Non-Local Field Theory
II.
1950 ~
Columbia

Seminar on Field Theories

Feb. 14, 1950

General Program

I. Free Field

(i) Definition of the Elementary Particles
— Elementary Non-local System
(ii) Lagrangian - Hamiltonian
formalism possible or not?
(iii) Zero mass case

II. Interaction

(i) Integral formalism — S-matrix
scheme — 4-dimensional reciprocal relations,
(ii) H-q.-scheme versus L-D-scheme
(iii) Question of Convergence —
Self-energy terms —
Vacuum Polarization
(iv) Existence or ~~non-existence~~ absence
of local wave function? Physical
Interpretation — Correlation
Amplitude
Analogy of Deuteron problem
(V) Problem of Causality — time
sequence
(Vi) Particular examples
nonlocal scalar + local spinor

Reciprocal Relations

(R.R1)

(i) One Dimensional R. R.

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} g(k)e^{ikx}dk \; ; \quad g(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{-ikx}dx$$

$$f(x) = 1 \qquad g(k) \overset{(\sqrt{2\pi})}{=} \delta(k) .$$

$$f(x) = \delta(x) \qquad g(k) = \frac{1}{\sqrt{2\pi}}$$

$$f(x) = \begin{cases} 1/2 & x>0 \\ -1/2 & x<0 \end{cases} \qquad g(k) = \frac{1}{\sqrt{2\pi}}\frac{1}{ik}$$

$$\Big(\because \quad \int_{-\infty}^{+\infty} e^{ikx}dk = 2\pi\,\delta(x)$$

$$\frac{\partial}{\partial x}\int_{-\infty}^{+\infty}\frac{e^{ikx}}{ik}dk = 2\pi\,\delta(x)$$

or if we define $f(x)$ by

$$f(x) \equiv \frac{1}{\cancel{\sqrt{2\pi}}\;2\pi}\int_{-\infty}^{+\infty}\frac{e^{ikx}}{ik}dk$$

$$\frac{\partial f(x)}{\partial x} = \delta(x)$$

$$\underset{(x<0)}{f(x)} = \int_{-\infty}^{x<0}\frac{\partial f(x)}{\partial x}dx + c = c ,$$

$$\underset{(x>0)}{f(x)} = \int_{-\infty}^{x>0}\frac{\partial f(x)}{\partial x}dx + c = 1 + c .$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{ikx}}{ik}dk = \frac{1}{2\pi}\int_{+\infty}^{-\infty}\frac{e^{-ikx}}{ik}dk$$

$$= -\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{ikx}}{ik}dk = -f(-x) ,$$

so that $1 + c = -c$ $\qquad \underline{c = -\frac{1}{2}}$ $\Big)$

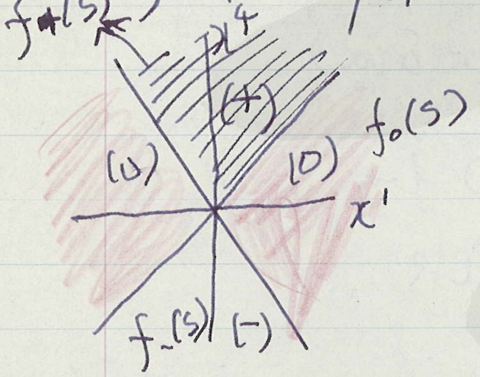
(ii) Four Dimensional R. R.

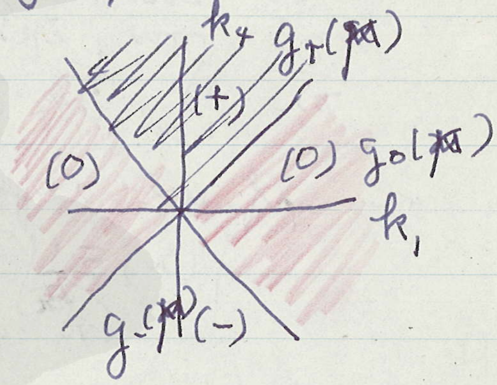
$$f(x_\mu) = \underbrace{\int_{-\infty}^{+\infty}\!\cdots\!\int}_{\left(\frac{1}{2\pi}\right)^2} g(k_\mu)\, e^{ik_\mu x^\mu} (dk_\mu)^4$$

$$g(k_\mu) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty}\!\cdots\!\int f(x_\mu)\, e^{-ik_\mu x^\mu} (dx^\mu)^4$$

$f(x_\mu) = 1$ $\qquad$ $g(k_\mu) = (2\pi)^2 \prod_{\mu=1}^{4} \delta(k_\mu)$

$f(x_\mu) = \prod_{\mu=1}^{4} \delta(x^\mu)$ $\qquad$ $g(k_\mu) = \frac{1}{(2\pi)^2}$





$s = \sqrt{x_\mu x^\mu}$ in (0) $\qquad$ $\mu = \sqrt{k_\mu k^\mu}$ in (0)

$s = \sqrt{-x_\mu x^\mu}$ in (+) and (−) $\qquad$ $\mu = \sqrt{-k_\mu k^\mu}$ in (+) and (−)

$f_+(s) = \frac{\varkappa J_1(\varkappa s)}{s}$ $\qquad$ $g_+(\mu) = \frac{2i}{}\delta(\mu^2 - \varkappa^2)$

$f_0(s) = 0$ $\qquad$ $g_0(\mu) = ~~\delta(\mu^2-\varkappa^2)~~ 0$

$f_-(s) = -\frac{\varkappa J_1(\varkappa s)}{s}$ $\qquad$ $g_-(\mu) = -\frac{2i}{}\delta(\mu^2 - \varkappa^2)$

~~$f_+(s) = \frac{\varkappa J_1(\varkappa s)}{4 s}$~~ $\Big\}$ ~~$g_+(\mu) = \frac{i}{2}\delta(\mu^2-\varkappa^2)$~~

~~$f_0(s) = 0$~~ $\Big\}$ ~~$g_0(\mu) = 0$~~ $\Big\}$

~~$f_-(s) = -\frac{\varkappa J_1(\varkappa s)}{4 s}$~~ ~~$g_-(\mu) = -\frac{i}{2}\delta(\mu^2-\varkappa^2)$~~

$R_1 R_2$

$$(\because\ D(x_\mu) = \left(\frac{1}{2\pi}\right)^2 i \iiint_{\substack{k_\mu k^\mu = \cancel{\mu}\varkappa^2 \\ k^4 > 0}} \frac{e^{ik_\mu x^\mu}}{|k^4|} dk_{\cancel{\mu}1}\, dk_2\, dk_3$$

$$-\left(\frac{1}{2\pi}\right)^2 i \iiint_{\substack{k_\mu k^\mu = \varkappa^2 \\ k^4 < 0}} \frac{e^{ik_\mu x^\mu}}{|k^4|} dk_1\, dk_2\, dk_3$$

$$k_1^2 + k_2^2 + k_3^2 - k_4^2 + \mu^2 = 0$$

$$|k_4|\, d|k_4| = \mu\, d\mu$$

$$d|k_4| = \frac{\mu\, d\mu}{|k_4|}$$

$$= \frac{1}{2|k_4|} d(\mu^2)$$

$$D(x_\mu) = \left(\frac{1}{2\pi}\right)^2 i \cdot \int_{(+)}\!\!\cdots\!\int g'_+(\mu)\, e^{ik_\mu x^\mu} (dk_\mu)^4$$

$$+ \left(\frac{1}{2\pi}\right)^2 i \int_{(-)}\!\!\cdots\!\int g'_-(\mu)\, e^{ik_\mu x^\mu} (dk_\mu)^4$$

$$= \left(\frac{1}{2\pi}\right)^2 i \int\!\cdots\!\int \frac{g'_+(\mu)}{2|k_4|} d(\mu^2)\, e^{ik_\mu x^\mu} (dk_i)^3$$

$$+ \left(\frac{1}{2\pi}\right)^2 i \int\!\cdots\!\int \frac{g'_-(\mu)}{2|k_4|} d(\mu^2)\, e^{ik_\mu x^\mu} (dk_\mu)^3$$

$$g'_+(\cancel{\mu}) = 2i\delta(\mu^2 - \varkappa^2)$$

$$g'_-(\mu) = -2i\delta(\mu^2 - \varkappa^2)$$

$$f'_+(s) = \frac{1}{\cancel{xx}} D(x_\mu) = \frac{1}{\cancel{xx}} \frac{1}{s} \frac{\partial F(s)}{\partial s}$$

$$= \frac{1}{\cancel{xx}} \frac{-\varkappa}{s} \frac{\partial J_0(\cancel{\varkappa}s)}{\partial(\varkappa s)} = \frac{1}{\cancel{x}s\cancel{x}}[\varkappa\cancel{x}\, J_1(\varkappa s)]$$

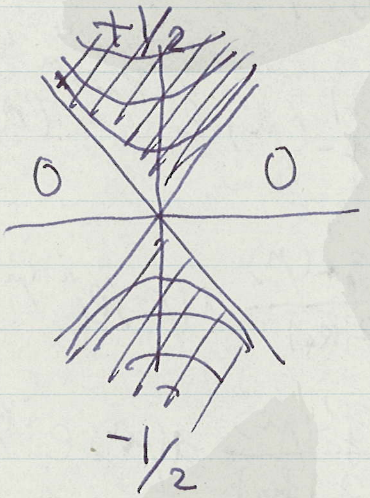
$$f'_0(s) = 0$$

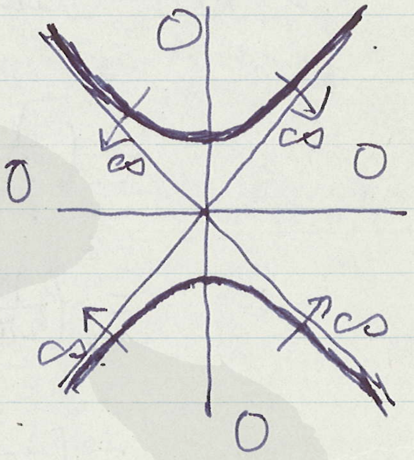
$$f'_-(s) = -\frac{1}{\cancel{x}s\cancel{x}}[\varkappa\cancel{x}\, J_1(\varkappa s)]\ )$$

$$\left.\begin{aligned} f_+(s) &= \frac{1}{2} \\ f_0(s) &= 0 \\ f_-(s) &= -\frac{1}{2} \end{aligned}\right\} \qquad \left.\begin{aligned} g_+(\mu) &= 2i\,\frac{\delta(k_\mu k^\mu)}{k_\mu k^\mu} \\ g_0(\mu) &= 0 \\ g_-(\mu) &= -2i\,\frac{\delta(k_\mu k^\mu)}{k_\mu k^\mu} \end{aligned}\right\}$$

More precisely

$$\left.\begin{aligned} f_+(s) &= \frac{1}{2}\lim_{\varkappa\to 0}\frac{2J_1(\varkappa s)}{\varkappa s} \\ f_0(s) &= 0 \\ f_-(s) &= -\frac{1}{2}\lim_{\varkappa\to 0}\frac{2J_1(\varkappa s)}{\varkappa s} \end{aligned}\right\} \qquad \left.\begin{aligned} g_+(\mu) &= 2i\lim_{\varkappa\to 0}\frac{\delta(\mu^2-\varkappa^2)}{\mu^2} \\ g_0(\mu) &= 0 \\ g_-(\mu) &= -2i\lim_{\varkappa\to 0}\frac{\delta(\mu^2-\varkappa^2)}{\mu^2} \end{aligned}\right\}$$





$$\left(\because \quad J_n(z) = \sum_{m=0}^{\infty}\frac{(-1)^m (z/2)^{n+2m}}{m!\,(n+m)!}\right.$$

$$J_1(z) = \frac{1}{2}z - \frac{z^3}{2\cdot 2^3} + \cdots$$

$$\left.\lim_{\varkappa\to 0}\frac{2J_1(\varkappa s)}{\varkappa s} = 1 - \lim_{\varkappa\to 0}\frac{(\varkappa s)^2}{2^3} + \cdots,\right)$$

R.R.3

$$f_+(s) = -\frac{\kappa}{s}\frac{dJ_0(\kappa s)}{d(\kappa s)} = -\frac{\kappa}{s}\{\delta(\kappa s) - J_1(\kappa s)\}$$

$$= -\frac{1}{s}\delta(s) + \frac{\kappa}{s}J_1(\kappa s)$$

$$= -2\delta(s^2) + \frac{\kappa}{s}J_1(\kappa s)$$

$$\left.\begin{aligned} f_+(s) &= -2\delta(s^2) + \frac{\kappa}{s}J_1(\kappa s) \\ f_0(s) &= 0 \\ f_-(s) &= \left\{\begin{aligned} &\cancel{\frac{\kappa}{s}\frac{dJ_0(\kappa s)}{d(\kappa s)}} \\ &2\delta(s^2) - \frac{\kappa}{s}J_1(\kappa s) \end{aligned}\right. \end{aligned}\right\} \quad \left\{\begin{aligned} g_+(K) &= 2i\delta(K^2-\kappa^2) \\ g_0(K) &= 0 \\ g_-(K) &= -2i\delta(K^2-\kappa^2) \end{aligned}\right.$$

$$\left.\begin{aligned} f_+(s) &= -\frac{2\delta(s^2)}{\kappa^2} + \frac{J_1(\kappa s)}{\kappa s} \\ f_0(s) &= 0 \\ f_-(s) &= \frac{2\delta(s^2)}{\kappa^2} - \frac{J_1(\kappa s)}{\kappa s} \end{aligned}\right\} \quad \left\{\begin{aligned} g_+(K) &= \frac{2i\delta(K^2-\kappa^2)}{\kappa^2} \\ g_0(K) &= 0 \\ g_-(K) &= -\frac{2i\delta(K^2-\kappa^2)}{\kappa^2} \end{aligned}\right.$$

$$\left.\begin{aligned} f'_+(s) &= \frac{2\delta(s^2-\kappa^2)}{\kappa^2} \\ f'_0(s) &= 0 \\ f'_-(s) &= -\frac{2\delta(s^2-\kappa^2)}{\kappa^2} \end{aligned}\right\} \quad \left\{\begin{aligned} g'_+(K) &= -\frac{2i\delta(K^2)}{\kappa^2} + \frac{\cancel{2}iJ_1(\kappa K)}{\kappa K} \\ g'_0(K) &= 0 \\ g'_-(K) &= \frac{2i\delta(K^2)}{\kappa^2} - \frac{\cancel{2}iJ_1(\kappa K)}{\kappa K}. \end{aligned}\right.$$

$$\left.\begin{aligned} f''_+(s) &= \frac{2(\delta(s^2-\kappa^2)-\delta(s^2))}{\kappa^2} + \frac{J_1(\kappa s)}{\kappa s} \\ f''_0(s) &= 0 \\ f''_-(s) &= -f''_+(s) \end{aligned}\right\}$$

$$\left.\begin{aligned} g''_+(K) &= \frac{2i(\delta(K^2-\kappa^2)-\delta(K^2))}{\kappa^2} + \frac{\cancel{2}iJ_1(\kappa K)}{\kappa K} \\ g''_0(K) &= 0 \\ g''_-(K) &= -g''_+(K) \end{aligned}\right\}$$

Self-Reciprocal Function

lim $\kappa \to 0$:

$$\left.\begin{aligned} F_+(S) &= 2\delta_+'(S^2) + \frac{1}{2} \\ F_0(S) &= 0 \\ F_-(S) &= -2\delta_+'(S^2) - \frac{1}{2} \end{aligned}\right\} \quad \left\{\begin{aligned} G_+(K) &= 2i\delta_+'(K^2) + 2i \\ G_0(K) &= 0 \\ G_-(K) &= -2i\delta_+'(K^2) - 2i \end{aligned}\right.$$

$\frac{i}{2}$

$-\frac{i}{2}$

$$\int f(x)\,\delta_+'(x)\,dx = \lim_{\substack{\varepsilon \to 0 \\ \varepsilon > 0.}} \frac{f(\varepsilon) - f(0)}{\varepsilon}$$

$$\left(\frac{1}{2\pi}\right)^2 \int F(S) \exp(-ik_\mu x^\mu)(dx^\mu)^4$$
$$= G(K) \qquad (dk_\mu)^4$$

$\underline{F^* = -F}$ $\qquad$ $\underline{E^* = E}$

$D_+ = \frac{1}{2}E + F$ $\qquad$ $D_- = \frac{1}{2}E - F$ $\qquad$ $D_+^* = D_-$

$$\left.\begin{aligned} (D_+)_+ &= 2\delta_+'(S^2) + 1 \\ (D_+)_0 &= 1/2 \\ (D_+)_- &= -2\delta_+'(S^2) \end{aligned}\right\} \quad \left\{\begin{aligned} (C_+)_+ &= \frac{1}{2}(\delta(K))^4 + 2i\delta_+'(K^2) + i \\ (C_+)_0 &= \frac{1}{2}(\delta(K))^4 \\ (C_+)_- &= +\frac{1}{2}(\delta(K))^4 - 2i\delta_+'(K^2) - i \end{aligned}\right.$$

$$\left.\begin{aligned} (D_-)_+ &= -2\delta_+'(S^2) \\ (D_-)_0 &= 1/2 \\ (D_-)_- &= 2\delta_+'(S^2) + 1 \end{aligned}\right\}$$

$\prod_{\mu=1}^{4} \delta(x^\mu) - 1$ $\qquad\qquad$ $1 + i\prod_{\mu=1}^{4} \delta(k_\mu)$

$1 + \prod_{\mu=1}^{4} \delta(x^\mu)$ $\qquad\qquad$ $-\prod_{\mu=1}^{4} \delta(k_\mu) + i$

$$f_+''(s) = \frac{2\{\delta(s^2-\kappa^2)-\delta(s^2)\}}{\kappa^2} + \frac{J_1(\kappa s)}{\kappa s}$$

$$g_+''(K) = \frac{2i\{\delta(K^2-\kappa^2)-\delta(K^2)\}}{\kappa^2} + \frac{2i\,J_1(\kappa K)}{\kappa K}$$

$$f_+'' = \quad 2$$

$$f_+(s) = -2\delta(s^2) + \frac{\kappa}{s} J_1(\kappa s) \qquad g_+(K) = 2i\delta(K^2-\kappa^2)$$

$$f_+'(s) = -2\delta(s^2) \qquad g_+'(K) = 2i\delta(K^2)$$

$$\lim_{\kappa\to 0} \frac{\overset{\kappa}{f_+}(s) - f_+'(s)}{\kappa^2} = \lim_{\kappa\to 0} \frac{\kappa J_1(\kappa s)}{\kappa s} \qquad \lim_{\kappa\to 0} \frac{g_+(K) - g_+'(K)}{\kappa^2}$$

$$= \frac{1}{2} \qquad\qquad = 2i\ \delta_+'(K^2)$$

$$2\delta(s^2) \qquad 2i\delta(K^2)$$

Volume in 4-Dimensional space

$s = \sqrt{c^2t^2 - r^2}$ $\tanh\chi = \frac{r}{ct}$

$r = s\sinh\chi$ ≥ 0

$ct = \pm s\cosh\chi$

$ct > 0$

$x = s\sinh\chi\sin\theta\cos\varphi$

etc.,

$ct = s\cosh\chi$

$0 \leq s \leq \infty$ $0 \leq \chi < \infty$

$$\int\cdots\int f(xyzct)(dx)^4$$

$$= \int_0^\infty\int_0^\infty\int_0^\pi\int_0^{2\pi} f(s,\chi,\theta,\varphi)\, s^3 \sinh^2\chi \sin\theta \, ds\, d\chi\, d\theta\, d\varphi$$

$$= 4\pi\int_0^\infty s^3 ds \int_0^\infty \sinh^2\chi\, d\chi$$

$$= \pi\int_0^\infty s^3 ds \int_0^\infty (e^{2\chi} + e^{-2\chi} - 2)\, d\chi$$

$$= \pi\int_0^\infty s^3 ds \cdot \left(\underset{\downarrow \atop \infty}{\sinh 2\chi} - 2\chi\right)_0^\infty$$

=

II. ~~Interaction of Local Scalar Field with Non-local Neutral Scalar~~ Field

Int. 1

Fourier transform of D-function

Yennie, Feb. 10, 1950

$$P\int \frac{k_0}{|k_0|}\,\delta'(k_\mu k^\mu)\exp(ik_\mu x^\mu)\,(dk)^4$$

$$= 2\pi\int_0^\infty\int_0^\pi\int_{-\infty}^{+\infty} k^2 dk \sin\theta\, d\theta\, dk_0\,\frac{k_0}{|k_0|}$$

$$\times\,\delta'(k^2-k_0^2)\exp(ikr\cos\theta - ik_0x_0)$$

$$= 2\pi\int_0^\infty\int_{-\infty}^{+\infty}\frac{k_0}{|k_0|}k^2\delta'(k^2-k_0^2)\exp(-ik_0x_0)$$

$$\times\,\frac{1}{ikr}\{\exp(ikr)-\exp(-ikr)\}\,dk\,dk_0$$

$$= \frac{\pi}{ir}\int_0^\infty\int_{-\infty}^{+\infty}\frac{k_0}{|k_0|}\exp(-ik_0x_0)\left(\exp(ikr)-\exp(-ikr)\right)$$

$$\times\,\frac{\partial}{\partial k}\delta(k^2-k_0^2)\,dk\,dk_0$$

$$= -\pi\int_0^\infty\int_{-\infty}^{+\infty}\frac{k_0}{|k_0|}\exp(-ik_0x_0)\left(\exp ikr+\exp -ikr\right)$$

$$\times\,\delta(k^2-k_0^2)\,dk\,dk_0 + \boxed{\text{integrated part}}$$

$$= -\frac{\pi}{2}\int_{-\infty}^{+\infty}\frac{dk_0}{k_0}\exp(-ik_0x_0)\left(\exp(ikr)+\exp(-ikr)\right)$$

$$P\int_{-\infty}^{+\infty}\frac{\exp(ix\omega)}{\omega}d\omega = \begin{cases}\pi i & x>0\\ -\pi i & x<0\end{cases}$$

For $x_0 > 0$:

$$\int = -\frac{\pi^2 i}{2}\begin{cases} 0 & \text{for } r > x_0 \\ -2 & \text{for } r < x_0 \end{cases}$$

For $x_0 < 0$

$$\int = -\frac{\pi^2 i}{2}\begin{cases} 0 & \text{for } r > |x_0| \\ 2 & \text{for } r < |x_0| \end{cases}$$

Hence

$$D = \frac{-i}{\pi^2} P \int (dk)^4 \frac{k_0}{|k_0|} \delta'(k^2) \exp(i k_\mu x^\mu)$$

$$E = \int (dk)^4 \delta_4(k) \exp(i k_\mu x^\mu)$$

$$\left.\begin{aligned} D_+ &= \frac{1}{2}(E + D) \\ D_- &= \frac{1}{2}(E - D) \end{aligned}\right\}$$

E : even function of x^μ $(\mu = 1, 2, 3, 4)$

D : even function of x_1, x_2, x_3 and odd function of x_4.

Integrals involving Bessel functions

+1

$$\int_0^\infty e^{-at} J_\nu(bt)\, t^{\mu-1}\, dt = \frac{(\frac{1}{2}b/a)^\nu\, \Gamma(\mu+\nu)}{a^\mu\, \Gamma(\nu+1)}\ {}_2F_1\left(\frac{\mu+\nu}{2}, \frac{\mu+\nu+1}{2};\ \nu+1;\ -\frac{b^2}{a^2}\right)$$

−1

for $\left\{\begin{array}{l} |b| < |a| \\ R(a+ib) > 0 \end{array}\right.$, $R(a-ib) > 0$

$$\int_0^\infty e^{-at} J_\nu(bt)\, t^{\mu-1}\, dt = \frac{(\frac{1}{2}b)^\nu\, \Gamma(\mu+\nu)}{\cancel{a^\mu}\, \Gamma(\nu+1)\,(a^2+b^2)^{\frac{1}{2}(\mu+\nu)}}\ {}_2F_1\left(\frac{\mu+\nu}{2}, \frac{1-\mu+\nu}{2};\ \nu+1;\ \frac{b^2}{a^2+b^2}\right)$$

$${}_2F_1(\alpha, \beta; \rho; z) = \sum_{n=0}^\infty \frac{(\alpha)_n (\beta)_n}{n!\,(\rho)_n} z^n$$

$$(\alpha)_n = \alpha(\alpha+1)\ldots(\alpha+n-1)$$

$$\int_0^\infty e^{-at} J_\nu(bt)\, t^\nu\, dt = \frac{(2b)^\nu\, \Gamma(\nu+\frac{1}{2})}{(a^2+b^2)^{\nu+\frac{1}{2}}\sqrt{\pi}} \qquad (R(\nu) > -\tfrac{1}{2})$$

$$\int_0^\infty e^{-at} J_\nu(bt)\, t^{\nu+1}\, dt = \frac{2a\cdot(2b)^\nu\, \Gamma(\nu+\frac{3}{2})}{(a^2+b^2)^{\nu+3/2}\sqrt{\pi}} \qquad (R(\nu) > -1)$$

Watson, Bessel Function p.385.

$$\int_0^\infty e^{-at} J_2(bt)\, t^2\, dt = \frac{(2b)^2\, \Gamma(2+\frac{1}{2})}{(a^2+b^2)^{2+\frac{1}{2}}\sqrt{\pi}}$$

$$\int_0^\infty e^{-at} J_1(bt)\, t^{1}\, dt = \frac{2b\, \Gamma(1+\frac{1}{2})}{(a^2+b^2)^{1+\frac{1}{2}}\sqrt{\pi}}.$$ ○

$$\int_0^\infty J_1(bt)\,t\,dt = \frac{2}{b^2}\,\frac{\Gamma(1+\frac{1}{2})}{\sqrt{\pi}}$$

$$g_+(\mu) = \int \frac{1}{2}\,\delta(\mu^2-x^2)\,x\,\cancel{\text{etc}},\,dx$$

$$= \frac{1}{4}$$

$$f_+(s) = \int \frac{xs\,J_1(xs)}{4s^2}\,x\,dx.$$

$$= \int \frac{(xs)^2\,J_1(xs)}{4s^2}\,d\,xs)$$

$$J_n(z) = \sum_{m=0}^{\infty} \frac{(-)^m(\frac{1}{2}z)^{n+2m}}{m!\,(n+m)!}$$

$$J_1(z) = \sum_{m=0}^{\infty} \frac{(-)^m(\frac{1}{2}z)^{2m+1}}{m!\,(m+1)!}$$

$$= \frac{1}{2}z - \frac{z^3}{2\cdot 2^3} + \cdots$$

$$\underbrace{\lim_{x\to 0}\frac{2J_1(xs)}{xs}}_{f(s)\;=} = 1 - \lim_{x\to 0}\frac{(xs)^2}{2^3} + \cdots$$

$$\frac{d^2 f}{ds^2} = \lim_{x\to 0}\frac{\partial^2\{2J_1(xs)/xs\}}{\partial s^2}$$

$$= 0 \quad \text{etc.}$$

$$g(k_\mu) = \left(\frac{1}{2\pi}\right)^2 \int \cdots \int D(x_\mu)\, e^{-ik_\mu x^\mu} (dx^\mu)^4$$

$$g_+(\mathbf{k}, k_4) = \left(\frac{1}{2\pi}\right)^2 \cdot \left\{ \int_{ct=0}^{\infty} \int_{r=0}^{ct} \int_0^{\pi} \int_0^{2\pi} e^{-i\mathbf{k}r - ik_4 \cancel{dt}\, ct} \cancel{dx\,dy\,dz\,d(ct)}\, d(ct)\cdot r^2\, dr \sin\theta\, d\theta\, d\varphi \right)$$

$$- \int_{ct=-\infty}^{0} \int_{r=0}^{c|t|} \int_0^{\pi} \int_0^{2\pi} \quad {}'' \quad \Big\}$$

$$= \left(\frac{1}{2\pi}\right)^2 \Big\{ \int_0^{\infty}$$

I. ~~Interacting~~ General theory of Fields.

Tuesday Jan. 10, 1950

Q. S-matrix in Nonlocal Field Theory ㊿(6)

Last time I discussed the generalization of fundamental notion of probability amplitude ~~a~~ in terms of "correlation amplitude".
However, it was also pointed out that the ~~no~~ most natural way of quantization of nonlocal field would not easily be reconciled with the above notion, because $n_{\underline{k}lm}$, for example, was ~~co~~ independent of x_μ and y_μ. Thus, as long as we ~~stick to the~~ do not change the method of quantization, it seems preferable to look for a formalism similar to S-matrix scheme.

