C25-P30

Interaction

Properties of Operators

III. ~~S-Matrix~~

which will be called the third class.

Among the operators of the second class, there are a particular family of operators, which can be expressed as sums of products of nonlocal field operators ~~such U, U*~~ and and operators of the first class. ~~Such operators will be called~~ A^III Any one of them can be written in the form

$$A^{III} = AU^*AU$$

$$A_0 + A_1U^* + A_2U^* + A_3U^*A_4U + \cdots$$

$$+ B_1\psi^*\gamma^4\psi + \cdots$$

$$+ C_1 U \psi^*\gamma^4\psi + \cdots,$$

where A_0, A_1, A_2 etc. $B_1, B_2 \cdots, C_1, C_2 \cdots$ are all operators of the first class.

~~A of the third class;~~

As will be shown in the ~~p~~ later one, the space-time average ~~of~~ of any operator ~~which~~ is defined

$$\bar{A} = \int\cdots\int (dx_\mu')^4 (dx_\mu'')^4 \langle x_\mu' | A | x_\mu'' \rangle,$$

~~which is still a submatrix~~,

of any operator A of the third class, is of

Obviously,

particular importance in connection with the physical interpretation of the formalism of nonlocal field theory, $\bar{\bar{A}}$ is a kind of matrix with rows and columns characterized by the distinct number of particles in all possible quantum states, It can be shown very generally that the space-time average $\bar{\bar{A}}$ of arbitrary operator A of the kinds class is an infinite matrix with follow property: The matrix elements of $\bar{\bar{A}}$ which form

(Thus, we can write more precise in the form

$$\left(n'(k,l,m)\cdots \middle| \bar{\bar{A}} \middle| n''(k,l,m)\cdots\right),$$

where $n'(k,l,m)$ are the numbers of particles (and $n''(k,l,m)$)

in the state k,l,m characterizing the row and column respectively of the submatrix.

Now it can be shown very generally that the matrix element of $\bar{\bar{A}}$ is different

from zero only if the row and the column are represented by the states of the total system of particles with the same total energy and total momentum. It should be notice that we mean by the total energy and the total momentum of the system the total sum of the energies of motions of centers of mass of particles present and that of the moments of motions of centers of mass of particles present, respectively.

In other word the matrix element $(\cdots|A|\cdots)$ is different from zero only if

$$\sum_{k,l,m} k_\mu n'(k,l,m) = \sum_{k,l,m} k_\mu n''(k,l,m),$$

where the summation covers all the possible states of particles of all types in consideration (of particles).

In order to ~~show~~ see that this is generally true, however, it is sufficient to ~~any of~~ consider simple ~~examples~~ cases, because look at the ~~derivation~~ way of ~~method of~~ proof can easily be extended to more general cases. For example, in ~~the~~ case A has a particular form

$$A_3 U^* A_4 U,$$

where A_3, A_4 are arbitrary operators of the ~~this~~ first class and U, U^* are non-local scalar field operators, it can ~~always~~ be decomposed into ~~a form~~ terms such as

$$U_I = \exp(i l^\mu p_\mu(k))\, a^*_{k,l,m} \exp\left(\frac{i}{\hbar} k_\mu x^\mu(k_4)\right) \exp i l' ;$$

$$U_{II} = \exp(i l^\mu p_\mu(k))\, a^*_{k,l,m} \exp i l'' ;$$

apart from the purely numerical factors, the space-time average of ~~each of~~ $\exp(i l^\mu p_\mu(k))$ ~~the~~ matrix elements $(x'_\mu | U_I | x''_\mu)$, $(x'_\mu | U_{II} | x''_\mu)$ ~~etc~~ is evidently equivalent to ~~the~~ integration ~~each~~

$C_{k,l,m}$

$$\int \cdots \int U_I(X_\mu, r_\mu)(dX_\mu^{(n)})^4 (dr_\mu)^4$$

for example

etc. Now, the dependence of $U_I(X_\mu, r_\mu)$, etc. on X_μ is only through the factor

$$\exp(-i k_\mu X^\mu) \exp(i k'_\mu X^\mu),$$

so that, after integration with respect to X_μ, there remain only terms with the factor

