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On the Theory of Elementary Particles. I.

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§1. Introduction

Present theory of elementary particles customa~~lly~~ ~~deals with~~ [ly start from] the quantized field, which is equivalent to the assembly of indefinite number of one sort of elementary particles. The field which should be quantized is sometimes nothing but the field wellknown in classical theory as in the case of the electromagnetic field, but it ~~has~~ may have sometimes no direct counterpart ~~m~~ in classical theory ~~in other cases such as~~ as in the case of the electron field, which appears first in ~~the~~ quantum mechanics as the wave function for an elementary particle. Hitherto it has been regarded as one of the greatest success ~~rather than the defect~~ of ~~the theory of~~ relativistic quantum mechanics that the two things, i. e. the material particle and the field of force, which had been entirely different in classical theory as well as in nonrelativistic quantum mechanics, could be ~~treated~~ coordinated by the method of field quantization. If , however, we want to go into the problem of the interaction of different sorts of elementary particles, we find at once the essential difference still remaining between matter and field. We consider, for instance, the interaction between two electrons usually as due to intermediary action of the electromagnetic field instead of direct interaction at distance. ~~Recent developement of the meson theory of nuclear forces seems to have made the situation more complicated, because the meson has been considered to be something between the field and and matter.~~

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(1) The content of this paper was read before the meetings of the Physical Society of Japan held at Kyoto University, Nov. 23, 1946 and at Nagoya University, March 1, 1947.

[2]

Under this standpoint we can go further
~~For this it seems to be~~
~~Thus~~ in order, there seems to be two ways too
for revising and completing the theory of elementary particles.

Now the wellknown divergence difficulties of the present theory have their origin in the very assumption of the coexistence of the field around the point particle. Although recent development of the meson theory of nucear forces seems to have made the situation more complicated because of the fact that the meson has been considered to be something with the dual nature, one of the main causes of newly added difficulties in the meson theory can still be ascribed to the point interaction between field and particle.(2) Various attempts hitherto made in order to remove these difficulties by many authors can be classified as follows:

One way is to
We can namely either
Namely
following thoroughly
further the general trend
of quantum mechanics of
treating all sorts of elementary particles

i) Reestablishment of the classical theory of action at distance by Wheeler[2] which starts from Dirac's formulation of the classical theory of the radiating electron.[3]

ii) Change of the method of quantization of the field by introducing ~~the do~~ the universal length in some way. Subtraction devices, S-matrix scheme, λ-limiting process, nonlinear field theory and direct quantization of particles the space-time frame work can all be included into this class.

iii) Elimination of the singularities ~~of the~~ by mutual compensation of two or more fields surrounding the particle such as in the cases of mixed field theory of Møler-Rosenfeld, Bopp's theory and various extension of these theories.

iv) Revision of the assumption of the point interaction between the field and matter, such as in the cases of extended source models.

Although each of these four methods of revision has its own advantage, any one of them cannot get rid of all defects of the present theory by itself. In this paper, the author wants to show that a natural extension ~~of~~ of the field concept, which has been so useful in physics for many years since, leads to a promising revision of the present theory, which can be regarded as a certain combination of the four methods above mentioned.

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(2) Wheeler,
(3) Dirac,
Sakata, 2, (194)

(2) ~~Relations between field and matter~~ Importance of the conflict between two concept field and matter was stressed also by Taketani, Prog. 3, 182 (1947); and

§2. An Extension of the Mixed Field Theory

It is One of the well-established empirical facts ~~is~~ that the charged particle of spin 1/2 such as the electron or the proton has the neutral partner such as the neutrino or the neutron, and that ~~some~~ various nuclear phenomena can be interpreted more simply ~~naturally~~ by assuming the transition between charged and neutral states of an elementary particle such as the nucleon or the light particle than by considering the charged and the neutral particles as entirely different things. Thus, if we assume the wave function of any one of these particles satisfies Dirac's wave equation of the form

neglect the mass difference between the charged states neutral and the

$$\left(\sum_{\mu=1}^{4}\alpha_\mu p_\mu + \beta mc\right)\psi = 0 \qquad (1)$$

in vacuum, where ~~denote the isotopic spin variables with and corresponding to the neutral and the charged states respectively,~~ the wave function ψ becomes a function of ~~x,y,z,t~~ x_1, x_2, x_3, x_4 and ρ_3, σ_3 as well as of τ_3. In the presence of other particles, it is usual to modify the operator in the wave equation ~~is modified~~ by the addition of the term characterizing the interaction of the original particle with other particles field originated by other particles ~~field~~. The most general form of this interaction operator for the original particle as well as for the can be expressed by a certain function of all the variables, which can always be reduced to a sum of linear functions of ρ's and σ's as well as of τ's. Thus, the wave equation takes the form

$$\left(\sum_{\mu=1}^{4}\alpha_\mu p_\mu + \beta mc + V\right)\psi = 0 \qquad (5)$$

~~where V~~ with

$$V = \sum_{\lambda,\mu,\nu=1}^{4}\rho_\lambda \sigma_\mu \tau_\nu V_{\lambda\mu\nu} \qquad (6)$$

where each of 64 coefficient $V_{\lambda\mu\nu}$ ~~are each~~ is a certain function of the particle ~~coordinates~~ variables x,y,z,t x_μ and p_μ ~~$p_x\ p_y\ p_z\ p_t$~~ and of the field variables.

