Lecture

Nov. 17, 1949)

Displacement Operator and S-Matrix in Nonlocal Field Theory

D ①

Last time I discussed about ~~the~~ various possibilities of extending Schrödinger differential equation itself so as to be applicable to the interaction of nonlocal fields. However, it turned out that there is a difficulty connected with the reinterpretation of differential dt.

[On the other hand, even if we want to start~~ed~~ from the integral form

$$\Psi(t) = \lim_{j} \prod_{j} K_j \, \Psi(0),$$

we must be very careful so as to avoid the appearance of "dt". So, first of all, we rewrite

$$\Psi(t_n) = \left\{ D(t_n, t_{n-1}) - D(t_n, t_{n-1}) \frac{i}{\hbar} \bar{H}'(t_{n-1})\, dt \right\}$$

$$\times \left\{ D(t_{n-1}, t_{n-2}) - D(t_{n-1}, t_{n-2}) \frac{i}{\hbar} \bar{H}'(t_{n-2})\, dt \right\}$$

$$\times \cdots$$

$$\times \left\{ D(t_j, t_{j-1}) - D(t_j, t_{j-1}) \frac{i}{\hbar} \bar{H}'(t_{j-1})\, dt \right\}$$

$$\times \cdots$$

$$\times \left\{ D(t_1, t_0) - D(t_1, t_0) \frac{i}{\hbar} \bar{H}'(t_0)\, dt \right\} \Psi(t_0)$$

where

$$\Psi(t_j) = D(t_j, t_{j-1}) \Psi(t_{j-1}) \qquad \text{with } t_j - t_{j-1} = dt$$

or

$$D(t_j, t_{j-1}) = (t_j | D | t_{j-1}) = \delta(t_j - t_{j-1} - \Delta t)$$

$$\int D(t_j, t_{j-1})\, dt_{j-1}\, D(t_{j-1}, t_{j-2}) = \delta(t_j - t_{j-2} - 2dt)$$

$$\equiv (t_j | D^2 | t_{j-2})$$

So, if we fix $t_n = t$, $t_0 = 0$

$$(t'|D^n|t'') = \delta(t' - t'' - \cancel{t_n}\ t)$$

$$\Psi(t') = \Psi(0) + (t'|D^{n-j+1}|t_{j-1}'')\overline{H}'(t_{j-1}'')\,dt$$
$$\times (t_{j-1}''|D^{j-1}|0)\,\Psi(0)$$

$+ \cdots$

$$= \cancel{\delta(t}\int (t'|D_t|t'')\,\Psi(t'')\,dt''$$

$$+\int_{t>\tau} d\tau \iint (t'|D_{t-\tau}|t''')\,\overline{H}'(t''')\,dt'''(t'''|D_\tau|t'')\,dt''$$

$$+\iint_{t>\tau+\tau'} d\tau\, d\tau' \int\cdots\int (t'|D_{t-\tau-\tau'}|t''')\,\overline{H}'(t''')\,dt'''(t'''|D_{\tau'}|t'''')$$
$$\times\ \overline{H}'(t'''')\,dt''''(t''''|D_\tau|t'')\,dt''$$

$+ \cdots$

~~So, for $t_n \to +\infty$ $t_0 \to -\infty$~~

~~$\Psi(\infty) = D_\infty + D_\infty \overline{H}' D_\infty$~~

~~$+ D_\infty \overline{H}' D_\infty \overline{H}' D_\infty$~~

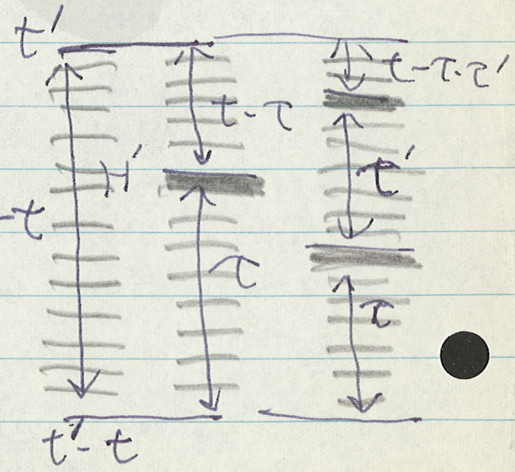
~~So, for if we integrate~~

~~if we write~~

~~$T = \frac{t}{2}$ and translate the origin $-t$~~

~~$\Psi(T)$ both t' and~~

Now t are arbitrary and if we integrate over t:



For this purpose, we have to go back again to the S-matrix formalism in usual field theory. Namely, we start from the Schrödinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = \bar{H}' \Psi ,$$

where Ψ is the function of time t and eigenvalues $N'_k \cdots M'_l \cdots \cdots$ * $\bar{H}'$ is a space integral of the interaction operator H' and is itself a submatrix, with rows and columns, which are characterized by $N'_k \cdots M'_l \cdots$, $\bar{\bar{H}}'$ ~~a~~ and $\bar{H}'$ ~~being~~ diagonal with respect to t, or including t as a parameter.

The integrated form for Ψ at time t in terms of $\Psi(0)$ is

$$\Psi(t) = \lim_{\substack{\Delta t \to 0 \\ n \to \infty}} K(t, t-\Delta t) K(t-\Delta t, t-2\Delta t) \cdots K(t-(n-1)\Delta t, 0)\Psi(0)$$

where $n\Delta t = t$,

$$K(t, t-\Delta t) = 1 - \frac{i\bar{H}'(t-\Delta t)}{\hbar}\Delta t$$

~~Now in nonlocal field theory, since H' is neither diagonal with respect to x, y, z nor with respect to t, we have to change~~

* For example, one may assume that there exist ~~spinor particles~~ one type of Fermions (N_k) and ~~scalar~~ one type of Bosons (M_l)

Suppose ~~that~~ there exists a wave functional Ψ in nonlocal field theory. Then

the ~~definition of Ψ~~ formalism as follows:
H' ~~is~~ consists of a sum of ~~different~~ products ~~operator~~ of field operators. Each product has a definite ~~di~~ factor, which displaces each of the arguments x_μ in Ψ by a fixed amount, whereas the total sum H' itself contain various terms, which displace~~s~~ x_μ in ~~as~~ different amounts.

Thus we find ~~a~~ very complicated links, which connect the wave functional Ψ at different space-time points and in fact connecting those points in space-like directions as well as those in time-like direction.

Now the operators $K(\xi)$ which appear~~ing~~ in the ~~integrated form of Ψ in~~ link connecting $\Psi(t)$ with $\Psi(0)$ in <u>local</u> field theory, ~~seem~~ be regarded as a very special class of operators, which contain displacements in time-like direction. ~~In fact~~

So we ~~can~~ guess that ~~a~~ consistent nonlocal field theory might be constructed, if we ~~could in some way connect~~ started ~~too~~ from the following considerations: