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Physics III
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Name H. Yukawa

Subject Non-localizable Field Theory. II.

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PHYSICS. III.
Nonlocalizable Field
Theory ~~and related topics~~
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N20

Previous works related to Nonlocalizable Field

(1) M. Markow

Über das "Vierdimensional-Ausgedehnte" Elektron in dem Relativistischen Quantengebiet.

(Jour. Phys. 2 (1940), 453)

(2) Born and Peng

(3) Heisenberg

Beobachtbare Größe

(4)

Possibilities of Generalization of Rel. Inv. Q. F. T.

Generalized Field Theory

Yukawa, On the Theory of Elementary Particles. I. (Prog. 2 (1947), 209)

Yukawa, Reciprocity in generalized Field Theory. (Prog. 3 (1948), 207)

The ~~field~~ quantized field theory ~~hither~~ above mentioned ~~is~~ relativistic invariant ~~so, obviously~~ seems to be ~~only~~ almost unique outcome of the following three postulates

i) special relativity principle should not be violated, i.e. ~~every~~ all physical quantities should be invariant. ~~Especially time~~ ~~ii) like and space In this connecti~~

An important property of Minkowski space underlying any relativistically invariant theory is that time-like direction and space-like direction on the one hand, past and future on the other hand, can be discriminated from each other in a relativistically invariant manner. ~~which~~

ii) principles of quantum mechanics should hold true, i.e. if we take ~~any~~ ~~one~~ two complete sets of independent mutually commutative observables A, B, ~~the any state of the~~ the probability amplitude that a result a obtained by observing A, provided ~~that it is certain~~ certainly a result b ~~will be~~ was or would be obtained, if B was or would be observed, is ~~gi~~ expressed by a, in general complex, transformation fn

$$\psi(a/b),$$

~~which~~ The ~~rule~~ ~~whi~~ equation, which this transformation should obey, ~~is, in general,~~ can be fn written in ~~the~~ a very general form

$$\cancel{L(A, B, C, \ldots)\psi(}$$

$$\sum_{a''} (a'|L_i|a'')\,\psi(a''/b) = 0, \qquad (1)$$
$$i = 1, 2, \ldots$$

where L_i ~~are~~ are functions of observables, which can be reduced to functions of A, B alone, if ~~A and B~~ two complet sets are ~~completely~~ ~~complementary~~ with each other.*

iii) ~~at least a couple of~~ Completely complementary sets of ~~observables~~ A, B ~~can be exist~~ should exist, ~~which are all ascribed to ev can be each pair~~ of which is ascribed to some point in four dimensional space. We call, according to Dirac's terminology (Phys. Rev. 73 (1948), 1092), ~~Principles~~ Postulate of Localizability.

* i.e. ~~when~~ if there is one-to-one correspondence between two sets of observables, with ~~corresponding~~ ~~pair~~ ~~pair~~ consisting of non-commutative pairs.

Now if we ~~consider~~ take all these three postulates into account, we obtain a theory ~~wit~~ of any physi~~cal~~ system with ~~a~~ completely complementary sets of observables localizable in Minkowsky space. In ~~order that~~ conformity with the requirement of relat. that no physical action can propagate with a velocity greater than c, so that the law of causality is not violated, ~~it~~ one can construct a complete set ~~of~~ A ~~observables~~ from the observables ascribed to points on any space-like surface*. Generalized Schrödinger equations (1) can now take a form

$$i\hbar\frac{\delta\Psi}{\delta\sigma}=H\Psi. \qquad (2)$$

This is nothing but Tomonaga-Schwinger theory**.

As ~~turned out by~~ made clear by recent development of quantum electrodynamics on these lines, ~~and~~ divergence difficulties ~~could~~ can not be removed, although it can be separated from finite observable term in a relativistically invariant way. This ~~consequence~~ is negative side of

* In addition to (2), ~~it is prescribed as~~ we need pre-arranged relations ~~with~~ between observables correspon-dency to field equations. ~~Any~~ extension by Tanikawa still implicitely formal contains this restriction

Although it ~~succeeded~~ seems successful in remove some of the difficulties

the theory was almost evident from the beginning, although the positive side was not.

Thus the old question arises again: Can we construct a consistent theory ~~by discarding at least one of three postulate~~ not violating any one of three postulate?
~~An obvious way is to introduce several~~
Attempts by Pais, Sakata and others are on this line, ~~but it seems almost impossible~~ do not seem complete by itself, because we introduce a new kind of particle in order to get rid of ~~one~~ a kind of divergence, unless ~~introducing~~ another kind of divergence, ~~accompanied by~~

Now ~~the~~ we can change the question as follows: Can we construct a consistent theory by discarding ~~at least~~ one of above three postulates.
One possibility is to go back from the field theory (Nahewirkungstheorie) to the theory ~~at distance~~ of action at distance (Fernwirkungstheorie) or some combination of field theory and action at distance theory.
~~Former kind of~~ An interesting theory of the former kind was developed a few years

~~by the~~ an

ago by Wheeler and Feynman[†]. Action at distance theory, ~~a term course is equivalent~~ means that we introduce in some way "an unquantized field", which can be described ~~by~~ not by mutually complementary observables, but by a set of c-numbers or ordinary space-time functions. According to Wheeler-Feynman theory, the ~~divergence~~ singularities in the ~~self-force~~ radiative reaction can removed by assuming ~~the~~ a dense absorbing matter ~~at~~ infinite distance, surround the whole system, so it can be considered as a sort of theory of "incomplete system"[††].

~~Recent development~~ A method of quantum electrodynamics recently developed by Feynman seems to have intimate connection with this idea, but I ~~donot~~ know ~~the detail~~ little about his new theory, because ~~no paper w~~ I hear only rumour and I can see no detailed paper.

~~Anyhow this kind of theory seems to go~~

[†] Wheeler and Feynman, R.M.P. 17 (1945), 157.

[††] Moreover it cannot easily be transformed into Hamiltonian form.

out of the ~~most~~ ~~ordinary~~ quantum mechanics in the narrow sense, ~~in~~ which only complete system is treated* and go nearer to ~~the~~ statistical mechanics.

~~To get out of the restrictions of special relativity~~ Above mentioned Dirac's attempt of starting ~~We me~~ from the generalized transformation function ~~on~~ arbitrary surface was ~~hindered by not only by~~ difficult to succeed, because it was hindered not only by restrictions of special relativity, but also by fundamental idea of q.m., and ~~we~~ ~~found~~ by the development of the invariant field theory we are more and more ~~clearly~~ convinced how intimately the special relativity theory and quantum mechanics are connected with each other. (See, however, Tanikawa)

Heisenberg's S-matrix theory† is clearly an attempt to go out of ~~quantum~~ traditional form of ~~quantum~~ mechanics and, ~~at the same time~~ (probably also out of the postulate of localizability.

† Heisenberg, ZS. f. Phys.
* If we want to make the system, we need infinite absorber at infinity.

General Theory of Nonlocalizable Fields, I.

Alternatively we can start from ~~the~~ discarding the localizability postulate. In quantum mechanics, space and time coordinates (~~t~~ t x y z) and energy-momentum operators are two things ~~which~~ complementary to each other. One or other of two set of things can be placed as common background for the representation of quantized fields. Thus we ~~can use~~ could adopt either the repres. in ordinary space or that in momentum-space.

Let ~~we can~~ us change the ~~point~~ stand point as follows: ~~We~~ Instead of considering the elementary particles or the quantized fields always in foreground and the space-time in background, let us consider both of them on the same footing.† Then the connection between these "~~observable~~" quantities can be expressed by some commutation relations, which corresp. to the field eq. for free particles. Here comes the postulate of reciprocity proposed by Born* in place of postulate of localizability. Thus each of the field quantities

x_μ, p_μ

$\boxed{A(x_\mu)}$

x_μ p_μ

A

* Born, Proc. Roy. Soc.

† ~~One may ask "what~~

~~are~~ becomes a matrix (in general non-diagonal) in the representation, in which space-time coordinates (or the momentum-energy variables) are diagonal. ~~My corresponds~~ can be a substitute for the ordinary quantized field quantities, by considering ~~the~~ each matrix element itself an operator.

Now, if we take up the ~~field~~ operators among infinite number of matrix elements* which are independent and commutative with each other, and ~~call~~ let us call any two complete sets of operators A, B respectively, ~~we can~~ there should exist the generalized ~~$\psi(a/b)$~~ transformation function

$$\psi(a/b),$$

where a, b represent respectively the ~~big~~ set of eigenvalues of A and B.

In ordinary field theory, each member of A (or B) is connected with a point of four-dimensional space, or more precisely ~~point~~: all of them are connected (one-to-one) with all points respectively on ~~the~~ space-like surface.

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* ~~They~~ ~~not~~ Each of them may be a matrix element itself or, some function of various matrix elements, (linear or quadratic etc.)

Here a new question arises: What is the substitute for Schrödinger equation for $\psi(a/b)$.

In order to answer this question, we have ~~first to consider the a small change of a.~~ we want to go into a simple example.

To fix the idea, ~~Let~~ us consider a set of field quantities A_μ ($\mu = 0, 1, 2, 3$)

~~There~~ We assume some commutation relations

$$\cancel{\sum_{\mu i} c_{\mu i}[p_\mu, A_i] = a A_j}$$

$$-i\hbar F_{\mu\nu} = [p_\mu, A_\nu] - [p_\nu, A_\mu]$$

$$[p^\mu, F_{\mu\nu}] = 0$$

which is equivalent to

$$[p_\mu [p^\mu, A_\nu]] = 0$$

provided that the generalized Lorentz condition

$$[p^\mu, A_\mu] = 0$$

holds for A_μ.*

We can write down the reciprocal commutation relations of the same form:

$$[x_\mu [x^\mu, A_\nu]] = 0,$$

$$[x^\mu, A_\mu] = 0. \quad **$$

* Yukawa, Prog. 2 (1947), 209.

** One may ask "What is x_μ, then?" I answer "There's no x_μ, which is completely separated from other things."

The first equations ~~are~~ clearly means that matrix elements are not zero ~~only when~~ in the representation, in which x's are diagonal, only ~~when~~ if $|x'_\mu - x''_\mu| = 0$, because

$$(x'|[x^\mu, A_\nu]|x'') = \underbrace{\cancel{x^{\mu'}}}_{(x^{\mu'}-x^{\mu''})}(x'|A_\nu|x'')$$

$$(x'|x_\mu[x^\mu, A_\nu]|x'') = |x'_\mu - x''_\mu|^2 (x'|A_\nu|x'')$$

The second equation can be represented by

$$(x'|[x^\mu, A_\mu]|x'') = (x^{\mu'} - x^{\mu''})(x'|A_\mu|x'')$$

$$= 0,$$

which can be interpreted as follows: ~~When~~ An electromagnetic inter~~action~~ $(x'|A_\mu|x'')$ ~~comes from~~ between two points x'_μ, x''_μ is always orthogonal to $x'_\mu - x''_\mu$. ~~Thus~~ Accordingly, if we take especially $(x'|A_0|x'') = 0$, ~~$(x'|A_\mu|x'')$~~ a three dimensional vector $(x'|A_k|x'')$ is perpendicular to the vector ~~A_k~~ $(x'_k - x''_k)$. More generally, if we decompose the vector $(x'|A_k|x'')$,

$$(x'|A_k|x'') = (\cdot|A^{(t)}_k|\cdot) + (\cdot|A^{(l)}_k|\cdot)$$

with the condition

$$(x'_k - x''_k)(x'|A^{(t)}_k|x'') = 0,$$ the above rel. reduces to

$$(x_k' - x_k'')(x'|A_k^{(l)}|x'') = (x_0' - x_0'')(x'|A_0|x'')$$

Thus if ~~$x_0'-x_0$~~ $x_\mu' - x_\mu''$ is a space-like vector, $x_0' - x_0'' = 0$ in some Lorentz frame, so that $(x'|A_k^{(l)}|x'') = 0$. On the contrary, if $x_\mu' - x_\mu''$ is a time-like vector, there is ~~a~~ some Lorentz frame, in which $|x_k' - x_k''| = 0$ and $(x'|A_0|x'') = 0$.

Now we have to show that these reciprocal commutation relations are ~~not~~ compatible with each other. A_ν ~~are~~ can be considered as functions of $\xi_\mu = x_\mu' - x_\mu''$ and $X_\mu = \frac{x_\mu' + x_\mu''}{2}$.*

$$(x'|[p_\mu, A_\mu]|x'') = \Big\{-i\hbar\frac{\partial}{\partial x_\mu'}(x'|A_\mu|x'')$$

$$-i\hbar\frac{\partial}{\partial x_\mu''}(x'|A_\mu|x'')\Big\}$$

$$= -i\hbar\frac{\partial}{\partial X_\mu}A_\mu(\xi, X).$$

$$* \quad \frac{\partial f(x', x'')}{\partial x_\mu'} = \frac{\partial f}{\partial \xi_\mu} + \frac{1}{2}\frac{\partial f}{\partial X_\mu}$$

$$\frac{\partial f}{\partial x_\mu''} = -\frac{\partial f}{\partial \xi_\mu} + \frac{1}{2}\frac{\partial f}{\partial X_\mu}$$

$$\therefore \frac{\partial f}{\partial \xi_\mu} = \frac{1}{2}\left(\frac{\partial f}{\partial x_\mu'} - \frac{\partial f}{\partial x_\mu''}\right) \qquad \frac{\partial f}{\partial X_\mu} = \frac{\partial f}{\partial x_\mu'} + \frac{\partial f}{\partial x_\mu''}$$

Similarly

$$(x'|[p_\mu[p^\mu, A_\nu]]|x'')$$

$$= \cancel{\frac{\partial}{\partial x_\mu}} \frac{\partial^2}{\partial X_\mu \partial X^\mu} A_\nu(\xi, X),$$

while the commutation relations between x_μ and A_ν restrict the form of function $A_\nu(\xi, X)$ with respect to ξ.

If we assume further

$$A_\nu(\xi, X) = \delta(\xi) A_\nu(X),$$

the field reduces to the ordinary localizable electromagnetic field.

~~So far there is~~ Thus there is no contradiction, as long as we are considering the generalized electromagnetic field in vacuum.

Our next tasks are ~~to~~

i) to quantize the field

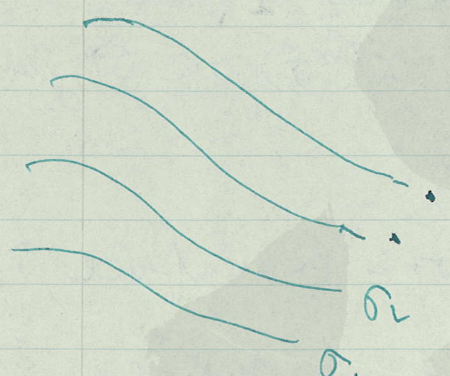
ii) to ~~consider~~ take the interaction with charged particle field into account,

~~iii)~~ which is equivalent to

ii)$_1$ introduce electron radius in relativistic way and to

ii)$_2$ establish the ~~relation~~ equation for the probability amplitude as the substitute for Schrödinger equation.

We want to consider ii) first. In localizable field theory, Schrödinger equation is necessary (when there's some interaction)* in order to determine the ~~chan~~ relation between two representations ~~in which~~ referring to field quantities (as diagonal matrices) at two different times. ~~#~~ The word "time" ~~can be extended as~~ should be understood in general as some "parameter" which is equivalent to time, such as the parameter ~~de fixing the location of~~ selecting a surface from an infinite sequence of space-like surfaces covering the whole 4-dimensional space. or, which



What is ~~the thing~~ corresponding to "time" in nonlocalizable field theory.

Interaction between fields ~~are~~ is something which changes the ~~intera~~ state.** ~~for the repres~~ In order to make the things clear let us go back again to the localizable field theory.

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* When there's no interaction, the relations between various representations ~~ca can b~~ can be determined from commutation relations.

** The representation of states referring two diff. "time" different from each other.

○ Localizable Field Theory

state　　　Heisenberg-Tomonaga representation

Ψ　　　$(a'(\sigma)/\ )$

no interaction; $(a'(\sigma)/\ )$ is independent of σ

$$\left(a'(\sigma)/a''(\sigma_0)\right) = \delta(a'(\sigma), a''(\sigma)) = \delta(a', a'')$$

interaction:

$$\frac{\delta}{\delta\sigma_P}(a'(\sigma)/\ ) = H_P\,(a'(\sigma)/\ )$$

$$(a'(\sigma)\,|H_P|\,a''(\sigma)) = \delta'(a'(\sigma), a''(\sigma)) \times (a'(P)|H|a''(P))$$

where

$$\delta' = \prod_{P' \neq P}{}' \delta(a'(P'), a''(P'))$$

○ nearly ~~Quasi~~ / ~~Non~~localizable Field Theory 1,

We ~~???~~ consider all points $x'(\sigma)$ on σ, and all the matrix elements of a

$$(x'(\sigma)\,|\,a\,|\,x''(\sigma''))$$

in which x' is always on a surface σ' and x'' on σ'',

We assume that there is some field quantity b, ~~whose~~ the matrix elements of which

$$(x'|b|x'') \equiv (\sigma'|b|\sigma'')$$

are all commutative with each other, if all x' lie on a space-like surface σ' and x'' on another space-like surface σ''. Then, ~~we can to construct a Schw-Tomonaga eq.~~ the state can be represented as a fcn

δ ~~σ'T|σ''~~ of $b'(\sigma'\sigma'')$.

$(b'(\sigma'\sigma'')/)$

and the S.D.T.-equation may take the form

$$\frac{\delta}{\delta\sigma_p'}(b'(\sigma'\sigma'')/) = \begin{cases} \cancel{(b'(\sigma'\sigma'')|H_p|b''(\sigma'\sigma''))} \\ H_p(b'(\sigma'\sigma'')/)^{*} \end{cases}$$

$(b'(\sigma'\sigma'')|H_p|b''(\sigma'\sigma'')) =$

② nearly localizable Field Theory 2,

The case 1, above considered, is already very complicated and the correspondence to the ordinary localizable field theory is not obvious, so we consider ~~still more~~ ~~as~~ a case 2, which still nearer to the ordinary theory. Although in this case also, ~~field quant~~ matrices ~~representing~~ for the field quantities are not diagonal in a representation, in which x's are diagonal, we assume we can find a quantity b (or a set of field quantities) ~~which is~~ any two of the diagonal elements of which are all commutative with each other as long

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* H $(x'_\mu|b|x'_\mu)$, $(x''_\mu|b|x''_\mu)$

as ~~they are to the points~~ the vector x'_μ ~~and~~ x''_μ is spacelike. ~~In this case~~, we can construct ~~the Schrödinger~~ Tomonaga eq.

$$\frac{\delta}{\delta\sigma_P}(\cancel{b'}$$

~~Moreover,~~ We assume further, that ~~all~~ the diagonal elements $(x'_\mu b | x'_\mu) = b(x'_\mu)$ ~~on~~ th x'_μ on any space-like surface σ constitute ~~on the~~ in all a complete set of operators, so that any state of the system can be represented as a function

$$(\cdots b(x'_\mu)\cdots/) \equiv (b(\sigma)/)$$

of $b(x'_\mu)$ ~~the~~, x'_μ running all over the surface σ.* Then there may exist S.T. eq.

$$\frac{\delta}{\delta\sigma_P}(\cancel{b'}(\sigma)/) = H_P(b'(\sigma)/)$$

Now we want to see whether the generalized theory for the electromagnetic field can be constructed so as to be in the category ~~just mentioned~~ of this kind.

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* In the case of the ordinary electron field theory, the density operators $\rho_{ij} = \psi^*_j(x'')\psi_i(x')$ ~~are not~~ can be considered as ~~something~~ ~~non~~ localizable quantities, but the charge density itself corresponds to b stated above.

Feature

General Theory of Nonlocalizable Fields. II.

Commutation relations between the field quantities A_ν and x_μ, p_μ as obtained in I. lead us to the consequence that $A_\nu(\xi, X)$ should satisfy ~~the~~ two sets of equations

$$\left.\begin{aligned} \frac{\partial^2}{\partial X_\mu \partial X^\mu} A_\nu(\xi, X) &= 0 \\ \frac{\partial A_\mu(\xi, X)}{\partial X_\mu} &= 0 \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} \xi_\mu \xi^\mu \cdot A_\nu(\xi, X) &= 0 \\ \xi^\mu A_\mu(\xi, X) &= 0 \end{aligned}\right\} \qquad (2)$$

From (1), one can expand A_ν in the form:

$$A_\nu(\xi, X) = \sum_{k} a_\nu(k, \xi)\, e^{ik_\mu X^\mu} \qquad (3)$$

with

$$\left.\begin{aligned} k_\mu k^\mu &= 0 \\ k_\mu a^\mu(k, \xi) &= 0 \end{aligned}\right\} \qquad (4)$$

Further, if we put (3) and (4) in (2), we obtain

$$\left.\begin{aligned} \xi_\mu \xi^\mu a_\nu(k, \xi) &= 0 \\ \xi^\mu a_\mu(k, \xi) &= 0 \end{aligned}\right\} \qquad (5)$$

Thus a_ν has in general the form:

General Tensor Theory of Nonlocalizable Fields. II.

Commutation relations between the field quantities A_ν and x_μ, p_μ as obtained in I, lead us to the consequence that $A_\nu(\xi, X)$ should satisfy ~~the~~ two sets of equations

$$\left.\begin{aligned} \frac{\partial^2}{\partial X_\mu \partial X^\mu} A_\nu(\xi, X) &= 0 \\ \frac{\partial A_\mu(\xi, X)}{\partial X_\mu} &= 0 \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} \xi_\mu \xi^\mu \cdot A_\nu(\xi, X) &= 0 \\ \xi^\mu A_\mu(\xi, X) &= 0 \end{aligned}\right\} \qquad (2)$$

From (1), one can expand A_ν in the form:

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$$\left.\begin{aligned} k_\mu k^\mu &= 0 \\ k_\mu a^\mu(k, \xi) &= 0 \end{aligned}\right\} \qquad (4)$$

Further, if we put (3) and (4) in (2), we obtain

$$\left.\begin{aligned} \xi_\mu \xi^\mu\, r a_\nu(k, \xi) &= 0 \\ \xi^\mu a_\mu(k, \xi) &= 0 \end{aligned}\right\} r \qquad (5)$$

Thus a_ν has in general the form:

$$k_\mu a^\mu(k,\xi) = 0$$

$$\left.\begin{aligned} a_\nu(k,\xi) &= \delta(\xi_\mu \xi^\mu)\, a'_\nu(k,\xi) \\ \xi^\mu a'_\mu(k,\xi) &= 0 \quad \text{for} \quad \xi_\mu\xi^\mu = 0, \end{aligned}\right\} (6)$$

~~Thus Two simple extreme ~~ ~~solutions of equations (5) or (6) are~~

~~i)~~ $\cancel{a_\nu(k,\xi) = \delta(\xi)\, a_\nu(k)}$ with $k_\mu a^\mu(k) = 0$

~~ii)~~ $\cancel{a'_\nu(k,\xi) = \xi_\nu\, a(k)}$

or $\cancel{a_\nu(k,\xi) = \dfrac{\xi_\nu \delta(\sqrt{\xi_\mu\xi^\mu})}{2\sqrt{\xi_\mu\xi^\mu}}\, a(k)}$

Simple solution of (4) (5) or (6) is evidently

$$a_\nu(k,\xi) = \delta(\xi)\, a_\nu(k)$$

with $k^\mu a_\mu(k) = 0$,
which is nothing but the ordinary electromagnetic potential.
In general $a'_\nu(k,\xi)$ should satisfy ~~two~~ conditions, when k_μ, ξ_μ are both four vectors with 0 absolute value †

$s^2 = \xi_\mu \xi^\mu$ ~~$2s\,ds =$~~

$$\delta(s^2)\, 2s\, ds = \delta(s)\, ds$$

$$\delta(s^2) = \frac{\delta(s)}{2s}$$

† i.e. zero vectors, isotropic vectors

$$\left.\begin{aligned} k^\mu a'_\mu &= 0 \\ \xi^\mu a'_\mu &= 0 \end{aligned}\right\} \qquad (\quad)$$

Thus for each ~~pair~~ of values of (ξ_μ, k_μ), in general there are two independent a'_μ corresponding to the polarization of the photon in ordinary ~~pure~~ quantum electrodynamics.* Namely, we can always choose the ~~reference~~ Lorentz frame ~~such that~~, for given k^μ such that

$$k^0>0\,; \quad k^1 = k^0 \quad k^2=k^3=0, \quad \xi^3=0.$$

Then the above equations ( ) become

$$\underline{a'_1 = -a'_0}$$

and

$$\xi^0 a'_0 + \xi^1 a'_1 + \xi^2 a'_2 = 0$$

or $$(\xi^0 - \xi^1)\, a'_0 + \xi^2 a'_2 = 0,$$

hence $$a'_2 = -\frac{\xi^0-\xi^1}{\xi^2}\, a'_0 \qquad (\quad)$$

Thus we can take a'_0 and a'_3 arbitrary.
If we write

$$\left.\begin{aligned} \xi^1 &= \xi^0 \cos\theta \\ \xi^2 &= \xi^0 \sin\theta \end{aligned}\right\} \qquad \text{or } a'_2 = \tan\tfrac{\theta}{2}\, a'_1,$$

$$a'_2 = -\frac{1-\cos\theta}{\sin\theta}\, a'_0 = -\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\, a'_0 = -\tan\tfrac{\theta}{2}\cdot a'_0$$

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* when ξ^μ is parallel to k^μ, ~~a~~ there are three independent a'_μ, ~~but namely~~ because

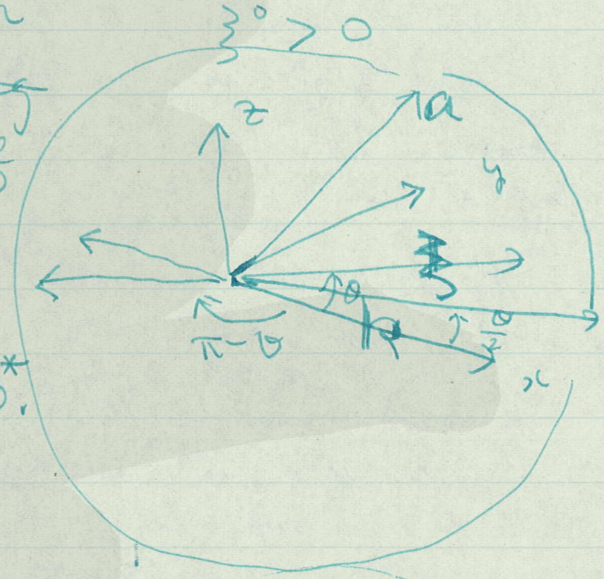
$$k^1 = k^0 \quad k^2=k^3=0 \quad \xi^2=\xi^3=0,$$

$$a'_1 = -a'_0,$$

so that a'_0, a'_2, a'_3 arbitrary, which is the ~~but a'_2~~ ~~can be fixed as~~ limiting case $\xi^2 \to 0$ in ( )

where θ ~~is the~~ or $\pi-\theta$ is the angle between two space vectors $\mathbf{k}$ and ξ, according as $\xi^0 \gtrless 0$. / for $k \geq 0$

Thus $\mathbf{a}$ is any vector on the plane ~~including~~ bisecting two plane $(\mathbf{k}, z)$ and $(\cancel{\mathbf{k}\,\xi}\ \xi, z)$, ~~or~~ ~~the side of~~ or $(\mathbf{k}, z)$ and $(-\xi, z)$ according as $\xi^0 \gtrless 0$.*



$k_0\xi^0 \gtrless 0$: $k_0\xi_0 > 0$.

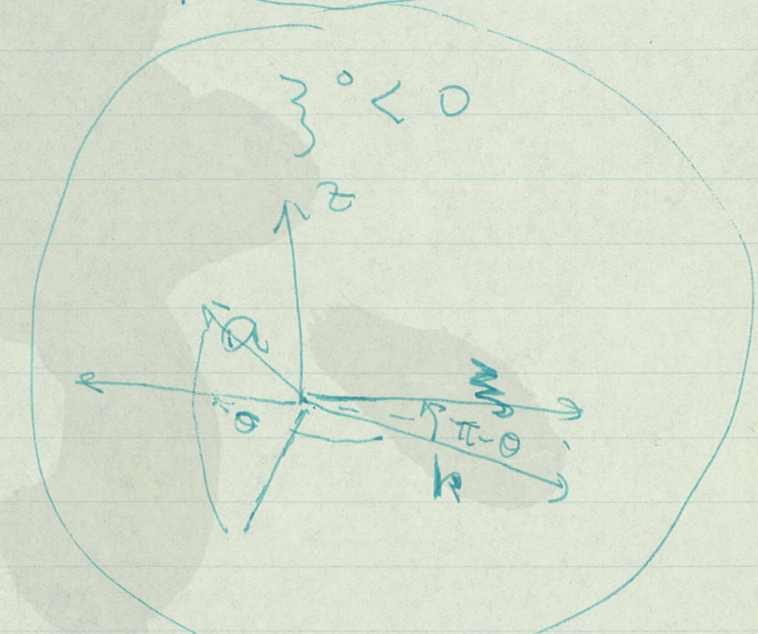
$$k_\mu \xi^\mu = k^\mu \xi_\mu$$

$$= k_0\xi^0 + \mathbf{k}\xi$$

$$= -k_0\xi_0 + k_0\xi_0 \cos\theta.$$

$$= -k_0\xi_0(1-\cos\theta)$$

$$= -2k_0\xi_0 \sin^2\frac{\theta}{2}$$



* z-axis is nothing but the ~~perpendicular line~~ normal ~~direction perpendicular~~ to the plane $(\mathbf{k}, \xi)$

Thus ~~for a_n~~ we obtain

$$a_1' = -a_0'$$

$$a_2' = -\tan\frac{\theta}{2} a_0' = \tan\frac{\theta}{2} a_1'$$

with a_0' and a_3' as certain function of $\xi_\mu K^\mu$, which is only non-zero invariant combination of ξ_μ and K_μ. *

For the simplest case, where a_0' and a_3' is independent of $\xi_\mu K^\mu$, –

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* As two independent solutions, we can take

$$a_1' = a_0', \quad a_2 = -\tan\frac{\theta}{2} a_0' \qquad a_3' = 0$$

and $\quad a_0' = a_1' = a_2' = 0 \qquad a_3' = a_3'(\xi_\mu K^\mu)$

So far, we confined our attention to pure electromagnetic field. ~~In~~ In order to take into account ~~the~~ other kinds of field such as the meson fields in broader sense, it seems necessary to extend our formalism to five dimension.[1)]Namely we consider 5-dim. space $(x_0, x_1, x_2, x_3, x_4)$ with the line element

We assume $ds^2 = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$.

~~Exactly~~ the ~~same equati~~ relations of the same form

$$\left.\begin{array}{l} [p_\mu[p^\mu A_\nu]] = 0 \\ [x_\mu[x^\mu A_\nu]] = 0 \end{array}\right\} \qquad (1)$$

~~as above~~

$$\left.\begin{array}{l} [p^\mu A_\mu] = 0 \\ [x^\mu A_\mu] = 0 \end{array}\right\} \qquad (2)$$

as above, with the only difference that μ, ν ~~are~~ now take the values 0, 1, 2, 3, 4. Thus we obtain

$$A_\nu(\zeta, X) = \sum_k a_\nu(k, \zeta)\, e^{ik_\mu x^\mu}, \qquad (3)$$

which can be rearranged as follows:

---

1) Yukawa, Prog. Theor. Phys. 3 (1948), 205.

$\xi \to r$

$$A_\nu(\xi, X) = \sum_{k_4} \left(\sum_{\substack{k \\ (\mu=0,1,2,3)}} a_\nu(k,\xi) e^{ik_\mu x^\mu} \right), \quad (4)$$

~~ea~~ the terms in a bracket for ~~corresponding~~ ~~to~~ a (fixed) value of k_4 corresponding to the field particle (~~mesons~~ photons and neutral mesons) with a mass

$$\mu = \frac{\hbar k_4}{c}, \quad \cancel{wh}$$

Now the coefficient (operators)

$$a_\nu(k,\xi) = \delta(\xi_\mu \xi^\mu)\, a'_\nu(k,\xi)$$

has a factor $\delta(\xi_\mu \xi^\mu)$, which is ~~not~~ different from zero for any ~~every~~ point

$$\sum_{\mu=0,1,2,3} \xi_\mu \xi^\mu = -\xi_4\xi^4 < 0.$$

In this case $a'_\nu(k,\xi)$ may be any function of $\sum_{\mu=0,1,2,3} \xi_\mu \xi^\mu = \xi_4\xi^4$, $\sum_{\mu=0,1,2,3} k_\mu k^\mu = -k_4 k^4$ and $\sum_{\mu=0}^{3} k_\mu \xi^\mu$ as far as they satisfy the relation

$$\left.\begin{aligned} k^\mu a'_\mu &= k^4 a'_4 \\ \xi^\mu a'_\mu &= \xi^4 a'_4 \end{aligned}\right\}$$

As a ~~spe~~ particularly simple example, we consider the case, where $a'_\nu(k,\xi)$ are all zero unless $k_4 = 0$, which corresponds to the fact that in X-space the electromagnetic wave propagate with the velocity c, just as in the ordinary ~~fiel~~ electrodynamics, but in ζ-space $a_i(k,\zeta)$ may be a function of $\zeta_\mu\zeta^\mu = -\zeta_4\zeta^4$, which ~~decre~~ falls off very rapidly with the increasing value of $|\zeta_\mu\zeta^\mu| = \zeta_4^2$. ~~for~~

On invariant Delta-fn (or covariant)

usually we consider an invariant integral of some invariant (or covariant in general) ~~integral~~ fn $f(k_0, k_1, k_2, k_3)$ and x_μ, where $k_0, k_1 \cdots k_3$ satisfying the condition

$$k_0^2 - k_1^2 - k_2^2 - k_3^2 = \kappa^2, \qquad (1)$$

κ being a positive constant,

$$F(x_\mu) = \iiint \frac{f(k_\mu, x_\mu)}{k_0} dk_1 dk_2 dk_3 .$$

This is invariant because

$$\frac{dk_1 dk_2 dk_3}{k_0} \qquad k_0 = +\sqrt{\kappa^2 + \sum_i k_i^2}$$

is proportional to the invariant surface element on three dimensional hyperbolic surface satisfying the equation (1).

Especially for

$$f(k_\mu, x_\mu) = \frac{1}{(2\pi)^3} \sin\left(\sum_i k_i x_i - k_0 x_0\right)$$

$$k_0 \geq 0,$$

$$D(x_0 x_1 x_2 x_3) = \frac{1}{(2\pi)^3} \iiint \frac{\sin\left(\sum_i k_i x_i - k_0 x_0\right)}{k_0} dk_1 dk_2 dk_3$$

is the wellknown D-fn with the properties:

$$\left.\begin{aligned} &D(0, x_1 x_2 x_3) = 0 \\ &\left(\frac{\partial D}{\partial x_0}\right)_{x_0 = 0} = -\delta(x_1 x_2 x_3) \end{aligned}\right\}$$

Now the only invariant ~~functions~~ combinations of $(k_\mu), (x_\mu)$ are

$$k_\mu k^\mu = \kappa^2 = \text{const.}$$

$$x_\mu x^\mu \qquad k_\mu x^\mu .$$

Invariant integral has the general form

$$\iiint \frac{f(x_\mu x^\mu, \; k_\mu x^\mu)}{k_0} dk_1 dk_2 dk_3$$

~~In order that~~ ~~If we further add the condition that~~ f

$$[a_\mu(k', \xi'), a_\nu(k'' \xi'')]$$

$$=$$

General Feature of Nonlocalizable Fields. III,

Quantized space-time structure
$(x, y, z, \underset{\overset{\shortparallel}{x_0}}{c\tau}\ ;\ p_x\ p_y\ p_z\ p_0)$
versus

System of Elementary Particles
$(\ A,\ \psi,\ \psi^\dagger\ \cdots\ )$

One electron system

observables $x\ y\ z\ ct\ ;\ p_x\ p_y\ p_z\ p_0$
$(\alpha, \beta) = (\sigma_x\ \sigma_y\ \sigma_z\ ;\ \rho_1\ \rho_2\ \rho_3)$

state $\psi,\ \psi^*$
$\psi(x\ y\ z\ ct\ ;\ \sigma_z,\ \rho_3)$

Indefinite number of elementary particles

back-ground $x\ y\ z\ ct\ (p_x \cdots p_0)$

observable, $\psi,\ \psi^*,\ \cdots$

state $\Psi,\ \Psi^*$

Structure of the universe

observables $\left\{ \begin{array}{l} x\ y\ z\ ct \qquad p_x\ p_y\ p_z\ p_0 \\ \psi,\ \psi^* \cdots \end{array} \right.$

state $\Psi,\ \Psi^*$

1. Scalar, neutral, 0-mass field:

$$\left.\begin{array}{l}[p^\mu[p_\mu, U]] = 0 \\ [x^\mu[x_\mu, U]] = 0\end{array}\right\}$$

$x^0 = ct \quad x^1 = x \quad x^2 = y \quad x^3 = z$

representation in which (x_μ) is diagonal:

$$(x'|x_\mu|x'') = x_\nu'\,\delta(x_0'-x_0'')\cdots\delta(x_3'-x_3'')$$

$$(x'|p_0|x'') = i\hbar\,\delta'(x_0'-x_0'')\,\delta(x_1'-x_1'') \times \delta(x_2'-x_2'')\,\delta(x_3'-x_3'')$$

$$(x'|p_1|x'') = -i\hbar\,\delta(x_0'-x_0'')\,\delta'(x_1'-x_1'') \times\delta(\ )\delta(\ )$$

$$p_0 = -\frac{i\hbar}{c}\frac{\partial}{\partial t} = -p^0$$

$$p_1 = -i\hbar\frac{\partial}{\partial x} = p^1$$

$$p_2 = -i\hbar\frac{\partial}{\partial y} = p^2$$

$$p_3 = -i\hbar\frac{\partial}{\partial z} = p^3$$

$$\int \delta'(x) f(x)\,dx = -f'(0)$$

$$\delta'(-x) = -\delta'(x)$$

$$(x'|p_0|x''')(x'''|U|x'') - (x'|U|x''')(x'''|p_0|x'') = \cancel{-i\hbar\left(\frac{\partial U}{\partial x^0}\right)}$$

$$+i\hbar\left(\frac{\partial U(x',x'')}{\partial x^{0\prime}} + \frac{\partial U(x'x'')}{\partial x^{0\prime\prime}}\right)$$

$$U(x'x'') = U(X, \xi)$$

$$X = \frac{1}{2}(x'+x'') \qquad \xi = x'-x''$$

$$\frac{\partial}{\partial X} = \frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}$$

$$\frac{\partial}{\partial \xi} = \frac{1}{2}\left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''}\right)$$

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial T^2}\right) U = 0$$

$$(\xi'_\mu - \xi''_\mu)(\xi'^\mu - \xi''^\mu) U = 0$$

$$U = (V)^{-\frac{1}{2}} \sum_{k_\mu} u^{(k_\mu, \xi)} \exp(ik_\mu X^\mu) \times (2k_0)^{-\frac{1}{2}}$$

with $k^\mu k_\mu = 0$.

U: real for X^μ;

or $U = (V)^{-\frac{1}{2}} \sum_{\substack{k_\mu \\ k_0 > 0}} u(k, \xi) \exp(ik_\mu X^\mu)$

$+ \sum_{\substack{k_\mu \\ k_0 < 0}} u^*(k, -\xi) \exp(ik_\mu X^\mu)$

because of ~~the~~ $\tilde{U}(X, \xi) = U(X, -\xi)^*$.

In the usual theory U is a diagonal matrix, so that $U(X, \xi) \neq 0$ only for $\xi^\mu = 0$. Thus if we write

$u(k, \xi) = u(k)$ for $\xi^\mu = 0$.

* The condition that U is a Hermite operator is

$$\tilde{U}(x', x'') = U(x'' x')$$

or $\tilde{U}(X, \xi) = U(X, -\xi)$

In this case, the usual formulation is ~~as~~ as follows[1]:

$$L = \frac{1}{2}\sum_{\mu}\left(\frac{\partial U}{\partial x_{\mu}}\frac{\partial U}{\partial x^{\mu}}\right)$$

$$T_{ik} = \frac{\partial U}{\partial x_{\mu}}\frac{\partial U}{\partial x_{\nu}} - L\, g_{\mu\nu}$$

$$U(x_{\mu}) = (V)^{-\frac{1}{2}} \sum_{k} (2k_0)^{-\frac{1}{2}} \{ u(k) \exp(i k^i x^i - k^0 x^0) + u^*(k) \exp(-i k^i x^i + k^0 x^0) \} \quad \text{with } k^0 > 0.$$

$$[u(k'), u^*(k'')] = \delta(k', k'') \quad *$$

We can extend the formalism in the following way:

$$[u(k', \xi'), u^*(k'', \xi'')] = \delta(k', k'') \times F(\xi', \xi''),$$

where F should be some invariant function of two four vectors ξ'_{μ}, ξ''_{μ}

(1) Pauli, Rev. Mod. Phys. 13 (1941), 203; Part II, section 1.