$$C_{k,l,m} \cdot C_{k',l,m},$$

the rows and columns certainly belong to the barycentral

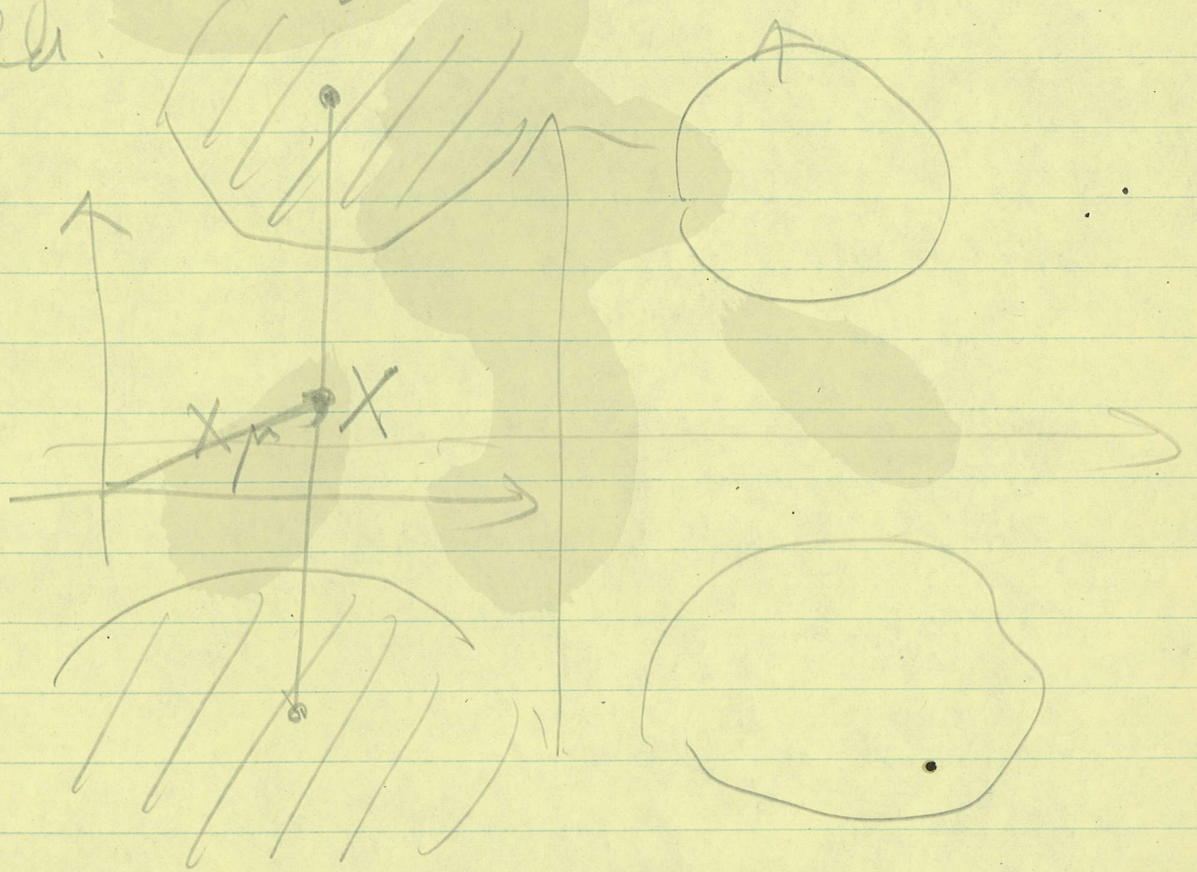
which is a diagonal submatrix and ~~This~~ particular case seems to be too simple, but the above argument can be at once to more general case. In fact ~~the author does not intend to do it is very~~ cumbersome to The only trouble is to write down complicated expressions explicitly, which ~~will be of little use, once~~ unnecessary, seems to be of little use. We author would like to spare with them particularly because of typographical difficulties.

~~if we~~

It is true even more generally that for any nonlocal operator A,

$$\bar{A}(\) = \int\int A(x, r)\, \rho(r)\, s(dx)^4 (dr^{4n})^4$$

~~is~~ a submatrix with rows and columns can be regarded characterized by distribution in number of particles and the matrix elements are different from zero only when conservation laws hold.



So $\rho(r_\mu)$ can be so chosen that $\rho(r_\mu)$ is different from zero only for

$$r_1^{4\prime} = x^{4\prime} - x^{4\prime\prime} \geq cT,$$

where T is a time interval very large compared with the collision time in question.