We start again from the Schrödinger equation for local field:

$$i\hbar\frac{\partial\Psi(n_k',t')}{\partial t}=\sum_{n_k''}(n_k'|\overline{H}(t')|n_k'')\Psi(n_k''t'),$$

which can be replaced by

$$i\hbar\frac{\partial\Psi(n_k't')}{\partial t}=\sum_{n_k''}\int_{-\infty}^{+\infty}(n_k't'|\overline{H}'|n_k''t'')dt''\Psi(n_k'',t'')$$

where

$$(n_k't'|\overline{H}'|n_k''t'')=(n_k'|\overline{H}''(t')|n_k'')\delta(t'-t'').$$

This, in turn, can be written in the integrated form

$$\Psi(n_k',t)=\overbrace{}^{\Psi(n_k',-\infty)}-\sum_{n_k''}\frac{i}{\hbar}\int_{-\infty-i0}^{t+i0}dt'(n_k't'|\overline{H}'|n_k''t'')dt''\Psi(n_k''t'')$$

We solve this integral $\underset{\text{for } t=+\infty}{\text{equation}}$ by successive approximation

0-th approx.

$$\Psi(n'_k, +\infty) = \Psi(n'_k, -\infty)$$

1-st approx.

$$\Psi(n'_k, +\infty) = \Psi(n'_k, -\infty) - \sum_{n''_k} \frac{i}{\hbar}(n'_k|\overline{\overline{H}}'|n''_k) \times \Psi(n''_k, -\infty)$$

where

$$(n'_k|\overline{\overline{H}}'|n''_k) = \iint_{-\infty}^{+\infty} dt'\,dt''\,(n'_k t'|\overline{H}'|n''_k t'')$$

more generally

$$\Psi(n'_k, t) = \Psi(n'_k, -\infty) - \sum_{n''_k} \int_{-\infty}^{+t} dt' \int_{-\infty}^{+\infty} dt''\,\underbrace{(n'_k t'|\overline{H}'|n''_k t'')}\,\Psi(n''_k, -\infty)$$

2nd approx.

$$\Psi(n'_k, +\infty) = \Psi(n'_k, -\infty)$$

$$- \sum_{n''_k} \frac{i}{\hbar}(n'_k|\overline{\overline{H}}'|n''_k)\,\Psi(n''_k, -\infty)$$

$$- \sum_{n''_k n'''_k} \frac{1}{\hbar^2} \iint_{-\infty\,-\infty}^{+\infty\,+\infty} (n'_k t'|\overline{H}'|n''_k t'')\,dt'\,dt''$$

$$\times \int^{t'}\!\!\int^{\infty}_{-\infty\,-\infty} (n''_k t'''|\overline{H}'|n'''_k t^{IV})\,dt'''\,dt^{IV}\,\Psi(n''_k, -\infty)$$

$$\sim \Psi(n'_k, +\infty) = \Psi(n'_k, -\infty)$$

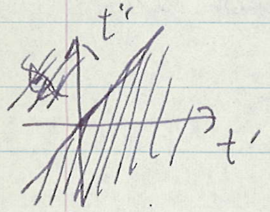
$$- \sum_{n''_k} \frac{i}{\hbar}(n'_k|\overline{\overline{H}}'|n''_k)\,\Psi(n''_k, -\infty)$$

$$- \sum_{n''_k} \frac{1}{\hbar^2}(n'_k|\overline{\overline{H}'\,\overline{\eta_+}\,\overline{H}'}|n''_k)\,\Psi(n''_k, -\infty),$$

where

$\cancel{\Psi(n_k'}$ $\cancel{\eta_+(t't'')}$

$$(n_k' t' | \overline{\eta}_+ | n_k'' t'') = (n_k' | 1 | n_k'') \, \eta_+ \cancel{(t' - t'')}$$



$$\cancel{\eta_+(t' - t'') = 1} \quad \text{for } t' - t'' > 0$$

$$= 0 \quad \text{for } t' - t'' < 0$$

Thus, the S-matrix can be defined ~~by~~

$$\Psi(n_k', t\infty) = \sum_{n_k''} (n_k' | S | n_k'') \, \Psi(n_k'', -\infty)$$

$$\cancel{S = \frac{i}{\hbar} \overline{\overline{H}}'}$$

$$S = 1 - \frac{i}{\hbar} \overline{\overline{H}}' - \frac{1}{\hbar^2} \overline{\overline{H}' \eta_+ \overline{H}'}$$

in the second approximation, and in general, we have

$$S = 1 - \frac{i}{\hbar} \overline{\overline{H}}' - \frac{1}{\hbar^2} \overline{\overline{H}' \overline{\eta}_+ \overline{H}'}$$

$$+ \left(\frac{i}{\hbar}\right)^3 \overline{\overline{H}' \overline{\eta}_+ \overline{H}' \overline{\eta}_+ \overline{H}'} + \cdots$$

Thus, ~~we can~~ this can be ~~naturally~~ extended to non-local field, by defining

$$\overline{\overline{H}}' = \iint (n_k' x' t' | H' | n_k'' x'' t'') (dx') (dt') (dx'') (dt'')$$

etc

$$\overline{\eta}_+ \cancel{(n'x't' | \eta_+ | n''x''t'')} \; (n'x' | \eta_+ | n''x'') = \begin{cases} (n'|1|n'') & \text{for } t' - t'' > 0 \\ 0 & \text{for } t' - t'' < 0 \end{cases}$$

Thus, we can calculate the scattering probabilities just as in the case of Heisenberg's S-matrix formalism. The difference is that we ~~ha~~ can now start from ~~the~~ a definite form of nonlocal field theory, whereas Heisenberg's theory ~~had no.~~ was completely arbitrary.

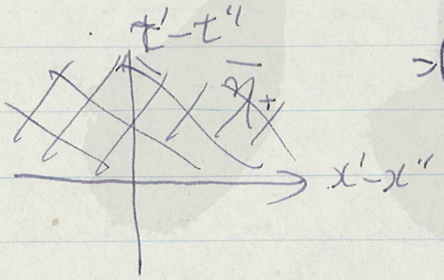
However, there still remains the question of relativistic invariance, because the time displacement operator η_+ is not relativistically invariant. We assume here that η_+ is replaced by the invariant ~~time-like~~ displacement operator D_+, where

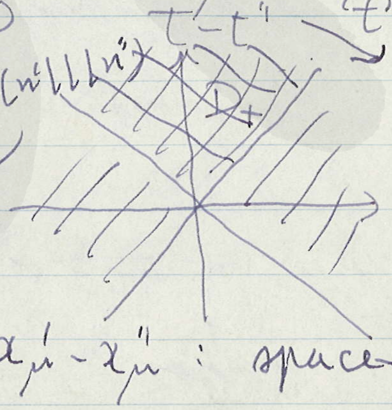
$(n'x't'|D_+|n''x''t'') = (n'|1|n'')$ for $x'_\mu - x''_\mu$: time-like and $t' > t''$ (~~$x'^4 - x''$~~)

$= 0$ $\quad t' < t''$

$= \frac{1}{2}(n'|1|n'')$

$x'_\mu - x''_\mu$: space-like





If we introduce two ~~of~~ simple invariant operators

$(n'x't'|E|n''x''t'') = (n'|1|n'')$ everywhere

and

$$(n'x't'|D|n''x''t'') = (n'|1|n'')$$

for $t'>t''$ and ~~for~~ $x_\mu'-x_\mu''$: time-like

$$= -(n'|1|n'')$$

for $t'<t''$ and $x_\mu'-x_\mu''$: time-like

$$= 0$$

for ~~t~~ $x_\mu'-x_\mu''$: space-like,

we have

$$D_+ = \frac{1}{2}(E+D)$$

$$D_- = \frac{1}{2}(E-D)$$

The above replacement of η_+ by D_+ is justify for the following reason.

$$S_2 \equiv \sum_{n''n'''} \iiiint (n't'|\bar{H}'|n''t'')(n''t''|\bar{\eta}_+|n'''t''')(n'''t'''|\bar{H}'|n^{IV},t^{IV}) \times$$

$$\times\, dt'\,dt''\,dt'''\,dt^{IV}$$

$$= \sum_{n''n'''} \int\cdots\int (n'x'|H'|n''x'')(n''x''|\eta_+|n'''x''')$$

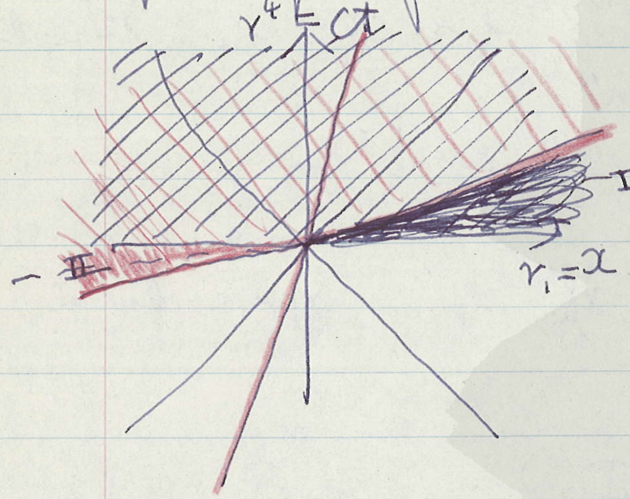
$$\times (n'''t'''|H'|n^{IV},t^{IV})(dx')^4(dx'')^4(dx''')^4(dx^{IV})^4$$

where

$$(n''x''|\eta_+|n'''x''') = \begin{cases} (n''|1|n''') & \text{for } t'-t''>0 \\ 0 & \text{for } t'-t''<0. \end{cases}$$

So the domain of integration, with respect to $r_\mu = x_\mu''-x_\mu'''$ covers the ~~($\tau = t''-t'''$~~ half space-time, whereas the domain of integration with respect to $X_\mu = \frac{1}{2}(x_\mu''+x_\mu''')$ covers the whole space-time.

If we perform a Lorentz Transformation,



domain I in r-space ~~in~~ is replaced by domain II ~~in r'-space~~) If we imagine another r'' ~~x''~~ Lorentz framework, just in the middle of old (r) and new (r') frameworks, the domain II is the mirror image of I with respect to ~~x~~ r_1''. So, if the remaining factor

$$\int\cdots\int (n'x'|H'|n''x'')\overset{(n''|1|n''')}{\wedge}(n''x'''|H'|n^{iv}x^{iv})$$

$$\times (dx')^4 (dX'')^4 (dx^{iv})^4$$

is invariant with respect to the reflection of the above type, S_2 is invariant with respect to the above Lorentz transformation.* More generally, if the ~~factor~~ integrand in S_2, ~~which must be mu to multiplied by on~~ after integration with respect to x', x^{iv} and X, is invariant with respect to ~~any~~ the reflection of arbitrary space-like axis, S_2 is invariant with respect to Lorentz group (without time reversal).

---

* In such a case, domain I can be repaced by II, or $\frac{1}{2}(I+II)$.

~~Therewith~~ In other words, η_+ can be replaced by an invariant displacement operator D_+, so that the relativistic invariance of S_2 is obvious.

More generally, we can define

$$S = 1 - \frac{i}{\hbar}\overline{\overline{H'}} + \left(\frac{-i}{\hbar^2}\right)^2 \overline{\overline{H'D_+H'}}$$

$$+ \left(\frac{-i}{\hbar}\right)^3 \overline{\overline{H'D_+H'D_+H'}} + \cdots$$

This definition of S automatically ~~also~~ guarantees the conservation of total energy and momentum, ~~for the to~~ because, for example, $\overline{R} = S - 1$ is ~~ax~~ space-time average of a nonlocal operator R defined by

$$R = \left(\frac{-i}{\hbar}\right)H' + \left(\frac{-i}{\hbar}\right)^2 H'D_+H'$$

$$+ \left(\frac{-i}{\hbar}\right)^3 H'D_+H'D_+H' + \cdots$$

By the way, R satisfies the equation

$$R = \left(\frac{-i}{\hbar}\right)H' + \left(\frac{-i}{\hbar}\right)\cdot H'D_+R .$$

Further, it can easily be proved that S is a unitary matrix. Namely

$$S^*S = SS^* = 1$$

provided that H' is an Hermitian operator. The proof is as follows

~~For~~

$$S^* = 1 + \frac{i}{\hbar}\bar{\bar{H}}' + \left(\frac{i}{\hbar}\right)^2 \overline{\overline{H'D_-H'}} + \left(\frac{i}{\hbar}\right)^3 \overline{\overline{H'D_-H'D_-H'}} + \cdots$$

because

$$D_+^* = D_- ,$$

$$S^*S = \left\{1 + \frac{i}{\hbar}\bar{\bar{H}}'' + \left(\frac{i}{\hbar}\right)^2 \overline{\overline{H'D_-H'}} + \left(\frac{i}{\hbar}\right)^3 \overline{\overline{H'D_-H'D_-H'}} + \cdots\right\}$$
$$\times \left\{1 + \left(-\frac{i}{\hbar}\right)\bar{\bar{H}}' + \left(\frac{-i}{\hbar}\right)^2 \overline{\overline{H'D_+H'}} + \left(\frac{-i}{\hbar}\right)^3 \overline{\overline{H'D_+H'D_+H'}} + \cdots\right\}$$
$$= 1 + \left(\frac{-i}{\hbar}\right)\left(\bar{\bar{H}}' - \cancel{\frac{i}{\hbar}}\,\bar{\bar{H}}'\right) + \left[\frac{i}{\hbar}\right]^2\left\{\overline{\overline{H'(D_+ + D_-)H'}} - \bar{\bar{H}}'\bar{\bar{H}}'\right\} + \left(\frac{-i}{\hbar}\right)^3\left\{\overline{\overline{H'D_+H'D_+H'}} - \bar{\bar{H}}'\cdot\overline{\overline{H'D_+H'}} + \overline{\overline{H'D_-H'}}\cdot\bar{\bar{H}}' - \overline{\overline{H'D_-H'D_-H'}}\right\}$$
$$= 1 + 0 + \left(\frac{-i}{\hbar}\right)^2 \overline{\overline{H'(\overset{0}{D_+ + D_- - E})H'}} + \left(\frac{-i}{\hbar}\right)^3\left\{\underset{0}{\underset{\parallel}{-\overline{\overline{H'D_-H'D_+H'}} + \overline{\overline{H'D_-H'D_+H'}}}}\right\} + \cdots$$

(50)
(10)

More generally

$$(1+\overline{\overline{R^*}})(1+\overline{\overline{R}}) = 1 + \overline{\overline{R^*}} + \overline{\overline{R}} + \overline{\overline{R^*}}\cdot\overline{\overline{R}}$$

~~$= 1 + \overline{\overline{R^*}} + \overline{\overline{R^*}} + \overline{\overline{R^* D_+ R}} + \overline{\overline{R^* D_- R}}$~~

~~$= 1 + \left(\frac{i}{\hbar}\right)\overline{\overline{H'}} + \left(\frac{-i}{\hbar}\right)\overline{\overline{H' D_+ R^*}}$~~

~~$+ \left(\frac{i}{\hbar}\right)\overline{\overline{H'}} + \left(\frac{i}{\hbar}\right)\overline{\overline{R^* D_- H'}}$~~

~~$+ R^* D_+$~~

$= 1,$

because

$$\overline{\overline{R^*}} + \overline{\overline{R}} = \left(\frac{-i}{\hbar}\right)\overline{\overline{H'}} + \left(\frac{-i}{\hbar}\right)\overline{\overline{H' D_+ R}}$$

$$+ \left(\frac{i}{\hbar}\right)\overline{\overline{H'}} + \left(\frac{i}{\hbar}\right)\overline{\overline{R^* D_- H'}}$$

$$= \left(\frac{-i}{\hbar}\right)\overline{\overline{H' D_+ R}} + \left(\frac{i}{\hbar}\right)\overline{\overline{R^* D_- H'}}$$

$$\overline{\overline{R^*}}\cdot\overline{\overline{R}} = \overline{\overline{R^*}}\cdot\left(\frac{-i}{\hbar}\right)\overline{\overline{H'}} + \overline{\overline{R^*}}\left(\frac{-i}{\hbar}\right)\overline{\overline{H' D_+ R}}$$

~~$= \left(\frac{i}{\hbar}\right)\overline{R^* D_- H' R}$~~

$$= \frac{i}{\hbar}\overline{\overline{H'}}\cdot\overline{\overline{R}} + \left(\frac{i}{\hbar}\right)\overline{\overline{R^* \cdot D_- H'}}\cdot\overline{\overline{R}}$$

$$=$$

→ And also ~~R~~ $SS^* = 1.$

Thus S-matrix above defined satisfies all the necessary conditions

(i) correspondence
(ii) invariance
(iii) conservation
(iv) unitarity

The most important requirement ~~question~~ ~~is~~ for S-matrix, which was not satisfied in usual theory, is
(V) finiteness.

This must be checked first of all in connection with the problem of self-energy. The second order term for the self-energy of each particle comes from

$$\left(\frac{-i}{\hbar c}\right)^2 \overline{\overline{H' D_t H'}}$$

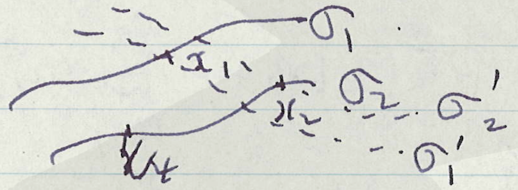
Lagrangian and Displacement Operator in S. Matrix

Z. Koba, 素粒子論研究.
Vol. 1, No. 3, p. 70 (1949).

$$U[\sigma_F, \sigma_A] = 1 + \left(\frac{-i\cancel{g}}{\hbar c}\right)\int_{\sigma_A}^{\sigma_F} H(x_1)\,d^4x_1$$
$$+ \frac{1}{2}\left(\frac{-i\cancel{g}}{\hbar c}\right)^2 \iint_{\sigma_A}^{\sigma_F} P(H(x_1), H(x_2))\,d^4x_1\,d^4x_2$$

$$\|$$

$$U^{(2)} = \frac{1}{4}\left(\frac{-i}{\hbar c}\right)^2 \iint \{\{H(x_1), H(x_2)\} + \varepsilon(x_1, x_2)[H(x_1), H(x_2)]\}$$
$$\times\, d^4x_1\, d^4x_2$$



$$H(x) \to H(x) + W(x, \sigma)$$

$$U^{(2)} = \frac{1}{4}\left(\frac{-i}{\hbar c}\right)^2 \iint \{\{\quad\} + \varepsilon(x, x_2)[H(x_1), H(x_2)]\}$$
$$\times\, d^4x_1\, d^4x_2$$
$$+ \left(\frac{-i}{\hbar c}\right)\int W(x_1, \sigma_1)\,d^4x_1$$

$$\frac{\delta W(x, \sigma_1)}{\delta \sigma_1(x_1')} - \frac{\delta W(x_1', \sigma_1)}{\delta \sigma_1(x_1)} = \frac{i}{\hbar c}[H(x_1), H(x_1')]$$

$$P^*(A(x), B(y)) = \begin{cases} A(x)B(y) & x-y: \text{f.l.} \\ \frac{1}{2}A(x)B(y) + \frac{1}{2}B(y)A(x) & x-y: \text{s.l.} \\ B(y)A(x) & \cancel{y}\, x-y: \text{p.l.} \end{cases}$$

$$P^*(A(x), B(y)) = \frac{1+\varepsilon^*(x\,y)}{2}A(x)B(y)$$
$$+ \frac{1-\varepsilon^*(x, y)}{2}B(y)A(x)$$

$$\varepsilon^*(x,y) = \begin{cases} 1 & x-y:\ \text{f.l.,} \\ 0 & x-y:\ \text{s.l.,} \\ -1 & x-y:\ \text{p.l.} \end{cases}$$

$$\Delta_F = \Delta^{(1)} + i\varepsilon\Delta$$

$$\rightarrow \Delta_F^* = \Delta^{(1)} + i\varepsilon^*\Delta$$

$$U[\infty, -\infty] = \exp\left\{\frac{i}{\hbar c}\int_{-\infty}^{+\infty} L(x)\, d^4x\right\}$$

$$= \frac{1}{n!}\left(\frac{i}{\hbar c}\right)^n \int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{\infty} P^*(L(x_1)\cdots L(x_n))\, d^4x_1\cdots d^4x_n.$$

S. Kanesawa and Z. Koba, Prog. Theor. Phys. 4 (1949), 297.

Jan. 19, 1950

The Last Seminar

3. Problem of Self-Energy. 1.

50/⑫

Now, we must see whether the S-matrix above obtained gives rise to the finite results. This is, of course, the most important point, because the main purpose of non-local field theory is to have a ~~the~~ relat. formalism, which is completely free from divergence.

The first problem is to ~~de~~ make sure the convergence of the self-energy ~~terms~~ parts in the ~~self-~~ second order term

$$\left(\frac{-i}{\hbar}\right)^2 \overline{\overline{H' D_+ H'}} = -\frac{1}{\hbar^2} \overline{\overline{H' D_+ H'}}.$$

As the simplest example, we take the case ~~ψ, ψ^*~~

$$H' = \sqrt{4\pi}\, g\, \psi^{**} U \psi$$

where ψ, ψ^* are ~~spinors~~ scalar ~~functions~~ operator for a local ~~spinor~~ scalar field and U is a nonlocal scalar operator for a nonlocal neutral scalar field. Then

$$\cancel{\psi}\, U = \int\cdots\int (dk)^4 (dl)^4\, \bar{u}(k_\mu, l_\mu) \exp(i k_\mu x^\mu)$$
$$\exp(i l^\mu p_\mu).$$

with

$$\bar{u}(k_\mu, l_\mu) = u(k_\mu, l_\mu)\, \delta(k_\mu k^\mu + \kappa^2)\, \delta(k_\mu l^\mu)$$
$$\times\, \delta(l_\mu l^\mu - \lambda^2)$$

$$\bar{u}^*(k_\mu, l_\mu) = \bar{u}(-k_\mu, -l_\mu)$$

$$[\bar{u}(k_\mu, l_\mu), \bar{u}^*(k'_\mu, l'_\mu)]$$

$$= \frac{k^4}{|k^4|}\cdot\delta(k_\mu k^\mu + \kappa^2)\,\delta(k_\mu l^\mu)\,\delta(l_\mu l^\mu - \lambda^2)$$

$$\times \prod_\mu \delta(k_\mu - k'_\mu)\,\delta(l_\mu - l'_\mu)$$

(or $[\bar{u}(k_\mu, l_\mu),\ \bar{u}(k'_\mu, l'_\mu)]$

$$= \frac{k^4}{|k^4|}\,\delta(k_\mu k^\mu + \kappa^2)\,\delta(k_\mu l^\mu)\,\delta(l_\mu l^\mu - \lambda^2)$$

$$\times \prod_\mu \delta(k_\mu + k'_\mu)\,\delta(l_\mu + l'_\mu)$$)

$$V = \int\cdots\int (dK)^4 (dL)^4\, \bar{v}(K,L) \exp(iK_\mu x^\mu)$$

$$\times \exp(iL^\mu p_\mu/\hbar)$$

$$\bar{v}(K,L) = v(K,L)\left(\frac{-\kappa}{2\pi\lambda}\right)\delta(K_\mu K^\mu + \kappa^2)$$

$$\times\cancel{\left(\frac{\kappa}{\lambda}\right)}\,\delta(K_\mu L^\mu)\,\delta(L_\mu L^\mu - \lambda^2)$$

lim $\lambda \to 0$

$$\left(\frac{-\kappa}{\lambda}\right)\delta(K_\mu L^\mu)\,\delta(L_\mu L^\mu - \lambda^2)$$

$$= \left(\frac{-\kappa}{\lambda}\right)\delta(\kappa L'^4)\,\delta(L'^2 - \lambda^2)$$

$$= 2\pi\,\cancel{\kappa}\ \delta(L'^4)\,\delta(L'_1)\,\delta(L'_2)\,\delta(L'_3)$$

$$\because \int \delta(\kappa L'^4)\,\kappa\, dL'^4 \cdot \delta(L'^2 - \lambda^2)\,\frac{1}{2\pi\lambda}\, dL'_1\, dL'_2\, dL'_3 = 1.$$

$\lambda \to 0$:

$$\bar{v}(K,L) = v(K,L)\,\delta(K_\mu K^\mu + \kappa^2)$$

$$\times(\delta(L))^4$$

or $$V = \int\cdots\int (dK)^4 v(K) \underset{\delta(K_\mu K^\mu + \kappa^2)}{\underline{\exp(iK_\mu x^\mu)}}$$

$$[\bar{v}(K,L), \bar{v}^*(K',L')]$$

$$= \frac{K^4}{|K^4|} \delta(K_\mu K^\mu + \kappa^2)\delta(K_\mu L^\mu)\delta(L_\mu L^\mu - \lambda^2)$$

$$\times \prod_\mu \delta(K_\mu - K'_\mu)\delta(L_\mu - L'_\mu)$$

$$[v(K,L)\overset{\delta(K_\mu K^\mu + \kappa^2)}{,} v^*(K',L')\delta(K'_\mu K'^\mu + \kappa^2)]$$

$$= \left(\frac{2\pi\lambda}{\kappa}\right)^2 \prod_\mu \delta(K_\mu - K'_\mu)\delta(L_\mu - L'_\mu)$$

$$\cancel{\delta(K'_\mu K'^\mu + \kappa^2)}\,\delta(K'_\mu L'^\mu)\delta(L'_\mu L'^\mu - \lambda^2)$$

$$\frac{K^4}{|K^4|}\delta(K_\mu K'^\mu + \kappa^2)$$

$$= -\left(\frac{2\pi\lambda}{\kappa}\right)\frac{K^4}{|K^4|}\delta(K_\mu K^\mu + \kappa^2)\prod_\mu \delta(K_\mu - K'_\mu)$$

$$\left[\sqrt{\frac{\kappa}{2\pi\lambda}}\, v(K)\delta(K_\mu K^\mu + \kappa^2),\right.$$

$$\left.\sqrt{\frac{\kappa}{2\pi\lambda}}\, v^*(K')\delta(K'_\mu K'^\mu + \kappa^2)\right]$$

$$= -\frac{K^4}{|K^4|}\delta(K_\mu K^\mu + \kappa^2)\prod_\mu \delta(K_\mu - K'_\mu)$$

$$L'^{(-K,\ \cancel{k}k,\ l,\ K'')} = \int\cdots\int \cdots \exp(-iK_\mu x^\mu) \cdots$$

$$\times \exp(ik_\mu x^\mu) \exp(il^\mu p_\mu/\hbar) \cdots$$

$$\times \exp(iK''_\mu x^\mu)$$

~~$= \int\cdots\int \cdots \exp -\frac{i}{2}(K_\mu - K''_\mu) x^\mu$~~

~~$\exp(il^\mu p_\mu/\hbar)$~~ $\exp \frac{i}{2} (K_\mu - K''_\mu) l^\mu$

~~$\exp -\frac{i}{2}(K_\mu - K''_\mu) x^\mu$~~

$$(\cdots x' | L' | \cdots x'') = \cdots \exp(-iK_\mu x'^\mu) \exp(ik_\mu x'^\mu)$$

$$\Pi\, \delta(x' - x'' + l) \quad \exp(iK''_\mu x''^\mu)$$

$$= \cdots \exp(-iK_\mu x'^\mu) \exp(ik_\mu x'^\mu)$$

$$\exp(iK''_\mu x'^\mu) \exp(iK''_\mu l^\mu)\, \Pi\, \delta(x' - x'' + l)$$

Problem of Self-Energy. 2

$$-\frac{1}{\hbar^2c^2}\overline{\overline{L' D_+ L'}} = -\frac{1}{2\hbar c^2}\{\overline{\overline{L'^*}}\}\cdot\overline{\overline{L'}} - \frac{1}{2\hbar^2 c^2}\overline{\overline{L'DL'}}$$

$$L' = \sqrt{4\pi}\, g V^* U V$$

$$U = \int\cdots\int (dk)^4 (dl)^4\, \bar{u}(k_\mu, l^\mu) \exp(i k_\mu x^\mu)$$
$$\times \exp(i l^\mu p_\mu)$$

$$V = \int\cdots\int (dK)^4 v(K)\,\delta(K_\mu K^\mu + \varkappa^2)$$
$$\times \exp(i K_\mu x^\mu)$$

$$V^* = \int\cdots\int (dK)^4 v^*(K)\,\delta(K_\mu K^\mu + \varkappa^2)$$
$$\times \exp(-i K_\mu x^\mu)$$

$$L' = \int\cdots\int \underline{v^*(K) \exp(-i K_\mu x^\mu)}\,\delta(K_\mu K^\mu + \varkappa^2)(dK)^4$$
$$\times\ \bar{u}(k, l)\exp(i k_\mu x^\mu)\underset{\exp(i l^\mu p_\mu/\hbar)}{}\ (dk)^4 (dl)^4 (dK')^4$$
$$\times\ v(K')\exp(i K'_\mu x^\mu)\,\delta(K'_\mu K'^\mu + \varkappa^2)$$

$$D_{\hbar} = \int\cdots\int D(L)\exp(i L^\mu p_\mu/\hbar)(dL)^4$$

$D_+(L^\mu) = -2$　　$D_+(L_\mu) = 1$

$D_0(L^\mu) = 0$　　$D_0(L_\mu) = 0$

$D_-(L^\mu) = +1$　　$D_-(L_\mu) = -1$

$$-\frac{1}{2\hbar^2}\overline{\overline{L'DL'}} = -\frac{4\pi g^2}{2\hbar^2c^2}\overline{\overline{V^*UVDV^*\underline{U}V}}$$

$$\left(\overline{\overline{v(K')}}\exp(ik'x)\right)$$

$$\overline{\overline{V^*UVDV^*\underline{U}V}} = \int\cdots\int v^*(K)\exp -iKx\cdot u(k,l)\exp ikx\exp ilp/\hbar$$

$$\times D(l'')\exp i\underline{l''p}/\hbar\cdot v^*(K'')\exp iK''x\cdot u^*(k',l')$$

$$\exp(-ik'x)\exp(il'p/\hbar)\,v(K''')\exp(iK'''x)$$

$$=\int\cdots\int\exp i(\cancel{k},K,k,K',K'',k',K''',l,l'',l')$$

$$\cdot v^*(K)\,u(k,l)\,v(K')\,D(l'')\,v^*(K'')$$

$$\underline{u(k',l')\,v(K''')}$$

$$\exp\tfrac{1}{2}iCx\ \exp iLp/\hbar\ \exp\tfrac{1}{2}iCx$$

$$=\int\cdots\int\underline{\exp i(\cancel{k},K,k,K',K'',k',K''',l,l'',l')}$$

$$\delta(-K\pm k\pm K'\pm K''\pm k'+K''')$$

$$v^*(K)\,\underline{u^*(k,l)}\,v(K')\,\underline{D(l'')}\,v^*(K'')$$

$$\underline{u(k',l')}\,v(K''')\,dK\,dk\,dK'\,dK''\,dk'$$

$$dl\,dl''\,dl'$$

$$=\int\cdots\int f(K,k,K',K'',k',K''')\,\delta(\Sigma K)$$

$$\times\,dK\,dk\,dK'\,dK''\,dk'$$

$$\times\ \delta(k-k')\,\delta(K'-K'')$$

$$=\int\cdots\int f(K''',k',K'',K'',k',K''')$$

$$\times(dK'')^4(dk')^4$$

$$=F(K''')$$

$$\int\cdots\int f(K'''k', K'''K'', k'K'')(dK'')^4(dk')^4$$

$$= \int\cdots\int f(K'''K'')(dK'')^4$$

$$= \int_{(+)}\cdots\int \frac{(dK'')^4}{\cancel{K'''K''}} f_+(K'''_4 K'')$$

$$+ \int_{(0)}\cdots\int \frac{(dK'')^4}{\cancel{K'''K''}} f_0(K'''K'')$$

$$+ \int_{(-)}\cdots\int \frac{(d\cdot K'')^4}{\cancel{K'''K''}} f_-(K'''K'')$$

~~五~~ $K'''K' = \bar{\bar{K}}'''_4 K^{4''}$.