* If we go back to x_{μ}-representation

$$i[U(x'_{\mu}), U(x''_{\mu})] = D(x'_{\mu} - x''_{\mu})$$

$$D(x_{\mu}) = \frac{1}{(2\pi)^3}\int (dk)^3 e^{ikx} \frac{\sin k_0 x_0}{k_0}$$

Tuesday Seminar

Remarks on Non-localizable Systems

(Nov. 30, 1948, Fuld Hall)

Recent development of quantum electrodynamics ~~by~~ due to beautiful works by Schwinger, Feynman and Dyson indicates ~~clearly~~ that on the one hand that ~~the~~ special theory of relativity and quantum mechanics are connected with each other more over intimately ~~connected~~, so that ~~it seems~~ ~~improbable~~ ~~probable~~ both of them will very probably retained in the future theory in some way. On the other hand it is also evident that we ~~need~~ still have to find out some ~~method~~ adequate method of removing ultraviolet catastroph. Many attempts have been made in this direction, but none of them seems satisfactory. I have been trying to extend the field theory by discarding the restriction that the field is localizable according to ~~the~~ Dirac's terminology, and instead of so-to-speak postulate of localizability I adopted the principle of reciprocity by Born as a guiding principle.

I already spoke a little about the non localizable field ~~at~~ at the theoretical seminar last time. I want to begin with recapitulating it, before proceeding further.

i) The new point of view, in which the space-time is no more a common background of whole description, but is considered as something on the equal footing as the field quantities, such as the number of elementary particles of definite kind.

ii) Commutation relations between field quantities, ~~and~~ space-time and momentum-energy in conforming ~~of~~ with the principle of reciprocity

iii) Introduction of new set of canonical coordinates ~~complex coordinates in~~ 8-dimensional phase space, ~~universal ~~ from ~~Space~~ which universal length and space-time displacement operators come out in a relativistically invariant way.

iv) It can be shown that the ~~electrodynamic~~ electromagnetic field is nearly localizable, the deviations appearing only for very high energy of order of μc^2, whereas the meson field is essentially nonlocalizable.

First Possibility

Introduction of Complex Space

(Based on the Lecture of Annual Meeting of P.S.J.: May 23, 1948 at Kyoto University)

~~two four vectors ξ_μ and ξ^*_μ~~

Hitherto I considered in a general way ~~the~~ the features of non-localizable field and it was found there may be some consistent theory in accord with the principle of reciprocity. The above general scheme contain too many things, which can ~~not~~ hardly have physical meaning. That is the field quantities u, u^* depend not only on k_μ, but also only ξ_μ. ~~In other~~ The appearance of extra variables ξ_μ means to ascribe extra degrees of freedom to the photon other than those characterized by ~~the~~ its momentum $\hbar \mathbf{k}$ and direction of polarization. Apparently there is an intimate correspondence between $u(k,0)$ and $u(k)$ of the ordinary field theory, but $u(k,\xi)$ for $\xi \neq 0$ seems to be something superfluous.

Thus our next task is to find a way of packing these $u(k,\xi)$ together such that the degrees of freedom for the photon are just its momentum and polarization, instead of discarding altogether $u(k,\xi)$ for $\xi \neq 0$, which brings us again to the usual theory.

Here we need something heuristic.

Now we look at the commutation relations between field quantities and (x, p), we find ~~immediately~~ that ~~the same form of~~ it will be useful to make a canonical transformation in (x, p)-space:

$$\left.\begin{aligned} \xi_\mu &= \frac{1}{\sqrt{2}}\left\{\frac{x_\mu}{\lambda} + i\frac{\lambda}{2\hbar} p_\mu\right\} \\ \xi_\mu^* &= \frac{1}{\sqrt{2}}\left\{\frac{x_\mu}{\lambda} - i\frac{\lambda}{2\hbar} p_\mu\right\} \end{aligned}\right\}$$

λ: universal length of the order of between $\frac{\hbar}{\mu c}$ and $\frac{\hbar}{Mc}$.

The commutation relations are

$$\xi_\mu \xi_\nu^* - \xi_\nu^* \xi^\mu = \delta_{\mu\nu}{}^*$$

The commutation relations between these coordinate variables with the field ~~e~~quantities are, by using the auxiliary quantities

$$\left.\begin{aligned} &[\xi_\mu, U] = iH_\mu^* \qquad [\xi_\mu^*, U] = iH_\mu^* \\ &[\xi_\mu^\mu + \xi_\mu^{\mu *}, H_\mu + H_\mu^*] = 0 \\ &[\xi_\mu^\mu - \xi_\mu^{\mu *}, H_\mu - H_\mu^*] = 0 \end{aligned}\right\}$$

or

---

* It is of some interest ~~that~~ the operators

$$n_\mu = \xi_\mu^* \xi^\mu$$

have the eigenvalues $0, 1, 2, \ldots$, so that the (x, p)-space may be said to have a discrete structure. It is tempting to consider the field quantities are functions of n_μ only, but it destruct the covariance because n_μ ~~alone are not~~ alone do not form a four vector.

$$\left.\begin{array}{l}[\xi^\mu, H_\mu] + [\xi^{\mu *}, H_\mu^*] = 0 \\ [\xi^\mu, H_\mu^*] + [\xi^{\mu *}, H_\mu] = 0\end{array}\right\} \quad \text{~~(a)~~ (1)}$$

Now we introduce an hypothesis that U is a ~~t~~ sum of functions each depending on ξ_μ or ξ_μ^* only. Thus

$$U(\xi, \xi^*) = \frac{1}{2}\{u(\xi) + u^*(\xi^*)\}^\dagger$$

Then we find at once that

$$iH_\mu = [\xi_\mu, u^*(\xi^*)]$$

is ~~contly~~ depend~~ent~~s only on ξ_μ^*, because

$$\xi_\mu (\xi^{\nu *})^n - (\xi^{\nu *})^n \xi_\mu = n(\xi^{\nu *})^{n-1},$$

so that for any $\underbrace{\text{f}^{\underline{n}} \text{ of } \xi^*}_{u^*(\xi^*)}$ which can be represented as power series, $[\xi_\mu\ u^*(\xi^*)]$ can be reduced to ~~a~~ $\text{f}^{\underline{\underline{ns}}}$ of ξ^* of only and indeed as the derivatives of u^* with respect to $\xi^{\mu *}$. Similarly

$$iH_\mu^* = [\xi_\mu^*, u(\xi)]$$

is a ~~f~~ $\text{f}^{\underline{\underline{ns}}}$ of ξ_μ only. So that the first of commutation relations (1) ~~reduces to~~ becomes

~~$[\xi^\mu, H_\mu] = \xi^\mu\{$~~

$$[\xi^\mu\ [\xi_\mu\ u^*(\xi^*)]] + [\xi^{*\mu}\ [\xi_\mu^*, u(\xi_\mu)]] = 0$$

† ~~Reality condi~~ Hermiticity condition is

$$U^*(\xi, \xi^*) = U^*(\xi^*, \xi),$$

u^* denoting the complex conjugate $\text{f}^{\underline{n}}$ to ~~of~~ u

the second ~~becoming~~ reducing to an identity.
Now ~~this equation~~ the left hand side of this eq. is a sum of two fns of ξ ~~each~~ depending ξ^*_μ (or ~~and~~) ξ_μ only ~~respectively~~, so that ~~two~~ each fn should be zero (or constant)

$$\left.\begin{array}{l}[\xi^\mu[\xi_\mu, u^*(\xi^*_\mu)]] = 0 \\ [\xi^{*\mu}[\xi^*_\mu, u(\xi_\mu)]] = 0\end{array}\right\}$$

Formal general solutions of these equations are

$$\left.\begin{array}{l}u(\xi_\mu) = (V)^{-\frac{1}{2}} \sum (2|k_0|)^{-\frac{1}{2}} a(k_\mu) e^{ik_\mu \xi^\mu} \\ u^*(\xi^*_\mu) = (V)^{-\frac{1}{2}} \sum\limits_{k_\mu} (2|k_0|)^{-\frac{1}{2}} a^*(k_\mu) e^{-ik_\mu \xi^{*\mu}}\end{array}\right\}$$

with $\quad k_\mu k^\mu = 0$

Now we ~~want~~ to find the correspondence of these formal solutions to the ordinary ~~fo~~ Fourier expansion of the scalar field.
First we consider the unquantized field:

$$e^{ik_\mu \xi^\mu} = e^{ik_\mu\left(\frac{x^\mu}{\lambda} + i\frac{\lambda}{\hbar} p^\mu\right)}$$

$$= e^{ik_\mu x^\mu/\lambda - \lambda k_\mu \frac{\partial}{\partial x_\mu}}$$

$$= 1 + \sum_{n=1}^{\infty} \frac{1}{n!}\left(ik_\mu \frac{x^\mu}{\lambda} - \lambda k_\mu \frac{\partial}{\partial x_\mu}\right)^n \quad (1)$$

~~Continued from Phys. II.~~

Now

$$e^{ik_\mu \xi^\mu} = e^{ik_\mu\left(\frac{x^\mu}{\lambda} + i\frac{\lambda}{\hbar} p^\mu\right)}$$

$$= \exp\left\{\frac{ik_\mu x^\mu}{\lambda} - \frac{\lambda}{\hbar} k^\mu p_\mu\right\},$$

in which the two factors commute with each other, because

$$\left[\frac{ik_\mu x^\mu}{\lambda}, -\frac{\lambda}{\hbar} k^\nu p_\nu\right]$$

$$= -\frac{i}{\hbar} k_\mu k^\nu [x^\mu, p_\nu] = -\frac{i}{\hbar} k_\mu k^\nu (i\hbar\, \delta_{\mu\nu})$$

$$= k_\mu k^\mu$$

and

$$k_\mu k^\mu = 0$$

for the field corresponding to the quanta with zero mass.

Thus we have:

$$e^{ik_\mu \xi^\mu} = e^{ik_\mu \frac{x^\mu}{\lambda}} e^{i\lambda k^\mu \frac{\partial}{\partial x^\mu}}$$

or

$$e^{ik_\mu x^\mu/\lambda} e^{\cancel{i}\lambda k^\mu \frac{\partial}{\partial x^\mu}} \psi(x^\mu)$$

$$= e^{ik_\mu x^\mu/\lambda} \psi(x^\mu + \lambda k^\mu) \cancel{e^{ik^\mu \frac{\partial}{\partial x^\mu}}}$$

Similarly

$$e^{ik_\mu \xi^{\mu *}} \psi(x^\mu) = e^{ik_\mu x^\mu/\lambda} \psi(x^\mu \underset{(-i)}{+} \lambda k^\mu).$$

† This is not the true matrix representation

~~In the representation, in which x^μ is diagonal~~

$$\cancel{(x'|e^{i\lambda k^\mu \frac{\partial}{\partial x^\mu}}|x'') = \delta(x^{\mu\prime} - x^{\mu\prime\prime} + i\lambda k^\mu)}$$

$$\cancel{(x'|e^{-i\lambda k^\mu \frac{\partial}{\partial x^\mu}}|x'') = \delta(x^{\mu\prime} - x^{\mu\prime\prime} - i\lambda k^\mu)} \quad \dagger$$

The total field U can be expanded in the form:

$$\cancel{U(\zeta,\zeta^*) = V^{-\frac{1}{2}}\sum (2|k_0|)^{-\frac{1}{2}} a(k^\mu)\frac{1}{2}\{\delta(x^{\mu\prime}-x^{\mu\prime\prime}-\lambda k^\mu) + \delta(x^{\mu\prime}-x^{\mu\prime\prime}+\lambda k^\mu)\} e^{ik}}$$

$$U(\zeta,\zeta^*) = V^{-\frac{1}{2}} \sum_{k_\mu} \underbrace{(2|k_0|)^{-\frac{1}{2}}}_{a(k_\mu)} \frac{1}{\cancel{2}} \overbrace{\{D_-(i\lambda k^\mu)}^{a_-(k^\mu)} + \underbrace{a_+ D_+(i\lambda k^\mu)}_{a^*(k_\mu)}\} a_k e^{ik_\mu x^\mu/\lambda},$$

where $D_-(i\lambda k^\mu)$ ~~and $D_+(\lambda k^\mu)$~~ are the operators with the matrix elements

$$(x'|D_-(i\lambda k^\mu)|x'') = \delta(x^{\mu\prime} - x^{\mu\prime\prime} + i\lambda k^\mu)$$

$$\cancel{(x'|D_+(\lambda k^\mu)|x'') = \delta(x^{\mu\prime} - x^{\mu\prime\prime} + \lambda k^\mu)}$$

$$D^*_-(i\lambda k^\mu) = D_+(-\lambda k^\mu), \quad D_-D_+ = D_+D_- = 1.$$

~~Next~~ Thus

$$D(\lambda k^\mu) = \frac{1}{2}\{D_-(\lambda k^\mu) + D_+(\lambda k^\mu)\}$$

is a Hermitian operator, and moreover

$$D(\lambda k^\mu) = D(-\lambda k^\mu)$$

Quantization of the Field

~~In the expansion~~ If we expand as above

$$U = V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}} \{ a_-(k_\mu) D_-(\lambda k^\mu) + a_+(k_\mu) D_+(\lambda k^\mu) \} e^{i k_\mu x^\mu / \lambda} ,$$

the condition ~~that~~ $U = U^*$, i.e. U is a Hermitian operator reduces to

$\cancel{U = U^*}$

$$U = U^*$$

$$\sum_{k_\mu} \{ a_-(k_\mu) D_-(\lambda k^\mu) + a_+(k_\mu) D_+(\lambda k^\mu) \} e^{i k_\mu x^\mu / \lambda}$$

$$= \sum_{k_\mu} \{ a_-^*(k_\mu) D_-^*(\lambda k^\mu) + a_+^*(k_\mu) D_+^*(\lambda k^\mu) \} e^{-i k_\mu x^\mu / \lambda}$$

$$= \sum_{k_\mu} \{ a_-^*(-k_\mu) D_-^*(-\lambda k^\mu) + a_+^*(-k_\mu) D_+^*(-\lambda k^\mu) \} e^{i k_\mu x^\mu / \lambda}$$

$$= \sum_{k_\mu} \{ a_-^*(-k_\mu) D_-(\lambda k^\mu) + a_+^*(-k_\mu) D_+(\lambda k^\mu) \} e^{i k_\mu x^\mu / \lambda} , \dagger$$

or

$$\left. \begin{aligned} a_-(k_\mu) &= a_-^*(-k_\mu) \\ a_+(k_\mu) &= a_+^*(-k_\mu) \end{aligned} \right\}$$

$\dagger$ $D_-^*(\lambda k^\mu) = D_+(\lambda k^\mu) = D_-(-\lambda k^\mu)$

or $D_-^*(-\lambda k^\mu) = D_-(\lambda k^\mu)$

and similarly $D_+^*(-\lambda k^\mu) = D_+(\lambda k^\mu)$

Thus U can be rewritten in the form

$$U = V^{-\frac{1}{2}} \sum_{k_\mu, (k_0>0)} (2k_0)^{-\frac{1}{2}} \left(\frac{1}{2}\right) \{ a_-(k_\mu) D_-(\lambda k^\mu) e^{ik_\mu x^\mu/\chi}$$
$$+ a_-^*(k_\mu) D_-(-\lambda k^\mu) e^{-ik_\mu x^\mu/\chi} + a_+(k_\mu) D_+(\lambda k^\mu) e^{ik_\mu x^\mu/\chi}$$
$$+ a_+^*(k_\mu) D_+(-\lambda k^\mu) e^{ik_\mu x^\mu/\chi} \}$$

or

$$U = V^{-\frac{1}{2}} \sum_{k_\mu, (k_0>0)} (2k_0)^{-\frac{1}{2}} \left(\frac{1}{2}\right) \left[\{ a_-(k_\mu) D_-(\lambda k^\mu) \right.$$
$$+ a_+(k_\mu) D_+(\lambda k^\mu) \} e^{ik_\mu x^\mu/\chi} + \{ a_-^*(k_\mu) D_-(-\lambda k^\mu)$$
$$\left. + a_+^*(k_\mu) D_+(-\lambda k^\mu) \} e^{-ik_\mu x^\mu/\chi} \right]$$

In the representation, in which x^μ is diagonal,

$$(x' | U | x'') = V^{-\frac{1}{2}} \sum_{k_\mu (k_0>0)} (2k_0)^{-\frac{1}{2}} \frac{1}{2} \left[\{ a_-(k_\mu) e^{ik_\mu x'^\mu/\chi} \right.$$
$$+ a_+^*(k_\mu) e^{-ik_\mu x'^\mu/\chi} \} \delta(x'^\mu - x''^\mu - \lambda k^\mu)$$
$$\left. + \{ a_+(k_\mu) e^{ik_\mu x'^\mu/\chi} + a_-^*(k_\mu) e^{-ik_\mu x'^\mu/\chi} \} \delta(x'^\mu - x''^\mu + \lambda k^\mu) \right]$$

In the limit $\chi \to 0$,

$$\frac{1}{2} \{ a_-(k_\mu) + a_+(k_\mu) \} \to u(k_\mu)$$

$$\frac{1}{2} \{ a_-^*(k_\mu) + a_+^*(k_\mu) \} \to u^*(k_\mu)$$

The difference between the old and new theories will be very clear by comparing the matrix schemes;

$$
\begin{array}{cc}
 & x'_\mu \\
x'_\mu & \left(\begin{array}{ccc} \ddots & \vdots & 0 \\ \cdots & \sum_{k_\mu}(\ )e^{ik_\mu x'^\mu/\lambda} & \\ 0 & & \ddots \end{array}\right)
\end{array}
$$

old theory

$$
\begin{array}{ccc}
 & x'_\mu-\lambda k'_\mu & x'_\mu+\lambda k'_\mu \\
x'_\mu & \multicolumn{2}{c}{\left(\begin{array}{cc} \ddots & \vdots \\ \sum(\ )e^{-ik_\mu x'^\mu/\lambda} + \sum(\ )e^{ik_\mu x'^\mu/\lambda} & \sum(\ )e^{ik_\mu x'^\mu/\lambda} + \sum(\ )e^{-ik_\mu x'^\mu/\lambda} \\ & \ddots \end{array}\right)}
\end{array}
$$

new theory

Thus the off diagonal distance (not four dimensional, but three or one dimensional) is larger, the larger the ~~wave~~ momentum

or energy of the photon concerned.
The off diagonal distance is of the order of universal length λ for the photon of wave length $\frac{\hbar}{\lambda}$. If we take $\lambda = \frac{e^2}{mc^2}$, the ~~energy~~ critical energy of the photon is $\frac{\hbar c}{\lambda} = \left(\frac{\hbar c}{e^2}\right) mc^2 = 137\, mc^2$. Below this critical energy, the deviation from the ordinary electrodynamics is very small, because the ratio of off diagonal distance λk_0 and the wave length $\frac{2\pi\lambda}{\cancel{2\pi} k_0}$ is ~~proportional to~~ equal to

$$\lambda k_0 : 2\pi\lambda / \cancel{2\pi} k_0 = \cancel{2\pi}\, k_0^2/2\pi$$

which is equal to $\frac{\alpha^2}{2\pi} = \frac{1}{2\pi}\left(\frac{1}{137}\right)^2$ for

$k_0 = \alpha$, corresponding to photon energy

$$\frac{k_0 \hbar c}{\lambda} = \frac{e^2}{\hbar c} \cdot \frac{\hbar c}{\frac{e^2}{mc^2}} = mc^2.$$

On the contrary, in the case of the meson field with the mass more than $200\, m_e$, off diagonal distance — in time direction — is ~~always~~ at least ~~two~~ of the same order of magnitude as λ, so that the meson field ~~is ess~~ should always be considered as essentially nonlocalizable, as presupposed by ~~Oppenheimer~~ various physicists, especially by

But the case of the meson field ~~with the~~ accompanied by quanta with rest mass can~~not~~ be ~~dealed with~~ treated in conformity with the principle of reciprocity ~~in~~ only in 5-dimensional space(1).

Before entering into the more complicated cases, we ~~want to~~ have ~~consider~~ to solve the problem of quantization in the ~~cas~~ simplest case of scalar neutral field with zero mass.

Our first task is to find correct commutation relations between $a_-(k_\mu)$, $a_+(k_\mu)$, $a_-^*(k_\mu)$, $a_+^*(k_\mu)$. As shown above, we cannot determine unambiguously ~~uniquely~~ the relations by ~~only~~ correspondence theoretical considerations only, because classical ~~$a_-(k_\mu)+a_+(k_\mu)$~~ ~~to~~ $u(k_\mu)$ corresponds to the sum of $a_-(k_\mu)$ and $a_+(k_\mu)$.

~~Let us We tentatively assume the commutation relations~~

$$\cancel{[a_-(k'_\mu), a_-^*(k''_\mu)] = \delta(k'_i, k''_i)^*}$$

$$\cancel{[a_+(k'_\mu), a_+^*(k''_\mu)] = \delta(k'_i, k''_i)^*},$$

(1) Yukawa, Prog. Theor. Phys. 3 (1948), 205.

* Here δ's are three dimensional δ-fns.

~~any other pair of variables being commutative with each other.~~

If we go back, however, to the original expression, this means

$$\cancel{U(\xi,\xi^*) = \frac{1}{2}\sum (2k_0)^{-\frac{1}{2}} a(k_\mu) e^{ik_\mu \xi_\mu}}$$

$$V^{-\frac{1}{2}}\frac{1}{2}.$$

$$U(\xi,\xi^*) = V^{-\frac{1}{2}}\cdot\frac{1}{2}\sum_{k_\mu (k_0>0)} (2k_0)^{-\frac{1}{2}}\{a_-(k_\mu)e^{ik_\mu\xi_\mu}$$

$$+ a_-^*(k_\mu)e^{-ik_\mu\xi_\mu} + a_+(k_\mu)e^{ik_\mu\xi_\mu^*}$$

$$+ a_+^*(k_\mu)e^{-ik_\mu\xi_\mu^*}\}, \quad \cancel{= \frac{1}{2}\{u(\xi) + u^*(\xi^*)\}}$$

~~so that~~

$$\cancel{[(\xi_\nu - \xi_\nu^*), U] = \frac{1}{2}\xi u(\xi).}$$

In order that $U(\xi,\xi^*)$ has the form

$$U(\xi,\xi^*) = \frac{1}{2}\{u(\xi) + u^*(\xi^*)\},$$

$$a_-(k_\mu) = a_+^*(k_\mu) = a(k_\mu)$$

$$a_-^*(k_\mu) = a_+^*(k_\mu) = a^*(k_\mu) \quad \text{say}$$

should hold, so that U reduces to

$$U(\xi,\xi^*) = V^{-\frac{1}{2}}\sum_{k_\mu(k_0>0)}(2k_0)^{-\frac{1}{2}}\{a(k_\mu)e^{ik_\mu\xi_\mu}$$

$$+ a^*(k_\mu)e^{-ik_\mu\xi_\mu^*}\}$$

and the commutation relations become

$$[a(k'_\mu), a^*(k''_\mu)] = \delta(k'_i, k''_i),$$

where $\delta(k'_i\ k''_i)$ is the three dimensional δ-fn in (k_1, k_2, k_3) space.

~~The In the limit of to~~ $\lambda \to 0$, ~~the~~

$$\cancel{u(x^\mu) = u^*(x^\mu)},$$

~~i.e. the field~~ which different from ~~ordinary~~ formalism

In the limit of $\lambda \to 0$

$$a(k_\mu) \to u(k_\mu)$$

$$a^*(k_\mu) \to u^*(k_\mu),$$

which correspond to the fact that the ordinary scalar field U should be hermitian operator.

From the ~~above~~ commutation relations

$$[\xi^\mu, \xi^*_\nu] = \delta_{\mu\nu},$$

we obtain

$$[\xi^\mu\ e^{-ik''^\nu\xi^*_\nu}] = -ik^{\mu''}e^{-ik^{\nu''}\xi^*_\nu}$$

$$[ik'_\mu\xi^\mu, e^{-ik^\nu\xi^*_\nu}] = k'_\mu k^{\mu''}e^{-ik^{\nu''}\xi^*_\nu}$$

$$[e^{ik'_\mu\xi^\mu}, e^{-ik^\nu\xi^*_\nu}] = k'_\mu k^{\mu''}e^{ik'_\mu\xi^\mu}\cdot e^{-i}$$

Total field U can be expanded in the form

$$U(\xi, \xi^*) = \frac{1}{2}\left(u(\xi) + u^*(\xi^*)\right)$$

$$= V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}} \frac{1}{2} \left\{ a(k_\mu) e^{ik_\mu \xi^\mu} + a^*(k_\mu) e^{-ik_\mu \xi^{\mu *}} \right\}$$

$$= V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}} \frac{1}{2} \left\{ a(k_\mu) e^{ik_\mu x^\mu} + a^*(k_\mu) e^{-ik_\mu x^\mu} \right\}$$

$$\times \, e^{i\lambda k^\mu \frac{\partial}{\partial x^\mu}} \qquad (1)$$

Now the operator $e^{i\lambda k^\mu \frac{\partial}{\partial x^\mu}}$ has the property:

$$e^{i k^\mu \frac{\partial}{\partial x^\mu}} \cdot \psi(x^\mu) = \psi(x^\mu + i k^\mu)$$

$$e^{i k^\mu \frac{\partial}{\partial x^\mu}} \cdot e^{i k_\mu x^\mu} = e^{i k_\mu x^\mu} \cdot e^{-\lambda k_\mu k^\mu}$$

Hence, if k_μ is a time like vector $\underbrace{\text{and}}_{\text{with } k_0 > 0}$ $k^0 > 0$ $(k_0 < 0)$,

$$-\lambda k_\mu k^\mu < 0,$$

so that, if we restrict the summation in (1) with respect to k_μ only ~~those k_μ with k_0 those~~ those with $k_0 < 0$, high energy

Second Possibility

Introduction of Real Coordinates in Phase Space

Instead of the combination

$$\left.\begin{aligned} \xi_\mu &= \frac{x_\mu}{\lambda} + i\frac{\lambda}{2\hbar} p_\mu \\ \xi^*_\mu &= \frac{x_\mu}{\lambda} - i\frac{\lambda}{2\hbar} p_\mu , \end{aligned}\right\}$$

we can alternatively consider a canonical transformation

$$\left.\begin{aligned} \eta_\mu &= \frac{x_\mu}{\lambda} + \frac{\lambda}{2\hbar} p_\mu \\ \zeta_\mu &= \frac{x_\mu}{\lambda} - \frac{\lambda}{2\hbar} p_\mu \end{aligned}\right\}$$

with the commutation relations *

$$\eta^\mu \zeta_\nu - \zeta_\nu \eta^\mu = -i\,\delta_{\mu\nu}$$

Field quantities ~~can be has~~ are assumed to have the form

$$U = \frac{1}{2}\left\{ v(\eta_\mu) + w(\zeta_\mu) \right\}$$

$$v(\eta_\mu) = V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}}\, b(k_\mu)\, e^{i k_\mu \eta^\mu}$$

$$w(\zeta_\mu) = V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}}\, c(k_\mu)\, e^{i k_\mu \zeta^\mu}$$

---

* $\eta^\mu \zeta_\nu - \zeta_\nu \eta^\mu = \left(\frac{x^\mu}{\lambda} + \frac{\lambda}{2\hbar} p^\mu\right)\left(\frac{x_\nu}{\lambda} - \frac{\lambda}{2\hbar} p_\nu\right) - \left(\frac{x_\nu}{\lambda} - \frac{\lambda}{2\hbar} p_\nu\right)\left(\frac{x^\mu}{\lambda} + \frac{\lambda}{2\hbar} p^\mu\right) = -\frac{1}{2\hbar}(x^\mu p_\nu - p_\nu x^\mu) - \frac{1}{2\hbar}(x_\nu p^\mu - p^\mu x_\nu) = -\frac{1}{\hbar}\delta_{\mu\nu}(-i\hbar) = -i\,\delta_{\mu\nu}$

Now
$$\left.\begin{aligned} e^{ik_\mu \eta^\mu} &= e^{ik_\mu x^\mu/\lambda}\cdot e^{\frac{i\lambda k^\mu p_\mu}{2\hbar}} \\ e^{ik_\mu \xi^\mu} &= e^{ik_\mu x^\mu/\lambda}\, e^{-i\lambda k^\mu p_\mu/2\hbar} \end{aligned}\right\}$$

~~Consider~~ any operator S, we have

$$\sum_{x''} (x'|e^{i\lambda k^\mu p_\mu/2\hbar}|x'')\,\cancel{S}\,(x''|S|x'')$$

$$= \cancel{x^{\mu''}+\lambda}\;\cancel{(x'|}\sum_n \frac{1}{n!}\left(\lambda k^\mu_2 \frac{\partial(x'|S|x'')}{\partial x^\mu}\right)\cancel{|x''|}$$

$$= (x^{\mu'}+\lambda k^\mu/2\,|S|\,x^{\mu''})$$

similarly

$$(x'|S\,e^{i\lambda k^\mu p_\mu/\hbar}|x'') = (x'|S|x^{\mu''}-\lambda k/2)$$

$$(x'|e^{-i\lambda k^\mu p_\mu/\hbar}S|x'') = (x'-\lambda k|S|x'')$$

$$(x'|S\,e^{-i\lambda k^\mu p_\mu/\hbar}|x'') = (x'|S|x''+\lambda k/2)$$

Thus we have
$$\left.\begin{aligned} (x'|e^{i\lambda k^\mu p_\mu/2\hbar}|x'') &= \delta(x'-x''+\lambda k/2) \\ (x'|e^{-i\lambda k^\mu p_\mu/2\hbar}|x'') &= \delta(x'-x''-\lambda k/2) \end{aligned}\right\}$$

~~Fro~~ Further from symmetry considerations, we assume v and w ~~have~~ the same form, so that

$$U = \frac{1}{2}\{v(\eta_\mu) + v(\xi_\mu)\}$$

$$= V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}} \cancel{b}\, b(k_\mu) e^{ik_\mu x^\mu}$$

$$\times \frac{1}{2}\{D\left(-\frac{\lambda k}{2}\right) + D\left(+\frac{\lambda k}{2}\right)\}$$

where D is the operator with the matrix el.

$$(x'|D(\frac{\lambda k}{2})|x'') \cancel{\frac{1}{2}\bar{D}} \delta(x' - x'' + \frac{\lambda k}{2})$$

In order that U is Hermitian operator

$$U^* = V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}} b^*(k_\mu) e^{-ik_\mu x^\mu}$$

$$\times \frac{1}{2}\{D(\frac{\lambda k}{2}) + D(-\frac{\lambda k}{2})\}$$

$$= U$$

so that

$$b(-k_\mu) = b^*(k_\mu),$$

and U reduces to the form

$$U = V^{-\frac{1}{2}} \sum_{k_\mu (k_0 > 0)} (2|k_0|)^{-\frac{1}{2}} \{b(k_\mu) e^{ik_\mu x^\mu} + b^*(k_\mu) e^{-ik_\mu x^\mu}\}$$

$$\frac{1}{2}\{D(\frac{\lambda k}{2}) + D(-\frac{\lambda k}{2})\}$$

If we compare our formalism with Heisenberg's subtraction device in the positron theory[1], we find that the difference is taking ~~ins~~ instead of space-time ~~i) either for~~ $U(x_\mu)$

i) either

$$\frac{1}{2}\{U(x_\mu + \tfrac{1}{2}\lambda_\mu) + U(x_\mu - \tfrac{1}{2}\lambda_\mu)\}$$

ii) or change the ~~coef.~~ Fourier coef. ~~in $U(x_\mu)$ by~~ $b(k_\mu)$, $b^*(k_\mu)$ into

$$\left.\begin{array}{l} b(k_\mu)\,\frac{1}{2}\{D(\frac{\lambda k}{2}) + D(-\frac{\lambda k}{2})\} \\ b^*(k_\mu)\,\frac{1}{2}\{D(\frac{\lambda k}{2}) + D(-\frac{\lambda k}{2})\} \end{array}\right\}.$$

In the case i), λ_μ should be a vector with the ~~magni~~ components of the order of magnitude λ, but we can say nothing about the direction of the vector λ_μ, whereas in ii) the whole procedure is ~~unambiguously defined~~ welldefined and evidently relativistically invariant.

[1] Heisenberg, ZS. f. Phys. 90 (1934), 209, Serber, Phys. Rev. 49 (1936), 545.

Further as in the usual theory, we assume the commutation relations

$$[b(k_\mu'),\ b^*(k_\mu'')] = \delta(k_\mu' - k_\mu''),$$

where $\delta(k_i', k_i'')$ is the three dimensional δ-fn in (k_1, k_2, k_3)-space.

If we consider U as a matrix in x-space, the matrix elements only those

$$\cancel{(x'|U|x')} \quad \text{~~(diagonal.~~}$$

$$(x'|U|x' - \frac{\lambda k}{2})$$
$$(x'|U|x' + \frac{\lambda k}{2})$$

are not zero, where k is any vector lying in the light-cone (zero-vector). Thus U is an operator connecting two points, ~~on the~~, the relative coordinates ~~of which~~ forming a zero-vector.

~~An ka~~ An alternative possibility is to assume U either to be a function of η only or ζ only. For example, if we take U as fn of η only:

$$U = V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}}\, b(k_\mu)\, D(\frac{\lambda k}{2}),$$

$e^{ik_\mu \eta_\mu}$

In order that the condition

$$U = U^*$$

$$= V^{-\frac{1}{2}} \sum_{k_\mu} (2|k_0|)^{-\frac{1}{2}} b^*(-k_\mu) D\left(\frac{-\lambda k}{2}\right) \times e^{-ik_\mu x^\mu}$$

$$b(-k_\mu) = b^*(k_\mu),$$

so that

$$U = V^{-\frac{1}{2}} \sum_{k_\mu (k_0 \geq 0)} (2k_0)^{-\frac{1}{2}} \left\{ b^*(k_\mu) D\left(\frac{\lambda k}{2}\right) \times e^{ik_\mu x^\mu} + b^*(k_\mu) D\left(\frac{-\lambda k}{2}\right) e^{-ik_\mu x^\mu} \right\},$$

Hereafter ~~we~~ use this simpler formalism.

If we write this

$$U = u + u^*$$

$$[u, u^*] = V^{-1} \sum_{k'} \sum_{k''} (2k_0')^{-\frac{1}{2}} (2k_0'')^{-\frac{1}{2}} \left[b(k_\mu') D\left(\frac{\lambda k'}{2}\right) e^{ik_\mu' x^\mu}, b^*(k_\mu'') D\left(\frac{-\lambda k''}{2}\right) e^{-ik_\mu'' x^\mu} \right]$$

$$= V^{-1} \sum_{k'k''} [b(k_\mu'), b^*(k_\mu'')] D\left(\frac{\lambda k'}{2}\right) D\left(\frac{-\lambda k''}{2}\right) \frac{1}{(2k_0')^{\frac{1}{2}} (2k_0'')^{\frac{1}{2}}} \cancel{e^{ik_\mu' x^\mu} e^{-ik_\mu'' x^\mu}}$$

$$= \cancel{\delta(k_\mu', k_\mu'') \frac{1}{2Vk_0'}}$$

$$\left[b(k_\mu')D\left(\frac{\Lambda k'}{2}\right)e^{ik_\mu' x^\mu},\ b^*(k_\mu'')D\left(-\frac{\Lambda k''}{2}\right)e^{-ik_\mu'' x^\mu} \right]$$

$$= b(k_\mu')D\left(\frac{\Lambda k'}{2}\right)e^{ik_\mu' x^\mu}\cdot b^*(k_\mu'')D\left(-\frac{\Lambda k''}{2}\right)e^{-ik_\mu'' x^\mu}$$

$$- b^*(k_\mu'')D \ \text{''}\ e \ \text{''}\ \ b(k_\mu') \ \text{''} \ \ \text{''}$$

$$+ b^*(k_\mu'')D\left(\frac{\Lambda k'}{2}\right)e^{ik_\mu' x^\mu} b(k_\mu')D\left(-\frac{\Lambda k''}{2}\right)e^{-ik_\mu'' x^\mu}$$

$$- b^*(k_\mu'')D\left(\frac{\Lambda k'}{2}\right)D\left(-\frac{\Lambda k''}{2}\right)b(k_\mu') e^{+ik_\mu' x^\mu}e^{-ik_\mu'' x^\mu}$$

$$+ b^*(k_\mu'')D\left(\frac{\Lambda k''}{2}\right)D\left(\frac{\Lambda k'}{2}\right)b(k_\mu') e^{-ik_\mu' x}e^{+ik_\mu'' x^\mu}$$

$$- b^*(k_\mu'')D\left(-\frac{\Lambda k''}{2}\right)e^{-ik_\mu'' x}\ b(k_\mu')D\left(\frac{\Lambda k'}{2}\right)e^{ik_\mu' x^\mu}$$

$$= \delta(k_\mu' k_\mu'') + \cdots$$

If we ~~assume~~ U function of ζ only, we obtain another field of similar type. Thus we can always expect ~~two kinds of scalar fields~~, the existence of the same type. The only difference is that $D\left(\frac{\Lambda k}{2}\right)$ and $D\left(-\frac{\Lambda k}{2}\right)$ are interchanged.

Lagrangian Formalism

The ~~H~~ commutation relations take the form in the simplest case, when ~~W~~ is a fn of η only:

$$[\xi^\mu[\xi_\mu U]] = 0$$

This suggest us to assume the Lagrangian integral (action fn)

$$W = \mathrm{Trace}\,\tfrac{1}{2}\{[\xi^\mu U][\xi_\mu U]\}$$

where Trace means the diagonal sum with respect to ~~out~~ matrix elements in ~~ret~~ any representation. For example, in the representation, in which x_μ are diagonal

$$[\xi_\mu U] = V^{-\frac{1}{2}} \sum \frac{-k_\mu}{(2k_0)^{\frac{1}{2}}}\Big\{ b(k_\mu)\, D\big(\tfrac{\lambda k_\mu}{2}\big) e^{+} - b^*(k_\mu)\, D\big(\tfrac{-\lambda k}{2}\big) e^{-} \Big\}$$

$$\mathrm{Trace}[\xi^\mu U][\xi_\mu U]$$

$$= \iiint - V^{-1} \sum_{k_\mu} \frac{k^\mu k_\mu}{2k_0}\{ b^*(k_\mu) b(k_\mu) + b(k_\mu) b^*(k_\mu)\}$$

, which is, in fact, zero, but we can use this formally

This procedure is the same as taking as Lagrangian

$$\mathcal{L} = \mathrm{Trace}\,\tfrac{1}{2}\{[\eta^\mu U][\eta_\mu U]\},$$

because it is proportional to

$$\mathrm{Trace}\{[(\eta^\mu - \zeta^\mu), U][\eta_\mu - \zeta_\mu, U]\}$$

$$= \mathrm{Trace}\{[\zeta^\mu U][\zeta_\mu U]\}$$

Thus the canonical variables can be defined as follows:

$$U^\dagger = [\eta_0 U]$$

$$H = \ldots\ldots$$

Semi-Tradition Method

As we have not yet the complete formalism general method of non-localizable field, we confine ourselves for the time being to the following semi-tradition method:

The scalar field:

$$U(\eta) = V^{-\frac{1}{2}} \sum_{k_\mu (k_0 > 0)} (2k_0)^{-\frac{1}{2}} \{ b(k_\mu) D(\lambda k_\mu) e^{ik_\mu x^\mu} + b^*(k_\mu) D(-\lambda k_\mu) e^{-ik_\mu x^\mu} \}$$

with

$$\left.\begin{aligned} \eta_\mu &= \frac{x_\mu}{\cancel{\lambda}} + \frac{\lambda}{\cancel{2\hbar}} p_\mu \\ \zeta_\mu &= \frac{x_\mu}{\lambda} - \frac{\lambda}{\hbar} p_\mu \end{aligned}\right\} \quad \begin{aligned} x_\mu &= \frac{\lambda}{2}(\eta_\mu + \zeta_\mu) \\ p_\mu &= \frac{\hbar}{2\lambda}(\eta - \zeta_\mu) \end{aligned}$$

$$[\eta_\mu, \zeta^\nu] = -2i\,\delta_{\mu\nu},$$

can be ~~considered as fn of~~ represented as matrix ~~$X_\mu = \frac{1}{2}(x'_\mu + x''_\mu)$~~

$$(x'|U|x'') = V^{-\frac{1}{2}} \sum_{k_\mu (k_0 > 0)} (2k_0)^{-\frac{1}{2}} \{ b(k_\mu)\, \delta(x' - x'' + \lambda k) \times e^{ik_\mu x'^\mu} + b^*(k_\mu)\, \delta(x' - x'' - \lambda k)\, e^{-ik_\mu x'^\mu} \}$$

Now, since $k_\mu x''^\mu = k_\mu x'^\mu$ for $x'^\mu - x''^\mu = \pm\lambda k^\mu$, if we go back to the previous consideration

$$\left.\begin{aligned} X &= \frac{x' + x''}{2} \\ r &= \frac{}{x' - x''} \end{aligned}\right\}$$

$$(x'|U|x'') = U(X, r)$$

$$= V^{-\frac{1}{2}} \sum_{k_\mu (k_0>0)} (2k_0)^{-\frac{1}{2}} \{ b(k_\mu)\delta(r+\lambda k) e^{ik_\mu X^\mu}$$

$$+ b^*(k_\mu)\delta(r-\lambda k) e^{-ik_\mu X^\mu} \}$$

and U satisfies the wave equation

$$\frac{\partial^2 U}{\partial X^\mu \partial X_\mu} = 0$$

for X^μ.*

This suggest us to deal X_μ as the substitute for the space-time coordinates x_μ in the usual theory. Thus the Lagrangian density takes the form†

$$L = \frac{1}{2}\left(\frac{\partial U}{\partial X^\mu}\frac{\partial U}{\partial X_\mu}\right)$$

$$U_\mu^+ = -\frac{\partial L}{\partial \frac{\partial U}{\partial X^0}} = \frac{\partial U}{\partial x^0}$$

$$H = U_\mu^+ \frac{\partial U}{\partial x^0} - L = \frac{1}{2} U_\mu^+ U_\mu^+ + \frac{1}{2}(\text{grad}\, U)^2$$

---

* Furthermore U satisfies

$$r_\mu r^\mu U = 0$$

because U is not zero only for $r_\mu = \pm\lambda k_\mu$.

