Prog. Vol 3, 207 (1948)

celled by the "counter-terms" in (1). $[P, \Pi]$'s...$[P, Q]$'s are found not to contribute to divergence.

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(1) Z. Koba and G. Takeda, Prog. Theor. Phys. **3** (1948), 98.
(2) Z. Koba and G. Takeda, Prog. Theor. Phys. in press. Cf. also T. Tati and S. Tomonaga, ibid. in press.

Reciprocity in Generalized Field Theory.

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The present author attempted recently to extend the field concept by discarding the restriction that the field quantities were functions of time and space coordinates alone.[1][2] Thus we obtained the operator equations for the generalized electromagnetic potentials A_ν in vacuum

$$[p_\mu, [p^\mu, A_\nu]]=0, \qquad (1)$$

which guaranteed the conservation of energy and momentum for the processes involving the emission or absorption of photons.

Now that the potentials A_ν depend not only on time and space coordinates x_μ, but also on energy-momentum coordinates p_μ, the equations (1) are not sufficient for characterizing the field in vaccuum. In other words, we need further commutation relations between A_ν and x_μ, which have been assumed to commute with one another in the ordinary field theory. In order to find appropriate relations recourse must be had to the well-established fact that the electromagnetic actions propagate with the velocity c in vacuum. Namely, two charged particles at world points x_μ' and x_μ'' respectively interact electromagnetically with each other, only when the condition

$$(x_\mu'-x_\mu'')(x'^\mu-x''^\mu)=0 \qquad (2)$$

is satisfied, so that we may assume the matrix elements

$$(x_\mu'|A_\nu|x_\mu'') \qquad (3)$$

for the electromagnetic potentials to be zero unless the condition (2) holds. This can in turn be expressed by the operator equations

$$[x_\mu, [x^\mu, A_\nu]]=0. \qquad (4)$$

These equations are nothing but those which can be obtained from (1) by replacing p_μ by x_μ. We find here a remarkable symmetry of physical laws with respect to spece-time and energy-momentum, as first noticed by Born under the name "reciprocity".[3]

In the case of the meson field the equations (1) should be replaced, for instance, by

$$[p_\mu, [p^\mu, A_\nu]]=-\mu^2c^2A_\nu, \qquad (5)$$

where A_ν now denote the potentials for the vector meson field with the rest mass μ. As the mesonic actions propagate with the velocity less than c, the condition (2) should be replaced by

$$(x_\mu'-x_\mu'')(x'^\mu-x''^\mu)<0 \qquad (6)$$

Thus, in order to generalize the equations (4) so as to include the meson field, it seems necessary to consider the five dimensional space $(x_0, x_1, x_2, x_3, x_4)$ with the line element

$$ds^2=-dx_0^2+dx_1^2+dx_2^2+dx_3^2+dx_4^2 \qquad (7)$$

The equations (1) and (4) retain their forms, if we extend the summation with respect to the suffix μ to the fifth coordinates p_4 and x_4 respectively and assume for $\nu=0, 1, 2, 3$ the additional conditions

$$[p_4, [p^4, A_\nu]]=\mu^2c^2A_\nu. \qquad (8)$$

Especially, when $\mu=0$ and

$$[x_4, [x^4, A_\nu]]=0, \qquad (9)$$

the equations reduce to those for the generalized electromagnetic field.

Thus, the further development of the generalized field theory may well be guided by the concept of "reciprocity". Detailed accounts will be given in later issue of this journal.[1]

(1) Yukawa, Prog. Theor. Phys. **2** (1947), 209. Part II of this paper will appear in the same journal.
(2) Another way of generalization by considering 5-dimentional space was proposed by Snyder, Phys. Rev. **71** (1947), 38; **72** (1947), 63.
(3) Born, Proc. Roy. Soc. A **165** (1938), 291.

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[N23 140]