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Quantum Theory of Non-local Fields

Part II. General Theory

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(Feb. , 1950)

Abstract

I, Introduction

II, Elementary Non-local Systems

Abstract

~~Non-local field~~ ~~quantized~~ operator
General properties of non-local fields were considered. ~~and~~ ~~Exact,~~ thus ~~types of irreducible fields~~ it was shown that irreducible fields could be classified by ~~the mass, radius four~~ the values of four constants, three of which ~~we~~ corresponded respectively to the mass, radius and magnitude of the internal angular momentum.
~~Further~~
in connection with the problem of the invariance of ~~non~~ with respect to ~~the~~ inhomogeneous Lorentz transformations, the whole group of
Further, ~~non-local~~ space-time displacement operators were introduced as ~~a~~ a particular kind of non-local operators.
As a tentative means of dealing with the interaction of non-local fields, an invariant matrix was defined ~~as~~ by the space-time integral of a certain invariant operator, which was a sum of products of non-local field operators and displacement operators. ~~It~~ Thus, it was shown that this matrix ~~had~~ satisfied all ~~all the following~~ conditions, which

Finally,

~~was~~ were required for the S-matrix in general. The question whether this matrix could be identified with the S-matrix of H was discussed ~~at the~~

The notion of elementary particle ~~in modern the~~ has been ~~inseparable~~ intimately connected with the procedure of ~~reduc~~ decomposing the quantized field into irreducible parts. If we extend the concept of the field itself, the definition of elementary particle will be changed accordingly. ~~Com~~

II. Elementary Non-local Systems

~~We start again~~ In order to redefine the elementary particle in non-local field theory, we must investigate the problem of reduction of an arbitrary non-local field into a mixture of irreducible fields ~~which are each of which transforms~~. We start again from ~~the~~ an arbitrary unquantized non-local scalar field U, which can be represented by an arbitrary matrix

$$(x'|U|x''), \qquad (1)$$

where x' and x'' stand for x_μ' and x_μ'' $(\mu=1,2,3,4)$ respectively. The matrix (1) can be regarded as before as a function $U(X, r)$ of two sets of ~~pa~~ real variables $X = \frac{1}{2}(x'+x'')$, $r = x'-x''$ as shown in Part I. Then, an arbitrary function $U(X,r)$ can be expanded in the ~~fac~~ form

$$U(X,r) = \int\cdots\int u(k_\mu, l)\exp(i k_\mu X^\mu) \prod_\mu \delta(r_\mu - l_\mu)(dk_\mu)^4(dl^\mu)^4, \qquad (2)$$

where $u(k,l)$ is an arbitrary function of two sets of parameters k_μ and l_μ. Now, if ~~you~~ we perform an arbitrary homogeneous Lorentz transformation

$$x_\mu' = a_{\mu\nu} x_\nu, \qquad (4)$$

$$U = \int\cdots\int u(k, r)\exp(i k_\mu X^\mu)(dk_\mu)^4 \qquad (2)$$

or ~~simple~~

Our problem is now to

It was expected that these simple fields might be examples of irreducible fields.

~~In~~ In order to have ~~more general~~

These must be

(1) In Part I, we ~~considered~~ confined ourselves to a particular types of non-local fields satisfying a set of operator equations. ~~This~~ We shall see later this

where x'_μ $(\mu = 1, 2, 3, 4)$ denote this time the space-time coordinates operators in the new coordinate system. Therewith ~~the~~ two sets of parameters X^μ, r are transformed into

$$X'_\mu = a_{\mu\nu} X_\nu \qquad r'_\mu = a_{\mu\nu} r_\nu \qquad (5)$$

and $U'(X, r)$ becomes

$$U(X', r') = \int \cdots \int u'(k', l') \exp(i k'_\mu X'^\mu)$$

$u'(k', l') = u(k, l)$

$$\times \prod_\mu \delta(r'_\mu - l'_\mu)(dk'_\mu)^4 (dl'^\mu)^4, \quad (6)$$

where k', l' are connected with k, l just as X', r' are connected with X, r. In order that (6) retains ~~has~~ the same form as (3) for arbitrary Lorentz transformation (4), ~~it is necessary~~ that ~~$u(k', l')$ is~~ the function of invariant quantities such as $k_\mu k^\mu$, $l_\mu l^\mu$ and $k_\mu l^\mu$ alone.* ~~In other words~~, This $U(X, r)$ can be written, in general, in the form