† Pauli, R.M.P. 13 (1941), 203, Part II.

$$L = \frac{1}{2}\iiiint \frac{\partial U^{(X,r)}}{\partial X^\mu}\frac{\partial U(X,-r)}{\partial X_\mu}(dr)^4$$

$$T_{\mu\nu} = \frac{1}{2}\iiiint\left\{\frac{\partial U^{(X,r)}}{\partial X^\mu}\frac{\partial U^{(X,-r)}}{\partial X^\nu} + \frac{\partial U^{(X,r)}}{\partial X^\nu}\frac{\partial U^{(X,-r)}}{\partial X^\mu}\right\} - L g_{\mu\nu}$$

$$T_{00} = \iiiint \frac{\partial U^{(X,r)}}{\partial X^0}\frac{\partial U^{(X,-r)}}{\partial X^0}dX^4 - \frac{1}{2}\iiiint\frac{\partial U^{(X,r)}}{\partial X^0}\frac{\partial U(X,-r)}{\partial X^0}dX^4$$
$$+\frac{1}{2}\iiiint(\mathrm{grad}\,U(X,r))(\mathrm{grad}\,U(X,-r))\,dX^4$$

$$\frac{\partial U}{\partial X^\mu} = V^{-\frac{1}{2}}\sum_{k_\mu(k_0>0)}(2k_0)^{-\frac{1}{2}}\,ik_\mu\left\{b(k_\mu)\delta(r+\lambda k)e^{ik_\mu X^\mu}\right.$$
$$\left. - b^*(k_\mu)\delta(r-\lambda k)e^{-ik_\mu X^\mu}\right\}$$

$U(X'',r'')$, $U(X',r')$, $\frac{\partial U(X',r')}{\partial X^{\mu'}}$, etc. are not commutative with each other only if

$$\left.\begin{array}{l}\delta(r'+\lambda k)\,\delta(r''-\lambda k)=0\\ \text{or}\ \delta(r'-\lambda k)\,\delta(r''+\lambda k)=0\end{array}\right\}$$

for same k, i.e. $r'=\lambda k$, $r''=-\lambda k$ or $r'=-\lambda k$, $r''=\lambda k$

~~$r'=r''=\pm\lambda k$~~

~~which means~~ Because

$$[U(X',r'),\ U(X'',r'')]$$

$$=V^{-\frac{1}{2}}\sum_{k_\mu(k_0>0)}[b(k_\mu),\ b^*(k_\mu)]\,\delta(r'+\lambda k)\delta(r''-\lambda k)\times\frac{e^{ik_\mu(X'^\mu-X''^\mu)}}{2k_0}$$

$$+ V^{-\frac{1}{2}} \sum_{k_\mu (k_0 > 0)} [b^*(k_\mu), b(k_\mu)] \delta(r' - \lambda k) \delta(r'' + \lambda k) \times \frac{e^{-ik_\mu (x'^\mu - x''^\mu)}}{2k_0}$$

Thus $U(x', r')$, $U(x'', r'')$ are commutative with each other, only if

$$\left.\begin{array}{ll} r' = -\lambda k' & r'' = \lambda k \\ \text{or } r' = \lambda k & r'' = -\lambda k \end{array}\right\}$$

~~and in these cases~~

$$\cancel{U(x', -\lambda k'), U(x''}$$

~~Energy~~ of the field: E.

$$\cancel{\frac{1}{c^2} E = \iiint T_{00} \, dx_1 dx_2 dx_3}$$

$$\cancel{= \sum \frac{1}{2k_0}}$$

Thus

$$L = \frac{1}{2} \iiiint \frac{\partial U(x, r)}{\partial x^\mu} \frac{\partial U(x, -r)}{\partial x_\mu} (dr)^4$$

$$= \frac{1}{2} \iiiint$$

where a product of the form $\delta(r + \lambda k')$ $\times \delta(r + \lambda k'')$ appears, so that the integration with respect to r gives $\delta(\lambda k'' - \lambda k')$

$$\left(\frac{\partial(X+\tfrac{1}{2}r|U|X-\tfrac{1}{2}r)}{\partial X}\right)\left(\frac{\partial(X-\tfrac{1}{2}r|U|X+\tfrac{1}{2}r)}{\partial X}\right)$$

and the ~~summation~~ double summation with respect to k', k'' reduces to a summation with respect to k'. ~~The result is~~ Namely

$$L=\frac{1}{2}\frac{\partial U(X,r)}{\partial X^\mu}\frac{\partial U(X,-r)}{\partial X_\mu} \quad *$$

$$=\frac{1}{2}V^{-1}\sum_{\substack{k'_\mu k''_\mu\,(k'_0>0)\\(k''_0>0)}}(2k'_0)^{-\frac{1}{2}}(2k''_0)^{-\frac{1}{2}}\, k'_\mu k^{\mu''}$$

$$\times\{b(k'_\mu)\delta(r_\mu+\lambda k'_\mu)e^{ik'_\mu X^\mu}-b^*(k'_\mu)\delta(r_\mu-\lambda k'_\mu)e^{-ik'_\mu X^\mu}\}$$

$$\times\{b(k''_\mu)\underset{(r_\mu-\lambda k''_\mu)}{\delta(-r_\mu+\lambda k''_\mu)}e^{ik''_\mu X^\mu}-b^*(k''_\mu)\underset{(r_\mu+\lambda k''_\mu)}{\delta(-r_\mu-\lambda k''_\mu)}e^{-ik''_\mu X^\mu}\}$$

$$\bar{L}=\iiiint L\,(dx)^4$$

$$=\frac{1}{2}V^{-1}\sum_{\substack{k'_\mu k''_\mu\\(k'_0>0)(k''_0>0)}}(2k'_0)^{-\frac{1}{2}}(2k''_0)^{-\frac{1}{2}}\,k'_\mu k^{\mu''}$$

$$\{\,b(k'_\mu)b(k''_\mu)\delta(\lambda k'_\mu+\lambda k''_\mu)e^{i(k'_\mu+k''_\mu)X^\mu}$$
$$-b^*(k'_\mu)b(k''_\mu)\delta(\lambda k''_\mu-\lambda k'_\mu)e^{-i(k'_\mu-k''_\mu)X^\mu}$$
$$-b(k'_\mu)b^*(k''_\mu)\delta(\lambda k'_\mu-\lambda k''_\mu)e^{i(k'_\mu-k''_\mu)X^\mu}$$

---

* The right hand side ~~corresponds~~ is proportional to the diagonal elements of

$$[p^\mu U][p_\mu U]$$

or $[\xi^\mu U][\xi_\mu U]$

in the representation, in which x_μ is diagonal.

$$+ b^*(k'_\mu)\, b^*_\mu(k''_\mu)\, \delta(\lambda k'_\mu + \lambda k''_\mu)\, e^{-i(k'_\mu + k''_\mu)x^\mu} \}$$
$$= 0.$$

$$T_{00} = \frac{1}{2\cancel{4}V} \sum_{k'_\mu (k'_0 > 0)} \cancel{\frac{k_0}{2i}}\, k_0 \{ b^*(k'_\mu) b(k'_\mu) + b(k'_\mu) b^*(k'_\mu) - \cancel{b(k'_\mu) b(-k'_\mu)} - \cancel{b^*(k'_\mu) b^*(-k'_\mu)}^{\,*} \}$$

Thus the energy density is ~~everywhere~~ positive definite constant, in x_μ-space ~~and~~ for everywhere.

~~$T_{\mu\nu} = 0$:~~ The striking difference with the ordi~~nary~~ry field theory is that there is no ~~field~~ fluctuation ~~in~~ vacuum field energy density. In the ordinary theory $\delta(r_\mu + \lambda k'_\mu)$ has the singularity concentrated to $r_\mu = 0$ for all k'_μ, whereas it is scattered all over the ~~r_μ-space~~ light cone in r_μ-space in this theory.

---

* Last two terms cancels because ~~$\delta(\lambda k'_\mu +$~~ $\delta(\lambda k'_0 + \lambda k''_0) = 0$ always for $k'_0, k''_0 > 0$.

Extension to Vector Field, Non-zero mass-field, spinor field.

1. Vector field

The extension to vector neutral zero-mass field is almost trivial.

$$\left.\begin{array}{l}[\xi_\mu[\xi^\mu, A_\nu(\eta^\mu)]] = 0 \\ [\xi^\mu, A_\mu] = 0\end{array}\right\}$$

$$A_\mu = V^{-\frac{1}{2}} \sum_{k_\mu (k_0>0)} (2k_0)^{-\frac{1}{2}} \{ a_\mu(k)\delta(r+\lambda k) e^{ikx}$$

$$+ a_\mu^*(k)\delta(r-\lambda k) e^{-ikx}\}$$

with the supplementary condition

$$k^\mu a_\mu(k) = 0.$$

2. Non-zero mass field

5-dimensional space

3. spinor field

There are two ways.

i) spinor field itself is considered as nonlocalizable

ii) spinor field is still localizable, only in the interaction with the Bose field appears the effect of nonlocalizability of the latter.

ii) can be maintained in so far as we ~~restrict our attention~~ there is no direct interaction between Fermi particles.

Hence we adopt the ~~latter~~ former alternative i).

usual Dirac eq.

$$(\alpha^\mu p_\mu + \beta mc)\psi(x^\mu) = 0 \qquad (1)$$

will change into

$$[\alpha^\mu p_\mu \cancel{+ \beta mc}, \psi] = \beta mc\psi \qquad (2)$$

for $\psi(x^\mu, p^\mu, \alpha^\mu, \beta \ldots)$. We assume again ψ is a function of η^μ,* ρ_3, σ_3, or four component fn of η^μ alone.

Then (2) takes the form

$$[\alpha^\mu \zeta_\mu, \psi] = 0,$$

where $[\zeta_4, \psi] = -\kappa_4 \psi$

with $\alpha^4 = \beta$.

* η^μ, ζ^μ now extends to η^4, ζ^4.

Now, as we are considering complex (charged) field,

$$\psi = \sum_{K_\mu (K_0 > 0)} \{ u(K_\mu) e^{iK_\mu \eta^\mu} + v^*(K_\mu) e^{-iK_\mu \eta^\mu} \}$$

with $K_\mu K^\mu = 0$ $\mu = 0, 1, 2, 3, 4.$
and K_4 is a ~~small~~ const

$$K_4 = \lambda \cdot \frac{mc}{\hbar}$$

or, if we write

$$\lambda = \frac{\hbar}{\mu c}$$

$$K_4 = \frac{m}{\mu},$$

which is of the order of 10^{-2} for the electron mass m.

$$(x' | H' | x'')$$

$$= (x' | \psi^*(\eta_\mu) A_\mu^{\alpha} \psi(\eta_\mu) | x'')$$

$$= \sum_{K'} \sum_{k} \sum_{K''} \sum_{x'''} \sum_{x^{IV}} u^*_{(K'_\mu)} a_\mu^{(k_\mu)} \alpha^\mu u(K''_\mu) e^{-iK'_\mu x'^\mu/\lambda}$$

$$\delta(x' - x''' - \lambda K') e^{+ik_\mu x'''^\mu/\lambda} \delta(x''' - x^{IV} + \lambda k)$$

$$e^{iK''_\mu x''^\mu/\lambda} \delta(x^{IV} - x'' + \lambda K'')^* + \cdots$$

$$\text{II}^* \quad \delta(x' - x''' - \lambda K') e^{ik_\mu x'''^\mu/\lambda}$$

$$= e^{ik_\mu (x'^\mu - \lambda K'^\mu)/\lambda} \delta(x' - x''' - \lambda K')$$

$$= \sum_{K'}\sum_{k}\sum_{K''}\sum_{x'''}\sum_{x^{iv}} u^*(K'_\mu)\, a_\mu(k_\mu)\, \alpha^\mu u(K''_\mu)$$

$$\times \delta\, e^{-iK'_\mu x'^\mu/\lambda} e^{+ik_\mu x'^\mu/\lambda} e^{-ik_\mu K'^\mu} \delta(x'-x'''-\lambda K')$$

$$\delta(x'''-x^{iv}+\lambda k)\, e^{iK''_\mu x^\mu/\lambda} \delta(x^{iv}-x''+\lambda K'')$$

$$+ \cdots$$

$$= \sum_{K'}\sum_{k}\sum_{K''}\sum_{x^{iv}} u^*(K'_\mu)\, a_\mu(k_\mu)\, \alpha^\mu u(K''_\mu)$$

$$e^{-iK'_\mu x'^\mu/\lambda}\, e^{ik_\mu x'^\mu/\lambda}\, e^{-ik_\mu K'^\mu}$$

$$\delta(x'-\lambda K'-x^{iv}+\lambda k)\, e^{iK''_\mu x^\mu/\lambda}$$

$$\times \delta(x^{iv}-x'+\lambda K'')$$

$$= \sum_{K'}\sum_{k}\sum_{K''} u^*(K'_\mu)\, a_\mu(k_\mu)\, \alpha^\mu u(K''_\mu)$$

$$e^{-iK'_\mu x'^\mu/\lambda}\, e^{ik_\mu x'^\mu/\lambda}\, e^{-ik_\mu K'^\mu}\, e^{iK''_\mu x''^\mu/\lambda}$$

$$\delta(x'-\lambda K' \pm x''+\lambda K''+\lambda k)$$

$$= \sum_{K'}\sum_{k}\sum_{K''} u^*(K'_\mu)\, a_\mu(k_\mu)\, \alpha^\mu u(K''_\mu)$$

$$e^{-iK'_\mu x'^\mu/\lambda}\, e^{ik_\mu x'^\mu/\lambda}\, e^{iK''_\mu x''^\mu/\lambda}$$

$$\times e^{-ik_\mu K'^\mu}\, e^{-iK''_\mu K'^\mu}\, e^{+iK''_\mu k^\mu}\, \delta(x'-\cancel{\lambda K}x'' -\lambda K'+\lambda K''+\lambda k)$$

diagonal element : $x' = x''$

$$K' = K'' + k$$

$$(x'|H|x) = \sum_{K'}\sum_{k}\sum_{K''} u^*(K'_\mu)\, a_\mu(k_\mu)\, \alpha^\mu u(K''_\mu)$$

$$e^{-iK'_\mu x'^\mu}\, e^{ik_\mu x'^\mu/\lambda}\, e^{iK''_\mu x'^\mu/\lambda}$$

$$e^{-ik_\mu K'^\mu}\, e^{-iK''_\mu K^{\mu\prime}}\, e^{iK''_\mu k^\mu} \qquad \delta(-\lambda K' + \lambda K'' + \lambda k)$$

$$= \sum_{K'}\sum_{K''} u^*(K'_\mu)\, a_\mu(k_\mu)\, \alpha^\mu u(K''_\mu)$$

$$\times\, e^{\cancel{-ik'_\mu x'}\, i(K'_\mu - k_\mu - K''_\mu)\, x'^\mu/\lambda}$$

$$\times\, e^{-i(K' - K'')K'}\, e^{-iK''K'}\, e^{iK''(K'-K'')}$$

$$e^{ik'K''} \left(\text{or } e^{-ikK'} \text{ or } e^{ikK''}\right)$$

$$= \sum_{K'}\sum_{K''} u^*(K'_\mu)\, a_\mu(k_\mu)\, \alpha^\mu u(K''_\mu)\, e^{ik'K''}\Big)$$

with $\frac{e^{ik'G} - e^{ik'}}{ik'}$ $\qquad K''_\mu + k_\mu = K'_\mu$

$$\frac{\partial\psi}{\partial x_0} = \iiint H(dx)^3 \psi$$

$$\psi(X, r)$$

$$\psi(X-\tfrac{1}{2}r) \quad \psi(X+\tfrac{1}{2}r)$$

What is the substitute for Schrödinger eq.?

~~time~~ generalization of time variable

$\rightarrow \zeta_0$?

canonical invariance

$$\frac{\partial}{\partial \zeta_0}$$

different field belong to different world ?? with the neutral distance λ, (canonical) ~~λ.~~

Thus if electron is ~~in ordinary space~~ localized in ordinary space x, the photons are ~~in a world~~ localized in a world with coordinates η.

Problems for the electron can be dealed with in x-space, whereas problems concerning photons should be dealed with in η-space. When one is localized the other is not localized in the same space. The interaction should be ~~such that~~ canonically invariant in this sense.

$$\sum_{K'}\sum_{K''}\int_{x''x^{VI}} e^{-iK'x'}\,\delta(x'-x''-K')\,e^{-ik'x''}\,\delta(x''-x'''-k')$$

$$e^{ik''x'''}\,\delta(x'''-x^{IV}+K'')\,e^{-iK''x^{IV}}\,\delta(x^{IV}-x^{V}-K''')$$

$$e^{ik''x^{V}}\,\delta(x^{V}-x^{VI}+k'')\,e^{iK^{IV}x^{VI}}\,\delta(x^{VI}-x^{VII}+K^{IV})$$

$$=\sum\sum\int_{x'''} e^{-iK'x'}\,\delta(x'-x'''+k'-K')\,e^{-ik'x'''}$$

$$e^{ik''x'''}\,\delta(x'''-x^{IV}+K'')\,e^{-iK''x^{IV}}\,\delta(x^{IV}-x^{V}-K''')$$

49 (1936), 545

$$e^{ik''x^{V}}\,\delta(x^{V}-x^{VI}+k'')\,e^{iK^{IV}x^{VI}}\,\delta(x^{VI}-x^{VII}+K^{IV})$$

$$=\sum\sum\int_{x^{IV}} e^{-iK'x'}\,\delta(x'-x^{IV}+K''-k'-K')$$

$$e^{-ik'x^{IV}}\,e^{ik'K''}\,e^{ik''x^{IV}}\,e^{-iK''x^{IV}}\,\delta(x^{IV}-x^{V}-K''')$$

$$e^{ik''x^{V}}\,\delta(x^{V}-x^{VI}+k'')\,e^{iK^{IV}x^{VI}}\,\delta(x^{VI}-x^{VII}+K^{IV})$$

$$=\sum\sum\int_{x^{V}} e^{-iK'x'}\,\cancel{e^{-ik}}\,\delta(x'-x^{V}-K'''+K''-k'-K')$$

$$e^{-ik'x^{V}}\,e^{-ik'K'''}\,e^{ik'K''}\,e^{ik''x^{V}}\,e^{ik''K''}\,e^{-iK''x^{V}}$$

$$e^{ik''x^{V}}\,\delta(x^{V}-x^{VI}+k'')\,e^{iK^{IV}x^{VI}}\,\delta(x^{VI}-x^{VII}+K^{IV})$$

$$=\sum\sum\int_{x^{VI}} e^{-iK'x'}\,\delta(x'-x^{VI}+k''-K'''+K''-k'-K')$$

$$e^{-ik'x^{VI}}\,e^{ik'k''}\,e^{-ik'K'''}\,e^{ik'K''}\,e^{iK''x^{VI}}\,e^{-iK''k''}$$

$$e^{ik''K'''}\,e^{-iK'''x^{VI}}\,e^{iK''k''}\,e^{ik''x^{VI}}\,e^{iK^{IV}x^{VI}}\,\delta(x^{VI}-x^{VII}+K^{IV})$$

$$e^{+\frac{1}{2}K_0 k^0} = e^{-\frac{1}{2}K^0 k^0}$$

$$U = u(\xi^\mu) + u^*(\xi^{\mu *})$$

$$\psi = \phi(\xi^\mu) + \chi^*(\xi^{\mu *})$$

$$\psi^* = \phi^*(\xi^{\mu *}) + \chi(\xi^\mu)$$

$k^0 > 0$ always

$$-K_\mu k^\mu = K^0 k^0 - \mathbf{K}\mathbf{k} \cancel{= \sqrt{2}\,Kk - \mathbf{K}\mathbf{k}}$$

$$K^0 = \sqrt{\kappa^2 + \mathbf{K}^2} \qquad \cancel{\geq (\sqrt{2}-1)Kk}$$

$$\cancel{k^0 = \sqrt{\kappa^2 + k^2}}$$

$\kappa = 0$: $\cancel{\mathbf{K} = \mathbf{k}}$

$k^0 = \mathbf{k}$

$\therefore$ $\mathbf{k} = 0$, $K^0 = \kappa$, $\therefore$ $K^0 k^0 = \kappa k^0$

κ : heavy particle $\kappa \gtrsim 1$ $\quad k^0 \gtrsim 1$.

κ : electron $\quad \kappa \sim \alpha = \frac{1}{137}$ $\quad k^0 \gtrsim \frac{1}{\alpha} = 137$.

$-\bar{K}$, δa

$$= \sum_{K}\sum_{k} e^{-ik'x}\,\delta(x'-x^{VII}+\underline{K}^{IV}+\underline{k}''-\underline{K}'''+\underline{K}''-\underline{k}'-K')$$

$$e^{-ik'x^{VII}}\, e^{ik'K^{IV}}\, e^{ik'k''}\, e^{-ik'K'''}\, e^{ik'K''}\, e^{iK''x^{VII}}\, e^{-iK''K^{IV}}$$

$$e^{-iK''k''}\, e^{iK''K'''}\, e^{-iK''x^{VII}}\, e^{iK''K^{IV}}\, e^{iK''k''}$$

$$e^{ik''x^{VII}}\, e^{-ik''K^{IV}}\, e^{iK^{IV}x^{VII}}$$

$$= \sum_{K}\sum_{k} e^{-ik'x'}\, e^{i(k'-\bar{K})x^{VII}}\,\delta(x'-x^{VII}-\bar{K})$$

$$\exp i(k'\underline{K}^{IV}+k'k''\overset{-ik'K'''}{+}k'K''-K''\underline{K}^{IV}-K''k''+K''K''' +K'''\underline{K}^{IV}+K'''k''-k''\underline{K}^{IV})$$

$$\cancel{=\sum\sum}$$

$$\cancel{\exp i(\bar{K}K^{IV}+k'K^{IV}-K'K^{IV}}$$

$$\bar{K} = -\underline{K}^{IV}-k''+\underline{K}'''-\underline{K}''+\underline{k}'+K'$$

$$\bar{K}+k''-K' = -K^{IV}+K'''-K''+k'$$

$$(x'|e^{\frac{i p}{\hbar}} f|x'') = (x'+1|f|x'')$$

$$\delta'(x'_\mu - x''_\mu)$$

$$\sum_n \frac{1}{n!}\,\delta'(x'-x'')\,\delta'(x''-x''')\cdots\delta'(x^n - x^{n+1})\,(x^{n+1}|f|x^{n+2})$$

$$= \sum_n \frac{1}{n!}\,\frac{\partial^n f(x', x^{n+2})}{\partial x^n}$$

$$= (x'+1|f|x^{n+2})$$

$$(x'|e^{-\frac{i\lambda}{2\hbar}k_\mu p^\mu}|x'')$$

$$(x'|e^{-\frac{i\lambda}{2\hbar}k_\mu p^\mu} f|x'') \qquad = \delta(x'_\mu - x''_\mu + \tfrac{i\lambda}{2}k_\mu)$$

$$= (x'_\mu + \tfrac{i\lambda}{2}k_\mu|f|x''_\mu)$$

$$f = f(x'_\mu)\,\delta(x'_\mu - x''_\mu)$$

$$(x'|e^{-\frac{i\lambda}{2\hbar}k_\mu p^\mu} f|x'') = f(x'_\mu + \tfrac{i\lambda}{2}k_\mu)\,\delta(x'_\mu + \tfrac{i\lambda}{2}k_\mu - x''_\mu)$$

$$f = e^{ik_\mu x'^\mu/\lambda} \to e^{ik_\mu x'^\mu/\lambda - \frac{1}{2}k_\mu k^\mu}$$

$$\delta(x'_\mu - x''_\mu)$$

$\underline{K'' = K''}$ $\quad \underline{x' = x'''}$ $\quad K = -K'' - k + k' + K' = 0$

$$\exp i\,(k'K^{IV} + k'k'' \cancel{- k'K''} \cancel{+ K''k''} - k''K^{IV})$$

$$\cancel{= \exp i\,(-k'k'' + k'K' + k'k'' + K'K'' - k''k' - k''K')}$$

$$\cancel{= \exp i\,(-k'k'' + k'K' + K'K'' - k''K')}$$

$$\overline{k' + K'}$$

$$= \underline{\underline{\exp i\,(k'K^{IV} + k'k'' - k''K^{IV})}}$$

$$\begin{array}{l} \quad\;\; e_1 \;\; e_2 \quad\;\; p_1 \\ \psi(1 \;\; 0, 0, 0 \cdots 1 \;\; 0 \cdots) \end{array}$$

$\psi($

$$\xi_\mu = \frac{x_\mu}{\lambda} - i\frac{\lambda}{2\hbar} p_\mu$$

$$\xi^*_\mu = \frac{x_\mu}{\lambda} + i\frac{\lambda}{2\hbar} p_\mu$$

$p_0 = -\frac{i\hbar}{c}\frac{\partial}{\partial t}$

$p^0 = \frac{i\hbar}{c}\frac{\partial}{\partial t}$

positive energy

$k_0 < 0$

$k^0 > 0$

$$[\xi^\mu, \xi^*_\nu] = -1.$$

$$U = u(\xi^\mu) + u^*(\xi^{\mu *})$$

$$= V^{-\frac{1}{2}} \sum_{k_\mu\,(k^0>0)} (2k^0)^{-\frac{1}{2}} \{ b(k_\mu) e^{ik_\mu \xi^\mu} + b^*(k_\mu) e^{-ik_\mu \xi^{\mu *}} \}$$

$$\frac{e^{+\frac{\lambda}{2\hbar}k_0 p^0}}{} = e^{+\frac{i\lambda}{2c}k_0\frac{\partial}{\partial t}}$$

$$\cancel{[e^{ik_\mu\xi^\mu}, \xi^*_\nu]} = e^{ik_\mu\xi^\mu}$$

$$e^{ik_\mu\xi^\mu} = e^{ik_\mu x^\mu/\lambda}\, e^{-\frac{\lambda}{2\hbar}k_\mu p^\mu}$$

$$[x_\nu, e^{-\frac{\lambda}{2\hbar}k_\mu p^\mu}] = -\frac{\lambda}{2\hbar}k_\nu (i\hbar)\, e^{-\frac{\lambda}{2\hbar}k_\mu p^\mu} = -\frac{i\lambda k_\nu}{2} e^{-\frac{\lambda}{2\hbar}k_\mu p^\mu}$$

$$[iK^\nu x_\nu, e^{-\frac{\lambda}{2\hbar}k_\mu p^\mu}] = \frac{\lambda K^\nu k_\nu}{2} e^{-\frac{\lambda}{2\hbar}k_\mu p^\mu}$$

$$[(iK^\nu x_\nu)^2, e^{-\frac{\lambda}{2\hbar}k_\mu p^\mu}] = (iK^\nu x_\nu)[iK^\nu x_\nu, e^{\cdots}]$$

$$+ [iK^\nu x_\nu, e^{\cdots}](iK^\nu x_\nu) = iK^\nu x_\nu, e^{\cdots}$$

In general:

$$\sum_{\sigma=1}^{4} K_\mu k^\mu = K^0 k^0 - \boldsymbol{K}\boldsymbol{\kappa} - \boldsymbol{K}\boldsymbol{k}$$

$$= \sqrt{K^2 + \boldsymbol{K}^2}\sqrt{\kappa^2 + \boldsymbol{k}^2} - \boldsymbol{K}\boldsymbol{\kappa} - \boldsymbol{K}\boldsymbol{k}$$

$$\left.\begin{array}{ll} K = 1, & \boldsymbol{K} = 1 \\ \kappa = 1, & \boldsymbol{k} = 1 \end{array}\right\} \quad K\kappa - \boldsymbol{K}\boldsymbol{k}$$

$$= K\kappa(1 - \cos\theta)$$

~~Lagrangian~~

$$[\xi^\mu u^*(\xi^*)][\xi_\mu u^*(\xi^*)] + [\xi^{\mu *} u(\xi)][\xi^*_\mu u(\xi)]$$

$$= a(k'_\mu)\cdot a^*(k''_\mu)[e^{ik'_\mu\xi^\mu}, e^{-ik''_\nu\xi^*_\nu}]$$

$$+ [a(k'_\mu), a^*(k''_\mu)]\, e^{ik'_\mu\xi^\mu} e^{-ik''_\nu\xi^*_\nu}$$

$$= a(k'_\mu)a^*(k''_\mu)\,(k'_\mu k^{\mu''})\, e^{ik'_\mu\xi^\mu} e^{-ik''_\nu\xi^*_\nu}$$

$$+ \quad \delta(k'_\mu, k''_\mu)$$

$$[q^2, p] = q[q, p] + [q\, p]q$$

$$[(ik'_\mu\xi^\mu)^2, e^{-ik''_\nu\xi^*_\nu}]$$

$$= ik'_\mu\xi^\mu\cdot(k'_\mu k^{\mu''})e^{\cdots}$$

$$+ e^{-ik''_\nu\xi^*_\nu}(k'_\mu k^{\mu''})\, ik'_\mu\xi^\mu$$

$$(x'|e^{ik'_\mu\xi^\mu}|x'') = \delta(x^{\mu'} - x^{\mu''} - \lambda k$$

$$\sum_{K} \sum_{k} \int_{x''} e^{-iK'x'} e^{-ik'x'} \delta(x'-x''-k') e^{iK''x''}$$

$$e^{-iK''x''} e^{ik''x''} \delta(x''-x'''+k'') e^{iK^{IV}x'''}$$

$$= \sum_{K} \sum_{k} \int_{x''} \Big\{ e^{-iK'x'} e^{-ik'x'} \delta(x'-x''+k''-k')$$

$$e^{iK''x'''} e^{-iK''k''} e^{-iK''x''} e^{iK''k''}$$

$$e^{ik''x'''} e^{iK^{IV}x'''} \Big\}$$

$$= \sum_{K} \sum_{k} e^{-iK'x'} e^{-ik'x'} \delta(x'-x''+k''-k')_{k'k''}$$

$$e^{iK''x''} e^{ik''x''} \exp\{-i\{K'k''-K''k'\}\}_{K^{IV}}$$

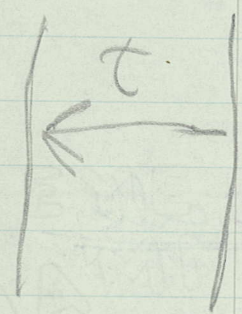
$(x'|L|x'')$ $(x''|L|x'')$

$x' > x''_0 > x''$

Generalized Transformation Functions in Nonlocalizable Field Theory

Dirac's g. t. f.

$$(q_t / q_T) = \exp\left(i\int_T^t L\,dt/\hbar\right)$$



localizable field:

$$\iiiint L(x_\mu)(dx_\mu)^4 \qquad (x'|L|x'')$$

$$x^{0\prime} > x^{0\prime\prime}$$

$$= \iiiint\cdot\iiiint (x'|\underbrace{L}_{(x')}|x'')\,dx''\,dx' \qquad \delta(x'-x'')$$

$$= \iiiint (x'|L^{(x')}|x')\,dx'$$

$\int P(L, L) \ldots)$ Dyson...

~~This is nothing but the~~ Diagonal sum, if we ignore the const infinite factor $\delta(0)$.

In order to avoid the infinity due to ~~the~~ δ-fn, we consider a large ~~box cube with the edges of the length Λ. We divide~~ consider ~~only such point~~ fourdimensional region with boundaries

$$x_\mu x^\mu = R^2$$

~~where~~

$$x_\mu x^\mu = -R^2$$

(two pairs of fourdimensional hyperbolas)

where R is a ~~pos~~ very large const with the dimension of length.

Then, if x_μ is in the region,

$$(x_\mu + \lambda k_\mu)(x^\mu + \lambda k^\mu) = x_\mu x^\mu + 2\lambda k_\mu x^\mu + \lambda^2$$

In nonlocalizable field theory, it is expected that the infinity $\delta(x'-x'')$ at $x'=x''$ is replaced by some finite factor, so that we ~~take~~ can replace

$$\iiiint (x'|L(x')|x')(dx')^4$$

by more general and ~~...~~ canonically invariant expression

$$\mathrm{Trace}\, L = \text{Diagonal Sum } L.$$

~~I~~nstead of

$$\exp\left(\frac{i}{\hbar c}\iiint L(x^{\mu})(dx)^4\right), \qquad (1)$$

we consider

$$\mathrm{Trace}\left\{\left(\frac{i}{\hbar c}\right)^n \frac{L^n}{n!}\right\} = \mathrm{Trace}\left(\exp\frac{i}{\hbar c}L\right)$$

~~Because~~ For $L^2, L^3, \ldots$, $\delta(\ldots)$ terms are

$$\cancel{\mathrm{Trace}(L^n) = \int\cdots\int (x'|L|x'')(x''|L|x''')\cdots(x^{(n)}|L|x')\,(dx')\ldots(dx^{(n)})}$$

~~a~~ evidently differ from

~~apparently~~ ~~Evidently~~ ~~Act~~ Indeed,

$$\left(\frac{i}{\hbar c}\right)^n \frac{\left(\iiiint L\,dx^4\right)^2}{2!} \quad \text{etc.,}$$

(~~T~~the most natural extention of (1) is formally

$$L^2 = (x'|L|x''')(x''|L|x^{iv})$$

$$(x'|x''')(x'''|x'')(x''|x^{iv}) \quad (x^{iv}|x^{v})(x^{v}|x^{vi})(x^{vi}|x^{vii})$$

$$\exp\left(\frac{i}{\hbar c}\,\mathrm{Trace}\,L\right),$$

But physical considerations that a ~~com~~ high order process can occur if the energy-momentum is conserved between only ~~the~~ initial and final state, the conservation of energy is not required in the intermediate state.

~~For example, corresponding to Compton effect,~~ the matrix element for L^2 ~~takes the form~~ has the factor of ~~has~~ the form

$$\iiiint \sum_{K's}\sum_{k's} e^{-iK'_\mu x'^\mu/\lambda}\,\delta(x'-x'''-\lambda K')\,e^{-ik'_\mu x'''^\mu/\lambda}$$
$x' x'' x^{iv} x^{v} x^{vi}$

$$\delta(x'''-x''-\lambda k')\,e^{iK''_\mu x''^\mu/\lambda}\,\delta(x''-x^{iv}+\lambda K'')$$

$$e^{-iK''_\mu x^{iv\mu}/\lambda}\,\delta(x^{iv}-x^{v}-\lambda K'')\,e^{ik''x^{v}/\lambda}$$

$$\delta(x^{v}-x^{vi}+\lambda k'')\,e^{iK''' x^{vi}/\lambda}\,\delta(x^{vi}-x^{vii}+\lambda K^{iv})$$

$$=\int_{x'''} e^{-iK'_\mu x'^\mu/\lambda}\,\delta(x'-x'''-\lambda k'-\lambda K')\,e^{-ik'_\mu x'''^\mu/\lambda}$$

$$e^{-iK''_\mu x'''^\mu/\lambda}\,\delta(x'''-x^{iv}+\lambda K'')\,e\ \cdots$$

$$=\int_{x^{iv}} e^{-iK'_\mu x'^\mu/\lambda}\,\delta(x'-x^{iv}+\lambda K''-\lambda k'-\lambda K')\,e^{-ik'_\mu x^{iv}/\lambda}$$

$$e^{ik_\mu K''/\lambda}\quad e^{-iK''_\mu x^{iv}/\lambda}\quad \delta(x^{iv}-x^{v}-\lambda K''')$$

$$e^{+iK'_\mu K''/\lambda}\quad e^{ik'_\mu K''/\lambda}\quad e^{-iK''_\mu K'''}$$

$$\exp\frac{i}{\lambda}\ ik_\mu K''\ \ k_\mu K''$$

Thus we finally arrive at a factor

$$\delta(x'-x^{VII}-\lambda K^{IV}-\lambda k''+\lambda K'''-\lambda K''+\lambda k'+\lambda K')$$

So for the diagonal element $x'=x^{VII}$ only ~~#~~ combinations satisfying

$$\lambda K^{IV}+\lambda k''-\lambda K'''+\lambda K''-\lambda k'-\lambda K'=0$$

When initially only one electron with the momentum-~~energy~~ K' and a photon k' exists, in intermediate states an ~~p~~ electron with the ~~same~~ momentum K''' can be emitted virtually only, if $K'''=K''$ ~~$=K'$~~

Now the summation with respect to ~~$K'''(=K''')$~~ K''

$$\sum_K\sum_k e^{-iK'x'}e^{i(K'-\bar{K})x^{VII}}\delta(x'-x^{VII}-\bar{K})$$

$$\exp i\,(k'K^{IV}+k'k''-k'K'''+k'K'-K'K^{IV}-K'k''$$
$$+K''K'''+K'''K^{IV}+K''k''-k''K^{IV})$$

$$\bar{K}=-K^{IV}-k''+K'''-K''+k'+K'$$

$K'=K'''$, $x'=x^{VII}$

$$\sum_K\sum_k\sum_{K''}\exp i\,(k'K^{IV}+k'k''-k''K^{IV})$$

Canonical Invariance

Lorentz transformation

$$x^{\mu'} = a_{\mu\nu} x^{\nu}$$ $\cancel{a_{\mu\kappa} a_{\nu\kappa}}$

$$p'_{\mu} = a^{\mu\nu} p_{\nu}$$

$$\left.\begin{array}{l} x^{\mu'} p'_{\mu} = x^{\mu} p_{\mu} \\ x^{\mu'} x'_{\mu} = x^{\mu} x_{\mu} \end{array}\right\}$$

$$a^{0i} = -a_{0i} \qquad a^{i0} = -a_{i0}$$

$$a^{ik} = a_{ik} \qquad a^{00} = a_{00}$$

$$a_{\mu\lambda} a^{\mu\nu} = \delta_{\lambda\nu}$$

$$\cancel{\varepsilon_{\mu\nu} a'_{\mu\nu} a_{\mu\lambda} = \delta_{\lambda\nu}} \qquad \varepsilon_{\mu\nu} = 0 \quad \mu \neq \nu$$

$$\cancel{\varepsilon_{00} = \varepsilon_{11} = \varepsilon_{22}} \qquad \varepsilon_{00} = -1 \quad \varepsilon_{11} = \cdots \varepsilon_{33} = 1$$

$$a^{\mu\nu} = \varepsilon_{\mu\kappa} \varepsilon_{\nu\lambda} a_{\kappa\lambda}$$

$x_i = (x, y, z)$ $\qquad p_i = i\hbar\,\mathrm{grad}$

$x_4 = i x_0 = ict,$ $\qquad p_4 = -ip_0 \left(= \frac{\hbar}{c}\frac{\partial}{\partial t} \right)$

$$\left.\begin{array}{l} x'_{\mu} = a_{\mu\nu} x_{\nu} \\ p'_{\mu} = a_{\mu\nu} p_{\nu} \end{array}\right\} \qquad \mu, \nu = 1, 2, 3, 4,$$

$$\underline{a_{\mu\kappa} a_{\mu\lambda} = \delta_{\kappa\lambda}}$$

$$\left.\begin{array}{l} \xi_{\mu} = \frac{1}{\sqrt{2}}\left\{ \frac{x_{\mu}}{\lambda} + i\frac{\lambda}{\hbar} p_{\mu} \right\} \\ \xi^{*}_{\mu} = \frac{1}{\sqrt{2}}\left\{ \frac{x_{\mu}}{\lambda} - i\frac{\lambda}{\hbar} p_{\mu} \right\} \end{array}\right\}$$

$$\left.\begin{array}{l} \xi'_{\mu} = a_{\mu\nu} \xi_{\nu} \\ \xi^{*\prime}_{\mu} = a_{\mu\nu} \xi^{*}_{\nu} \end{array}\right\} \qquad [x_{\mu}, p_{\nu}] = -i\hbar\,\delta_{\mu\nu}$$

$$2[\xi_{\mu}, \xi^{*}_{\nu}] = \cancel{-\frac{i}{\hbar}(x_{\mu} p_{\nu} - x_{\nu} p_{\mu})}$$

$$\left(\frac{1}{\lambda^2}[x_{\mu}, x_{\nu}] - i\frac{\lambda}{\hbar}[x_{\mu}\, p_{\nu}] + i\frac{\lambda}{\hbar}[p_{\mu}, x_{\nu}] + \left(\frac{\lambda}{\hbar}\right)^2 [p_{\mu}, p_{\nu}] = 2\,\delta_{\mu\nu} \right.$$

Thus Lorentz transformation is reduced to ~~complex~~ *

$$\left.\begin{aligned} \xi'_\mu &= a_{\mu\nu}\xi_\nu \\ \xi'^*_\mu &= a_{\mu\nu}\xi^*_\nu \end{aligned}\right\} \quad (L)$$

with the coeff. satisfying the cond.

$$(L, i) \quad a_{\mu\varkappa}a_{\mu\lambda} = \delta_{\varkappa\lambda}, \qquad (L, 1)^R$$

(L ii) a_{i4}, a_{4i} $(i = 1, 2, 3)$. ~~being~~ are pure imaginary, others being real. (L, 2)

Further ξ_μ, ξ^*_μ satisfy the commutation relations

$$[\xi_\mu, \xi^*_\nu] = \delta_{\mu\nu}^* \qquad (C)$$

As the result of Lor. transf.

$$[\xi'_\mu, \xi'^*_\nu] = a_{\mu\varkappa}\cancel{b}a_{\nu\lambda}[\xi_\varkappa, \xi^*_\nu]$$

$$= a_{\mu\varkappa}a_{\nu\varkappa}$$

Now from

$$a_{\mu\varkappa}a_{\mu\lambda} = \delta_{\varkappa\lambda}$$

we construct ~~a~~ a inverse transf. $b_{\mu\nu}$:

$$b_{\mu\lambda}a_{\lambda\nu} = \delta_{\mu\nu}$$

~~so that~~ or $\cancel{a_{\mu\lambda}b_{\lambda\nu} = \delta_{\mu\nu}}$

$\cancel{b_{\lambda\mu}a_{\mu\varkappa}a_{\mu\lambda} =}$

---

* $[\xi_\mu, \xi_\nu] = [\xi^*_\mu, \xi^*_\nu] = 0$.

$$\cancel{a_{\mu\kappa} b_{\kappa\nu} \cdot a_{\mu\lambda} b_{\lambda\tau} = \delta_{\kappa\lambda} b_{\kappa\nu} b_{\lambda\tau}}$$

or

$$b_{\kappa\mu} a_{\mu\lambda} = \delta_{\kappa\lambda}.$$

$$a_{\mu\kappa} = b_{\kappa\mu}.$$

$$\therefore \quad b_{\kappa\mu} b_{\lambda\mu} = \delta_{\kappa\lambda}.$$

$$\therefore \quad [\xi'_\mu, \xi'^*_\nu] = \delta_{\mu\nu}$$

So canonical commutation relations (C) ~~∴ [~~ are Lorentz invariant.

