Letter to the Editor of The Physical Review

S-Matrix in Non-local Field Theory

Hideki Yukawa
Columbia University, New York, New York*

January 23, 1950

Recently the present author discussed certain types of quantized non-local fields, which were supposed to correspond to assemblies of elementary particles with finite radii.(1) In order to deal with the system consisting of two non-local fields, or a local field and a non-local field, which interact with each other, one must first find the substitute for the Schrödinger equation for the total system. However, it is naturally anticipated that the S-matrix in local field theory, which was obtained as the result of integration of the Schrödinger equation by successive approximations, may well find a counterpart in non-local field theory, whereas the physical interpretation of the Schrödinger wave functional itself, if it exists in non-local field theory, may be quite different from that in local field theory.

In fact, the S-matrix in non-local field theory can be obtained by a straightforward extension of the usual formalism. An arbitrary non-local operator A can be represented by a matrix $(n', x' \,|A|\, n'', x'')$, with rows and columns characterized by n', x' and n'', x'' respectively, where each of n', n'' stands for the distribution in numbers of particles in all possible quantum states

---

* On leave of absence from Kyoto University, Kyoto, Japan

and each of x', x'' stands for a set of eigenvalues of four space-time operators x_μ. Further, we define $\bar{\bar{A}}$ for an arbitrary operator A by

$$(n'|\bar{\bar{A}}|n'') = \int\!\cdots\!\int (n', x'|A|n'', x'')(dx')^4(dx'')^4 \tag{1}$$

Then, the S-matrix with matrix elements (n'|S|n'') can be written symbolically in the form

$$S = 1 + \left(\frac{i}{\hbar}\right)\overline{\overline{L'}} + \left(\frac{i}{\hbar}\right)^2 \overline{\overline{L'D_+L'}} + \left(\frac{i}{\hbar}\right)^3 \overline{\overline{L'D_+L'D_+L'}} + \cdots\cdots \tag{2}$$

in non-local field theory as well as in local field theory, where L' is an invariant operator characterizing the interaction and can be expressed as a sum of products of non-local and local field quantities. In particular case of the interaction between local fields, matrix elements of L' reduce to the form

$$(n', x'|L'|n'', x'') = (n'|L'(x')|n'') \prod_{\mu=1}^{4} \delta(x'_\mu - x''_\mu), \tag{3}$$

where (n'|L'(x')|n'') is the matrix element for the Lagrangian density for the interaction in the usual theory. D_+ is an invariant displacement operator with the matrix elements

$$(n', x'|D_+|n'', x'') = 1, \quad 1/2, \text{ and } 0, \tag{4}$$

according as x'-x'' is a future-like vector, a space-like vector and a past-like vector respectively. We can show very generally that the matrix S as defined by (2) fulfills all requirements for the S-matrix. Firstly, it is obviously relativistically invariant.

Secondly, one can prove that it is unitary. Thirdly, the matrix element (n' |S| n'') is different from zero, only if the final and initial states, which are characterized by n' and n'' respectively, have the same total energy and momentum.[2]

As for the finiteness of the matrix S, it is not easy to draw a general conclusion. However, we can show that, for example, the self-energy of a spinor particle interacting with the non-local scalar field is finite due to the appearance of the form factor, as far as the third term of S in (2) is concerned. Further investigations are needed in order to settle the question of convergence in non-local field theory.

Detailed accounts will be given in the later issue of this journal.

---

(1) H. Yukawa, Phys. Rev. 76, 300 (1949); 76, 1731 (1949); 77, 219 (1950).

(2) We mean by the total energy and momentum the sums of energies and momenta respectively of the translational motion of the existing particles.