$$U(X, r) = \int \cdots \int w(K, L, M)\, \delta(k_\mu k^\mu - K)$$

$$\delta(l_\mu l^\mu - L)\, \delta(k_\mu l^\mu - M),\ \underbrace{\exp(i k_\mu X^\mu)}_{u(k,l)} \times$$

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* If we confine our attention to ~~the~~ ~~those~~ the subgroup of ~~the homogeneous~~ Lorentz group, which does not include the reversal of time, $u(k, l)$ may depend also on $k_4/|k_4|$ provided that k_μ is a time-like vector and similarly for l_μ. Accordingly $w(K, L, M)$ may also depend on $\frac{k_4}{|k_4|}$, $\frac{l_4}{|l_4|}$ for time-like k_μ and l_μ.

$$\times \prod_{\mu=1}^{4} \delta(r_\mu - l_\mu) \cancel{(dk_\mu)^4 (dl^\mu)^4\, dK\, dL\, dM,} \quad (7)$$

where ~~the integration with~~ respect to

$$\cancel{u'(k',l') = u(k,l)}$$

either of the two ~~condition~~ requirements must be satisfied:

(i) Either $u(k,l)$ is a function k and l, which ~~retains~~ retains its form under ~~an~~ an arbitrary homogeneous Lorentz transform. In other words, it is required then

$$u'(k',l') = u(k,l) \quad (7)$$

for an arbitrary transformation

$$k'_\mu = a_{\mu\nu} k_\nu, \quad l'_\mu = a_{\mu\nu} l_\nu, \quad (8)$$

so that u must be the function of invariant quantities such $k_\mu k^\mu$, $l_\mu l^\mu$ and $k_\mu l^\mu$ alone.* Thus, $U(X,r)$ can be written, in general, in the form

$$U(X,r) = \int\cdots\int w(K,L,M)\, \delta(k_\mu k^\mu - K)\, \delta(l_\mu l^\mu - L)\, \delta(k_\mu l^\mu - M)\, \cancel{f(K,L,M)} \exp(ik_\mu X^\mu) \prod_{\mu=1}^{4} \delta(r_\mu - l_\mu)\, (dk_\mu)^4 (dl_\mu)^4\, dK\, dL\, dM, \quad (9)$$

where ~~the arb~~ $w(K,L,M)$ is an arbitrary function of real parameters K, L, M ** and the integrations with respect to K, L and M extend from $-\infty$ to $+\infty$.* This case ~~will~~ has

nothing to do with the quantized non-local field, because there is no room for the ~~[illegible]~~ application of the method of s.q. ~~[illegible]~~,
~~[illegible]~~ (∵) ~~[illegible]~~ $u(k,l)$ is not a mere function of k and l, but an ~~assembly of~~ ensemble of ~~operations, which~~ quantities, which are subject to the procedure of the second quantization. In other words, if we go over to the ~~quantize~~ quantization of the field U, $u(k,l)$ might be reinterpreted as an ensemble of creation and annihilation operators ~~in connection with~~ concerning the quanta associated with the non-local field U.
In such a case, the requirement of invariance is ~~stated as~~ simply to identify $u(k,l)$ $(\equiv u'(k',l'))$
~~$u'(k',l') = u(k,l)$~~)
with the creation or annihilation ~~operator~~ operator for a particle in the quantum state characterized by k, l. The only effect of ~~the~~ a coordinate transformation is to give ~~a different~~ a new ~~name~~ notation $u'(k',l')$ ~~to~~, which is different from the old notion $u(k,l)$, to the same operator. ~~In other words,~~ ~~the~~ The difference comes only from the change of the names for the same quantum quantum state ~~of~~ due to the change of the reference system.