Now we consider more general linear canonical transformations, ~~satisfying (L)', but the condition (L, i), but not (L, ii).~~ instead of (L, ii)

$$\left.\begin{aligned} \cancel{\xi'_\mu = a_{\mu\nu} \xi_\nu} \\ \cancel{\xi'^*_\mu = \tilde{a}^*_{\mu\nu} \xi^*_\nu} \end{aligned}\right\} \quad (L)'$$

~~where $\tilde{a}^*_{\mu\nu}$ are complex conjugate to $a_{\mu\nu}$, satisfying the condition~~

$$\cancel{[\xi'_\mu, \xi'^*_\nu] =} \ .$$

which leave both $\xi^*_\mu \xi_\mu + \xi_\mu \xi^*_\mu$ and $[\xi_\mu, \xi^*_\nu]\ (= \delta_{\mu\nu})$ invariant.

It is evidently

$$\left.\begin{aligned} \xi'_\mu &= a_{\mu\nu}\xi_\nu \\ \xi'^*_\mu &= a_{\mu\nu}\xi^*_\nu \end{aligned}\right\} \qquad (L')$$

with

Lagrangian Scheme of Non-localizable Fields, I.

Scalar Field:

$$[p^\mu U][p_\mu U] = p^\mu (U[p_\mu U]) - U\, p^\mu [p_\mu U]$$

$$= -U[p^\mu[p_\mu U]] + [p^\mu, (U[p_\mu U])]$$

$$\mathrm{Trace}\,[p^\mu U][p_\mu U] = \mathrm{Trace}\{-U[p^\mu[p_\mu U]]\}$$

Thus $[p^\mu U][p_\mu U]$ corresponds to Lagrangian density, $\mathrm{Trace}[p^\mu U][p_\mu U]$ to space-time integral of Lagrangian density.

If we assume U to satisfy

$$[p^\mu[p_\mu U]] = 0,$$

then $\mathrm{Trace}\,[p^\mu U][p_\mu U] = 0$, but $[p^\mu U][p_\mu U]$ is not zero. ~~the latter has only non-diagonal elements different from 0 in the representation~~

$$\cancel{[p^\nu_\mu U][p_\nu U] + [p_\nu U][p_\mu U]}$$

This corresponds to the fact that in classical theory for the scalar field ~~from the~~ the field ~~wave~~ equation

$$\frac{\partial^2 U(x_\mu)}{\partial x_\mu \partial x^\mu} = 0$$

leads to

$$\iiiint \frac{\partial U}{\partial x_\mu}\frac{\partial U}{\partial x^\mu}\, dv^{(4)} = 0$$

provided that

$$\lim_{S\to\infty} \iiint_S \left(U \frac{\partial U}{\partial x_\mu}\right) dS_\mu^{(3)} = 0 , \quad *$$

because

$$\iiiint_V \frac{\partial U}{\partial x_\mu}\frac{\partial U}{\partial x^\mu}\, dv^{(4)} = \iiint_S \left(U \frac{\partial U}{\partial x_\mu}\right) dS_\mu^{(3)}$$

$$- \iiiint_V U \frac{\partial^2 U}{\partial x_\mu \partial x^\mu}\, dv^{(4)}$$

The condition * corresponds to the identity

$$\mathrm{Trace}\left[p_\mu (U[p^\mu U])\right] = 0$$

or $\mathrm{Trace}\,(p_\mu U[p^\mu U]) = \mathrm{Trace}\,(U[p^\mu U]\, p_\mu)$, $**$

This is satisfyed ~~provided that~~ whenever the traces on both sides have finite (consequently equal) values.

~~$**$ The condition corresponding to this is implicitly that just an identity~~ Trace

$$** \ \mathrm{Trace}\{[p^\mu U][p_\mu U]\} = \mathrm{Trace}\,(p^\mu U[p_\mu U]) - (U p^\mu [p_\mu U])$$

$$= \mathrm{Trace}\,(U[p_\mu U]\, p_\mu) - \mathrm{Trace}\,(U p^\mu [p_\mu U])$$

if $\mathrm{Trace}[p_\mu (U[p^\mu U])] = 0$, which is satisfyed for any converging Trace.

The variation principle in classical field theory:

$$\delta \iiiint \frac{\partial U}{\partial x_\mu} \frac{\partial U}{\partial x^\mu} dv^{(4)} = 0$$

or

$$\iiiint_V \frac{\partial \delta U}{\partial x_\mu} \frac{\partial U}{\partial x^\mu} dv^{(4)}$$

$$= \iiint_S \delta U \frac{\partial U}{\partial x_\mu} dS_\mu^{(3)} - \iiiint_V \delta U \frac{\partial^2 U}{\partial x_\mu \partial x^\mu} dv^{(4)}$$

$$= 0$$

can be transformed into ~~the~~ more general form in the nonlocalizable field:

$$\cancel{\frac{\partial(SUS^{-1})}{\partial x_\mu} \frac{\partial(SUS^{-1})}{\partial x^\mu} dv^{(4)}}$$

$U \to U + \varepsilon[S, U]$ ε: very small

$$\mathrm{Trace}\,\{[p_\mu, [S, U]][p^\mu, U] + [p_\mu U][p^\mu [S U]]\} = 0,$$

where S is an Hermitian operator which commutes with p_μ.

~~Trace~~ Now

$$[A[BC]] = A[BC] - [BC]A$$
$$= ABC - ACB - BCA + CBA$$
$$[B[CA]] = BCA - BAC - CAB + ACB$$
$$[C[AB]] = CAB - CBA - ABC + BAC$$

$$\therefore [A[BC]] + [B[CA]] + [C[AB]] = 0$$

Especially when A and B are commutative,

$$[A[BC]] = -[B[CA]]$$

$$\therefore\ [p_\mu[S,U]] = [S[p_\mu U]]$$

$$\therefore\ \mathrm{Trace}\{[p_\mu[SU]]\cdot[p^\mu \cancel{S} U] + [p_\mu U][p^\mu[SU]]\}$$

$$\cancel{= \mathrm{Trace}\{[S[p_\mu U]][S[p^\mu U]]\}}$$

$$\cancel{= \mathrm{Trace}\{(S[p_\mu U] - [p_\mu U]S)}$$

$$= \mathrm{Trace}\{[S[p_\mu U]][p^\mu U] + [p_\mu U][S[p^\mu U]]\}$$

$$= \mathrm{Trace}\{S[p_\mu U][p^\mu U] - [p_\mu U]S[p^\mu U]$$
$$+ [p_\mu U]S[p^\mu U] - [p_\mu U][p^\mu U]S\}$$

$= 0$, which is an identity

More generally, if S is not commutative with p^μ,

$$[p_\mu[SU]]\cancel{[p^\mu U]} = [S[p_\mu U]]$$
$$- [U[p_\mu S]]$$

$$\therefore\ \mathrm{Trace}\{[p_\mu[SU]]\cdot[p^\mu U] + [p_\mu U][p^\mu[SU]]\}$$

$$= -\mathrm{Trace}\{[U\underbrace{[p_\mu S]}_{p_\mu S - S p_\mu}]\cdot[p^\mu U] + [p_\mu U][U[p^\mu S]]\}$$

$$= -\mathrm{Trace}\{U[p_\mu S][p^\mu U] - [p_\mu S]U[p^\mu U]$$
$$+ [p_\mu U]U[p^\mu S] - [p_\mu U][p^\mu S]U\}$$

$\cancel{= -\mathrm{Trace}\{}$

~~now~~

$\cancel{= \mathrm{Trace}\{U [p_\mu [p^\mu U]] S - p_\mu [p^\mu U] S U}$

$= -\mathrm{Trace}\{ U [[p_\mu S], [p^\mu U]]$

$+ U [[p_\mu S], [p^\mu U]] \} = 0$

or $\mathrm{Trace}\{ U [[p_\mu S], [p^\mu U]] \} = 0$

or $\mathrm{Trace}\{ U (p_\mu S [p^\mu U] - S p_\mu [p^\mu U]) \}$

$= \mathrm{Trace}\{ S ([p^\mu U] U p_\mu - p_\mu [p^\mu U] U) \}$

$= \mathrm{Trace}\{ S ([p^\mu U] [U p_\mu] + \cancel{[p^\mu U] p_\mu U}$

$- [p_\mu [p^\mu U]] U - \cancel{[p^\mu U] p_\mu U}) \} = 0$

$\cancel{[p^\mu U][U p_\mu] = p^\mu U}$

~~if~~ or $\underline{\mathrm{Trace}\{ S [p^\mu U] [p_\mu U] \}}$

$\underline{= -\mathrm{Trace}\{ S ([p_\mu [p^\mu U]] U) \}}$

Complex field

$$\mathrm{Trace}\{[\gamma_\mu, \tilde{U}]\,[\gamma^\mu, U]\}$$

$$U \to 1 + \varepsilon[S, U]$$

$$\tilde{U} \to 1 + \varepsilon[S, \tilde{U}]$$

$$\mathrm{Trace}\, U[\gamma_\mu [\gamma^\mu U]]$$

$$U \to 1 + \varepsilon[S, U]$$

$$\mathrm{Trace}\{[S,U][\gamma_\mu[\gamma^\mu U]] + U[\gamma_\mu[\gamma^\mu,[S\,U]]]\} = 0$$

$[\gamma_\mu, S] = 0$:

$$\mathrm{Trace}\{[S,U][\gamma_\mu[\gamma^\mu U]] + U[S[\gamma_\mu[\gamma^\mu U]]]\} = 0$$

$$\mathrm{Trace}\{[S,U][\gamma_\mu[\gamma^\mu U]] + US[\gamma_\mu[\gamma^\mu U]] \mp U[\gamma_\mu[\gamma^\mu U]]S\}$$

$\cancel{= \mathrm{Trace}\{}\ = 0$ identically

Discussions with Dr. Pais

~~$L = \mathrm{Trace}\{[p_\mu U][p^\mu U]\}$~~

(Jan. 10, 1949)

~~four~~ ~~dimension~~ formulation of nonloc. field with rest mass

$$[p^\mu [p_\mu u]] + \frac{\kappa^2}{4} u = 0$$

$$[x^\mu [x_\mu u]] + \lambda^2 u = 0$$

$$\left.\begin{aligned} \eta_\mu &= \left(\frac{x_\mu}{2\lambda} + \frac{p_\mu}{\kappa}\right) \\ \zeta_\mu &= \left(\frac{x_\mu}{2\lambda} - \frac{p_\mu}{\kappa}\right) \end{aligned}\right\} \qquad \left.\begin{aligned} x_\mu &= \lambda(\eta_\mu + \zeta_\mu) \\ p_\mu &= \frac{\kappa}{2}(\eta_\mu - \zeta_\mu) \end{aligned}\right\}$$

$[\eta^\mu, \zeta_\mu] = \mathrm{const.}$

$$[(\eta^\mu \pm \zeta^\mu), [(\eta_\mu \pm \zeta_\mu), u]] + u = 0$$

$$[(\eta^\mu + \zeta^\mu)[(\eta_\mu + \zeta_\mu), u]] + \quad u = 0$$

~~$[\eta_\mu [\eta_\mu, u]] + [\zeta_\mu [\zeta_\mu, u]] =$~~

$$[\eta^\mu [\zeta_\mu, u]] + [u, [\eta^\mu, \zeta_\mu]]$$
$$+ [\zeta_\mu [u, \eta^\mu]] = 0,$$

$$[\eta^\mu [\zeta_\mu, u]] = [\zeta_\mu [\eta^\mu, u]]$$

$$[\eta^\mu [\eta_\mu, u]] + [\zeta^\mu [\zeta_\mu, u]]$$
$$\mp [\eta^\mu [\zeta_\mu, u]] \mp [\zeta^\mu [\eta_\mu, u]] + u = 0$$

$$[\eta^\mu[\zeta_\mu, u]] = [\zeta_\mu[\eta^\mu, u]] = 0$$

$$[\eta^\mu[\eta_\mu, u]] + [\zeta^\mu[\zeta_\mu, u]] + u = 0$$

If we assume ~~$u_\eta(\eta_\mu)\, u_\zeta($~~

$$u = u_\eta u_\zeta$$

$$\cancel{\eta^\mu([\zeta_\mu u_\eta] u_\zeta \ldots - u_\zeta[}$$

$$u_\eta \zeta_\mu u_\zeta$$

$$[\eta^\mu[\zeta_\mu u]] = \eta^\mu(\zeta_\mu u_\eta u_\zeta - u_\eta u_\zeta \zeta_\mu)$$

$$- (\zeta_\mu u_\eta u_\zeta - u_\eta u_\zeta \zeta_\mu)\eta^\mu$$

$$= \eta^\mu \zeta_\mu u_\eta u_\zeta - u_\eta \underline{\eta^\mu u_\zeta} \zeta_\mu$$

$$- \underline{\zeta_\mu u_\eta}\, \underline{u_\zeta \eta^\mu} + u_\eta \underline{\zeta_\mu}\, \underline{u_\zeta \eta^\mu}$$

$$= \cancel{\zeta_\mu \eta^\mu u_\eta u_\zeta} - u_\eta \zeta_\mu \eta^\mu u_\zeta$$

$$\zeta_\mu u_\eta \eta^\mu u_\zeta$$

$$- \zeta_\mu u_\eta u_\zeta \eta^\mu + u_\eta \zeta_\mu u_\zeta \eta^\mu$$

$$= \zeta_\mu u_\eta [\eta^\mu u_\zeta] - u_\eta \zeta_\mu [\eta^\mu \zeta_\zeta]$$

$$= [\zeta_\mu u_\eta][\eta^\mu u_\zeta] = 0.$$

$$\underline{F_\mu(\eta_\nu)\, G^\mu_\mu(\zeta_\nu) = 0}$$

If $G^{\mu}_{\mu}(\zeta_\nu)$ is not zero at a point ζ_ν
~~If~~ By performing some suitable Lorentz transformation, we can make
$$G^{\mu}_{\mu}(\zeta_\nu) = 0, \underbrace{}_{\mu \neq \rho} \cancel{\text{except } G_\rho(\zeta_\nu)}$$
so that
$$F_\rho(\eta_\nu) = 0.$$
in this system, but $F_\mu(\eta_\nu)$ $(\mu \neq \rho)$ may be different from 0. In other words F_μ, G^{μ}_{μ} are orthogonal with each other irrespective of η_ν, ζ_ν.
This is only true, when F_μ, G^μ are both const. This in turn result in the restriction that ~~ζ_μ~~ u_η, u_ζ are respectively linear functions of η and ζ, but if so,
$$[\zeta_\mu, \sum a_\rho \eta^\rho_\rho] \cdot [\eta^\mu, \sum b_\tau \zeta_\tau] = 0$$
$$\sum_\mu a_{\rho\mu} b_{\rho\mu} = 0$$
On the other hand, only linear combination which is invariant, is
$$\eta^\rho \zeta_\rho \quad \text{or} \quad \sum a_\mu b_\mu = 4 \neq 0$$

From $\{\eta^\mu[\zeta_\mu, u]\} = [\zeta_\mu[\eta^\mu, u]]$,
it follows that, if we expand u in power ~~$[\zeta_\mu, u] = F(\zeta)$~~ of η and rearrange:

$$u = \sum_n F_n(\eta)G_n(\zeta),$$ ~~with~~ F_n being homogeneous

we obtain

$$\sum_n [\zeta_\mu F_n][\eta^\mu G_n] = 0,$$

which essentially a pro

General solution of

$$[p^\mu [p_\mu U]] = 0$$

$$[x^\mu [x_\mu, U]] = 0$$

$$\frac{\partial}{\partial X} = \frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}, \qquad \frac{\partial}{\partial \xi} = \frac{1}{2}\left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''}\right)$$

$$\left.\begin{array}{l} \left(\frac{\partial^2}{\partial X^2} + \cdots - \frac{1}{c^2}\frac{\partial^2}{\partial T^2}\right) U = 0 \\ \cancel{(\xi'_\mu - \xi''_\mu)(\xi'^\mu - \xi''^\mu)} \underset{(\xi_\mu \xi^\mu)}{} U = 0 \end{array}\right\}$$

$$U(X, \xi) = \sum_k u(k, \xi) \exp(i k_\mu X^\mu)$$

$$\xi_\mu \xi^\mu \cdot u(k, \xi) = 0$$

~~$\xi_\mu \xi$~~ $\therefore \quad u(k, \xi) = \delta(\xi_\mu \xi^\mu)\, u'(k, \xi)$

especially, if we take

$$u(k, \xi) = u(k)\,\delta(\xi \pm \lambda k),$$

it reduces to the case, in which U is a function of η or ζ alone.

$\cancel{\eta^\mu = x_\mu}$

$$\frac{\partial^2 u}{\partial \zeta_\mu \partial \eta^\mu} = 0 \;\underset{\text{yes}}{\overset{?}{\rightleftarrows}}\; u = F(\eta) + G(\zeta)$$

$$u = \eta^\mu \zeta_\mu \;:\; \frac{\partial^2 u}{\partial \zeta_\mu \partial \eta^\mu} \neq 0.$$

Linearization

$$[p_\mu, U] = \kappa' F_\mu \quad (1)$$
$$[x_\mu, U] = \lambda' G_\mu \quad (2)$$
$$[p^\mu, F_\mu] = \kappa'' U \quad (3)$$
$$[x^\mu, G_\mu] = \lambda'' U \quad (4)$$
$$[p^\mu, G_\mu] = 0 \quad (5)$$
$$[x^\mu, F_\mu] = 0 \quad (6)$$

From (5), (6) $\cancel{G_\mu(x)}$, $\cancel{F_\mu(p)}$

2

Fifth Dimension as mass change operator.

$$\left.\begin{array}{l}[p^\mu[p_\mu U]] = 0 \\ [x^\mu[x_\mu U]] = 0\end{array}\right\} \quad \mu = -1, 0, 1, 2, 3.$$

$$U = \sum_{k_\mu} u(k_\mu) e^{ik_\mu \eta^\mu}$$

$$= \sum_{k_\mu} u(k_\mu) e^{ik_\mu x^\mu/\lambda} \cdot e^{ik^\mu p_\mu/\varkappa}$$

$$\sum_{x'''} (x'|p_\mu|x''')(x'''|F|x'') - \sum_{x'''} (x'|F|x''')(x'''|p_\mu|x'')$$

$$= \sum (i\hbar)\delta(x_0'-x_0''')\cdots\delta'(x_\mu'-x_\mu''')\cdots(x'''|F|x'')$$

$$\cancel{+}\ i\hbar\sum(x'|F|x'')\ \delta(x_0'''-x_0'')\cdots\delta'(x_\mu'''-x_\mu'')\cdots$$

$$= -i\hbar\frac{\partial (x'|F|x'')}{\partial x'} - i\hbar\frac{\partial (x'|F|x'')}{\partial x''}$$

$$(x'|F|x'') = F(X, r)$$

$$X = \frac{1}{2}(x'+x'') \qquad r = x'-x''$$

$$\sum_{x'''}\{(x'|p_\mu|x''')(x'''|F|x'') - (x'|F|x''')(x'''|p_\mu|x'')\}$$

$$= \frac{\partial F(X, r)}{\partial X}$$

$$(x'|e^{ik^\mu p_\mu/\varkappa}\cdot F|x'') = \sum_n \frac{1}{n!}\left(k^\mu\frac{\hbar}{\varkappa}\frac{\partial}{\partial x^{\mu\prime}}\right)^n F$$

$$= F(x^{\mu\prime} + \frac{\hbar}{\varkappa}k^\mu, x^{\mu\prime\prime})$$

$$(x'|e^{ik^\mu p_\mu/\chi}|x'') = D(x^{\mu\prime} + \frac{\hbar}{\chi}k^\mu - x'')$$

$$\sum_{x'''}(x'|e^{ik^\mu p_\mu/\chi}|x''')(x'''|F|x'') = (x' + \frac{\hbar}{\chi}k \,|F|\, x'')$$

$$\sum_{x'''}(x'|F|x''')(x'''|e^{ik^\mu p_\mu/\chi}|x'') = (x' \,|F|\, x'' - \frac{\hbar}{\chi}k)$$

$$\cancel{U = \sum_{k_{-1}} e^{ik_{-1}x^{(-1)}/\chi} \sum_{k_\mu\,(\mu\neq-1)} u(k_\mu)\, e^{ik_\mu x^\mu/\chi}}$$

$$\cancel{e^{ik_{-1}p_{-1}/\chi}} \quad e^{ik^\mu p_\mu/\chi}$$

$$U = \sum_{k_\mu\,(\mu\neq-1)} u(k_\mu)\, e^{ik_\mu x^\mu/\chi}\, e^{ik^\mu p_\mu/\chi} \cdot \sum_{k_{-1}} e^{ik_{-1}x^{-1}/\chi} \cdot e^{ik^{-1}p_{-1}/\chi}$$

$$k_{-1}k^{-1} = -k_\mu k^\mu$$

$$\sum_{p''_{-1}}(p'_{-1}|e^{ik_{-1}x^{-1}/\chi}|p''_{-1}) \cancel{(p''_{-1}|F|p''_{-1})}$$

$$= D(p'_{-1} - \frac{\hbar}{\chi}k_{-1} - p''_{-1})$$

$$\cancel{U} =$$

$$(x', p'_{-1}|\, e^{ik_\mu x^\mu/\chi}\, e^{ik^\mu p_\mu/\chi}\, e^{ik_{-1}x^{-1}/\chi}\, e^{ik^{-1}p_{-1}/\chi}\, |x'', p''_{-1})$$

$$= \sum_{x^{\mu\prime\prime\prime}} e^{ik_\mu x^{\mu\prime}/\chi} D(x^{\mu\prime} + \frac{\hbar}{\chi}k^\mu - x^{\mu\prime\prime}) \cdot \Theta \quad \cancel{D(\ldots)}$$

$$\times D(p'_{-1} - \frac{\hbar}{\lambda} k_1 - p''_{-1})\, e^{ik^{-1} p''_{-1}/\kappa}$$

$$k_{-1} = \pm\sqrt{-k_{\mu} k^{\mu}} = \pm\frac{\mu c}{\hbar}.$$

$$\times D(p'_{-1} \mp \frac{\hbar}{\lambda}\frac{\mu c}{\hbar} - p''_{-1})\, e^{\pm i\frac{\mu c}{\hbar} p''_{-1}/\kappa}$$

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Feb. 26, 1949

On nonlocalizable field theory

i) Example
reciprocity equations
phase space
ii) Lagrangian
self-reciprocity of Born
Trace
Feynman's method
(R.M.P. 20 (1948), 367, Especially
8. Operator Algebra.
S-matrix theory

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Green

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Stueckelberg, Rivier
Pauli

Five Dimensional Dual Spaces

$[p^\mu[p_\mu U]] = -m^2 U$ real mass

$[p^\mu\{p_\mu U\}] = m^2 U$ imaginary mass

~~$p^\mu[p_\mu U$~~ real mass:

$$\left.\begin{aligned}[p_\mu U_5] &= m U_\mu = [p_5 U_\mu] \\ [p^\mu U_\mu] &= -m U = -[p^5 U_5]\end{aligned}\right\}$$

fifth dimensional: $\underline{p_5 = p^5}$: space-like

imaginary mass:

$$\left.\begin{aligned}[p_\mu U_5] &= m U_\mu = [p_5 U_\mu] \\ \{p^\mu U_\mu] &= m U_5 = -[p^5 U_5]\end{aligned}\right\}$$

fifth dimension: $\underline{p_5 = -p^5}$: time-like

Five dimensional dual spaces:

○ particle — pseudoparticle

or

five dimensional invariant Δ-function

For zero mass particle there is no distinction between particle and pseudoparticle five dimensional invariant D-fn

$$D(x_1 x_2 x_3; x_4, x_5) = \frac{2\pi}{L^3} \sum_{k_i} \Big\{ \frac{e^{i\Sigma}}{ik_0}$$

invariant volume ~~in~~ on ~~five dimensional space~~ three dimensional
~~on~~ hyperbolic surface in four dimensional space

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2 \pm dx_5^2$$

$$ds^2 = dk_1^2 + dk_2^2 + dk_3^2 - dk_4^2$$

~~$k_4 dk_4 =$~~

dk $\quad k_1^2 + k_2^2 + k_3^2 - k_4^2 = \varkappa^2$

$$k_1 dk_1 + k_2 dk_2 + k_3 dk_3 - k_4 dk_4 = 0$$

$$ds^2 = dk_1^2 + dk_2^2 + dk_3^2 - \frac{k_1^2}{k_4^2} dk_1^2 - \frac{k_2^2}{k_4^2} dk_2^2$$

$$- \frac{k_3^2}{k_4^2} dk_3^2 - \frac{2k_1k_2}{k_4^2} dk_1 dk_2 - \cdots - \cdots$$

$$\begin{vmatrix} 1 - \frac{k_1^2}{k_4^2} & \cancel{2}k_1k_2 & \cancel{2}k_1k_3 \\ \cancel{2}k_1k_2 & 1 - \frac{k_2^2}{k_4^2} & \cancel{2}k_2k_3 \\ \cancel{2}k_1k_3 & \cancel{2}k_2k_3 & 1 - \frac{k_3^2}{k_4^2} \end{vmatrix} = -\frac{k_1^2 + k_2^2 + k_3^2 - k_4^2}{k_4^2}$$

$$= -\frac{\varkappa^2}{k_4^2}$$

real mass: $k_1^2+k_2^2+k_3^2-k_4^2+\kappa^2=0$ (2)

$k_1 = k\sin\theta\cos\varphi = \kappa\sinh\chi\sin\theta\cos\varphi$

$k_2 = k\sin\theta\sin\varphi = \kappa\sinh\chi\sin\theta\sin\varphi$

$k_3 = k\cos\theta = \kappa\sinh\chi\cos\theta$

$k_4 = \pm\sqrt{k^2+\kappa^2} = \pm\kappa\cosh\chi$

~~$k_4 \geq 0$~~
$\chi=(0,\infty)$
~~$k_4 \leq 0$~~
~~$\chi=(-\infty,0)$~~

$ds^2 = dk_1^2+dk_2^2+dk_3^2-dk_4^2$

$dk_1 = \kappa\cosh\chi\sin\theta\cos\varphi\cdot d\chi + \kappa\sinh\chi\cos\theta\cos\varphi\cdot d\theta - \kappa\sinh\chi\sin\theta\sin\varphi\, d\varphi$

$dk_2 = \kappa\cosh\chi\sin\theta\sin\varphi\, d\chi + \kappa\sinh\chi\cos\theta\sin\varphi\, d\theta + \kappa\sinh\chi\sin\theta\cos\varphi\, d\varphi$

$dk_3 = \kappa\cosh\chi\cos\theta\, d\chi - \kappa\sinh\chi\sin\theta\, d\theta$

$dk_4 = \pm\kappa\sinh\chi\, d\chi$

$ds^2 = \kappa^2\cdot(d\chi^2 + \sinh^2\chi\, d\theta^2 + \sinh^2\chi\sin^2\theta\, d\varphi^2)$

$(d\tau)^3 = \begin{vmatrix} \kappa^2 & & \\ & \kappa^2\sinh^2\chi & \\ & & \kappa^2\sinh^2\chi\sin^2\theta \end{vmatrix}^{1/2} d\chi\, d\theta\, d\varphi = \kappa^3\sinh^2\chi\sin\theta\, d\chi\, d\theta\, d\varphi$

$x_1=x_2=0 \quad x_3=r \quad x_4=ct$

$k_1x_1+k_2x_2+k_3x_3-k_4x_4 = \kappa r\sinh\chi\cos\theta - \kappa ct\cosh\chi$

$k_4>0$.

$$\int_0^\infty\int_0^\pi\int_0^{2\pi} \exp\{i\kappa(r\sinh\chi\cos\theta - ct\cosh\chi)\}\cdot\kappa^3\sinh^2\chi\sin\theta\, d\chi\, d\theta\, d\varphi$$

$$= 2\pi \cancel{\varkappa^3} \int_0^\infty \int_0^\pi \exp\{i\varkappa(r \sinh\chi \cos\theta - ct \cosh\chi\} \cdot \sinh^2\chi \cdot \sin\theta \, d\theta \, d\chi$$

$$\cos\theta = -x$$

$$= 2\pi\varkappa^3 \int_0^\infty \int_{-1}^{+1} \exp\{-i\varkappa \, r \sinh\chi \, x - i\varkappa ct \cosh\chi\} \sinh^2\chi \cdot dx \cdot d\chi$$

$$= 2\pi\varkappa^3 \int_0^\infty \left[\frac{\exp\{-i\varkappa r \sinh\chi - i\varkappa ct \cosh\chi\}}{-i\varkappa r} - \frac{\exp\{i\varkappa r \sinh\chi - i\varkappa ct \cosh\chi\}}{-i\varkappa r} \right] \sinh\chi \, d\chi$$

$r < c|t|$; $\quad s = \sqrt{c^2t^2 - r^2}$

$$r = s \cdot \sinh\omega \qquad \omega \geq 0$$

$$ct = s \cdot \cosh\omega$$

$$\pm \sinh\omega \sinh\chi + \cosh\omega \cosh\chi$$

$$= \cosh(\chi \pm \omega)$$

$$2\pi\varkappa^3 \int_0^\infty \left[\frac{\exp(-i\varkappa s \cdot \cosh(\chi + \omega))}{-i\varkappa r} - \frac{\exp(-i\varkappa s \cdot \cosh(\chi - \omega))}{-i\varkappa r} \right] \sinh\chi \cdot d\chi$$

$$\text{I} \quad 2\pi\varkappa^3 \int_0^\infty \left[\frac{\exp(i\varkappa s \cosh(\chi + \omega))}{i\varkappa r} - \frac{\exp(i\varkappa s \cosh(\chi - \omega))}{+i\varkappa r} \right] \sinh\chi \cdot d\chi$$

(3)

$r > c|t|$:　　$s = \sqrt{r^2 - c^2 t^2}$

$r = s \cosh \omega$

$ct = s \sinh \omega$

$$2\pi\kappa^3 \int_0^\infty \left[\frac{\exp\{-i\kappa r \sinh\chi - i\kappa ct \cosh\chi\}}{-i\kappa r} - \frac{\exp\{i\kappa r \sinh\chi - i\kappa ct \cosh\chi\}}{-i\kappa r} \right] \sinh\chi \, d\chi$$

$$= 2\pi\kappa^3 \int_0^\infty \left[\frac{\exp(-i\kappa s \sinh(\chi+\omega))}{-i\kappa r} - \frac{\exp(i\kappa s \sinh(\chi-\omega))}{-i\kappa r} \right] \sinh\chi \, d\chi$$

$$2\pi\kappa^3 \int_0^\infty \left\{ \frac{\exp\{-i\kappa r \sinh\chi - i\kappa ct \cosh\chi\}}{-i\kappa r} - \frac{\exp\{i\kappa r \sinh\chi - i\kappa ct \cosh\chi\}}{-i\kappa r} \right\} \sinh\chi \, d\chi$$

$$= -\frac{2\pi\kappa}{r} \frac{\partial}{\partial r} \int_0^\infty \left[\exp\{-i\kappa r \sinh\chi - i\kappa ct \cosh\chi\} + \exp\{i\kappa r \sinh\chi - i\kappa ct \cosh\chi\} \right] d\chi$$

$$\cosh\omega \sinh\chi \pm \cancel{\cos} \sinh\omega \cosh\chi$$

$$\frac{e^{\omega}+e^{-\omega}}{2} \cdot \frac{e^{\chi}-e^{-\chi}}{2} + \frac{e^{\omega}-e^{-\omega}}{2} \frac{e^{\chi}+e^{-\chi}}{2}$$

$$= \frac{e^{\omega+\chi} - e^{-\omega-\chi}}{2} \text{ or } \frac{e^{\chi-\omega} - e^{-\chi+\omega}}{2} = \sinh(\chi \pm \omega)$$

$$= -\frac{2\pi\kappa}{r}\int_{-\infty}^{+\infty} \exp(-i\kappa r\cdot\sinh\chi - i\kappa ct\cosh\chi)\,d\chi$$

$r < c|t|$ $\qquad s = \sqrt{c^2t^2 - r^2}$

$r = s\cdot\sinh\omega \qquad ct = s\cosh\omega$

$\omega \geq 0$

$$-\frac{2\pi\kappa}{r}\int_{-\infty}^{+\infty} \exp(-\overset{i}{\cancel{\chi}}\kappa s\cosh\chi)\,d\chi.$$

$$= -\frac{4\pi\kappa}{r}\int_{0}^{\infty} \cos(-\cancel{\chi}\kappa s\cosh\chi)\,d\chi$$

$r > c|t|$ $\qquad s = \sqrt{r^2 - c^2t^2}$

$r = s\cosh\omega \qquad ct = s\sinh\omega$

$$-\frac{2\pi\kappa}{r}\int_{-\infty}^{+\infty} \exp(-\overset{i}{\cancel{\chi}}\kappa s\sinh\chi)\,d\chi$$

$$= -\frac{4\pi i\kappa}{r}\int_{0}^{\infty} \cancel{\exp}\,\sin(-\cancel{\chi}\kappa s\sinh\chi)\,d\chi$$

imaginary mass: $k_1^2+k_2^2+k_3^2-k_4^2-\varkappa^2=0$

$$k_1 = k\sin\theta\cos\varphi = \varkappa\cosh\chi\sin\theta\cos\varphi$$
$$k_2 = \cancel{k\sin\varkappa\sinh}\chi = \varkappa\cosh\chi\sin\theta\sin\varphi$$
$$k_3 = \varkappa\cosh\chi\cdot\cos\theta$$
$$k_4 = \varkappa\sinh\chi$$
$$-\infty < \chi < +\infty$$

$$ds^2 = dk_1^2+dk_2^2+dk_3^2-dk_4^2.$$

$$dk_1 = \varkappa\sinh\chi\sin\theta\cos\varphi\cdot d\chi + \varkappa\cosh\chi\cos\theta\cos\varphi\, d\theta - \varkappa\cosh\chi\sin\theta\sin\varphi\, d\varphi$$
$$dk_2 = \varkappa\sinh\chi\sin\theta\sin\varphi\, d\chi + \varkappa\cosh\chi\cos\theta\sin\varphi\, d\theta + \varkappa\cosh\chi\sin\theta\cos\varphi\, d\varphi$$
$$dk_3 = \varkappa\sinh\chi\cos\theta\, d\chi - \varkappa\cosh\chi\sin\theta\, d\theta$$
$$dk_4 = \varkappa\sinh\chi\, d\chi$$

$$ds^2 = \varkappa^2(d\chi^2+\cosh^2\chi\, d\theta^2+\cosh^2\chi\sin^2\theta\, d\varphi^2)$$

$$(d\tau)^3 = \varkappa^3\cosh^2\chi\sin\theta\, d\chi\, d\theta\, d\varphi$$

$$x_1 = x_2 = 0,\quad x_3 = r \qquad x_4 = ct$$
$$k_1x_1+k_2x_2+k_3x_3-k_4x_4 = \cancel{\varkappa r}$$
$$= \varkappa r\cosh\chi\cdot\cos\theta - \varkappa ct\sinh\chi$$

$$\int_{-\infty}^{+\infty}\int_0^{\pi}\int_0^{2\pi} \exp\{i\varkappa(r\cancel{\sin}\cosh\chi\cos\theta - ct\sinh\chi)\}\, \varkappa^3\cosh^2\chi\sin\theta\, d\chi\, d\theta\, d\varphi$$

$$= 2\pi\kappa^3 \int_{-\infty}^{+\infty}\int_0^{\pi} \exp\{i\kappa(r\cosh\chi\cos\theta - ct\sinh\chi)\} \cdot \cosh^2\chi\, d\chi \sin\theta\, d\theta$$

$$= 2\pi\kappa^3 \int_{-\infty}^{+\infty}\int_{-1}^{+1} \exp\{-i\kappa r\cosh\chi\cdot x - i\kappa ct\sinh\chi\} \cosh^2\chi\, d\chi\, \cancel{\sin\theta\, d\theta}\, dx$$

$$= 2\pi\kappa^3 \int_{-\infty}^{+\infty} \left[\frac{\exp(-i\kappa r\cosh\chi - i\kappa ct\sinh\chi)}{-i\kappa r} - \frac{\exp(i\kappa r\cosh\chi - i\kappa ct\sinh\chi)}{-i\kappa r}\right]\cosh\chi\, d\chi$$

$$= \frac{2\pi\kappa^2}{r}\int_{-\infty}^{+\infty}\left[\frac{\exp(i\kappa r\cosh\chi - i\kappa ct\sinh\chi}{i} - \frac{\exp(-i\kappa r\cosh\chi - i\kappa ct\sinh)}{i}\right]\cosh\chi\, d\chi$$

$$= -\frac{2\pi\kappa}{r}\frac{\partial}{\partial r}\int_{-\infty}^{+\infty}\left[\exp(i\kappa r\cosh\chi - i\kappa ct\sinh\chi) + \exp(-i\kappa r\cosh\chi - i\kappa ct\sinh\chi)\right]d\chi$$

$r < c|t|$; $\quad r = s\sinh\omega \qquad \omega \geq 0$

$\qquad ct = s\cosh\omega \qquad t > 0.$

$$r\cosh\chi - ct\sinh\chi = s\sinh(\omega-\chi) = -s\sinh(\chi-\omega)$$

$$r\cosh\chi + ct\sinh\chi = s\sinh(\omega+\chi)$$

⑤

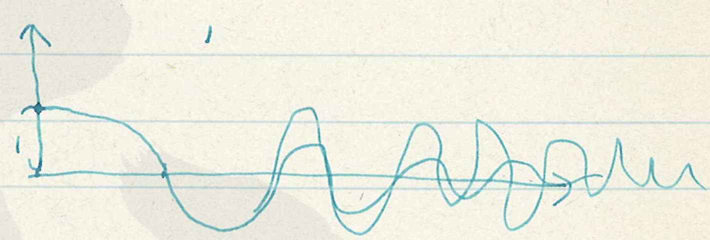
$$\int_{-\infty}^{+\infty}\Big[\exp(i\kappa r\cosh\chi - i\kappa ct\sinh\chi) + \exp(-i\kappa r\cosh\chi - i\kappa ct\sinh\chi)\Big]\,d\chi$$

$$= \int_{-\infty}^{+\infty}\Big[\exp(-i\kappa s\sinh(\chi-\omega)) + \exp(-i\kappa s\sinh(\chi+\omega))\Big]\,d\chi$$

$$= 2\int_{-\infty}^{+\infty}\exp(-i\kappa s\sinh\chi)\cdot d\chi$$

$$= 2\int_{-\infty}^{+\infty}\cos(-\kappa s\sinh\chi)\,d\chi$$

$$\cancel{\frac{\partial}{\partial s}\int_{-\infty}^{+\infty}\exp(-i\kappa s\sinh\chi)\,d\chi}$$

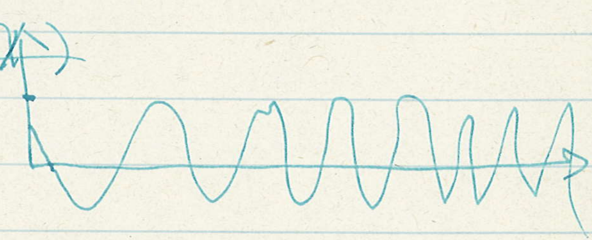


$$\cancel{= -i\kappa\int_{-\infty}^{+\infty}\exp}$$

$$= 4\int_{0}^{\infty}\cos(\kappa s\sinh\chi)\,d\chi$$

$$\cancel{\sin(\pi - \kappa s\sinh\chi)}$$

$$\cancel{= 0}\quad J_0(\cancel{\kappa s})$$



$r > c|t|$:

$$\int_0^\infty \overset{?}{\underset{\cos}{\sin}}(\kappa s\cosh\chi)\,d\chi$$

$$\cancel{= J_0(\kappa s}$$

Thus for pseudoparticle field, there is no function corresponding to D-fn ~~in the cas~~ for particle field. This is because the hyperbolic surface ~~of the~~ three dimension for imag. particle is one sheeted in contrast to ~~that~~ the fact that the hyp. surf for real particle is two sheeted. Namely, in the latter case, we can take two invariant functions independently,† whereas in the former case essential ~~only~~ one function remains invariant, which is even with respect to time reversal.

which is relativistically ~~[illegible]~~ covariant

Now, any field (localizable) can be decomposed into real particle fields and imaginary particle fields with different masses. The commutation relations between field quantities can be represented by linear combinations of invariant D-fns for real and imaginary particles and their derivatives.

