Abstract of the paper to be presented at 295th meeting of the American Physical Society in Chicago, November 25, 26, 1949.

S-Matrix in Nonlocal Field Theory

Hideki Yukawa

Columbia University, New York, N. Y.

The problem of the interaction between nonlocal fields, which have been investigated by the author,[1] is dealt with so as to give a definite content to Heisenberg's S-matrix scheme. The space-time average

$$\overline{S} = \int \cdots \int (x'_\mu | S | x''_\mu)(dx'_\mu)^4 (dx''_\mu)^2$$

of the matrix elements $(x'_\mu | S | x''_\mu)$ for an arbitary operator S, which can be constructed from nonlocal field quantities, is itself an operator, which can be represented by a matrix with rows and columns characterized by the number of particles of various kinds in various states. The matrix element of $\overline{S}$ is different from zero, only if the row and the column belong to the same values of the total kinetic energy and the total momentum. Thus the conservation laws for collision processes are automatically guaranteed by defining the S-matrix as the space time average $\overline{S}$ of the matrix elements of a certian nonlocal operator S. The fundamental equation for S can be obtained by taking into account the correspondence to the usual local field theory in the limit of vanishing radii of particles and has the form

$$S - 1 = W + W D_+ (S - 1),$$

where W is an invariant operator characterizing the interaction of fields in question and D_+ is an operator, which is invariant and independent of field operators with the properties similar to δ_+-function in usual theory.

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(1) H. Yukawa, Prog. Theor. Phys. 2, 209 (1947); 3, 205, 452 (1948); Phys. Rev. 76, 300 (1949); 77, (1950), in press.

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$$S - 1 = W + WD_+(S-1),$$

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Abstract of the paper to be presented at 292nd meeting of the American Physical Society in Washington, D.C., April 28, 29, 30, 1949.

On Nonlocalizable Field Theory

Hideki Yukawa

Institute for Advanced Study, Princeton

Possibility of extending present theory of quantized field to more general case, in which field quantities are not localizable, was discussed recently in connection with Born's idea of reciprocity.(1) In order to develope any consistent theory of two or more nonlocalizable fields interacting with each other, it is necessary to avoid explicit use of Schroedinger equation or any substitute for it, which determines the change of probability amplitude during an infinitesimal time interval. An integral formalism, which was suggested by Dirac and developed recently by Feynman,(2) can be extended so as to cover nonlocalizable field theory. Namely, we can start from the postulate that probability amplitude between measurements of any two sets of field quantities is given by relevant matrix element in the representation of an unitary operator $W = \exp iS/\hbar$ with S = const. Trace L, where L is an operator corresponding to Lagrangian density in classical field theory. When there is no interaction between fields, W should reduce to the identi~~cal~~ty operator corresponding to the fact that the distribution in number of any kind of particle associated with any kind of free field remains the same for ever. This is actually the case for free fields previously considered. The interaction between fields ~~which~~ give~~s~~ rise to scattering in very general sense, ~~which~~ which is reproduced in the corresponding matrix representation of the operator W.

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(2) Feynman, Rev. Mod. Phys. 20, 367 (1948); Talks at Institute for Advanced Study, Princeton.

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Telephone to Darrow

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$$W = \sum_n 1/n! \ \mathrm{Trace}\cdot (iL)^n$$

$$W = \underbrace{\mathrm{Trace}}_{\text{const.}} \left(\exp \frac{iL}{\hbar}\right)$$

$$W = \text{const}\cdot \mathrm{Trace}\, S \text{ with } S = \exp\frac{iL}{\hbar}$$

March 10, 1949

Dear Dr. Darrow:

I sent to you abstract of the paper to be presented at Washington meeting, but I suppose it reached you later than the dead line, because the delay must have delayed due to short postage, which I noticed after I had posted at Syracuse on March 2nd. Of course, I do neither expect that my abstract will be included in the program, nor it will be printed in Bulletin. The only thing that I ask you is that, if it happens by any chance that the abstract is going to be piblished in Bulletin, please correct the following error in the manuscript:

Abstract of the paper entitled "On Nonlocalizable Field Theory" by
Hideki Yukawa

Trace ρ

11 th line: change S = const. Trace L into S = const. L

Trace Lρ

I am very sorry to trouble you on such a thing.

Yours sincerely,

H. Yukawa

Hideki Yukawa