~~II~~ III. ~~Intr~~ Classification of ~~Possible Models for the~~ Elementary Systems* ~~in Nonlocal Field Theory~~

The most general nonlocal scalar field can be written as

$$(x'|U|x'') = U(X_\mu, r_\mu),$$

$$= \int \cdots \int \bar{u}(k_\mu, l_\mu) \exp(i k_\mu X^\mu)$$
$$\times \delta(r_\mu + l_\mu)(dk_\mu)^4 (dl_\mu)^4.$$

or

$$U = \int \cdots \int \bar{u}(k_\mu, l_\mu) \exp(i k_\mu x'^\mu/2)$$
$$\exp(i l^\mu p_\mu/\hbar) \exp(i k_\mu x'^\mu/2)$$
$$(dk_\mu)^4 (dl_\mu)^4$$

or

$$U = \int \cdots \int \bar{u}'(k_\mu, l_\mu) \exp(i k_\mu x^\mu)$$
$$\exp(i l^\mu p_\mu/\hbar)(dk_\mu)^4 (dl_\mu)^4$$

or

$$= \int \cdots \int \bar{u}''(k_\mu, l_\mu) \exp(i l^\mu p_\mu/\hbar)$$
$$\exp(i k_\mu x^\mu)(dk_\mu)^4 (dl_\mu)^4$$

where

$$\left.\begin{aligned} \bar{u}'(k_\mu, l_\mu) &= \bar{u}(k_\mu, l_\mu) \exp(i k_\mu l^\mu/2) \\ \bar{u}''(k_\mu, l_\mu) &= \bar{u}(k_\mu, l_\mu) \exp(-i k_\mu l^\mu/2) \\ \bar{u}'(k_\mu, l_\mu) &= \bar{u}''(k_\mu, l_\mu) \exp(i k_\mu l^\mu) \end{aligned}\right\}$$

Now the above general field can be decomposed into several parts, ~~w~~ each of which transforms

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* T. D. Newton and E. P. Wigner, Rev. Mod. Phys. 21 400 (1949)

First we consider ~~one~~ homogeneous Lorentz transformations.

~~among~~ ~~by themselves~~ in itself by Lorentz transformations.

By a Lorentz transformation

$$x'_\mu = a_{\mu\nu} x_\nu ,$$

$$\exp(i k_\mu x^\mu) \quad \text{and} \quad \exp(i l^\mu p_\mu/\hbar)$$

transform respectively into

$$\exp(i k'_\mu x'^\mu) \quad \text{and} \quad \exp(i l'^\mu p'_\mu/\hbar),$$

where

$$k'_\mu = a_{\mu\nu} k_\nu , \qquad l'_\mu = a_{\mu\nu} l_\nu .$$

The only independent ~~quantities~~ functions of k'_μ, l'_μ, which are invariant under homog. Lorentz group, are

$$k_\mu k^\mu = k'_\mu k'^\mu \qquad l_\mu l^\mu = l'_\mu l'^\mu$$

$$k_\mu l^\mu = k'_\mu l'^\mu .$$

$$(\because \; k_\mu l_\nu \cdot k^\nu l^\mu = k_\mu l^\mu \cdot k_\nu l^\nu .)$$

Thus, each part of the field, which ~~correspond to~~ is associated with a set of the definite values of $k_\mu k^\mu$, $l_\mu l^\mu$ and $k_\mu l^\mu$, constitute a irreducible ~~part~~ field. ~~That~~ Irreducible fields or the elementary systems in nonlocal field theory, can be classified ~~as follows:~~ first ~~by~~ according to the signs of $k_\mu k^\mu$ ~~of~~ and $l_\mu l^\mu$:

	$k_\mu k^\mu$	$l_\mu l^\mu$	~~$k_\mu l^\mu$~~
I.	$-\kappa^2$	λ^2	
II.	$-\kappa^2$	$-\lambda^2$	
III.	κ^2	λ^2	
IV.	κ^2	$-\lambda^2$	