~~They cont~~ The characteristic of the particles with real masses and ~~lin.~~ ~~D-~~ commutation infinitesimal relations is evidently that they can be identified with particles in that in classical

† They can be so chosen that one is even and one is odd with respect to time reversal, the latter being the well known D-fn.

limit, they ~~reduce to the we~~ leave precisely ~~the proper~~ reproduces the properties of the particle with a real mass.
On the contrary, the imaginary particle field or real particle field with non-infinitesimal commutation relation has no direct connection with the ~~cla~~ classical particle concept.
Moreover if these field ever exist it will violate the fundamental concept of the relativity theory in that they may propagate the action with the velocity larger than c.
However, D-fns (even) has nearly infinitesimal** for larger values of mass (~~a~~ absolute value of imaginary mass)

$$N_0(z) = -\frac{2}{\pi}\int_0^\infty \cos(z\cosh\varphi)\,d\varphi \qquad y>0.$$

$$H^{(2)}_0(z) = i\frac{2}{\pi}\int_0^\infty e^{-iz\cosh\varphi}\,\cancel{\cosh}\,d\varphi$$

$y<0$ (Heine)

$$i H_0^{(1)}(ix) = -iH_0^{(2)}(-ix)$$

** Kanai, $D_1(\cancel{x}r,t) \propto \frac{1}{r}H_1^{(1)}(i\kappa r)$
for $t=0$ $\sim \frac{1}{r^{3/2}}e^{-\kappa r}$

Pauli,

Invariant Quantities in Rel. Field Theory

$$\int_0^\infty f(\varkappa^2) \int_0^\infty \frac{k^2\,dk}{\sqrt{k^2+\varkappa^2}}$$

quadratic divergence

$$f(\varkappa^2) = \delta(\varkappa^2 - m^2)$$

$$f(\varkappa^2) = \delta(\varkappa^2 - m_1^2) - \delta(\varkappa^2 - m_2^2)$$

$$\int_0^\infty \left(\frac{k^2}{\sqrt{k^2+m_1^2}} - \frac{k^2}{\sqrt{k^2+m_2^2}} \right) dk$$

$$= \int^\infty \left(-\frac{1}{2}\frac{m_1^2}{k} + \frac{1}{2}\frac{m_2^2}{k} \right) dk$$

logarithmic div.

$\varkappa^2 > 0$: $D - f\frac{1}{2}$ infinitesimal particle field

$D_1 - f\frac{1}{2}$ pseudoparticle field

$\varkappa^2 < 0$ $D_1 - f\frac{1}{2}$ pseudoparticle field

either Negative Energy Particle

or

Pseudoparticle (only appearing in the intermediate states)

On the system of Particles and Pseudoparticles

As pointed out by Pauli *, the question of gage invariance in Schwinger's formalism can be settled down only by introducing some additional prescription ~~for the~~ ~~to~~ in the way of computing quantities, which by itself, ~~inde~~ are indeterminate. ~~Namely~~ For example, the additional current due to the presence of an electromagnetic potential A_ν is given by

$$\langle j_\mu(x) \rangle = -4e^2 \int K_{\mu\nu}(x-x') A_\nu(x') d^4x' \quad (1)$$

with

$$K_{\mu\nu}(x) = \frac{\partial \bar{\Delta}}{\partial x_\mu} \frac{\partial \Delta^{(1)}}{\partial x_\nu} + \frac{\partial \bar{\Delta}}{\partial x_\nu} \frac{\partial \Delta^{(1)}}{\partial x_\mu} - \left(\frac{\partial \bar{\Delta}}{\partial x_\lambda} \frac{\partial \Delta^{(1)}}{\partial x_\lambda} + m^2 \bar{\Delta} \Delta^{(1)} \right) \delta_{\mu\nu}, \quad (2)$$

where

$$\bar{\Delta}(x) = -\frac{1}{2} \Delta(x) \varepsilon(x). \;**$$

The condition of gage invariance ~~is~~ with respect to the transformation

$$A_\nu \to A_\nu + \frac{\partial f}{\partial x_\nu}$$ is

$$\frac{\partial K_{\mu\nu}}{\partial x_\nu} = 0. \quad (3)$$

---

* Letter to Schwinger
Rev. Mod. Phys.

** Schwinger. Phys. Rev. 74 (1948), 1439; 75 (1949), 651.

However,

$$\frac{\partial K_{\mu\nu}}{\partial x_\nu} = -\delta(x)\frac{\partial \Delta^{(1)}}{\partial x_\mu},$$

in which $\frac{\partial \Delta^{(1)}}{\partial x_\mu}$ has a singularity of the type $\frac{x_\mu}{(x_\lambda x_\lambda)^2}$ at the light cone, so that $\delta(x)\frac{\partial \Delta^{(1)}}{\partial x_\mu}$ is non-defined and the condition (3'') is not an identity.

Divergences in the Mixed Field Theory

Feldman: (Umezawa-Kawabe (素粒論研究 4))

v: $\delta_{\mu} j_\mu(x) = \frac{\alpha}{2\pi}\left(\frac{1}{2}\log\frac{1}{\gamma\varepsilon} + 1\right) J_\mu(x)$

$\qquad - \frac{\alpha}{4\pi m^2}\left(\frac{1}{3}\log\frac{1}{\gamma\varepsilon} + \frac{1}{10}\right)\Box J_\mu(x)$

S: $\delta j_\mu(x) = -\frac{\alpha}{3\pi}\log\frac{1}{\gamma\varepsilon} J_\mu(x) - \frac{\alpha}{120\pi m^2}\Box J_\mu(x)$

el: $\delta j_\mu(x) = -\frac{\alpha}{3\pi}\log\frac{1}{\gamma\varepsilon} J_\mu(x) - \frac{\alpha}{15\pi m^2}\Box J_\mu(x)$

$\Delta^{(1)}\bar{\Delta}$:

$$\bar{\Delta} \simeq \int d\alpha\, e^{i\alpha\lambda + im^2/4\alpha}$$

$$\Delta^{(1)} = \int d\beta \quad \cdots\cdots$$

$\alpha + \beta = \frac{1}{z} \qquad \alpha - \beta = \frac{y}{z}$

$$\int_\varepsilon^\infty dz\, \frac{e^{iz}}{z|z|}$$

Umezawa-Kawabe: Quadratic term in vector case, which is non-gauge invariant and ~~an~~ ambiguous.
Feldman already dropped this term.

Finite Distance Operator, Pais

$$L = \iint \frac{\partial U(x)}{\partial x_\mu} F(x-x') \frac{\partial U(x')}{\partial x'_\mu} (dx)^4 (dx')^4$$

$$F(x-x') = \cosh(\lambda^2 \Box)$$

$$L = -\iiint (dk)^4 (dk')^4 (dk'')^4 \iint k_\mu U(k) e^{ikx}$$

$$\times\ F(k''_\mu k''^\mu) e^{ik''(x-x')} k'^\mu U(k') e^{ik'x'} (dx)^4 (dx')^4$$

$$= \frac{-1}{(2\pi)^8} \iiint (dk)^4 (dk')^4 (dk'')^4 \; k_\mu U(k) \delta(k+k'') F(k''_\mu k''^\mu)$$

$$\delta(k''-k') k'^\mu U(k')$$

$$= \frac{1}{(2\pi)^8} \iiiint k_\mu k^\mu U(k) U(-k) \; F(k_\mu k^\mu)(dk)^4$$

$$= \frac{1}{(2\pi)^8} \underbrace{\iiint}_{dK} K\, F(K)\, U(k,K)\, U(-k,K) \frac{(dk)^3}{|k_0|}$$

March 11, 1949

On the Theory of Pseudoparticle in connection with the problem of Stability of Elementary Particles.

~~Free~~ Wave ~~field~~ function for a free particle is represented by a plane wave

$$e^{\frac{i}{\hbar} p_\mu x^\mu} \qquad (1)$$

$x_0 = -x^0$

with $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$,

$p_0 = -E/c$, $p_1 = p_x$, $p_2 = p_y$, $p_3 = p_z$.

$p^0 = +E/c$

For a particle with the rest mass m ~~the relation~~ well-known

$$p_\mu p^\mu + m^2c^2 = 0 \qquad (I)$$

restrict possible values for the energy ~~and~~ E and momentum p_x, p_y, p_z. Furthermore, ~~the~~ condition (I) ~~guarantee the well~~ is consistent with the fundamental fact that every kind of physical action propagates with the velocity not greater than light velocity c. ~~Theory of elementary particles is essentially based on the~~ In physics, we ~~have been~~ always dealing with the particles with ~~the~~ a definite mass, which is connected with ~~its~~ momentum and energy by the relation (I). The failure of consistent theory of elementary particles has been looked upon as ~~the lack~~ due to the lack of ~~some fundamental rule~~ our knowledge concerning the intrinsic properties ~~and as well as~~ of elementary particles as well as their mode of interaction, In this connection, I propose ~~have~~ to introduce in the theory of elementary particles, ~~an object, which is in many respect contrary to the ordinary particle~~ an object, which is in many respects contrary to the ordinary particle. ~~The wave~~ For brevity, we call it a pseudoparticle, ~~which~~ a free pseudo-particle ~~is~~ with a pseudomass ~~μ~~ can be represented by a plane wave of the same form as (1), but the relation between the momentum and energy is

$$p_\mu p^\mu - \mu^2 c^2 = 0 \qquad (3)$$

instead of (2). Thus ~~a pseudoparticle has~~ the absolute value of ~~a~~ the momentum of a pseudoparticle is always equal to or larger than μc, while the ~~energy and~~ absolute value for the energy can be as small as 0. ~~Once we assume the existence of this pseudoparticle~~

The pseudoparticle has many properties, which are so ~~two~~ strange that ~~few~~ one ~~may~~ will be very sceptical as to the existence of such a thing in nature. ~~Before apologizing the introduction of pseudoparticle, these strange properties are mentioned~~ enumerated:

First of all, ~~1.~~ pseudoparticle has always a velocity larger than light velocity c, so that it can propagate the action from one place to another ~~with~~ faster than light signal. This ~~alone is at~~ seems to be in striking contradiction with ~~the~~ the fundamental notion of special relativity, and this alone seems to be enough for excluding the possibility of existence of the pseudoparticle.

~~However, if we assume that the~~ pseudo ~~mass of μ of the pseudoparticle is very large~~ enough~~, for instance of the order of the nucleon mass M, it we can expect that it will be created only when it is by a process between elementary particles, in which only when the momentum change~~

Moreover, a pair of pseudoparticles with the opposite momentum of the same magnitude μc* will be created simultaneous in vacuum,

* with opposite charge, if they are charged

without violating the conservation laws.
In other words, if a ~~a~~ group of terms ~~like~~ with ~~a~~ the factor

$$a^*_{(+)} e^{\frac{i}{\hbar} p^{(+)}_\mu x^\mu} \quad a^*_{(-)} e^{\frac{i}{\hbar} p^{(-)}_\mu x^\mu} \qquad (4)$$

~~with the~~ satisfying the condition

$$p^{(+)}_\mu + p^{(-)}_\mu = 0, \qquad (5)$$

~~will~~ appears in Lagrangian or Hamiltonian for pseudoparticle, ~~that it is~~ these terms are coupled together ~~the term~~ so as to ~~form~~ give infinite probability of creation of pseudoparticles in vacuum. This can be avoided by multiply (4) by a factor

$$p^{(+)}_\mu p^{(-)\mu} + \mu^2 c^2 = 0$$

Spin and Statistics of Pseudoparticle
(Introduction to the Theory of Elementary Particles Vol I, p188 ~ 193.

$$D(x_1 x_2 x_3, x_0) = \frac{2\pi}{L^3} \sum_{\substack{k_i \\ (k \geq \kappa)}} \left\{ \frac{e^{i(\sum_i k_i x_i - k_0 x_0)}}{ik_0} \cancel{\pm} \frac{e^{-i(\sum \cdots)}}{ik_0} \right\}$$

not invariant

~~or~~ t. for limit $L \to \infty$:

$$D = \frac{1}{4\pi^2} \iiint_{(k \geq \kappa)} \left\{ \frac{e^{i(\sum_i k_i x_i - k_0 x_0)}}{ik_0} \cancel{\pm} \frac{e^{-i(\cdots)}}{ik_0} \right\} dk_x dk_y dk_z$$

$$= \frac{1}{4\pi^2} \iiint_{(k \geq \kappa)} \frac{2\sin(\sum_i k_i x_i - k_0 x_0)}{k_0} dk_x dk_y dk_z$$

$$\sum_i k_i^2 - k_0^2 = \kappa^2$$

$$\left(\sum_{i=1}^3 \frac{\partial^2}{\partial x_i^2} - \frac{\partial^2}{\partial x_0^2} + \kappa^2 \right) D = 0$$

~~$D(x_1 x_2 x_3, 0) =$~~

$$D(r,t) = \frac{1}{4\pi^2} \lim_{K \to \infty} \int_\kappa^K \int_0^\pi \int_0^{2\pi} \frac{2\sin(kr\cos\theta - k_0 ct)}{\cancel{k^2}\ k_0} k^2 dk \sin\theta \, d\theta \, d\varphi$$

$$= \frac{1}{\pi r} \left\{ \int_\kappa^\infty \frac{\cos(kr - k_0 ct)}{k_0} k\,dk - \int_\kappa^\infty \frac{\cos(kr + k_0 ct)}{k_0} k\,dk \right\}$$

$$= \frac{1}{r} \frac{\partial F(r,t)}{\partial r} \qquad \left(\neq \frac{1}{s} \frac{dF(s)}{ds} \right)$$

not invar.

$$F(r,t) = \frac{1}{\pi}\left\{\int_{\kappa}^{\infty} \frac{\sin(kr - k_0ct)}{k_0}\,dk - \int_{\kappa}^{\infty} \frac{\sin(kr + k_0ct)}{k_0}\,dk\right\}$$

$$\cancel{= \frac{1}{\pi}\Big\{\int}$$

$r < |ct|$ or $\cancel{r > ct > \kappa}$

$s = \sqrt{c^2t^2 - r^2}$; $\begin{cases} r = s\,\sinh\varphi \\ ct = s\,\cosh\varphi \end{cases}$

$$\begin{cases} k_0 = \kappa\,\sinh\chi \quad * \\ k = \kappa\,\cosh\chi \quad (k \geq \kappa) \end{cases}$$

$$F(r,t) = \frac{1}{\pi}\left\{\int_0^{\infty} \sin(s\kappa\,\sinh(\chi - \varphi))\,d\chi - \int_0^{\infty} \sin(s\kappa\,\sinh(\chi + \varphi))\,d\chi\right\}$$

$\chi \to -\chi'$

$$= \frac{1}{\pi}\left\{-\int_0^{-\infty} \sin(s\kappa\,\sinh(\chi' + \varphi))\,d\chi' - \int_0^{\infty} \sin(s\kappa\,\sinh(\chi + \varphi))\,d\chi\right\}$$

$\neq 0$ not invariant

$*$ $kr \pm k_0ct = s\kappa\{\cosh\chi\,\sinh\varphi \pm \sinh\chi\,\cosh\varphi\}$

$$= s\kappa\left\{\left(\frac{e^{\chi}+e^{-\chi}}{2}\cdot\frac{e^{\varphi}-e^{-\varphi}}{2}\right) \pm \left(\frac{e^{\chi}-e^{-\chi}}{2}\cdot\frac{e^{\varphi}+e^{-\varphi}}{2}\right)\right\}$$

$$= s\kappa\left\{\pm\frac{e^{\chi\pm\varphi}-e^{-\chi\mp\varphi}}{2}\right\} = s\kappa\,\sinh(\varphi \pm \chi) = \pm s\kappa\,\sinh(\chi \pm \varphi)$$

$$D_1(x_1 x_2 x_3, x_4) = \frac{2\pi}{L^3} \sum_{\substack{k_i \\ (k \gtrsim \kappa)}} \left\{ \frac{e^{i(\sum_i k_i x_i - k_0 x_0)}}{k_0} + \frac{e^{-i(\sum_i k_i x_i - k_0 x_0)}}{k_0} \right\}$$

$$D_1 = \frac{1}{4\pi^2} \iiint_{k \gtrsim \kappa} \frac{2\cos(\sum_i k_i x_i - k_0 x_0)}{k_0} dk_x dk_y dk_z$$

$$= \frac{1}{\pi r} \int_\kappa^\infty \frac{\sin(kr + k_0 c\tau)}{k_0} k\,dk$$

$$+ \int_\kappa^\infty \frac{\sin(kr - k_0 c\tau)}{k_0} k\,dk$$

$$= \frac{1}{r}\frac{\partial F}{\partial r}$$

$$F(r,t) = \frac{1}{\pi}\left\{ \int_\kappa^\infty \frac{\cos(kr - k_0 c\tau)}{k_0} dk \right.$$

$$\left. + \int_\kappa^\infty \frac{\cos(kr + k_0 c\tau)}{k_0} dk \right\}$$

$$= \frac{1}{\pi}\left\{ \int_0^\infty \cos(-s\kappa \sinh(\chi - \varphi))\,d\chi \right.$$

$$\left. + \int_0^\infty \cos(s\kappa \sinh(\chi + \varphi))\,d\chi \right\}$$

$$= \frac{1}{\pi}\left\{ -\int_0^{-\infty} \cos(+s\kappa \sinh(\chi' + \varphi))\,d\chi' \right.$$

$$\left. + \int_0^\infty \cos(s\kappa \sinh(\chi + \varphi))\,d\chi \right\}$$

$$\begin{vmatrix} \cancel{-\frac{k_0^2-k_1^2}{k_0^2}}\; -1+\frac{k_1^2}{k_0^2} & \frac{k_1k_2}{k_0^2} & \frac{k_1k_3}{k_0^2} \\ \frac{k_1k_2}{k_0^2} & \cancel{-\frac{k}{k_0}}\; -1+\frac{k_2^2}{k_0^2} & \frac{k_2k_3}{k_0^2} \\ \frac{k_1k_3}{k_0^2} & \frac{k_2k_3}{k_0^2} & -1+\frac{k_3^2}{k_0^2} \end{vmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc} y & -x & \end{array} \\ = & \begin{vmatrix} x^2-1 & xy & xz \\ xy & y^2-1 & yz \\ xz & yz & z^2-1 \end{vmatrix} \end{array}$$

$$= \frac{1}{y}\begin{vmatrix} -y & xy & xz \\ x & y^2-1 & yz \\ 0 & yz & z^2-1 \end{vmatrix}$$

$$= \cancel{y}\left\{ -(y^2-1)(z^2-1) + y^2z^2 \right.$$

$$\left. - \frac{x}{\cancel{y}}\, x\cancel{y}(z^2-1) + x^2z^2 \right.$$

$$= -1 + y^2 + z^2 + x^2 = \frac{k_1^2+k_2^2+k_3^2-k_0^2}{k_0^2} = \frac{k^2}{k_0^2}$$

Spin and Statistics of Pseudoparticle

李平倍 詳解 上巻　p. 188 ~ 193

$$D(x_1 x_2 x_3, x_0) = \frac{2\pi}{L^3} \sum_{\substack{k_i \\ \cancel{(k_0 > \kappa)} \\ (k > \kappa) \\ \cancel{(k_0 > 0)}}} \left\{ \frac{e^{i(\sum_i k_i x_i - k_0 x_0)}}{i|k_0|} \right\} + \frac{e^{-i(\cdots)}}{i|k_0|}$$

$$= \frac{1}{4\pi^2} \iiint_{k>\kappa \quad k_0>0} \frac{2\cos(\sum_i k_i x_i - k_0 x_0)}{k_0} \, dk_x dk_y dk_0$$

$$\left(\sum_{i=1}^{3} \frac{\partial^2}{\partial x_i^2} - \frac{\partial^2}{\partial x_0^2} + \kappa^2 \right) D = 0$$

$$\underbrace{D}_{(r,t)} = \frac{1}{4\pi^2} \int_{\kappa}^{K} \int_0^{\pi} \int_0^{2\pi} \frac{2\cos(kr\cos\theta - k_0 c t)}{k_0} \, k^2 dk \sin\theta \, d\theta d\varphi$$

$$-\cos\theta = x \qquad \sin\theta d\theta = dx$$

$$= \frac{1}{\pi r} \int_{\kappa}^{\infty} \frac{k^2 dk}{k_0} \int_{-1}^{+1} \cos(krx - k_0 c t) \, dx$$

$$= \frac{1}{\pi r} \int_{\kappa}^{\infty} \frac{k\, dk}{k_0} \left\{ \cancel{\cos} \sin(kr - k_0 c t) + \sin(kr + k_0 c t) \right\}$$

$$= \frac{1}{r} \frac{\partial F}{\partial r}$$

$$F(r,t) = -\frac{1}{\pi} \left\{ \int_{\kappa}^{\infty} \frac{\cos(kr - k_0 c t)}{k_0} dk + \int_{\kappa}^{\infty} \frac{\cos(kr + k_0 c t)}{k_0} dk \right\}$$

$r < c|t|$.

$s = \sqrt{c^2t^2 - r^2}$

$t > 0$: $\left.\begin{array}{l} r = s \sinh\varphi \\ ct = s\cosh\varphi \end{array}\right\}$

$\left.\begin{array}{l} k_0 = \varkappa \sinh\chi \\ k = \varkappa\cosh\chi \end{array}\right\}$

$$F(r,t) = \frac{-1}{\pi}\left\{\int_0^\infty \cosh\{s\varkappa \sinh(-\chi+\varphi)\}\,d\chi\right.$$

$$\left. + \int_0^\infty \cosh\{s\varkappa \sinh(\chi+\varphi)\}\,d\chi\right\}$$

$$= \frac{-1}{\pi}\left\{-\int_0^{-\infty} \cosh\{s\varkappa \sinh(\chi+\varphi)\}\,d\chi\right.$$

$$\left. + \int_0^\infty \cosh\{s\varkappa \sinh(\chi+\varphi)\}\,d\chi\right\}$$

$$= \frac{-1}{\pi}\int_{-\infty}^{+\infty} \cosh(s\varkappa\sinh\chi)\cdot d\chi \neq 0$$

$r > c|t|$: $r = s\cosh\varphi$ $ct = s\sinh\varphi$ $(t > 0)$

$$F(r,t) \; \cancel{=} = \frac{-1}{\pi}\int_{-\infty}^{+\infty} \cos(s\varkappa\cosh\chi)\,d\chi \neq 0$$

Thus, there's no invariant D-fn, which is 0 ~~for time-like region~~ everywhere in time-like region.

$*$ $$kr - k_0ct = s\varkappa(\cosh\chi\sinh\varphi - \sinh\chi\cosh\varphi)$$

$$= s\varkappa\left(\frac{e^\chi + e^{-\chi}}{2}\cdot\frac{e^\varphi - e^{-\varphi}}{2} - \frac{e^\chi - e^{-\chi}}{2}\cdot\frac{e^\varphi + e^{-\varphi}}{2}\right)$$

$$= s\varkappa\,\frac{e^{-\chi+\varphi} - e^{\chi-\varphi}}{2} = s\varkappa\sinh(-\chi+\varphi)$$

$$kr + k_0ct = s\varkappa\sinh(\chi+\varphi)$$

D and D_1-fn for particle.

$$D = \frac{1}{s}\frac{dF}{ds}$$

$$\left.\begin{array}{ll} s = \sqrt{c^2t^2 - r^2} & \text{for } c|t| > r \\ s = \sqrt{r^2 - c^2t^2} & \text{for } c|t| < r \end{array}\right\}$$

$$F = -J_0(\varkappa s) \quad \text{for } ct > r$$

$$F = 0 \quad \text{for } r > ct > -r$$

$$F = J_0(\varkappa s) \quad \text{for } ct < -r$$

$$J_0(\varkappa s) = \int_0^\infty \sin(\varkappa s \cosh \chi)\, d\chi.$$

$$D_1 = \frac{1}{r}\frac{\partial F_1}{\partial r} \quad \left(= \frac{1}{s}\frac{\partial F}{ds}\right)$$

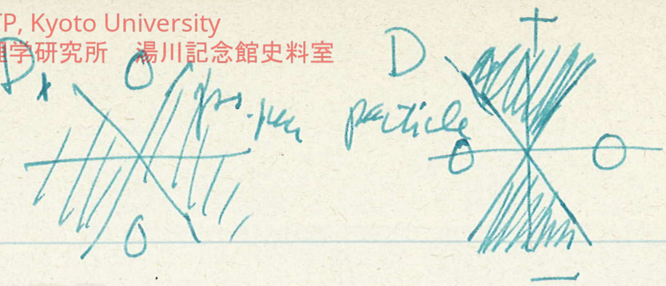
$$F_1 = -\frac{1}{\pi}\left(\int_0^\infty \frac{\cos(kr - k_0ct)}{k_0}\, dk + \int_0^\infty \frac{\cos(kr + k_0ct)}{k_0}\, dk\right)$$

$r < c|t|$: $\quad s = \sqrt{c^2t^2 - r^2}$

$$\left.\begin{array}{l} r = s \sinh \varphi \\ t>0: \quad ct = s \cosh \varphi \end{array}\right\} \qquad \left.\begin{array}{l} k_0 = \varkappa \cosh \chi \\ k = \varkappa \sinh \chi \end{array}\right\}$$

$$F_1 = -\frac{1}{\pi}\left(+\int_0^\infty \cos\{s\varkappa \cosh(-\chi + \varphi)\}\, d\chi + \int_0^\infty \cos\{s\varkappa \cosh(\chi + \varphi)\}\, d\chi\right)$$

$$= -\frac{1}{\pi}\left(\int_{-\infty}^\infty \cos\{s\varkappa \cosh(\chi \cancel{+\varphi})\}\, d\chi\right)$$



$r > c|t|$.

$$\left.\begin{aligned} r &= s\cosh\varphi \\ ct &= s\sinh\varphi \end{aligned}\right\} \qquad \begin{aligned} k_0 &= \kappa\cosh\chi \\ k &= \kappa\sin\chi \end{aligned}$$

~~$t>0$~~

$$F_1(r,t) = -\frac{1}{\pi}\iint_0^\infty$$

D_1-fn ~~for~~ of particle for $s = \sqrt{c^2t^2 - r^2}$ $(c|t| > r)$
is equal to
D-fn of pseudoparticle for
$$s = \sqrt{r^2 - c^2t^2} \quad (c|t| < r).$$
and vice versa

$\because$ $\cancel{\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right.}$

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \kappa^2\right)D_1 = \cancel{\frac{\partial^2}{\partial r^2} + \cdots r}$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rD_1) - \frac{1}{c^2}\frac{\partial^2 D_1}{\partial t^2} - \kappa^2 D_1$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial D_1}{\partial r}\right) + \frac{1}{r}\frac{\partial D_1}{\partial r} - \cdots$$

$$= \frac{\partial^2 D_1}{\partial r^2} + \frac{2}{r}\frac{\partial D_1}{\partial r} - \frac{1}{c^2}\frac{\partial^2 D_1}{\partial t^2} - \kappa^2 D_1$$

$c|t| > r$:

$$\frac{\partial D_1}{\partial r} = -\frac{r}{s}\frac{\partial D_1}{\partial s}$$

$$\frac{\partial^2 D_1}{\partial r^2} = -\frac{1}{s}\frac{\partial D_1}{\partial s} -$$

In other words, even ~~we~~ if we consider a field U satisfying

$$\left(\frac{\partial^2}{\partial x_\mu \partial x^\mu} + \varkappa^2\right) U = 0,$$

commutation relations are necessary ~~not~~ infinitesimal neither in the ordinary sense nor in the sense, in which ~~space~~ the rôles space and time ~~are~~ interchange. In other words, a particle is localized in space, but is extended in ~~time~~ time, whereas pseudoparticle ~~can neither~~ is always extended ~~both~~ space and time directions*, because we can not construct a ~~co~~variant commutation relations between field quantities, ~~which is~~ which are infinitesimal ~~both~~ either in space or ~~and~~ in time direction.

~~The~~ spinor ~~parti~~ field satisfying

$$(\gamma^\mu p_\mu + mc)\psi = 0, \qquad (1)$$

~~we can~~ with

$$\gamma_\mu \gamma^\nu + \gamma^\nu \gamma_\mu = -\delta_{\mu\nu}, \qquad (2)$$

corresponds to Dirac particle with p_μ satisfying

$$p_\mu p^\mu + m^2c^2 = 0 \qquad (3)$$

* photons are particular in that ~~it~~ the field is not only localized in space, but also concentrated in light cone.

We can write alternatively

$$\gamma^1 = \gamma_1 = \beta\alpha_x = i\rho_2\sigma_x$$
$$\gamma^2 = \gamma_2 = \beta\alpha_y = i\rho_2\sigma_y$$
$$\gamma^3 = \gamma_3 = \beta\alpha_z = i\rho_2\sigma_z$$
$$\gamma^4 = -\gamma_4 = \beta = \rho_3$$

$$\beta(\gamma^\mu p_\mu + mc)\psi = 0$$

or $(\boldsymbol{\alpha}\mathbf{p} - p^4 + \beta mc)\psi = 0$

$$(\rho_1\boldsymbol{\sigma}\mathbf{p} - p^4 + \rho_3 mc)\psi = 0$$

~~Alternat~~ Instead of (1), we assume

$$\gamma'_\mu\gamma'_\nu + \gamma'_\nu\gamma'_\mu = \delta_{\mu\nu} \qquad (1)'$$

$$\gamma'^1 = \gamma'_1 = i\rho_1\alpha_x = \rho_3\sigma_x$$
$$\gamma'^2 = \gamma'_2 = i\rho_1\alpha_y = \rho_3\sigma_y$$
$$\gamma'^3 = \gamma'_3 = i\rho_1\alpha_z = \rho_3\sigma_z$$
$$\gamma'^4 = -\gamma'_4 = i\rho_2$$

$i\rho_3$ $\quad \beta(\rho_3\boldsymbol{\sigma}\cdot\mathbf{p} - i\rho_2 p^4 + mc)\psi = 0$

or $(\boldsymbol{\sigma}\mathbf{p} - \rho_1 p^4 + \rho_3 mc)\psi = 0$ *

$$\bar\psi(\boldsymbol{\sigma}\mathbf{p} \pm \rho_1 p^4 + \rho_3 mc) = 0.$$

$$\frac{\partial s_\mu}{\partial x_\mu} = \frac{\partial S^\mu}{\partial x^\mu} = \mathrm{div}(\bar\psi\boldsymbol{\sigma}\psi) + \frac{1}{c}\frac{\partial}{\partial t}(\bar\psi\rho_1\psi) = 0$$

$$\mathbf{S} = \bar\psi\boldsymbol{\sigma}\psi \qquad S^4 = \bar\psi\rho_1\psi.$$

~~* $(i\rho_3\boldsymbol{\sigma}\mathbf{p} - \rho_2 p^4 + \rho_1 mc)\psi = 0$~~

Commutation Relations in Space-Time for Nonlocalizable Field Quantities (i)

$$[U(X_\mu, r_\mu), U(X'_\mu, r'_\mu)]$$

$$= \sum_{k_\mu} \sum_{k'_\mu} (2k_0)^{-\frac{1}{2}} (2k'_0)^{-\frac{1}{2}} \Big[b(k_\mu)\delta(r_\mu + \lambda k_\mu) \exp(ik_\mu X^\mu/\lambda),$$

$$b^*(k'_\mu)\delta(r'_\mu - \lambda k'_\mu) \exp(-ik'_\mu X'^\mu/\lambda) \Big]$$

$$+ \sum \sum (2k_0)^{-\frac{1}{2}} (2k'_0)^{-\frac{1}{2}} \Big[b^*(k_\mu)\delta(r_\mu - \lambda k_\mu)$$

$$\times \exp(-ik_\mu X^\mu/\lambda),\ b(k'_\mu)\delta(r'_\mu + \lambda k'_\mu) \exp(ik'_\mu X'^\mu/\lambda) \Big]$$

$$= \sum_{k_\mu} \frac{1}{2k_0} \{ \delta(r_\mu + \lambda k_\mu)\delta(r'_\mu - \lambda k_\mu) \exp\{ik_\mu(X^\mu - X'^\mu)/\lambda\}$$

$$- \delta(r_\mu - \lambda k_\mu)\delta(r'_\mu + \lambda k_\mu) \exp\{-ik_\mu(X^\mu - X'^\mu)/\lambda\} \}$$

$$\neq 0 \quad \text{for } r_\mu + r'_\mu = 0.$$

$$\Big(-r_\mu = +r'_\mu = \lambda k_\mu : \quad \frac{1}{2k_0} \exp\{ik_\mu(X^\mu - X'^\mu)/\lambda\}$$

$$r_0 < 0,\ r'_0 > 0 \qquad = \frac{c\,\delta(r_\mu + r'_\mu)}{2\sqrt{r_1^2 + r_2^2 + r_3^2}} \exp\{-ir_\mu(X^\mu - X'^\mu)\} \Big)$$

$$\Big(r_\mu = -r'_\mu = \lambda k_\mu : \quad r_0 > 0,\ r'_0 < 0 \qquad \{-i(r_\mu X^\mu + r'_\mu X'^\mu)\} \Big)$$

$$-\frac{1}{2k_0} \exp\{-ik_\mu(X^\mu - X'^\mu)/\lambda\}$$

$$= -\frac{c\,\delta(r_\mu + r'_\mu)}{2\sqrt{r_1^2 + r_2^2 + r_3^2}} \exp\{-ir_\mu(X^\mu - X'^\mu)\} \Big)$$

$$[U(x'_\mu, x''_\mu), U(x'''_\mu, x''''_\mu)]$$

$$= \frac{c\,\delta(x'_\mu - x''_\mu + x'''_\mu - x^{(iv)}_\mu) \exp\{-i(x'_\mu x''^\mu \overbrace{}^{-x''_\mu x'''^\mu} + x'''_\mu x''''^\mu - x''''_\mu x^{(iv)\mu})\}}{2\sqrt{(x'_1 - x''_1)^2 + (x'_2 - x''_2)^2 + (x'_3 + x''_3)^2}}$$

General Conditions for the Commutation Relations

$$[U(x'_\mu, x''_\mu), U(x'''_\mu, x^{(IV)}_\mu)] = F(x'_\mu, x''_\mu; x'''_\mu, x^{(IV)}_\mu)$$

i) $F(x'_\mu + y_\mu, x''_\mu + y_\mu; x'''_\mu + y_\mu, x^{(IV)}_\mu + y_\mu)$
$= F(x'_\mu, x''_\mu; x'''_\mu, x^{(IV)}_\mu)$

ii) $F(x''_\mu, x'_\mu; x^{(IV)}_\mu, x'''_\mu)$
$= -\tilde{F}(x'_\mu, x''_\mu; x^{(IV)}_\mu, x'''_\mu)$

iii) $F(x'''_\mu, x^{(IV)}_\mu; x'_\mu, x''_\mu)$
$= -F(x'_\mu, x''_\mu; x'''_\mu, x^{(IV)}_\mu)$

From i) $F(x'_\mu, x''_\mu; x'''_\mu, x^{(IV)}_\mu)$ contains $(x'_\mu, x''_\mu; x'''_\mu, x^{IV}_\mu)$ only in the combinations

$$\left.\begin{aligned} x^{12}_\mu &= x'_\mu - x''_\mu \\ x^{13}_\mu &= x'_\mu - x'''_\mu \\ x^{14}_\mu &= x'_\mu - x^{(IV)}_\mu \end{aligned}\right\} \quad \left(\begin{aligned} x^{34}_\mu &= x^{14}_\mu - x^{13}_\mu \\ x^{24}_\mu &= x^{14}_\mu - x^{12}_\mu \\ x^{23}_\mu &= x^{13}_\mu - x^{12}_\mu \end{aligned}\right)$$

Especially ~~fo~~ in the case of localizable field F differs from 0 only for

$$x'_\mu = x''_\mu \qquad x'''_\mu = x^{(IV)}_\mu ,$$

so that F can be considered as a fn of $x^{13}_\mu = x'_\mu - x'''_\mu$ alone

(ii)

Alternatively F can be considered as functions

$$r_\mu = x_\mu^{12} = x'_\mu - x''_\mu$$
$$s_\mu = x_\mu^{34} = x_\mu^{4} - x_\mu^{(III)}$$
$$Z_\mu = X_\mu - Y_\mu = \frac{1}{2}(x'_\mu + x''_\mu) - \frac{1}{2}(x'''_\mu + x_\mu^{(IV)})$$

If we assume the vacuum equations:

$$\frac{\partial F}{\partial X_\mu \partial X^\mu} = 0$$
$$\frac{\partial F}{\partial Y_\mu \partial Y^\mu} = 0$$
$$r_\mu r^\mu F = 0$$
$$s_\mu s^\mu F = 0$$

Thus

$$F = \delta(r_\mu r^\mu)\delta(s_\mu s^\mu)\, f(r_\mu s^\mu, r_\mu Z^\mu, s_\mu Z^\mu)$$

Conditions ii) and iii) can be written

ii)' $\cancel{F(-r_\mu, -s_\mu)}$

$$F(X_\mu, -r_\mu; Y_\mu, -s_\mu) = -\tilde{F}(X_\mu, r_\mu; Y_\mu, s_\mu)$$

iii)' $F(Y_\mu, s_\mu; X_\mu, r_\mu)$

$$= -F(X_\mu, r_\mu; Y_\mu, s_\mu)$$

or if we consider F

$$F \doteq F(Z_\mu, r_\mu, s_\mu),$$

then ii)'' $\tilde{F}(Z_\mu, -r_\mu, -s_\mu)$

$= -\tilde{F}(Z_\mu, r_\mu, s_\mu)$

iii)'' $\tilde{F}(-Z_\mu, s_\mu, r_\mu)$

$= -\tilde{F}(Z_\mu, r_\mu, s_\mu)$

Further, if we consider

iv)'' $\tilde{F}(Z_\mu r^\mu, Z_\mu s^\mu, r_\mu s^\mu)$

which is zero unless ~~with~~ $r_\mu r^\mu = s_\mu s^\mu = 0$,

then

ii)''' $\tilde{F}(-Z_\mu r^\mu, -Z_\mu s^\mu, r_\mu s^\mu)$

$= -\tilde{F}(Z_\mu r^\mu, Z_\mu s^\mu, r_\mu s^\mu)$

iii)''' $\tilde{F}(-Z_\mu s^\mu, -Z_\mu r^\mu, r_\mu s^\mu)$

$= -\tilde{F}(Z_\mu r^\mu, Z_\mu s^\mu, r_\mu s^\mu)$

Moreover, the condition

$$\frac{\partial^2 \tilde{F}}{\partial Z_\mu \partial Z^\mu} = 0$$

is ~~$\tilde{F}$~~ automatically satisfied by the conditions that ~~$\tilde{F}$~~

iv)''' only for $r_\mu r^\mu = s_\mu s^\mu = 0$

$\tilde{F}(Z_\mu r^\mu, Z_\mu s^\mu, r_\mu s^\mu) \neq 0$

Thus $\tilde{F}(Z_\mu r^\mu, Z_\mu s^\mu, r_\mu s^\mu)$

$= \delta(r_\mu r^\mu)\,\delta(s_\mu s^\mu)\, f(Z_\mu r^\mu, Z_\mu s^\mu, r_\mu s^\mu)$

satisfying ii)''', iii)'''.

E. Kanai, Some Remarks on the Non-infinitesimal Commutation Relations
(Prog. Theor. Phys. 2 (1947), 135)

complex scalar field

$$\frac{\partial\psi}{\partial x^4} = \pi \qquad \frac{\partial \pi}{\partial x^4} = (\Delta - \varkappa^2)\psi$$

$$[\psi(\vec{x},t), \psi^*(\vec{x}',t)] = f_1(\vec{x}-\vec{x}')$$

$$[\pi(\vec{x},t), \pi^*(\vec{x}',t)] = f_2(\vec{x}-\vec{x}')$$

$$[\pi(\vec{x},t), \psi^*(\vec{x}',t)] = f_3(\vec{x}-\vec{x}')$$

$$\text{other } [\ \ ] = 0$$

$$f_3^*(-\vec{x}) = -f_3(\vec{x})$$

$$f_2(\vec{x}) = -(\Delta - \varkappa^2) f_1(\vec{x})$$

$$f_1^*(-\vec{x}) = f_1(\vec{x})$$

$$\frac{\partial f_1(\vec{x})}{\partial x^i} + x^i(\Delta - \varkappa^2) f_1(\vec{x}) = 0$$

$$x^i f_3(\vec{x}) = 0$$

(1) $f_1 = f_2 = 0 \qquad f_3 = \delta\text{-fn}$

(2) $f_1 = f(R) \qquad R^2 = x^2 + y^2 + z^2$

$$f = g\int e^{R\zeta + \frac{\varkappa^2}{4\zeta}}\, d\zeta$$

For example: $f = -\frac{\pi\varkappa}{2\sqrt{R}}\, g\, H_1^{(1)}(i\varkappa\sqrt{R})$

$$f(\infty) = \sqrt{\frac{\pi\varkappa}{2}}\, g\, e^{-\varkappa\sqrt{R}}/R^{\frac{3}{4}}$$

$$[\psi(\vec{x}), \psi'(\vec{x}')] = \delta(\vec{x}, \vec{x}')$$

$$[\pi(\vec{x}), \pi'(\vec{x}')] = -(\Delta - \varkappa^2)\,\delta(\vec{x}, \vec{x}')$$

other $[\ \ ] = 0$

$$\left.\begin{aligned} \psi' &= \int \rho(\vec{x}, \vec{x}')\,\psi^*(\vec{x}')\,d\vec{x}' \\ \pi' &= \int \rho(\vec{x}, \vec{x}')\,\pi^*(\vec{x}')\,d\vec{x}' \end{aligned}\right\}$$

$$\bar{H} = i\hbar c \int \rho(\vec{x}', \vec{x}'')\,(\psi^*(\vec{x}')\,\pi(\vec{x}'') - \pi^*(\vec{x}'')\,\psi(\vec{x}'))\,d\vec{x}'\,d\vec{x}''$$

$$\int f_1(\vec{x} - \vec{x}')\,\rho(\vec{x}' - \vec{x}'')\,d\vec{x}' = \delta(\vec{x}, \vec{x}'')$$

$$\bar{H} = \hbar c \sum_{k} \omega_k (A'(\vec{k})\,A(\vec{k}) - B'(\vec{k})\,B(\vec{k}))$$

Fermi ~~stat~~ statistics !!!
Lagrangian ?