$$\int_{(+)}\cdots\int (dK''_i)^3 \cdot \frac{dK^{4''}}{\cancel{K'''_4 \kappa K^{4''}}} f_+(\kappa K^{4''})$$

$$+ \int_{(0)}\int (dK''_i)^3 \cdot \underline{dK^{4''}} \, f_0(\kappa K^{4''})$$

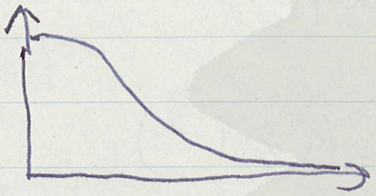
$$+ \int_{(-)}\int (dK''_i)^3 \, f_-(-\kappa K^{4''}) \qquad \frac{1}{(K^{4''})\,((K^{4''})^2+\kappa^2)}$$

$$\propto \int_0^\infty (K^{4''})^3 \, \cancel{f_+(\kappa} \, w_+(K^{4''})\, dK^{4''}$$

$$+ \int_{-\infty}^0 (K^{4''})^3 \, w_-(K^{4''})\, dK^{4''}, \qquad \frac{1}{(K^{4''})\,((K^{4''})^2+\kappa^2)}$$

$$\int_0^\infty \frac{\kappa \, dK^{4''}}{(K^{4''})^2+\kappa^2} = \overset{\kappa}{\cancel{\tan}}\arctan\left(\frac{K^{4''}}{\kappa^2}\right)$$

$$= \cancel{\tfrac{1}{2}}\ \cancel{\kappa}\ \pi/2$$



$$\int_0^\infty \frac{dx}{1+x^2} = \pi/2$$

$$y = \tan x \qquad \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x}{\cancel{x}\cos x}\right)$$

$$\frac{dy}{dx} = +1 + \frac{\sin^2 x}{\cos^2 x} = \cancel{\quad} 1 + y^2$$

$$\frac{dy}{1+y^2} = dx,$$

$$\cancel{K}^4 \int \frac{k^2 \, dk}{k^2+\kappa^2}\cdot\frac{\sin k\lambda}{k\lambda} = \int \frac{k \sin k\lambda}{k^2+\kappa^2}\, dk$$

Self-energy of a Dirac Particle in usual Q. E. D. (Perturbation Theory)

Feynman, Phys. Rev. 74 (1948), 1430

$$\Delta\mu_0 = \frac{e^2}{2\pi^2\mu}\int d\omega\, dK\, \delta(\omega^2-K^2)$$

$$\times\left\{\frac{2\mu^2 - E_0E_f + P_0P_f}{E_f(E_f-E_0+\omega)} + \frac{2\mu^2 - E_0E_f + P_0P_f}{E_f(E_f+E_0+\omega)}\right\}$$

$$\simeq \frac{e^2}{\cancel{2}\pi^2}\mu^2\int d\omega\, dK\, \delta(\omega^2-K^2)\left\{\frac{1}{E_f(E_f-E_0+\omega)}\right.$$

$$\left.+\frac{1}{E_f(E_f+E_0+\omega)}\right\}$$

$$\simeq \frac{e^2\mu^2}{\cancel{2}\pi^2}\int\frac{\mu\,\cancel{d\omega}\,dK}{\omega^3}\cdot\cancel{E_{\omega}} \simeq \frac{e^2\mu^3}{\pi^2}\log_{K\to\infty}\frac{K}{K_0}$$

H. Umezawa and R. Kawabe, Prog. 4 (1949), 461

$$\Delta E^{(1)} = \frac{e^2\mu^2}{2\pi E_p}\left[\int_0^\infty\frac{dw}{1+w} + \frac{1}{2}\int_0^\infty\frac{w}{1+w}dw\right.$$

$$\left.-\frac{1}{2}\int_0^\infty\frac{w^2}{(1+w)^2}dw\right]$$

$$= \frac{e^2\mu}{E_p}\frac{3\mu}{2\pi}\left[\frac{1}{2}\log(1+w) - \frac{1}{6}\right]_{w\to\infty}$$

$$(E_{p-l}+l)^2 - E_p^2 = \mu^2 w$$

$$\left(\quad 2(l_\mu P^\mu - l^\mu) = \mu^2 w\quad\right) \qquad x = \cos\theta$$

$$l = \frac{\mu^2 w}{2}\cdot 1\Big/\{(1+w)\mu^2+p^2\}^{1/2} - px$$

A. Pais, Verh. Kon. Ac. Amsterdam 19(1947).

$$W(\mathbb{P}) = W(0)\sqrt{1-\beta^2}$$

$v^*(K)\exp(-iKx)\,u(k,l)\exp(ikx)\exp(il\,p/\hbar)$

$v(K')\exp(iK'x)\exp(iL\,p/\hbar)\,v^*(K')\exp(-iK'x)$

$u^*(k,l')\exp(-iK''x')\exp(-il'p/\hbar)\,v(K'')\exp(iK''x)$

$\rightleftarrows\ \overline{v^*(K)u(k,l)v(K')}\,v^*(K')u^*(k,l')v(K'')$

$\exp\{i(K''-K)X\}\exp\{i(K'+k-K)(l'-l-L)\}$

$\exp(iK'l)\exp(-iK'l')\ \cancel{\delta(x)}$

$= \overset{v^*(K)}{\prod_\mu \delta(K_\mu, K''_\mu)}\cdot\ \overset{v(K)}{\exp(iKl)\exp(-iKl')}$

$\exp\{i(K'+k-K)L\}$

(i) K'

$\frac{M}{\Lambda}\frac{\partial J_0(\Lambda M)}{\partial\Lambda}\exp(iKl)\cdot\exp(-iKl')$

$\exp\{i(k-K)L\}$

(ii) L'

$\frac{M}{\Lambda}\frac{\partial J_0(\Lambda M)}{\partial\Lambda}\left[\cancel{\int_0^\infty \sin\{\Lambda\bar{k}\sinh(\chi-\varphi)\}d\chi}\right.$

$\cancel{\int_0^\infty \sin\{\Lambda\bar{k}\sinh(\chi+\varphi)\}d\chi}$

$\frac{1}{l}\frac{\partial}{\partial l}\left[\int_0^\infty \frac{\sin(\bar{k}_i l_i - k_4 l^4)+\sin(\bar{k}_i l_i + k_4 l^4)}{k_4}dk\right]$

$\sin\{\bar{k}\Lambda(\cancel{\cosh\chi}\sinh\chi\cos\varphi-\cosh\chi\sin\varphi)\}$

$+\sin\{\bar{k}\Lambda(\sinh\chi\cos\varphi+\cosh\chi\sin\varphi)\}$

$$\int\cdots\int \frac{L^4}{|L^4|} \exp(iKL)(dL)^4 \propto \frac{K_4}{|K_4|}\delta'(K_\mu K^\mu)$$

$$\int\cdots\int \exp(iKl)\,\overset{\delta(K_\mu l^\mu)}{\delta(l^2-\lambda^2)}\; l^2 dl \sin\theta d\theta d\varphi$$
$$\times\, dl_4$$

$$\cancel{\propto \iiint \exp(iKl)}\;\delta(l^2-\lambda^2) \propto \frac{\sin K_k l}{K_k l}\, l^2 dl$$

$$K_{k4} = kK/\kappa - K_4 k_4/\kappa$$

$$(K_k)^2 = (K_{k4})^2 - M^2$$
$$= M^2 k_4^2/\kappa^2 - M^2$$
$$= \left(\frac{M}{\kappa}\right)^2 k^2$$

$$\propto \frac{\sin \frac{M}{\kappa} k\lambda}{\frac{M}{\kappa} k\lambda}$$

$$\propto \frac{\sin K'_k\lambda}{K'\lambda}$$

K'_k: absolute value of the space component of K_μ in the coordinate system in which $k'_\mu = a_{\mu\nu}k_\nu = 0$ for $\mu = 1, 2, 3$.

Charged Scalar — Non-local Neutral Scalar

Yennie, Feb. 9, 1950

$$V = \left(\frac{\hbar}{c(2\pi)^3}\right)^{1/2} \int (dk)^4 \, v(k_\mu) \exp(i k_\mu x^\mu) \times \delta(k_\mu k^\mu + \kappa^2)$$

$$\delta(k_\mu k^\mu + \kappa^2)[v^*(k_\mu), v(k_\mu')] = \frac{k_4}{|k_4|} \Pi \delta(k_\mu - k_\mu')$$

$k_4 < 0$; $\quad N(k_\mu) \propto v^*(k_\mu) v(k_\mu)$

$k_4 > 0$; $\quad N(k_\mu) \propto v(k_\mu) v^*(k_\mu)$

$$U(x, r) = \left(\frac{\hbar}{c(2\pi)^3} \frac{4\kappa}{\lambda}\right)^{1/2} \int (dk)^4 \, u(k_\mu, r_\mu)$$

$$\times \exp(i k_\mu x^\mu) \delta(k_\mu k^\mu + \kappa^2) \delta(r_\mu r^\mu - \lambda^2) \delta(r_\mu k^\mu)$$

$$[U(x, r)]^* = U(x, -r)$$

or $\quad u^*(k_\mu, r_\mu) = u(-k_\mu, -r_\mu)$

$$\delta(k_\mu k^\mu + \kappa^2) \delta(r_\mu r^\mu - \lambda^2) \delta(k_\mu r^\mu)$$

$$\times \{u^*(k_\mu, r_\mu), u(k_\mu', r_\mu')] = \frac{k_4}{|k_4|} \Pi \delta(k_\mu - k_\mu') \delta(r_\mu - r_\mu')$$

$k_4 < 0$; $\quad M(k_\mu, r_\mu) \propto u(-k_\mu, -r_\mu) u(k_\mu, r_\mu)$

$k_4 > 0$; $\quad M(k_\mu, r_\mu) \propto u(k_\mu, -r_\mu) u(-k_\mu, r_\mu)$

(2)

$$(n'; \overset{x'}{\cancel{X+\tfrac{1}{2}(y'-y'')}}) \,|\, L'(-K, \overset{k}{\vee} l, K'') \,|\, n''; x'')$$

$$= \cdots \exp(-iKx')\exp(ikx')\exp(K''x')$$

$$\exp(iK''l)\ \Pi\,\delta(x'-x''+l)$$

$$= \cdots \exp i(-K+k+K'')x' \cdot \exp(iK''l)$$

$$\Pi\,\delta(x'-x''+l).$$

$$(n', X+\tfrac{1}{2}(y'-y'') \,|\, L'(-K, k, l, K'') \,|\, n''', \tfrac{1}{2}(y'+y''+r'))$$

$$= \cdots \exp i(-K+k+K'')X$$

$$\exp\{\tfrac{1}{2}i(-K+k+K'')y'\} \quad ○$$

$$\exp\{-\tfrac{1}{2}i(-K+k+K'')y''\}$$

$$\exp(iK''l)\ \Pi\,\delta(X-\underline{y''}-\tfrac{1}{2}r'+l)$$

$$(n''', \tfrac{1}{2}(y'+y''-r') \,|\, L'(-K'', -k, -l', K') \,|\, n'', X-\tfrac{1}{2}(y'-y''))$$

$$= \cdots \exp i(-K''-k+K')X$$

$$\exp(-\tfrac{1}{2}i(-K''-k+K')y') \quad ○$$

$$\exp(\tfrac{1}{2}i(-K''-k+K')y'')$$

$$\exp(-iK''l')\ \Pi\,\delta(\cancel{X}-X+\underline{y'}-\tfrac{1}{2}r'\cancel{+}-l')$$

$$\iint dy'dy'' =$$

(1)

$$\{(n'x'|L'|n'''x''')\tfrac{1}{2}(x'''|D|x^{iv})(n'''x^{iv}|L'|n''x'')\}$$

$$=\{(n', X+\tfrac{1}{2}r|L'|n''', X'+\tfrac{1}{2}r')\tfrac{1}{2}D(r')(n''', X'-\tfrac{1}{2}r'|L'|n'', X-\tfrac{1}{2}r)\}$$

where

$$X=\tfrac{1}{2}(x'+x'') \qquad r=x'-x''$$

$$X'=\tfrac{1}{2}(x'''+x^{iv}) \qquad r'=x'''-x^{iv} \Big\}$$

$$X'+\tfrac{1}{2}r=y' \qquad X'-\tfrac{1}{2}r=y''$$

$$X'=\tfrac{1}{2}(y'+y'') \qquad r=y'-y'' \Big\}$$

$$X+\tfrac{1}{2}r= X+\tfrac{1}{2}(y'-y'')$$

$$X'+\tfrac{1}{2}r'= \cancel{X}\ \tfrac{1}{2}(\cancel{r}\ y'+y''+r')$$

$$X'-\tfrac{1}{2}r'= \tfrac{1}{2}(y'+y''-r')$$

$$X-\tfrac{1}{2}r= X-\tfrac{1}{2}(y'-y'')$$

$$\{\quad\}=\{(n', X+\tfrac{1}{2}(y'-y'')|L'|n''', \tfrac{1}{2}(y'+y''+r')\}$$

$$\times\tfrac{1}{2}D(r')(n''', \tfrac{1}{2}(y'+y''-r')|L'|n'', X-\tfrac{1}{2}(y'-y''))\}$$

$$(x'|L'|x'')= L'(X, r)$$

$$\{\quad\}=\{(n'|L'(\tfrac{1}{2}X+\tfrac{1}{2}y'+\tfrac{1}{4}r', X-y''-\tfrac{1}{2}r')|n''')$$

$$\times\tfrac{1}{2}D(r')(n'''|L'(\tfrac{1}{2}X+\tfrac{1}{2}y''-\tfrac{1}{4}r', -X+y'-\tfrac{1}{2}r')|n'')\}$$

$(?)$

$-K, k, l, K''$　　$+K'', -k, l', K'$

(n'', n'')

$\{\quad\} = \int\cdots\int (n', n''')\cdot$ ~~$\exp i(-K+k+K'')X$~~ $\exp\{i(K'-K)X\}$

$\exp(+\frac{1}{2}i\{(-K+k+K'') + (-K'+k+K'')\}y'$

$\exp(-\frac{1}{2}i\{(-K+k+K'') + (-K'+k+K'')\}y''$

$\exp(iK''l)\exp(-iK''l')\,\Pi\,\delta(X - y'' - \frac{1}{2}r' + l)$

$\Pi\,\delta(-X + y' - \frac{1}{2}r' - l')\frac{1}{2}D(r')\,dX\,dr'\,dy'\,dy''$

$$= \int\int \overbrace{(n', n''')}^{-K,\,k,\,l,\,K''}\overbrace{(n'', n'')}^{K'',\,-k,\,-l',\,K'} \exp(i(K'-K)X)$$

$\exp\frac{1}{2}i\{(-K+k+K'') + (-K'+k+K'')\}r'$

$\exp(\frac{1}{2}i)\{(-K+k+K'') + (-K'+k+K'')\}l$

$\exp(\frac{1}{2}i)\{(-K+k+K'') + (-K'+k+K'')\}l'$

$\exp(iK''l)\exp(-iK''l')\frac{1}{2}D(r')$

$dX\,dr'$

$= \int (\quad)(\quad)\,\delta(K'-K)$

$\exp(-iKl)\cdot\exp(-iKl')\frac{1}{2}D(r')$

$\exp i(k+K''-K)r'\,dr'$

$$\propto (\quad)(\quad)\,\delta(K'-K)\left(\frac{\sin\frac{M}{\varkappa}k\lambda}{\frac{M}{\varkappa}k\lambda}\right)^2$$

$$\int \exp i(k+K''-K)r'\,dr'\frac{1}{2}D(r')$$

$$\propto (\quad)(\quad)\,\delta(K'-K)\left(\frac{\sin\frac{M}{\varkappa}k\lambda}{\frac{M}{\varkappa}k\lambda}\right)^2$$

$$\times\,(4\pi)^2 i\frac{(k_4+K''_4-K_4)}{|k_4+K''_4-K_4|}\,\delta'((k+K''-K)_\mu(k+K''-K)^\mu)$$

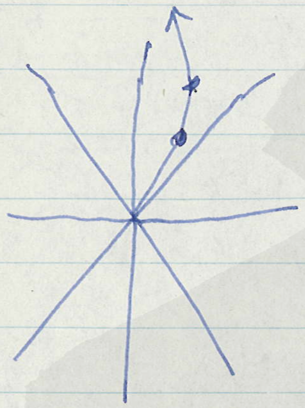
Self Energy of the Vacuum

$$(4\pi)^2 i \int \cdots \int f(\cancel{k} K, k, K'') \left(\frac{\sin K'_k \lambda}{K'_k \lambda} \right)^2$$

$K^4 > 0$
$K''^4 > 0$
$k^4 > 0$

$$\frac{k_4 + K''_4 + K_4}{|k_4 + K''_4 + K_4|} \delta'\left((k + K'' + K)_\mu (k + K'' + K)^\mu\right)$$

$$\times \, dk \, dK \, dK''.$$



$f(K, k, K'')$:

$$a^{(+)*} b^* a^{(-)*}$$

$$\pm a^{(-)*} b^* a^{(+)*} = 2 a^{(+)*} b^* a^{(-)*}$$

+ : bosons
− : fermions

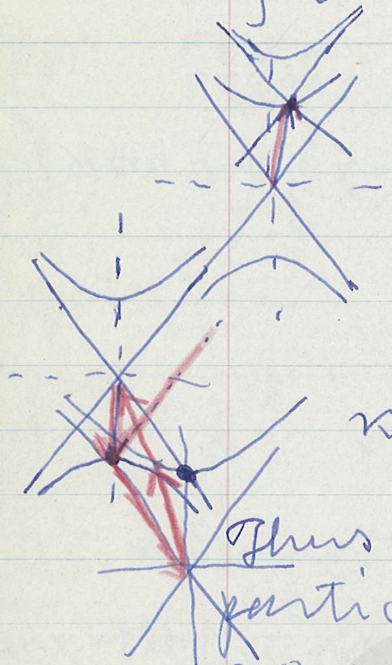
$f\delta' = 0$ everywhere

Thus, the energy of the vacuum is zero, unless both types of interacting particles have rest mass zero.

In other words, any particle with zero mass other than the photon must be neutral, in order that the self-energy of the vacuum must be zero (at least up to the second approximation)

Self Energy of a particle

$f(K, k, K'')$



$$(k+K'')_\mu (k+K'')^\mu$$

$$= k_\mu k^\mu + K''_\mu K''^\mu + 2 k_\mu K''^\mu$$

$$= -\kappa^2 - M^2 - \frac{2\kappa M(1 \pm \beta\gamma\cos\theta)}{\sqrt{1-\beta^2}\sqrt{1-\gamma^2}}$$

$$\kappa^2 + M^2 + \frac{2\kappa M}{\sqrt{1-\beta^2}\sqrt{1-\gamma^2}} \rightarrow \frac{2\kappa M\beta\gamma}{\sqrt{1-\beta^2}\sqrt{1-\gamma^2}}$$

$$> M^2$$

Thus the self-energy of a particle is zero, unless it is ~~some~~ coupled with another type of particles ~~to and both have~~ zero mass. with

The same argument applies to the ~~particle~~ wi Boson coupled with

JOINT THEORETICAL PHYSICS SEMINARS

Room 303, Palmer Physical Laboratory

4:30 PM

Princeton University
Princeton, New Jersey

Friday, May 5, 1950

Dr. D. Bohm, Princeton University

Subject: "Plasma Oscillations and the Interaction of Electrons"

Friday, May 12, 1950

Dr. H. Yukawa, Columbia University

Subject: "On Non-Local Field Theory"

Friday, May 19, 1950

Dr. A. Wightman, Princeton University

Subject: "The Theoretical Interpretation of Some Experiments on Mesons"

On Non-Local Field Theory

Joint Theoretical Physics Seminars
Princeton University
May 12, 1950

I. General Properties of Non-Local Operators

By a non-local ~~field~~ operator we mean any operator A, which could be represented by a matrix $(x'|A|x'')$, where x', x'' stand for two sets of values of space-time coordinates x_μ. Alternatively, A could be regarded as a function of x_μ and p_μ, which satisfy the ~~$p_\mu =$~~ canonical commutation relations

$$(x'|p_\nu|x'') = -i\hbar\delta'(x'^\nu - x''^\nu)\prod_{\mu\neq\nu}\delta(x'^\mu - x''^\mu)$$

$$[x^\mu, p_\nu] = i\hbar\delta_{\mu\nu}.$$

Now, it is clear that any matrix $(x'|A|x'')$ could be regarded as a function

$$A(X, r)$$

of

$$X_\mu = \frac{1}{2}(x'_\mu + x''_\mu)$$

$$r_\mu = x'_\mu - x''_\mu,$$

$$\int \delta'(x'-x'')\Psi(x'')dx'' = \frac{\partial\Psi(x')}{\partial x'}$$

which, in turn, could be expanded in the form

$$A(X, r) = \int\cdots\int a(k_\mu, l_\mu)\exp(ik_\mu X^\mu)\,\delta(r_\mu - l_\mu)$$
$$\times (dk_\mu)^4 (dl_\mu)^4.$$

In other words, any nonlocal operator A of x_μ, p_μ can be expanded in the form

$$A(x_\mu, p_\mu) = \int\cdots\int a(k_\mu, l_\mu)\exp\left(\frac{i}{2}k_\mu x^\mu\right)$$
$$\times \exp\left(-\frac{i}{\hbar}l_\mu p^\mu\right)\exp\left(\frac{i}{2}k_\mu x^\mu\right)(dk)^4(dl)^4$$

because

$$\delta(r_\mu - l_\mu) = \delta(x'_\mu - x''_\mu - l_\mu)$$

is the matrix representation of the operator

$$\exp\left(-\frac{i}{\hbar} \cancel{p} l_\mu p^\mu\right),^{*}$$

because

$$\int \delta(x'_\mu - x''_\mu - l_\mu)\, \Psi(x''_\mu) = \Psi(x'_\mu - l_\mu)$$

on the one hand, and

$$\int (x' | \exp\left(-\frac{i}{\hbar} l_\mu p^\mu\right) | x'')\, \Psi(x''_\mu)(dx'')^4$$

$$= \int (x' | \prod_\mu \sum_n \frac{\left(-\frac{i}{\hbar} l_\mu p^\mu\right)^n}{n!} | x'')\, \Psi(x'')(dx'')^4$$

$$= \prod_\mu \sum_n \left(-l_\mu \frac{\partial}{\partial x'_\mu}\right)^n \Big/ n!\ \Psi(x'_\mu) = \Psi(x'_\mu - l_\mu).$$

$*$ ~~$\int (x' | \exp\left(\frac{i}{\hbar} l_\mu p^\mu\right) | x'')\, \Psi(x'')$~~ $(dx'')^4$

~~$= \int \exp\{l_\mu \delta'(x'_\mu - x''_\mu) \prod_{\nu \neq \mu} \delta(x'_\nu - x''_\nu)\}$~~ $\Psi(x'')(dx'')^4$

$$= \int 1 + l_\mu \delta'(x'_\mu - x''_\mu$$

$$\int (x' | p^\mu | x'')\, \Psi(x'')(dx'')^4 = -i\hbar \frac{\partial \Psi(x'_\mu)}{\partial x'_\mu}$$

$$\int (x' | \prod_\mu \sum_n \frac{(i\hbar l_\mu p^\mu)^n}{n!} | x'')\, \Psi(x'')(dx'')^4 = \prod_\mu \sum_n \left(l_\mu \frac{\partial}{\partial x'_\mu}\right)^n \Big/ n! \cdot \Psi(x'_\mu)$$

$$= \Psi'(x'_\mu + l_\mu)$$

We can write also

$$A(x_\mu, p_\mu) = \int\cdots\int a(k_\mu, l_\mu)\exp\left(-\frac{i}{2}k_\mu l^\mu\right)$$
$$\exp(ik_\mu x^\mu)\exp\left(-\frac{i}{\hbar}l_\mu p^\mu\right)(dk)^4(dl)^4$$

or

$$= \int\cdots\int a(k_\mu, l_\mu)\exp\left(\frac{i}{2}k_\mu l^\mu\right)$$
$$\exp\left(-\frac{i}{\hbar}l_\mu p^\mu\right)\exp(ik_\mu x^\mu)(dk)^4(dl)^4$$

because

$$\exp\left(\frac{1}{2}ik_\mu x'^\mu\right)\exp\left(\frac{1}{2}ik_\mu x''^\mu\right)\delta(x'_\mu - x''_\mu - l_\mu)$$
$$= \exp\left(\frac{1}{2}ik_\mu x''^\mu\right)\exp\left(\frac{1}{2}ik_\mu(x'^\mu - l^\mu)\right)\delta(x'_\mu - x''_\mu - l_\mu)$$
$$= \exp\left(\frac{1}{2}ik_\mu(x''^\mu + l^\mu)\right)\exp\left(\frac{1}{2}ik_\mu x''^\mu\right)\delta(\;''\;)$$

In other words, any non-local operator can be formally expanded into double Fourier series (or integral), in spite of the fact that x_μ, p^μ_μ are not commutative.

