}. (airplain)

$\lambda_{Pb} = 310 \pm 30$ - - - 310 ± 50

$\lambda_C = 140 \pm 20$ $\lambda_{Fe} = 200 \pm 10$

Tin lot

Echo-lake, Ithaca } 133 ± 7

Cocconi

absorption of star producing particle.

$L_{air} = 135 \pm 4$ 22 km (baloon)

$L_{Pb} = 360 \pm 5$ 315 ± 20 327

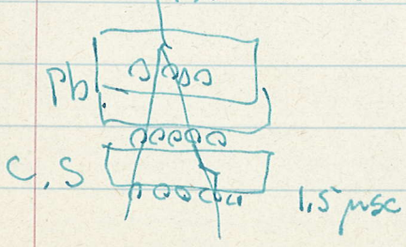
$L_{Al} = 166 \pm 20$

$L_C =$ 140 ± 30

no transition effect excludes π, φ, p mesons

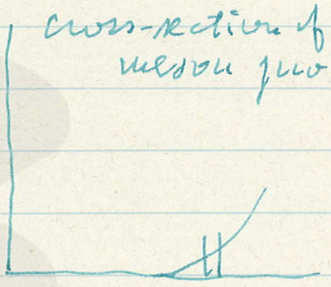
contribution of π no more than 10%

Piccioni { $\pi \to \mu \to e$, $\mu \to e$ C.S ≅ 1, 2



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

decaying
star particle ; all π-meson



$$I' = \frac{N_s}{\sigma' N} = \frac{1.7 \cdot 10^{-4}}{\sigma' N}$$

$$I = 2\pi J_0 \left\{ e^{-h/135} + \left(\frac{h}{135}\right) E_i(-h/135) \right\} \simeq 7.1 \cdot 10^{-4}\ \mathrm{cm^{-2}\,sec^{-1}}$$

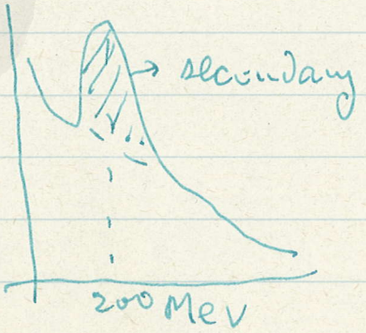
$\sigma' \simeq 5.8 \cdot 10^{-24}$ air

not only primary, second nucleons contribute to star production.

15

Goldenberg - Wuthsegsen
elastic collision. not production of meson
Primary proton (<30 Bev) hits Ag-nucleus

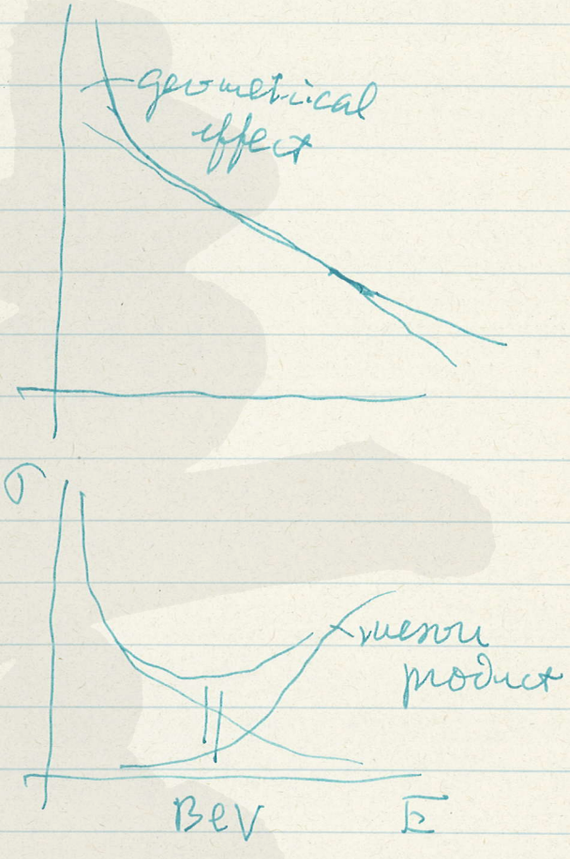
Anderson
upper atmosphere
protons
multiplicity ~10



Schein
transition effect

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Cocconi



meson production is multiple.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Harding, Lattimore, Li, Perkins
Absorption of Star-Producing Radiation under Ice
(Nature 163 (1949), 319, Feb. 26)

$I = I_0 \exp(-h/\ell)$

$\ell =$ 160 gm./cm^2) in air
140 gm/cm^2
320 gm/cm^2 in lead
200 gm/cm^2 in ice

Assuming the cross-section for star production proportional to geometrical cross-section of the nucleus, we obtain

$\sigma_{Oxygen} = 0.15$ barns

Thus we conclude that the stars are produced by stable particles, their range depending on the atomic weight of the absorber.

Meson Masses ...
Born, Rodriguez ibid. 320.

$L_n^{(1)}(x) \simeq 0$

$m = b\sqrt{x}/c$. $\quad ab = \pi$.

$a = e^2/\mu c^2$ $\quad \mu$: electron mass

$\frac{m}{\mu} = \frac{\pi c}{e^2}\sqrt{x}$

$x = 2 \longrightarrow \frac{m}{\mu} \simeq 194$

1.26 4.73
0.93 3.31
⋮ ⋮ ⋮ ⋱

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Theoretical Seminar

Feb. 18, 1949

Prof. Gamow, Origin of Elements and Galaxies

$$\frac{dl}{dt} = \sqrt{\frac{8\pi G}{3}\rho l^2 - A^2}$$

$A^2 > 0$ closed
$A^2 < 0$ open

$$\rho_{mat} = \frac{1}{l^3} \qquad \rho_{rad} = \frac{aT^4}{c^2}$$

$$\frac{1}{l}\frac{dl}{dt} = \sqrt{\frac{8\pi G}{3}\rho - \frac{A^2}{l^2}}$$

$\rho \sim 10^{-30}$ $\qquad A = \cancel{\alpha l^*} = \alpha l^*$; $l^* = 10^{10}$ cm.

heat capacity

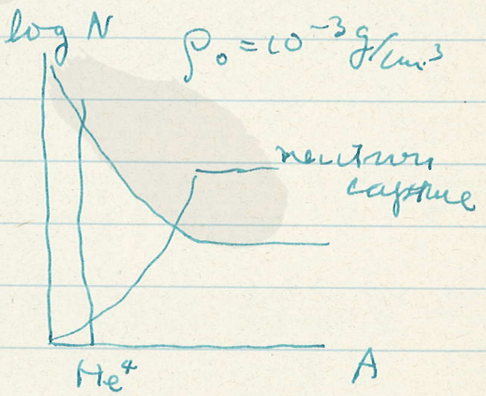
matter: $n = 10^{-6}$ $\qquad kn = 10^{-16} \cdot 10^{-6} = 10^{-22}$

radiation: $4aT^3 = 10^{-15}$

$$T = \sqrt[4]{\frac{3c^2}{32Ga}}\,\frac{1}{t^{1/2}} = \frac{1.15 \cdot 10^{10}\,{}^\circ K}{t^{1/2}}$$

$$\frac{aT^4}{c^2}$$

Alpha ~ Gamow



Fermi: lightest elements

$n \to p + e$

$n + p \to D + \gamma$

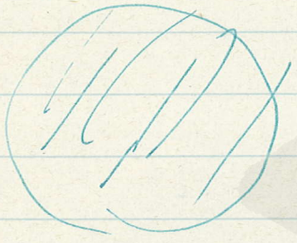
⋮

$\rho_0 = 10^{-3}$ g/cm³.

$He^4 + T^3 \to L^7 + h\nu$

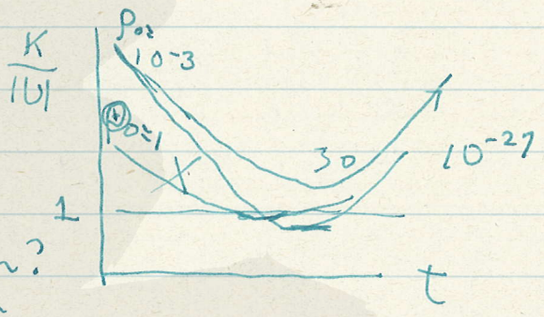
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Galaxy



$K+U+H<0$

$K<|U|$



Isothermal Distribution?

Exchange of momenta in radial direction only.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

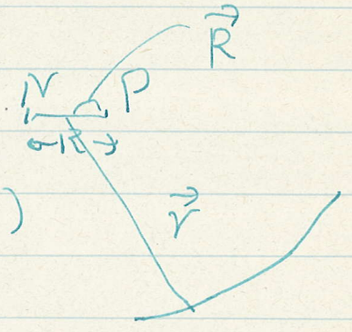
Science Feb. 24, 1949

A. Bohr, ^2H - Hyperfine Structure

Bethe

$$\frac{R}{a}$$

$$\vec{R} = a\left(1 + \frac{r_0}{a} + \left(\frac{r_0}{a}\right)^2 + \cdots\right)$$



$$\frac{e^2}{2mc}\vec{\alpha}\left(\vec{P}\frac{1}{|r-R|} + \cdots\right)$$

$$\vec{\alpha}\cdot\left\{\vec{D}\frac{1}{r}\times(\vec{R}\times\vec{P})\right\}$$

$$\psi(r-R)\chi(R)$$

Latter: Berkeley Experiments.

380 α.
190 D.

negative mesons

$$E = \frac{e^2}{2mc^2}(Hp)^2 \quad \text{nonrelat.}$$

$$E = k\,m\left(\frac{R}{m}\right)^n \quad \text{empirical}$$

313 ± 16 me

284 ± 2%

($\frac{1}{500} \to 160$ me)

only heavy mesons are ejected

~~Elem~~ Energy Spectrum

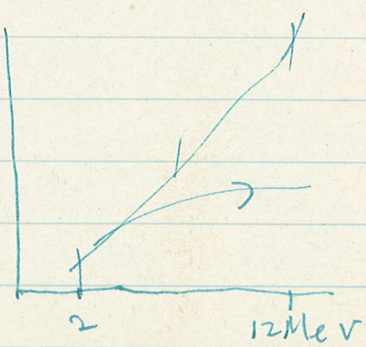
Cross-section

4 ~ 6 MeV.

