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Quantum Theory of Nonlocal Fields.

Part II. Interaction between Fields.

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Abstract

Footnote to Part I

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I.

Let us consider a system consisting of two ~~kinds of~~ scalar nonlocal fields U and V, ~~which~~ which can be expanded in the form (see equation (42) and (49))

$$(1)\left\{\begin{aligned} U &= \sum_{\underline{k}}\sum_{l,m}\sqrt{\left(\frac{2\pi}{L}\right)^3\frac{\lambda}{4\kappa\sqrt{k^2+\kappa^2}}}\{a(\underline{k},l,m)U(\underline{k},l,m) \\ &\qquad + b^*(\underline{k},l,m)U^*(\underline{k},l,m)\} \\ U^* &= \sum_{\underline{k}}\sum_{l,m}\sqrt{\left(\frac{2\pi}{L}\right)^3\frac{\lambda}{4\kappa\sqrt{k^2+\kappa^2}}}\{b(\underline{k},l,m)U(\underline{k},l,m) \\ &\qquad + a^*(\underline{k},l,m)U^*(\underline{k},l,m)\} \\ V &= \sum_{\underline{k}'}\sum_{l,m}\sqrt{\left(\frac{2\pi}{L}\right)^3\frac{\lambda'}{4\kappa'\sqrt{k'^2+\kappa'^2}}}\{c(\underline{k}',l,m)V(\underline{k}',l,m) \\ &\qquad + b^*(\underline{k}',l,m)V^*(\underline{k}',l,m)\} \end{aligned}\right.$$

etc.

There exist mutually commutative operators x_μ, $n^+(\underline{k},l,m)$, $n^-(\underline{k},l,m)$, $\nu^+(\underline{k}',l,m)$, $\nu^-(\underline{k}',l,m)$. In the representation, in which all these operators are represented by diagonal matrices, ~~[illegible]~~ the probability amplitude for the total system, assuming that it exists, is given by a function

$$\Psi(x'_\mu,\ n'(\underline{k},l,m),\ m'(\underline{k},l,m),\ \text{etc})$$

of eigenvalues x'_μ, $n'(\underline{k},l,m)$, $m'(\underline{k},l,m)$, $\nu'(\underline{k},l,m)$, $\mu'(\underline{k},l,m)$ of the operators x_μ etc.

Then $|\Psi|^2$ ~~could~~ might ~~can~~ be interpreted as the probability of finding, at space-time point x_μ, $n'(k, l, m)$ etc particles ~~of each state of motion~~ kind. This interpretation is very odd strange, because the particles can no more be localized at a single point. ~~—~~ In local field theory, such oddness is automatically ~~excluded~~, avoided, because $n'(k)$ defined by

$$
(2)\quad \left\{
\begin{aligned}
n'(k) &= \int a^*(x') \exp(i k_\mu x'_\mu)(dx')^3 \\
&\times \int a(x'') \exp(-i k_\mu x''_\mu)(dx'')^3 \\
&= \int\!\!\int a^*(x') a(x'') \exp(\cancel{i}\, k_\mu (x'_\mu - x''_\mu)) \\
&\qquad (dx')^3 (dx'')^3
\end{aligned}
\right.
$$

is not commutative with the operators x_μ. (More precisely, we start from $a(x)$ ~~and $a^*(x)$~~ the assumption U is commutative with x_μ, so that $a(x')$ is proportional to the diagonal element of U, ~~in the represen~~ except for the factor $\prod_\mu \delta(x'_\mu - x''_\mu) \cdot \exp(i k_\mu x'_\mu)$, in the representation, in which x_μ are diagonal. Thus not $a(x')$ but the original field operator U, is ~~not~~ the operator, which ~~in the sense it can be~~ is on the same footing as x_μ or p_μ.) ~~but the original field operator U~~

Similarly, in nonlocal field theory, a, b, x etc are all so-to-speak submatrices, which can be obtained by ~~of~~ omitting from the original field operators the factors depending on x_μ and p_μ. Thus it is ~~meaningless~~ ~~to~~ not legitimate to talk about x_μ, x^+, ~~xx~~ x^- etc as commutative operators. On the other hand, ~~We must have started from~~ ~~In other words,~~ instead ~~an operator~~ ~~if we started from~~ an operator with ~~of~~ the form matrix elements

$$a(k, l, m) \prod_\mu \delta(x'_\mu - x''_\mu) \qquad (3)$$

has no single meaning in connection with the nonlocal field.

However, we can define a new operator S, which is a certain function of U, U^*, V, V^* as well as of x_μ, p_μ and which has ~~xx~~ submatrices with rows and columns characterized by n', m' etc. This operator cannot immediately be connected with the macroscopic measurements, but only ~~the~~ some kind of space-time average of whole matrix elements (not of submatrix elements) can be ~~compared with~~ interpreted physically in connection with the actual observations.

