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Possible Types of Nonlocalizable Fields.*

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In the previous letter (1) it was pointed out that the field concept ~~can~~ could be extended in conformity with the idea of reciprocity ~~due to Born~~ ~~(2)~~ to the case, in which the field quantities were no more functions of time and space coordinates alone.(2) The simplest example of such nonlocalizable fields is the scalar, neutral and zero-mass field in vacuum.(3) The commutation relations between the scalar potential, space-time coordinates x_μ and momentum-energy operators p_μ takes the form

$$[x^\mu, p_\nu] = i\hbar\,\delta_{\mu\nu}, \qquad (1)$$

$$[x_\mu[x^\mu, U]] = 0, \qquad (2)$$

$$[p_\mu[p^\mu, U]] = 0. \qquad (3)$$

Compatibility of these relations can easily be proved by using matrix representation, in which x_μ is diagonal. Namely, the matrix elements $(x'_\mu|U|x''_\mu)$ can be considered as a function $U(X_\mu, r_\mu)$ of $X_\mu = 1/2(x'_\mu + x''_\mu)$ and $r_\mu = x'_\mu - x''_\mu$. Thus (2) and (3) reduce to

$$r_\mu r^\mu U(X_\mu, r_\mu) = 0 \qquad (2)'$$

and

$$\frac{\partial^2 U(X_\mu, r_\mu)}{\partial X^\mu \partial X_\mu} = 0 \qquad (3)'$$

respectively, so that the compatibility is self-evident.

Now there are various possible types of fields which satisfy the relations (2) and (3). Apart from the trivial case of the ordinary localizable field, there are ~~simplest cases two types~~ two simple types of fields among them. The first type can be obtained by ~~first~~ performing a linear canonical transformation

(4)

$$\left.\begin{aligned}\eta_\mu &= \left(\frac{x_\mu}{\lambda} + \frac{\lambda}{\hbar} p_\mu\right)\\ \zeta_\mu &= \left(\frac{x_\mu}{\lambda} - \frac{\lambda}{\hbar} p_\mu\right)\end{aligned}\right\} \tag{4}$$

where λ is a constant with the dimension of length. The new dimensionless operators η_μ, ζ_μ satisfy the commutation relations

$$[\eta^\mu, \zeta_\nu] = -2i\,\delta_{\mu\nu}. \tag{5}$$

If we assume further that the field U is a function of η_μ alone, the commutation relations (2) and (3) reduce to

$$[\zeta_\mu[\zeta^\mu, U]] = 0 \tag{6}$$

which has the solution of the form

$$U(\eta^\mu) = \sum_{k_\mu\,(k_0>0)} (2k_0)^{-1/2}\{b(k_\mu)\exp(ik_\mu\eta^\mu) + b^*(k_\mu)\exp(-ik_\mu\eta^\mu)\} \tag{7}$$

where k_μ is a null vector satisfying $k_\mu k^\mu = 0$ and the summation with respect to k_μ is restricted to $k_0 > 0$, so that the condition of Hermiticity is satisfied.

Now by using the definition (4) and the relation

$$[k_\mu x^\mu, k^\nu p_\nu] = i\hbar k_\mu k^\mu = 0 \tag{8}$$

we can write

$$\exp(ik_\mu\eta^\mu) = \exp(ik_\mu x^\mu/\lambda)\cdot\exp(i\lambda k^\mu p_\mu/\hbar) \tag{9}$$

If we go back again to the representation, in which U is regarded as a function of X_μ and r_μ, $\exp(i\lambda k^\mu p_\mu/\hbar)$ is nothing but

$$\delta(r_\mu + \lambda k_\mu) \equiv \delta(r_0+\lambda k_0)\,\delta(r_1+\lambda k_1)\,\delta(r_2+\lambda k_2)\,\delta(r_3+\lambda k_3) \tag{10}$$

Thus $U(X_\mu, r_\mu)$ takes the form

$$U(X_\mu, r_\mu) = \sum_{k_\mu (k_0>0)} (2k_0)^{-1/2} \{ b(k_\mu)\,\delta(r_\mu + \lambda k_\mu) \exp(i k_\mu X^\mu/\lambda) + b^*(k_\mu)\,\delta(r_\mu - \lambda k_\mu) \exp(-i k_\mu X^\mu/\lambda) \} \qquad (10)$$

Second quantization can be performed by considering $b(k_\mu)$, $b^*(k_\mu)$ as noncommutative operators satisfying usual commutation relations

$$\left.\begin{aligned} [b(k'_\mu), b(k''_\mu)] &= [b^*(k'_\mu), b^*(k''_\mu)] = 0 \\ [b(k'_\mu), b^*(k''_\mu)] &= \delta(k'_\mu, k''_\mu) \end{aligned}\right\} \qquad (11)$$

The second and slightly more complicated type of field can be obtained by introducing complex canonical operators

$$\begin{aligned} \xi_\mu &= 1/\sqrt{2}\,(x_\mu/\lambda - i\lambda p_\mu/\hbar) \\ \xi^*_\mu &= 1/\sqrt{2}\,(x_\mu/\lambda + i\lambda p_\mu/\hbar) \end{aligned} \qquad (12)$$

satisfying the relations

$$[\xi^\mu, \xi^*_\nu] = -\delta_{\mu\nu} \qquad (13)$$

and by assuming the field to have the form

$$U(\xi_\mu, \xi^*_\mu) = 1/2\,\{u(\xi_\mu) + u^*(\xi^*_\mu)\} \qquad (14)$$

Extension of above arguments to the case of vector field corresponding to the electromagnetic field is obvious. ~~The first example~~ The vector field similar to U in (7) or (10) can be regarded as a relativistically invariant way of introducing universal length λ in the formalism of the positron theory[4], whereas the field similar to (14) can be regarded as a relativistic cut-off procedure with the exponential cut-off factor. In order to extend the above considerations to the fields with non-zero mass, it is necessary to introduce five dimensional space as shown in the previous letter[1][5]. The case of coexistence of two or more interacting fields is under investigation.

3

(*) ~~Main contents~~ A part of this letter was read before the Annual Meeting of Physical Society of Japan at Kyoto University, May 23, 1948.

(**) On leave of absence from Kyoto University.

(1) Yukawa, Prog. Theor. Phys. **2**(1948), 205.

(2) Very recently, Prof. Born informed me kindly that he had pointed out the self-reciprocity of the ordinary localizable electromagnetic field in ~~a lecture~~ Phys. Soc. Camb. Conf. Rep. 1947.

(3) The word " nonlocalizable" is used as an extension of the terminology by Dirac, Phys. Rev. **73**(1948), 1092.

(4) Heisenberg, ZS. f. Phys. **90**(1934), 209; Serber, Phys. Rev. **49**(1936), 545. This procedure can also be regarded as a way of replacing λ-limiting process by non-zero λ.

(5) Watanabe, ~~Prog. Theor. Phys. in press.~~ Phys. Rev. **74** (1948), ~~in press~~ 1864