Theor. Cross-section

$E^{1/2}(E-E_0)^4 f(E)\,dE$.

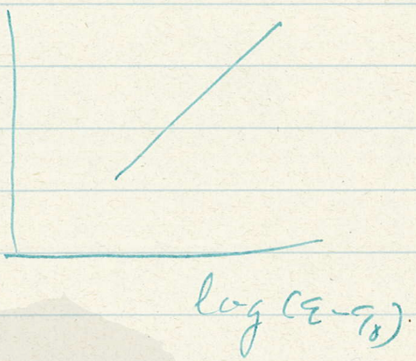
↓ 45 MeV.



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$\sigma = 1.8 \times 10^{-29}\ cm^2$ α-C-collision

α-energy		meson-number
380	100%	339
	90	164
	80	38
268	70	7

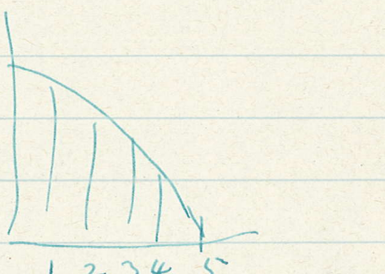


$$\varepsilon = \left(\frac{E}{4} + 5^- + 5^-_1\right)^2$$

$\varepsilon_0 = m_\pi c^2$?

negative

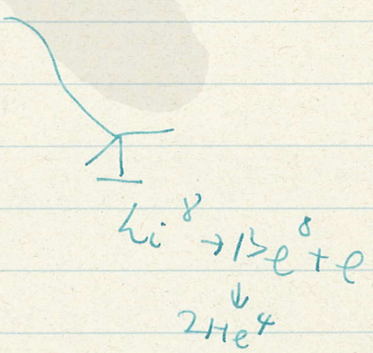
60 or heavy mesons:
73% make stars.
27% no stars
no π-μ-decay



$C_6^{12} + \pi^- \rightarrow 2He_2^4 + H_1^3 + n_0^1$

$O_8^{16} + \pi^- \rightarrow 3He + H_1^3 + n_0^1$

Li^8: peculiar to heavy-ion and π^--stars.
not in proton stars



heavy positive mesons:
relative number of π^- : π^+

Be, C	4 : 1	Σ^- n, p	no positive
Al	15 : 1		
Cu:			

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

π^+ mass 282 m_e
215 m_e

π-μ-decay : 5 MeV μ

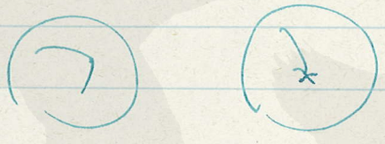
mass ratio $\frac{m_\pi}{m_\mu} = 1.31 \pm .01$

1.311 ± .006

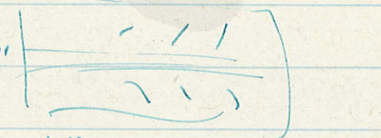
~~ν~~ (m_ν, m_{μ_0}) ~ 0 ~ 15 m_e

half life : $\geq 1.1 \times 10^{-8}$ sec,
mean life ?

Proton beam 350 MeV
4 ~ 6 MeV 40 times
neutron beam → 270 MeV. (mean)
Be.



Synchrotron 330 MeV. electrons 3"
γ-ray → π-mesons
35 : 30 stars
10 ~ 20 MeV.



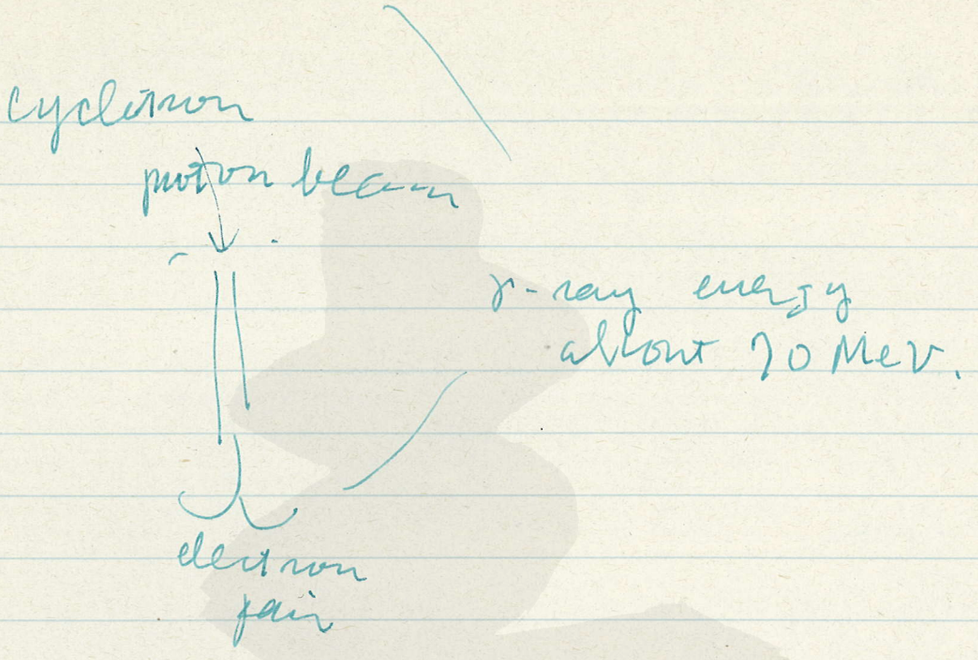
15 ~ 20 Mesons

$\pi \to \mu$-decay : < 10%

p-p : Panofsky

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

5



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Uhlenbeck　　March 8, 1949

Ferromagnetism

Kac – ~~Bohm~~ Berlin

Phase transition is conceivable only we go to the limit of infinitely many molecules.

Ising

$$Q_N = \underbrace{\sum}_{x_p = \pm 1} e^{H \sum x_p x_q}$$

2 dimensional lattice

K. B.

$$Q_N = \frac{1}{A_N (\sqrt{N})} \int \cdots \int_{\sum x_i^2 = N} dx_1 \cdots dx_N \, e^{H \sum x_p x_q}$$

Feldmen　　March 12, 1949

Pauli's letter to Schwinger

von Neumann, Electronic Computer

March 15, 1949

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Sommerfeld Festschrift
(Z.S. f. Naturforschung, Bd 3a
Heft 8~11

G. Wentzel, Über die Feldgleichungen in quantisierten Feldtheorien. 430

H. A. Bethe, Bemerkungen über die Wasserstoff-Eigenfunktionen in der Diracschen Theorie 470

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

G. Wentzel: Helv. Phys. Acta 22 (1949) 101: March 18, 1949

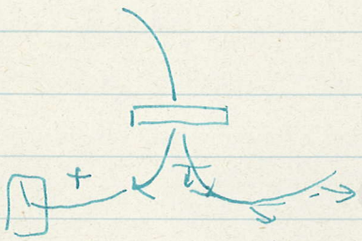
Zur Frage des Spins des π-Mesons.

Wentzel: Polarization

of Vector Meson

nucleon-nucleon collision

anisotropy of decay μ-mesons.



$$\phi_{p\lambda}(\vec{r},\sigma) = u_{p\lambda}(\vec{r},\sigma) + \sum_{p'\lambda'} u_{p'\lambda'}(\vec{r},\sigma)(p'\lambda'|R|p\lambda)$$

$$S = 1 + R$$

$$S^*S = 1, \qquad R + R^* + RR^* = 0$$

$$\phi^*_{p'\lambda'} H(k) \phi_{p_0\lambda_0}$$

$$H(k) = g_v H_v(k) + g_t H_t(k)$$

$$H_v(k) = e^{-i\vec{k}\vec{r}}\, \vec{a}^*_k \left\{ \vec{\alpha} - \frac{\vec{k}}{m} + \left(\frac{\omega_k}{m} - 1\right)\frac{(\vec{\alpha}\vec{k})\vec{k}}{k^2} \right\} \times \left(\frac{m}{\omega_k}\right)^{1/2} \tau$$

$$H_t(k) = e^{-i\vec{k}\vec{r}}\, \vec{a}^*_k \left\{ \vec{\alpha} + i\left[\frac{\vec{k}}{\omega_k} \times \vec{\sigma}\right] + \left(\frac{m}{\omega_k} - 1\right)\frac{(\vec{\alpha}\vec{k})\vec{k}}{k^2} \right\} \times \beta\, i \left(\frac{\omega_k}{m}\right)^{1/2} \tau .$$

$$\phi^*_{p\lambda} H(k) \phi_{p_0\lambda_0} = \sum_{p'\lambda'} (p\lambda|R^*|p'\lambda')(p'\lambda'|H|p_0\lambda_0) + \sum_{p''\lambda''} (p\lambda|H|p''\lambda'')(p''\lambda''|R|p_0\lambda_0) + \cdots$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$E_0 = \sqrt{M^2 + p_0^2} \to M + m$, $\frac{k}{\omega_n}$, $\frac{p}{E_p} \ll v_0 = \frac{p_0}{E_0}$

$(p'\lambda' | e^{-i\vec{k}\vec{\alpha}}\, \vec{\alpha}\cdot\tau | p_0\lambda_0)$, $p' \simeq p_0 - k$

$\cong (p_0\lambda' | \vec{\alpha}\tau | p_0\lambda_0)$

$= \vec{v}_0 (\lambda' | \tau | \lambda)$

$(p'\lambda' | e^{-i\vec{k}\vec{\alpha}}\, \vec{\alpha}\beta\dot{\alpha}\tau | p_0\lambda_0)$

