DEPARTMENT OF PHYSICS

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突然手紙を差上げますが失礼の段おゆるし下さい。実は先生より先日荒木教授の許に送られた手紙の写しが中村氏のところ迄届いたのを拝見しましたところ、核力ポテンシャルの正確な計算について Van Hove とか云う人のやって居られることがどうやら小生のものに似ているように思れますので、もう少し詳しいことが知りたいと存じます。

小生の結果は昨年10月の学会で報告し、素粒子論研究第2号にも載せておきましたが、計算の方法は摂動論を使わず、接觸変換に operational method（素粒子論研究第1号、及び昨年夏先生にお渡ししました note）を併せ用いるもので、完全に相対論的に問題を取扱い、nucleon の recoil 及び retardation が式の中に取入れられて居ります。

核力は scalar meson では $\frac{1}{r}$、pseudoscalar では $\frac{1}{r^2}$、vector 及び pseudovector では $\frac{1}{r^3}$ ですが、特に実際の transition (scattering) に於しては pseudoscalar の場合も $\frac{1}{r}$ になることは先生のお便りと一致するように思われます。たゞ direct interaction はちょっと調べたところでは cancel しないような気がしたので深く調べずにそのまゝになって居りました。核力に $\frac{1}{r^2}$、$\frac{1}{r^3}$ が ~~現れ~~ 残っていることは、Dirac 方程式のような1次の微分方程式に於てはやはり具合が悪いように思われるので、あまり気乗りがせずその後計算を進めて居りません。しかしそちらで同じような結果が出ているのならばもう少し検討して見たいと存じ

ますので、なるべく詳しくお知らせ願えれば幸いと存じます。なおこれに関し progress に送った letter の写しを御参考までに同封致します。
それから、小生は目下 Tomonaga の理論に基いて nucleon の magnetic moment の計算を進めて居りますが、これについては既に America で結果が得られている由、その概要は洩れ聞いていても詳細な点が分り兼ねますので、これももしできれば甚だ勝手ながらお知らせ願いたいと存じます。

最後に先生の generalized wave equation について小生の最近考えましたことをお知らせして御批判を仰ぎたいと存じます。これは素粒子論研究第2号に簡単に書きました通り、第3量子化法という概念を導入して operator に関する方程式を解かうという考えです。即ち第2量子化した波動函数の積は常に星のついたものを、星のつかないものの左に置くようにして zero pt fluctuation をはっきり分離するという建前をとれば、かような量（operator）自身を1つの operand と考えて、それを変化させる新しい operator（第3量子化された operator）を導入することができます。かような第3量子化の formalism は第2量子化と完全に parallel に組立てることができ、従って更に第4、第5、… と進むこともできます。これを generalized wave equation

$$[p_\mu[p^\mu, A]] = 0 \qquad (1)$$

に応用する場合には、Aをpとqの函数（$[p,q]=-i$ ~~$[q,p]=1$~~）で、pを常にqの左におくように整頓したもの：　$A = f(p)g(q)$ と考えます。~~すると~~、1般にqの積 q^n にpを乗じた結果を整頓すれば…

$$p\times q^n = pq^n$$

$$q^n\times p = q^{n-1}[qp] + q^{n-2}[qp]p + q^{n-3}[qp]p^2 + \cdots + [qp]p^{n-1} + pq^n = nq^{n-1}[qp] + pq^n$$

$$\therefore\ [p, q^n] = nq^{n-1}[qp] = inq^{n-1} = i\frac{\partial}{\partial q}q^n \qquad (2)$$

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従って (1) は

$$-\frac{\partial}{\partial q_\mu}\cdot\frac{\partial}{\partial q^\mu} f(p_\lambda)\cdot g(q_\nu) = f(p_\lambda)\cdot\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q^\mu} g(q^\nu) = 0 \qquad (3)$$

となり，これは普通の波動方程式です．故にその基本解は

$$g(q^\nu) = e^{i(M_\nu q^\nu)}, \qquad M_\nu M^\nu = 0 \qquad (4)$$

です．同様にして (1) に reciprocal な式

$$[q_\mu[q^\mu A]] = 0 \qquad (5)$$

は

$$[q, p^n] = i\frac{\partial}{\partial p}p^n \qquad (6)$$

を使って

$$-\frac{\partial}{\partial p_\mu}\frac{\partial}{\partial p^\mu}\cdot f(p_\lambda)\,g(q_\nu) = 0 \qquad (7)$$