Next, we can define non-local scalar, vector, ~~and~~ spinor etc. in the usual way. ~~Name~~ For example, by a Lorentz transformation

~~$x'^{\mu} = a^{\mu\nu}x^\nu$~~

$$x'_\mu = a_{\mu\nu}x_\nu,$$

a covariant non-local vector A_μ transforms as

$$A'_\mu = a_{\mu\nu}A_\nu \qquad \text{etc.}$$

~~General~~ An arbitrary non-local operator, say a scalar A, could be decomposed into irreducible parts with respect to the (homogeneous) Lorentz transformation in the following manner: Suppose that

$$\exp(i k_\mu x^\mu)$$

is transformed into the form

$$\exp(i k'_\mu x'^\mu),$$

Then $$k_\mu k^\mu = k'_\mu k'^\mu$$

must hold always. Similarly, we have

$$l_\mu l^\mu = l'_\mu l'^\mu$$

and also

$$k_\mu l^\mu = k'_\mu l'^\mu.$$

Thus, those terms in the expansion of A, which correspond to ~~the~~ a set of values of $k_\mu k^\mu$, $l_\mu l^\mu$, $k_\mu l^\mu$, are transformed among themselves. Thus any one of the irreducible parts of a scalar non-local operator can be written as

$$\int \cdot \int a(k_\mu, l_\mu) \exp(i k_\mu x^\mu) \exp(-\frac{i}{\hbar} l_\mu p^\mu)$$
$$\delta(k_\mu k^\mu \pm K)\, \delta(l_\mu l^\mu \pm L)\, \delta(k_\mu l^\mu - M)$$
$$(dk)^4 (dl)^4$$

where ~~each~~ K, ~~A~~ L, M can, each take any real number whatever. ~~However, in~~ Thus, one can classify the irreducible non-local operators ~~by K, L, M~~ as follows:

K	L	M	
+	+	0, ±	(time-like)
+	0	0, ±	(local field)
+	−	0, ±	(space-like)
0	+	0, ±	
0	0	0, ±	
0	−	0, ±	

$K<0$ ~~must~~ is to be excluded, because it correspond to the case of imaginary mass

Further, we consider the transformation of a non-local operator with respect to inhomogeneous Lorentz transformation of the form

$$x'_\mu = a_{\mu\nu}(x_\nu + b_\nu)$$
$$= a_{\mu\nu} x_\nu + b'_\mu$$

By this p_μ is transformed as

$$p'_\mu = a_{\mu\nu} p_\nu .$$

Correspondingly,

$$\left.\begin{aligned} X'_\mu &= a_{\mu\nu}(X_\nu + b_\nu) \\ \cancel{x} r'_\mu &= a_{\mu\nu} r_\nu \end{aligned}\right\}$$

Thus, the factor

$$\exp iK_\mu X^\mu \quad \text{or} \quad \exp ik'_\mu x^\mu$$

is transformed into

$$\exp(iK'_\mu X'^\mu)\exp(-iK''_\mu b^\mu)$$

or

$$\exp(ik'_\mu x'^\mu)\exp(-ik''_\mu b^\mu)$$
$$(-ik'_\mu b'^\mu)$$

whereas

$$\delta(r_\mu - l_\mu) \quad \text{or} \quad \exp(-\tfrac{i}{\hbar} l_\mu p^\mu)$$

is transformed into

$$\delta(r'_\mu - l'_\mu) \quad \text{or} \quad \exp(\tfrac{-i}{\hbar} l'_\mu p'^\mu).$$

In order to guarantee the invariance of A, one must assume ~~that~~

(i) either that $a(k_\mu, l_\mu)$ themselves transforms as

$$a'(k'_\mu, l'_\mu) = a(k_\mu, l_\mu)\cdot\exp(ik_\mu b^\mu)$$

(ii) or that $a(k_\mu, l_\mu)$ are all zero ~~exp~~ except for $k_\mu = 0$.

The former (i) implies that the coefficient $a(k_\mu, l_\mu)$ themselves ~~must be independent~~ represent in some way or other the new degrees of freedom, which ~~is compati~~ turns out to ~~hav~~ correspond to the particle aspect of the non-local field.* Namely, ~~if you~~ we can assume that $a(k_\mu, l_\mu)$ are operators satisfying the commutation relations of the type

$$[a(k_\mu, l_\mu), a(k'_\mu, l'_\mu)] = f(k_\mu, l_\mu; k'_\mu, l'_\mu)$$

etc.

---

* The same argument can, of course, be applied to the usual local field operators.

where f is an invariant function of k_μ, l_μ, k'_μ, l'_μ. In particular, if A is an hermitian operator

$$a(-k_\mu, -l_\mu) = a^*(k_\mu, l_\mu)$$

and, if f contains a factor

$$\prod_\mu \delta(k_\mu + k'_\mu)\,\delta(l_\mu + l'_\mu),$$

the above commutation relations would be invariant with respect to inhomogeneous Lorentz transformations. Also the operator

$$a^*(k_\mu, l_\mu)\cdot a(k_\mu, l_\mu)$$

is invariant, which could be interpreted as the number of particles in the quantum state (k_μ, l_μ). Thus, ~~we arr~~ (i) is reduced essentially to the quantized ~~field~~ non-local field as discussed in Part I.*

The latter alternative (ii) implies that A is a function of x_μ (or p_μ) alone. Since $\delta(x_\mu - l_\mu)$ or $\exp(-i l_\mu p^\mu/\hbar)$ is a particular kind of displacement operator, which displaces ~~the~~ the argument of an arbitrary fn $\Psi(x'_\mu)$ by an amount $-l_\mu$, (ii) represents, in general, an invariant displacement operator~~s~~. We may ~~say that~~ call it ~~is~~ a ~~particular~~ kind of unquantized non-local field.

Among such displacement operators, two simple examples are of particular importance in connection with the problem of interaction of fields.

One is the operator E represented by the matrix

$$(x'|E|x'') = E(X, r)$$

which is independent of x', x'' or independent of X, r. It should be noticed that this is not an identity operator, which has the matrix element

$$\prod_{\mu} \delta(r_{\mu}) .$$

The other is the operator P ~~represented by~~ with the matrix elements

$$\begin{aligned}(x'|P|x'') &= +1 \qquad \text{for } x'-x'' : \text{f.l.}\\ &= 0 \qquad \text{for } x'-x'' : \text{s.l.}\\ &= -1 \qquad \text{for } x'-x'' : \text{p.l.}\end{aligned}$$

It is useful to introduce the notations

$$\left.\begin{aligned} P_+ &= \tfrac{1}{2}(E+P), & P^* &= -P\\ P_- &= \tfrac{1}{2}(E-P), & P_+^* &= P_-\\ & & P_-^* &= P_+ \end{aligned}\right\}$$

P_+ is identical with the operator D_+ in my letter*. We ~~avoid~~ do not the use of the symbol D_+ hereafter, in order to avoid the

---

*

possible confusion with the wellknown invariant D-functions.

* Before going over to the problem of interaction, it should be noticed that the ~~quantized~~ non-local field of type (i) can be further ~~redu~~ decomposed into irreducible parts. Namely, ~~there is no rest~~ the coefficient $a(k_\mu, l_\mu)$ can be any function of l_μ, whereas ~~it cannot be~~ the same is not true for the dependence a on k_μ. Thus, a can be expanded into power series of l_μ, the coefficients of expansions now depending only on k_μ. Then, the terms containing a definite power of l_μ transforms among themselves ~~This ans~~ constituting a irreducible part. This amount to the same thing to take out such terms as correspond to the same l in the expansion of non-local field into series of spherical harmonics.
So the irreducible parts of the scalar field non-local behaves just as the higher spin local field, as pointed out recently by Fierz;* as far as the free field is concerned. However, even in the

---

*

framework of free fields, we can clearly see an essential difference between non-local and local fields in the case of spinor field.

April 26, 1950.

M. Fierz, Non-local Fields
(Phys. Rev. 78 (1950), 184)

Fierz!

Equivalence of

$$\left(\frac{\partial^2}{\partial x_\mu \partial x^\mu} - \varkappa^2\right) U(x^\mu) = 0$$

or

$$\left.\begin{aligned} &\left(\frac{\partial^2}{\partial X_\mu \partial X^\mu} - \varkappa^2\right) U(X_\mu, r_\mu) = 0 \\ &r_\mu \frac{\partial U(X_\mu, r_\mu)}{\partial X^\mu} = 0 \end{aligned}\right\} (N.L.)$$

and

$$\left.\begin{aligned} &\Box A_{ik\ldots l} = \varkappa^2 A_{ik\ldots l} \\ &\frac{\partial}{\partial X_i} A_{ik\ldots l} = 0 \end{aligned}\right\} (L.)$$

Rest system: (N.L) $u(\theta . \varphi)$

$$u(0,0,0,-\varkappa; r_\mu)\,\delta(r_\mu r^\mu - \lambda^2)\,\delta(\varkappa r_4) \times \exp(-i\varkappa X^4)$$

$$= \sum_{l,m} c(0,0,0,-\varkappa; l,m)\, P_l^m(\theta,\varphi) \cdot \delta(r^2 - \lambda^2) \times \delta(\varkappa r_4) \times \exp(-i\varkappa X^4)$$

moving system: (N.L)

$$u(\theta', \varphi')\,\delta(r'_\mu r'^\mu - \lambda^2)\,\delta(\varkappa r'_4)\,\exp(-i\varkappa X'^4)$$

Thus, $r^\mu \to r'^\mu$

$$P_l^m(\theta,\varphi)\{m = -l \cdots +l\} \to P_l^{m'}(\theta',\varphi')\{m' = -l, \cdots +l\}$$

	total spin
$l=0$:	0
$l=1$:	1
$l=2$:	2
⋮	

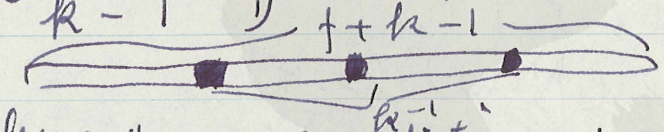
Local Field

Fierz, über die relativistische Theorie kräftfreier Teilchen mit beliebigem Spin (Helv. Phys. Acta., XII (1939), 3)

$$\underbrace{A_{ik\ldots l}}_{f} = \overset{\circ}{A}_{ik\ldots l} \cdot e^{ik^ix_i}$$

$\overset{\circ}{A}_{ik\ldots l}$: symmetric tensor in 4-dimensional space.

$\begin{pmatrix} f+k-1 \\ k-1 \end{pmatrix}$, components for $k=4$.



Supplementary condition:

$$A_{ii\ldots l} = 0 \ (5) \longrightarrow \begin{pmatrix} f-2+k-1 \\ k-1 \end{pmatrix} \text{ relations}$$

~~$\left(\frac{\partial A_{ik\ldots l}}{\partial x_i} = 0\right)$~~.

$$\begin{pmatrix} f+k-1 \\ k-1 \end{pmatrix} - \begin{pmatrix} f-2+k-1 \\ k-1 \end{pmatrix}$$

$$= \begin{pmatrix} f+3 \\ 3 \end{pmatrix} - \begin{pmatrix} f+1 \\ 3 \end{pmatrix} = \frac{(f+3)!}{3!\,f!} - \frac{(f+1)!}{3!\,(f-2)!}$$

$$= \frac{(f+1)\cancel{!}}{3!\,\cancel{f!}}\left\{(f+3)(f+2) - f(f-1)\right\} = \frac{(f+1)\cancel{!}}{3!\,\cancel{f!}}\left\{6f+6\right\} = (f+1)^2$$

Fünf 2

Due to the condition

$$\frac{\partial A_{ik\ldots l}}{\partial x_i} = 0,$$

in rest system $A_{ik\ldots l} = \overset{\circ}{A}_{ik\ldots l}\, e^{\pm i\varkappa x_4}$

$$(\varkappa)\overset{\circ}{A}_{4k\ldots l} = 0,$$

so that we may assume

$$i, k, \ldots l = 1, 2, 3.$$

$$\binom{f+2}{2} - \binom{f}{2} = \frac{(f+2)!}{2!\,f!} - \frac{f!}{2!(f-2)!}$$

$$= \frac{f!}{2!(f)!}\{(f+2)(f+1) - f(f-1)\}$$

$$= \frac{1\cancel{f(f+1)}}{2}\{4f+2\} = \underline{\underline{2f+1}}$$

These 2f+1-components transforms linearly by space-rotation because (L) and (S) are obviously invariant with respect to rotation. This ~~could be identified wi~~ is isomorphic as the irreducible representation of rotational group $\mathfrak{D}_f$.

This just corresponds ~~to the part of~~ in expansion of non-local field (at rest) to the part corresponding to $l = f$, in that both transform in the same way with respect to the space-rotation.

If we include all waves with the same l, the correspondence between N.L and L is established in such a way that both fields transform similarly with respect to Lorentz transformations.

This is what Fierz has shown in his letter. However, there is an essential difference, even in the framework of the theory of free fields, ~~Namely~~ if we started from the nonlocal spinor field.

Namely, in this case, the nonlocal spinor field was assumed to satisfy the equations

$$\left.\begin{aligned} \beta_\mu [x^\mu, \psi] + \lambda \psi = 0 \\ \gamma^\mu [p_\mu, \psi] + mc \psi = 0 \end{aligned}\right\} (N, L)$$

with

$$\left.\begin{aligned} \gamma^1 = i\rho_2\sigma_1 \quad \cdots\cdots \quad \gamma^4 = \rho_3 \\ \beta_1 = \rho_3\sigma_1 \quad \cdots\cdots \quad \beta_4 = -i\rho_2 \end{aligned}\right\}$$

Alternatively, we could ~~write as~~ replace the first one by

$$\frac{1}{6} \sum \varepsilon_{\kappa\lambda\mu\nu} \gamma^\kappa \gamma^\lambda \gamma^\mu [x^\nu, \psi] + i\lambda\psi = 0.$$

In any case we have

$$\left.\begin{aligned} \gamma^\mu \frac{\partial \psi(X_\mu, r_\mu)}{\partial X^\mu} + i\kappa \psi(X_\mu, r_\mu) = 0 \\ \beta_\mu r^\mu \psi(X_\mu, r_\mu) + \lambda \psi(X_\mu, r_\mu) = 0 \end{aligned}\right\} (N, L)'$$

Fierz 3

ψ have, of course, four components and if we expand, as before, in the form

$$\psi(X_\mu, r_\mu) = u(k_\mu, r_\mu)\,\delta(k_\mu k^\mu + \varkappa^2)$$

$$\times\, \delta(r_\mu r^\mu - \lambda^2)\,\delta(k_\mu r^\mu)\exp(ik_\mu X^\mu),$$

u must satisfy

$$(r_\mu r^\mu - \lambda^2)\,u(k_\mu, r_\mu) = 0.$$

Thus, we can again use angular variables θ, φ in the rest system and expand u into series of $P_l^m(\theta, \varphi)$ in rest system. Each term of the expansion corresponding to a fixed value of l is equivalent to the mixture of two types of states corresponding respectively to

$$j = l - \tfrac{1}{2} \qquad j = l + \tfrac{1}{2}.$$

However, there are two sets of states corresponding to the same value of j. with

$$l = j' - \tfrac{1}{2} \text{ or}$$

$$l = j + \tfrac{1}{2} \quad \text{and} \quad l = (j+1) - \tfrac{1}{2},$$

$$(j' = j+1)$$

They are essentially different from each other in contrast to the case of local field considered by Fierz.

II. S-matrix for Interacting Fields*

In order to handle the interacting non-local field, we have first to find the substitute for Schrödinger equation or ~~any ro~~ its general form in n. T. F. We don't no yet whether ~~there is~~ a differential form of Schrödinger eq. can find its counterpart in non-local field theory, but we can ~~anticipate~~ expect that an integral form may ~~find its~~ be extended to our case with less difficulties.

An arbitrary ~~operator~~ non-local operator A could be represented by a matrix

$$(n', x' | A | n'', x''),$$

with rows and columns characterized by n', x' and n'', x'' respectively, where each of the symbols n', n'' stands for the distribution in numbers of particles in all possible quantum state, while each of x' and x'' stands for a set of eigenvalues of four space-time operators x_μ. Further, we define $\{A\}$ for an arbitrary operator A by

$$(n' | \{A\} | n'') = \int \cdots \int (n', x' | A | n'', x'') (dx')^4 (dx'')^4.$$

By the help of these notations, we can easily extend the wellknown S-matrix for the ~~non-lo~~ local field

---

[*] H. Yukawa, Phys. Rev. 77 (1950), 849.

~~$(n'|S|n'')=$~~

~~$\Psi(\sigma_F) = \Psi(\sigma_I) + \left(\frac{-i}{\hbar c}\right)\int_I^F (\sigma_F|H|\sigma_I)$~~

$$\Psi_F(n') = \Psi_I(n') + \left(\frac{-i}{\hbar c}\right)\sum_{n''}\int_I^F (n'|H(x'_\mu)|n'')\,(dx'_\mu)^4\,\Psi_I(n'')$$

$$+\left(\frac{-i}{\hbar c}\right)^2 \sum_{n''}\sum_{n'''}\int\cdot\cdot\int (n'|H(x'_\mu)|n'')\,\varepsilon_+(x'_\mu, x''_\mu)(n''|H(x''_\mu)|n''') \times (dx'_\mu)^4 (dx''_\mu)^4\,\Psi_I(n''')$$

$$+\cdots$$

where $\varepsilon_+(x'_\mu, x''_\mu) = +1$ or 0 according as x'_μ is ~~so~~ later or earlier than x''_μ, ~~in~~ with respect to the coordinate system in question. In the limit $F \to +\infty$, $I \to -\infty$, we have

$$(n'|S|n'') = \delta(n', n'') + \left(\frac{-i}{\hbar c}\right)\sum_{n''}\int\cdot\cdot\int (n'|H'(x'_\mu)|n'')\,(dx'_\mu)^4$$

$$+\left(\frac{-i}{\hbar c}\right)^2 \sum_{n'', n'''}\int\cdot\cdot\int (n'|H'(x'_\mu)|n'')\,\varepsilon_+(x'_\mu, x''_\mu) \times (n''|H'(x''_\mu)|n''')(dx'_\mu)^4 (dx''_\mu)^4$$

$$+\cdots$$

It should be noticed that not only $\varepsilon_+(x'_\mu, x''_\mu)$, but also ① the form of $H(x'_\mu)$ itself depends, in general, on the choice of the coordinate system, ~~so as~~ in such a way that T. S. eq is both invariant and integrable.

Now in order to extend the above S-matrix to non-local field, we take advantage of the correspondence

$$H'(x'_\mu)\prod_\mu \delta(x'_\mu - x''_\mu) \leftarrow (x'_\mu | H' | x''_\mu)$$

or more precisely

$$(n'|H'(x'_\mu)|n'')\prod_\mu \delta(x'_\mu - x''_\mu) \leftarrow (x'_\mu, n' | H | x''_\mu, n'').$$

Then the above. S ~~can~~ be written as

$$(n'|S|n'') = \delta(n', n'') + \left(\frac{-i}{\hbar c}\right)\sum_{n''}\int\cdots\int (n'|H'(x'_\mu)|n'')$$
$$\times \prod_\mu \delta(x'_\mu - x''_\mu)(dx'_\mu)^4(dx''_\mu)^4$$
$$+\left(\frac{-i}{\hbar c}\right)^2 \sum_{n'', n''', n''''}\int\cdots\int (n'|H(x'_\mu)|n'')\prod_\mu \delta(x'_\mu - x''_\mu)$$
$$\times\ \varepsilon_+(x''_\mu, x'''_\mu)\underset{\prod\delta(n'', n''')}{\underline{(n'''}}|H'(x'''_\mu)|n^{iv})\prod_\mu \delta(x'''_\mu - x^{iv}_\mu)$$
$$\times (dx'_\mu)^4(dx''_\mu)^4(dx'''_\mu)^4(dx^{iv}_\mu)^4$$
$$+ \cdots\cdots.$$

This suggests us the following form for S-matrix ~~for the~~ in non-local field theory:

$$(n'|S|n'') = \delta(n',n'') + \left(\frac{-i}{\hbar c}\right)\sum_{n''}\int_{x'} (x'n'|H'|x''n'')(dx')^4(dx'')^4$$

$$+ \left(\frac{-i}{\hbar c}\right)^2 \sum_{n''n'''n^{iv}} \int\cdots\int (x'n'|H'|x''n'')(x''n''|\epsilon_+|x'''n''')$$

$$\times (x'''n'''|H'|x^{iv}n^{iv})(dx')^4(dx'')^4(dx''')^4(dx^{iv})^4$$

$$+ \cdots\cdots$$

However, ~~this f expression~~ it is not likely that this expression is invariant in the case of non-local fields, because the proof of invariance in the case of local fields is based on the fact that ~~field quantities~~ H(x') and H(x'') are commutative for any two space-like points x' and x''.

So ~~it is~~ we had better to adopt an obviously invariant form for S-matrix in the framework of local field before going over to non-local field. This question was discussed very recently by Koba*. When the interaction term ~~is~~ L' in the Lagrangian depends on the derivatives of field quantities, the interaction Hamiltonian

---

*Z. Koba, Prog. Theor. Phys. 5 (1950), 139.

takes the form in M.T.F

$$H' = -L' + H'',$$

where H'' depends explicitly on the choice of family of space-like surfaces σ's. L' is proportional to the first power of the coupling constant, whereas H'' is prop. to the second power. So the terms in S, which are proportional to the square of the coupling constant, ~~and~~ can be summarized as

$$S^{(2)} = \frac{1}{4}\left(\frac{+i}{\hbar c}\right)^2 \int \cdot\cdot \int [L'(x'), L'(x'')]_+ (dx')^4 (dx'')^4$$

$$+ \frac{1}{4}\left(\frac{i}{\hbar c}\right)^2 \int \cdot\cdot \int \varepsilon(x', x'') [L'(x'), L'(x'')] (dx')^4 (dx'')^4$$

$$+ \left(\frac{-i}{\hbar c}\right) \iint H''(x', \sigma)(dx')^4,$$

where $\varepsilon(x', x'') = 1$ for x' later than x''
$\qquad\qquad\qquad -1$ for x' earlier than x''

~~The note~~ the condition ~~In order~~ that, this is independent of the choice of σ's, is satisfied, if we take instead of ε the matrix $(x'|P|x'')$ and $H'' = 0$.* With this, ε_+ in $(x'|S|x'')$ is replaced by P_+ and we can simply take $-L'$ in place of H'. Thus, we obtain

---

* More precisely, $(x'|P|x'')$ is to be considered as the limit of a function of $x' - x''$, which ~~is~~ has the discontinuity on the cone, (which is slightly outside the light cone.

$$(n'|S|n'') = \delta(n', n'')$$

$$+\left(\frac{i}{\hbar c}\right)\sum_{n''}\int (x'n'|L'|x''n'')(dx')^4(dx'')^4$$

$$+\left(\frac{i}{\hbar c}\right)^2\sum_{n''n'''n^{iv}}\iiint (x'n'|L'|x''n'')(x''n''|P_+|x'''n''')$$

$$\times\ (x'''n'''|L'|x^{iv}n^{iv})(dx')^4\cdot(dx^{iv})^4$$

$$+ \cdots$$

We can write it symbolically as

$$S = 1 + \left(\frac{i}{\hbar c}\right) \cancel{*}\{L'\}\cancel{*}$$

$$+\left(\frac{i}{\hbar c}\right)^2 \{L'P_+L'\}$$

$$+\left(\frac{i}{\hbar c}\right)^3\cdot\{L'P_+L'P_+L'\}$$

$$+ \cdots$$

This can certainly applied to the non-local field as well as to the local field and indeed fulfills all the requirements for the S-matrix:

(i) It is obviously relativistically invariant, if ~~the~~ L' itself is rel. inv.

(ii) It is unitary, because

$$S^* = 1 + \left(\frac{i}{\hbar c}\right)\{L'\}$$

$$+\left(\frac{-i}{\hbar c}\right)^2\{L'P_-L'\} + \left(\frac{-i}{\hbar c}\right)^3\{L'P_{\pm}L'P_{\pm}L'\} + \cdots,$$

provide that L' is a Hermitian operator.

$$S^*S = 1 + \left(\frac{1}{\hbar c}\right)^2 \{L'\}\{L'\}$$
$$-\left(\frac{1}{\hbar c}\right)^2 \left\{\{L'P_+L'\} + \{L'P_-L'\}\right\}$$
$$+\left(\frac{-i}{\hbar c}\right)^3 \left\{ \{L'\}\{L'P_+L'\} \pm \{L'P_-L'\}\{L'\}\right.$$
$$\left. + \{L'P_-L'P_-L'\} - \{L'P_+L'P_+L'\}\right\}$$
$$+ \cdots$$

The second and the third terms cancel with each other, because of the relation

$$P_+ + P_- = E$$

and

$$\{AEB\} = \{A\}\{B\}$$

for any two non-local operators A, B. *
Similarly, we can show that $\left(\frac{-i}{\hbar c}\right)^3$-terms cancel etc. Thus, we are left with

$$S^*S = 1 \quad \text{and also} \quad SS^* = 1 .$$

(iii) The matrix element $(x'|S|x'')$ is different from zero, only if the initial and final states, which are characterised by the distributions x'' and x' respectively have the same total momentum and energy. This could be proved in the following

---

* $$\int\!\cdots\!\int (x'|A|x'')(x''|E|x''')(x'''|B|x^{iv})(dx')^4(dx'')^4(dx''')^4(dx^{iv})^4$$
$$= \iint (x'|A|x'')(dx')^4(dx'')^4 \iint (x'''|B|x^{iv})(dx''')^4(dx^{iv})^4 .$$

manner.

Any term in S (or more precisely, S−1) can be regarded as the space-time average $\{A\}$ of an operator A, which can be expanded into double Fourier series

$$A = \sum_{K_\mu, L_\mu} a(K_\mu, L_\mu) \exp(iK_\mu x^\mu) \exp(-\tfrac{i}{\hbar} L_\mu p^\mu),$$

or

$$= \sum_{K_\mu, L_\mu} a'(K_\mu, L_\mu) \exp(\tfrac{i}{2} K_\mu x^\mu) \exp(\quad) \exp(\tfrac{i}{2} K_\mu x^\mu)$$

or

$$(x'|A|x'') = \sum_{K_\mu L_\mu} a'(K_\mu, L_\mu) \exp(\tfrac{i}{2} K_\mu x'^\mu)$$

$$(x'|\exp(-\tfrac{i}{\hbar} L_\mu p^\mu)|x'') \exp(\tfrac{i}{2} K_\mu x''^\mu)$$

$$= \sum_{K_\mu, L_\mu} a'(K_\mu, L_\mu) \exp(i K_\mu X^\mu) \times \prod_\mu \delta(r_\mu - L_\mu)$$

Therefore

$$\{A\} = \int\!\!\cdot\!\!\int_{K_\mu L_\mu} \left(\frac{1}{2\pi}\right)^4 \prod_\mu \delta(K_\mu) \int\!\!\cdot\!\!\int_{(dr_\mu)^4} a'(K_\mu, L_\mu) \prod_\mu \delta(r_\mu - L_\mu)$$

or $\{A\}$ contains only such terms, for which

$$K_\mu = 0.$$

Now A is, in general, a sum of products of several non-local quantized field operators and of displacement operators. The latter has nothing to do with our ~~problem~~ present problem, because ~~they~~ can always be reduced to the function of r_μ alone,

(10)

On the other hand, any none-local q.-f. operator has a ~~form~~ general form

$$\text{const.}\begin{cases} a(k_\mu)\exp(ik_\mu x^\mu) \\ a^*(k_\mu)\exp(ik_\mu x^\mu) \end{cases}$$

according as $k_4 < 0$ or $k_4 > 0$, so that the term $K_\mu = \sum_i k_\mu^{(i)}$ ~~contain a~~ has two factors corresponding to

$$\prod_i a^{(i)}(k_\mu^{(i)}) \quad \text{and} \quad \exp(iK_\mu x^\mu)$$

where $a^{(i)}$ could be either annihilation operator or a creation operator. Thus, for $K_\mu = 0$, the net result of operation of $\prod_i a^{(i)}(k_\mu^{(i)})$ is the change in number of particles with energies and momentum $k_\mu^{(i)}$, such that the total energy and momentum are not altered by this operation.