$= [\vec{v}_0 \times (\lambda' | \vec{\sigma}\tau | \lambda_0)]$

$\phi^*_{p\lambda} H \phi_{p_0\lambda_0} \cong g_v (\vec{a}^*_k \vec{v}_0)(p\lambda | R^* \tau | p_0\lambda_0)$

$+ g_t [\vec{a}^*_k \times \vec{v}_0](p\lambda | R^* \vec{\sigma}\tau | p_0\lambda_0)$

$g_t = 0$; $\vec{v}_0 \updownarrow \vec{a}_k$

$g_v = 0$; $\vec{v}_0 \leftrightarrow \vec{a}_k$

π-μ-decay:

$\pi^+ \to \mu^+ + \nu$

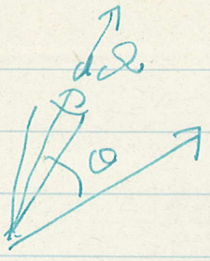
$u^*_{\mu p\lambda} K u_{\nu p_0\lambda_0}$

$K = \gamma_v K_v + \gamma_t K_t$

$K_v = (\vec{\alpha}\cdot\vec{\alpha})$

$K_t = (\vec{\alpha}\cdot\vec{\alpha})\beta i$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室



const. $d\Omega \{1 \mp v_\mu \cos 2\theta\}$

$-$: vector

$+$: tensor

$v_\mu = .27$

$\frac{1+v_\mu}{1-v_\mu} = 1.74$

{Effect of magnetic and electric fields

$F_{\nu\mu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}$ $\quad \partial_\mu = \frac{\partial}{\partial x_\mu} - ieA_\mu$

$\partial_\mu(\partial_\mu \psi_\nu - \partial_\nu \psi_\mu) - m^2 \psi_\nu - i\gamma e F_{\nu\mu} \psi_\mu = 0$

$g = 1 + \gamma$

$\partial_\mu \varphi = m^2 \varphi$

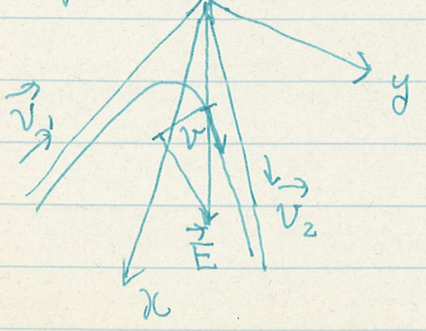
$\psi_\mu = \varphi \cdot a_\mu$

$\frac{\partial \vec{a}}{\partial t} = -\frac{e}{2mc}(1+\gamma)[\vec{H} \times \vec{a}]$

$\vec{s} = i[a \times a^*]$

Laporte:

In nonrelativistic region, depolarization effect is small. Positive π-meson



$|\vec{E} \times \vec{v}| < E_x \cdot v$

$\leqq E_x \cdot v_1$

$\delta \leqq \frac{ge}{2m} v_1 \sin\alpha \int dt\, E_x$

$\frac{m}{e} |v_2 - v_1|$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\delta \leqq g v_1^2 \sin\alpha \sin\frac{\alpha}{2}$$

$$\Delta^2 = \sum \delta^2$$

$$d(\Delta^2) = N \cdot v\,dt \int_0^{2\pi} d\alpha \int dp\, p\, \delta^2$$

$$d(\Delta^2) \leqq N \cdot v\,dt\, g^2 \frac{\pi}{2} \left(\frac{Ze^2}{m}\right)^2 \log\left(1+\frac{R^2}{\rho_1^2}\right)$$

$$\rho_1 = \frac{Ze^2}{m v_1^2}$$

Born approximation

$$g^2 \to \gamma^2$$

$$-dv = N v\,dt \cdot \frac{4\pi Z e^4}{m_e m v^3} \log\gamma$$

$$d(\Delta^2) \lesssim |dv|\, v^3 \frac{Z m_e}{m} .$$

$$\Delta^2 \lesssim v_i^4 \cdot \frac{Z m_e}{m}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Wigner　　March 22, 1949

Shell Source Model of Nuclei

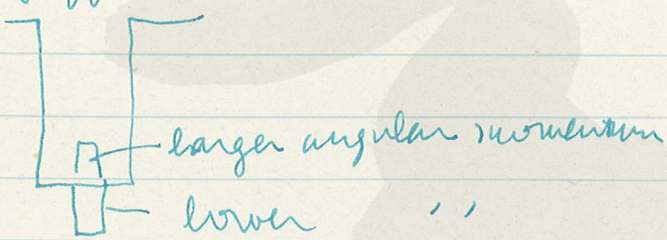
1. Feenberg,
2. Nordheim,
3. Meyer,

Bethe

2　8　10　20 | 28　40　50　82　126

$(1s)^2(2p)^6(2s)^2(3d)^{10}$ | $\cdot\cdot(3p)\ (4f)\ (3s)(4d)(5g)$

Guggenheim and Flasser

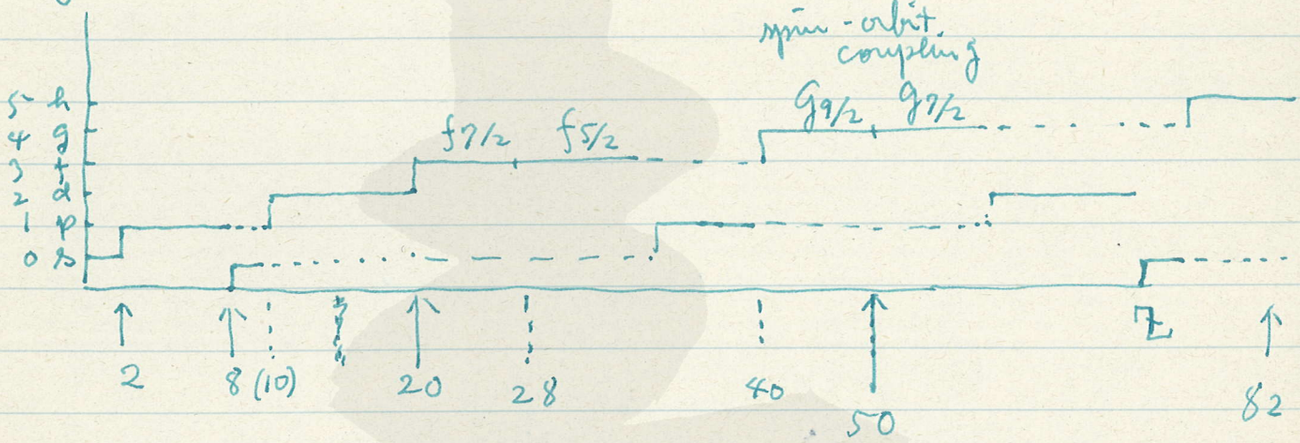


1. Spin, Magnetic Moment
2. β-decay
3. Power

Deuteron;　S-state;
Tensor force

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Meyer:



Nordheim:

β-energy

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar

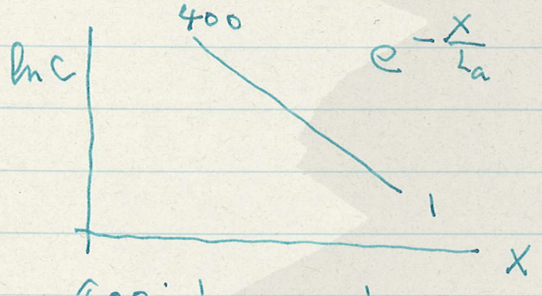
Rossi, Present State of Cosmic Ray Problems

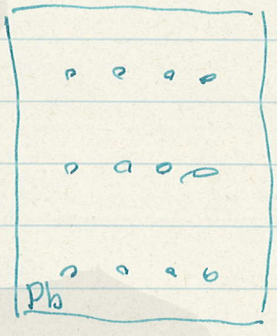
March 24, 1949

1. Electromagnetic Interaction

2. Nuclear Interaction

Jonossy
Variation with altitude





400 : 1　　$L_a = 118 \pm 2$

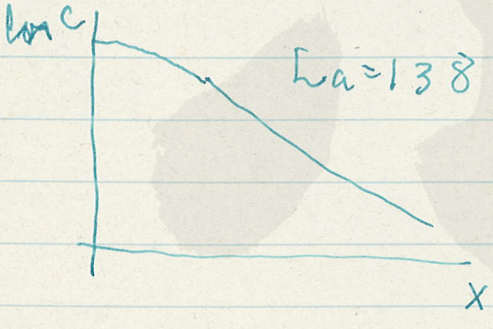
not by μ-meson

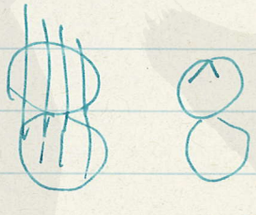
proton-neutron ○
π-meson ×

6 : 1



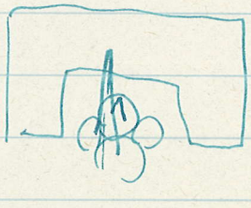
Ionization chamber



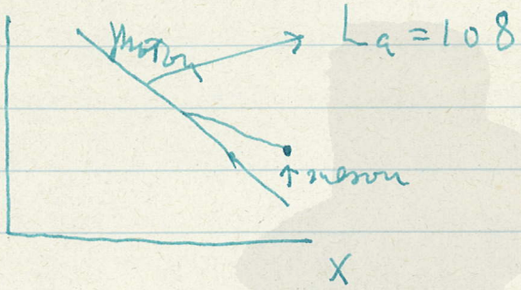


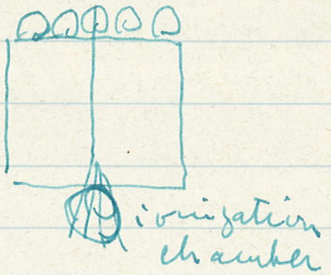
2 : 98

lead shield
absorption small



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室





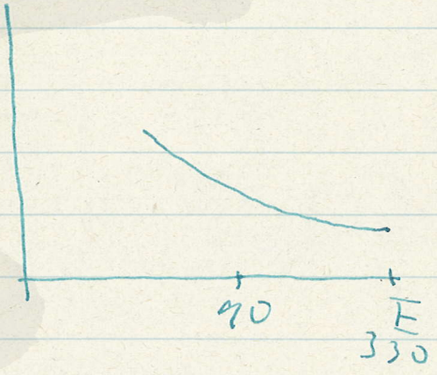
Photographic Plates

Proton → 6 mesons

n-p: 8×10^{-26}

n-n: $\frac{1}{4}$ ··

n-nuclear(air) scattering
~ 138 per star



production of π-meson

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

~~September~~ March 25, 1949

Case, Application of S. T. method to meson theory

Vector meson theory

$$\kappa G_{\mu\nu} = \frac{\partial \phi_\nu}{\partial x_\mu} - \frac{\partial \phi_\mu}{\partial x_\nu} - \Gamma_{\mu\nu} \qquad \Gamma_{\mu\nu} = \frac{g}{\kappa}\bar{\psi}\sigma_{\mu\nu}\psi$$