となりますから

$$f(p_\lambda) = e^{i(N_\lambda p^\lambda)}, \qquad N_\lambda N^\lambda = 0. \qquad (8)$$

故に

$$A = e^{i(N_\lambda p^\lambda)}\cdot e^{i(M_\nu q^\nu)} \qquad (9)$$

が一つの基本解となります．これの matrix element は energy-momentum が保存され，且し light cone の上にあるような2つの状態の間でだけ値を有するということは，これを $e^{ik_\alpha x^\alpha}$ に operate して見れば分ります．即ち

$q_\alpha = x_\alpha$，$p_\alpha = \frac{1}{i}\frac{\partial}{\partial x_\alpha}$ と置きかえれば

$$e^{iN_\lambda p^\lambda}\cdot e^{iM_\nu q^\nu}\cdot e^{ik_\alpha x^\alpha} = e^{i(k_\alpha+M_\alpha,\ x^\alpha+N^\alpha)} \qquad (10)$$

となり，$\vec{k}$ という energy-momentum の粒子が $\vec{x}$ という点で $\vec{M}$ という光を吸收して，$\vec{x}+\vec{N}$ という点の $\vec{k}+\vec{M}$ という状態に移ることを意味します．　A の一般解は (9) の解の linear combination ですから，いろんな $\vec{M}, \vec{N}$ の transition の表わすことになります．　なお解としては p と q の順序を入れ替えたものをとっても勿論差支えないわけです．

先生のお考えでは p と q とは普通の意味の共軛量のように思われますが，Snyder のような形の関係をつけてもやはり同様に（q_α 及びその共軛量の函数を用いて）解くことができると思います．この方面について少し考察して見るつもりで居ります．もし上のようにして non-commutable な量の函数に対する方程式を解くことができるならば

空間の量子化ということもあながち数学的に恐れなくてもよいという気が致します.

以上の考えについて先生の御批判.御教示を仰ぎたいと存じます.
尚量子化法自体については目下詳しく formulation をまとめて居りますが, ほんの outline だけは最近やはり letter として Progress に送りましたので, その写しをも同封致します.

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This is meant to be private communication.

On the Relativistic Formulation of the Perturbation Theory

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The method of the relativistic canonical transformation has proved powerful in dealing with the problems of the self energy and the reaction of the radiation field in Tomonaga's theory of self-consistent subtraction.[1] This method consists in the transformation

$$\Psi = U_1 \Psi_1 \qquad (1)$$

$$U_1 = \exp\left[-i\varepsilon \int^t H^{(1)} dt'\right], \qquad (2)$$

where $\varepsilon H^{(1)}$ is the interaction whose effect we are going to consider. By this transformation the new Hamiltonian becomes

$$H^{(2)} = -\frac{i}{2}\varepsilon^2 \left[H^{(1)}, \int^t H^{(1)} dt'\right] \qquad (3)$$

within the order ε^2. In $H^{(2)}$ occurs integrations containing the invariant D-functions as a result of the commutation of the quantized wave fields. Now, as is well known in the operational calculus

$$\int^t dt' e^{iEt'} x = e^{iEt} \cdot \frac{1}{d/dt + iE}\, x \qquad (4)$$

If we write for short $\int^t dt' \equiv \int$, $\frac{d}{dt} \equiv \delta$, so that $\int\delta = \delta\int = 1$, and take the upper limit of integration to be zero, we easily get the relation

$$\int D(x_0)_\zeta = -\frac{1}{4\pi} \cdot \frac{1}{r} \exp\left(-r\sqrt{\kappa^2 + \delta^2}\right). \qquad (5)$$

For example, in the case of the interaction of the electron and the radiation field:

$$H^{(1)} = \varepsilon S^\mu A_\mu, \quad S^\mu = i\psi^\dagger \gamma_\mu \psi, \qquad (6)$$

the transformed Hamiltonian is

$$H^{(2)} = -\frac{i}{2}\varepsilon^2 \{ s^\mu [A_\mu, \int A_\nu'] s^{\nu'} - \int A_\nu' [s_\nu', s_\mu] A_\mu \}, \tag{7}$$

