Oct. 18, 1949

Dear Mr. Bryans:

Recently I heard from you that my paper "Quantum Theory of Nonlocal Fields. Part I. Free Fields" were scheduled to appear in Jan. 1 (1950) issue of the Physical Review. After I had sent the manuscript to the editor, I noticed that some short additional remarks were indispensable. I am very sorry to trouble you on this matter, but I shall be very happy if you would be kind enough to attatch the enclosed additional remarks at the end of the above paper before the galley proof will be prepared, because it will be very troublesome both for you and for me to change the galley proof afterwards. Your cooperation will be highly appreciated.

Sincerely yours,

H ideki Y ukawa

Remarks on Nonlocal Spinor Field

The problem of invariance of the relation (55) with respect to improper Lorentz transformation can be solved without introducing extra components to the spinor field. Namely, we take advantage of the antisymmetric tensor of the fourth rank with the components $\varepsilon_{\kappa\lambda\mu\nu}$, which are +1 or -1 according as $(\kappa, \lambda, \mu, \nu)$ are even or odd permutations of (1, 2, 3, 4) and 0 otherwise. Further we take into account the relations

$$i\beta_\nu = \gamma^\kappa\gamma^\lambda\gamma^\mu \tag{78}$$

where $(\kappa, \lambda, \mu, \nu)$ are even permutations of (1, 2, 3, 4). Then (55) can be written in the form

$$\frac{1}{6}\sum_{\kappa\lambda\mu\nu} \varepsilon_{\kappa\lambda\mu\nu}\gamma^\kappa\gamma^\lambda\gamma^\mu[x^\nu, \psi] + i\lambda\psi = 0, \tag{79}$$

which is obviously invariant with respect to the whole group of Lorentz transformation. The invariance can be proved more explicitly by associating a linear transformation

$$\psi' = S\psi \tag{80}$$

with each of the Lorentz transformation (70), where S is a matrix with four rows and columns satisfying the relations

$$S\gamma^\mu S^{-1} = a_{\mu\nu}\gamma^\nu \tag{81}$$

It should be noticed, however, that the relation (79) is a unification of the relations (55) and (67) rather than the simple reproduction of (55), because (79) must be identified with (67) in the coordinate system, which is connected with the original coordinate system by an improper Lorentz transformation with the determinant -1.

Letter to the Editor of Physical Review

Remarks on Nonlocal Spinor Field

Hideki Yukawa

Columbia University, New York, New York

In a recent letter to the editor,[1] it was shown that quantized nonlocal fields could be so constructed as to represent assemblies of particles with the definite mass and radius. In a paper, which will appear very soon,[2] detailed accounts ~~are made~~ is given together with the elucidation of most of the points, on which the author was not very sure when he wrote the above letter.[1] However, there is still one point, which seems to the author to be unsatisfactory. Namely, in the case of nonlocal spinor field, we assumed the commutation relation

$$\beta_\mu [x^\mu, \psi] + \lambda \psi = 0 \tag{1}$$

between the space-time operators x^μ and the nonlocal spinor operator ψ, in addition to the commutation relation

$$\gamma^\mu [p_\mu, \psi] + mc\, \psi = 0 \tag{2}$$

between ψ and the space-time displacement operators p_μ. Further, we assumed that γ^μ, β_μ, which were matrices with four rows and columns, were defined by

$$\left.\begin{array}{llll} \gamma^1 = i\rho_2\sigma_1, & \gamma^2 = i\rho_2\sigma_2, & \gamma^3 = i\rho_2\sigma_3, & \gamma^4 = \rho_3 \\ \beta_1 = \rho_3\sigma_1, & \beta_2 = \rho_3\sigma_2, & \beta_3 = \rho_3\sigma_3, & \beta_4 = -i\rho_2 \end{array}\right\} \tag{3}$$

Now the difficulty was that, in contrast to (2), the relation (1) was not invariant with respect to the improper Lorentz transformation with the determinant -1, but was to change itself into the form

$$\beta_\mu [x^\mu, \psi] - \lambda \psi = 0 \tag{4}$$

In the paper mentioned above,[2] a way of removing this difficulty was indicated, but was very unsatisfactory in that the number of components of the spinor ψ was to be increased from 4 to 8 without any immediate physical interpretation for the extra degree of freedom. It came to the author's notice very recently that the following alternative way was far more acceptable in that no extra components of the spinor was introduced. Namely, we take advantage of the antisymmetric tensor of the fourth rank with the components $\varepsilon_{\kappa\lambda\mu\nu}$ which are +1 or -1 according as $(\kappa, \lambda, \mu, \nu)$ are even or odd permutations of (1,2,3,4) and 0 otherwise.[3] Further we take into account the relations

$$i\beta_\nu = \gamma^\kappa \gamma^\lambda \gamma^\mu \tag{5}$$

where $(\kappa, \lambda, \mu, \nu)$ are even permutations of (1,2,3,4). Then (1) can be written in the form

$$\frac{1}{6}\sum_{\kappa\lambda\mu\nu} \varepsilon_{\kappa\lambda\mu\nu}\gamma^\kappa\gamma^\lambda\gamma^\mu[x^\nu, \psi] + i\lambda\psi = 0, \tag{6}$$

which is obviously invariant with respect to the whole group of Lorentz transformations. However, the invariance of (6) can be proved more explicitly by transforming ψ, while the matrices γ^μ are assumed to retain their prescibed forms as defined by (3) independent of the coordinate system. Namely, we can associate a linear transformation

$$\psi' = S\psi \tag{7}$$

with each of the Lorentz transformation

$$x'_\mu = a_{\mu\nu} x_\nu, \tag{8}$$

where S is a matrix with four rows and columns satisfying the relations

$$S\gamma^\mu S^{-1} = a_{\mu\nu}\gamma^\nu. \tag{9}$$

If we insert (7), (8) and (9) in (6) and take advantage of the fact that $\varepsilon_{\kappa\lambda\mu\nu}$ are components of a tensor of the fourth rank, we obtain the commutation relation

$$\frac{1}{6}\sum_{\kappa\lambda\mu\nu} \varepsilon'_{\kappa\lambda\mu\nu}\gamma^\kappa\gamma^\lambda\gamma^\mu[x'^\nu, \psi'] + i\lambda\psi' = 0, \tag{10}$$

which has the same form as (6).

It should be noticed, however, that the relation (6) is to be regarded as a unification of (1) and (4) rather than the mere reproduction of (1), because (6) must be identified with (4) in the coordinate system, which is connected with the original coordinate system by an improper Lorentz transformation with the determinent -1.

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(1) H. Yukawa, Phys. Rev. 76, 300 (1949).
(2) H. Yukawa, Phys. Rev. 77, Jan. 1 (1950).
(3) The antisymmetric tensor $\varepsilon_{\kappa\lambda\mu\nu}$ was useful for unifying the scalar and pseudoscalar fields as well as the vector and the pseudovector fields as shown by M. Schoenberg, Phys. Rev. 60, 468 (1941).