$$\kappa \phi_\nu = \frac{\partial}{\partial x_\mu} G_{\mu\nu} + \Gamma_\nu/\kappa \qquad \Gamma_\nu = i g \bar{\psi}\gamma_\nu\psi$$

$$\mathcal{L} = -\frac{\kappa^2}{2}\left[\frac{G_{\mu\nu}G_{\mu\nu}}{2} + \phi_\mu\phi_\mu\right] + \Gamma_\nu\phi_\nu$$

Specialize σ:

$$i\hbar c \frac{\delta \Psi}{\delta \sigma} = \left(-\Gamma_\mu \phi_\mu - \frac{\Gamma_4^2}{2\kappa^2}\right)\Psi$$

$$\to \left(-\Gamma_\mu \phi_\mu + n_\mu n_\nu \Gamma_\mu \Gamma_\nu / 2\kappa^2\right)\Psi$$

$$\phi_4 = \frac{i c}{\lambda^2}\frac{\partial \pi_i}{\partial x_i}$$

P. S. Theory

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar April 19

Vallarta

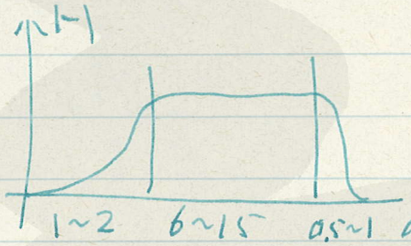
Energy Spectrum: $E^{-2.8}$ etc.

Galactic Rotation

Sideral Time Dependence

	+	−
0.04%		
6.5 h	1	1

Solar Effect



Proton energy up to 7.3 BeV

Cygnation (cygnus = swan)

Tunnel of allowed zone

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar　　March 29, 1949

Rohrlich, High Energy Nucleon-Nucleon Scattering

central force

$$\sigma = \frac{4\pi}{k^2} \sum_L \sin^2 \delta_L \cdot (2L+1)$$

tensor force

$$\sigma \gtrless \frac{4\pi}{k^2} \sum_j \frac{2j+1}{3} \left(\sin^2 \delta_j^{\alpha} + \sin^2 \delta_j^{\beta} + \sin^2 \delta_j^{\gamma} \right)$$

Pais-Power　　$\tau \to 2\mu + \pi \to$ decay

$\tau^- \to (\pi^-) + 2\mu$　　X

$(\tau^+) \to \pi^+ + 2\mu$　　X

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Seminar

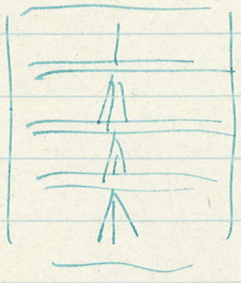
April 2.

Steinberger

Schiff: Bremsstrahlung

soft comp. accompanying nuclear component

Fretter:



3~3 showers

60% soft particles

neutral mesons or Bremsstrahlung?

$$2\pi \rho_E |\psi_{\text{running out}} A \psi_{\text{plane}}|^2$$

total energy radiated W:

$$\frac{W}{E} = \frac{4}{3}\frac{\alpha}{\pi}\left[\ln \frac{E}{\mu c^2} - 19/2\right]$$ spin 0

$$\frac{W}{E} = \frac{11\alpha}{4680\pi}\left(\frac{E}{\mu c^2}\right)^4$$ spin 1

average energy $\frac{1}{3}E$

cut-off : $\sim \frac{E^2}{(\mu c^2)^2}$

.5~5

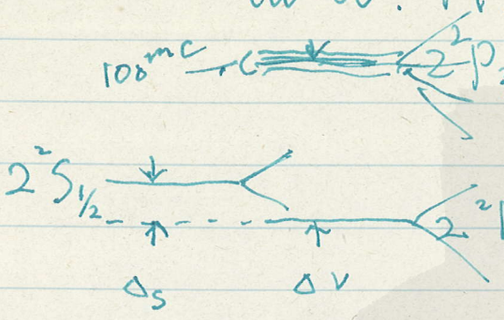
Oppenheimer: neutral meson

According to Serber the intensity of γ-ray increases ~~on the~~ rapidly as the meson intensity increases.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Kellogg meeting : May 12, 1949

Lamb. H, He^+ fine-structure



$$\Delta_s = \frac{\alpha^3}{2\pi} Z^4 R_y \log \frac{a mc^2}{\eta R_y Z^2} + \Delta'$$

(H) $\Delta\nu = \frac{\alpha^2}{16} R_y \left(1 + \frac{\alpha}{\pi}\right)$ $\sim 10,960\ mc$

$\Delta_s \cong 1051\ mc + \Delta'$

$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.031} = \frac{1}{137.042}$

R.F.

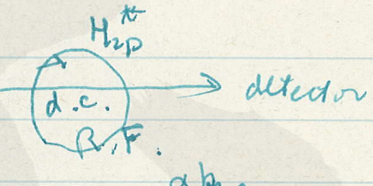
metastable $\tau_{2s} = \frac{1}{7}$ sec

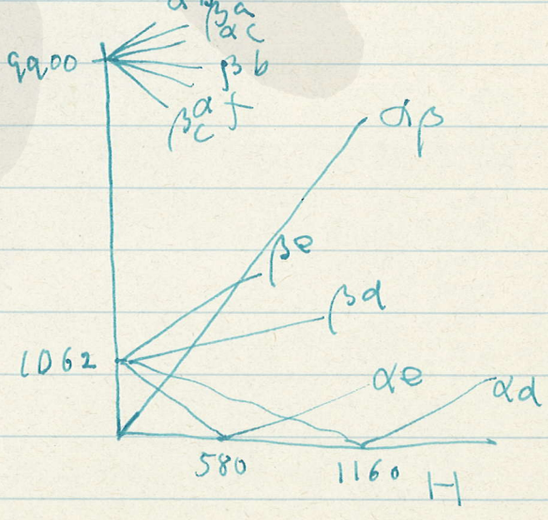
$\tau_{2p} = 1.6 \cdot 10^{-7}$ sec

beam

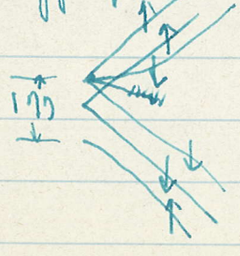
$H_2 \xrightarrow[RT]{} H_{1s} \xrightarrow[\text{electron bombardment}]{} H_{2s}^*$

$h\nu \rightarrow H_2 \rightarrow H_{1s} \rightarrow$





hyperfine structure



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

^2H

^1H $\quad \Delta_s = 1060.6$

$\Delta = 1062.2$

1060.9

He^+ $\quad \Delta_s = 14.0 \pm .2 \times 10^3$ mc

Theor. Seminar May 13, 1949

Placzek, Slow Neutron Scattering

coherent scattering

incoherent scattering

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Miss Morette, Creation of Mesons
by Nucleon-Nucleon Collision

Relativistic Calculations

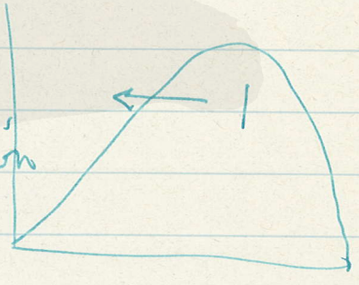
$\sigma = f\, 10^{-26}\ cm^2$ for one meson

for total energy $2Mc^2$ in centre of mass system, which corresponds to $\frac{1}{2}Mc^2$ 500 MeV for laboratory system.

Energy dependence is $E^{1.8}$

Maximum of distribution is between $\frac{1}{2}E_{max}$ and E_{max}.

The effect of coupling ~~reduces the~~ shifts the distribution to lower energy.



There is certainly effect of pair creation in intermediate states.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Sneddon, Touschek (Univ. Glasgow)

Production of Mesons by Electrons

(Nature 163, 524 (1949))

$e + P \to e' + P' \to e' + N + \pi^+$

vector meson:

$$\sigma = \frac{3}{\pi}\left(\frac{1}{137}\right)^2 \frac{g^2}{\hbar c}\left(\frac{\hbar}{m_\pi c}\right)^2 I(E)$$

$E/m_\pi c^2$	1.25	1.5	2.0	4.0
$10\,I(E)$	2.4	21	200	2000

neglecting recoil

$E \sim 300$ MeV $\sigma \sim 10^{-33}$ cm^2

$e + N \to e + P + \pi^- \to e' + P' + \pi^-$

heavier elements: multiplying factor

$A\,Z$ for π^+

$A(A-2)$ for π^-

$E \sim 300$ MeV $\sigma \sim 10^{-29}$ cm^2

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所 湯川記念館史料室

Theory of H. S. W. Massey

Non-central interactions between neutron and proton

(Univ. College, London)

(Proc. Roy. Soc. A 146 135 (1934))

Ramsey (1947) Proc. Roy. Soc.