Thus, the ~~most~~ important ~~questi~~ problem is not set up the fundament equation for this operator S. For reasons, which will be ~~shn~~ indicated below, we assume a form

$$i[p_\mu, S] = (J_\mu S' \cancel{\pm S} J_\mu^*) \qquad (4)$$

where J_μ are ^form^ interaction operators, which constitute a form vector. If we perform a ~~canonical~~ unitary transformation

$$S^* = TS'T^{-1} \qquad (5)$$

where T is a unitary operator, we obtain from (4),

$$[p_\mu, TS'T^{-1}] = p_\mu TS'T^{-1} - TS'T^{-1}p_\mu$$

$$= [p_\mu, T]S'T^{-1} + T\,\cancel{S'}[p_\mu, S'T^{-1}]$$

$$= [p_\mu, T]S'T^{-1} + T[p_\mu, S']T^{-1} + TS'[p_\mu, T^{-1}]$$

$$\cancel{= [p_\mu, T]S'T^{-1} +}$$

$$= i(J_\mu TS'T^{-1} \cancel{\pm} TS'T^{-1}J_\mu^*)$$

$$[p_\mu, S'] = iJ_\mu' S' \cancel{\pm i S'} J_\mu^{*\prime}$$

$$+ \{T^{-1}[p_\mu, T]S' + S'[p_\mu, T^{-1}]T\}$$

$$J_\mu' = T^{-1}J_\mu T.$$

$$T^{-1}[p_\mu, T]S' + S'[p_\mu, T^{-1}]T$$

$$= J_\mu' S' = T^{-1} J_\mu S T$$

or

$$[p_\mu, T]T^{-1}S + ST[p_\mu, T^{-1}] = J_\mu S \qquad (4)$$

So, if we can find T such that

$$\cancel{T^{-1}[p_\mu, T]S' = J_\mu T.}$$

$$\cancel{[p_\mu, T^{-1}] = (T^{-1} J_\mu^*}$$

$$\cancel{\text{or } [T^*, p_\mu] = T^* J_\mu^*}$$

or $J_\mu' = T^{-1}[p_\mu, T] + S'[p_\mu, T^{-1}]TS'^{-1}$

for $\mu = 1, 2, 3$, (4) is reduced to

$$[p_4, S'] = i(H' S' - S' H')$$

$$[p_\mu, S'] = 0,$$

where

$$H' = J_4' - T^{-1}[p_4, T] - S'[p_4, T^{-1}]TS'^{-1}$$

$$\cancel{H' = J_4 + iT^{-1}[p_4, T]}$$

Now if we express S as power series of U, U^* & V, V^*, and further expand U, U^*, V, V^* themselves, each term of expansion has a factor of the form

$$C(k_\mu^{(1)}, k_\mu^{(2)}, \dots l_\mu^{(1)}, l_\mu^{(2)}, \dots)$$

$$\times \exp i\Big\{\sum_{\substack{k^4>0 \\ i}} k_\mu^{(i)} x^\mu - \sum_{\substack{k^4>0 \\ j}} k_\mu^{(j)} x^\mu\Big\} \exp i\Big\{\sum_i l^{\mu(i)} p_\mu - \sum_j l^{\mu(j)} p_\mu\Big\}$$

So, if we ~~intro~~ ~~out~~ go over to the matrix representation, in which x^μ are diagonal, ~~they are it has the form~~ and then ~~to~~ regard it as a function of X_μ and Y_μ, it has the ~~form~~ factor

$$\exp\left(i\sum_i k_\mu^{(i)} X^\mu - \sum_j k_\mu^{(j)} X^\mu \right)$$

~~and~~ the space-time average of S can be defined as

$$\bar{S} = \iint (x'|S|x'')(dx')^4(dx'')^4$$

$$= \iint S(X, r)\,(dX)^4(dr)^4.$$

After integration,
Each term has the factor

$$\prod_\mu \delta\left(\sum_i k_\mu^{(i)} - \sum_j k_\mu^{(j)} \right),$$ ~~total kinetic~~

~~which means the conservation of energy-momentum for the total system, where~~.
~~that $\bar{S}$, as~~.

$\bar{S}$ can be regarded as a matrix, with row and columns characterized by

$n'(k, l, m)$ etc

and the δ-functions ~~above~~ considered ~~indicated~~ guarantees the conservation of energy-momentum for the total system, if $\bar{S}$ is interpreted as something connected with the transition probability of transition due to the interaction.

The fundamental equations ( ) can be written in the form

$$-i\hbar \frac{\partial S(X, r)}{\partial X^\mu} = (J_\mu S - S J_\mu^*)$$

$$\sum_{k',k''} (\Sigma k' - \Sigma k'') S_{k';k''}(x', x'') = \int (dx''')^4 J_\mu(x' x''') S(x''', x'')$$

$$- \int (dx''')^4 S(x' x''') J_\mu^*(x''', x'')$$

$$\sum_{k',k''} (\Sigma k' - \Sigma k'') S_{0,k'k''}(x', x'') = 0$$

$$\sum_{k'k''} (\Sigma k' - \Sigma k'') S_{1,k',k''}(x', x'') = \int (dx''')^4 J_\mu(x' x''') \sum S_{0,k'k''}(x''' x'')$$

$$- \cdots$$

Moreover, S must be ~~a unit operator when there is no interaction, so that~~

~~$[x^\mu, S] = 0$.~~

~~or $\gamma_\mu \frac{\partial}{\partial X_\mu} S(X, r) = 0$.~~

so chosen as to satisfy the asymptotic condition

$S(x' x'') = 0$ for $x'_4 < x''_4$

and $\cancel{x'_\mu x'^\mu}$ $(x'_\mu - x''_\mu)(x'^\mu - x''^\mu) < 0$

Q

In $\exp i\{\sum_j k_\mu^{(j)} x^\mu\}$, the four vector $\sum_j k_\mu^{(j)}$ can be always ~~divided into two parts~~ written as the difference of two time-like vectors, each corresponding to the sum of energy-momentum of particles created and annihilated respectively.