Lagrangian scheme for nonlocalizable fields. 1

In classical field theory of scalar field, we start from

$$\delta \int \cdots \int \frac{\partial U}{\partial x_\mu} \frac{\partial U}{\partial x^\mu} dv^{(4)} = 0 \qquad (1)$$

$$\int_V \cdots \int \frac{\partial U}{\partial x_\mu} \frac{\partial U}{\partial x^\mu} dv^{(4)} = \iint_S \int \left(U \frac{\partial U}{\partial x_\mu} \right) ds_\mu^{(3)}$$

$$- \int_V \cdots \int U \frac{\partial^2 U}{\partial x_\mu \partial x^\mu} dv^{(4)}$$

$$\delta \int \cdots \int \frac{\partial U}{\partial x_\mu} \frac{\partial U}{\partial x^\mu} dv^{(4)} = 2 \int \cdots \int \frac{\partial \delta U}{\partial x_\mu} \frac{\partial U}{\partial x^\mu} dv^{(4)}$$

$$= 2 \int\int\int \left(\delta U \frac{\partial U}{\partial x^\mu} \right) ds_\mu^{(3)} - 2 \int \cdots \int \delta U \frac{\partial^2 U}{\partial x_\mu \partial x^\mu} dv^{(4)}$$

Thus, if

$$\lim_{S \to \infty} \iiint_S \delta U \frac{\partial U}{\partial x_\mu} ds_\mu^{(3)} = 0,$$

$$\frac{\partial^2 U}{\partial x_\mu \partial x^\mu} = 0$$

should hold at every point x^μ.

In generalized field theory, ~~of scalar fo~~ we start from

$$\mathrm{Trace}\, [p^\mu, U][p_\mu, U] \cancel{=}$$

and corresponding to (1), we assume

$$\mathrm{Trace}\,[p^\mu, U+\delta U][p_\mu, U+\delta U]$$
$$= \mathrm{Trace}\,[p^\mu, U][p_\mu, U]$$

or

$$\mathrm{Trace}\,[p^\mu, U][p_\mu, \delta U]$$
$$+ \mathrm{Trace}\,[p^\mu, \delta U][p_\mu U] = 0$$

or

$$\mathrm{Trace}\,[p^\mu, U](p_\mu \delta U - \delta U p_\mu)$$
$$+ \mathrm{Trace}\,(p^\mu \delta U - \delta U p^\mu)[p_\mu U] = 0$$

or

$$\mathrm{Trace}\{[[p^\mu, U]p_\mu]\delta U$$
$$+[[p_\mu, U]p^\mu]\delta U\} = 0$$

or $$\underline{[p_\mu[p^\mu, U]] = 0}$$

The same relations hold ~~for~~ in more general case:

$$\mathrm{Trace}\,[\xi^\mu U][\xi_\mu U] = \text{stationary}.$$

~~and~~ From

$$\left.\begin{aligned}&\mathrm{Trace}\,[p^\mu, U][p_\mu U] = \text{stationary}\\ &\mathrm{Trace}\,[x^\mu, U][x_\mu U] = \text{stationary}\end{aligned}\right\},$$

it follows that for any new set of canonical variables

$$\left.\begin{aligned}&\mathrm{Trace}\,[p'^\mu, U'][p'_\mu U'] = \text{stationary}\\ &\mathrm{Trace}\,[x'^\mu, U'][x'_\mu, U'] = \text{stationary}\end{aligned}\right\}$$

because the Trace is invariant with respect to any canonical transformation.

In matrix representation

$$\mathrm{Trace}\,[p^\mu, U][p_\mu, U]$$

$$= \{p'^\mu (p'^\mu|U|p''^\mu) - (p'^\mu|U|p''^\mu)\, p''^\mu\}$$

$$\times \{p''_\mu (p''^\mu|U|\cancel{p'''^\mu}\,p'^\mu) - (\cancel{p''^\mu}|U|\cancel{p'^\mu})\, \cancel{p'''_\mu}\,p'_\mu\}$$

$$= -(p'^\mu - p''^\mu)(p'_\mu - p''_\mu)(p'^\mu|U|p''^\mu)(p''^\mu|U|p'^\mu)$$

According to Born's reciprocity arguments(1) Lagrangian should be invariant under ~~the~~ linear reciprocal transformation(2)

$$\cancel{p'_\mu =}$$

$$\left.\begin{aligned} x'_\mu &= a_{\mu\nu} x_\nu + b_{\mu\nu} p_\nu \\ p'_\mu &= c_{\mu\nu} x_\nu + d_{\mu\nu} p_\nu \end{aligned}\right\}$$

which ~~leaves~~ satisfies

$$x'_\mu x'^\mu + p'_\mu p'^\mu = x_\mu x^\mu + p_\mu p^\mu$$

This can alternative considered as a Lorentz transformation in complex Minkowski space with the coordinates

$$\xi_\mu = \frac{1}{\sqrt{2}}\{x_\mu + i\, p_\mu\} \qquad \tilde{\xi}_\mu = \frac{1}{\sqrt{2}}\{x_\mu - i\, p_\mu\},$$

(1) Born,

(2) We use universal units $\hbar$, c and μ where μ is a ~~mass~~ universal mass probably between m_e and M.

which leaves
$$\tilde{\xi}_\mu \xi^\mu = \xi^\mu \tilde{\xi}_\mu - 4$$
invariant.

The action function in classical ~~fi~~ field theory will thus be replaced by
const. Trace$\{[p^\mu U][p_\mu U] + [x^\mu U][x_\mu U]\}$,
or, if we assume that U ~~is~~ has the form
$$U = u(\xi) + \tilde{u}(\tilde{\xi}) \quad (*)$$
$$S = \mathrm{Trace}\{[\tilde{\xi}^\mu, u(\xi)][\tilde{\xi}_\mu, u(\xi)] + [\xi^\mu, \tilde{u}(\tilde{\xi})][\xi_\mu, \tilde{u}(\tilde{\xi})]\}$$
corresponds to classical action fn.,
because we get field equations
$$\left.\begin{aligned} &[\tilde{\xi}^\mu[\tilde{\xi}_\mu u(\xi)]] = 0 \\ &[\xi^\mu[\xi_\mu u(\tilde{\xi})]] = 0 \end{aligned}\right\}$$
immediately as condition of stationarity of S with respect to arbitrary variation of u and $\tilde{u}$.

(*) For general U, field equations cannot be deduced by variation of a single action function.

Now, as long as we consider one field U, commutation relations and the special assumption such as

$$U = u(\xi) + \tilde{u}(\tilde{\xi})$$

is sufficient to determine statistical relations between ~~of~~ various quantities ~~connected with~~ by postulating that

[I] number of particles ~~in~~ with definite value of k_μ ~~in each~~ never changes. In other words, the probability amplitude

$$\langle \cdots n'(k_\mu) \cdots | \cdots n''(k_\mu) \cdots \rangle = e^{i\gamma(\cdots n', \cdots n'' \cdots)} \delta(\cdots n' \cdots | \cdots n'' \cdots),$$

where γ is a real number depending on $\cdots n' \cdots \cdots n'' \cdots$. Or ~~more specially~~ we can start from more special assumption

$$\langle \cdots n'(k_\mu) \cdots | \cdots n''(k_\mu) \cdots \rangle = \langle \cdots n'(k_\mu) \cdots | 1 | \cdots n''(k_\mu) \cdots \rangle .$$

Next, when there ~~are~~ two or more mutually interacting fields, we postulate that

[II] the corresponding probability amplitude is given by

$$\langle \cdots n'(k_\mu) \cdots | \cdots n''(k_\mu) \cdots \rangle = \langle \cdots n' \cdots | W | \cdots n'' \cdots \rangle$$

where

$$W = e^{iS/\hbar}$$

$$S = \mathrm{Trace}\, L$$

L, curvein term, corresponds to Lagrangian density in usual field theory.
In the case of scalar field consider above ~~技~~ L has the form

$$L = [p^\mu U][p_\mu U] + [x^\mu U][x_\mu U]$$
$$+ \ldots\ldots$$
$$+ L_I$$

where L_I is the interaction term of ~~other~~

Interacting Nonlocalizable Fields

The characteristic of a free field is, in general, that the ~~number of~~ corresponding free particles have a definite value of energy-momentum, (and also mass) ~~and~~ so that the ~~number of~~ distribution of particles in ~~pro~~ energy-momentum-mass space does not change.

Interaction between fields is something which changes this distribution, or more generally speaking, something which changes the probability of possible distribution of number of particles.

In other words, any one distribution is never mixed with other distribution as long as there is no interaction. The interaction gives rise to mixing. ~~and~~ the whole information about the result of mixing can be expressed by mixing matrix

$$\langle \cdots n'(k_n) \cdots | \cdots n''(k_n) \cdots \rangle$$

This mixing matrix is equal to

$$\langle \cdots n'(k_n) \cdots | 1 | \cdots n''(k_n) \cdots \rangle$$

as long as there is no interaction between fields.

We postulate that, when there is interaction the mixing matrix, ~~it~~ takes the form

$$\langle \cdots n' \cdots | W | \cdots n'' \cdots \rangle$$

where W is an unitary operator of the form

$$W = e^{iS/\hbar}$$

with

$$S = \mathrm{Trace}\ L.$$

L, in turn, corresponds to Lagrangian density in usual field theory.

$$W = \exp(iS/\hbar)$$

$$S = \mathrm{Trace}\, L$$

$$L = \cancel{[\xi^\mu[\xi_\mu U]] + [\xi^{*\mu}[\xi^*_\mu U]]}$$

$$[\xi^\mu U][\xi_\mu U] + [\xi^{*\mu} U][\xi^*_\mu U]$$

$$U = \sum_{k_\mu} b(k_\mu) \exp(i k_\mu \xi^\mu) + \sum_{k_\mu} b^*(k_\mu) \exp(i k_\mu \xi^{*\mu})$$

$$L \propto \sum_{k'_\mu k''_\mu} b^*(k'_\mu)\, b^*(k''_\mu)\, k'_\mu k''_\mu \exp i(k'_\mu + k''_\mu)\xi^{*\mu} + \ldots$$

$$\mathrm{Trace}\, L \propto$$

$$\mathrm{Trace}\, L = \mathrm{Tr}[\xi^\mu U][\xi_\mu U]$$

$$= \sum_{x_\mu r_\mu} \Big[\sum_{k_\mu (k_0 > 0)} \Big\{ (2k_0)^{-\frac{1}{2}} \Big(b(k_\mu)\, \delta(r_\mu + \lambda k_\mu) \left\{ k^\mu \right) \exp(i k_\mu x^\mu / \lambda) + b^*(k_\mu)\, \delta(r_\mu - \lambda k_\mu) \left\{ -k^\mu \right) \exp(-i k_\mu x^\mu / \lambda) \Big\} \Big]$$

$$\times \Big[\sum_{k'_\mu (k'_0 > 0)} \Big\{ (2k'_0)^{-\frac{1}{2}} \Big\{ b(k'_\mu)\, \delta(-r_\mu + \lambda k'_\mu)\, k'_\mu \exp(i k'_\mu x^\mu / \lambda) + b^*(k'_\mu)\, \delta(-r_\mu - \lambda k_\mu) \left\{ -k'_\mu \right\} \exp(-i k'_\mu x^\mu / \lambda) \Big\} \Big\} \Big]$$

$$\propto \sum_{r_\mu} \sum_{k_\mu (k_0 > 0)} (2k_0)^{-1} \left\{ b(k_\mu)\, b^*(+k_\mu) + b^*(k_\mu)\, b(k_\mu) \right\} k^\mu k_\mu$$

$$= 0.$$

March 2, 1949

Integral Formalism for nonlocalizable Field

$$W = \exp \frac{i}{\hbar} \text{const. Trace } L.$$

free field

$$L = \frac{\lambda^2}{\hbar^2}[p^\mu U][p_\mu U] + \frac{1}{\lambda^2}[x^\mu U][x_\mu U]$$

$$\text{Trace } L = \overbrace{\left\{-\frac{\lambda^2}{\hbar^2} U[p^\mu[p_\mu U]] - \frac{1}{\lambda^2} U[x^\mu[x_\mu U]]\right\}}^{\text{Trace}}$$

$$= 0,$$

interaction: two scalar fields

$$L = \frac{\lambda^2}{\hbar^2}[p^\mu U][p_\mu U] + \frac{1}{\lambda^2}[x^\mu U][x_\mu U]$$

$$+ \frac{\lambda^2}{\hbar^2}[p^\mu V][p_\mu V] + \frac{1}{\lambda^2}[x^\mu U][x_\mu U]$$

$$+ gUVU$$

$$\text{(I)} \quad \frac{\lambda^2}{\hbar^2}[p^\mu[p_\mu U]] + \frac{1}{\lambda^2}[x^\mu[x_\mu U]] = gVU$$

If U ~~is a s~~ satisfies (I), ~~to~~
Trace $L = 0$.

If U satisfies instead of (I), free field equations

$$\text{Trace } L \overset{\text{Trace}}{=} gUVU.$$

2

$$V = \sum \cdot\cdot \exp i k_\mu x^\mu/\lambda \cdot \exp i k_\mu p^\mu \lambda/\hbar$$

$$U = \sum \cdot\cdot \exp i K_\mu x^\mu/\lambda$$

$$UVU = \sum\sum\sum \cdot\cdot \exp i K'_\mu x^\mu/\lambda \exp i k_\mu x^\mu/\lambda$$
$$\times \exp i k_\mu p^\mu \lambda/\hbar \exp i K''_\mu x^\mu/\lambda$$

$$(x' | UVU | x'')$$

$$= \sum\sum\sum \cdot\cdot \exp i K'_\mu x'^\mu/\lambda \exp i k_\mu x'^\mu/\lambda$$
$$\times \delta(x' - x'' - k\lambda) \exp i K''_\mu x''^\mu/\lambda$$

$$= \sum \exp i(K'_\mu + K''_\mu + k_\mu) x'^\mu/\lambda$$
$$e^{-i k K''} \delta(x' - x'' - k\lambda)$$

$=$

$$(p' | e^{k_\mu p^\mu \lambda/\hbar} | p'') f(p'')$$

$$= \delta(p', p'') e^{k_\mu p''^\mu \lambda/\hbar} f(p'')$$

$$f(p'') = e^{i k_\mu p''^\mu \lambda/\hbar}$$

$$= \delta(p', p'') e^{i(k_\mu - i k_\mu) p''^\mu \lambda/\hbar}$$

$$g(x'') = \int e^{i x''_\mu p''^\mu/\hbar} f(p'') \propto \delta(x'')$$

$$f(p'') = 1,$$

$$g(x'') = \int_{-\infty}^{\infty} e^{i(x''_\mu - i k_\mu \lambda) p''^\mu/\hbar} dp''$$

$k_\mu \lessgtr 0$.

$$= \int_0^\infty e^{i(x''_\mu - i k_\mu \lambda) p''^\mu/\hbar} dp''$$

$$\int_0^\infty e^{(ix-\varepsilon)y} dy = -\frac{1}{ix-\varepsilon}$$

$$\int_{-\infty}^{0} e^{(ix+\varepsilon)y} dy = +\frac{1}{ix+\varepsilon}$$

Feynman's procedure of introducing
ε !!!

Pauli's letter;
Feynman, P.R, 74 (1948), 1430,

Schönberg, P.R. 74 (1948), 738
(Electrodynamics)
748 (meson theory)
Freezing ? universal length
cut-off

General Formalism ~~Hints to Possible Form~~ of N.L.F.T. March 9.

Recent attempts by S.[1] T.[2] ~~and~~ to construct ~~to~~ develope quantum electrodynamics in perfectly relativistic manner arrived at ~~the~~ a remarkable result that all the divergences in q.e.d. can be reduced to ~~a~~ two infinities ~~coeff~~ in the mass and charge renormalizations. Very recently Pauli[3] pointed out that these two divergences and the ambiguity in the ~~calc~~ determination of photon self-energy can be consistently removed by using ~~a~~ "regulator" $\rho(x)$, which satisfies the conditions

$$\int_{-\infty}^{+\infty} \rho(x)\,dx = \int_{-\infty}^{+\infty} x\,\rho(x)\,dx = 0,$$

where x represents the ~~mass~~ square of the mass (of some imaginary field quanta coexisting with the ordinary photons). This is actually a generalization of ~~considerations~~ assumptions by Feynman[4] and Stueckelberg and Rivier[5].

(5) S. and R., P.R. 74 (1948), 218; 986.

(1) T.

(2) S.

(3) Letter to Schwinger. dated Jan. 24, 1949

(4) F. P.R. 74, 1430

Now, if we want to ~~get for~~ construct a model of interacting fields corresponding to ~~these~~ any assumptions, we suffer always from the appearence of negative energy quanta.(6) In order to get rid of this serious difficulty, one has to ~~assume~~ ~~consider~~ either

i) to change the standpoint and take advantage of the action at distance theory as attempted by Feynman(7), ~~or~~ or

ii) to ~~change~~ generalize the field concept so as to ~~cover~~ unite ~~represent~~ ~~the mixture in the fr the~~ things which look like mixture of ~~field~~ quanta of different masses into one field.(8)

~~In the present~~ According to the opinion of the author, the latter alternative seems to be ~~far~~ more ~~to~~ promising, because ~~if~~ we can ~~want to~~ include the ~~f~~ meson field in the scheme, it ~~seem~~ will be easier to.

(7) F.

(8) Y.

(6) ~~In order to make~~ For the purpose of making self-energies of the electron and the photon finite, ~~we~~ one needs only to assume the coexistence of positive energy neutral and charged scalar fields as shown by Pais, Sakata and others. This is not the case, however, for charge renormalization.

~~However, the latter alternative~~ Among various possibilities, probably the most comprehensive one is to extend the usual field to the nonlocalizable field. However, in nonlocalizable field theory, we can no more stick to the usual formulation of q. m. which ~~starts~~ starts from the Schrödinger differential equation for ~~the wave~~ probability amplitude or any substitute for it, which serves to determine the change of prob. amp. during an infinitesimal time interval. On the other hand, ~~An~~ an alternative formalism, so to speak integral formalism suggested by Dirac[9] and developed by Feynman[10], ~~will be the more convenient starting~~ can be extended ~~to as~~ so as to cover the non-local systems.

The starting point is to ~~define~~ find the substitute for the probability amplitude between measurements with respect to a ~~to~~ system of nonlocalizable fields interacting with each other. ~~According to~~

(9) Dirac
(10) Feynman

~~Dirac and Feynman~~,
Now for a particle with the coordinate q, the probability amplitude

$$(q_2, t_2 / q_1, t_1)$$

is the solution of Schröd. eq.
~~which is the Schröd. eq.~~

$$i\hbar \frac{\partial}{\partial t_2}(q_2, t_2 / q_1, t_1) = H^{(t_2)}(q_2, t_2 / q_1, t_1)$$

with the initial condition

$$(q_2, t_1 / q_1, t_1) = \delta(q_2, q_1)$$

Thus

$$(q_2, t_1 + dt / q_1, t_1)$$

$$= e^{-\frac{iH(t_1)dt}{\hbar}} \delta(q_2, q_1).$$

Or we may write

$$\cancel{L(2,1)} = L(q_2 t_2) = E_2 - H(q_2 t_2)$$

$$\cancel{L(q_2, t_2; q_1, t_1)}$$

$$L(p_2, E_2, q_2, t_2) \int (q_2\, t_2 / q_1, t_1) = 0.$$

$$\cancel{p_1, E_1, q_1, t_1)}$$

or for very small $t_2 - t_1 = dt$

$$\lim_{dt \to 0} \cancel{e^{i\hbar L(p_2 E_2, q_2 t_2 + dt)} \delta(q_2\, t_1 + dt / q_1, t_1)} = \lim_{dt\to 0} (q_2\, t_1^{+dt} / q_1 t_1)$$

$$= \int \delta(q_2\, t_2 ; q_1\, t_1)\, dt_2$$

$$\left(1 \cancel{\mp} + \frac{i L\, dt}{\hbar}\right)_{q_2 t_2} (q_2 t_2/q_1 t_1) = (q_2 t_2/q_1 t_1)$$

$$\cancel{= \delta(q_2, q_1)}$$

$$\sum_{q_2} (q_3| e^{+i\int L dt/\hbar} |q_2)(q_2 t_2/q_1 t_1)$$

$$= (q_3 t_1/q_1 t_1)$$

$$= \delta(q_3, q_1)$$

or

$$(q_2| e^{i\int_{t_1}^{t_2} L dt/\hbar} |q_1)$$

$$= \delta(q_2, q_1)$$

apart from the ~~although there are~~ ambiguities as to the definition of exp., which will be discussed later in detail.

Thus for the ~~free~~ particle (or moving in any external ~~force~~ potential field), ~~the probability amplitude between two set of similar observables at t.~~ satisfying the wave eq.

$$L \psi = 0.$$

$e^{i\int L dt/\hbar}$ is a unit matrix.

This is quite trivial. But ~~we can say more generally, if there is a... operator~~

~~L~~

Now, if we divide H into two parts

$$H = H_0 + H'$$

and ~~q~~ the ~~observable~~ time dependence of $q(t)$ is ~~so~~ defined, say new $Q(t)$

such that,

$$\cancel{i\hbar \frac{\partial}{\partial t}(q\,t/q't') = H(q\,t/q't')},$$

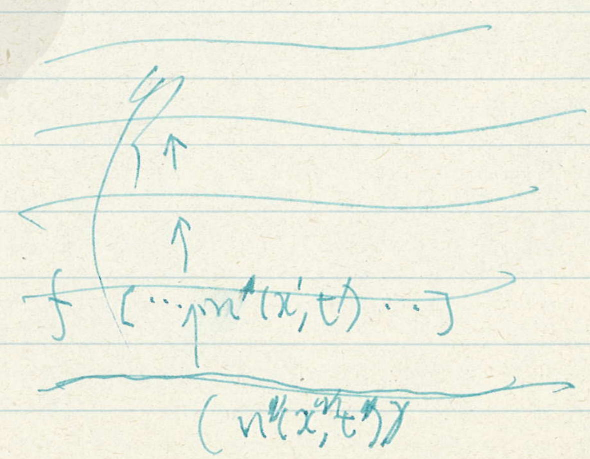
or

$$i\hbar \frac{\partial Q(t)}{\partial t} = [Q(t), H_0(t)]$$

$$\cancel{i\hbar \frac{\partial}{\partial t}(q,t/q',t')}$$

$$i\hbar \frac{\partial}{\partial t}(Q,t/Q't')$$

$$= \{[Q, H_0] + i\hbar \frac{\partial}{\partial t}\}(Q,t/Q't')$$



$$\cancel{\text{抜}}\, f'\{n'(x't')\} = \{n'(x't')|\{n(x,t)\}\, f\{n(x,t)\}$$

~~free~~ $(n'(x't')|n(xt))$ can be considered as a representation of an ~~matrix~~ operator, ~~whose diagon~~ If we represent this operator with other set of observables $n(k,t)$ ~~dia~~ as diagonal matrices,

$$f'[n'(k't')] = (n'(k't')|n(k,t)) \times f[n(k,t)].$$

Further that ~~n(k~~ the distribution of $n(k,t)$, or the functional form of f', does not change with time,

$$(n'(k't')|n(k,t))$$

is a unit matrix ~~with respect to the arguments~~ $n'(k')$, $n(k)$ irrespective of t and t'. If we go back to $n(x,t)$ and $n'(x',t')$, ~~in~~ ~~in general~~

$$(n'(x't')|n(xt))$$

may not be a unit matrix, because the transformation functions

$$(n'(x't')/n'(k't')) \text{ and } (n(xt)/n(kt))$$

may be different from each other in general. ~~But if they have the same form, the operator represented by these matrices are~~ ~~is~~ ~~in itself a unit operator.~~

~~This is certainly the case, if we can chose the for free field~~

For example, for ~~an~~ assembly of similar particle without mutual interaction, the distribution ~~(...)~~ with respect to $n(k)$, ~~is certainly~~ where k ~~so~~ denote the momentum and energy of the particle, does not change. However, the distribution in space will change.

In nonlocalizable field theory, we consider x_μ, p_μ as operators. In this case ~~so that~~, if we ~~find~~ can find a set of parameters k, which ~~is characteristic to~~ characterizes the mutually independ solutions (operators) of the free field equations, we can expect that the distribution of particles ~~with~~ respect to $n(k)$ does not change, so that

$$(\ldots n'(k')\ldots | \ldots n(k)\ldots) \qquad (1)$$

is still a unit matrix.

This can be ~~...~~ a representation of W with respect $n(k)$, where

$$W = \exp iS/\hbar$$

$$S ~~\alpha~~ \text{Trace } L,$$

provided that for free fields which satisfy free field equations,

$$\mathrm{Trace}\ L = 0 \qquad (2)$$

and k is a set of parameters which characterizes the mutually independent solutions of the free field equations.

If there is an interaction between fields, free fields ~~equations~~ will not make Trace L zero, so that (1) will not be a unit matrix, (1) is in general a non-diagonal matrix.

In order to go back to space-time representation, we have to ~~use representation of field quantities~~ confine our attention to certain regions of values of k, for which the effect of nonlocalizability is negligible.

Interaction of Electron and Electromagnetic Field in nonlocalizable field theory

We start from the assumption that the electron field is localizable as usual satisfying the ordinary Dirac equation

$$i\hbar\,\gamma_\mu \frac{\partial\psi}{\partial x_\mu} + mc\psi = 0, \qquad (1)$$

whereas the electromagnetic field is nonlocalizable satisfying commutation relations

$$\left.\begin{aligned} [p^\mu [p_\mu A_\nu]] &= 0 \\ [x^\mu [x_\mu A_\nu]] &= 0 \end{aligned}\right\} \qquad (2)$$

with the Lorentz condition

$$[p^\mu A_\mu] = 0 \qquad (3)$$

and its reciprocal condition

$$[x^\mu A_\mu] = 0 \qquad (4)$$

If there is no interaction between two fields, Lagrangian density operator takes the form

$$L = i\hbar c\,\psi^\dagger \gamma_\mu \frac{\partial\psi}{\partial x_\mu} + \psi^\dagger mc^2\psi + \frac{1/\kappa^2}{8\pi\hbar}[p^\mu A^\nu][p_\mu A_\nu] + \frac{1}{8\pi\lambda^4}[x^\mu A^\nu][x_\mu A_\nu] \qquad (5)$$

Then

$$\mathrm{Trace}\ L = 0$$

for ψ, ψ^{T}, $A_{\mu\nu}$ satisfying (1) ~ (4)

Now ~~most~~ probably the general ~~solution~~ form ~~for~~ for $A_{\mu\nu}$ satisfying (2), (3), (4) is ~~of the form~~

$$A_{\nu} = \sum_{k_\mu, l_\mu} a_\nu(k_\mu, l_\mu) e^{ik_\mu x^\mu} e^{il_\mu p^\mu}$$

$$+ \sum_{k_\mu, l_\mu} b_\nu(k_\mu, l_\mu) e^{il_\mu p^\mu} e^{ik_\mu x^\mu}$$

(according to Nambu)

with the restriction

$$\left.\begin{array}{l} k_\mu k^\mu = 0 \\ l_\mu l^\mu = 0 \\ a_\mu k^\mu = 0 \\ a_\mu l^\mu = 0 \end{array}\right\}$$

We assume further that $a_\nu(k_\mu, l_\mu)$ is different from 0, only when k_μ is proportional to l_μ, i.e. we assume

$$l_\mu = \pm \frac{\lambda^2 k_\mu}{\hbar}, \quad a_\nu\left(k_\mu, \pm\frac{\lambda^2 k_\mu}{\hbar}\right) = a_\nu(k_\mu, \varepsilon k_\mu)^{*} \ \cancel{= a_\nu(k_\mu, \pm)}$$

In order that ~~A_ν is a Hermitian operator~~

$$A_\nu = \sum_{\substack{k_\mu, \pm \\ \varepsilon^2 = \pm \lambda^2/\hbar}} a_\nu(k_\mu, \varepsilon) e^{ik_\mu x^\mu} e^{i\varepsilon k_\mu p^\mu}$$

* Then two terms in A_ν become identical with ~~eq~~ each other, because $k_\mu p^\mu$ and $k_\mu x^\mu$ commute.

is a Hermitian operator, i.e. in order that

$$A_\nu^* = \sum a_\nu^*(k_\mu \varepsilon)\, e^{-ik_\mu x^\mu} e^{-i\varepsilon k_\mu p^\mu}$$

is equal to A_ν, it is necessary that

$$a_\nu(k_\mu, \varepsilon) = a_\nu^*(-k_\mu, \varepsilon)$$

Thus

$$A_\nu = \sum_{k_\mu, \varepsilon, (k_0 > 0)} \Big(a_\nu(k_\mu, \varepsilon)\, e^{ik_\mu x^\mu} e^{i\varepsilon k_\mu p^\mu} + a_\nu^*(k_\mu, \varepsilon)\, e^{-ik_\mu x^\mu} e^{-i\varepsilon k_\mu p^\mu} \Big)$$

and ~~A_ν is Herm~~ two parts of A_ν corresponding to $\varepsilon = +\lambda^2 \cancel{k_0}/\hbar$ and $\varepsilon = -\lambda^2 \cancel{k_0}/\hbar$ is separately Hermitian.

A slightly more general assumption is ~~thus~~ possible: Instead of $\sum_\varepsilon a_\nu(k_\mu \varepsilon)\, e^{\cdots} e^{\cdots}$, we can take

$$\int_{-\infty}^{+\infty} a_\nu(k_\mu, \varepsilon)\, e^{ik_\mu x^\mu} e^{i\varepsilon k_\mu p^\mu}\, d\varepsilon$$

Another way of generalization is to consider localizable field in five dimensional space. For instance, we can assume

$$A_\nu = \sum_{k_\mu} a_\nu(k_\mu, \cancel{k_5 \varepsilon})\, e^{ik_\mu x^\mu},$$

where $\mu = 1, 2, 3, 4, 5$. However this a separate subject, and will be considered in other place.

now

$$(x'|\, e^{ik_{,\mu}x^\mu}\, e^{i\varepsilon k^\mu p_\mu} \,|x'')$$

$$= e^{ik_{,\mu}x^{\mu\prime}}\,\delta(x',x'')\cdot\delta(x^{\mu\prime}-x^{\mu\prime\prime}+\varepsilon k^\mu)$$

so that this is an operator connecting a quantity on the right at $x^{\mu\prime}+\varepsilon k^\mu$ with another quantity on the left at $x^{\mu\prime}$. Especially, if $\varepsilon k^0>0$, (or $\varepsilon k_0<0$), it connects the quantity on the right with another quantity on the left at earlier time, while, if $\varepsilon k^0<0$ (or $\varepsilon k_0>0$), it connects ~~that~~ one on the right with another on the left at later time.

The field A_ν can thus be divided and into two parts, one with $\varepsilon k^0>0$, the other with $\varepsilon k^0<0$. The former gives the time order from left to right, while the latter gives the time order from right to left.

Now, if we go over to the case of interacting fields, it is tempting to substitute

$$\exp \frac{i}{\hbar}\int L\,dt \qquad (1)$$

in q.m. of particles by

$$\exp \frac{i}{\hbar}\ \cancel{\mathrm{Trace}}\ S$$

$$S = \mathrm{const.}\ \mathrm{Trace}\ L$$

where L is the Lagrangian ~~density~~ density operator in nonlocalizable field theory.

But, ~~things~~ is not so promising in this single form, because for the interaction between Dirac el. and el.mg. field, as for example the interaction term ~~has~~ gives the form

$$L_I = -e\,\psi^\dagger A_\mu \gamma^\mu \psi$$

gives

$$\mathrm{Trace}\ L_I = 0,$$

~~As~~. This corresponds to the fact that there is no ~~real~~ first order process satisfying conservation laws. Instead of (1), we can take in local field theory,

$$\prod_j \exp \frac{i}{\hbar}\ \frac{\mathrm{Trace}}{\cancel{\mathrm{const.}}}(L\,\rho_j) \qquad (1)'$$

where ρ_j ~~are~~ is the operator characterising the j-th ~~space~~ small space-time region, ~~if~~ i.e. we take ρ_j diagonal

diagonal

in space-time representation with non-zero
elements only in j-th space-time region.
In nonlocalizable field theory,
we have to ~~take~~ ρ_j assume ρ_j
nondiagonal, the off diagonal distance
extending to $\lambda \sim 1/K$, where K is the
maximum wave number considered.
Thus we are now in a very ~~[illegible]~~
strange situation that, if we define
the ~~space time~~ boundary of space-time
region

NATIONAL

Tumbler

No. T5882E—11 x 8½—Theme No. 2

TURNS QUICKLY　LIES FLAT

Preliminary Works
prior to Nonlocalizable
Field Theory

Name H. Yukawa, Princeton

Subject Mainly Nonlocalizable Field Theory. I.

MADE IN U. S. A.

On the Radius of the Elementary Particle

May 28, 1949 (I)

zero mass:

$$\left.\begin{array}{l}[x^\mu, p_\nu] = i\hbar\delta_{\mu\nu} \\ [x_\mu[x^\mu, U]] = 0 \\ [p_\mu[p^\mu, U]] = 0\end{array}\right\} \quad (\mathrm{I})$$

with mass m $\quad \kappa = \frac{mc}{\hbar}$ *

and radius λ

$$\left.\begin{array}{l}[x^\mu, p_\nu] = i\hbar\delta_{\mu\nu} \\ [x_\mu[x^\mu, U]] = \lambda^2 U \\ [p_\mu[p^\mu, U]] = -m^2c^2 U\end{array}\right\} \quad (\mathrm{II})$$

$$(x'_\mu|U|x''_\mu) = U(X_\mu, r_\mu) \qquad \left.\begin{array}{l}X_\mu = \frac{1}{2}(x'_\mu + x''_\mu) \\ r_\mu = x'_\mu - x''_\mu\end{array}\right\}$$

~~$x_\mu x^\mu$.~~

$$\left.\begin{array}{l}(r_\mu r^\mu - \lambda^2)U = 0 \\ \left(\frac{\partial^2 U}{\partial X_\mu \partial X^\mu} - \kappa^2\right)U = 0\end{array}\right\} \quad (\mathrm{II})'$$

$$U = \sum_{k_\mu} u(r_\mu, k_\mu) e^{ik_\mu X^\mu}$$

with $k_\mu k^\mu + \kappa^2 = 0$.

$$u(r_\mu, k_\mu) \left(= \delta(r_\mu r^\mu) f_{\pm}(r_\mu, k_\mu)\right)$$

$$= \sum_{l_\mu} b(k_\mu) \cdot \delta(r_\mu + l_\mu)$$

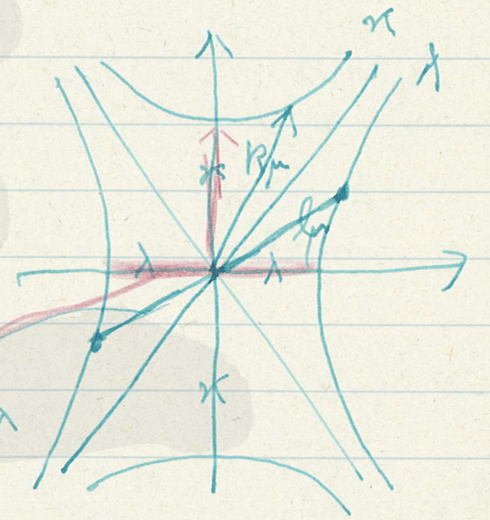
with $\begin{cases} l_\mu l^\mu = 0 \\ k_\mu l^\mu = 0 \end{cases}$

* Pais,

$$w(X_\mu, r_\mu) = \iint \delta(k_\mu k^\mu + \varkappa^2)\,\delta(k_\mu l^\mu)\,\delta(l_\mu l^\mu - \lambda^2)$$
$$\times f(k_\mu)\,\delta(r_\mu + l_\mu)\, e^{ik_\mu X^\mu} (dk)^4 (dl)^4 . \qquad (\mathrm{III})$$

The sphere with radius λ with the centre at X_μ:

$$r_\mu r^\mu = \lambda^2$$
$$r_0 = 0;\ r_1^2 + r_2^2 + r_3^2 = \lambda^2$$



This corresponds to the (density) ~~charge~~ distribution charge of the particle at rest. When it is moving with the velocity corresponding to k_μ $\left(v = \frac{\sqrt{k_1^2 + k_2^2 + k_3^2}}{k_0}\right)$ the charge ~~distribution~~ is distributed on the ~~sphere~~ three dimensional closed surface, which can be obtained ~~by~~, from the above sphere by Lorentz transformation (Transformation into a new reference system ~~moving with the particle~~ in which the particle is at rest)

Thus we find a very natural and simple model ~~for the rigid~~ class for the elementary particle with mass and radius, which just correspond to the rigid spherical model of Lorentz in the classical electron theory. It is surprising that the nonlocalizable field quantity describing the model has a particular simple and invariant form.

the field operator (III) can be written in terms of x, p-operators very simply:

$$U(x_\mu, p_\mu, b(k_\mu))$$

$$= \iint \delta(k_\mu k^\mu + \varkappa^2)\, \delta(l_\mu l^\mu - \lambda^2)\, \delta(k_\mu l^\mu)$$

$$\times\, b(k_\mu)\, e^{i l^\mu p_\mu} \cdot e^{i k_\mu x^\mu}\, (dk)^4 (dl)^4$$

In contrast to the case of zero mass, there is no such simple combination as

$$\eta_\mu = \frac{x_\mu}{\lambda} + \lambda p_\mu .$$

~~However~~ However, it should be noticed that k_μ and l_μ are always mutually reciprocal, just as the reciprocal lattice in three dimension. If we want to express the whole formulation in five dimensional space, $(x_1, x_2, x_3, x_4, x_5)$ and $(p_1, p_2, p_3, p_4, p_5)$ are mutually reciprocal de Sitter space with the line elements

$$ds^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - (dx_4)^2 + (dx_5)^2$$

$$dq^2 = (dp_1)^2 + (dp_2)^2 + (dp_3)^2 - (dp_4)^2 + (dp_5)^2$$

respectively. Thus the radius and the mass of an elementary particle can be considered as the eigenvalues of ~~mu~~ mutually conjugate operators x_5 and p_5.

spinor particle:

$$\left.\begin{array}{l}\gamma^\mu\{p_\mu,\psi\}+mc\psi=0\\ \gamma'_\mu[x^\mu,\psi]+\lambda\psi=0\end{array}\right\}$$

$$\left.\begin{array}{l}\gamma^1=\beta\alpha_x=i\rho_2\sigma_x\\ \gamma^2=\beta\alpha_y=i\rho_2\sigma_y\\ \gamma^3=\beta\alpha_z=i\rho_2\sigma_z\\ \gamma^4=\beta=\rho_3\end{array}\right\}$$

$$\left.\begin{array}{l}\gamma'_1=i\rho_2\alpha_x=\rho_3\sigma_x\\ \gamma'_2=i\rho_2\alpha_y=\rho_3\sigma_y\\ \gamma'_3=i\rho_2\alpha_z=\rho_3\sigma_z\end{array}\right\}$$

$$\gamma'_4=-i\rho_2$$

$(\gamma^\mu\gamma'_\nu=\gamma'_\nu\gamma^\mu$ for $\mu\neq\nu$, $\gamma^\mu\gamma'_\nu=-\gamma'_\nu\gamma^\mu$ for $\mu=\nu)$

$$=-\rho_1$$

$$\left.\begin{array}{l}k_\mu k^\mu+\varkappa^2=0\\ l_\mu l^\mu-\lambda^2=0\end{array}\right\}*$$

condition of compatibility
$$k_\mu l^\mu=0$$

$*\ \psi\propto u\,\delta(k_\mu+l_\mu)\,e^{ik_\mu x^\mu}$

$$(\gamma^\mu k_\mu+\varkappa)\,u=0$$
$$(\gamma'_\mu l^\mu-\lambda)\,u=0$$

$$(\gamma'_\mu l^\mu-\lambda)(\gamma^\mu k_\mu+\varkappa)u=(\gamma^\mu k_\mu+\varkappa)(\gamma'_\mu l^\mu-\lambda)u$$
$$k_\mu l^\mu=0$$

Pais　　Private Conversation　　Feb. 28

$$U = \sum_{k_\mu} \left(u^+(k_\mu) e^{i k_\mu x^\mu} + u^-(k_\mu) e^{-i k_\mu x^\mu} \right)$$

$$\Delta\varphi \, \Delta x = \lambda^2 k_\mu$$

$$[x_\mu, u^\pm(k_\mu)] = i\lambda^2 k_\mu u^\pm(k_\mu)$$

$$[x_\mu, U] = i\lambda^2 \cancel{k_\mu U} \, U'$$

$$i\lambda^2 [x_\mu, U'] = +\lambda^4 \cancel{k} \kappa^2 U$$

March 14
1949

Nonlocalizable Field for particle
with rest mass

Previously we assumed that the nonlocalizable field for ~~the~~ particle with rest mass ~~was~~ could be described by ~~an~~ introducing fifth dimension.
Another way, which makes it unnecessary to introduce fifth dimension, is to ~~consider~~ apply reciprocity principle ~~as~~ in four dimensional ~~phase~~ space follows;
The commutation relations between the scalar field U and the momentum-energy operator ~~p~~ p^μ is assume to be

$$[p^\mu[p_\mu U]] = -m^2c^2 U \qquad (1)$$

as usual, ~~but~~ that ~~fo~~ between U and the space-time coordinates is not assume to be

$$[x^\mu[x_\mu U]] = \lambda^2 U \qquad (2)$$

instead of

$$[x^\mu[x_\mu U]] = -\lambda^2 U, \qquad (2)'$$

Whereas (2)' ~~which is fo~~ is the direct counterpart of (1) in five dimensional formulation,[(1)] (2) can not so simply connected with (1) in five dimensional formalism, because ~~the~~ (2) ~~is~~ can only be related to five dimensional space, in which the line element

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2 - dx_5^2 \qquad (3)$$

---

(1) Yukawa, Prog. Theor. Phy. 3 (1948),

instead of

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2 + dx_5^2 \quad (3)'$$

satisfying (2)

Now the matrix elements of U in the representation, in which $x\mu$ is diagonal, are different from zero, only if the row x' and column x'' satisfy the relation

$$(x'_\mu - x''_\mu)(x'^\mu - x''^\mu) = \lambda^2. \qquad (4)$$

In other words U is an operator connecting to spacelike points. This is not in contradiction with fact that ~~U~~ the quanta associated with this field have the momentum and energy satisfying the relation

$$p_\mu p^\mu + m^2c^2 = 0, \qquad (5)$$

~~Among general~~.