Rarita-Schwinger (1941)

$$\left.\begin{matrix} -\frac{1}{3}(\tau_1\tau_2) \\ -\frac{1}{2}(1+\tau_1\tau_2) \\ 1 \end{matrix}\right\}\left\{-\frac{\hbar^2}{M}\frac{a}{r_2^2}\left[1+\frac{1}{2}g(\sigma_1\sigma_2-1)\right.\right.$$

$$\left.\left.+\gamma\left(\frac{3(\sigma_1 r)(\sigma_2 r)}{r^2}-\sigma_1\sigma_2\right)\right\}V(r/r_0)\right.$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

D. H. Perkins, Nuclear Capture of μ-Mesons

Nature 163 (1949), 682.

$\mu^- + P \rightarrow N + \nu$

$\pi \rightarrow \mu + \nu$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Gregory : Cosmic Rays

May 2, 1949

0 · 1 air shower

3600 1000 stopped 2350 230

meson protons

19 : 19 , 50

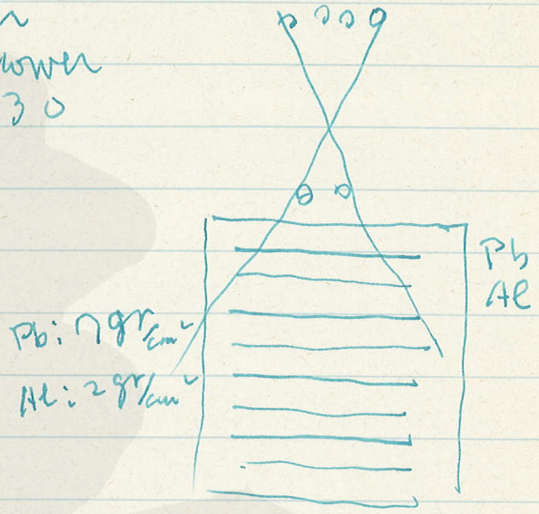
nuclear events

50

LAS

14

7Pb 7Al



no penetrating particle stopped in Pb.

σ_G $\cancel{P}$

$$\frac{P}{M} = \frac{N_p}{2350} \qquad I_p = 8.6 \cdot 10^{-4}$$

$\cong$ 4 ~ 5%

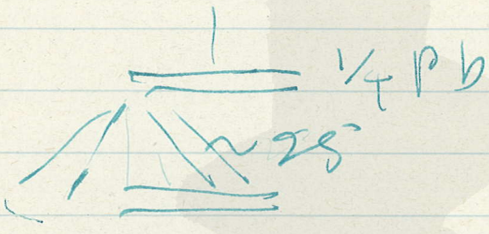
— 4 BeV $I = 10^{-1}$

700 gr

— > 350 MeV $I_M(\sigma)$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

F. Oppenheimer:



π-meson: 198
30~40 MeV

114 disint

22 +
176 −

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Bakerian lecture

Nuclear Explosions

Sir George Thomson

Imperial College of Science and Techn.
London

(Proc. Roy. Soc. 196 (1944), 311)

Blau and Wambacher, S.B. Akad. Wiss. Wien, 146 (1937), 469, 623.

Stetter and Wambacher, ~~Z.f. Phys.~~ Phys. Z. 39 (1938), 883; 40 (1934), 702.

The plates are not sensitive to neutrons (> 8 MeV), protons (> 100 MeV) and ~~apb~~ alphas (> several thousand MeV)

Scotch proverb: Fools and bairns should not see a thing half done.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Nuclear disintegration produced by cosmic rays

Harding, Lattimore and Perkins

(Proc. Roy. Soc. 196 (1949), 325)

(e) Range of Nuclear Forces

Heisenberg, Nuovo Cimento 15 (1938), 31

Bagge, Phys. Z. S. 21 (1944), 461

Rosenfeld, Nuclear Forces. Vol I.

Gaussian potential

$$\log_{10} n(E) = \text{const} - \left[0.029 \left(\frac{1}{\chi d}\right)^2 + 0.0096 \times \left(\frac{2r_0}{d}\right)^2\right] E$$

for protons (> 30 MeV)

$$\chi^{-1} = (2.0 \pm 0.3) \times 10^{-13} \text{ cm}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Galant, Cosmic Ray Experiments
May 21, 1949

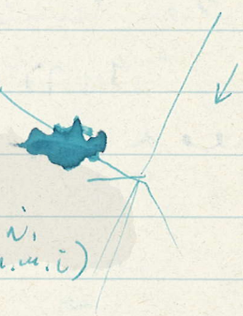
altitude	fm			cut-off
7700 ft	7%	9 prongs		
(60,000)?	1/3	9 prongs	(Minesota)	2.5 BeV
(93,000)	40~60%		(Cuba)	8 BeV

Cuba, minimum ionization plates.

collimated star

	mc^2	high altitude
25 (m.i)	~1	10%
50 (n.m.i)	0.1~1	50% (m.i. + n.m.i)

h.



lowest number of prongs 8

average 15

highest number of prongs 32



©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

(Bernaldini, Phys. Rev. 74 (1948), 1878))

George and Jason, Nuclear Disintegrations in Photographic Plates exposed to Cosmic Rays under Lead Absorbers. (Proc. Phys. Soc. 62A (1949), 243)

star intensity

$$I = I_0 \exp(-h/R)$$

$R = 140\ gm/cm^2$.

(Perkins, Nature 159 (1947); 126; 160 (1947) 299; 707.)

lead absorber thickness upto 28 cm.

altitude 3,457 m (Jungfraujoch)

mean range in lead of N-radiation:

$$R_{Pb} = 310 \pm 20\ gm/cm^2.$$

$$R_{air} = 150 \pm 7\ gm/cm^2$$

energy required for average star

~ 100 MeV.

× (photon star (Baldwin, Klaiber, P.R. 70 (1946), 259; 73 (1948), 1156) is ruled out

× proton star not more than 5%.

× meson star / ~~only appears when~~ caused by σ-mesons coming to rest), which is only 2%.

× meson stars due to capture of σ-mesons in flight is also small in number)

○ neutron star

$a = \pi r^2$ $\qquad l = A^{1/3}/N\pi r_0^2\ gm/cm^2$

$l_{air} = 130\ gm/cm^2$ $\qquad l_{Pb} = 310\ gm/cm^2$.

$l_{Pb}/l_{air} \approx 2.4$ (theor), 2.1 ± 0.2 (observ)

(constancy of cross-section over few hundred MeV)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Harding, Origin of Cosmic Ray Stars
(Phil. Mag. 40 (1949), 513)

$R = \frac{\text{No. of recoils upwards}}{\text{No. } \quad \text{downwards}}$

$$= \frac{\frac{1}{2}\int_{\theta_0}^{\pi/2} \cos\theta\, d\theta}{\frac{1}{2}\int_{-\pi/2}^{\theta_0} \cos\theta\, d\theta} = \frac{1-\sin\theta_0}{1+\sin\theta_0}$$

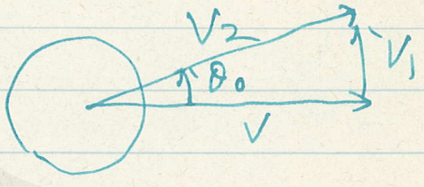
$$= \frac{102}{197}$$

(V_2: downward velocity of excited nucleus)
(V_1: random velocity)

$\sin\theta_0 = \frac{V_2}{V_1}$ $\Rightarrow V_2 = 0.27\, V_1$

correction of direction of incidence

$$V_2 = \frac{4}{3} \times 0.27\, V_1 = 0.37\, V_1$$



average range : 4.3
mean velocity : $V = 3.3 \times 10^8$ cm/sec.

$V_2 = 3.2 \cdot 10^8$ cm/sec

$V_1 = 1.2 \cdot 10^8$ cm/sec

mean momentum transfer : $\cong 175$ MeV/c (heavy nuclei)
$\cong 160$ " (light nuclei)

mean excitation energy : $\cong 190$ MeV (heavy n.)
of kinetic energy U $\cong 130$ " (light n.)

X1. neutron brought to rest : m_0: rest mass of neutron

$$pc = \sqrt{2m_0c^2U + U^2}$$ p : momentum transfer

$U \sim 190$ MeV $\Rightarrow$ $p \cong 600$ MeV/c.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

2. Neutron with velocity near c, (without capture)

× 3. photons

○ 4. Unstable particle

$m_0 c^2 = \sqrt{E^2 - p^2 c^2}$

$m_0 c^2 \cong 74\ \mathrm{MeV}$

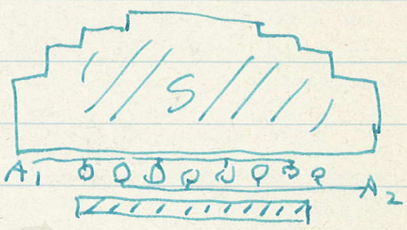
$m_0 \sim 150\ m_e$

○ 5. Meson with heavier mass (for example, 500 MeV/c^2) is given captured by in the ~~the~~ upper part of nuclei.

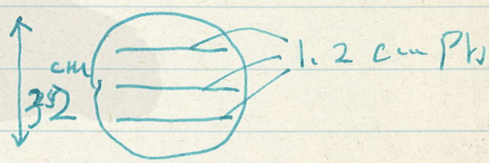
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

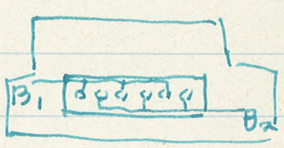
Nuclear Interactions of the Particles produced in Cosmic Ray Bursts

L) Rovati, Mura, Salvini and Tagliaferri
(Nature 163 (1949), 1004)
University of Milan.
Testa Grigia (3,500 m)
one discharge in each of groups A_1, A_2, B_1, B_2.



Successive multiple production: the most energetic of the particles produced may successively give rise to new bursts. Cross-section for the production of new bursts





$$\sigma = \frac{4 \times 10^{-25}}{p} \text{ cm}^2 \text{ per nucleus Pb}$$

p: probability that a burst produced within the plate may still appear in the cloud chamber with heavily ionizing particles. ~ 0.4.