~~However, the invar quantities~~
Thus, if we compare the two representations (3) and (6) for the same field operator U, ~~we find~~ one-to-one correspondence between each of $u(k,l)$ and each of $u(k',l')$. The requirement of invariance is ~~only~~ fulfilled, ~~is established~~ if and only if we can establish this one-to-one correspondence for any ~~arbitrary Lorentz tran~~ ~~the whole group of homogeneous~~ any Lorentz transformation whatever.

Now, since $k_\mu k^\mu$, $l_\mu l^\mu$ and $k_\mu l^\mu$ are not changed by ~~the~~ any Lorentz transformation, the correspondence is still established, if the domain of integration ~~in~~ on the right hand side of (3) is restricted to ~~the~~ definite values K, L, M of these invariant quantities $k_\mu k^\mu$, $l_\mu l^\mu$ and $k_\mu l^\mu$ respectively. In such a case, (3) ~~can be~~ $U(X,r)$ ~~takes the form~~ can ~~alterna~~ be written in the form

$$U(X,r) = \int\cdots\int u(k,l)\exp(ik_\mu X^\mu)\prod\delta(\ )$$

$$\delta(k_\mu k^\mu - K)\,\delta(l_\mu l^\mu - L)\,\delta(k_\mu l^\mu - M)$$

$$(dk_\mu)^4\,(dl^\mu)^4. \qquad (10)$$

It is now clear that ~~the example of~~ the scalar non-local field, which was dealt with in detail ~~ed~~ in Part I., is a particular example corresponding to the case

$$K \equiv -\kappa^2 \qquad L \equiv +\lambda^2, \qquad M = 0. \quad (11)$$

However, we can further reduce the field, corresponding to each set of value of K, L, M, into irreducible systems. ~~Namely~~, For example, if ~~you~~ we expand the coefficient $u(k, l)$ into power series of l_μ, we have

$$u(k, l) = u^{(0)}(k) + l^\mu u^{(1)}_\mu(k)$$
$$+ (~~k_\mu l^\mu~~)^2 \, l^\mu l^\nu u^{(2)}_{\mu\nu}(k) + \cdots, \qquad (12)$$

~~since $k_\mu l^\mu$ is the only invariant contraction of k_μ and l_μ.~~

where $u^{(0)}$; $u^{(1)}_\mu$, $u^{(2)}_{\mu\nu}$ etc must be a scalar, a vector and a tensor etc. By a Lorentz transformation, l^μ are transformed linearly ~~and~~ so that ~~$u^{(0)}(k)$~~ each of $u^{(0)}(k)$, $u^{(1)}_\mu(k)$, $u^{(2)}_{\mu\nu}(k)$ etc must be transformed ~~and~~ by themselves. Thus, we are left with irreducible systems characterized ~~by~~ respectively by the ~~oper~~ operators $u^{(0)}(k)$, $u^{(1)}_\mu(k)$ etc. One may imagine that the ~~order to~~ above argument remains true, if one interchange k and l. ~~This is not so,~~ But, if we further require the invariance of the formalism

$k_\mu X' = c'_{\mu\nu} k_\nu \left(c'_{\mu\nu} (X'^{\nu} - b'^{\nu}) \right)$

$u(k,l) \exp(i k_\mu X^\mu) = u(k,l) \exp(-i(k'_\mu b'^\mu)) \exp(i k'_\mu X'^\mu)$

II. II. 4.

with respect to the whole group of Lorentz transformation including the inhomogeneous L.T in addition to the h. L. T. ~~above discussed.~~ Namely, ~~The invariance~~ by an inhomogeneous L. T.

$$x'_\mu = a_{\mu\nu}(x_\nu + b_\nu)$$

or

$$x'_\mu = a_{\mu\nu} x_\nu + b'_\mu$$

with $b'_\mu = a_{\mu\nu} b_\nu$,

X and r are transformed into

$$\left.\begin{aligned} X'_\mu &= a_{\mu\nu}(X_\nu + b_\nu) \\ r'_\mu &= a_{\mu\nu} r_\nu \end{aligned}\right\}$$