Alternatively, we can write

$$\bar{A} = \mathrm{Trace}\, A D_{\hbar c T}$$

$$= \sum_{x' x''} (x' | A | x'') (x'' | D_T | x')$$

where D_T is an invariant, (field displacement independent) operator with the matrix elements $(x' | D_T | x'')$, which are different from zero only when $x' - x''$ is in time direction and

$$x^{4\prime} - x^{4\prime\prime} \leq cT,$$

(20)

IV. S-Matrix in N.L.F.T.

Now we can go into the question of the interaction of fields. The first question, with which we meet, is whether there is the Schrödinger equation or any substitute for it in N.L.F.T. It is difficult to answer this question directly. However, irrespective of the existence or absence of such an equation, it is possible for our case to define the S-matrix in the sense of Heisenberg and Møller. Suppose that there exist at the beginning a number of particles with internal structure in various quantum states. After a long time, the number of particles and the distribution in various quantum states will be changed due to the interaction between particles. The question now is to determine the probability that arbitrary distribution in number of particles in different quantum states, provided that the initial distribution is known.

It is clear that the distribution will not be changed if the particles are independent of each other. This can expressed mathematically by the statement that there exists a ~~matrix~~ diagonal matrix with rows and columns ~~each being~~ characterized by the distribution in number of particles of various types in various states, ~~and~~ The ~~absolute~~ square of absolute value of ~~these~~ the diagonal element must ~~be~~ each be equal to 1, corresponding to the fact that the probability of finding ~~the~~ original distribution is 1. ~~if there is no interaction between~~ after a long time One can simplify the statement further by assuming that this diagonal matrix is the unit matrix, (submatrix) The effect of the interaction is then amount to change this matrix from the unit matrix to some nondiagonal matrix $\overline{S}$. ~~This matrix S must satisfy, at least, the following two conditions~~

The square of the absolute value of the matrix element of $\bar{S}$ is supposed to represent the probability that the distrib. corresponding to the row will be realized, when the system is initially in the state characterized by the column.

In order that such an interpretation is permissible, the matrix S must satisfy, at least, the following three conditions:

(i) $\sum_j \langle i|\bar{S}|j\rangle \langle j|\bar{S}^*|i\rangle = \sum_j \bar{S}_{ij}\bar{S}^*_{ji} = 1$

independent of i.

(ii) The matrix element of S must be zero, unless the total energy and momentum of the state of the system corresponding to the row is equal to those corresponding to the column.

The latter condition is satisfied for any $\bar{S}$ defined by

$$\langle i|\bar{S}_{ij}|j\rangle = \int\cdots\int (x'_i|\bar{S}_{ij}|x'')\rho(x'-x'')\times(dx')^4(dx'')^4$$

$$= \int\int S_{ij}(X,r)\rho(r)(dX)^4(dr)^4,$$

(iii) $\bar{S}^*_{ij}\bar{S}_{ji}$ must be real. inv.

where S is an arbitrary invariant operator, which can be expressed as a sum of products of field operators, and $\rho(r_\mu)$ is any invariant function of r_μ.
$\bar{S}$ can alternatively written as

$$\bar{S} = \text{Trace } S \rho D_T$$

where D_T is an operator depending only on p_μ, so that it can be written in the form

$$D_T = \int\cdots\int D_T(l_\mu) \exp(i l^\mu p_\mu/\hbar)(dl_\mu)^4$$

or

$$(x'|D_T|x'') = D_T(x, r) = \frac{1}{\Sigma}\rho(\pm r_\mu)$$

with $\rho(r_\mu)$ being an invariant fn of r_μ.

(Trace $S\rho)_{ij} = \int\cdots\int (x'|S_{ij}|x'')(x''|\rho|x')(dx')^4(dx'')^4$

$$= \int\cdots\int S_{ij}(X, r)\rho(-r)(dX)^4(dr)^4$$

The condition (iii) is also satisfied, if ρ is an invariant operator depending only on p_μ, because the form of $\bar{S}$ as the function of n_k' and n_k'' does not change by Lorentz transformation.

The condition (i) is not identical with the more familiar condition

$$\bar{S}^* \bar{S} = 1,$$

the latter being much stronger requirement to $\bar{S}$.

Our main task is to find an operator S, so as to satisfy that $\bar{S}$ satisfies the condition (iii) and reduces to the usual probability amplitude or the wave matrix in Møller's sense in the limit of local fields.