$\sigma \sim 10^{-24}$ cm^2

geom. cross-sect. of Pb $\sim 2.5 \times 10^{-24}$ cm^2

Large percentage of events are produced by the secondary generated in the absorbers. Scattering is very rare, almost none.
(Wilson Cimento)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Production of Cosmic Ray Stars

George Thomson

(Phil. Mag. 40 (1949), 589)

(1) Number of stars diminishes with depth x as $e^{-x/L}$ $L = 160$ gr./cm^2

(Perkins, Nature 160 (1947), 707)

(2) Similar exponential diminution occurs if the plates are under ice or lead.

(3) Transition effect is doubtful.

(4) Absorption under ice and Pb (and air) is proportional to $A^{2/3}$.

(5)

Heitler and Janossy:

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

○ Mechanism of π-meson disintegrations
By D. H. Perkins,
Phil. Mag. 40 (1949), 601.

J. Evans and E. P. George
○ Observation of Nuclear Disintegrations below Ground
Nature 164 (1949), 20.

sea level: 1.5 ± 0.07 stars per c.c. per day
60 m water depth: 0.01 ± 0.004 " "
ratio: 150 ± 60
(radiation responsible for the majority of stars above ground is absorbed with a range ~ 160 gm/cm² and would give rise to about 10^{-17} stars/c.c./day at 60 m depth)
μ-nucleon scattering?
$\sigma_n = 0.5 \times 10^{-28}$ cm²
mean energy 10^{10} eV
(Coulomb force: $\sigma = 10^{-30}$ cm² for recoil proton with 100 MeV)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Wilson, Proc. Roy. Soc. A. 174 (1940), 73.
Code, Phys. Rev. 59 (1941), 229
Schutt, " 61 (1942), 6.
at sea level, scattering of mesons:
$\sigma \sim 5 \times 10^{-28}$ cm^2.

Other possibilities:
(i) electrons or photons produced by μ-mesons: cross-section for photo-disint. of nuclei: $\sigma_e = 7 \times 10^{-26}$ cm^2.
(ii) neutral particles produced by the μ-mesons or in the atmosphere by energetic primary.

○ Structure of Cosmic-Ray Air Showers (Nature 164 (1949), 21) by Pro J. R. Prescott and C. B. O. Mohr.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

D. Basu, Relativistic Scattering of Neutrons by Protons. (Proc. Roy. Irish Academy 52 (1949), 127)

charged vector, pseudoscalar fields

E.R. $\quad Q = \frac{1}{5\pi}\frac{g^4}{(\hbar c)^4}p_0^2$

N.R. $\quad Q = \frac{3}{32\pi}\frac{g^4}{(\hbar c)^4}M^2\left\{\frac{10\mu^2}{\mu^2+4p_0^2} - \frac{\mu^4}{p_0^2(\mu^2+2p_0^2)}\log x\right\}$

$x = \frac{\mu^2+4p_0^2}{\mu^2} \qquad E_0 = \sqrt{M^2+p_0^2}$

$\frac{\chi^2 g^2}{4\pi\hbar c} = 0.13$ for $\mu/M = 1/10$

$\frac{\chi^2 g^2}{4\pi\hbar c} = 0.242$ for $\mu/M = 1/5$

p_0/M	0	0.25	0.50	1
E_{pion} (c.m.)	0	28	110	390
E_{pion} (l.s.)	0	117	470	1880
$Q_{1/10}$ (10^{-24} cm^2)	(2.11)	0.144	0.117	0.642
$Q_{1/5}$ (10^{-24} cm^2)	(0.458)	0.090	0.052	0.149

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

vector (charged or neutral) without δ-fn

$$Q = \frac{1}{32\pi}\frac{g^4}{(\hbar c)^4}\begin{cases}\frac{160}{27}p_0^2 & (\Sigma.R)\\ \frac{16}{3}M^2 & (N.R.\ p_0 \gg \mu)\end{cases}$$

V. with δ-fn

$$Q = \frac{1}{4\pi}\frac{g^4}{(\hbar c)^4}\begin{cases}\frac{2}{3}p_0^2 & \because\\ 2M^2 & \because\end{cases}$$

p.sc. without δ-fn

$$Q = \frac{1}{24\pi}\frac{g^4}{(\hbar c)^4}\begin{cases}2p_0^2 & \because\\ 3M^2 & \because\end{cases}$$

p.sc. with δ-fn

$$Q = \frac{1}{12\pi}\frac{g^4}{(\hbar c)^4}\begin{cases}p_0^2 & \because\\ 3M^2 & \because\end{cases}$$

V. (c or n.) without δ-fn

$$Q = \frac{1}{16\pi}\frac{g'^4}{(\hbar c)^4}\cdot\begin{cases}4\mu^2 & \\ M^2\frac{\mu^2}{p_0^2} & \end{cases}$$

with δ-fn

$$Q = \frac{1}{4\pi}\frac{g'^4}{(\hbar c)^4}\begin{cases}\frac{1}{3}p_0^2 & \\ M^2 & \end{cases}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

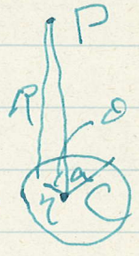
A Property of the Yukawa Potential

I. N. Sneddon and C. K. Thornhill
(Proc. Camb. Phil. Soc. 45 (1949), 318)

C. J. Bouwkamp, Physica 13 (1947), 501.

mutual energy:

$$W = \int_0^a d\eta \int_0^{\pi} 2\pi\eta^2 \sin\theta\, \rho(\eta) \frac{\partial v(r)}{\partial\theta}$$



$$r^2 = \eta^2 - 2\eta R\cos\theta + R^2$$

$$V(r) = \int_{r_0}^{r} r\, v(r)\, dr$$

$$\frac{\partial V}{\partial\theta} = r v \frac{\partial r}{\partial\theta} = R v \eta \sin\theta$$

$$W = \frac{2\pi}{R}\int_0^a \eta\rho(\eta)\{V(R+\eta) - V(R-\eta)\}d\eta$$

$$\underline{V(R+\eta) - V(R-\eta) = V'(R) f(\eta).}$$

$$W = \frac{2\pi}{R} V'(R) \int_0^a \eta\rho(\eta) f(\eta)$$

$$= m(a)\, v(R)$$

$$m(a) = 2\pi\int_0^a \eta\rho(\eta) f(\eta)\, d\eta$$

$$\underline{\frac{V'''(R)}{V'(R)} = \frac{f''(\eta)}{f(\eta)} = \lambda^2 \quad (\text{say})}$$

$$D(D+\lambda)(D-\lambda)V(R) = 0$$

$$V(R) = A + Be^{\lambda R} + Ce^{-\lambda R}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$v(r) = \frac{1}{r}\frac{dV}{dr}$$

$$V(r) = -\frac{e^{-\lambda r}}{r} \qquad (y)$$

Thus the only forms of interaction for which two spheres attract each other as if reduced masses are concentrated at their centres are those which are embraced in the Yukawa law (y).

~~$v(r) = -e$~~

$$f(\eta) = \frac{2}{\lambda}\sinh\lambda\eta$$

$$m(a) = \frac{4\pi}{\lambda}\int_0^a \eta\,\rho(\eta)\sinh\lambda\eta\,d\eta$$

$$w = \int_0^a d\rho \int_0^{2\pi} d\theta\, 2\pi\rho^2 \sin\theta \frac{e^{-\lambda r}}{r}$$

$$\sin\theta\frac{e^{-\lambda r}}{r} = -\frac{1}{\lambda\rho R}\frac{d}{d\theta}(e^{-\lambda r})$$

$$w = \frac{2\pi e^{-\lambda R}}{\lambda R}\int_0^a (e^{\lambda\rho} - e^{-\lambda\rho})\,\rho\,d\rho$$

$$= (2\pi a)^{3/2}\frac{I_{3/2}(\lambda a)}{\lambda^{3/2}}\frac{e^{-\lambda R}}{R}$$

$$I_{3/2}(z) = \sqrt{\frac{2}{\pi z}}\left(\cosh z - \frac{\sinh z}{z}\right)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

E.H. Burhop and H.V. Yeadon, University College, London

Scattering of high-energy neutrons by protons and the law of interaction between nucleons

(Proc. Roy. Soc. A 197 (1949), 505) ~523

neutron energy 83 ~ 250 MeV

Born approximation

non-central

spherical well

Yukawa well (1.18×10^{-13} cm)

I. Sym. Th.

$$V = \frac{\hbar^2}{M r_0^2} \tau_1 \tau_2 \left[\frac{a}{12} (\sigma_1 \sigma_2 + 3) + \frac{a\gamma S_{12}}{3} + \frac{g}{4} (\sigma_1 \sigma_2 - 1) \right] V(r/r_0)$$

II. charged

$$\frac{\hbar^2}{M r_0^2} \frac{1 + \tau_1 \tau_2}{2} \left\{ \frac{a}{4} (\sigma_1 \sigma_2 + 3) + a\gamma S_{12} + \frac{g}{4} (\sigma_1 \sigma_2 - 1) \right] V(r/r_0)$$

III. neutral

$$-\frac{\hbar^2}{M r_0^2} \left[\frac{a}{4} (\sigma_1 \sigma_2 + 3) + a\gamma S_{12} - \frac{g}{4} (\sigma_1 \sigma_2 - 1) \right] \times V(r/r_0)$$

Even state

$$^1V_{even} = -\frac{\hbar^2}{M r_0^2} g \, V(r/r_0)$$

$$^3V_{even} = -\frac{\hbar^2}{M r_0^2} \cdot a \cdot (1 + \gamma S_{12}) V(r/r_0)$$

Hu and Massey, P.R.S. 196 (1949), 1402

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所, 湯川記念館史料室

Nature 154 (1944), 567

W. Heisenberg, Max Planck Institute for Physics, Göttingen

Production of Meson Showers

Heisenberg, Cosmic Rays (1943)
equations 38 ~ 40.