the first term of which becomes, in virtue of (5),

$$-\left(\frac{\varepsilon^2}{8\pi^2}\right) s^\mu(\mathbf{r}) \{ \exp(-|\mathbf{r}-\mathbf{r}'|\delta)/|\mathbf{r}-\mathbf{r}'| \} s_\mu(\mathbf{r}') = -\frac{\varepsilon^2}{8\pi} s^\mu(\mathbf{r}) \frac{1}{|\mathbf{r}-\mathbf{r}'|} s_\mu(\mathbf{r}', t - |\mathbf{r}-\mathbf{r}'|), \tag{8}$$

or ~~the~~ the Møller's interaction between electrons. We also get the Breit's formula by putting instead of A_μ the transverse wave

$$A_{tr} = A - \iint \mathrm{grad}\,\mathrm{div}\,A. \tag{9}$$

Applications of this methods to the nuclear forces yields following results. Using Kemmer's notation,[2] the nuclear potential is given by

$$\begin{aligned}
&\text{scalar: } -(gQ^2/2) \int w\, V_{ret}\, w'\, d\tau \\
&\text{pseudoscalar: } -(Q/2)(f+2\mu g)^2 \int \bar{s}\, V_{ret}\, \bar{s}'\, d\tau + (Q/2)\, g(f+2\mu g)\, i\frac{d}{dt} \int \bar{t}_0\, V_{ret}\, \bar{s}'\, d\tau, \\
&\text{vector: } -(Q/2) \int \{(g-2\mu f) v_\alpha + f x_\alpha\} V_{ret} \{(g-2\mu f) v_\alpha' + f x_\alpha'\}\, d\tau \\
&\qquad + (Qf/2)\, i\frac{d}{dt} \int u_{i0}\, V_{ret} \{(g-2\mu f) v_i' + f x_i'\}\, d\tau \\
&\text{pseudovector: } -(Q/2) \int \{(f \bar{t}_\alpha + g \bar{x}_\alpha) V_{ret} (f \bar{t}_\alpha' + g \bar{x}_\alpha') + 4f^2\mu^2 \bar{s}\, V_{ret}\, \bar{s}'\}\, d\tau \\
&\qquad + (Q/2)\, i\frac{d}{dt} \int \{-2\mu f^2\, \bar{t}_0\, V_{ret}\, \bar{s}' + fg\, \bar{u}_{0i}\, V_{ret}\, \bar{t}_i' + g^2 \bar{u}_{0i}\, V_{ret}\, \bar{x}_i'\}\, d\tau
\end{aligned} \tag{10}$$

$$V_{ret} = \exp(-|\mathbf{r}-\mathbf{r}'|\sqrt{\kappa^2+\delta^2})/|\mathbf{r}-\mathbf{r}'|, \quad x_\alpha = -i(\psi^\dagger \partial_\alpha \psi - \partial_\alpha \psi^\dagger \psi),$$
$$\bar{x}_\alpha = -i(\psi^\dagger \gamma^5 \partial_\alpha \psi - \partial_\alpha \psi^\dagger \gamma^5 \psi)$$

κ = meson mass, μ = nucleon mass, Q = isotopic spin operator.
The first two potentials have a singularity only of order $1/r^4$, and $1/r^2$ respectively, while for the remaining it is of order $1/r^3$. Delta functions are neglected.[3] Especially for elastic scattering the $1/r^2$ term again drops in the first case. Thus the non-relativistic formula hitherto used seems unjustifiable because it does not take into account the motion of the nucleon. Detailed discussions will appear soon.

(1) Prog. Theo. Phys. 3 (1948), 211. Phys. Rev. 74 (1948), 224.

(2) Proc. Roy. Soc. A. 166 (1938), 127.

(3) A similar idea has been reached by Tanikawa, Kagaku 18 (1948), 439 (in Japanese), and a lecture at the Symposium on Elementary Particles, Oct. 16-17, in Tokyo.

THIS is meant to be private communication

Second Configuration Space and Third Quantization

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Various divergence difficulties in quantized field theory are closely connected with the zero point fluctuation of the field. Formally, it is due to the fact that a quantity composed of q numbers has sometimes a non-vanishing expectation value even in the lowest state considered. To dispose of this zero point fluctuation, we may decompose field quantities into creation and annihilation operators, and rearrange products of these operators in such a way that creation operators (with asterisks) stand always to the left of annihilation operators (without asterisks), thereby separating pure fluctuation terms from the rest. This idea, or essentially the same as this, was applied with brilliant success to quantum electrodynamics by Schwinger[1] and Tomonaga.[2] If we make the convention that every quantity composed of quantized fields should be well ordered in the manner above mentioned, multiplication of two such quantities will require ~~in general~~ a re-arrangement.