$\rho_4 = \sigma_4 = \tau_4 = 1$ and

$$\alpha_4 = 1 \quad \alpha_1 = \rho_1\sigma_1 \quad \alpha_2 = \rho_1\sigma_2 \quad \alpha_3 = \rho_1\sigma_3 \quad \beta = \rho_3 \qquad (2)$$

and

$$p_\mu = -i\hbar\frac{\partial}{\partial x_\mu} \qquad (3)$$

with

$$x_1 = x \quad x_2 = y \quad x_3 = z \quad x_4 = ct, \qquad (4)$$

where τ_1, τ_2, τ_3 ~~are~~ denote isotopic spin variables, ~~with~~ $\tau_3 = \pm 1$ denoting the n. and c. states resp.

(6) can alternatively be written into the form

$$= (\tau_1 + i\tau_2)\sum_{\lambda,\mu} \rho_\lambda \sigma_\mu V^{(1)}_{\lambda\mu} + (\tau_1 - i\tau_2)\sum V^{(2)} + (1+\tau_3)\sum V^{(3)} + (1-\tau_3)\sum V^{(4)} \qquad (7)$$

In this expression the charged fields are included in first two and last two terms respectively.

In this way we can decompose the most general field acting on the particle first into the charged and the neutral fields, each of which in turn can be further decomposed in the usual manner into scalar, vectro, pseudoscalar and spëudovector fields, so that we ~~have~~ arrive at a the most general mixed field theory of the mixed fields containing 16 sorts of fields, ~~each~~ An essential difference between these ~~fields~~ and the ordinary fields is that the former are not necesarily functions of ~~x,y,z and t~~ x_μ alone, but they may depend on p_μ also. Such an extension of the fieldconcept was already made by Markow in the case of electromagnetic field.(4) He assumed a certain commutation relations between the coordinates and the field quantities ~~in order to~~ so as to obtain the damping effect. We want to consider, however, the general properties of the field as represented by operators of the above sense, before we go into the particular form of these operator to be characterized, for example, by the commutation relations.

which included in the last term of (7).

We start from the simplest case of the electromagnetic field. We ... by takeing into account another wellestablished fact that the transition of an elementary particle form one charged state to another is accompanied by the emission or absortption of a photon with the energy and momentum the proper ... satisfying the conservation laws. ~~If we denote~~ the electromagnetic potentials ~~by~~ A_ν ~~they~~ are certain functions of x_μ in the ordinary field theory, so that they can be represented by matrices of the form

$$(p'_\mu|A_\nu|p''_\mu) = (2\pi\hbar)^{-4}\int\cdots\int e^{-ip'_\mu x'_\mu/\hbar} A_\nu(x'_\mu)\,\delta(x'_\mu - x''_\mu)\, e^{ip''_\mu x''_\mu/\hbar}(dx'_\mu)^4(dx''_\mu)^4$$

$$= (2\pi\hbar)^{-4}\int\cdots\int e^{-i(p'_\mu - p''_\mu)x'_\mu/\hbar}(dx'_\mu)^4 \qquad (8)$$

in p-space and conversely we have

$$A_\nu(x'_\mu) = \int (p'_\mu|A_\nu|p''_\mu)\, e^{i\pi'_\mu x'_\mu/\hbar}(d\pi'_\mu)^4 \qquad (9)$$

In order that the electromagnetic field, when quantized, is equivalent to an assembly of photons, the matrices $(p'_\mu|A_\nu|p''_\mu)$ should have the form

$$(p'_\mu|A_\nu|p''_\mu) = A_\nu(\pi'_\mu)\,\delta(\pi'_\mu\pi'^\mu) \qquad (11)$$

where

$$\pi'_\mu\pi'^\mu = \pi'^2_1 + \pi'^2_2 + \pi'^2_3 - \pi'^2_4. \qquad (12)$$

where

$$\pi'_\mu = p'_\mu - p''_\mu \qquad (10)$$

(4) Markow

Now we can extend the expression (11) without missing the expectation that the electromagnetic field will still represent an assembly of photons, when it is quantized. We assume namely a more general form

where the matrix elements are no more ~~a~~ functions of alone, corresponding to the situation that the potentials A are not functions of x alone. In~~o~~ order that A transform like a four vector under orentz transformation, each matrix element should transform to a linear combination of the corresponding matrix elements in the new coordinate system in the sam eway as a four vector. Thus ~~i~~should ~~be~~ ~~a~~ take a form

where the coefficients are functions of invariants .

Alternatively we can start form the D'Alembertian for the ordinary electromagnetic potential

instead of the expression (9). Inp-space the equati ons ( ) are represented in the form

or

which is equivalent to the operator equations

where

Thus we can ~~take~~ define the electric and magnetic field

by
by

respectively

and can assume the field equation for the extended field to be of the form

Above consideration can be easily generalized to the case of the extenion of the field, which is accompanied by particles of finite rest mass. In the case of the vector field, for example, the field equations can be transformed to the ~~form~~ simultaneous operator equations

where are four vector and six vector operators respectively. In this way, we obtain the commutation relations between the momentum and energy pperators and the field variables as a natural extension of the classical field equations. While in the case of the ordinary field the field equations are the only restriction, in the case we have to consider further the commutation relations between the space-time operators and the field variables.

$$[p_\mu, [p^\mu, A_\nu]] = 0$$

$$\rightarrow \int \cancel{e^{-i(p'-p'')x/\hbar}} \, ((p'_\mu - p''_\mu))^2 (p'_\mu | A_\nu | p''_\mu) = 0$$

$$(p'_\mu | A_\nu | p''_\mu) = 0 \quad \text{for} \quad (p'_\mu - p''_\mu)^2 \neq 0$$

$$\frac{p'_\mu + p''_\mu}{2} = P_\mu$$

$$A_\nu(P_\mu, \pi_\mu) = A'_\nu(P_\mu, \pi_\mu)\, \delta(\pi_\mu \pi^\mu)$$