meson spectrum produced

$$dI = a\,dk_0/k_0 \qquad (\hbar = c = 1)$$

$\frac{dI}{k_0}$: differential intensity

$$\Sigma = a\int_\kappa^\varepsilon dk_0/k_0 = a \log \varepsilon/\kappa$$

$$a = \varepsilon/\log \varepsilon/\kappa$$

Σ : total energy available in c-system

$$n = a\int_\kappa^\varepsilon dk_0/k_0^2 \approx \varepsilon/\kappa \log \varepsilon/\kappa$$

n : average number of mesons

$$\Sigma = \sqrt{2M(M+E)} - 2M_\kappa - \Delta\Sigma$$

$$\Delta\Sigma \sim \kappa^2/M$$

$$n \propto E^{1/2}$$

primary proton spectrum $dE/E^{2.7}$

$$\bar{\varepsilon} \approx 1.42\sqrt{2M(M+E_0)} - 2M - 2\Sigma$$

$M/\kappa \approx 6.5$:

$\varepsilon \approx 2.9M$ $n \approx 6$ for $E_0 = 5^- M$

ϑ : direction of meson in r-system

$\cos\vartheta \approx 1 - \frac{8}{n^2}$ for $n > 4$

(visible number of mesons : $\frac{2}{3}n$ in symmetric theory)

Maximum intensity $\rightarrow$ $E = 10M$ (400 MeV)

lies at 400 MeV for

Shower containing n-mesons: maximum at $65\,n$ MeV

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

μ-meson spectrum:

$$dn \sim a p_0 / p_0^{-2.17} / \log(4.3 p_0/\kappa).$$

positive excess:

$$(M p_0/\kappa^2)^{-1/2} \cdot 0.14 \log(0.65 M p_0/\kappa^2)$$

(Bernardini: positive excess decreases as $p_0^{-1/2}$)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

R. Huby, Capture of Negative Mesons
(Univ. Liverpool)
(Phil. Mag. 40 (1949), 685

Time of trapping of a negative meson in lowest Bohr orbit.

Ferretti, Nuovo Cimento 5 (1948), 325.

π-mesons:

μ-mesons:

heavier mesons:

R.L. Rosenberg, Loss of Energy of Slow Negative Mesons in Matter
(ibid. 759)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

(Euler and Kockel, Naturwiss 23 (1935), 246)

H. Y. Tzu (Manchester)

On the Interaction between Mesons

(Phil. Mag. 40 (1949), 717)

The interaction between mesons transmitted by the sea of nucleons in negative energy states is investigated for low energy mesons. Range of interaction: meson Compton wavelength, singularity: R^{-5}, total cross-section for scattering of mesons by mesons: $10^{-29}\,cm^2$. The existence of a closed stationary or quasi-stationary assembly of mesons is unlikely.

(Scattering matrix element increases as the square of the momentum of the scattering particles.

Neutral, pseudoscalar mesons are considered.

In general, the interaction term is

$$H_s \sim G^n \qquad n > 2$$

G: field quantities, More generally,

$$H_s \sim (G_1 + G_2)^n$$

G_1, G_2: Two wave packets.

$$(p'|V_n|p'') \sim p^n$$

$$V_n(x) = (x'|V_n|x'') \sim \int \exp\left(-\frac{i}{\hbar c} p x'\right) p^n \times \exp\left(\frac{i}{\hbar c} p x''\right) dp$$

$(\times \delta)$

$$V_n(x) \sim \frac{1}{R^{n+3}} \int \exp\left(-\frac{i}{\hbar c} pR\right)(pR)^n d(pR)$$

R^{-m}: $m - n = 3$. (Coulomb scattering p^{-2}. $n = -2$ $m = 1$)

(meson theory R^{-3}, $\frac{1}{R}\delta'(R)$)

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Double Stars with Relativistic Particles from Cosmic Rays

By Prof. M. Cosyns, Mrs. C. C. Dilworth, Dr. G. P. S. Occhialini and Prof. M. Schönberg.
Centre de Physique Nucléaire, Université Libre de Bruxelles.

Kodak NT4, 400 microns thick, Pic du Midi

Fig. 1.



β-decay of light elements (such as He^6, C^{10} with the life-time of the order of 1 sec.)

Fig. 2. β-decay

Fig. 3.



electron emission during the capture of a meson by an atom?

Fig. 4.

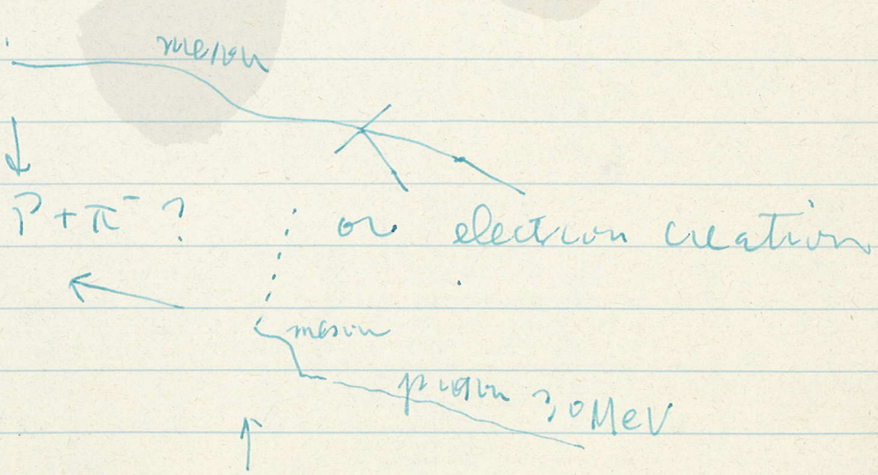


Fig. 5.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

G. B. Harding and D. H. Perkins

Production of Heavy Mesons in Cosmic Ray Stars.

(Nature 164 (1949), 285)

glacier at Jungfraujoch

(See Nature 163, 319, 1949.)

Assumptions

(1) Rate of production of mesons at a given depth is proportional to the star intensity at that depth.

(2) Meson-producing radiation and mesons to produced travel vertically downwards

(3) loss of energy of the mesons by ionization processes only need be taken into account.

Star intensity at a depth x:

$$S_x = S_0 \exp(-x/\lambda) \qquad \lambda = 200 \text{ gm/cm}^2$$

The range spectrum of mesons

$$q(x)\,dx = \frac{1}{\alpha}\frac{dP}{dx}\exp(-x/\lambda)\,q(x)\,dx$$

α: average multiplicity

$P(x) =$ $\frac{\text{number of mesons coming to rest}}{\text{stars at depth } x}$

$$q(x) = \exp\left[\int_0^x \frac{E_0\,dx'}{c\rho\tau_0(E+\bar{E}_0)}\right] \qquad \bar{E}_0 = 140 \text{ MeV}$$

$$\tau_0 \sim 6\times10^{-9} \text{ sec.}$$

Mean range $\bar{R} = 100 \text{ gm/cm}^2$ (energy 250 MeV)

$q(x)$: very small for $x > 300 \text{ gm/cm}^2$ (energy 600 MeV)

©2022 YITP, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

THE INSTITUTE FOR ADVANCED STUDY
PRINCETON, NEW JERSEY

May 20, 1949

Dr. Yukawa

There will be a general informal discussion of the year's work starting at 2:00 P.M. on Saturday, May 28th. Following this, there will be drinks and supper at Olden Manor; wives and husbands are welcome.

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Last Seminar

May 28, 1949

(1)

Pais-Uhlenbeck,

Two Masses:

$$(\gamma_\mu \partial_\mu + \varkappa e^{-\lambda \gamma_\mu \partial_\mu})\psi = 0$$

$$e^{\alpha \frac{d}{dx}} f(x) \to f(x+\alpha)$$

$$(\gamma_\mu \partial_\mu + \varkappa \cosh \lambda \gamma_\mu \partial_\mu + \varkappa \sinh \lambda \gamma_\mu \partial_\mu)\psi = 0$$

$$\gamma_\mu \partial_\mu + \varkappa \cosh \lambda \sqrt{\Box} + \varkappa \frac{\gamma_\mu \partial_\mu}{\sqrt{\Box}} \sinh \lambda \sqrt{\Box})\psi = 0$$

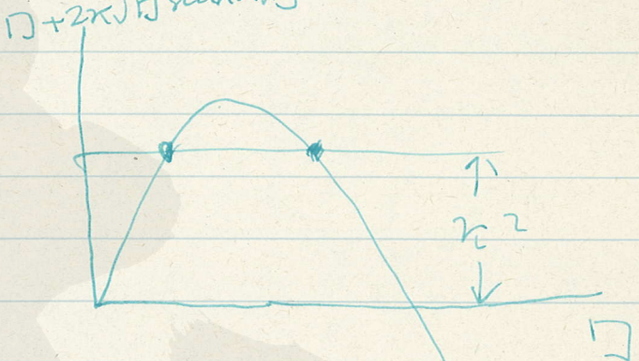
$$\{\Box \left(1 + \frac{\varkappa}{\sqrt{\Box}} \sinh\right)^2 - \varkappa^2 \cosh^2\}\psi = 0$$

$$\left(\Box + 2\varkappa \sqrt{\Box} \sinh \lambda \sqrt{\Box} + \varkappa^2\right)\psi = 0$$

$$\psi = e^{i k_\mu x_\mu}$$

$$\Box = -k_\mu^2 = M^2$$

$$\lambda \sim \frac{\hbar}{4\pi mc}$$



Finite distance Operator

$$e^{\alpha \frac{d^2}{dx^2}} f(x) = \frac{1}{\sqrt{4\pi\alpha}} \int d\xi\, f(\xi)\, e^{-\frac{(x-\xi)^2}{4\alpha}}$$