(iv) Another requirement, which is to be fulfilled by S, is that S reduces to the usual S-matrix in the limit of local field theory. This seems to be obvious from our procedure of constructing it. However, some caution is needed, because of the discontinuity of P, ~~or~~ P_+ ~~or~~ P_- on the light cone. Let us consider a simple scattering problem of between two ~~parti~~ scalar particles, which are described by a scalar field V, V^*

and ~~which interact with another~~ a scalar real field U. The interaction Hamiltonian is assumed to be

$$L' = g V^{*} V U.$$

The second order scattering matrix is

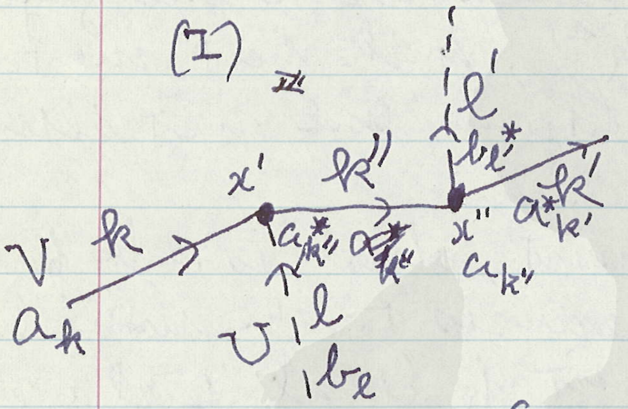
$$S^{(2)} = \left(\frac{+i}{\hbar c}\right)^2 \int \{L' P_+ L'\}$$

$$= \left(\frac{i}{\hbar c}\right)^2 \frac{1}{2}\left\{\{L'\}\{L'\} + \{L' P L'\}\right\}$$

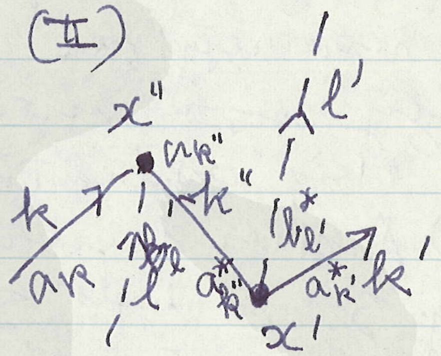
In the case of local fields, this is reduced to

$$S^{(2)} = \frac{1}{2}\left(\frac{i}{\hbar c}\right)^2 \int\cdots\int L'(x')\,(x'|P|x'')\,L'(x'')\,(dx')^4 (dx'')^4$$

(I)



(II)



(I) $L'(x') = \cancel{\text{const.}}\; C_1\, a_k \cdot a^*_{k''}\, b_l \dfrac{1}{\sqrt{k_0 k''_0 l_0}}$

$\times \exp i(k_\mu - k''_\mu + l_\mu) x'^\mu$

$L'(x'') = \cancel{\text{const}}\; \tilde{C}_2\, a_{k''}\, a^*_{k'}\, b^*_{l'} \dfrac{1}{\sqrt{k''_0 k'_0 l'_0}}$

$\times \exp i(k''_\mu - k'_\mu - l'_\mu) x''^\mu$

(II) $L'(x') = C_2\, a^*_{k''}\, a^*_{k'}\, b^*_{l'} \dfrac{1}{\sqrt{k''_0 k'_0 l'_0}}$

$\times \exp -i(+k''_\mu + k'_\mu + l'_\mu) x'^\mu$

⑪

$$L'(x'') = \tilde{c}_1 a_{k'} a_{k''}^{\cancel{*}} b_l \frac{1}{\sqrt{k_0 k_0'' l_0}}$$

$$\times \exp i(k_\mu + k''_\mu + l_\mu) x''^\mu$$

For $c_1 = \tilde{c}_1$, $c_2 = \tilde{c}_2$ (or $c_1 = -\tilde{c}_1$, $c_2 = -\tilde{c}_2$)

$$S^{(2)} = \frac{1}{2}\left(\frac{i}{\hbar c}\right)^2 c_1 c_2 \frac{1}{\sqrt{k_0 k_0' l_0 l_0'}} \frac{1}{k_0''}$$

$$\times \Big[\int\cdot\cdot\int \exp i\{(k_\mu - k''_\mu + l_\mu) x'^\mu + (k''_\mu - k'_\mu - l'_\mu) x''^\mu\} \\ (x'|P|x'')(dx')^4(dx'')^4$$

$$+ \int\int \exp i\{-(k''_\mu + k'_\mu + l'_\mu) x'^{\mu}_{\cancel{\mu}} + (k_\mu + k''_\mu + l_\mu) x''^\mu\} \\ (x'|P|x'')(dx')^4(dx'')^4 \Big]$$

$$\Big[\quad\Big] = \int\cdot\cdot\int \Big\{ \exp i\{(k_\mu - k''_\mu + l_\mu) \overset{X+\frac{1}{2}r}{x'^\mu} + (k''_\mu - k'_\mu - l'_\mu) \overset{X-\frac{1}{2}r}{x''^\mu}\}$$

$$- \exp i\{(k_\mu + k''_\mu + l_\mu) \overset{X+\frac{1}{2}r}{x'^\mu} - (k''_\mu + k'_\mu + l'_\mu) \overset{X-\frac{1}{2}r}{x''^\mu}\}\Big\}$$

$$\times \overset{P(r)}{(x'|P|x'')}(dx')^4(dx'')^4$$

$$= \int\int\int\int \{\exp i(k_\mu + l_\mu - k'_\mu - l'_\mu) X^\mu\}(dX)^4$$

$$\times \int\int\int\int \Big\{ \exp i\{\tfrac{1}{2}(k_\mu + l_\mu + k'_\mu + l'_\mu) - k''_\mu\} r^\mu$$

$$- \exp i\{\tfrac{1}{2}(k_\mu + l_\mu + k'_\mu + l'_\mu) + k''_\mu\} r^\mu \Big\} P(r)(dr)^4$$

$$= \left(\frac{1}{2\pi}\right)^4 \prod_\mu \delta(k_\mu + l_\mu - k'_\mu - l'_\mu)$$

$$\times \int\cdot\cdot\int \Big[\exp\{i(k_\mu + l_\mu - k''_\mu) r^\mu\} - \exp\{i(k_\mu + l_\mu + k''_\mu) r^\mu\}\Big]$$

$$\times P(r)(dr)^4$$

$$P(r_\mu) = \int\cdots\int P(j_\mu)\exp(i j_\mu r^\mu)(dj^\mu)^4$$

The above integral is

$$\int\cdots\int (dj_\mu)^4 P(j_\mu)\int\cdots\int\Big[\exp i(k_\mu + l_\mu - k_\mu'' + j_\mu)r^\mu$$

$$- \exp i(k_\mu + l_\mu + k_\mu'' + j_\mu)r^\mu\Big](dr^\mu)^4$$

$$= \frac{1}{(2\pi)^4}\int\cdots\int (dj_\mu)^4 P(j_\mu)\Big[\prod_\mu \delta(k_\mu + l_\mu - k_\mu'' + j_\mu)$$

$$- \prod_\mu \delta(k_\mu + l_\mu + k_\mu'' + j_\mu)\Big]$$

In the usual theory

$$P(j_\mu) = \frac{\cancel{j_4}\ 2}{|j_4|\cancel{+|j_4|}} = \frac{2j_4}{i(j_4)^2} = \frac{2}{i j_4}$$

$$\times\frac{1}{(2\pi)^3}\prod_i \delta(j_i)$$

$$\frac{1}{8\pi^4 i}\cdot\frac{1}{(2\pi)^3}\cdot\Big\{\prod_i \delta(k_i + l_i - k_i'')\frac{1}{-(k_4 + l_4 - k_4'')}$$

$$- \prod_i \delta(k_i + l_i + k_i'')\frac{1}{-(k_4 + l_4 + k_4'')}\Big\}$$

$$= \frac{1}{8\pi^4(2\pi)^3 i}\Big\{\prod_i \delta(k_i + l_i - k_i'')\frac{1}{k_0 + l_0 \pm k_0''}$$

$$- \prod_i \delta(k_i + l_i + k_i'')\frac{1}{k_0 + l_0 + k_0''}\Big\}$$

$$\sum_{k''} = \frac{1}{8\pi^4(2\pi)^3 i}\Big\{\frac{1}{k_0 + l_0 \pm\sqrt{(k_n + l_n)^2 + \kappa^2}}$$

$$- \frac{1}{k_0 + l_0 + \sqrt{(k + l)^2 + \kappa^2}}\Big\}$$

$$\sum_{k''} = \text{,,} \left\{ \frac{2\sqrt{(k+l)^2+\varkappa^2}}{-k_\mu l^\mu + \lambda^2} \right\}$$

$$S^{(2)} \propto \frac{1}{\sqrt{k_0 k_0' l_0 l_0'}} \frac{1}{-k_\mu l^\mu + \lambda^2}$$

In our theory

$$P(j_\mu) = \frac{1}{\pi^2 i} \frac{j_4}{|j_4|} \delta'(j_\mu j^\mu)$$

$$x = j^2 - j_4^2 \qquad F(x) = f(j)$$

$$dx = 2j\,dj$$

$$\int F(x)\,\delta(x)\,dx = F(0)$$

$$\int f(r) \frac{\delta(j-j_4)+\delta(j+j_4)}{2j} 2j\,dj = \begin{cases} f(j_4) & j_4 > 0 \\ f(-j_4) & j_4 < 0 \end{cases}$$

Hence

$$\delta(x) = \frac{\delta(j-j_4)+\delta(j+j_4)}{2j}$$

$$\delta'(x) = \frac{dj}{dx} \frac{d}{dj} \left(\frac{\delta(j-j_4)+\delta(j+j_4)}{2j} \right)$$

$$= \frac{1}{4j^2} \left(\delta'(j-j_4) + \delta'(j+j_4) \right)$$

$$- \frac{1}{4j^3} \left(\delta(j-j_4) + \delta(j+j_4) \right)$$

Thus, the ~~above~~ integral ~~is~~

$$\int \cdots \int \underbrace{(dk'')^3 \int \cdots \int (dj_\mu)^4 P(j_\mu)}_{\frac{1}{\sqrt{k_0 k_0' l_0 l_0'}} \quad \cancel{k_0''} \quad \cancel{\partial(k^2} \; - \; \frac{1}{k_0''}} \int \cdots \int [\qquad]$$

$k''_4 = k_4 + l_4 + j_4$

$k''^2 -$

$$\int \frac{1}{\sqrt{k_0 k_0' l_0 l_0'}} \frac{1}{k_0''}$$

$$= \int \cdots \int \underline{\underline{(dk'')}}^3 (dj)^4 \frac{1}{\pi^2 i} \frac{j_4}{|j_4|} \left\{ \frac{1}{4j^2} \left(\delta'(j - j_4) + \delta'(j + j_4) \right) \right.$$

$$\left. - \frac{1}{4j^3} \left(\delta(j - j_4) + \delta(j + j_4) \right) \right\}$$

$$\times \left\{ \prod_\mu \delta(k_\mu + l_\mu - \underline{\underline{k''_\mu}} + j_\mu) - \prod_\mu \delta(k_\mu + l_\mu + \underline{\underline{k''_\mu}} + j_\mu) \right\}$$

$$= \left(\frac{1}{2\pi} \right)^4 \frac{1}{\pi^2 i} \int \cdots \int (dj)^4 \frac{j_4}{|j_4|} \left[\frac{1}{4j^2} \left(\delta'(j - j_4) + \delta'(j + j_4) \right) \right.$$

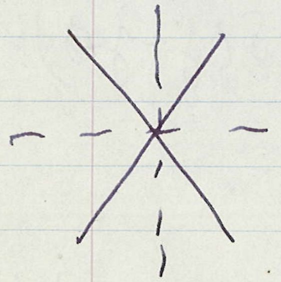
$$\left. - \frac{1}{4j^3} \left(\delta(j - j_4) + \delta(j + j_4) \right) \right\} \frac{1}{\sqrt{k_0 k_0' l_0 l_0'}}$$

$$\times \left\{ \frac{\delta(k_4 + l_4 + j_4 - \overbrace{\qquad\qquad}^{\sqrt{(k_i + l_i + j_i)^2 + \kappa^2}}}{|k_4 + l_4 + j_4|} - \frac{\delta(k_4 + l_4 + j_4 + \sqrt{\qquad\qquad})}{|k_4 + l_4 + j_4|} \right\}$$

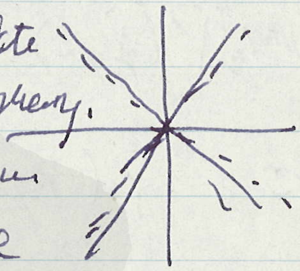
$k_\mu + l_\mu + j_\mu = k''_\mu \qquad |k''_4|^2 = k''_i k^{i''} + \kappa^2$

~~$|k''_4| =$~~

$(k_\mu + l_\mu + j_\mu)(k^\mu + l^\mu + j^\mu) + \kappa^2 = 0$.



As indicated above, there is some difficulty in our formalism and ~~the~~ immediate way of getting rid of it is to ~~consider that~~ define P-operator as a limit of an operator, which extends a little bit outside the light cone, in order to establish a complete correspondence with the usual field theory.



V) The most important problem is undoubtedly that of the convergence.

Let us start from the simplest case of the complete vacuum. ~~[illegible]~~ ~~We~~ The definition of vacuum, of course, depends on what kinds of particles or fields ~~we must~~ there ~~assume~~ exist in nature. ~~We k~~ Suppose that we confine our attention to

(i) a spinor field $\psi, \psi^\dagger$

(ii) a scalar U or vector field A_μ

interacting with the Lagrangian

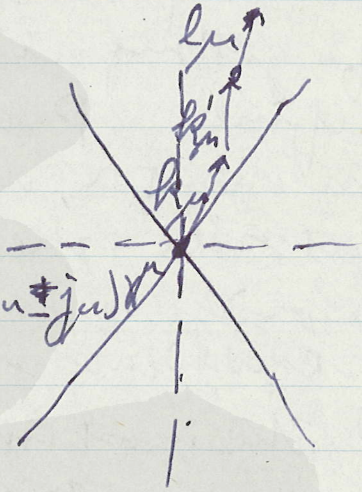
$$L' = \psi^\dagger \psi U \quad \text{or} \quad \psi^\dagger \gamma^\mu \psi A_\mu .$$

Then we must assume the virtual existence of pair of spinor particles and a boson in the intermediate state, which gives rise to the infinite self energy ~~in~~ for the pure vacuum.

In the lowest order, it comes from

$$\{L'PL'\},$$

~~in~~ which has a factor



$$\int\cdots\int (dx_\mu)^4 P(j_\mu)\cdot \exp i(k_\mu+k'_\mu+l_\mu \pm j_\mu)x^\mu$$

$$\propto \quad P(k_\mu+k'_\mu+l_\mu)$$

$=$ $P(k_\mu+k'_\mu+l_\mu)$ is always zero, provided that at least one of the particles has a rest mass, which different from zero. In other words, if we assume that all the charged particles (electrons, protons, charged mesons) have non-zero rest masses, then the self-energy of pure vacuum is zero up to the second order.

Next lowest order is fourth order with $\{L'P_+L'P_+L'P_+L'\}$

$$= ~~\{L'\}\{L'P_+L'P_+L'\}~~ + \{L'PL'P_+L'P_+L'\}$$

$$= \{L'P'L'PL'P_+L'\} + \{L'PL'PL'P_+L'\}$$

$$= \{L'PL'PL'PL'\}$$

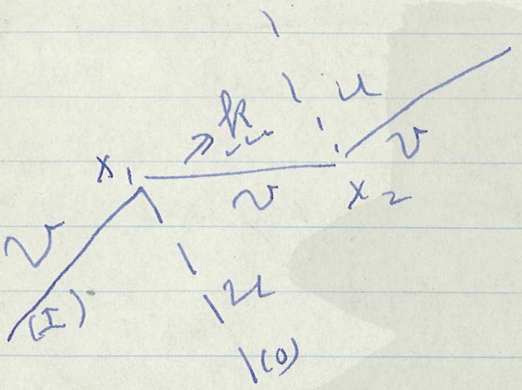
$$= 0$$

Seminar

April 11, 1950

(Yennie)

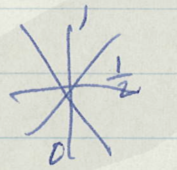
Yennie: $\eta \to D$



$H_I = g V^* V U$

$$S^{(2)} = \left(\frac{i}{\hbar c}\right)^2 \int H(x_2)(x_2|D_+|x_1) H(x_1)$$
$$(dx_2)^4 (dx_1)^4$$

D_+



$\eta_+ \quad \frac{+1}{0}$

$$S^{(2)} = \frac{1}{2}\left(\frac{i}{\hbar c}\right)^2 \int (x_2|D|x_1)\{H(x_2), H(x_1)\} \cdots$$

$$\propto \int (x_2|D|x_1) V^*(x_2) V(x_1) U(x_2) U(x_1)$$
$$[V(x_2), V^*(x_1)](dx_1)^4 (dx_2)^4$$

$$A(x) = D(x) \cdot \Delta(x)$$

$$= \int f_A(k) e^{ikx} (dk)^4$$

$$f_A(k) = \int f_\Delta(k - p) f_D(p)(dp)^4$$

$$= \int \frac{-1}{(2\pi)^3} \delta\{(k-p)^2 + \kappa^2\} \frac{(k-p)_0}{|(k-p)_0|}$$

$$\times \frac{-i}{\pi^2} \frac{p_0}{|p_0|} \delta'(p^2)$$

$$= \frac{1}{8\pi^4}\left\{\frac{1}{\underset{\sim}{k}^2+\kappa^2} - \frac{1}{2\underset{\sim}{k}^2}\right\} \qquad (D)$$

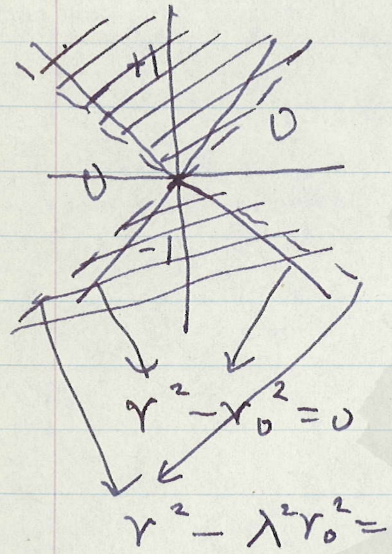
$$\therefore = \frac{1}{8\pi^4}\cdot\frac{1}{\underset{\sim}{k}^2+\kappa^2} \qquad (\eta)$$

$$\underline{-2k_\mu l^\mu + \lambda^2}$$

May 2, 1950

Yennie, D-operator as the limit of Dλ-operator

Yennie 2



$$D_\lambda = \frac{-i}{\pi^2}\int \frac{p_0}{|p_0|}\,\delta'(p^2)\exp\{i(\underline{p}\,\underline{r} - \lambda p_0 r_0)\} \times (dp)^4$$

$$= \frac{-i}{\pi^2\lambda}\int \frac{p_0}{|p_0|}\,\delta'\!\left(p^2 - \frac{p_0^2}{\lambda^2}\right) \exp(i p_\mu r^\mu)(dp)^4$$

$\lambda > 1.$

$$f(k) = \frac{\hbar}{ic}\,\frac{1}{8\pi^5}\int \delta[(k-p)^2 + \kappa_0^2]\,\frac{(k-p)_0}{|k-p|_0}\,\frac{p_0}{|p_0|} \times \delta'\!\left(p^2 - \frac{p_0^2}{\lambda^2}\right)(dp)^4$$

k : time-like, take $\underline{k} = 0$, $k^4 = k_0$

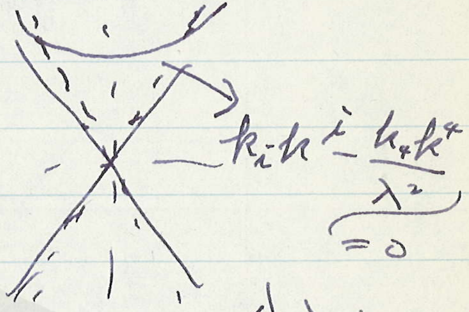
$$f(k) = \frac{\hbar}{ic\lambda}\,\frac{1}{4\pi^4}\Big\{\iint -dp\,dp_0\Big[\delta(p^2 + \kappa_0^2 - (k_0 - p_0)^2\Big] \cdot \frac{k_0 - p_0}{|k_0 - p_0|}\,\frac{p_0}{|p_0|}\,\delta\!\left(p^2 - \frac{p_0^2}{\lambda^2}\right)$$

$$+ 2p^2\,\delta'[p^2 + \kappa_0^2 - (k_0 - p_0)^2]\,\frac{k_0 - p_0}{|k_0 - p_0|}\,\frac{p_0}{|p_0|} \times \delta\!\left(p^2 - \frac{p_0^2}{\lambda^2}\right)\Big\}$$

Now, what will happen, if we define P as the limit of operator above considered? Keep first the upper limit of integration of k_μ, k'_μ, l_μ, so that $k_\mu+k'_\mu+l_\mu$ is always inside the ~~cone~~ shifted cone in k-space for P.

This is always possible.

Then the self-energy of the vacuum ~~of~~ is identically zero, before going over to $\lambda \to 1$.



$\lambda > 1.$

$\lambda - 1 \ll 1.$

Although before ~~the~~ $\lambda \to 1$, the modified-P is not relativistically invariant, in the limit $\lambda \to 1$ it is certainly.

Now, what will be the self-energy of ~~one~~ a particle in vacuum.

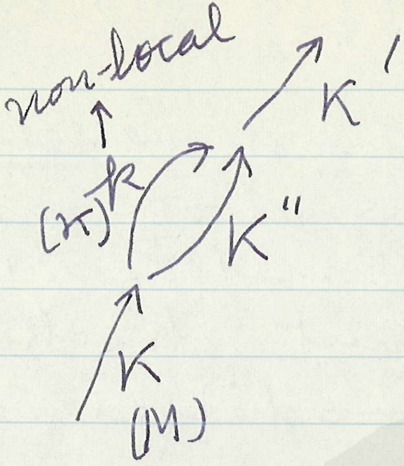
~~$\{x'|L'$~~

$$\{L' P L'\} = \int\cdots\int (x'|L'|x''')(x'''|P|x^{iv})(x^{iv}|L|x'') \times (dx')^4 (dx'')^4 (dx''')^4 (dx^{iv})^4$$

$$\left.\begin{aligned} X &= \tfrac{1}{2}(x'+x'') & r &= x'-x'' \\ X' &= \tfrac{1}{2}(x'''+x^{iv}) & r' &= x'''-x^{iv} \end{aligned}\right\}$$

$$\int\cdots\int (X+\tfrac{1}{2}r|L'|X'+\tfrac{1}{2}r')P(r')(X'-\tfrac{1}{2}r'|L'|X-\tfrac{1}{2}r) \, dX\, dX'\, dr\, dr'$$

$$\propto \prod_\mu \delta(K'_\mu - K_\mu) \cdot \iint \left(\frac{\sin \frac{M}{\kappa} k\lambda}{\frac{M}{\kappa} k\lambda}\right)^2 \exp i(k+k''-k)r'\,dr' \; P(r')\, dk$$


non-local
(x)k
K'
K''
K
(M)

Self-Reciprocal Displacement Operators

$$f_+(s) = -\frac{\varkappa}{s}\frac{dJ_0(\varkappa s)}{d(\varkappa s)} = -\frac{1}{s}\delta(s) + \frac{\varkappa}{s}J_1(\varkappa s)$$

$$= -2\delta(s^2) + \frac{\varkappa}{s}J_1(\varkappa s)$$ (future) inside

$$f_0(s) = 0$$ outside

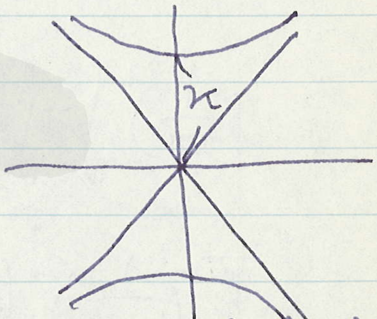
$$f_-(s) = -f_+(s)$$ ~~sin~~ inside (past)

$s^2 = x_\mu x^\mu$ outside $s^2 = -x_\mu x^\mu$ inside

$$g_+(K) = 2i\delta(K^2 - \varkappa^2)$$

$$g_0(K) = 0$$

$$g_-(K) = -2i\delta(K^2 - \varkappa^2)$$



$$f_+^{(1)}(s) = -\frac{2\delta(s^2)}{\varkappa^2} + \frac{J_1(\varkappa s)}{\varkappa s}$$

$$f_0^{(1)}(s) = 0$$

$$f_-^{(1)}(s) = -f_+^{(1)}(s)$$

$\sim$

$$g_+^{(1)}(K) = \frac{2i\delta(K^2-\varkappa^2)}{\varkappa^2}$$

$$g_0^{(1)}(K) = 0$$

$$g_-^{(1)}(K) = -\frac{2i\delta(K^2-\varkappa^2)}{\varkappa^2}$$

$$f_+^{(2)}(s) = \frac{2\delta(s^2-\varkappa^2)}{\varkappa^2}$$

$$f_0^{(2)}(s) = 0$$

$$f_-^{(2)}(s) = -f_+^{(2)}(s)$$

$\sim$

$$g_+^{(2)}(s) = -\frac{2i\delta(K^2)}{\varkappa^2} + i\frac{J_1(\varkappa K)}{\varkappa K}$$

$$g_0^{(2)}(s) = 0$$

$$g_-^{(2)}(s) = -g_+^{(2)}$$

$$f_+^{(1)}(s) + f_+^{(2)}(s) = \frac{2\left(\delta(s^2-\varkappa^2) - \delta(s^2)\right)}{\varkappa^2} + \frac{J_1(\varkappa s)}{\varkappa s}$$

$$f_0^{(1)}(s) + f_0^{(2)}(s) = 0$$

$$g_+^{(1)} + g_+^{(2)} = \frac{2i\left(\delta(K^2-\varkappa^2) - \delta(\varkappa^2)\right)}{\varkappa^2} + \frac{iJ_1(\varkappa K)}{\varkappa K}$$

$$g_0^{(1)} + g_0^{(2)} = 0$$

$\lim r \to 0$:

$$F_+(S) = -2\delta_+'(S^2) + \frac{1}{2} \qquad G_+(K) = -2i\delta_+'(K^2) + \frac{i}{2}$$

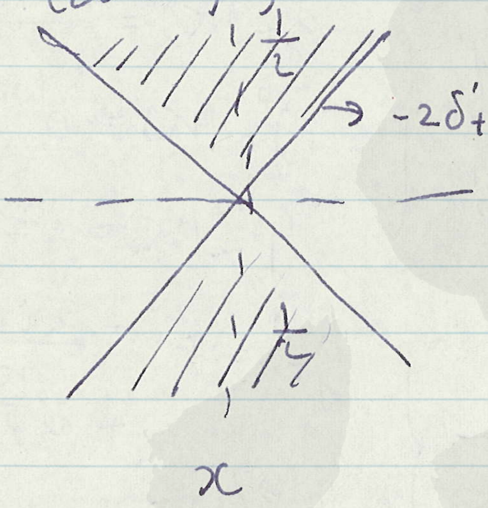
$$F_0(S) = 0 \qquad G_0(K) = 0$$

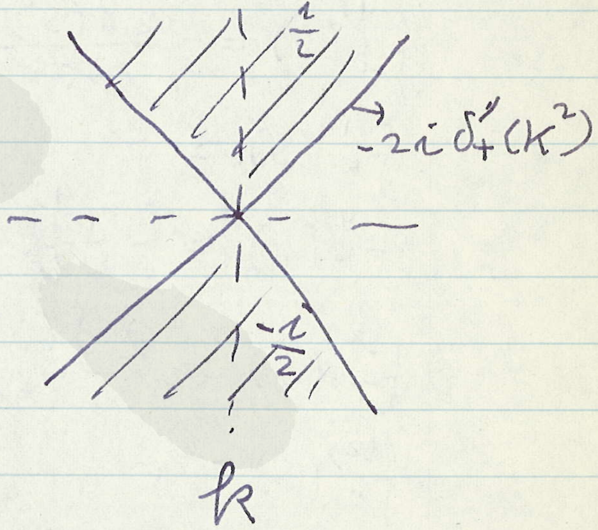
$$F_-(S) = -F_+(S) \qquad G_-(K) = -G_+(K)$$

$$-\int f(x)\,\delta_+'(x)\,dx = \lim_{\substack{\varepsilon \to 0 \\ \varepsilon > 0}} \frac{f(\varepsilon) - f(0)}{\varepsilon}$$

$$\left(\frac{1}{2\pi}\right)^2 \int \cdots \int F(S) \exp(-i k_\mu x^\mu)(dx^\mu)^4 = G(K)$$

or $$\left(\frac{1}{2\pi}\right)^2 \int \cdots \int G(K) \exp(i k_\mu x^\mu)(dx^\mu)^4 = F(S)$$





III.