Now let us regard a product of quantized fermion fields $\psi_1^* \psi_2^* \cdots \psi_n^* \psi_{1'} \psi_{2'} \cdots \psi_{n'}$, where the suffixes denote space-time as well as spin coordinates, as a <u>state</u> or a <u>wave function</u> specified by these coordinates, and write for short

$$\psi_1^* \psi_2^* \cdots \psi_n^* \psi_{1'} \psi_{2'} \cdots \psi_{n'} \equiv (1, 2, \cdots n; 1', 2', \cdots n') \tag{1}$$

Multiplication by ψ_r^* or ψ_r either from the left or from the right will yield, after rearrangement, the following result:

$$\begin{aligned}
\overrightarrow{\psi}_r^{\,*}(1,2,\cdots n; 1'2'\cdots n') &= (r,1,2,\cdots n; 1',2',\cdots n') \\
\overrightarrow{\psi}_r(1,2,\cdots n; 1',2',\cdots n') &= \sum_{i=1}^{n} (-1)^{i-1}\{r,i\}(1,2,\cdots i-1, i+1,\cdots; 1',2',\cdots n') \\
&\quad + (-1)^n (1,2,\cdots; r,1',2',\cdots n'), \quad \{r,i\} \equiv [\psi_r, \psi_i^*]_+, \\
\overleftarrow{\psi}_r(1,2,\cdots n; 1',2',\cdots n') &= (1,2,\cdots n; 1',2',\cdots n', r) \\
\overleftarrow{\psi}_r^{\,*}(1,2,\cdots n; 1',2',\cdots n') &= \sum_{i'=n'}^{1} (-1)^{n'-i'}\{i',r\}(1,2,\cdots n; 1',2',\cdots i'-1, i'+1,\cdots n') \\
&\quad + (-1)^{n'}(1,2,\cdots n, r; 1',2',\cdots n')
\end{aligned} \tag{2}$$

If we introduce such operators that create (+) or annihilate (−) the fields ψ^* and ψ, and denote them by $\overset{+*}{\psi}, \overset{-*}{\psi}, \overset{+}{\psi}$, and $\overset{-}{\psi}$, Eqs.(1) may be written as

$$\overrightarrow{\psi}_r^{\,*} = \overset{+}{\psi}_r^{\,*}, \qquad \overrightarrow{\psi}_r = \overset{+}{\psi}_r + \sum_i \{r,i\}\, \overset{-}{\psi}_i^{\,*},$$
$$\overleftarrow{\psi} = \overset{\widetilde{+}}{\psi}_r, \qquad \overleftarrow{\psi}_r^{\,*} = \overset{\widetilde{+}}{\psi}_r^{\,*} + \sum_i \overset{\widetilde{-}}{\psi}_i \{i,r\}. \tag{3}$$

We have included in the above defined operators the sign functions, which necessitates thw distinction between ψ and $\tilde{\psi}$. They are connected by simple relations:

$$\overset{\pm}{\psi}_r \overset{\pm}{\psi}_s = -\overset{\widetilde{\pm}}{\psi}_r \overset{\widetilde{\pm}}{\psi}_s, \qquad \overset{\pm}{\psi}_r^{\,*} \overset{\pm}{\psi}_s^{\,*} = -\overset{\widetilde{\pm}}{\psi}_r^{\,*} \overset{\widetilde{\pm}}{\psi}_s^{\,*}, \qquad \overset{\pm}{\psi}_r \overset{\pm}{\psi}_s^{\,*} = -\overset{\pm}{\psi}_r \overset{\pm}{\psi}_s^{\,*},$$
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Further defining new operators by

$$\overset{+}{\Psi}_r^{\,*} \equiv \overset{+}{\psi}_r^{\,*}, \qquad \overset{-}{\Psi}_r^{\,*} \equiv \sum_i \{r,i\}\, \overset{-}{\psi}_i^{\,*},$$
$$\overset{+}{\Psi}_r \equiv \overset{+}{\psi}_r, \qquad \overset{-}{\Psi}_r \equiv \sum_i \overset{-}{\psi}_i \{i,r\}, \tag{5}$$

which obey the same commutation relations as the original wave field:

$$\{\overset{-}{\Psi}_r^{\,*}, \overset{+}{\Psi}_s^{\,*}\} = \{\overset{-}{\Psi}_r, \overset{+}{\Psi}_s\} = \{r,s\}, \tag{6}$$