Accordingly, we have

~~$u'(k', l') =$~~

$$k'_\mu = a_{\mu\nu} k_\nu \qquad l'_\mu = a_{\mu\nu} l_\nu$$

and

$$u'(k', l') = \exp(i k'_\mu b'^\mu) u(k, l)$$

$$= \exp(-i k_\mu b^\mu) u(k, l).$$

The above decomposition (12) still holds, if we assume ~~Thus Still the above proc~~ this ~~implies~~

$$\left.\begin{aligned} u'^{(0)}_a(k') &= \exp(-i k_\mu b^\mu)\, u^{(0)}_a(k) \\ l'^\mu u'^{(1)}_\mu(k') &= \exp(-i k_\mu b^\mu)\, l^\mu u^{(1)}_\mu(k) \end{aligned}\right\}$$

etc.

whereas a similar procedure fails, if we interchange k and l.

It should be noticed that the expansion (12) ~~decomp.~~

is essential the same as the expansion into series of ~~harmonics~~ spherical in Part II, I.

~~As we~~ Thus the simplest elementary system corresponding to $u^0(k)$ is just the one, which was considered in a previous article.(1) The method of second quantization can easily be applied to this system. Namely, we assume relat. invariant commutation relations*

$$\delta(k^2+\varkappa^2)[u(k),\ u(k')] = -C \cdot \frac{k_4}{|k_4|} \prod_\mu \delta(k_\mu + k'_\mu),$$

~~for~~ which is equivalent to the usual comm. relation

$$[u(k_i),\ u^*(k'_i)] = C \cdot \frac{\delta(k_i - k'_i)}{\sqrt{k_4^2+\varkappa^2}},$$

where

$$u^*(-k_\mu) = u(-k_\mu)$$

with $k_4 = -\sqrt{k^2+\varkappa^2}$. $\varkappa^2 = K > 0$.

The expression (10) ~~can~~ reduces now to

$$U(X,r) = \iiint \frac{C^{1/2}}{(k^2+\kappa^2)^{1/4}} \{a(k)\exp(ik_\mu X^\mu) + a^*(k)\exp(-ik_\mu X^\mu)\}(dk_i)^3 \int\cdots\int \prod_\mu \delta(r_\mu - l_\mu)$$

$$\delta(k_\mu l^\mu)\,\delta(l_\mu l^\mu - \lambda^2)(dl_\mu)^4$$

with $\lambda^2 = \Lambda > 0$, $M = 0$.

(1) H. Yukawa, Phys. Rev. 76 (1949), 300.

* We consider real field and write $u(k)$ instead of $u^{(0)}(k)$.

The non-local spinor field can be decomposed into irreducible parts in the similar manner. ~~Namely,~~ We start from the ~~relations (172), (173) in Part I, i.e.~~

$$\cancel{\gamma^\mu\left(\frac{\partial\psi(X_\mu, r_\mu)}{\partial X^\mu}\right) + i\varkappa\psi(X_\mu, r_\mu) = 0},$$

$$\cancel{\beta_\mu r^\mu \psi(X_\mu, r_\mu) + \lambda\psi(X_\mu, r_\mu) = 0},$$

~~for the spinor operator ψ, which is regard as a mat a set of four w functions of X_μ and r_μ. If we expand~~

$$\cancel{\psi(X_\mu, r_\mu) =}$$

spinor operator ψ, which is equivalent to a set of four functions $\psi_i(X_\mu, r_\mu)$ of X_μ and r_μ $(i = 1, 2, 3, 4)$. ψ_i can be expanded in the form

$$\psi_i(X_\mu, r_\mu) = \int\cdots\int u_i(k_\mu, l^\mu)\exp(ik_\mu X^\mu)\prod_\mu \delta(r_\mu - l_\mu)\,(dk_\mu)^4(dl^\mu)^4.$$