Lagrangian Formalism

There are apparently three ways to ~~get rid of the difficulty~~ deal with the problem of interaction.

(i) Lagrangian method – direct integration of field equations

(ii) S-matrix formalism with η-function

(iii) S-matrix formalism with P-operator.

The difficulty of constructing a Lagrangian formalism lies in the fact that, if ~~yo~~ we start

$$\delta \iint (x'|\mathfrak{L}|x'')\, dx' dx'' = 0,$$

we can only obtain one equation for the real scalar field, for example.

Seminar
May 4, 1950

Bloch, Lagrangian

$$L(X) = \int \mathcal{L}(X, r)\, dr$$

$$I_\Omega = \int_\Omega L(X)\, dX$$

$$\delta I_\Omega = 0$$

$$I_\Omega = \mathrm{Tr.}\, J_\Omega \mathcal{L}$$

$$= \int dx' (x' | J_\Omega \mathcal{L} | x')$$

$$= \iint \mathcal{L}(X, r) J_\Omega (X, -r)\, dX\, dr$$

$$J_\Omega(X, -r) = \begin{cases} +1 & X \text{ in } \Omega \\ 0 & x \text{ not in } \Omega \end{cases}$$

$$\mathrm{Tr}\, [p_\mu, J_\Omega] A = \mathrm{Tr}\ J_\Omega [A, p_\mu]$$

$$= \int_\Omega dX\, dr \frac{\partial A}{\partial X^\mu} = \int_\Sigma d\sigma_\mu \int dr . A(X, r)$$

$$\mathcal{L} = \sum \{ u^{(i)} u^{(j)} + u^{(i)} [u^{(j)}, p_\mu] \}$$

$$\delta \mathcal{L} = \sum \mathcal{L}_1 \delta u^{(i)} \mathcal{L}_2 + \sum \mathcal{L}_1^\mu [\delta u, p_\mu] \mathcal{L}_2^\mu$$

$$\delta I_\Omega = \mathrm{Tr}\, \mathcal{L}_2 J_\Omega \mathcal{L}_1 \delta u + \overset{\mathrm{Tr}}{\sum} \mathcal{L}_2^\mu J_\Omega \mathcal{L}_1^\mu [\delta u, p_\mu]$$

$$\mathrm{Tr} \{ \mathcal{L}_2^\mu J_\Omega \mathcal{L}_1^\mu [\delta u, p_\mu] \} = \mathrm{Tr}\, [p_\mu, \mathcal{L}_2^\mu J_\Omega \mathcal{L}_1^\mu] \delta u$$

$$\delta I_\Omega = \delta I_1 + \delta I_\Sigma$$

$$\delta I_1 = \mathrm{Tr} \sum \mathcal{D}_{(i)} \delta u^{(i)}$$

$$\delta I_\Sigma = 0$$

$\mathcal{D}_{(i)} = 0$; field equations

素

Last Talk at Institute, Princeton Aug. 1949.

S-matrix in Nonlocal Field Theory ①

W. Heisenberg, ZS. f. Phys. 120 (1943), 513; 673; 123 (1944), 93.
C. Møller, Det Kgl. Danske Vidensk. Selsk. Mat.-Fys. Medd. 23 (1945), Nr. 1, 22, (1946), Nr. 19.

$S = e^{i\eta}$: Lorentz invariance

$S^{\dagger}S = SS^{\dagger} = 1$

Connection between S-matrix and cross-section

$$S = 1 + R$$

$$(\mathbb{k}'|R|\mathbb{k}^0) = \delta(W'-W^0)(\mathbb{k}'|U|\mathbb{k}^0)$$

$$= \delta(\mathbb{K}'-\mathbb{K}^0)\,\underline{\delta(W'-W^0)(\mathbb{k}'|U_{\mathbb{K}^0W^0}|}\,(\mathbb{k}^0)$$

$$U = -2\pi i V \Psi$$

~~$S = S\,\delta(\mathbb{K}'-\mathbb{K}^0)\,\delta(W'-W^0) = 1$~~

$$S = 1 \underbrace{+ \delta}_{-2\pi i}(\mathbb{K}'-\mathbb{K}^0)\,\delta(W'-W^0)(\mathbb{k}'|V\Psi|\mathbb{k}^0)$$

$$S = 1 - 2\pi i(\mathbb{k}'|V\Psi|\mathbb{k}^0)$$

$$(E-W')\Psi(\mathbb{k}_1' \cdots \mathbb{k}_{n'}')$$

$$= \sum_{n''}\int(\mathbb{k}_1' \cdots \mathbb{k}_{n'}'|V|\mathbb{k}_1'' \cdots \mathbb{k}_{n''}'')\,d\mathbb{k}_1'' \cdots d\mathbb{k}_{n''}''$$

$$\times \Psi(\mathbb{k}_1'' \cdots \mathbb{k}_{n''}'')$$

$$\Psi \to (\mathbb{k}_1' \cdots \mathbb{k}_{n'}'|\Psi|\mathbb{k}_1^0 \cdots \mathbb{k}_{n^0}^0)$$

$$\left(\begin{aligned} R^{+}R &= 4\sin^{2}\frac{\eta}{2}. \\ Q &= \frac{4\pi}{k_2^{0\,2}} \sum_{l'=0}^{\infty} (2l'+1) \sin^{2}\frac{\eta'}{2} \end{aligned} \right)$$

$$dQ_{\substack{sc.\\em.}} = 4\pi^{2} \frac{k_2'^{2} \left| \overset{k_2', k_3'}{\widehat{(k_1' | U_{K^0W^0} | k_1^0 k_2^0)}} \right|^{2} d\Omega\, dk_3'}{\sqrt{(u_1^0 - u_2^0)^2 - |u_1^0 \times u_2^0|^2} \cdot |(u_2' - u_1') e_2'|}$$

$$\Psi(k_1' \cdots k_n') \equiv \Psi(k')$$

$$(E - W')\Psi(k') = \int (k'|V|k'')\, dk''\, \Psi(k'')$$

$$\Psi(k') = (k'|V\Psi) \left(\frac{1}{E - W'} + \lambda\,\delta(E - W') \right]$$

$$(W^0 - W')(k'|\Psi|k^0) = \int (k'|V|k'')\, dk''(k''|\Psi|k^0)$$

$$\Psi(k', k^0) = (k'|U|k^0) \left(\frac{1}{W^{(0)} - W'} + \lambda\,\delta(W^{(0)} - W') \right)$$

$$\Psi(k', k^0) = \delta(k', k^0) + \delta_{+}(W' - W^0)(k'|U|k^0)$$

V X

$$\delta_{\pm}(W' - W^0) = \frac{\pm 1}{2\pi i (W' - W^0)} + \frac{1}{2}\delta(W' - W^0)$$

$$U(k', k^0) = V(k', k^0) + V(k', k'')\delta_{+}(W'' - W^0)(k''|U|k^0)$$

~~not yet~~

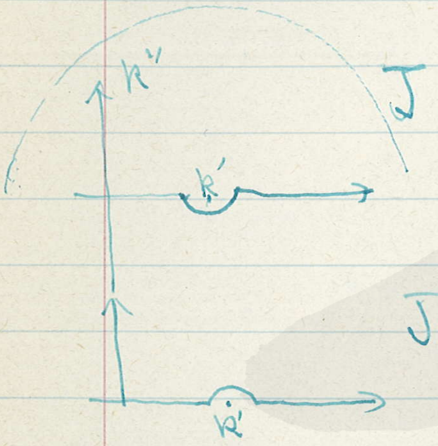
$$\left(V \Rightarrow \iiint (x'|V|x'')\, dx'\, dx'' \; ? \right)$$

$$\longrightarrow \iiint V(x, r)\, \underline{(dx)^{3}}\, (dr)^{4}$$

$$\delta_+(k) = \frac{1}{2\pi}\int_0^\infty e^{ikt}dt$$

$$\delta_-(k) = \frac{1}{2\pi}\int_{-\infty}^0 e^{ikt}dt$$

$$\delta = \delta_+ + \delta_- \qquad \delta_- = \delta_+^*$$



$$J_+ = \int dk'' e^{ik''r}\delta_+(\cancel{k}k'-k'')$$

$$= -\frac{2\pi i}{r}k'e^{ik'r}$$

$$J_- = \frac{2\pi i}{r}k'e^{-ik'r}$$

$$\delta_+(k) = -\frac{1}{2\pi ik} \qquad Im(k) > 0.$$

$$\delta_-(k) = \frac{1}{2\pi ik} \qquad Im(k) < 0.$$

$$R(k',k^0) = \delta(W'-W^0)(k'|U|k^0)$$

$$= \delta(W'-W^0)\,V(k',k^0)$$

$$+ \delta(W'-W^0)V(k',k'')\delta_+(W''-W^0)\,\times(k''|U|k^0)$$

$$R(k',k^0) = \bar{V}(k'k^0) + \bar{V}(k',k^c)\cancel{(k''|R|k}\;R_+(k'',k^0)$$

$$\text{now}\ \bar{V}(k',k^0) = \iiint (x'|V|x'')(dx'dx'')^4 \ .\ !!!$$

$$R_+(k'',k^0) = \int_0^\infty dx^4\, e^{i(k_4'-\sum k_4^{(0)})x^4} \cdots$$

Conservation of Energy-Momentum

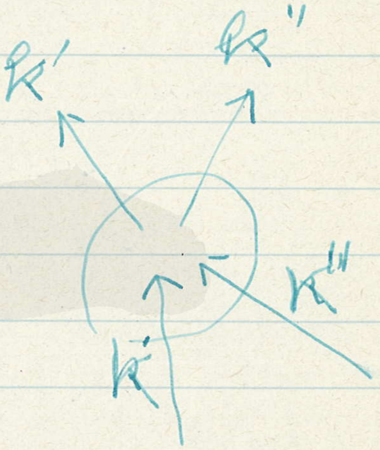
$$e^{\pm i(k'_\mu - k''_\mu)}$$

$$\sum \bigcirc \{ e^{-ik'_\mu x^\mu} \cdot e^{-il'_\mu p^\mu}$$

$$\times e^{-ik''_\mu x^\mu} e^{-il''_\mu p^\mu}$$

$$\times e^{+iK'_\mu x^\mu} e^{+iL'_\mu p^\mu}$$

$$\times e^{iK''_\mu x^\mu} e^{iL''_\mu p^\mu} \}$$



$$= \sum \bigcirc \bigcirc \exp\{-i(k'_\mu + k''_\mu - K'_\mu - K''_\mu) x^\mu\}$$

$$\rightarrow \sum \bigcirc f(r_\mu) \exp \frac{i}{2}(k'_\mu + k''_\mu - K'_\mu - K''_\mu) X^\mu$$

$$\int_{X^4=0}^{\infty}\iiint \bigcirc (dr_\mu)^4 (dX^\mu)^4$$

$$= \int_0^\infty \exp \frac{i}{2}(k^{(4)\prime} + k^{(4)\prime\prime} - K^{(4)\prime} - K^{(4)\prime\prime}) X^4 \times dX^4$$

$$\propto \delta_+(k^{(4)\prime} + k^{(4)\prime\prime} - K^{(4)\prime} - K^{(4)\prime\prime})$$

$$(W^0-W')\,(\mathfrak{k}'|\Psi|\mathfrak{k}^0)=\int(\mathfrak{k}'|V|\mathfrak{k}'')\,d\mathfrak{k}''\,(\mathfrak{k}''|\Psi|\mathfrak{k}^0)$$

$$(\mathfrak{k}'|\Psi|\mathfrak{k}^0)=\int(\mathfrak{k}'|V|\mathfrak{k}'')\,d\mathfrak{k}''\,(\mathfrak{k}''|\Psi|\mathfrak{k}^0)$$
$$\times\left(\frac{1}{W^0-W'}+\lambda\,\delta(W^0-W')\right)$$

$$(\mathfrak{k}'|\Psi|\mathfrak{k}^0)=\delta(\mathfrak{k}',\mathfrak{k}^0)+\delta_+(W'-W^0)\,(\mathfrak{k}'|U|\mathfrak{k}^0)$$

$$\left.\begin{aligned}U&=-2\pi i\,V\Psi.\\ \overline{U}&=\delta(W'-W^0)\,U\end{aligned}\right\}\ S=1+\overline{U}$$

$$\int(\mathfrak{k}'|V|\mathfrak{k}'')(\mathfrak{k}''|\Psi|\mathfrak{k}^0)\,d\mathfrak{k}''$$
$$=(\mathfrak{k}'|V|\mathfrak{k}^0)+\int(\mathfrak{k}'|V|\mathfrak{k}'')$$
$$\times\,\delta_+(W''-W^0)\,(\mathfrak{k}''|U|\mathfrak{k}^0)\,d\mathfrak{k}''$$

$$(\mathfrak{k}'|U|\mathfrak{k}^0)=(\mathfrak{k}'|V|\mathfrak{k}^0)+$$
$$+\int(\mathfrak{k}'|V|\mathfrak{k}'')\,\delta_+(W''-W^0)\,\underbrace{(\mathfrak{k}''|U|\mathfrak{k}^0)}_{d\mathfrak{k}''}$$

$$(W'\mathfrak{k}'|\overline{U}|W^0\mathfrak{k}^0)=(W'\mathfrak{k}'|\overline{V}|W^0\mathfrak{k}^0)$$
$$+\int\delta(W'-W^0)\,(\mathfrak{k}'|V|\mathfrak{k}'')\,\delta_+(W''-W^0)(\mathfrak{k}''|U|\mathfrak{k}^0)$$
$$\times\,d\mathfrak{k}''$$
$$\|$$
$$\int\delta(W'-W'')\,(W'\mathfrak{k}'|V|\mathfrak{k}'')\,\delta(W''-W^0)\,\delta_+(W''-W^0)$$
$$(\mathfrak{k}''|U|\mathfrak{k}^0)\,d\mathfrak{k}''\,dW''$$

$$(W'\mathfrak{k}'|\overline{U}|W^0\mathfrak{k}^0)=(W'\mathfrak{k}'|\overline{V}|W^0\mathfrak{k}^0)$$
$$+\int(W'\mathfrak{k}'|\overline{V}|W''\mathfrak{k}'')\,\delta_+(W''-W^0)\,(W''\mathfrak{k}''|\overline{U}|W^0\mathfrak{k}^0)\,d\mathfrak{k}''\,dW''$$

$$(W'k'|S|W^0k^0) - \delta(W',W^0)\delta(k',k^0)$$

$$= (W',k'|\bar{V}|W^0k^0) + \int (W'k'|\bar{V}|W''k'') \times$$

$$\times \delta_+(W''-W^0)(~~W~~k''|~~U~~S|W^0k^0)\frac{dk''}{dW''}$$

$$- \int (W'k'|\bar{V}|W''k'')\delta_+(W''-W^0)~~(W''k''+~~$$

$$\delta(W''-W^0)\delta(k''-k^0)dW''dk''$$

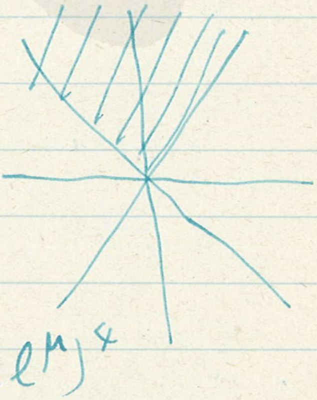
$$(n'_k, x'|\bar{U}|n''_k, x'') = (n'_k, x'|\bar{V}|n''_k, x'')$$

$$+\sum_{n'''_k}\iint_{x_4''' > x_4''} (n'_k x'|\bar{V}|n'''_k, x''')\overset{?}{~~\delta_+(~~}\rightarrow f(x''', x'')$$

$$\times (n'''_k x'''|\bar{U}|n''_k, x'') ~~d~~ dx'''$$

$$X_4 = \frac{1}{2}(x_4''' + x_4'') \qquad r_4 = x_4''' - x_4''.$$

$$\int_{x'}\underbrace{\int_{x'''}\int_{x''}}_{X,r}{}_{X_4=0}^{X_4=\infty} (n'_k, x'|\bar{V}|n'''_k, X+\frac{r}{2})\,\bar{U}(n''_k, X+\frac{r}{2})$$



$$\int f(x''', x'')$$

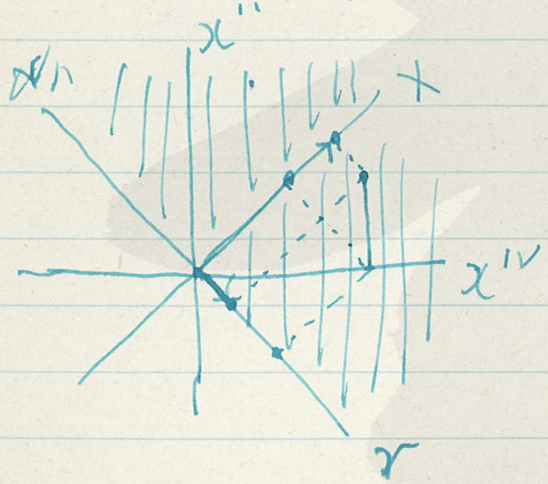
$$\rightarrow \iint\cdots e^{il^\mu p_\mu/\hbar}(dl^\mu)^4$$

$$\underline{l_\mu l^\mu ~~\lessgtr~~ 0, \quad l_4 > 0}$$

$$\sum_{w'''k'''} \int\cdots\int (dx')^4 (dx''')^4 (w'k'x'|\bar{V}|w'''k'''x''')$$

$$\times \int_{X^4=0}^{\infty}\cdots\int (dx^{IV})^4 (dx'')^4 (w'k'x'|\bar{U}|w'''k'''x^{IV})$$

$$X^4 = (x^4)^{IV} + (x^4)''$$



$x'' + x^{IV} \geq 0.$

Heitler-Stueckelberg's theory
(Wentzel, Rev. Mod. Phys. 19 (1947), 1)

$$\bar{K}(1 + \tfrac{1}{2}\bar{R}) = \frac{1}{2\pi i}\bar{R}$$

$$\bar{K} = (q|K|q')\delta(E_q - E_{q'})$$

$$K = \sum_{n=0}^{\infty} T^n H'$$

$$(q|T|q') = \frac{(q|H'|q')}{E_0 - E_{q'}}$$

Sept. 1, 1949 Columbia University

Even in nonlocal field theory, we can assume the existence of $\Psi(n'(x, x; \underline{k}, l, m); x'_\mu)$ just as in ordinary field theory. The ~~dif~~ essential difference between two cases is, however, that in local field Ψ can be transformed by a linear transf. into the form $\Psi(n'(x'_\mu); x'_4)$, where

$$U\cancel{\psi}(x'_\mu) = \sum_{\underline{k}} \exp i k_\mu x'^\mu \cdot a^*(\underline{k}) \cdot \sqrt{N_k}$$

$$U^*\cancel{\psi}^*(x'_\mu) = \sum_{\underline{k}} \exp(-ik_\mu x'^\mu) \cdot a^*(\underline{k}) \sqrt{N_k}$$

$$n(x'_\mu) = \sum_{\underline{k}'} \sum_{\underline{k}''} \cancel{a^*} \cdot a^*(\underline{k}'') a(\underline{k}') \exp i(k'_\mu - k''_\mu) x'^\mu \sqrt{\cancel{N_{k'}} N_{k''}}$$

$$\cancel{\psi^*(x'_\mu)}$$

$$\iiint n(x'_\mu)(dx'_\mu)^3 = \sum_{\underline{k}'} \cancel{a^*(\underline{k}')}\, n(\underline{k}') \cdot \cancel{N_{k'}}$$

In nonlocal field theory, there is no quantity such as $n(x'_\mu)$, because the relation between U and a is now

$$U(x'_\mu, x''_\mu) = \sum \exp i k_\mu \left(\frac{x'^\mu + x''^\mu}{2}\right) \times P_l^m(\theta, \varphi) \times a(\underline{k}, l, \ldots) \sqrt{N_{\underline{k}, l \ldots}}$$

Thus, $n(\underline{k}'', l'', \ldots)$ can be considered as ~~the operator~~ an observable corresponding to the number of particles in the state $\underline{k}, l, m$; but ~~it c~~ there is not such a observable as to correspond to the number of particles at a definite point.

However, the quantity

$$\int U^*_{(X_\mu, r_\mu)} U(X_\mu, -r_\mu) \cancel{(dr_\mu)^4} r_\mu^2 d\theta\, d\varphi = \int |U(X_\mu, r_\mu)|^2 \cancel{(dr_\mu)} \cancel{\sin\theta}\, d\theta\, d\varphi$$

in which each term of U, U* is divided by ~~the~~ a suitable delta fn such as $\delta(r_4) \times \delta(r-\lambda)$, can be interpreted as the ~~cent~~ number of particles, ~~which lies in~~ with the centre of mass lying at ~~the~~ the point X_μ, ~~altho~~ although this quantity is no more relativistically invariant

Now in local field theory, we can define ~~the~~ any stationary ~~system~~ state by a fn.

$$\Psi(n'(\underline{k})).$$

Particular~~ly~~, we can a stationary ~~system~~ state

$$\Psi(n'(\underline{k}), n^{(0)}(\underline{k})),$$

by starting from

Columbia

Sept. 20, 1949 ⑥

$$(W'k'|\bar{U}|W^0k^0) = (W'k'|\bar{V}|W^0k^0)$$

$$+\int \delta(W'-W^0)(k'|V|k'')\delta_+(W''-W^0)(k''|U|k^0)$$

$$dk''$$

$$\overset{?}{\parallel}$$

$$\int \delta(W'-W'')\delta(W''-W^0)dW''\int(k'|V|k'')$$

$$\times\ \delta_+(W''-W^0)(k''|U|k^0)dk''$$

$$(W'k'|\bar{V}|W^0k^0) = \int (dx')^4(dx'')^4\cdot\cancel{V}(x'|V|x'')$$

$$\int \delta(W'-W^0)(k'|V|k'')\delta_+(W''-W^0)(k''|U|k^0)dk''$$

$$= \int\cdots\int (x'|V|x''')\quad(x'''|U|x'')(dx')^4(dx'')^4(dx'')^4$$

$$\varepsilon(x''+x'')$$

$$\int\cdots\int(x'|VU|x'')(dx')^4(dx'')^4$$

$$U = V + V\varepsilon U$$

$$U = V + U^{(1)} + U^{(2)} + \cdots$$

$$U^{(1)} = V\varepsilon V$$

$$U^{(2)} = V\varepsilon V\varepsilon V$$

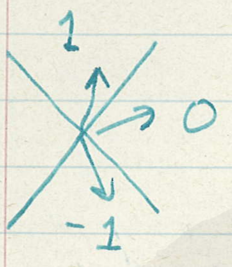
Invariant Operators Ω and Θ

$$(x'|\Omega^{(0)}|x'') = \Omega^{(0)}(X, r) = \text{const} \equiv 1$$

independent of X, r or x', x''.

$$(x'|\Theta^{(1)}|x'') = \Theta^{(1)}(X, r)$$

r-space



$$\left.\begin{array}{ll} = 1 & \text{for } -X_4 = X^4 > 0, \quad r_\mu r^\mu < 0 \\ \;\;\; 0 & \text{for } X_\mu X^\mu > 0 \\ \;\; -1 & \text{for } -X_4 = X^4 < 0 \quad r_\mu r^\mu \cancel{>} < 0 \end{array}\right\}$$

$$\Delta^{(\pm)} = \frac{1}{2}(\Delta^{(0)} \pm \Delta^{(1)}$$

$$\Omega\cancel{\Theta}(X, r) \propto \int\cdots\int \exp(ik'_\mu x'^\mu)\cdot\exp(-ik''_\mu x''^\mu) (dk'_\mu)^4(dk''_\mu)^4$$

$$= \int\cdots\int \exp i\overset{k_\mu}{(k'_\mu - k''_\mu)} X^\mu \cdot \exp i\underset{K_\mu}{\frac{(k'_\mu + k''_\mu)}{2}} \cdot r^\mu \quad (dk_\mu)^4(dK_\mu)^4$$

$$\Theta(X, r) \propto \int\cdots\int \exp iK_\mu X^\mu \cdot (dK_\mu)^4$$

$$\times \left\{ \underset{(K_\mu > 0 \;\; K_\mu K^\mu < 0)}{\int\cdots\int} \exp(iK_\mu X^\mu)(dK_\mu)^4 - \underset{(K_\mu < 0,\; K_\mu K^\mu < 0)}{\int\cdots\int} \exp(iK_\mu X^\mu)(dK_\mu)^4 \right\}$$

⑰

$$\overline{R} = \mathrm{Trace}\, R \overset{\Delta^{(0)}}{\Omega} = \int\cdots\int (x' | \Omega | x''') (x''' | R | x'') (dx')^4 (dx'')^4$$

$$= \int\cdots\int (x' | R | x'') (dx')^4 (dx'')^4$$

$$\mathrm{Trace}\, R \overset{\Delta^{(0)}}{\textcircled{H}} = \int\cdots\int (x' | R | x'') \varepsilon (x'' - x''') (dx')^4 (dx'')^4$$

$$\mathrm{Trace}\, R \overset{\Delta^{(0)}}{\Omega} = \mathrm{Trace}\, V \Omega + \mathrm{Trace}\, V R \textcircled{H}\, \Delta^{(+)}$$

$$\overline{V} \doteq \mathrm{Trace}\, V \Omega$$

$$\overline{R} = \overline{V} + \mathrm{Trace}\, V R \textcircled{H}$$

$$\Delta^{(+)} \quad \Delta^{(0)}$$

$$S = 1 + \overline{R}$$

$S, \overline{R}, \overline{V}$: (Submatrix) matrices with rows and columns characterized by $n^+(k, l, m)$ etc …

~~S or $\overline{R}$~~ assumption: R is a function of V, Ω and $\textcircled{H}$. $\Delta^{(+)}$

~~~~ $$R = V + R'$$

$$\mathrm{Trace}\, R' \Omega = \mathrm{Trace}\, V \cdot V \overset{\Delta^{(+)}}{\textcircled{H}} + \mathrm{Trace}\, V \cdot R' \textcircled{H}$$

local field : V : local operator

$$(x' | V | x'') = V(X) \prod_\mu \delta(r_\mu) = V(x') \prod_\mu \delta(x'_\mu - x''_\mu)$$

$$\mathrm{Trace}\, V \cdot V \textcircled{H} = \int\cdots\int V(x') \prod \delta(x' - x'') V(x'') \prod \delta(x'' - x''') (x''' | \textcircled{H} | x') (dx')^4 (dx'')^4 (dx''')^4$$

$$\Delta^{(+)} V \Delta^{(0)} \qquad \Delta^{(+)}$$
~~~~

~~$\mathrm{Trace}\ V\cdot V\cdot\textcircled{H} = \int\cdots\int V(x')V(x')\delta(x'-x'')$~~

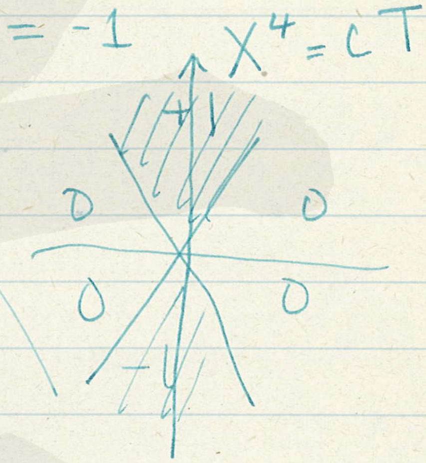
~~$\times\ (x''|\textcircled{H}^{(+)}(x')(dx')^4(dx'')^4$~~

~~$= \int\cdots\int V(x')V(x')(x'|\textcircled{H}|x')(dx')^4$~~

~~$(x'|\textcircled{H}^{(+)}|x'') = \textcircled{H}^{(+)}(X)$~~

~~$\cancel{\lim_{x^4\to\infty}}\ (x'|\textcircled{H}|x') = 1$~~ ~~fo~~

~~$\lim_{x^4\to-\infty}\ (x'|\textcircled{H}|x') = -1$~~



$$\mathrm{Trace}\ V\Delta^{(+)}V\Delta^{(0)}$$

$= \cancel{\mathrm{Tr}\int\int}\ V(x')\delta(x'-x'')\Delta^{(+)}(x'',x''')V(x''')\delta(x'''-x^{(iv)})$