~~The Each of these four cases can further be decomposed classified~~

Among these four cases, cases III and IV has no direct correspondence with the motion of elementary particles in usual theory, because ~~III and IV cases~~ the ~~idea~~ motion of particles with imaginary masses contradict the ~~very idea~~ fundamental principles ~~basis~~ of special relativity theory. Thus, we ignore the case with positive $k_\mu k^\mu$ and instead

	$k_\mu k^\mu$	$l_\mu l^\mu$	important example $k_\mu l^\mu = 0$ structure
I.	$-\kappa^2$	λ^2	..... space-like
II.	$-\kappa^2$	0	... $k_\mu l^\mu = 0$: local field st.
III.	$-\kappa^2$	$-\lambda^2$	... time-like ($k_\mu l^\mu = \pm\kappa\lambda$)
IV.	0	λ^2	... s.l.s.
V.	0	0	
VI.	0	$-\lambda^2$	... time-like st.

~~Among~~ Each of these six ~~cases~~ classes can further be classified by the value of $k_\mu l^\mu$, which may of class I

either positive, negative or 0. In Part I, we considered the particular case with

$$k_\mu k^\mu = -\varkappa^2, \quad l_\mu l^\mu = \lambda^2, \quad k_\mu l^\mu = 0$$

in detail and found that it had an intimate connection with the rigid sphere model in classical ~~theory~~ electrodynamics. The ~~case~~ particular case of class II with

$$k_\mu k^\mu = -\varkappa^2, \quad l_\mu l^\mu = 0, \quad k_\mu l^\mu = 0$$

reduces to the ordinary local field corresponding to the point particles. However, the case ~~se~~ in class III with ~~magneto~~

$$\cancel{k_\mu l^\mu} \quad k_\mu k^\mu = -\varkappa^2, \quad l_\mu l^\mu = -\lambda^2$$

$$k_\mu l^\mu = 0$$

is ~~not only~~ a little interesting, ~~because~~ in that

$$(k_\mu - l_\mu)(k^\mu - l^\mu) = -\varkappa^2 - \lambda^2$$

a ~~k_μ~~ more generally, for any vector

~~$a k_\mu$~~

$$j_\mu = a k_\mu + b l_\mu$$

$$j_\mu j^\mu = -a^2\varkappa^2 - b^2\lambda^2 < 0,$$

i.e. j_μ is time-like always.

~~However~~ must be~~cause~~ excluded, because in rest system with respect to a given set of values of k_μ,

$k'_i = 0 \quad k'_4 = \pm\varkappa$, so that $l'_4 = 0$.

l'_μ is ~~not a time~~ a space-like vector.

~~So l_μ must be 0~~ This is a contradiction.

We can show easily that $|k_\mu l^\mu| \geq \varkappa\lambda$.

The case with

$$k_\mu k^\mu = -\kappa^2 \qquad l_\mu l^\mu = -\lambda^2$$

$$k_\mu l^\mu = + \text{ or } - \kappa\lambda$$

is of particular interest, because in rest system with $k_1 = k_2 = k_3 = 0$ $k_4 = \pm\kappa$,

$$l^4 = \pm(\pm\lambda), = \pm\lambda.$$

or k_μ and l_μ are parallel or antiparallel with each other.

~~Th~~ This correspond to the particle model, which has no structure, ~~so~~ as to give rise to internal motion. The only difference from the usual particle is that the action may be advanced or retarded due to the occurence of the factor

$$e^{il^\mu p_\mu/\hbar} = e^{\pm ik^\mu p_\mu/\hbar}.$$

This case ~~is par~~ has the advantage ~~that the~~ particular ~~for~~ as a possible model for spinor particles such as the electron and the proton. The reason is that the model with finite radius gives rise to internal rotation and it may contradict ~~the~~ Pauli exclusion principle, ~~if we accept~~ unless we change the method of quantization.