~~General~~ a solution of the equations (1) and (2) is

$$e^{ik_\mu x^\mu} \cdot e^{il^\mu p_\mu/\hbar} \qquad (6)$$

where k_μ and l_μ satisfy

$$\left.\begin{aligned} k_\mu k^\mu &= -\kappa^2 \\ l_\mu l^\mu &= \lambda^2 \end{aligned}\right\} \quad \kappa = \frac{mc}{\hbar} \quad (7)$$

respectively. Two factors in (6) are commutative with each other, only if

$$k_\mu l^\mu = 0. \qquad (8)$$

For $\varkappa = \lambda = 0$, (8) ~~is~~ reduces to
$$k_\mu l^\mu = kl\cos\theta \mp kl = 0$$
so that $\theta = 0$ or π, which implies
$$l^\mu = \text{const}\cdot k^\mu,$$
as previous considered.
In more general case, (8) can be written in the form
$$kl\cos\theta \mp \sqrt{k^2+\varkappa^2}\sqrt{l^2-\lambda^2} = 0$$
or $$\cos\theta = \pm \frac{c}{v}\cdot\frac{c}{u},$$
where
$$\frac{v}{c} = \frac{k}{\sqrt{k^2+\varkappa^2}} < 1$$
$$\frac{u}{c} = \frac{l}{\sqrt{l^2-\lambda^2}} > 1.$$
Thus for give value of k_μ ~~or k~~ or v/c,
$$\frac{u}{c}\cos\theta = \frac{l\cos\theta}{\sqrt{l^2-\lambda^2}} \left(= \pm \frac{c}{v}\right)$$
should have a definite value. Especially
$$\frac{l}{\sqrt{l^2-\lambda^2}} \geq \frac{c}{v} = \frac{\sqrt{k^2+\varkappa^2}}{k},$$
or
$$l_{max}^2 = l_{max}^2\left(\frac{c}{v}\right)^2 - \lambda^2\left(\frac{c}{v}\right)^2$$
or
$$l_{max} = \sqrt{\frac{\lambda^2\left(\frac{c}{v}\right)^2}{\left(\frac{c}{v}\right)^2-1}} = \frac{\lambda}{\sqrt{1-\left(\frac{v}{c}\right)^2}},$$
$$= \lambda\cdot\frac{\sqrt{k^2+\varkappa^2}}{\varkappa}$$

which is smaller for ~~the~~ smaller k or v.
Especially for $k=0$. $l_{max}=\lambda$,
and θ is arbitrary. In order ~~that~~ to
get a linear combination of (6), which
only depend on k_μ and still invariant,

$$e^{ik_\mu x^\mu}\int_{l^\mu (k_\mu l^\mu = 0)} c(kl^\mu)\, e^{il^\mu p_\mu/\hbar},$$

for $k=0$: $(l_0=0)$ $c(k_\mu l^\mu)$: ind. dep of l^μ

$$\int e^{il^\mu p_\mu/\hbar} \propto \frac{e^{i\lambda p/\hbar}}{p}$$

This is ~~actua~~ roughly equivalent to
giving a radius λ to the quantum
at rest.

For spinor field, a new difficulty arises
as shown below. For vector field,
there's no such difficulty.

Nonlocalizable Spinor Field

~~To For spinor field, satisfying (1) with the coefficients obeying the condition (2)', a pseudoparticle with p_μ~~

~~$$p_\mu p^\mu - m^2c^2 = 0 \qquad (3)'$$~~

~~corresponds.~~

Now we want to extend the above considerations to nonlocalizable ~~fields~~ spinor fields:

$$\left.\begin{array}{l} \gamma^\mu [p_\mu, \psi] + mc\,\psi = 0 \\ \gamma'_\mu [x^\mu, \psi] + \lambda\,\psi = 0 \end{array}\right\}$$

We assume a solution of the form

$$\psi \propto e^{ik_\mu x^\mu}\, e^{il^\mu p_\mu/\hbar}$$

$$(\gamma^\mu k_\mu + \varkappa)\,\psi = 0 \qquad \varkappa = \frac{mc}{\hbar}$$

$$(\gamma'_\mu l^\mu \mp \lambda)\,\psi = 0$$

$$(i\rho_2\,\sigma\mathbf{k} + \rho_3 k_4 + \varkappa)\,\psi = 0$$

$$(\rho_1\,\sigma\mathbf{k} + k_4 + \rho_3\varkappa)\,\psi = 0$$

~~$$(\sigma\mathbf{l} - \rho_1 l_4 - \rho_2\lambda)\,\psi = 0$$~~

~~$$\rho_3\,\sigma\mathbf{l} - i\rho_2$$~~

$$(\rho_3\,\sigma\mathbf{l} \pm i\rho_2 l_4 - \lambda)\,\psi = 0$$

$$i\rho_2\rho_3(\mathbf{k}\mathbf{l} - k_4 l_4) = 0$$

$$\underline{k_\mu l^\mu = 0}$$

$$\{\gamma^\mu[p_\mu,\psi]+mc\}\psi=0$$

$$\{\gamma'_\mu[x^\mu,\psi]+\lambda\}\psi=0$$

γ^μ:
$$\left.\begin{array}{l}\gamma^1=\gamma_1=i\rho_2\sigma_x\\ \gamma^2=\gamma_2=i\rho_2\sigma_y\\ \gamma^3=\gamma_3=i\rho_2\sigma_z\end{array}\right\}$$
$$\gamma^4=-\gamma_4=\rho_3$$

ρ_3: $i\rho_2\sigma[p,\psi]-\rho_3[p^4,\psi]+mc\psi=0$

$$\rho_1\sigma[p,\psi]-[p^4,\psi]+\rho_3 mc\psi=0$$

γ'_μ:
$$\left.\begin{array}{l}\gamma'^1=\gamma'_1=\rho_3\sigma_x\\ \gamma'^2=\gamma'_2=\rho_3\sigma_y\\ \gamma'^3=\gamma'_3=\rho_3\sigma_z\end{array}\right\}$$
$$\gamma'^4=-\gamma'_4=i\rho_2$$

ρ_3 $\rho_3\sigma[x,\psi]-i\rho_2[X^4,\psi]+\lambda\psi=0$.

$$\sigma[X,\psi]-\rho_1[X^4,\psi]+\underset{(\rho_3)?}{\lambda}\psi=0.$$

$$\psi\propto e^{ik_\mu x^\mu}e^{il^\mu p_\mu/\hbar}\qquad \varkappa=\frac{mc}{\hbar}$$

$$(i\rho_2\sigma k+\rho_3k_4+\varkappa)\psi=0$$
$$-k^2+k_4^2-\varkappa^2=0,$$
$$(\rho_3\sigma l-i\rho_2l^4+\lambda)\psi=0$$
$$l^2-(l^4)^2+\lambda^2=0$$

Condition of Compatibility:

$$(\rho_3 \sigma \mathbf{l} - i\rho_2 l^4 + \lambda)(\overset{i\rho_2 \sigma \mathbf{k} +}{\rho_3 k_4 + \kappa})$$

$$= (i\rho_2 \sigma \mathbf{k} + \rho_3 k_4 + \kappa)(\rho_3 \sigma \mathbf{l} - i\rho_2 l^4 + \lambda)$$

$$\mathbf{k}\mathbf{l} + k_4 l^4 = k_\mu l^\mu = 0.$$

Particle

Ψ { scalar — $\beta = \rho_3$

vector — $\alpha = \rho_1 \sigma$, 1

tensor — $i\beta\alpha = \rho_2 \sigma$, $\beta\sigma = \rho_3 \sigma$

pseudovector — σ , ρ_1

pseudoscalar — ρ_2 } ψ

Pseudoparticle λ : pseudoscalar

Ψ { scalar — ρ_3

vector — σ, $\cancel{\rho_1}$

tensor

pseudovector — σ , ρ_1

pseudoscalar — ρ_2 $\quad \lambda\rho_3$ }

Beck, Rev. Mod. Phys. 17 (1945), 187.
Equations for density operators for the electron.

$$(\gamma^\mu k_\mu + \kappa)\psi = 0$$

$$(\gamma'_\mu l^\mu + \lambda\gamma'_5)\psi = 0 \qquad \gamma'_5 = i\rho_1 ?$$

$$(\gamma'_\mu l^\mu + \lambda\gamma'_5)(\gamma^\mu k_\mu + \kappa) = (\gamma^\mu k_\mu + \kappa)(\gamma'_\mu l^\mu + \lambda\gamma'_5)$$

$$2\rho_1 k_\mu l^\mu + 2\lambda \underbrace{k_\mu \cancel{\rho_2} \gamma'_5\gamma^\mu}_{-\kappa}$$

$$\rho_1 \underbrace{k_\mu(l^\mu + \lambda\gamma^\mu)}_{-\kappa\lambda} = 0.$$

$$k_\mu l^\mu - i\kappa\lambda = 0$$

$k_4 = \kappa$, $k_1 = k_2 = k_3 = 0$. $\therefore$ $\kappa l^4 - i\kappa\lambda = 0$

$$l_1^2 + l_2^2 + l_3^2 = 0,$$

$$\boxed{l^4 = \lambda}$$

p

λ: ps.sc.

γ

$$\left.\begin{array}{l} \gamma'_\mu (x^\mu, \psi(\lambda)) + \lambda\psi(\lambda). \\ \gamma'_\mu (x^\mu, \psi(\lambda)) - \lambda\psi(-\lambda). \end{array}\right\}$$

$$\gamma_\nu' [x^\nu (\gamma^\mu [p_\mu, \psi] + mc\psi)] + \lambda (\gamma^\mu [p_\mu, \psi] + mc\psi)$$

$$= [\gamma^\mu p_\mu, (\gamma_\mu^{\mu\prime} [x^\mu, \psi] + \lambda\psi)] + mc(\gamma_\mu' [x^\mu, \psi] + \lambda\psi)$$

$$\gamma_\nu' \gamma^\mu [x^\nu [p_\mu, \psi]]$$

$$= \gamma^\mu \gamma_\nu' [p_\mu [x^\nu, \psi]]$$

$$[x^\nu [p_\mu, \psi]] + [p_\mu [\psi, x^\nu]]$$

$$+ [\psi [x^\nu, p_\mu]] = 0$$

$$[x^\nu [p_\mu, \psi]] = [p_\mu [x^\nu, \psi]] + [\underset{\shortparallel}{[x^\nu, p_\mu]}, \psi]$$

$$= [p_\mu [x^\nu, \psi]]$$

$$(\gamma_\nu' \gamma^\mu - \gamma^\mu \gamma_\nu') [p_\mu [x^\nu, \psi]] = 0,$$

$$\shortparallel$$

$$\bigcirc \quad \delta_{\mu\nu} \quad \underline{[p_\mu [x^\mu, \psi]] = 0}$$

June 1, 1949

Density Distribution inside the Elementary Particle. (D.1)

Starting from the relations

$$\left.\begin{aligned}[x_\mu[x^\mu U]] &= \lambda^2 U\\ [p_\mu[p^\mu U]] &= -m^2c^2 U\\ [p_\mu[x^\mu U]] &= 0,\end{aligned}\right\}$$

which can be reduced to

$$\left.\begin{aligned}(r_\mu r^\mu - \lambda^2)\, U(X_\mu, r_\mu) &= 0\\ \left(\frac{\partial^2}{\partial X_\mu \partial X^\mu} - \kappa^2\right) U(X_\mu, r_\mu) &= 0\\ r^\mu \frac{\partial}{\partial X^\mu} U(X_\mu, r_\mu) &= 0,\end{aligned}\right\}$$

~~U~~ where $X_\mu = \frac{1}{2}(x'_\mu + x''_\mu)$, $r_\mu = x'_\mu - x''_\mu$.

$$U(X_\mu, r_\mu) = \sum_{k_\mu} u(k_\mu, r_\mu)\, e^{ik_\mu X^\mu}$$

$$\left.\begin{aligned}(r_\mu r^\mu - \lambda^2)\, u &= 0\\ (k_\mu k^\mu + \kappa^2)\, u &= 0\\ k_\mu r^\mu\, u &= 0\end{aligned}\right\}$$

$$u(k_\mu, r_\mu) = b(k_\mu)$$

for r_μ satisfying

$$r_\mu r^\mu = \lambda^2 \qquad k_\mu r^\mu = 0.$$

or

$$U = \int\cdots\int b(k_\mu)\, \cancel{\delta(k_\mu, l^\mu}\, \delta(k_\mu k^\mu + \kappa^2)\, \delta(l_\mu l^\mu - \lambda^2)\, \delta(k_\mu l^\mu)\, e^{il^\mu p_\mu}\, e^{ik_\mu X^\mu}$$

$$U^* = \int\cdots\int b^*(k_\mu)\, \delta(k_\mu k^\mu + \kappa^2)\, \delta(l_\mu l^\mu - \lambda^2)\, \delta(k_\mu l^\mu)\, e^{-il^\mu p_\mu}\, e^{-ik_\mu X^\mu}$$

$[A_1B_1, A_2B_2] = A_1B_1A_2B_2 - A_2B_2A_1B_1$

$$[U^*, U^*] = \int\cdots\int [b(k_\mu) e^{il^\mu p_\mu/\hbar} e^{ik_\mu x^\mu}, b^*(k'_\mu) e^{-il'^\mu p_\mu/\hbar} e^{-ik'_\mu x'^\mu}]$$

$$\times \delta(k_\mu k^\mu + \kappa^2)\delta(l_\mu l^\mu - \lambda^2)\delta(k_\mu l^\mu)(dk)^4(dl)^4$$

$$\times \delta(k'_\mu k'^\mu + \kappa^2)\delta(l'_\mu l'^\mu - \lambda^2)\delta(k'_\mu l'^\mu)(dk')^4(dl')^4.$$

$$= \int\cdots\int \{b(k_\mu) b^*(k'_\mu) [e^{il^\mu p_\mu/\hbar} e^{ik_\mu x^\mu}, e^{-il'^\mu p_\mu/\hbar} e^{-ik'_\mu x^\mu}]$$

$$+ \delta(k_\mu - k'_\mu)\} \delta(k_\mu k^\mu + \kappa^2)\delta(l_\mu l^\mu - \lambda^2)\delta(k_\mu l^\mu)$$

$$\times (dk)^4 (dl^4) \delta(k'_\mu k'^\mu + \kappa^2)\delta(l'_\mu l'^\mu - \lambda^2)\delta(k'_\mu l'^\mu)(dk')^4(dl')^4.$$

$$e^{il^\mu p_\mu/\hbar} e^{ik_\mu x^\mu} \cdot e^{-il'^\mu p_\mu/\hbar} e^{-ik'_\mu x^\mu} = e^{ik'_\mu x^\mu} \cdot e^{-ik'_\mu x^\mu}$$

$$\mathrm{I} = e^{-ik'_\mu l^\mu} e^{il^\mu p_\mu/\hbar} e^{-il'^\mu p_\mu/\hbar}$$

$$= e^{i(k_\mu - k'_\mu)x^\mu} e^{-i(l^\mu - l'^\mu)p_\mu/\hbar} e^{-ik'_\mu l^\mu}$$

$$e^{-il'^\mu p_\mu/\hbar} e^{-ik'_\mu x^\mu} \cdot e^{il^\mu p_\mu/\hbar} e^{ik_\mu x^\mu}$$

$$\mathrm{II} = = e^{-ik'_\mu x^\mu} e^{ik_\mu x^\mu} e^{-il'^\mu p_\mu/\hbar} e^{il^\mu p_\mu/\hbar} e^{-ik_\mu l'^\mu}$$

$$\mathrm{I} - \mathrm{II} = e^{i(k_\mu - k'_\mu)x^\mu} e^{-i(l^\mu - l'^\mu)p_\mu/\hbar}$$

$$\times \{e^{-ik'_\mu l^\mu} - e^{-ik_\mu l'^\mu}\}$$

$$= \quad \cdots \quad \cdots \{e^{iKl} - e^{-iKl'}\}$$

$$K = k - k'$$

$[A_1, B_1] = [A_1, B_2] = [A_2 B_1] = [A_2 B_2] = 0$

$[A_1B_1, A_2B_2] = A_1B_1A_2B_2 - A_2B_2A_1B_1$

$= A_1A_2[B_1, B_2] + A_1A_2B_2B_1 - A_2A_1B_2B_1$

$= A_1A_2[B_1, B_2] + [A_1, A_2]B_2B_1$

(D, 2)

Thus the commutation relations between U, U^* is very complicated. Also UU^* and U^*U themselves are fairly complicated. However, such terms in UU^*, U^*U, which are independent of x^μ, ~~have~~ are much simpler. Namely

$$(U^*U)_{density} = \cancel{\iint n(k_\mu)(dk)^4} \iint \left\{ \begin{matrix} b^*(k)\,b(k) \\ b(k)\,b^*(k) \end{matrix} \right\} \delta(k_\mu k^\mu + \kappa^2)$$

$$(UU^*)_{density} = \cancel{\iint n(k_\mu)+1\,(dk^4)}$$

$$\delta(l_\mu l^\mu - \lambda^2)\,\delta(k_\mu l^\mu)\,\delta(l'_\mu l'^\mu - \lambda^2)\,\delta(k_\mu l'^\mu)$$

$$\times\, e^{i(l^\mu - l'^\mu)p_\mu/\hbar}\,(dk)^4 (dl)^4 (dl')^4$$

k_μ: $k_1 = k_2 = k_3 = 0 \quad k_4 = \pm\kappa$.

$$l_1^2 + l_2^2 + l_3^2 = l_1'^2 + l_2'^2 + l_3'^2 = \lambda^2$$

$$l_4 = l_4' = 0.$$

$$\sum_{k_\mu k^\mu = -\kappa^2} \begin{pmatrix} b^*(k)\,b(k) \\ b(k)\,b^*(k) \end{pmatrix} \delta(l_\mu l^\mu - \lambda^2)\,\delta(k_\mu l^\mu)\,\delta(l'_\mu l'^\mu - \lambda^2) \times \delta(k_\mu l'^\mu)\, e^{i(l^\mu - l'^\mu)p_\mu/\hbar}(dl)^4 (dl')^4$$

$$\int_{l^2 = l'^2 = \lambda^2} e^{i(l^\mu - l'^\mu)p_\mu/\hbar}\, \cancel{(dl)^3 (dl')^3} \left(\frac{\lambda}{2\kappa}\right)^2 \cdot \sin\theta\, d\theta\, d\varphi \cdot \sin\theta' d\theta' d\varphi'$$

$$\| $$

$$e^{i\frac{\lambda}{\hbar}(p_x \sin\theta\cos\varphi + p_y \sin\theta\sin\varphi + p_z\cos\theta) - (\cdots)}$$

$$(U^*U)_{density} = \int\cdots\int \frac{\delta(k_\mu - k'_\mu)}{\delta(k'_\mu k'^\mu + \kappa^2)} \left(\underline{\qquad U^* \qquad} \times \cdot\, U \qquad \right)$$

$$\cancel{\delta(k_\mu l^\mu)}$$

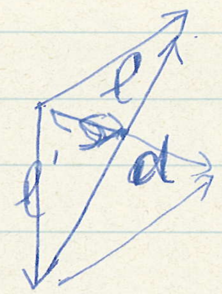
$$\int \underbrace{\delta(l_\mu l^\mu - \lambda^2)}_{\delta(l^2 - \lambda^2)}\, \delta(\overset{\pm\kappa l_4}{k''_\mu l^\mu})\, \underbrace{dl_1 dl_2 dl_3 dl_4}_{l^2 dl \sin\theta\, d\theta\, d\varphi\, dl_4} = \frac{\lambda}{2\kappa} \int_{l=\lambda}$$

$$l^2 dl = \frac{d(l^2)}{2}\cdot l$$

$l_1 - l_1' = d_1 \qquad \frac{1}{2}(l_1 + l_1') = s_1$

$l_2 - l_2' = d_2 \qquad \frac{1}{2}(\cancel{l_1}+l_2') = s_2$

$l_3 - l_3' = d_3 \qquad \frac{1}{2}(l_3 + l_3') = s_3$



$0 = d_1 = \lambda(\sin\theta\cos\varphi - \sin\theta'\cos\varphi')$

$0 = d_2 = \lambda(\sin\theta\sin\varphi - \sin\theta'\sin\varphi')$

$d = d_3 = \lambda\,\underline{(\cos\theta - \cos\theta')}$

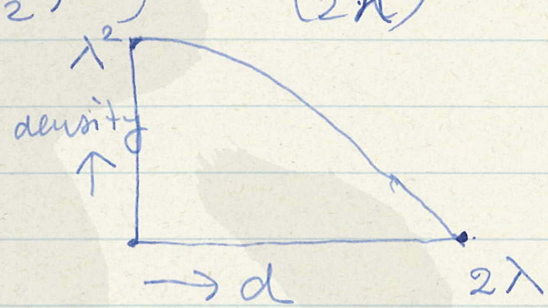
$s_1 = \cancel{\sqrt{\left(\frac{d}{2}\right)^2 - \lambda^2}} \quad \sqrt{\lambda^2 - \left(\frac{d}{2}\right)^2}\,\sin\alpha\ \cancel{\cos\alpha}\cos\beta$

$s_2 = \quad \sqrt{\lambda^2 - \left(\frac{d}{2}\right)^2}\,\sin\alpha\sin\beta$

$s_3 = \quad \sqrt{\lambda^2 - \left(\frac{d}{2}\right)^2}\,\cancel{\sin}\cos\alpha$

$e^{id p_z/\hbar}\left(\lambda^2 - \left(\frac{d}{2}\right)^2\right)\cdot 4\pi\left(\frac{\lambda}{2\pi}\right)^2$

$(x'|D|x'') \neq 0$ only for
$|x' - x''| \leq 2\lambda$



or $(X + \frac{1}{2}r|D|X - \frac{1}{2}r)$
$\neq 0$
only for $\left|\frac{r}{2}\right| < \lambda$.

Symmetric and Antisymmetric Products of Operators

(S.A. 1)

Antisymmetric Products of two operators

Commutator : $[A, B] \equiv AB - BA$

A. P. of three operators

$$[A, B, C] \equiv \sum_P \varepsilon_P P \cdot ABC$$
$$\equiv ABC - ACB + BCA - BAC + CAB - CBA$$
$$= A[BC] + B[CA] + C[AB]$$
$$= [BC]A + [CA]B + [AB]C ,$$

from which Jacobi's identity

$$[A[BC]] + [B[CA]] + [C[AB]] = 0$$

is obtained immediately.

$$[A, B, C] = -[B, A, C] = \cdots = +[B, C, A] = \cdots$$

A.P. of four operators

$$[A, B, C, D] \equiv \sum_P \varepsilon_P P \cdot ABCD$$
$$= \cancel{AB[CD] +} A[B, C, D]$$
$$* - B[A, C, D] + C[A, B, D] - D[A, B, C]$$
$$= [A, B, C] D * - [A, B, D] C + [A, C, D] B$$
$$\pm * [A, B, D] A$$

$$\therefore [A, [B, C, D]] \mp [B, [A, C, D]] + [C [A, B, D]] - [D [A, B, C]] = 0 .$$

$$[A, B, C, D] = AB[C, D] + AC[D, B] + AD[B, C]$$
$$- BA[C, D] \pm BC[D, A] - BD[A, C]$$
$$+ CA[BD] + CB[D, A] + CD[AB]$$
$$- DA[BC] - DB[C, A] - DC[AB]$$
$$= [A, B][C, D] + [A, C][D, B] + [AD][BC]$$
$$+ [B, C][A, D] + [B, D][C, A] + [CD][AB]$$
$$= ([A, B], [C, D]) + ([A, C], [D, B]) + ([AD], [BC])$$

where $(A,B) = AB + BA$.

$$\begin{cases} [A,[B,C]] = ABC - ACB - BCA + CBA \\ [A,(B,C)] = -ABC - BCA + ACB - CBA \end{cases}$$

Symmetric Product

Two operators $(A,B) = AB + BA$

$$(A,B) + [A,B] = 2AB$$

$$(A,B) - [A,B] = 2BA$$

Three operators

$$(A,B,C) = \sum_P PABC$$

$$(A,B,C) + [A,B,C] = \sum_P (1+\varepsilon_p)\, P\cdot ABC$$

$$(A,B,C) - [A,B,C] = \sum_P (1-\varepsilon_p)\, P\cdot ABC$$

$$\left.\begin{aligned} (A,B,C) = {} & A(B,C) + (B,C)A \\ & + B(C,A) + (C,A)B \\ & + C(A,B) + (A,B)C \end{aligned}\right\} = A(BC) + B(CA) + C(AB) + \cdot$$

$$= (A,(B,C)) + (B,(C,A)) + (C,(A,B))$$

four operators

$$(A,B,C,D) = A(B,C,D) + B(A,C,D) + C(A,B,D) + D(A,B,C)$$

$$= A(B,(CD)) + \cancel{A(C(} + A(C,(BD)) + A(D,(BC)) + \cdots$$

$$= ((AB),(CD)) + ((AC),(BD)) + ((AD),(BC))$$

$$2[A,B,C] = (A,[BC]) + (B,[CA]) + (C,[AB])$$

(S.D.R)
(S.A.2)

$$[A,[B,C]] + [A,(B,C)] = 2(ABC - BCA)$$
$$= 2[A, BC]$$
$$[A,(BC)] - [A[B,C]] = 2[A, CB]$$

Six Dimensional Representation of Nonlocalizable Fields.

$$\left.\begin{aligned} x_\mu x^\mu &= x_1^2 + x_2^2 + x_3^2 - x_4^2 + x_5^2 - x_6^2 \\ p_\mu p^\mu &= \underbrace{p_1^2 + p_2^2 + p_3^2}_{\text{space}} - \underset{\text{time}}{p_4^2} + \underset{?.}{p_5^2} - \underset{\text{size}}{p_6^2} \end{aligned}\right\}$$

momentum　energy　mass　?

$$[x^\mu, p_\nu] = i\hbar\,\delta_{\mu\nu}$$
$$[x^\mu, U] = G^\mu$$
$$[p_\mu, U] = F_\mu$$

$$\left.\begin{aligned}[x_\mu, G^\mu] &= 0 \\ [p_\mu, G^\mu] &= 0\end{aligned}\right\} \qquad \left.\begin{aligned}[x^\mu, F_\mu] &= 0 \\ [p^\mu, F_\mu] &= 0\end{aligned}\right\}$$

(i) Complex field with definite mass and size

$$[x^5, U] = [p_6, U] = 0$$
$$[x^6, U] = \pm\lambda U \qquad [p_5, U] = \pm mcU$$

(ii) ?

$$\left.\begin{aligned}[x^5, U] &= \pm\lambda' U \\ [x^6, U] &= \pm\lambda U\end{aligned}\right\} \lambda > \lambda' \qquad \left.\begin{aligned}[p_5, U] &= \pm mcU \\ [p_6, U] &= \pm m'cU\end{aligned}\right\} m, m'$$

(iii) Real field, U, V: Hermitian (with mass)

$$\left.\begin{aligned}[x^5, U] &= \pm\lambda V \\ [x^6, V] &= \mp\lambda U\end{aligned}\right\} \qquad \left.\begin{aligned}[p_5, U] &= \pm mcV \\ [p_5, V] &= \mp mcU\end{aligned}\right\}$$

There must be always two kinds of neutral particles with the same mass and the same transformation properties.

Relation to Formulation in Four Dimensional Space.

(i) $+\lambda$, $+m$, U: complex field.

$$[x^\mu, U] = G^\mu \qquad [x^5, U] = \lambda U$$
$$[p_\mu, U] = F_\mu \qquad [p_5, U] = mc\,U$$
$$\left.\begin{array}{l}[x_\mu\, G^\mu] = \lambda^2 U = 0 \\ [p_\mu, G^\mu] = 0\end{array}\right\} \qquad \left.\begin{array}{l}[x^\mu, F_\mu] = 0 \\ [p^\mu, F_\mu] + m^2c^2 U = 0\end{array}\right\}$$

(iii) U, V: real field $\pm\lambda$, $\pm m$.
→ complex fields. $U(=U+iV)$, $U^*(=U-iV)$

(iv) Electromagnetic field is very ~~particular~~ peculiar in many respects.

$$[x_\mu, A_\nu] = G_{\mu\nu} \qquad \Big\{\; G_{\mu\nu} = -G_{\nu\mu}$$
$$[p_\mu\, A_\nu] = F_{\mu\nu} \qquad F_{\mu\nu} = -F_{\nu\mu}$$

$$[x^\mu, G_{\mu\nu}] = 0$$
$$[p^\mu, F_{\mu\nu}] = 0$$
$$\boxed{\left.\begin{array}{l}[x^\mu, A_\mu] = 0 \\ [p^\mu, A_\mu] = 0.\end{array}\right\}}$$

$$\left\{\begin{array}{l}F_{\mu\nu} = [p_\mu A_\nu] - [p_\nu A_\mu] \\ G_{\mu\nu} = [x_\mu A_\nu] - [x_\nu A_\mu]\end{array}\right.$$

$$\boxed{\begin{array}{l}\left.\begin{array}{l}[p^\mu [p_\mu A_\nu]] = 0 \\ [x^\mu [x_\mu A_\nu]] = 0\end{array}\right\} \\ [x^\mu [p_\mu A_\nu]] = 0\end{array}}$$

Gravitational field

$$F_{\mu\nu} = [p_\mu A_\nu] + [p_\nu A_\mu]$$
$$G_{\mu\nu} = [x_\mu A_\nu] + [x_\nu A_\mu]$$
$$\left.\begin{array}{l}[x^\mu, G_{\mu\nu}] = 0 \\ [p^\mu, F_{\mu\nu}] = 0\end{array}\right\}$$

[Cambridge Berkeley June, 1949]

Cambridge Meeting

June 16, 1949

Schwinger, Q.E.D.: localizability

Bartlett, He wave equation.

Welton, A Field Formalism without Self-Action

Dirac, 1932 P.R.S.

L. Goldstein and M. Goldstein, On Exchange Energy in He_3 Assemblies.

[Corben]

Green:

Berkeley June 29, 30, 1949

Serber, Berkeley Experiments

1. Production of Mesons and Photons by ~~high~~ energy ~~collisi~~ photons.

$\pi^+/\pi^- = 3$ theoretically

$P + N \to \pi^+, \pi^-$

$P + P \to \pi^+, \pi^+$

Spectrum of γ-rays can be well accounted for by assuming

$$\begin{cases} \pi^{(0)} = \gamma + \gamma \\ m_\pi^{(0)} = m_\pi \end{cases}$$

2. P-P-scattering at 31 MeV.

p-d scattering very small, almost S-scattering.

No potential can be adequate.

3. N-P-scattering

4. $\pi^- \begin{cases} 5 \text{ stars} \\ 15 \text{ no decay particles} \end{cases}$ about 5%

(except possibly electron)

~~Theoretical Physics Seminar~~

5. 1/4-size model of Bevatron.

6. Bevatron

7.

Seminar: Theoretical Physics, Summer sessions. Phys. Dept. 3 pm. June 29.

Yukawa: Non-local field theory.

Particle — Field

i) dimensionless particle — quantized local field

ii) particle of finite dimension — quantized nonlocal field

internal motion: Oppenheimer, Serber

$$[x^\mu p_\nu - x^\nu p_\mu, U] = [x^\mu p_\nu, U] - [x^\nu p_\mu, U]$$

$$= x^\mu[p_\nu, U] + [x^\mu, U]p_\nu$$

$$- x^\nu[p_\mu, U] - [x^\nu, U]p_\mu$$

$$= \frac{1}{2}\{x^\mu[p_\nu, U] + [p_\nu, U]x^\mu\} + \frac{1}{2}[x^\mu[p_\nu, U]]$$

$$- \frac{1}{2}\{x^\nu[p_\mu, U] + [p_\mu, U]x^\nu\} - \frac{1}{2}\{x^\nu[p_\mu, U]]$$

$$\cancel{\xrightarrow{-i\hbar}\left(x^\mu \frac{\partial U}{\partial x^\nu} - x^\nu \frac{\partial U}{\partial x^\mu}\right) + \frac{1}{2}i\hbar}$$

$$+ \frac{1}{2}\{[x^\mu, U]p_\nu + p_\nu[x^\mu, U]\}$$

$$+ \frac{1}{2}\{[x^\mu, U]p_\nu]$$

$$- \frac{1}{2}\{[x^\nu, U]p_\mu + p_\mu[x^\nu, U]\}$$

$$- \frac{1}{2}\{[[x^\nu, U], p_\mu]$$

Berkeley, June 30, 1949

Rotational Degrees of Freedom

$$[x_\mu p_\nu - x_\nu p_\mu , U] = \frac{1}{2}\{x_\mu[p_\nu, U] + [p_\nu, U]x_\mu\}$$

$$+\frac{1}{2}\cancel{[x_\mu[p_\nu, U]]} - \frac{1}{2}\{x_\nu[p_\mu, U] + [p_\mu, U]x_\nu\}$$

$$-\frac{1}{2}\cancel{[x_\nu[p_\mu U]]}$$

$$+\frac{1}{2}\{[x_\mu, U]p_\nu + p_\nu[x_\mu, U]\} + \frac{1}{2}\cancel{[[x_\mu, U]p_\nu]}$$

$$-\frac{1}{2}\{[x_\nu, U]p_\mu + p_\mu[x_\nu, U]\} - \frac{1}{2}\cancel{[[x_\nu, U], p_\mu]}$$

$$\to -i\hbar\left(x_\mu \frac{\partial U}{\partial x^\nu} - x_\nu \frac{\partial U}{\partial x^\mu}\right)$$

$$\cancel{+} - \frac{1}{2} i\hbar\left(r_\mu \frac{\partial U}{\partial x^\nu} - r_\nu \frac{\partial U}{\partial x^\mu}\right)$$

$$+\cancel{\frac{1}{2}} - \frac{1}{2} i\hbar\left(\frac{\partial}{\partial r^\nu}(r_\mu U) - \frac{\partial}{\partial r^\mu}(r_\nu U)\right)$$

$$+\frac{1}{2} i\hbar\left(\frac{\partial}{\partial x^\nu}(r_\mu U) - \frac{\partial}{\partial x^\mu}(r_\nu U)\right)$$

$$= -i\hbar\left(x_\mu \frac{\partial U}{\partial x^\nu} - x_\nu \frac{\partial U}{\partial x^\mu}\right)$$

$$-\cancel{\frac{1}{2}} i\hbar\left(r_\mu \frac{\partial U}{\partial r^\nu} - r_\nu \frac{\partial U}{\partial r^\mu}\right)$$

$$\cancel{\frac{1}{2}}\left[r_\rho \frac{\partial}{\partial r^\sigma} - r_\sigma \frac{\partial}{\partial r^\rho}, r^\mu r_\mu\right] = 2 r_\rho r_\mu \delta_{\mu\sigma} - 2 r_\sigma r_\mu \delta_{\mu\rho}$$

$$= 0$$

$$\cancel{\frac{1}{2}}\left[r_\rho \frac{\partial}{\partial r^\sigma} - r_\sigma \frac{\partial}{\partial r^\rho}, r^\mu \frac{\partial}{\partial x^\mu}\right] = \cancel{\frac{1}{2}}\left(r_\rho \frac{\partial}{\partial x^\sigma} - r_\rho \frac{\partial}{\partial x^\rho}\right)$$

$$r_\rho \frac{\partial U}{\partial x^\sigma} - r_\sigma \frac{\partial U}{\partial x^\rho} = 0$$

$$(l_\rho k_\sigma - l_\sigma k_\rho)\, U(k, l) = 0$$

$k_1 = k_2 = k_3 = 0 \qquad k_4 = \pm\varkappa.$

$$(l_\rho k_\sigma - l_\sigma k_\rho) = \begin{pmatrix} 0 & 0 & 0 & l_1 k_4 \\ 0 & 0 & 0 & l_2 k_4 \\ 0 & 0 & 0 & l_3 k_4 \\ -l_1 k_4 & -l_2 k_4 & -l_3 k_4 & 0 \end{pmatrix}$$

Assumption A.

$$\cancel{k_4}\; x_\rho \frac{\partial U}{\partial x^\sigma} - x_\sigma \frac{\partial U}{\partial x^\rho} = 0$$

for $\rho, \sigma \neq 4$ for U at rest.

and

B (i) $\quad r_\rho \dfrac{\partial U}{\partial r_\sigma} - r_\sigma \dfrac{\partial U}{\partial r_\rho} = 0$

for $\rho, \sigma \neq 4$ for U at rest

$$\left.\begin{aligned} l_1 &= l \sin\theta \cos\varphi \\ l_2 &= l \sin\theta \sin\varphi \\ l_3 &= l \cos\theta \\ l_4 &= 0 \end{aligned}\right\}$$

$u(k_\mu, l^\mu)$

$\rightarrow u(\cancel{k_\mu}\, 0, 0, 0, \pm\varkappa;\, l_1, l_2, l_3, 0)$

$l = \lambda.$ spherical symmetry

$\rightarrow u(\cancel{0, 0, 0, \pm\varkappa;}\,(\varkappa, \lambda).$

B (ii)

Nucleon-Meson

Ann Arbor July 21

8-Component Spinor Field

Footnote 6. in A is not correct

$$\gamma'_\mu [x'^\mu, \psi] + \lambda \tau_3 \psi = 0$$

$$\gamma^\mu [p_\mu, \psi] + mc\psi = 0$$

proper Lorentz transformation $\| a_{\mu\nu} \| = 1$*

$x^\mu \to -x^\mu$

$p_\mu \to -p_\mu$ $\quad \psi \to \gamma'_1 \gamma'_2 \gamma'_3 \gamma'_4 \psi.$

improper Lorentz transformation $\| a_{\mu\nu} \| = -1$

$x'_1 \to -x'_1$ $\quad \psi \to \gamma'_1 \psi$

$p_1 \to -p_1$

$\lambda \tau_3 \psi \to -\lambda \tau_3 \psi.$

$\underline{\psi \to \gamma'_1 \tau_1 \psi.}$

identification of $\tau_3 = 1$ and -1 with neutral and charged states of spinor particle ?

$(1, \tau_1)\gamma^\mu$: vector $\qquad (\tau_2, \tau_3)\gamma^\mu$: ps. vector

$(1, \tau_1)\gamma'_\mu$: ps. vector $\qquad (\tau_2, \tau_3)\gamma'_\mu$: vector

---

* rotation $\begin{cases} x' = x\cos\omega + y\sin\omega \\ y' = -x\sin\omega + y\cos\omega \end{cases}$

$$\psi' = \left(\cos\frac{\omega}{2} + \gamma'_1\gamma'_2 \sin\frac{\omega}{2}\right)\psi$$

$x' = -x,\ y' = -y$: $\quad \psi' = \gamma'_1 \gamma'_2 \psi$

$$[x^{\nu}, A_{\mu}\psi] = x^{\nu}A_{\mu}\psi - A_{\mu}x^{\nu}\psi$$
$$+ A_{\mu}x^{\nu}\psi - A_{\mu}\psi x^{\nu}$$
$$= [x^{\nu}, A_{\mu}]\psi + A_{\mu}[x^{\nu}, \psi]$$

$$[\gamma'_{\nu}x^{\nu}, \gamma^{\mu}A_{\mu}\psi] = \gamma'_{\nu}x^{\nu}\gamma^{\mu}A_{\mu}\psi - \gamma'_{\nu}\gamma^{\mu}A_{\mu}x^{\nu}\psi$$
$$+ \gamma'_{\nu}\gamma^{\mu}A_{\mu}x^{\nu}\psi - \gamma^{\mu}\gamma'_{\nu}A_{\mu}x^{\nu}\psi$$
$$+ \gamma^{\mu}\gamma'_{\nu}A_{\mu}x^{\nu}\psi - \gamma^{\mu}\gamma'_{\nu}A_{\mu}\psi x^{\nu}$$
$$= \gamma'_{\nu}\gamma^{\mu}[x^{\nu}, A_{\mu}]\psi$$
$$+ [\gamma'_{\nu}, \gamma^{\mu}]A_{\mu}x^{\nu}\psi$$
$$+ \gamma^{\mu}\gamma'_{\nu}A_{\mu}[x^{\nu}, \psi]$$
$$=$$

July 21 ~22, 1949
Michigan Symposium

On the Rigid Sphere Model
for the Elementary Particle
By H. Yukawa

(i) ~~Rotational deg.~~ Internal rotation
(ii) 8-component Spinor Field

(iii) Mass zero case

$$k_\mu l^\mu = 0. \qquad l_\mu \propto k_\mu$$

$$l_\mu = \frac{a}{\cancel{k_0}} k_\mu .$$

Informal Talk at the Inst. f. Adv. Study, Princeton, on Aug. 26, 1949.