$\times\ \Delta^{(0)}(x^{(iv)},x')(dx')^4(dx'')^4(dx''')^4(dx^{(iv)})^4$

$$= \int\cdots\int V(x')\Delta^{(+)}(x',x''')V(x''')\delta(x'''-x^{(iv)})$$
$$(dx')(dx''')(dx^{(iv)})$$

$$= \int\cdots\int V(x')\Delta^{(+)}(x',x''')\ V(x''')(dx')^4(dx''')^4$$

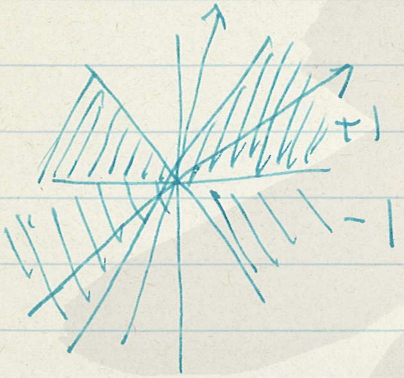
$x' = X+\frac{1}{2}r,\quad x''' = X-\frac{1}{2}r$

$$= \int\cdots\int_{r_\mu r^\mu<0,\ r^4>0} V(x')\ V(x''')(dx')^4(dx''')^4$$

$$= \frac{1}{2}\int\cdots\int V(x') \, V(x'')(dx')^4(dx'')^4$$

$$+ \frac{1}{2}\int\cdots\int \cancel{\varepsilon(} V(x') \, \varepsilon(x' - x'') \, V(x'')(dx')^4(dx'')^4$$

$$\varepsilon(\overbrace{x' - x''}^{r}) = \begin{cases} +1 & \cancel{x} \; r_\mu r^\mu < 0, \; r^4 > 0 \\ 0 & r_\mu r^\mu > 0, \\ -1 & r_\mu r^\mu < 0, \; r^4 < 0 \end{cases}$$



If $V(x') \, V(x'')$ is ~~an~~ ~~even~~ symmetric with respect to the reflection ~~with me.~~

$$x_1' - x_1'' \rightarrow x_1'' - x_1', \quad \text{etc,}$$

it will be symmetric with respect to the reflection

$$(x_1')' - (x_1')'' \rightarrow (x_1')'' - (x_1')'$$

and the integrations over the region /// and \\\ (with + and − sign) compensate with each other and the net result is

$$\frac{1}{2}\int\cdots\int V(x')(dx')^4 \int\cdots\int V(x'')(dx'')^4$$

$$+ \frac{1}{2}\int\cdots\int_{(x^4)' > (x'')^4} V(x') \, V(x'')(dx')^4(dx'')^4$$

$$\bar{R} = \mathrm{Trace}\ R\Delta^{(0)} = \mathrm{Trace}\ V\Delta^{(0)} + \mathrm{Trace}\ V\Delta^{(+)}R\Delta^{(0)}$$

$$\bar{R}^* = \mathrm{Trace}\ \cancel{R}^{+}\Delta^{(0)}R^* = \mathrm{Trace}\ \Delta^{(0)}V$$

$$+ \mathrm{Trace}\ \Delta^{(0)}R^*\Delta^{(-)}V$$

~~$\bar{R}^* \bar{R}^* \Delta + \bar{R} =$~~

$$\bar{R} = \iint (x'|R|x'')(dx')(dx'')$$

$$= \iint (x'|V|x'')(dx')(dx'')$$

$$+ \int\cdots\int (x'|V|x'')(x'''|R|x^{iv})(dx')(dx'')(dx''')(dx^{iv})$$

$(x^4)'' > (x^4)'''$

$$\bar{R}^* = \iint (x'|R^*|x'')(dx')(dx'')$$

$$= \iint (x'|V^*|x'')(dx')(dx'')$$

$$+ \int\cdots\int (x'|R^*|x'')(x'''|V|x^{iv})(dx')(dx'')(dx''')$$

$$(dx^{iv})$$

$(x^4)'' > (x^4)'''$

$$\int\cdots\int A(x'x'')\Delta^{(0)}(x''x''')B(x'''x^{iv})(dx')(dx'')(dx''')$$

$$= \int A(x'x'')\ B(x''x')(dx')(dx''')(dx^{iv})$$

$$= \int A(x'x'')$$

⑨

Pauli, meson theory, p. 50

$$(k|\Phi|0) = (k|1|0) + (k|F|0)$$

$$(k|F|0) = (k|f|0)\left[\frac{1}{E_k - E_0} + i\pi\delta(E_k - E_0)\right]$$

$$(k|f|0) = (k|H|0) + (k|HF|0)$$

$$f = H + HF$$

$$f^+ = H^+ + (HF)^+ = H + F^+H$$

$$(k|F^+|0) = -(k|f^+|0)\left[\frac{1}{E_k - E_0} + i\pi\delta(E_0 - E_k)\right.$$

$$F^+f = F^+H + F^+HF$$

$$f^+F = HF + F^+HF$$

$$F^+f - f^+F = F^+H - HF = f^+ - f$$

$$(A, \cancel{k}_{\beta E_0}|\underline{f}^+ - \underline{f}|0, \cancel{k}_0)\,\cancel{4} = -2i\pi\int_{A'}(A|\underline{f}^+|A')(A'|\underline{f}|0)$$

$$\underline{f}^+ - \underline{f} + 2\pi i\,\underline{f}^+\underline{f} = 0$$

$$\underline{S}^+\underline{S} = (\underline{1} - 2\pi i\,\underline{f}^+)(\underline{1} + 2\pi i\,\underline{f}) = 1$$

Lecture (Extra)

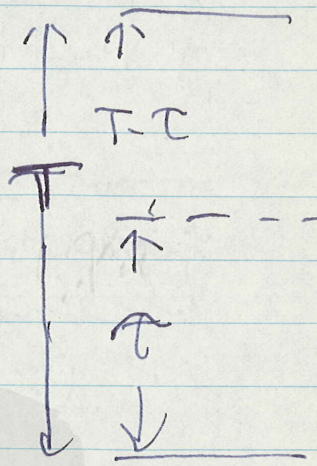
Nov. 18, 1949

D ③

$$S = D_\infty + D_\infty \bar{H}' D_\infty + D_\infty \bar{H}' D_\infty \bar{H}' D_\infty + \dots$$

~~S~~ where

$$D_\infty = \lim_{T\to\infty} \frac{1}{T}\int_0^\infty D_t\, dt$$



Now

$$D_t = \exp(-i l^4 p_4/\hbar)$$

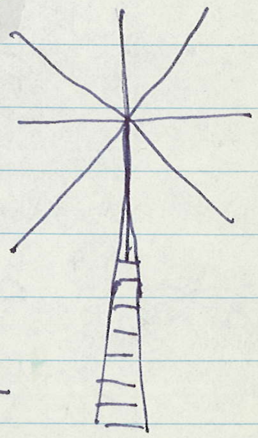
$$(x^{4'}|D_t|x^{4''}) = \delta(x^{4'} - x^{4''} - l^4)$$

$$D_\infty = \lim_{T\to\infty}\frac{1}{cT}\int_0^{cT} \exp(-i l^4 p_4/\hbar)\,(dl^4)$$

~~$= \lim_{T\to\infty}\frac{1}{cT}\int_0^{cT} \exp(i l_4 p_4/\hbar)\, dl_4$~~

or:

$$D_\infty = \lim_{T\to\infty}\frac{1}{cT}\int_0^{cT} \exp(-i l^\mu p_\mu/\hbar)\, \delta(l_1)\delta(l_2)\delta(l_3)\ (dl^\mu)^4$$



We have further to ~~transform~~ change the ~~eq~~ definition of S, so that it is more obviously relativistic. D_∞ is not relativistic in that ~~a particular time~~ it is a sum of displacements of different amounts in one particular time direction. Furthermore, $\bar{H}'$ is not relativistic in that it is ~~an space~~ obtained by integration in a particular space. Also the product of the form

$$D_{\infty}\bar{H}'D_{\infty}$$

~~is not yet the pro~~ does not yet have the ~~form~~ of the product of nonlocal operators, because

$$(x'|D_{\infty}|x'')\cancel{\int\cdots\int(x_i'''|\bar{H}'|x^{iv})dx''dx^{iv}}$$

$$\cancel{\bar{H}'(x_i''', x_i'''')\delta(x_i'', x_i''')\delta(x_i'''', x_i^{iv})}$$

$$(dx')^3(dx'')$$

$$(x'|D_{\infty}\bar{H}'D_{\infty}|x'')$$

$$=\iint(x_4'|D_{\infty}|x_4''')\int(x_\mu'''|H'_{(x_4''')}|x_\mu^{iv})(dx_i''')^3(dx_i^{iv})^3$$

$$(x_4'''|D_{\infty}|x_4'')dx_4'''$$

Instead, ~~of using D_{∞} common to all space points x_i''' and x_i^{iv}, we use~~ we can insert the space displacement operator D_s:

$$\left(\text{~~with x_4 D_{∞}~~}\quad D_s=\int\cdots\int\delta(l_4)\exp(-il^\mu p_\mu/\hbar)(dl^\mu)^4\right)$$

$$D_{\infty}D_sH'D_sD_{\infty}$$

Now $D_{\infty}D_s$ consist of all displacements with $l^4\geq 0$. It can be divided into

timelike part and spacelike part.

The ~~that~~ space-like will change by Lorentz transformation. Thus, we take up only the time-like part, so that the product now

D ④

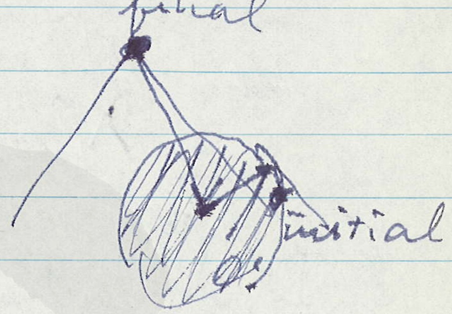
becomes invariant. This correspond to the postulate that the wave function Ψ at any space time point is influence by the values of it at those points, which ~~are~~ belongs to the past.

~~$S =$~~ $D H' D$ $D_\times$ Similarly, ~~D_∞ inside~~ the first term of

S can be replaced by $D_\infty D_s$ from relativistic argument. Thus we have finally

$$S = D + DH'D + DH'DH'D + \cdots,$$

which consists of almost exclusively of operators connecting the future with the past, but not completely, if H' is an nonlocal operator.



The displacement operator D can be regarded as the ~~ext~~ counterpart of δ_+ function in usual theory.
In fact, in usual theory

$$\Psi(R', R^0) = \delta(R', R^0) \cancel{+} - 2\pi i\, \delta_+(W' - W^0)\,(R'|V|R'')\,\Psi(R'', R^0)$$

which can be compared with

$$\overline{\overline{S}} = 1 \cancel{+} \frac{i}{\hbar}\overline{DH'S}$$

in our theory. When $D = D_\infty D_s$ and $(x'|S|x'')$ is independent of space parameters x'_ν, this reduces to the above formula

Invariant Time-like ~~Opera~~
Properties of Displacement Operator.

Now D is defined as

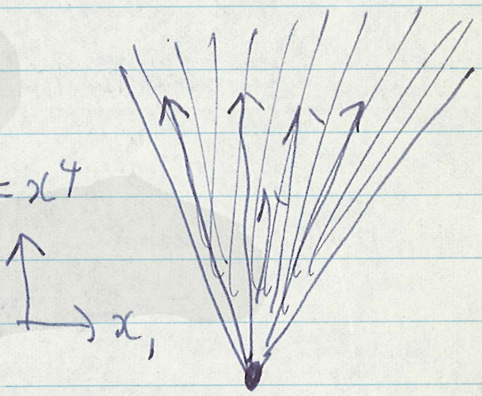
$$D = \frac{1}{V_T}\int\cdots\int_{\substack{l_\mu l^\mu \leq 0 \\ l^4 \geq 0}} (dl)^4 \exp(-i l^\mu p_\mu/\hbar)$$

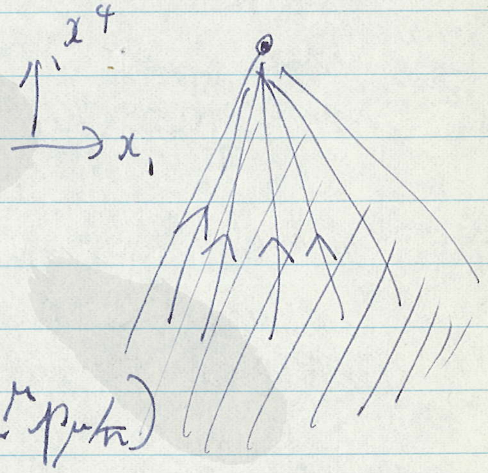
$l_\mu l^\mu \leq 0$
$l^4 \geq 0$.

~~Pro~~

$$D^2 = \frac{1}{V_T^2}\int\cdots\int_{\substack{l_\mu l^\mu \leq 0 \\ l^4 \geq 0}} (dl)^4 \exp(-i l^\mu p_\mu/\hbar) \quad ct = x^4$$

$$\times \iint_{\substack{l'_\mu l'^\mu \leq 0 \\ l'^4 \geq 0}} (dl')^4 \exp(-i l'^\mu p_\mu/\hbar)$$





$$L^\mu = l^\mu + l'^\mu$$

$$\lambda^\mu = \frac{1}{2}(l^\mu - l'^\mu)$$

$$D^2 = \left(\frac{1}{V_T^2}\right)\int\cdot\int (dL)^4 (d\lambda)^4 \exp(-i L^\mu p_\mu/\hbar)$$

$$L^\mu L_\mu = (l_\mu + l'_\mu)(l^\mu + l'^\mu) = l_\mu l^\mu + l'_\mu l'^\mu + \underbrace{2\, l_\mu l'^\mu}_{\text{alway } (-) \text{ because } l_4 l'^4 < 0}$$

$\leq - l^4 l'^4$

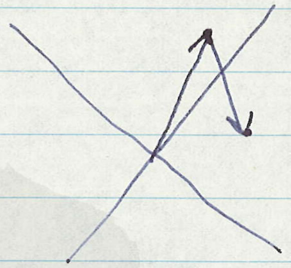
or $L^\mu L_\mu \leq 0$ $\quad L^4 \geq 0$

$$D^2 = \frac{1}{V_T}\int\cdot\int (d\lambda)^4 \cdot D = \frac{1}{V_T} D \cdot V_{TS}$$

D ⑤

$$\lambda^\mu\lambda_\mu = \frac{1}{4}(l^\mu - l'^\mu)(l_\mu - l'_\mu)$$

$$= \frac{1}{4}\{\underset{(-)}{l^\mu l_\mu} + \underset{(-)}{l'^\mu l'_\mu} - \underset{(+)}{2\, l^\mu l'_\mu}\}$$



$$D^*D = \int\cdots\int (dL)^4 (d\lambda)^4 \exp(-i\lambda^\mu p_\mu/\hbar)$$

$$= \cancel{}\frac{1}{V^2}V_T \underset{\underline{TS}}{\int\cdots\int}(d\lambda)^4 \underset{\overset{\|}{V_{TS}\cdot E}}{\exp(-i\lambda^\mu p_\mu/\hbar)}$$

$$= \frac{V_T}{V_T^2}\cancel{}V_{TS}\cdot E$$

$$= \frac{V_{TS}}{V_T}E.$$

$$\cancel{D}\ D' = \sqrt{\frac{V_T}{V_{TS}}}\,D \qquad D'D' = \left(\frac{V_T}{V_{TS}}\right)^2\left(\frac{V_{TS}}{T}\right)D = D'$$

$$\cancel{E' = \sqrt{\frac{V_T}{V_{TS}}}E} \qquad D'^*D' = \cancel{\frac{V_{TS}}{V_T}E''}\ \frac{V_T}{V_{TS}}E$$

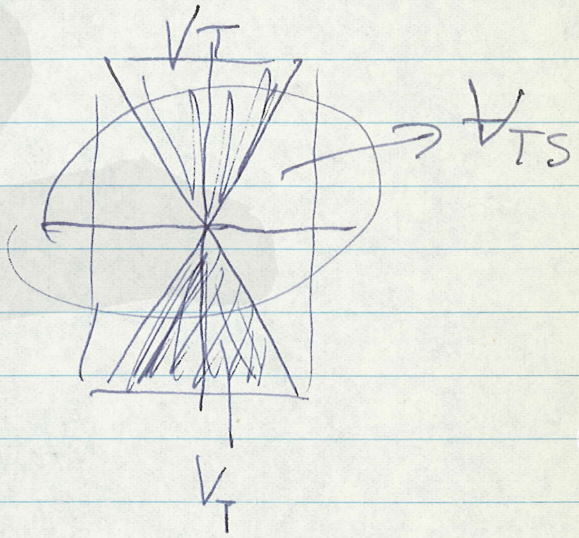
$$D' = \frac{1}{V_{TS}}\underset{l_\mu l^\mu \leq 0,\ l^4 > 0.}{\int\cdots\int}(dl)^4 \exp(-il^\mu p_\mu/\hbar)$$

$$E'' = \frac{\cancel{V_T}\,1}{V_{TS}}\int\cdots\int (dl)^4 \exp(-il^\mu p_\mu/\hbar)$$

$$V_{TS} \fallingdotseq \int\cdots\int (dl^\mu)^4$$

$$V_T = \lim_{\substack{T\to\infty \\ \chi_0\to\infty}} \frac{\pi}{4}(cT)^4 \{\sinh(2\chi_0) - 2\}$$

$$V_{TS} \overset{?}{\cong} 6 V_T$$



$$\overline{\overline{D}} = \frac{1}{V_T} \int\cdots\int (dx) \underset{\substack{l_\mu l^\mu \leq 0 \\ l^4 \geq 0}}{\int\cdots\int} (dl^4)$$

$$= V_{TS}$$

$$\overline{\overline{D^2}} = \frac{V_{TS}}{V_T}\overline{\overline{D}} = \frac{V_{TS}^2}{V_T} \qquad \overline{\overline{E}} = V_{TS}$$

$$\overline{\overline{\left(\frac{D}{V_{TS}}\right)}} = 1 \qquad \overline{\overline{\left(\frac{D}{V_{TS}}\right)^2}} = \frac{1}{V_T}.$$

$$\overline{\overline{\left(\frac{D^*}{V_{TS}}\right)\left(\frac{D}{V_{TS}}\right)}} = \frac{\overline{\overline{E}}}{V_T V_{TS}} = \frac{1}{V_T}.$$

N.D. 1

Normalization of Displacement Operator

$x^4_{\alpha} = ct = -x^{\cancel{\mu}}_4 ;\quad x_1 = x ;\quad x_2 = y ;\quad x_3 = z ;$

$$\int_0^{\Lambda} d\Lambda^2 \iiiint_{x^4_{\alpha}>0} \delta(x_\mu x^\mu + \cancel{\lambda}\Lambda^2)(dx^\mu_{\mu})^4$$

$$= \int_{x^4_{\cancel{\mu}}>0}\cdots\int (dx^\mu_{\cancel{\mu}})^4 \cdot \cancel{\left[\frac{1}{2\lambda}\right]_{\lambda=-x_\mu x^\mu}} ,\ \text{only if } \overset{-\Lambda^2 >}{x_\mu x^\mu} > 0 \quad \text{otherwise } 0,$$

$$\alpha = \int_{x_0>0}\cdots\int (dx^\mu_{\cancel{\mu}})^4 \cancel{\left(\frac{1}{2\sqrt{x_\mu x^\mu}}\right)} ,\ \text{only if } -\Lambda^2 > x_\mu x^\mu > 0.$$

$\Lambda > \sqrt{-x_\mu x^\mu} > 0$

$s = \sqrt{-x_\mu x^\mu}$

$r = \sqrt{x_1^2 + x_2^2 + x_3^2} = s\, \overset{\sinh\chi}{\cancel{\cosh\varphi}} \geq 0$

$x^4_{\alpha} = ct = s \cosh \cancel{\varphi}\,\chi > 0$

$$\tanh\chi = \frac{r}{ct} = \frac{e^{\chi} - e^{-\chi}}{e^{\chi} + e^{-\chi}}$$

~~$\cosh\chi =$~~

$0 \leq s \leq \Lambda \qquad\qquad 0 \leq \varphi \leq 2\pi$

$0 \leq \chi < \infty \qquad\qquad 0 \leq \theta \leq \pi$

$x = s \sinh\chi \sin\theta \cos\varphi$

$y = s \sinh\chi \sin\theta \sin\varphi$

$z = s \sinh\chi \cos\theta$

$ct = s \cosh\chi$

$$(dx^\mu_{\cancel{\mu}})^4 = \begin{vmatrix} \sinh\chi \sin\theta \cos\varphi & \sinh\chi \sin\theta \sin\varphi & \sinh\chi \cos\theta & \cosh\chi \\ s\ s(\chi)\, s(\theta)\, c(\varphi) & s\ s(\chi)\, s(\theta)\, s(\varphi) & s\ c(\chi)\, c(\theta) & s\, s(\chi) \\ s\ s(\chi)\, c(\theta)\, c(\varphi) & s\ s(\chi)\, c(\theta)\, s(\varphi) & -s\, s(\chi)\, s(\theta) & 0 \\ -s\ s(\chi)\, s(\theta)\, s(\varphi) & s\ s(\chi)\, s(\theta)\, c(\varphi) & 0 & 0 \end{vmatrix}$$

$\times\ ds\, d\chi\, d\theta\, d\varphi$

$$= s^3 \sinh\chi^2 d\chi \sin\theta \, ds \, d\chi \, d\theta \, d\varphi$$

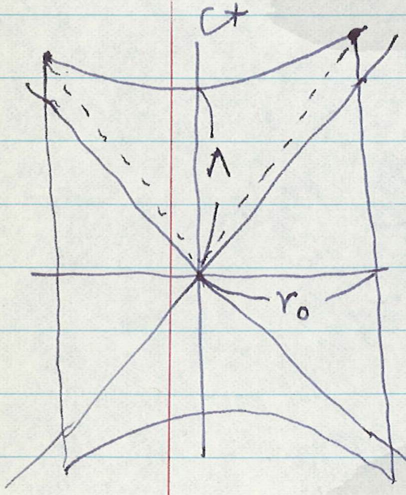
$$\int\cdots\int (dx^\mu)^4 = \int_0^\Lambda s^3 ds \int_0^\infty \sinh^2\chi \, d\chi \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\varphi$$

inside
upper half cone

$$= 4\pi \cdot \frac{\Lambda^4}{4} \cdot \frac{1}{4}\left\{\sinh(2\chi_0) - 2\chi_0\right\}\Big|_{\chi_0\to\infty} \;{}^{*}$$

$$= \frac{\pi}{4} \cdot \Lambda^4 \cdot \left\{\sinh(2\chi_0) - 2\chi_0\right\}_{\chi_0\to\infty}$$

$$\sinh 2\chi_* = \frac{1}{2}\left(\frac{e^{2\chi_*} - e^{-2\chi_*}}{2}\right)$$



$$= \left(\frac{e^{\chi_*} + e^{-\chi_*}}{2}\right)\left(\frac{e^{\chi_*} - e^{-\chi_*}}{2}\right)$$

$$= \sinh\chi_* \cdot \cosh\chi_*$$

$$= \frac{r \cdot ct}{(c^2t^2 - r^2)}$$

$$= \frac{r}{ct} \cdot \frac{1}{1 - \left(\frac{r}{ct}\right)^2}$$

~~sinh~~ $\left(\frac{r}{ct}\right)_{max} = R.$ ~~$\gg 1$~~ $\cong 1.$

$$\sinh 2\chi_0 = R\,\frac{1}{1-R^2} \gg 1.$$

---

* $\sinh^2\chi = \frac{1}{4}\left(e^{+2\chi} - 2 + e^{-2\chi}\right)$

$$\int_0^{\chi_0} \sinh^2\chi \, d\chi = \frac{1}{4}\left(\frac{e^{2\chi_0} - 1}{2} - 2\chi_0 \; \cancel{+} - \frac{e^{-2\chi_0} - 1}{2}\right)$$

$$= \frac{1}{4}\left\{\sinh 2\chi_0 - 2\chi_0\right\}$$

(N, D 2)

$$\cancel{\chi_0} \cong$$

$$\tanh \chi_0 = \left(\frac{r}{ct}\right)_{max} = R$$

or $$\frac{e^{\chi_0} - e^{-\chi_0}}{e^{\chi_0} + e^{-\chi_0}} = \frac{1 - e^{-2\chi_0}}{1 + e^{-2\chi_0}} = R$$

or $$1 - 2e^{-2\chi_0} = R.$$

$$e^{-2\chi_0} = \frac{1-R}{2} \qquad e^{2\chi_0} = \frac{2}{1-R}$$

$$\cancel{\log}\ 2\chi_0 = \log \frac{2}{1-R}.$$

$$\sinh 2\chi_0 - 2\chi_0 = \frac{R}{1-R^2} - \log \frac{2}{1-R}$$

$$= \frac{1}{1-R^2} - \underbrace{\frac{1-R}{1-R^2}}_{\frac{1}{1+R}} - \log \frac{2}{1-R} \cong \underline{\frac{1}{1-R^2}}. - \log \frac{1}{1-R}$$

$$V_{FS} = \lim_{R\to 1} \frac{\pi}{4} \Lambda^4 \frac{1}{1-R^2}$$

Lecture
Nov. 22, Tuesday, 1949

D 6

Last time I arrived at ~~a~~ the expression for the operator, which connects the wave function ~~at~~ in the past with that in the future and which may ~~have meaning~~ be of importance in nonlocal field theory, ~~although~~ in spite of the ~~st~~ situation that the existence of wave function itself ~~is may be doubtful.~~ ~~can~~ is rather unlikely.

~~We~~ I would like to ~~repeat the~~ recapitulate the process of deducing the final expression, because so few of the… were present last Friday.

It may be more satisfactory to proceed in the opposite way ~~way slightly~~ ~~different from that the previous one~~.

Suppose that even ~~in~~ in nonlocal field theory, ~~there~~ is it is mathematically permissible to consider ~~Schrödinger~~ a function of distribution ~~in~~ number n_{ke} of particles in various q.s. as well as of space-time coordinates x_μ's, because n_{ke} and x_μ are operators commutative with each other.

Let us write it as

$$\Psi(\cdots n'_{kl}\cdots, x'_\mu).$$

However, the physical mean of Ψ is by no means obvious, particularly because we are dealing with particles with finite extension, so that the observation of the existence or non-existence of particle at a (definite ~~particular~~) point ~~at a~~ in space ~~def~~inite at a definite time instant will not be possible in general. So, if we could give Ψ a physical meaning at all, it ~~will~~ might be as follows:

$$|\Psi(\cdots n'_{kl}\cdots; x'_\mu)|^2$$

is, ~~when~~ the probability, that …. the distribution in number of particles in various states is $\cdots n'_{kl}\cdots$ and the measurement was localized ~~performed~~ at x'_μ. Thus, the probability that the distribution in number of particles is $\cdots n'_{kl}\cdots$ irrespective of the location of measurement is

$$\int\cdots\int(dx'_\mu)^4|\Psi(\cdots n'_{kl}\cdots; x'_\mu)|^2 = \mathrm{Tr}\,\rho(\cdots n'_{kl}\cdots)$$

~~Trace $\tilde{\Psi}(\cdots n'_{kl}\cdots; x'_\mu)\Psi$~~

On the other hand, the probability that the measurement was localized at x'_μ

is

$$\sum_{\cdots n'_{kl}\cdots} |\Psi(\cdots n'_{kl}\cdots ; x'_\mu)|^2$$

Now, ~~say~~ in quantum mechanics, we always started from ~~th~~ Schrödinger function Ψ depen~~dent~~ing on time variable x'_4 and other variables $q'_1, q'_2, \ldots$, ~~which was~~ the physical meaning of Ψ was

$$|\Psi(q'_1 q'_2 \cdots ; x'^{(0)}_4)|^2$$

denote the probability that the results of measurements ~~$q'_1 q'_2 \cdots$~~ of $q_1 q_2 \cdots$ were $q'_1 q'_2 \cdots$, when the measurement were performed at a definite instant $x'^{(0)}_4$. This can be included in the above general function Ψ as a particular case, in which Ψ has a factor

$$\delta(x'_4, x'^{(0)}_4),$$

where x'_4 is the variable and $x'^{(0)}_4$ is a particular value of x'_4 corresponding to the exact time of measurement. Corresponding to the normalization

$$\sum_{q'_1 q'_2 \cdots} |\Psi(q'_1 q'_2 \cdots ; x'^{(0)}_4)|^2 = 1$$

~~in usual~~ for usual Ψ, $\overset{4x}{*}$ ~~we have for our generalized Ψ, the normalization~~

$$\sum_{n'_{kl}} \int \cdots \int \tilde{\Psi}^{(r)}(\cdots n'_{kl}, \cdots ; x'_\mu)\, \Psi^{(s)}(\cdots n'_{kl} \cdots ; x'_\mu)\,(dx'_\mu)^4 = \begin{cases} \delta_{rs} \\ \delta(r,s) \end{cases}$$

However, this does not ~~which~~ reduces to the above ~~former usual~~ expression, ~~if~~ ~~Ψ~~ which

$$\Psi \equiv \Psi(\cdots n'_{kl} \cdots ; x'_\mu)\, \delta(x'_4, x_4'^{(0)}),$$

because of the appearance of $\delta^2(x'_4, x_4'^{(0)})$ in $|\Psi|^2$ instead of $\delta(x'_4, x_4'^{(0)})$.