Lecture VII
Oct. 27

	$k_\mu k^\mu$	$l_\mu l^\mu$	$k_\mu l^\mu$	Particular case
I	$-\kappa^2$	λ^2	any value	$k_\mu l^\mu = 0$ (def. radius)
II	$-\kappa^2$	0	any value	$k_\mu l^\mu = 0$ (local field)
III	$-\kappa^2$	$-\lambda^2$	$\lvert k_\mu l^\mu \rvert \geq \kappa\lambda$	$\lvert k_\mu l^\mu \rvert = \kappa\lambda$
IV	0	λ^2		
V	0	0		
VI	0	$-\lambda^2$		

We would like to discuss ~~the~~ particular in case III with $|k_\mu l^\mu| = \kappa\lambda$ more in detail

$$U = \int\cdots\int (dk_\mu)^4 (dl_\mu)^4\, \bar{u}(k_\mu, l_\mu) \exp(ik_\mu x^\mu) \exp(il^\mu r_\mu/\kappa)$$

where

$$\bar{u}(k_\mu, l_\mu) = u(k_\mu, l_\mu)\,\delta(k_\mu k^\mu + \kappa^2)\,\delta(k_\mu l^\mu + \varepsilon\kappa\lambda)^{1+\varepsilon} \times \delta(l_\mu l^\mu + \lambda^2)$$

ε may be either +1 or −1.

For any fixed value of k_μ, we can perform the Lorentz transformation, so that k_μ ~~becomes~~ reduce, in new coordinate system to

$$k_1 = k_2 = k_3 = 0 \qquad k_4 = \pm\kappa.$$

In this system, we perform the integration with respect to l_μ. Now

$$\int \delta(\pm\kappa l_4 \pm \kappa\lambda)\,\delta(l_1^2 + l_2^2 + l_3^2)\, \cancel{f(l_\mu)}\, f(0,0,0,\pm\kappa; l_\mu) \times (dl_\mu)^4$$

$$= \frac{1}{\kappa}\, 2\pi l \cdot f(0,0,0,\pm\kappa;\, 0,0,0,\pm\lambda)$$

$$l = \sqrt{l_4^2 - \lambda^2} \to 0 \longrightarrow \frac{dm}{\sqrt{l_\mu l^\mu + \lambda^2}}$$

~~Although~~

~~Although~~ this gives $U=0$ as such, we can still multiply in advance a factor

$$\frac{1}{\sqrt{k_\mu k^\mu+\lambda^2}}, \quad \times \frac{\varkappa}{2\pi}$$

Then

$$u(k_\mu)$$

$$U=\int\cdots\int (dk_\mu)^4\, u(k_\mu, \pm\frac{\lambda}{\varkappa}k_\mu)\exp(ik_\mu x^\mu)$$
$$\times \exp\left(\pm\frac{i\lambda k^\mu p_\mu}{\varkappa\hbar}\right)$$

$$\pm i\frac{\lambda p_\mu}{\varkappa\hbar}=\pm\frac{\lambda}{\varkappa}\frac{\partial}{\partial x^\mu}$$

The commutation relations will be

$$[u(k_\mu), u^*(k'_\mu)] = \prod_\mu \delta(k_\mu, k'_\mu)$$
$$[u(k_\mu), u(k'_\mu)] = 0$$
etc.

Extension to vector and spinor field is very simple. For example, in the case of spinor field, we need only to postulate

$$\gamma^\mu[p_\mu, \psi]+mc\,\psi=0$$

$$\gamma_\mu[x^\mu, \psi]+\lambda\,\psi=0.$$

Then we obtain immediately

$$[p^\mu[p_\mu, \psi]]+m^2c^2\psi=0$$
$$[x_\mu[x^\mu, \psi]]+\lambda^2\psi=0$$

and

$$[x^{\mu}[p_\mu, \psi]] - \cancel{\kappa}\, \kappa c \lambda \psi = 0$$

or

$$\left.\begin{aligned} k_\mu k^\mu + \kappa^2 &= 0 \\ l_\mu l^\mu + \lambda^2 &= 0 \\ k_\mu l^\mu + \kappa\lambda &= 0 \end{aligned}\right\}$$

For such cases, which correspond to elementary systems with extension in time direction, we may need ~~can expect~~ ~~more closer non~~ come less drastic change of present formalism, because U is an operator which increases or decreases the proper time of the ~~probability~~ particle by $\frac{\lambda}{c}$.