Eqs.(2) become

$$\overrightarrow{\psi}_r^{\,*} = \overset{+}{\Psi}_r^{\,*}, \qquad \overrightarrow{\psi}_r = \overset{+}{\Psi}_r + \overset{-}{\Psi}_r^{\,*},$$
$$\overleftarrow{\psi}_r = \overset{\widetilde{+}}{\Psi}_r, \qquad \overleftarrow{\psi}_r^{\,*} = \overset{\widetilde{+}}{\Psi}_r^{\,*} + \overset{\widetilde{-}}{\Psi}_r. \tag{7}$$

Similar operators can be defined for a boson field, φ^* and φ , provided that we regard $\frac{1}{\sqrt{n_1!\,n_2!\cdots}}\varphi_1^*\varphi_2^*\cdots\varphi_n^*\varphi_{1'}\varphi_{2'}\cdots\varphi_{n'}$ as a state, where n_1, n_2, etc. are the number of times it contains φ_1^*, φ_2^*, etc. respectively. Corresponding to Eqs.(1) to (5), we get

$$\overrightarrow{\varphi}_r^{\,*} = \overset{+}{\varphi}_r^{\,*} = \overset{+}{\Phi}_r^{\,*}, \qquad \overrightarrow{\varphi}_r = \overset{+}{\varphi}_r + \sum_i [r,i]\, \overset{-}{\varphi}_i^{\,*} = \overset{+}{\Phi}_r + \overset{-}{\Phi}_r^{\,*}$$
$$\overleftarrow{\varphi}_r = \overset{+}{\varphi}_r = \overset{+}{\Phi}_r, \qquad \overleftarrow{\varphi}_r^{\,*} = \cancel{\mp}\, \overset{+}{\varphi}_r^{\,*} + \sum_i \overset{-}{\varphi}_i [i,r] = \overset{+}{\Phi}_r^{\,*} + \overset{-}{\Phi}_r$$
$$[\overset{-}{\Phi}_r^{\,*}, \overset{+}{\Phi}_s^{\,*}] = [\overset{-}{\Phi}_r, \overset{+}{\Phi}_s] = [r,s] = [\varphi_r, \varphi_s^*]\,. \tag{8}$$

Now an equation of motion for the transformation matrix $U[C, C_0]$ defined by $\Psi[C] = U[C, C_0]\,\Psi[C_0]$ is of the form

$$i\frac{\delta}{\delta C}U = HU, \qquad -i\frac{\delta}{\delta C}U^* = U^*H \tag{9}$$

in Tomonaga's super-many time formalism. This may be regarded as an equation of motion for the wave function U if the operator H is expressed in terms of the above mentioned operators:

$$i\frac{\delta}{\delta C}U = \vec{H}U, \qquad -i\frac{\delta}{\delta C}U^* = \overleftarrow{H}U^* \tag{10}$$

In the same manner, an equation of Heisenberg type

$$-i\frac{\delta}{\delta C}A = [H, A] \tag{11}$$

goes over into an equation of Schroedinger type

$$-i\frac{\delta}{\delta C}A = (\vec{H} - \overleftarrow{H})A \equiv \mathcal{H}A \tag{12}$$

Moreover, it is possible to make $\vec{H}$ or $\overleftarrow{H}$ Hermitian by a simple transformation, and the resulting Hamiltonian very much resembles the original one. On these grounds we should like to call the present formalism the method of the third quantization. A quantity composed of operators of the second quantization (for short, q-2 numbers) may be interpreted as a wave function in the second configuration space on which, when transformed to the third quantization representation, the q-3 numbers ($\Psi^{\pm *}$, $\Psi^{\pm}$, etc.) can operate.

Application of this method to the theory of Schwinger and Tomonaga enables one to carry out the canonical transformation in a quick way and gives a clear perspective of the whole theory. It also enables one to solve easily the generalized wave equation proposed by Snyder[3] and Yukawa.[4]

A paper dealing with mathematical refinement and various applications is now being worked out.

(1) J. Schwinger, Phys. Rev. 73 (1948), 415.

(2) S. Tomonaga, Phys. Rev. 74 (2248), 224.

(3) H. Snyder; Phys. Rev. 72 (1947), 68.

(4) H. Yukawa, Prog. Theo. Phys. 2 (1947), 209; ibid. 3 (1948), 205.