This can be decomposed into parts in an invariant way by giving each of $k_\mu k^\mu$, $l_\mu l^\mu$ and $k_\mu l^\mu$ a definite value respectively. Each part can further be expanded into the power series of l^μ as before:

$$u_i(k_\mu, l^\mu) = \cancel{\Sigma}\, u_i^{(0)}(k_\mu) + l^\mu u_{i,\mu}^{(0)}(k_\mu) + \cdots\cdots$$

$u_i^{(0)}(k_\mu)$ ~~is a~~ represents the simple irreducible (nonlocal) spinor field. The next ~~simple~~ terms $l^\mu u_{i\mu}^{(1)}(k_\mu)$ have 3×4 independent terms, because l^μ can ~~take~~ be in any of three independent directions. If we take the ~~positive~~ positive energy states alone, thus there are 6 independent ~~[illegible]~~ quantum states for a given set of values of ~~k~~ momentum ~~k_μ~~ k_μ. They can be decomposed into two parts with 2 components and 4 components respectively, just as in the ~~case of~~ theory of higher spin particles in local field theory.(1)
The former corresponds to the states of resultant spin 1/2 and the latter to ~~that~~ that of spin 3/2.

More precisely, if we first go over to the coordinate system, ~~a~~ which is moving with the particle with the wave vector k_μ, ~~the field quantity~~ $l^\mu u_{i\mu}^{(1)}(k_\mu)$ is different from zero only for those values of l^μ satisfying

$$\left.\begin{aligned} l_1 &= \lambda \sin\theta\cos\varphi \\ l_2 &= \lambda\sin\theta\sin\varphi \\ l_3 &= \lambda\cos\theta \\ l_4 &= 0 \end{aligned}\right\}$$

(1) Fierz, Phys. Rev. 77 (1950), in press.

with

$$k_1 = k_2 = k_3 = 0 \qquad |k_4| = \varkappa .$$

The irreducible part corresponding to the resultant spin 1/2 has the form

$$\cancel{\begin{pmatrix} c_1 \sin\theta\, e^{-i\varphi} \\ c_2 \sin\theta\, e^{i\varphi} \\ 0 \\ 0 \end{pmatrix}},$$

~~where c_1, c_2~~

$$v_1 \begin{pmatrix} \cancel{\sin\theta\, e^{-i\varphi}}\ \cos\theta \\ \sin\theta\, e^{i\varphi} \\ 0 \\ 0 \end{pmatrix} + v_2 \begin{pmatrix} \sin\theta\, e^{-i\varphi} \\ \cancel{\sin\theta\, e^{i\varphi}}\ \cos\theta \\ 0 \\ 0 \end{pmatrix},$$

for positive energy states, where v_1, v_2 are coefficients, which must be quantized according to F.D. statistics.

The other part corresponding to the resultant spin 3/2 has the form

$$v_1' \begin{pmatrix} \sin\theta\, e^{i\varphi} \\ \cos\theta \\ 0 \\ 0 \end{pmatrix} + v_2' \begin{pmatrix} \cos\theta \\ \sin\theta\, e^{-i\varphi} \\ 0 \\ 0 \end{pmatrix} + v_3' \begin{pmatrix} \sin\theta\, e^{-i\varphi} \\ \cos\theta \\ 0 \\ 0 \end{pmatrix}$$

$$+ v_4' \begin{pmatrix} \sin\theta\, e^{i\varphi} \\ \cos\theta \\ 0 \\ 0 \end{pmatrix}.$$

It is very interesting that, apart from the difference in internal structure, there are two kinds of non-local spinor fields, which have both resultant spin ½. The first one is spherical symmetric with respect to

$r_\mu = x'_\mu - x''_\mu$, $({}^2S_{1/2})$ whereas the second one is not $(P_{1/2})$. This opens the possibility of asigning the latter field to nucleons, so that the deuteron q. m. m be understood as due to the spacial asymmetry of the ~~nucleon field~~ individual nucleon itself rather than the asymmetry in the interaction of ~~nucleons~~ between nucleons.