Born: $e^{-\lambda^2 \Box} \Box A_\mu = -S_\mu$

$$(1 - \lambda^2 \Box)\Box A_\mu = 0.$$

(Pais, Uhlenbeck) Phys. Rev. 1939

rel. cut off
compensation

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

$$\mathcal{L} = -\hbar \bar{\psi} E^{(\lambda_\mu D_\mu^-)} (\gamma_\mu D_\mu^- + \kappa_0) \psi$$

$$- \frac{1}{i} \frac{\partial A_\mu}{\partial x_\nu} f(\lambda \Box) \frac{\partial A_\mu}{\partial x_\nu}$$

$$D_\mu^- = \frac{\partial}{\partial x_\mu} - \frac{i\varepsilon}{\hbar} A_\mu$$

f : integral function (no zeros)

f : has simple zero's on real positive axis (no other zeros)

$$f = \prod_1^\infty \left(1 + \frac{\Box}{\omega_k^2}\right) \left(e^{g(\Box)}\right) \qquad g(\Box) = 0 \text{ for the moment}$$

$$\sum \frac{1}{\omega_k^2} : \text{convergent}$$

F integral

simple 0's on real neg axis.

product repres. with the same property as f.

$$F(\lambda \gamma_\mu \partial_\mu) = \prod_{k=1}^\infty \left(1 + \frac{\lambda \gamma_\mu \partial_\mu}{\kappa_k}\right)$$

$$\sum \frac{1}{\kappa_k} = \text{const.}$$

Model

$$\mathcal{L} = -q f(D^2)(1 + D^2) q \qquad D = \frac{d}{dt}$$

$$= -q F(D^2) q$$

$$F(D^2) = \prod_{k=1}^\infty \left(1 + \frac{D^2}{\omega_k^2}\right)$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Ostrogadski: $\mathcal{L}(q, q^{(1)}, q^{(2)}, \cdots q^{(N)})$

$$\left.\begin{aligned} q &= Q_1 \\ q^{(1)} &= Q_2 \\ &\vdots \\ q^{(N-1)} &= Q_N \end{aligned}\right\}$$

$$P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}}\right) + \frac{d^2}{dt^2}\left(\frac{\partial^2 L}{\partial \dddot{q}}\right) - \cdots$$

$$P_2 = \frac{\partial L}{\partial \ddot{q}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dddot{q}}\right) + \cdots$$

$$P_i = \frac{\delta L}{\delta q^{(i)}}$$

$$H = \sum_1^N P_k \dot{Q}_k - L$$

$$= \sum_1^{N-1} P_k Q_{k+1} + P_N \dot{Q}_N - L$$

$$P_N = \frac{\delta L}{\delta q^{(N)}} \quad \text{etc.}$$

$$Q_k = \underset{k \text{ missing}}{\Pi'}\left(1 + \frac{D^2}{\omega_n^2}\right) q$$

$$\mathcal{L}(q, \dot{q}, \ddot{q} \cdots)$$

$$\bar{\mathcal{L}}(q, \dot{q}, \ddot{q} \cdots) = G\mathcal{L} + \frac{dX}{dt}$$

$$\mathcal{L} = -q F(D^2) q$$

$$\bar{\mathcal{L}} = -\sum_1^\infty \eta_k Q_k \left(1 + \frac{D^2}{\omega_k^2}\right) Q_k$$

$$\cong -\sum_1^\infty \eta_k q \{\Pi'\}^2 \left(1 + \frac{D^2}{\omega_k^2}\right) q$$

$$\cong -\sum_1^\infty \eta_k q \frac{F^2(D^2)}{1 + \frac{D^2}{\omega_k^2}} q$$

$$(c = 1) \quad \sum_1^\infty \frac{\eta_k}{1 + \frac{D^2}{\omega_k^2}} = \frac{1}{F(D^2)}$$

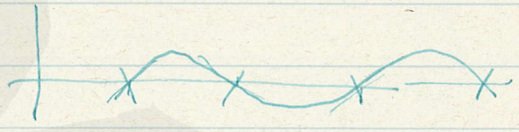
©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Mittag-Leffler:

$$\frac{1}{F(D^2)} = \sum \frac{\omega_k^2}{F'(-\omega_k^2)} \frac{1}{1+\frac{D^2}{\omega_k^2}}$$

$$\eta_k = \frac{\omega_k^2}{F'(-\omega_k^2)}$$

$$\xi = \Sigma \eta_k Q_k$$



Conservation laws:

$$F(\gamma_\mu D_\mu^-) = F_1(\sqrt{D_\mu^{-2}}) + \gamma_\mu D_\mu^- F_2(\sqrt{D_\mu^{-2}})$$

$$\bar{F}(\gamma_\mu D_\mu^- + \gamma_{10})\Psi = 0$$

$$\bar{\Psi} = 0$$

$$\left(F_1(\sqrt{D^{+2}}) - \gamma_\mu D_\mu^+ F_2(\sqrt{D^{+2}})\right)(\gamma_\mu D_\mu^+ - \kappa)\Psi = 0$$

$$j_\mu = \Sigma \xi_k \underbrace{\bar{\Psi}_k \gamma_\mu \Psi_k}_{j_\mu^{(k)}}$$

$$\Box Q_\mu^0 = -J_\mu^0$$

$$J_\mu^0 = i\epsilon \sum_0^\infty \eta_k \bar{\Psi}_k \gamma_\mu \Psi_k$$

$$\Box Q_\mu^k - \mu_k^2 Q_\mu^k = J_\mu^k$$

$$J_\mu^k = \eta_k J_\mu^0 \qquad k \geq 1.$$

Elm. field: $\xi_0 = \frac{1}{c}$ $\quad c = f(0)$

$$\sum_{k\geq 1} \xi_k = \frac{f(0)}{f'(\mu_k^2)} \cdot \frac{1}{\mu_k^2}$$

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Electrons: $\eta_0 = 1$

$C = F(-x_0)$

$\eta_k = -\dfrac{F(-x_0)}{F'(-x_k)} \dfrac{1}{x_k - x_0}$ $\quad k \geq 1$

1. Electron self-Energy:

Pauli $\quad \sum C_i = 0$

$\bar{D}_0(x) \to \bar{D}_0(x) + \sum_s \zeta_s \mu_s^4 \bar{D}_s(x)$

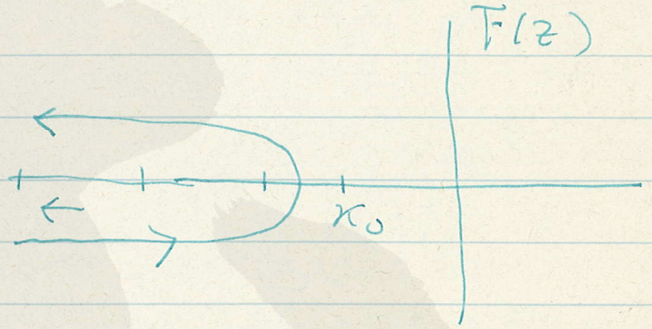
$\sum \zeta_s \mu_s^4 = -1.$

2. Photon Self Energy:

(a) $\sum C_i' M_i'^2 = 0$

(b) $\sum C_i' = 0$ automatic

(a) $\oint dz \dfrac{z - x_0}{F(z)} = 0$



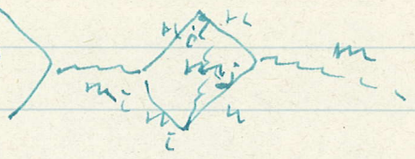
Nonlocalizable Field

$\Psi(x) \to \Psi$ ("interval")

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Steinberger, Regulator

(a) $\begin{cases} (i) \sum c_i = 0 \\ (ii) \sum c_i m_i = 0 \\ (iii) \sum c_i m_i^2 = 0 \end{cases}$ scalar meson $\to 2\gamma$.



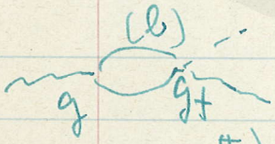
(b) $\begin{cases} \sum c_i \ln m_i = 0 \\ \sum m_i c_i \ln m_i = 0 \\ \sum m_i^2 c_i \ln m_i = 0 \end{cases}$ for $m_i \to \infty$

(a) $\pi^{(0)}$ vector, ps. vector $\to 10^{-3}$ sec. $\to 3\gamma$.
$\pi^{(0)}$ sc. $\to 1$ sec. $\to 4\gamma$
ps sc. $\to 10^{-14}$ sec. $\to 2\gamma$
→ ps. sc
→ ps. vector

(i) $\left(\frac{m_e}{m}\right)^8$

(ii)

(i) × equivalance



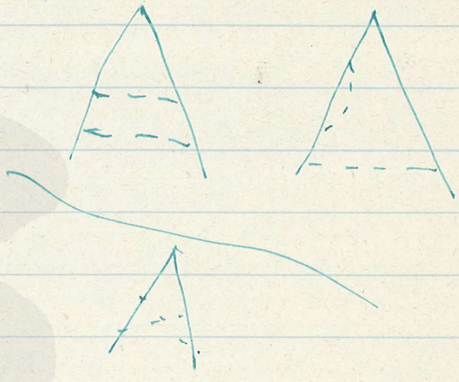
$\pi^{(\pm)} \to e^{(\pm)} + \nu$

10^{-2} sec ~ 100 sec

©2022 YHAL, YITP, Kyoto University
京都大学基礎物理学研究所　湯川記念館史料室

Karplus, Magnetic moment (4)

$$+\frac{2.049\,\alpha^2}{\pi^2}$$



Rohrlich, Self-Stress.

$$\Theta_{11} = \frac{1}{3}\left(\Theta_{\mu\mu} - \Theta_{44}\right)$$

$$\Theta_{44} = -H_{self}$$

Without
With) regulator

$$\frac{\alpha}{2\pi}$$

$$\left.\begin{aligned} &\Sigma c_i = 0 \\ &-\Sigma c_i \ln\frac{M_i}{\kappa} = \ln \end{aligned}\right\} \frac{\alpha}{2\pi} \cancel{\left(3\ln + \frac{3}{4}\right)}$$