(i) Nonlocal Scalar Field
Normalization
(ii) Spinor Field
8-Component Spinor
(iii) S-matrix scheme
Energy conservation Law
δ_+, δ_- - fn. time integral (X^4).

$$r_\mu' = a_{\mu\nu} r_\nu$$

$$(a_{\mu\nu}) = \begin{pmatrix} 1+\frac{k_1^2}{K^2} & \frac{k_1k_2}{K^2} & \frac{k_1k_3}{K^2} & -\frac{k_1}{\kappa} \\ \frac{k_1k_2}{K^2} & 1+\frac{k_2^2}{K^2} & \frac{k_2k_3}{K^2} & -\frac{k_2}{\kappa} \\ \frac{k_1k_3}{K^2} & \frac{k_2k_3}{K^2} & 1+\frac{k_3^2}{K^2} & -\frac{k_3}{\kappa} \\ -\frac{k_1}{\kappa} & -\frac{k_2}{\kappa} & -\frac{k_3}{\kappa} & \frac{k_4}{\kappa} \end{pmatrix}$$

$$\left.\begin{aligned} r_1' &= \lambda \sin\theta' \cos\varphi' \\ r_2' &= \lambda \sin\theta' \sin\varphi' \\ r_3' &= \lambda \cos\theta' \end{aligned}\right\}$$

$$(r_4' = 0)$$

$$u(k, \theta', \varphi') = \sum_{l,m} c(\cancel{k}, l, m)\, P_l^m(\theta', \varphi')$$

normalization

$$\begin{pmatrix} \frac{k_1}{k} & \frac{k_2}{k} & \frac{k_3}{k} & 0 \\ -\frac{\sqrt{k_2^2+k_3^2}}{k} & \frac{k_1 k_2}{k\sqrt{k_2^2+k_3^2}} & \frac{k_1 k_3}{k\sqrt{k_2^2+k_3^2}} & 0 \\ 0 & -\frac{k_3}{\sqrt{k_2^2+k_3^2}} & \frac{k_2}{\sqrt{k_2^2+k_3^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{k_1 k_4}{\kappa k} & \frac{k_2 k_4}{\kappa k} & \frac{k_3 k_4}{\kappa k} & -\frac{k}{\kappa} \\ -\frac{\sqrt{k_2^2+k_3^2}}{k} & \frac{k_1 k_2}{k\sqrt{k_2^2+k_3^2}} & \frac{k_1 k_3}{k\sqrt{k_2^2+k_3^2}} & 0 \\ 0 & -\frac{k_3}{\sqrt{k_3^2+k_3^2}} & \frac{k_2}{\sqrt{k_2^2+k_3^2}} & 0 \\ -\frac{k_1}{\kappa} & -\frac{k_2}{\kappa} & -\frac{k_3}{\kappa} & \frac{k_4}{\kappa} \end{pmatrix}$$

$$
\begin{pmatrix}
1+\frac{k_1^2}{K^2} & \frac{k_1k_2}{K^2} & \frac{k_1k_3}{K^2} & -\frac{k_1}{\kappa} \\
\frac{k_1k_2}{K^2} & 1+\frac{k_2^2}{K^2} & \frac{k_2k_3}{K^2} & -\frac{k_2}{\kappa} \\
\frac{k_1k_3}{K^2} & \frac{k_2k_3}{K^2} & 1+\frac{k_3^2}{K^2} & -\frac{k_3}{\kappa} \\
-\frac{k_1}{\kappa} & -\frac{k_2}{\kappa} & -\frac{k_3}{\kappa} & \frac{k_4}{\kappa}
\end{pmatrix}
$$

$$\frac{1}{K^2} = \frac{1}{k^2}\left(\frac{k_4}{\kappa}-1\right)$$

$$= \frac{1}{k^2}\frac{\left(\left(\frac{k_4}{\kappa}\right)^2-1\right)}{\left(\frac{k_4}{\kappa}\right)+1} = \frac{1}{\kappa(k_4+\kappa)}$$

$$K = \sqrt{\kappa(k_4+\kappa)}$$

$$2\frac{k_1k_2}{K^2} + \frac{k_1k_2\,k^2}{K^4} - \frac{k_1k_2}{\kappa^2}$$

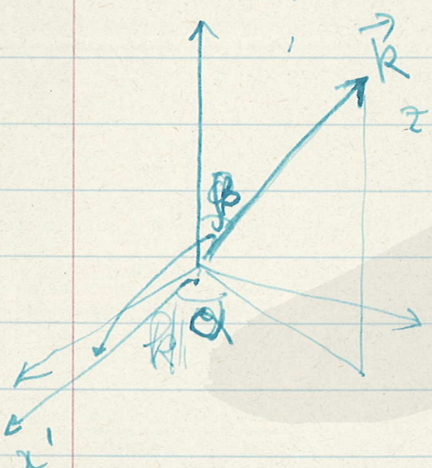
$$= \frac{k_1k_2}{\kappa^2(k_4+\kappa)^2}\left\{-2\kappa(k_4+\kappa) + k^2 + (k_4+\kappa)^2\right\}$$

$$+\,k_4^2-\kappa^2$$

$$k_1 : k_2 : k_3 : k_4 = \frac{mv_x}{\hbar\sqrt{1-\frac{v^2}{c^2}}} : \frac{mv_y}{\hbar\sqrt{1-\frac{v^2}{c^2}}} : \frac{mv_z}{\hbar\sqrt{1-\frac{v^2}{c^2}}} : \frac{\pm mc}{\hbar\sqrt{1-\frac{v^2}{c^2}}}$$

$$k'_\mu = a_{\mu\nu} k_\nu$$

$$a_{\mu\nu} = \Big(\frac{k_1}{k} \quad \frac{k_2}{k} \quad \frac{k_3}{k}$$



$$\begin{pmatrix} \frac{\beta_x^2}{\beta\sqrt{1-\beta^2}} & \frac{\beta_x\beta_y}{\beta\sqrt{1-\beta^2}} & \frac{\beta_x\beta_z}{\beta\sqrt{1-\beta^2}} & \pm\frac{\beta_x}{\sqrt{1-\beta^2}} \\ -\frac{\beta_x\beta_y}{\beta\sqrt{1-\beta^2}} & \frac{\beta_y}{\beta\sqrt{1-\beta^2}} & \frac{\beta_y\beta_z}{\beta} & \pm\frac{\beta_y}{\sqrt{1-\beta^2}} \\ & & & \pm\frac{\beta_z}{\sqrt{1-\beta^2}} \\ & & & \pm\frac{1}{\sqrt{1-\beta^2}} \end{pmatrix}$$

$$k''_\mu = b_{\mu\nu} k'_\nu$$

$$k''_1 = b_{14} k'_4 \qquad b_{14} = \cancel{\pm\frac{c}{v_x}} \quad \pm\frac{v_x}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$k''_2 = \qquad b_{24} = \pm\frac{c}{v_y} \quad \pm\frac{v_y}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$k''_3 = \qquad b_{34} = \pm\frac{c}{v_z} \quad \pm\frac{v_z}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$k''_4 = \qquad b_{44} = \pm\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$a_{22} \qquad a_{23}$$

$$y' k = y\frac{k_2}{\sqrt{k_2^2+k_3^2}} + z\frac{k_3}{\sqrt{k_2^2+k_3^2}}$$

$$z' = -y\frac{k_3}{\sqrt{k_2^2+k_3^2}}\,a_{32} + z\frac{k_2}{\sqrt{k_2^2+k_3^2}}\,a_{33}$$

$$k_2' = \sqrt{k_2^2+k_3^2}$$

$$k_3' = 0$$

$$k_1'' = k_1\frac{k_1}{k} + \sqrt{k_2^2+k_3^2}\,\frac{\sqrt{k_2^2+k_3^2}}{k} = k$$

$$k_2'' = -k_1\frac{\sqrt{k_2^2+k_3^2}}{k} + \sqrt{k_2^2+k_3^2}\,\frac{k_1}{k} = 0$$

$$\times\begin{pmatrix} \frac{k_1}{k} & \frac{\sqrt{k_2^2+k_3^2}}{k} & 0 & 0 \\ -\frac{\sqrt{k_2^2+k_3^2}}{k} & \frac{k_1}{k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{k_2}{\sqrt{k_2^2+k_3^2}} & \frac{k_3}{\sqrt{k_2^2+k_3^2}} & 0 \\ 0 & -\frac{k_3}{\sqrt{k_2^2+k_3^2}} & \frac{k_2}{\sqrt{k_2^2+k_3^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{k_4}{\sqrt{k_4^2-k^2}} & 0 & 0 & -\frac{k}{\sqrt{k_4^2-k^2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{k}{\sqrt{k_4^2-k^2}} & 0 & 0 & \frac{k_4}{\sqrt{k_4^2-k^2}} \end{pmatrix} \times$$

$$
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{k_2}{\sqrt{k_2^2+k_3^2}} & -\frac{k_3}{\sqrt{k_2^2+k_3^2}} & 0 \\
0 & \frac{k_3}{\sqrt{k_2^2+k_3^2}} & \frac{k_2}{\sqrt{k_2^2+k_3^2}} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\times
\begin{pmatrix}
\frac{k_1}{k} & -\frac{\sqrt{k_2^2+k_3^2}}{k} & 0 & 0 \\
\frac{\sqrt{k_2^2+k_3^2}}{k} & \frac{k_1}{k} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

$$
=\begin{pmatrix}
\frac{k_1}{k} & -\frac{\sqrt{k_2^2+k_3^2}}{k} & 0 & 0 \\
\frac{k_2}{k} & \frac{k_1 k_2}{k\sqrt{k_2^2+k_3^2}} & -\frac{k_3}{\sqrt{k_2^2+k_3^2}} & 0 \\
\frac{k_3}{k} & \frac{k_1 k_3}{k\sqrt{k_2^2+k_3^2}} & \frac{k_2}{\sqrt{k_2^2+k_3^2}} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

$$
=\begin{pmatrix}
\frac{1}{k^2}\left\{\frac{k_1^2 k_4}{x}+(k_2^2+k_3^2)\right\} & \frac{k_1 k_2}{k^2}\left(\frac{k_4}{x}-1\right) & \frac{k_1 k_3}{k^2}\left(\frac{k_4}{x}-1\right) & -\frac{k_1}{x} \\
\frac{k_1 k_2}{k^2}\left(\frac{k_4}{x}-1\right) & \frac{1}{k^2}\left\{\frac{k_1^2 k_4}{x}+(k_1^2+k_3^2)\right\} & \frac{k_2 k_3}{k^2}\left(\frac{k_4}{x}-1\right) & -\frac{k_2}{x} \\
\frac{k_1 k_3}{k^2}\left(\frac{k_4}{x}-1\right) & \frac{k_2 k_3}{k^2}\left(\frac{k_4}{x}-1\right) & \frac{1}{k^2}\left\{\frac{k_3^2 k_4}{x}+(k_1^2+k_2^2)\right\} & -\frac{k_3}{x} \\
-\frac{k_1}{x} & -\frac{k_2}{x} & -\frac{k_3}{x} & \frac{k_4}{x}
\end{pmatrix}
$$

Interacting fields

~~$\gamma_\mu'[x^\mu, \psi] + (a A^{(0)}_\mu + a A^{(1)}_\mu \tau_1)\gamma^\mu \psi \cdot A_\mu + (a_2\tau_2 + a_3\tau_3)$~~
$\gamma_\mu' \psi \cdot$

$$D_r \psi = \gamma_\mu'[x^\mu, \psi] + \lambda \tau_3 \psi + (a' A^\mu + b' B^\mu \tau_1)\gamma_\mu' \psi + (c' C^\mu \tau_2 + d' D^\mu \tau_3)\gamma_\mu' \psi = 0$$

$$D_X \psi = \gamma^\mu [p_\mu, \psi] + mc\psi + (a A_\mu + b B_\mu \tau_1)\gamma^\mu \psi + (c C_\mu \tau_2 + d D_\mu \tau_3)\gamma^{\mu\prime} \psi = 0$$

$$D_r \psi = \{ \gamma_\mu' \gamma^\mu + \lambda\tau_3 + (a' \overset{\to X-\frac{r}{2}}{A^\mu} + b' B^\mu \tau_1)\gamma_\mu' + (c' C^\mu \tau_2 + d' D^\mu)\gamma_\mu \}\psi = 0$$

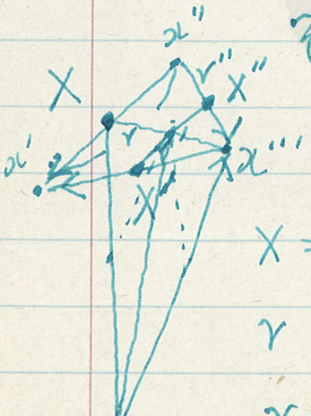
$$D_X \psi = \{ \gamma^\mu \frac{\partial}{\partial X^\mu} + mc + (a \overset{\to X-\frac{r}{2}}{A_\mu} + b B_\mu \tau_1)\gamma^\mu + (c C_\mu \tau_2 + d D_\mu \tau_3)\gamma^{\mu\prime} \}\psi = 0$$

$$(D_X D_r - D_r D_X)\psi$$

$$= \gamma^{(1)}_{(1)} \gamma^\mu \frac{\partial}{\partial X^\mu} \quad a' \frac{\partial A^\mu}{\partial X^\mu}$$

A^μ: local field
$(x'|A^\mu \psi|x'') = (x'|A^\mu|x') \times (x'|\psi|x'')$
$A^\mu \psi(X, r) = A^\mu(X - \frac{r}{2}) \psi(X, r)$

commutativity of A^μ etc and ψ.



$$(x'|A^\mu \psi|x'') = (x'|A^\mu|x''')(x'''|\psi|x'')$$

$$A^\mu \psi(X, r) = A^\mu(X', r')\psi(X'', r'')$$

$$X = \frac{x' + x''}{2}, \quad r = x' - x'' \qquad X' = \frac{x' + x'''}{2}, \quad r' = x' - x''' \qquad X'' = \frac{x''' + x''}{2}, \quad r'' = x''' - x''$$

$$r = r' + r'' \qquad X' - X'' = \frac{r}{2}$$

$$x''' = X' + X'' - X$$

Natural unit $\hbar, c, \lambda_0$

$$L = \mathrm{Trace}\Big\{ \psi^\dagger \gamma_\mu' [x^\mu, \psi] + \psi^\dagger \lambda \tau_3 \psi$$

$$+ \psi^\dagger \gamma^\mu [p_\mu, \psi] + \psi^\dagger \overset{\varkappa}{\cancel{ha}} c\, \psi$$

$$+ \psi^\dagger \gamma^\mu A_\mu \psi + \Big\}$$

Interaction of Nonlocal Spinor Field and Local vector Field

$$\left.\begin{aligned} &\gamma'_\mu[x^\mu,\psi]+\lambda\tau_3\psi+a'A^\mu\gamma'_\mu\psi=0\\ &\gamma^\mu[p_\mu,\psi]+mc\psi+aA_\mu\gamma^\mu\psi=0\end{aligned}\right\}$$

$$(x'|A^\mu\psi)x'')=(x'|A^\mu|x')(x'|\psi|x'')$$

$$A^\mu\psi(X,r)=A^\mu(X+\frac{r}{2})\psi(X,r)$$

$$\left.\begin{aligned} &\{\gamma'_\mu r^\mu+\lambda\tau_3+a'A^\mu(X+\frac{r}{2})\gamma'_\mu\}\psi(X,r)=0\\ &\{\gamma^\mu\frac{\partial}{\partial X^\mu}+mc+aA_\mu(X+\frac{r}{2})\gamma^\mu\}\psi(X,r)=0\end{aligned}\right\}$$

$$\{2\gamma^\mu\frac{\partial}{\partial X^\mu}+a\gamma^\mu A_\mu(X+\frac{r}{2})-a'\frac{\partial A^\mu}{\partial X^\mu}\}\psi=0$$

$$\cancel{\gamma^\mu A_\mu(X-\frac{r}{2})}$$

$$\gamma'_\mu[x^\mu,\psi]+\lambda\tau_3\psi+a'\gamma'_\mu\langle A^\mu,\psi\rangle=0 \quad *$$

$$\gamma^\mu[p_\mu,\psi]+mc\psi+a\gamma^\mu\langle A_\mu\psi\rangle=0$$

$$\{2\gamma^\mu\frac{\partial}{\partial X^\mu}+a\gamma^\mu(A_\mu(X-\frac{r}{2})+A_\mu(X+\frac{r}{2}))-a'(\frac{\partial A^\mu(X-\frac{r}{2})}{\partial X^\mu}+\frac{\partial A^\mu(X+\frac{r}{2})}{\partial X^\mu})\}\psi=0$$

the Spinor particle can neither stay at rest, nor move with const. vel., when it interacts with another field.

$$*\ \langle A,B\rangle=AB+BA$$

(x, r) $(x, -r)$

$x + \frac{r}{2}$ $\quad x + \frac{r}{2}$

$\cancel{\partial}_\mu \quad \cancel{p}$

$[x_\mu [x^\mu, A_\nu]] + e \psi^\dagger \gamma_\nu \psi = 0$

$[p_\mu [p^\mu, A_\nu]] + \cancel{i\hbar^2 e}$
$+ e \quad \psi^\dagger \gamma_\nu \psi = 0$

$[p_\mu [p^\mu ([x_\lambda [x^\lambda A_\nu]])]$

$\cancel{= \frac{\partial}{\partial x}}$

$x - \frac{r}{2}$

$A_\nu (x + \frac{r}{2})$

Normalization of Nonlocal Field ①

$$U(X_\mu, r_\mu) = \int\cdots\int (dk)^4 (dl)^4 u(k_\mu, l_\mu)\,\delta(k_\mu k^\mu + \varkappa^2)$$
$$\times\, \delta(l_\mu l^\mu - \lambda^2)\,\delta(k_\mu l^\mu)\, \exp(i k_\mu X^\mu) \prod_\mu \delta(r_\mu + l_\mu)$$
$$dk_1 dk_2 dk_3 dk_4 \quad dl^1 dl^2 dl^3 dl^4$$

$$U = \int\cdots\int (dk_\mu)^4 (dl^\mu)^4 u(k_\mu, l_\mu)\,\delta(k_\mu k^\mu + \varkappa^2)$$
$$\times\, \delta(l_\mu l^\mu - \lambda^2)\,\delta(k_\mu l^\mu)\,\exp(i k_\mu x^\mu)\,\exp\left(\frac{i}{\hbar} l^\mu p_\mu\right)$$

$$[u(k_\mu, l_\mu),\, u^*(k'_\mu, l'_\mu)] = \delta(k_\mu, k'_\mu)\,\delta(l_\mu, l'_\mu)$$

$\delta(k_\mu k^\mu + \varkappa)\,\delta(\varkappa l')$ $\quad \delta\ \delta\ \delta$

$$U = \sum_{k^4_4 = \pm\sqrt{k^2+\varkappa^2}} \int\cdots\int dk_1 dk_2 dk_3 \frac{u(k_\mu, l_\mu)}{2k_4^4}\,\delta(l_\mu l^\mu - \lambda^2)\,\delta(k_\mu l^\mu)\,(dl^\mu)^4$$
$$\times\, \exp(i k_\mu x^\mu)\,\exp\left(\frac{i}{\hbar} l^\mu p_\mu\right)$$

$$\delta(l_\mu l^\mu - \lambda^2)\,\delta(k_\mu l^\mu)\,(dl)^4$$
$$= \delta(l'_\mu l'^\mu - \lambda^2)\, l'^2 \underset{dl'}{} \sin\theta' d\theta' d\varphi'\, \delta(k'_\mu l'^\mu)\, dl'^4 \quad \varkappa l'_4$$
$$= -\delta(l' - \lambda)\,\frac{l'}{2}\, dl' \sin\theta' d\theta' d\varphi'\, \delta(l'_4)\, dl'_4 \frac{1}{\varkappa}.$$

$$U = -\sum_{k^4 = \pm\sqrt{k^2+\kappa^2}} \int\cdots\int dk_1 dk_2 dk_3 \frac{u(k_\mu; \theta', \varphi')}{2k_4} \frac{\lambda}{2\varkappa} \sin\theta' d\theta' d\varphi'$$
$$\times\, \exp(i k_\mu x^\mu)\,\exp\left(\frac{i}{\hbar} l'^\mu p'_\mu\right)$$

$$\left.\begin{aligned} k^4_4 &= \pm\sqrt{k^2+\varkappa^2} \\ l'_1 &= \lambda \sin\theta' \cos\varphi' \quad \text{etc} \\ l'_4 &= k_\mu l^\mu/\varkappa = 0 \\ p'_\mu &= a_{\mu\nu} p_\nu . \end{aligned}\right\} \text{for } k^4 > 0.$$

$k_1 = \frac{2\pi}{L} n$

$\frac{\hbar}{L}\left(\frac{L}{2\pi}\right)^3 \cancel{(2\pi)^3} dk_1 dk_2 dk_3$

~~$u(k_\mu, \theta', \varphi') =$~~

$$U = \iiint \frac{\lambda\, dk_1 dk_2 dk_3}{4\kappa\sqrt{k^2+\kappa^2}} \Big[u(k_1 k_2 k_3\ \theta' \varphi') \cancel{\exp}\ \cancel{\exp \tfrac{i\lambda}{\hbar}(\sin\theta' \cdots}$$

$$\exp i(k_1 x_1 + k_2 x_2 + k_3 x_3 - \sqrt{k^2+\kappa^2}\, x_4)$$

$$+ v^*(+k_1 + k_2 + k_3\ \theta' \varphi') \exp -i(k_1 x_1 + \cdots - \sqrt{k^2+\kappa^2}\, x_4) \Big]$$

$$\sin\theta' d\theta' d\varphi' \exp \frac{i\lambda}{\hbar}(\sin\theta' \cos\varphi' p_1' + \sin\theta' \sin\varphi' \cdot p_2' + \cos\theta' \cdot p_3') \Big]$$

$$u(k_1 k_2 k_3, \theta', \varphi') = \sum_{l,m} u(k_1 k_2 k_3, l, m)\, P_l^m(\theta', \varphi')$$

$$v^*(k_1 k_2 k_3\ \theta' \varphi') = \sum_{l,m} v^*(k_1 k_2 k_3, l, m)\, \tilde{P}_l^m(\theta', \varphi')$$

$$U = \left(\frac{2\pi}{L}\right)^3 \sum_{\substack{k_1 k_2 k_3 \\ l,m}} \frac{\lambda}{4\kappa\sqrt{k^2+\kappa^2}} \Big\{ u(k, l, m)\, U_{klm} + v^*(k, l, m)\, U^*_{klm} \Big\}$$

$$U_{klm} = \exp(k_1 x_1 \cdots - \sqrt{\ }\, x_4) \times \iint \sin\theta' d\theta' d\varphi' P_l^m(\theta', \varphi') \exp \frac{i\lambda}{\hbar}(\sin\theta' \cos\varphi' p_1' + \cdots)$$

$$U^*_{klm} = \exp i(-k_1 x_1 - \cdots + \sqrt{\ }\, x_4) \times \iint \sin\theta' d\theta' d\varphi' \tilde{P}_l^m(\theta', \varphi') \exp(-\frac{i\lambda}{\hbar})(\cdots)$$

$$[u(k, l, m), u^*(k', l', m')] = C\, \delta_{k_1 k_1'} \delta_{k_2 k_2'} \delta_{k_3 k_3'} \delta_{ll'} \delta_{mm'}$$

②

l_μ
k_μ : fixed

$\cancel{\delta(\ )\delta(\ )\delta(\ )}$ $\delta(\ )\delta(\ )\delta(\ )$

$$\int\cdots\int[u(k_\mu, l_\mu), u^*(k'_\mu l'_\mu)](dk')^4(dl')^4 = 1.$$

θ', φ'

$$\sum_{k'_4=7}\int\cdots\int[u(k_\mu, l_\mu), u^*(k'_\mu l'_\mu)]\cancel{\delta(k_\mu k^\mu + \varkappa^2)\delta(l_\mu l^\mu - \lambda^2)}$$

$$\cancel{\delta(k_\mu l^\mu)}\cdot\frac{1}{2k'_4}\frac{\lambda}{2\varkappa}\cdot dk'_1 dk'_2 dk'_3 \cancel{d\theta} \sin\theta' d\theta' d\varphi' = 1$$

$$\sum_{k'l'm'}[u(k, l, m), u^*(k', l', m')] = C$$

$$\int\cdots\int(dk)^4(dl)^4[u(k,l,m), u(k'_\mu, l'_\mu)]\delta(\ )\delta(\ )\delta(\ )$$

$\iint u(k,l,m)$ / $P_l^m(\theta,\varphi)$.

$$\left(\frac{L}{2\pi}\right)^3\cdot\frac{4\varkappa\sqrt{k^2+\varkappa^2}}{\lambda\cancel{\sqrt{k^2}}\cancel{2}}P_l^m(\theta', \varphi') = C$$

$$C = \left(\frac{2\pi}{L}\right)^3\frac{\lambda}{4\varkappa\sqrt{k^2+\varkappa^2}}$$

$$N_{k} = \sqrt{\left(\frac{2\pi}{L}\right)^3\frac{\lambda}{4\varkappa\sqrt{k^2+\varkappa^2}}}$$

$$U = \sum_{k,l,m} N_k\{a(k,l,m)U_{klm} + b^*(k,l,m)U^*_{klm}\}$$

$$U^* = \sum_{k,l,m} N_k\{a^* \quad U^* + \quad b\,U\}$$

$$[a(k,l,m), a^*(k'l'm')] = \delta_{kk'}\delta_{ll'}\delta_{mm'}$$

$$[u(k_\mu, l_\mu), u^*(k'_\mu, l'_\mu)]\,\delta(k'_\mu k'^\mu + \varkappa^2)$$
$$\times\,\delta(l'_\mu l'^\mu - \lambda^2)\,\delta(k'_\mu l'^\mu) = \delta(k,k')\,\delta(l,l')$$

$$[\bar{u} = u \cdot \delta \cdot \delta \cdot \delta.$$

$$[\bar{u}(k,l), \bar{u}^*(k',l')] = \delta(k,k')\,\delta(l,l')$$
$$\times\,\delta(k,\varkappa)\,\delta(\cancel{k},\lambda)\,\delta(kl)\ \cancel{\delta(k',\varkappa)\,\delta(k',\lambda)\,\delta(k'l')}$$

$$\lambda \to 0\quad \frac{2\varkappa}{\lambda}\,\delta(l'_\mu l'^\mu - \lambda^2)\,\delta(k'_\mu l'^\mu) \to \text{—}\ \delta(l,l')\,\delta($$

The other type of ~~commun~~

If we ~~assume~~ start from the commutation relations

in stead of (37), we arrive at the wellknown contradiction, in the limit of $\lambda \to 0$ which prohibits the ~~local~~ elementary ~~scalar field~~ particles with spin 0 ~~pro~~ from obeying Fermi statistics.

③

$$\int\!\cdots\!\int [\bar{u}(k_\mu, l_\mu), \bar{u}^*(k'_\mu, l'_\mu)] (dk'_\mu)^4 (dl'_\mu)^4$$

$$= \delta(k_\mu k^\mu + \varkappa^2)\,\delta(l_\mu l^\mu - \lambda^2)\,\delta(k_\mu l^\mu)$$

or

$$\int\!\cdots\!\int [u(k_\mu, l_\mu), u^*(k'_\mu, l'_\mu)]\,\delta(k'_\mu k'^\mu + \varkappa^2)\,\delta(l'_\mu l'^\mu - \lambda^2)$$

$$\times\,\delta(k'_\mu l'^\mu)\,(dk'_\mu)^4 (dl'_\mu)^4 = 1$$

$$\sum_{k,l^2,\Theta,\Phi}\!\!\iint [u(k_\mu, l_\mu)\, u^*(k'_\mu, l'_\mu)] \left(\frac{2\pi}{L}\right)^3 \cancel{\lambda \sin} \left(\cancel{\frac{2\pi}{L}}\right)^3 \frac{\lambda \sin\Theta\, d\Theta\, d\Phi}{4\varkappa\sqrt{k^2+\varkappa^2}}$$

$$= 1.$$

$$\cancel{[u(k_\mu, l_\mu), u^*(k'_\mu, l'_\mu)] =}$$

$$\sum_{l,m}\iint [u(k, l, m)\, P_l^m(\Theta, \Phi)\, \cancel{\sin\Theta\, d\Theta\, d\Phi} \left(\frac{2\pi}{L}\right)^3 \frac{\lambda \sin\Theta'\, d\Theta'\, d\Phi'}{4\varkappa\sqrt{k^2+\varkappa^2}}$$

$$[u^*(k, l, m)]$$

$$\cancel{u(k_\mu, l_\mu)}\; P_l^{m*}(\Theta', \Phi') \qquad \delta(\Theta - \Theta', \Phi - \Phi')$$

$$\delta(\vartheta - \Theta, \varphi - \Phi) = \sum_{l,m} Y_{lm}^*(\Theta, \Phi)\, Y_{lm}(\vartheta, \varphi)$$

$$\int \delta(\vartheta - \Theta, \varphi - \Phi)\, d\omega = 1.$$

$$\int |Y_{lm}|^2\, d\omega = 1.$$

$$Y_{lm}(\theta, \varphi) = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{(2l+1)}{2} \frac{l - |m|!}{l + |m|!}} \frac{(1-x^2)^{\frac{|m|}{2}}}{2^l \cdot l!} \frac{d^{l+|m|}}{dx^{l+|m|}} (x^2-1)^l \cdot e^{im\varphi}$$

$$x = \cos\theta$$

$$[u(\underline{k},l,m),\ u^*(\underline{k},l,m)]\left(\frac{2\pi}{L}\right)^3\frac{\lambda}{4\kappa\sqrt{k^2+\kappa^2}}$$

$$=1,$$

$$a(\underline{k},l,m)=\sqrt{\left(\frac{2\pi}{L}\right)^3\cdot\frac{\lambda}{4\kappa\sqrt{k^2+\kappa^2}}}\cdot u$$

$$a^*(\underline{k},l,m)=\sqrt{\quad ,, \quad}\cdot u^*$$

$$[a(\underline{k},l,m),\ a^*(\underline{k}',l',m')]=\delta(k,k')\,\delta(l,l')\,\delta(m,m')$$

Nonlocal Spinor Field

①

$$\gamma^\mu [p_\mu, \psi] + mc\,\psi = 0$$

$$\beta_\mu [x^\mu, \psi] + \omega\lambda\,\psi = 0$$

$$\beta_\mu \gamma^\nu [x^\mu [p_\nu, \psi]] = 0$$

$$\gamma^\nu \beta_\mu [p_\nu, [x^\mu, \psi]] = 0$$

$$[\beta_\mu, \gamma^\nu][x^\mu [p_\nu, \psi]] = 0$$

$$[\beta_\mu, \gamma^\nu] = c\,\delta_{\mu\nu}$$

$$\gamma^1 = i\rho_2\sigma_1, \quad \gamma^2 = i\rho_2\sigma_2, \quad \gamma^3 = i\rho_2\sigma_3, \quad \gamma^4 = \rho_3$$

$$\beta_1 = \rho_3\sigma_1, \quad \beta_2 = \rho_3\sigma_2, \quad \beta_3 = \rho_3\sigma_3, \quad \beta_4 = -i\rho_2$$

$$[\beta_\mu, \gamma^\nu] = \rho_1$$

Transformation properties of ψ.

W. Pauli, Annales de l'Institut Henri Poincaré. VI (1936), 109.

E. L. Hill and R. Landshoff, Rev. Mod. Phys. 10 (1938), 87.

$$\left(\sum_k \beta_k \pi_k + i m_0 c\right)\psi = 0$$

$$x_4 = ict, \qquad \pi_k = -i\hbar \frac{\partial}{\partial x_k}$$

$$\beta_1 = i\alpha_4\alpha_1 = -\rho_2\sigma_1 \qquad \beta_2 = i\alpha_4\alpha_2 = -\rho_2\sigma_2$$

$$\beta_3 = i\alpha_4\alpha_3 = -\rho_2\sigma_3 \qquad \beta_4 = \alpha_4 = \rho_3$$

$$[\alpha_i, \alpha_k]_+ = 2\delta_{ik}$$

Rotation:

$$\chi_1' = \cos\tfrac{1}{2}\xi\, e^{i\frac{1}{2}(\eta+\zeta)}\chi_1 + i\sin\tfrac{1}{2}\xi\, e^{-i\frac{1}{2}(\eta-\zeta)}\chi_2$$

$$\chi_2' = i\sin\tfrac{1}{2}\xi\, e^{i\frac{1}{2}(\eta-\zeta)}\chi_1 + \cos\tfrac{1}{2}\xi\cdot e^{-i\frac{1}{2}(\eta+\zeta)}\chi_2$$

$$\chi_3' = \cos\tfrac{1}{2}\xi\, e^{i\frac{1}{2}(\eta+\zeta)}\chi_3 + i\sin\tfrac{1}{2}\xi\, e^{-i\frac{1}{2}(\eta-\zeta)}\chi_4$$

$$\chi_4' = i\sin\tfrac{1}{2}\xi\, e^{i\frac{1}{2}(\eta-\zeta)}\chi_3 + \cos\tfrac{1}{2}\xi\cdot e^{-i\frac{1}{2}(\eta+\zeta)}\chi_4.$$

	x	y	z
x'	$\cos\zeta\cos\eta - \sin\zeta\cos\xi\sin\eta$	$\cos\zeta\sin\eta + \sin\zeta\cos\xi\cos\eta$	$\sin\eta\sin\xi$
y'	$-\sin\zeta\cos\eta - \cos\zeta\cos\xi\sin\eta$	$-\sin\zeta\sin\eta + \cos\zeta\cos\xi\cos\eta$	$\cos\eta\sin\xi$
z'	$\sin\xi\sin\eta$	$-\sin\xi\cos\eta$	$\cos\xi$

Lorentz transf.

$$\chi_1' = \cosh\tfrac{1}{2}\vartheta\cdot\chi_1 + \sinh\tfrac{1}{2}\vartheta\cdot\chi_4$$

$$\chi_2' = \cosh\tfrac{1}{2}\vartheta\cdot\chi_2 + \sinh\tfrac{1}{2}\vartheta\,\chi_3$$

$$\chi_3' = \cosh\tfrac{1}{2}\vartheta\,\chi_3 + \sin\tfrac{1}{2}\vartheta\,\chi_2$$

$$\chi_4' = \cosh\tfrac{1}{2}\vartheta\,\chi_4 + \sin\tfrac{1}{2}\vartheta\,\chi_1$$

$$\cosh\vartheta = (1-\beta^2)^{-\frac{1}{2}} \qquad \sinh\vartheta = \beta(1-\beta^2)^{-\frac{1}{2}}$$

$$x' = \frac{x-\beta ct}{\sqrt{1-\beta^2}} \qquad t' = \frac{t-\beta x/c}{\sqrt{1-\beta^2}}$$

Reflection

R_0: $x\to -x$, $y\to y$, $z\to z$, $t\to t$

(ρ_3) $\chi_1\to\chi_1$, $\chi_2\to\chi_2$, $\chi_3\to-\chi_3$, $\chi_4\to-\chi_4$

R_t: $x\to x$, $y\to y$, $z\to z$, $t\to -t$

(ρ_2) $\chi_1\to\chi_2^*$, $\chi_2\to-\chi_1^*$, $\chi_3\to\chi_4^*$, $\chi_4\to-\chi_3^*$

$$x'_\mu = a_{\mu\nu} x_\nu \quad \rightarrow \quad \psi' = S\psi.$$

proper Lorentz transformation

$\|a_{\mu\nu}\| = +1$ $\qquad \psi' = S\psi.$

improper Lorentz transformation

$\|a_{\mu\nu}\| = -1$ $\qquad \psi' = \tau_1 S \psi.$

Classification of Lorentz transformations:

(i) Pure Rotation: = product of even number
$e_r - e_t$ of reflections.

(ii) Rotation - Reflection: = product of
$o_r - e_t$ even number of time-reversals and ~~odd number of~~ space-reflections

(iii) Time-Reversal - ~~Space~~ Rotation Reflection: = product
$e_r - o_t$ of even number of space reflection and odd number of time reversal.

(iv) Time-Reversal - Rotation-Reflection:
$o_r - o_t$

$$\frac{\partial x'_4}{\partial x_4} \neq 0 \qquad >0 \qquad <0$$

$$\frac{\partial(x'_1 x'_2 x'_3)}{\partial(x_1 x_2 x_3)} \neq 0 \qquad >0 \qquad <0$$

$$\frac{\partial(x'_1 x'_2 x'_3 x'_4)}{\partial(x_1 x_2 x_3 x_4)} \neq 0 \qquad +1 \qquad -1$$

prop. Lor. transf. $x'_\mu = a_{\mu\nu} x_\nu$, $\|a_{\mu\nu}\| = 1$.

$\psi' = S\psi$

improp. Lor. transf. $\|a_{\mu\nu}\| = -1$.

$\psi' = \omega_i S \psi$

$$\gamma^\mu [p_\mu, \psi] + mc\psi = 0$$
$$\beta_\mu [x^\mu, \psi] + \omega_3 \lambda \psi = 0$$

$x'_\mu = a_{\mu\nu} x_\nu$: $\psi' = (\omega_1)^n S \psi$
$\| a_{\mu\nu} \| = (-1)^n$.

$$\gamma^\mu \frac{\partial \psi(X_\mu, r_\mu)}{\partial X^\mu} + i\kappa \psi(X_\mu, r_\mu) = 0$$

$$\beta_\mu r^\mu \psi(X_\mu, r_\mu) + \omega_3 \lambda \psi(X_\mu, r_\mu) = 0$$

$$\psi(X_\mu, r_\mu) = \bar{u}(k_\mu, r_\mu) \exp(i k_\mu X^\mu)$$

$$\gamma^\mu k_\mu \bar{u} - \kappa \bar{u} = 0$$
$$\beta_\mu r^\mu \bar{u} + \omega_3 \lambda \bar{u} = 0$$

$$\left.\begin{aligned} (k_\mu k^\mu + \kappa^2) \bar{u} &= 0 \\ (r_\mu r^\mu - \lambda^2) \bar{u} &= 0 \\ k_\mu r^\mu \bar{u} &= 0 \end{aligned}\right\}$$

The second quantization can be performed by assuming commutation relations of the type (37)' between field quantities.

~~Et~~ Each $\bar{u} = u(k_\mu, r_\mu)\, \delta(k_\mu k^\mu + \kappa^2)\, \delta(r_\mu r^\mu - \lambda^2) \times \delta(k_\mu r^\mu)$

Each of eight components of $\bar{u}$ can be expanded in the same way as ~~in~~ the scalar operator $\bar{u}$ in the preceding sections. Detailed analysis of ~~the~~ account will be made ~~in~~ ~~the~~ part II of

① Interaction between N.L.F.

this paper. At any rate it is now clear that there ~~is~~ ~~are~~ exist nonlocal scalar, vector and spinor fields corresponding ~~respectively~~ to the assembly of particles with the mass, radius and the spins 0, 1 and ½ respectively.

V. Interaction ~~of~~ between Nonlocal Fields.

Now ~~the~~ we have to go into the problem of the interaction between two nonlocal fields or between ~~a~~ nonlocal and local fields. The first question, with which we are met here, is whether we can ~~find~~ start from Schrödinger ~~equation~~ for the total system or ~~or any sub~~ any substitute for it, thus retaining the most essential feature of quantum mechanics. We know that Schrödinger equation for the probability amplitude, which is not relativistic in that it ~~refers to~~ is obviously a differential equation with respect to ~~a particular~~ the one time variable alone, can be ~~extended~~ ~~reformu~~ ~~incorporated~~ extended ~~in the relativistic theory of local fields~~ ~~many-time formalism~~ ~~without~~ to a relativistic form as ~~shown~~ well known in Dirac's many-time formalism and

Tomonaga-Schwinger's super-many-time formalism, as long as we are dealing with local fields satisfying ~~the~~ infinitesimal commutation relations. ~~In other words, This is mainly because~~ On the contrary, however, whenever ~~once~~ we introduce either the nonlocalizability of either the field itself or that of ~~of the~~ interaction with other fields, the clean-cut distinction between space-like and time-like direction ~~to~~ is impossible because the interaction ~~opera~~ term, which is the product of at least three ~~two~~ field operators contains, in general, such ~~the~~ operators ~~as connecting two points~~, the displacement operators in the time-like direction as well as those in the space-like direction. Thus, even if there exists ~~we assume can start from~~ an equation of Schrödinger type, it cannot be solved uniquely by ~~assigning~~ giving the initial condition at a certain time.

Under these circumstances, it is recommended to have recourse to more general formalism, such as the S-matrix scheme, which was proposed by Heisenberg ( ). ~~In~~ other words, ~~Thus,~~ Namely, we had ~~ourselves by~~ better start from the integral formalism rather than the differential formalism. In local field theory, ~~they are proved to be equivalent in~~ the integral formalism such as that developed by Feynman( ) ~~are~~ was ~~proved to be equivalent~~ can be deduced from ~~differential~~ the ordinary formalism ( )( )

In nonlocal field theory, it may well happen that there ~~is no~~ remains only some kind of integral formalism. ~~It part II, it will.~~ In fact, it will be shown in Part II. that S-matrix scheme is very well fitted to the nonlocal field theory.

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