~~Instead,~~ ~~Ψ~~ In order to remove this difficulty, one must ~~start~~ have from generalized ~~the density matrix~~ Ψ, ~~which~~ so as to reduces to the usual Ψ, when the former had the factor

$$\sqrt{\delta(x'_\mu, x_\mu'^{(0)})}.$$

Now, we operate an operator D_{l_μ}, which is represented by the matrix

$$(x'|D_{l_\mu}|x'') = \delta(x'_\mu - x''_\mu \mp l_\mu),$$

on $\Psi(\cdots n'_{kl} \cdots ; x'_\mu)$, the result is

$$D_l \Psi(\cdots n'_{kl} \cdots ; x'_\mu)$$

$$= \Psi(\cdots n'_{kl} \cdots ; x'_\mu - l_\mu)$$

For example

* provided that we are dealing with discrete states. When we are dealing with continuous states characterized by the parameter α_i

$$\sum_{n'_{kl}} \int\cdots\int_{(dx'_\mu)^4} \tilde{\Psi}(\cdots n'_{kl}\cdots; x'_\mu; \alpha'_i)\, \Psi(\cdots n'_{kl}\cdots; x'_\mu; \alpha''_i) = \prod_{i=1}^{l} \delta(\alpha'_i, \alpha''_i)$$

In particular, when all Ψ has the form

$$\Psi \doteq \Psi(\cdots n'_{kl}\cdots; x'_i, \alpha'_1, \alpha'_2, \alpha'_3 \cdots \alpha'_{l-1}) \cdot \delta(x'_4, x'^{(0)}_4) \qquad \alpha'_l = x'^{(0)}_4$$

$$\sum_{n'_{kl}} \int\cdots\int_{(dx'_\mu)^4} \tilde{\Psi} \cancel{\Psi(dx'_\mu)}^4 (\cdots n'_{kl}\cdots, x'_\mu; \alpha'_i)\, \Psi(\cdots n'_{kl}\cdots x'_\mu; \alpha''_i)\, \delta(x'_4, x'^{(0)}_4)\, \delta(x'_4, x''^{(0)}_4)$$

$$= \prod_{i=1}^{l-1} \delta(\alpha'_i, \alpha''_i)\, \delta(x'^{(0)}_4, x''^{(0)}_4)$$

or

$$\sum_{n'_{kl}} \int\cdots\int (dx'_\mu)^3\, \tilde{\Psi}(\cdots n'_{kl}\cdots x'_\mu; \alpha'_i)\, \Psi(\cdots n'_{kl}\cdots x'_\mu; \alpha''_i)$$

$$= \prod_{i=1}^{l-1} \delta(\alpha'_i, \alpha''_i)$$

Now, we operate an operator $D_{l\mu}$, which is defined by the matrix

$$(x' | D(l_\mu) | x'') = \delta(x'_\mu - x''_\mu - l_\mu)$$

on $\Psi(\cdots n'_{kl}\cdots; x'_\mu)$, the result is

$$D_l \Psi(\cdots n'_{kl}\cdots; x'_\mu)$$
$$= \Psi(\cdots n'_{kl}\cdots; x'_\mu - l_\mu)$$

For example, if we started from

$$\Psi(\cdots n_{kl}' \cdots ; x_\mu') = \Psi(\cdots n_{kl}' \cdot)\prod_\mu \delta(x_\mu', x_\mu'^{(0)})$$

$$D_l\,\Psi(\cdots n_{kl}' \cdots ; x_\mu') = \Psi(\cdots n_{kl}' \cdots) \times \prod_\mu \delta(x_\mu' - l_\mu, x_\mu'^{(0)})$$

$$= \Psi(\cdots n_{kl}' \cdots)\prod_\mu \delta(x_\mu', x_\mu'^{(0)} + l_\mu)$$

Thus, $D(l_\mu)$ is an operator, which displaces the ~~measuring~~ (average) location of measurement by an amount l_μ.

The solution of Schrödinger equation

$$i\hbar \frac{\partial \Psi}{\underset{\to \cancel{x_4}\, x^4}{\partial (ct)}} = \bar{H}/c\,\Psi$$

is of the general form (in local field theory)

$$\Psi(\cdots n_k \cdots ; \overset{\to x^4}{ct}) = \Psi(\cdots n_k \cdots ; \underset{\downarrow x^4}{\overset{\nearrow t}{ct}} - \underset{X^4}{\overset{T}{cT}})$$

$$+ \int_{\cancel{ct-cT}\; x^4 - X^4}^{x^4 \swarrow ct} \frac{-i\bar{H}(t_1^{x})}{\hbar c}\, dt_1^{x}\; \Psi(\cdots n_k \cdots , t - T)$$

$$+ \int_{t-T,\; x^4 - X^4}^{x^4\; t} \frac{-i\bar{H}(t_1^{x_1})}{\hbar}\, dt_1^{x_1} \int_{t-T,\; x^4 - X^4}^{t_1 \to x^4_{(1)}} \frac{-i\bar{H}(t_2^{x_2})}{\hbar}\, dt_2^{x_2}\; \Psi(\cdots n_k \cdots , t - T)$$

$$-\frac{i\bar{H}}{\hbar c} = i\bar{L}$$

$$+ \cdots$$

By the help of the operator ~~$D_T = D(l_\mu)$~~ ~~with $l_1 = l_2 = l_3 = 0$, $l_4 = cT$. ($l^4 = -cT$).~~

~~$(x' | D_T | x'') = \prod_i \delta(x'_\mu - x''_\mu)$.~~

$$(\cdots n'_k \cdots t' | D_T | \cdots n''_k \cdots t'') = \prod_k \delta_{n'_k n''_k} \cdot \delta(t' - t'' - T),$$ the above ~~relation~~ solution can be written as

$$\{\Psi\}(\cdots n_k \cdots, t) = \{D_T \Psi\}(\quad \cdots \quad)$$

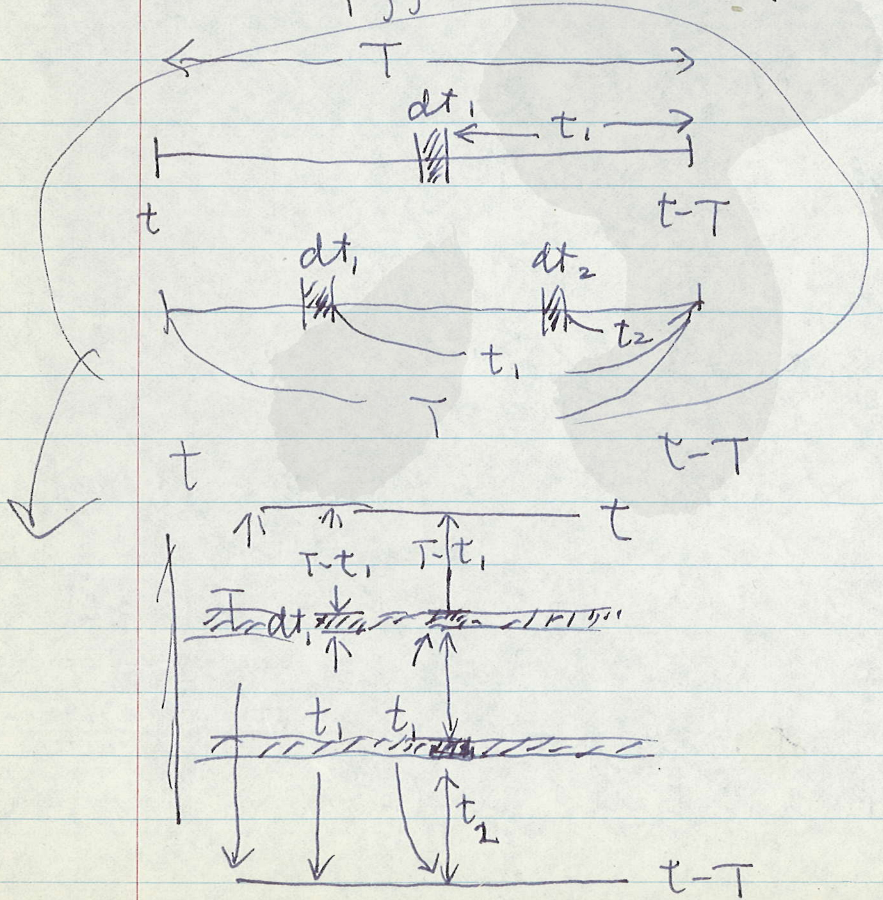
$$+ \{\int_{0}^{T} D_{T-t_1} \left(-\frac{i H(t_1)}{\hbar c} \underset{\substack{\downarrow \\ x^4_{(1)}}}{dt_1}\right) D_{t_1} \Psi\}(\quad \cdots \quad)$$

$$D_{X^4 - x^4_{(1)}}$$

$$+ \{\iint \cdot dt_1 dt_2 \, D_{T-t_1} \left(-\frac{i H(t_1)}{\hbar c} dt_1\right) D_{t_1 - t_2} \left(-\frac{i H(t_2)}{\hbar c} dt_2\right)$$

$$D_{t_2} \Psi\}(\quad \cdots \quad)$$

$$+ \quad \cdots$$



~~D_T~~

In order to extend this formula to nonlocal field, we have ~~to~~ first to consider D_T etc. as operators represented by matrices with rows and columns characterized by $n'_k \cdots$ ~~$ct' = x'_4$~~ $x^{4\prime}_{(1)}$ as well as

$\cancel{-_4}\,\overline{L}(x^4)$

x_1', x_2', x_3', and to replace ~~$H'(t)$~~ by

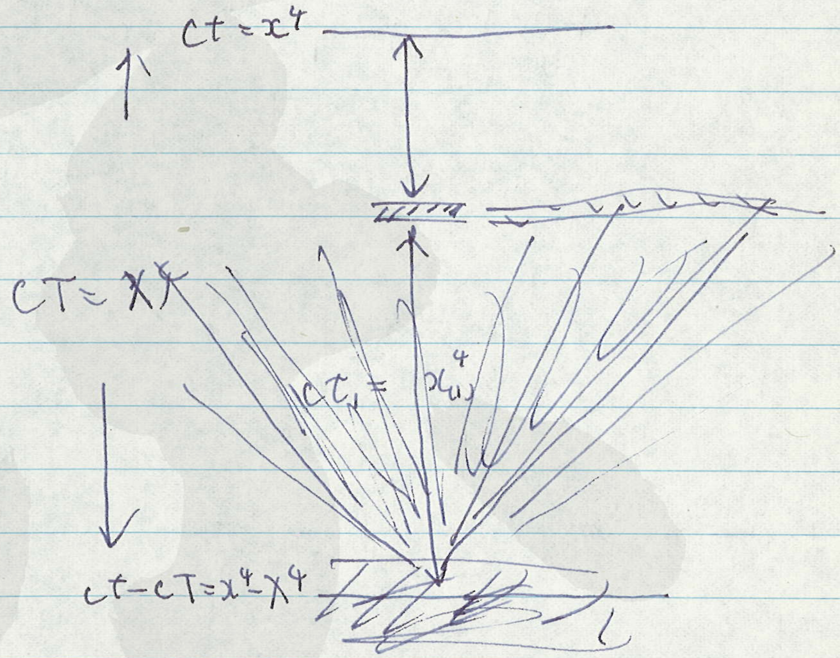
$$\iint(\cdot n_k' \cdots x_i' x_4' \,\underset{L}{|}\, \cdot n_k'' \cdots x_i'' x_4'')(dx_i')^3(dx_i'')^3$$

and also replace $\Psi(\cdots n_k' t')$ by $\Psi(\cdots n_k' \cdots x_\mu')$. Thus we obtain

$$\underbrace{\Psi}_{(\cdots x')} = {}^{(x')}D^{\int^{x_4''}}_{X^4 - x^4_{(1)}}\Psi(x'') + i\int_0^{X^4} {}^{(x')}D^{|(x'')}_{X^4 = x^4_{(1)}} \underbrace{\iint(\cdot|L|\cdot)}_{(dx'')^4(dx')^4} {}^{(x'')}D^{|x'^4}_{x^4_{(1)}}\, dx^4_{(1)}$$

$$-\iint_0^{X^{(4)}}\int_0^{x^4_{(1)}} dx^4_{(1)}\, dx^{(4)}_2\, D_{X^4 - x^4_{(1)}} \iint(\cdot|L|\cdot)\, D_{x^{(4)}_{(1)} - x^4_{(2)}} \iint(\cdot|L|\cdot)\, D_{x^{(4)}_{(2)}}$$

$\Psi(n_k, x_\mu)$

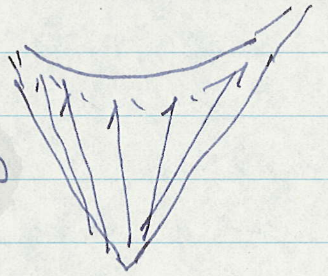


Now in nonlocal field theory, the operator L itself contains the space-time displacement operators, so that ~~the~~ on the right hand side of the above relation, not only $\Psi(\cdots)$ at $x'^4 - X^4$, but also Ψ at various time instants x''^4, which differ from $x'^4 - X^4$.

DLO

At the same time, it should be noticed that ~~D_{x^4}~~ operators such ~~as~~ as D_{x^4} are no more pure time-displacement operators, ~~because~~ it but the ~~product~~ sum of products of ~~the~~ fixed amount of time-displacement operators and arbitrary amount of space-displacement. Obviously such operators are not relativistically invariant. So we propose to replace them by a <u>time-like</u> displacement <u>operators</u>.

We propose further alteration that in the limit of $cT \cancel{=} x^4 \to \infty$



$$\Psi = D\Psi + iDLD\Psi$$

$$- DLDLD\Psi$$

This ~~is~~ <u>approximately</u> connects the future with the past in the sense that D, DLD are ~~o~~perator with matrix elements $(x'|D|x'')$, which are <u>~~at most~~ always</u> ~~different from~~ zero, <u>almost always</u> ~~if~~ unless

$$x^{4'} > x^{4''}.$$

Thus, we may say that

$$S = D + iDLD - DLDLD + - \cdots$$

is the operator connecting past with future.

If S is operatrated on Ψ, which is constant over a large space-time region, $S\Psi$ ~~consists of those~~ is the superposition of those states, which have the same total energy and total momentum*.

If, on the contrary, Ψ is not const. but depend on x_μ explicitly, it can be expanded into Fourier series, and the conservation of energy and momentum is ~~guara~~ reestablished, only when the extra momentum-energy due to Ψ itself ~~is~~ taken into account. ~~In other words,~~ This corresponds to the situation that we are dealing now with the system ~~f~~ influence by external forces. In other words, ~~the~~ those Ψ which are constant over space-time region correspond to state of system of free particles, whereas non-constant Ψ to ~~state of~~ bound states in the broad sense. those states, which include

---

* This is because the factor ΔL can ~~all be~~ always be shifted to ~~the end of~~ the right hand side end without changing the space-time dependence of the potl. $\exp(ik_\mu x_\mu)$.

S.11

~~D13~~

S-Matrix and its Physical Interpretation ~~of S~~.

~~when Ψ is const. with respect to x'_μ, $S\Psi$ is also const. because~~

$$\cancel{\{}(n'_{kl}, x'_\mu | S | n''_{kl}, x''_\mu)\cancel{\}^2}$$

can be interpreted as the correlation amplitude and

$$|(n'_{kl}, x'_\mu | S | n''_{kl}, x''_\mu)|^2$$

the correlation between ~~of~~ two sets of values ~~results~~ of ~~r~~ n'_{kl} and n''_{kl}, which can be obtained as the results of measurements of n_{kl} at points x'_μ and x''_μ respectively.

Lecture

Nov. 29, Tuesday, 1949 D I

Displacement Operators D II

Last time I have written down the ~~f~~ expression

$$\Psi(t') = \int (t'|D_T|t'')\,\Psi(t'')\,dt''$$

$$+\int (t'|D_{T-t_1}|t'')\,dt''\left(\frac{-iH'(t'')}{\hbar}dt_1\right)^{\delta(t''-t''')} dt'''(t'''|D_{t_1}|t^{IV})\,\Psi(t^{IV})\,dt^{IV}$$

$+ \cdots$

$$D_T = \delta(t'-t''-T)$$

$$\Psi(t') = \Psi(t'-T) + \int -\frac{iH'(t'-T+t_1)\,\delta(t'-T+t_1-t''')\,dt_1}{\hbar}\int dt''' \int (t'''|D_{t_1}|t^{IV})\,dt^{IV}\,\Psi(t^{IV})$$

$+ \cdots$

$$= \Psi(t'-T) + \int -\frac{iH'(t'-T+t_1)}{\hbar}dt_1\,(t'-T+t_1|D_{t_1}|t^{IV})\,dt^{IV}\,\Psi(t^{IV})$$

$+ \cdots$

$$= \Psi(t'-T) + \int -\frac{iH'(t'-T+t_1)\,dt_1}{\hbar}\cdot\Psi(t'-T)$$

$+ \cdots$

or ~~$D_T = \int D_{T-t_1}\cdot\frac{iH'}{\hbar}dt_1\cdot D_{t_1} + \cdots$~~

~~$\rightarrow D_\infty = \lim_{T\to\infty}\frac{1}{T}\int_0^T D_t\,dt$~~

$$D_\infty + \oint D_\infty\left(\frac{-iH'}{\hbar}\right)D_\infty$$

In general, when H' is not diagonal, this will be replaced by

$$D_T - \int D_{T-t_1}\left(\frac{iH'}{\hbar}\right) D_{t_1}\,dt_1 + \cdots$$

with matrix element

$$\delta(t'-t''-T) - \int_0^T i(t'-T+t_1|$$

$$-\int_0^T\int^{-t''}\int \delta(t'-T+t_1)\,dt'''\,(t'''|\frac{iH'}{\hbar}dt_1|\,t^{IV})\,dt^{IV}\delta(t^{IV}-t''-t_1)+\cdots$$

$$= \delta(t'-t''-T) - \frac{i}{\hbar}\int_0^T (t'-T+t_1|H'|t''+t_1)\,dt_1$$

$+ \cdots$

$$\Psi(t') = \Psi(t'-T) - \frac{i}{\hbar}\int_0^T (t'-T+t_1|H'|t''+t_1)\,dt_1\,dt''\Psi(t'')$$

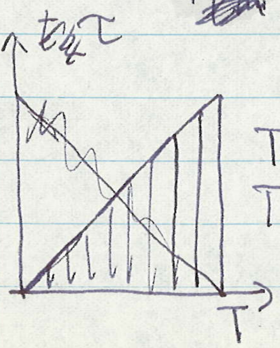
So, ~~in the second term~~, only those values of $\Psi(t'')$, for which

$$t'-T+t_1 \simeq t''+t_1$$

or $t'-T \simeq t''$,

contribute to the second term.

$$\Psi(t') = \frac{1}{T}\int_0^T \Psi(t'-t)\,dt - \frac{i}{\hbar}\frac{1}{T}\int_0^T dt\int_0^{t_1}(t'-t+\tau|\overline{H'}|t''+\tau)\,d\tau\,dt''\Psi(t'')$$



$T \ge t_1 \ge t_2 \ge 0$

$T \ge t - \tau \ge 0$

$t_1 = t - \tau$

$t_2 = \tau$

$$dt_1\,dt_2 = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} dt\,d\tau$$

$= ~~dt_1 dt_2~~\ dt\,d\tau$

P 2

D,13

$$\Psi(t') = \frac{1}{T}\int_0^T \Psi(t'-t)\,dt$$

$$- \frac{i}{\hbar}\,\frac{1}{T}\int_0^T\!\!\int_0^T dt_1\,(t'-t_1|\bar{H}'|\underbrace{t''+t_2)\,dt_1\,dt_2}_{\Psi(t'')}$$

$$(t'|\bar{D}_T|t'') = \frac{1}{T}\int_0^T \delta(t'-t''-t)\,dt$$

$$= \frac{1}{T} \quad \text{for } T \geq t'-t'' \geq 0$$

$$0 \quad \text{for } \left.\begin{matrix} t'-t'' > T \\ t'-t'' < 0 \end{matrix}\right\}$$

$$\Psi = \bar{D}_T \Psi$$
$$+ \bar{D}_T \bar{H}' \bar{D}_T \Psi$$

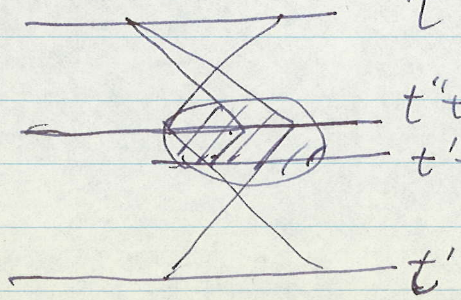
$$S = \bar{D}_T + \bar{D}_T \bar{H}' \bar{D}_T + \bar{D}_T \bar{H}' \bar{D}_T \bar{H}' \bar{D}_T^{\times}$$
$$+ \cdots$$

---

* ~~$\iiint_0^T D_{t_1} H' D_{t_2} H' D_{t_3}\,dt_1\,dt_2\,dt_3$~~

~~$= D_{T-t_1} H' D \quad H' D_{t_3}$~~

~~By this procedure,~~



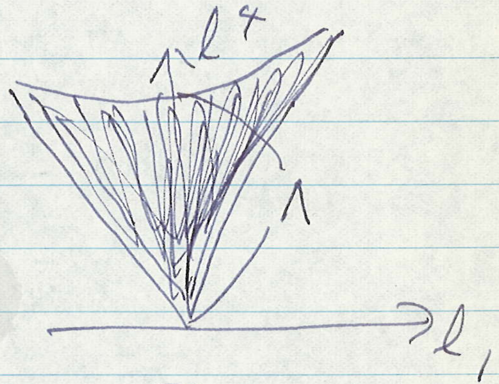
Now, if we consider S as a nonlocal operator, it must be replaced by

$$S = \underbrace{\bar{\bar{D}}_{TS}}_{\bar{\bar{D}}_{TS}+} H' \bar{\bar{D}}_{TS}$$

where

$$\bar{\bar{D}}_{TS} = \frac{1}{V}\iiiint \delta(x'_\mu - x''_\mu - l_\mu)(dl_\mu)^4$$

with $-\Lambda^2 \le l_\mu l^\mu \le 0$
$l^4 \ge 0$



$D = \bar{\bar{D}}_{TS}$

or simply

$$S = D + DH'D + DH'DH'D + \cdots$$

$$DH'S = S - D$$

$$\underline{S = D + DH'S}$$

Last Lecture ~~of~~ before departure to Stockholm

Dec. 1, 1949

q. 1

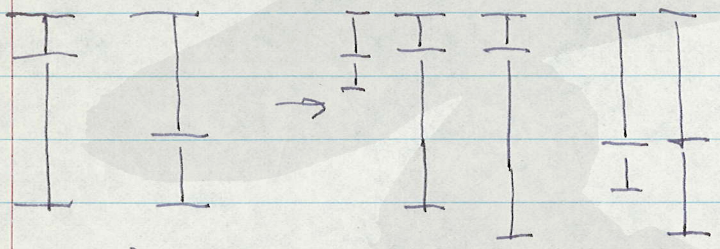
Questions to be solved

(i) Relativistic Generalization of the Operator

$$S = \frac{1}{T}\bar{D}_T + \underset{\wedge\,\frac{1}{T}}{}\left(\frac{-i}{\hbar}\right)\bar{D}_T\,\bar{H}'\,\bar{D}_T + \left(\frac{-i}{\hbar}\right)^2\underset{\wedge\,\frac{1}{T}}{}\bar{D}_T\bar{H}'\bar{D}_T\,\bar{H}'\bar{D}_T$$

where

$$(n'_k, t'|\bar{D}_T|n''_k, t'') = \prod_k \delta_{n'_k n''_k}\int_0^T \delta(t'-t''-t)\,dt$$



$$\int^{(T)} D_{T-t}\,\bar{H}'\,D_t\,dt \qquad \overset{(T\sim 2T)}{\bar{D}_T\,\bar{H}'\,\bar{D}_T}$$

$$\bar{D}_T = \int_0^T dt\,\exp(-i\,t\,p_t/\hbar)$$

$$[t, p_t] = i\hbar \qquad [x^4, p_4] = i\hbar$$

$$cT = k\Lambda, \quad ct = l^4 \qquad p_t = c\,p_4$$

or

$$\bar{D}_{k\Lambda} = c\bar{D}_T = \int_0^{k\Lambda} \exp(-i\,l^4 p_4/\hbar)\,dl^4$$

Now, ~~we want~~ in order to reformulate S so that it is the sum of products of nonlocal operators, it is necessary to introduce the displacement operator

$$(n'_k, \underbrace{t'}_{x'y'z'} | \overline{\overline{D}}_{T} | ~~\cancel{\ }~~ \underbrace{n''_k, t''}_{x''y''z''}) = \prod_k \underbrace{\delta(n'_k, n''_k)}_{\delta(x'_i - x''_i)} \int_0^T \delta(t' - t'' - t)\, dt$$

or

$$\cancel{(n'_k, x'_\mu | \overline{\overline{D}}_\Lambda | n''_k, x''_\mu)} \underbrace{\ }_{\overline{\overline{D}}_\Lambda = c D_T} = \prod_k \delta(n'_k, n''_k) \int_0^\Lambda \underbrace{\exp(-il^4 p_4/\hbar)}_{dl^4}$$

~~or~~

$$\cancel{\overline{\overline{D}}_\Lambda = \frac{1}{L^3} \prod_k \delta(n'_k, n''_k) \iiiint_0^\Lambda \exp(-il^\mu p_\mu/\hbar)(dl^\mu)^4}$$

~~$0 \leq l^4 \leq \Lambda$~~

~~$-L \leq l_1, l_2, l_3 \leq +L$~~

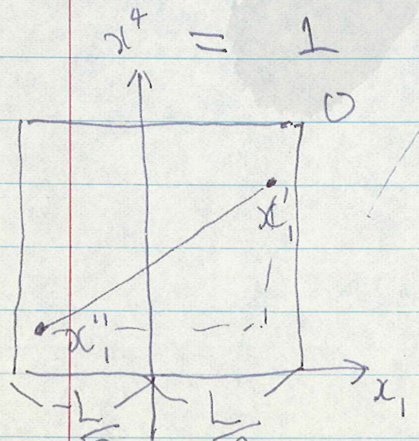
~~because~~

$$\cancel{(x'_1 | \int \exp(-il^1 p_1/\hbar)\, dl^1 | x''_1)}$$

$$\cancel{= \int_{-L}^{+L} \delta(x'_1 - x''_1 - l_1)\, dl_1}$$

~~$= 1$ if $-L \leq x'_1 - x''_1 \leq +L$~~

~~0 otherwise~~



On the other hand

$$(n'_k, t'|\overline{H}'|n''_k t'') = \int\cdots\int (n_k\, x'_\mu|H'|n''_k, x''_\mu)\,(dx'_i)^3 (dx''_i)^3$$

$$= \overline{\overline{D}}_L \cdot H' \overline{\overline{D}}_L$$

$$(n'_k, x'_\mu|\overline{\overline{D}}_L|n''_k\, x''_\mu) = \prod_k \delta(n'_k, n''_k)$$

$$\overline{\overline{D}}_L = \iiint_{-L}^{+L} \exp(-il^i p_i/\hbar)(dl^i)^3$$

because

$$(x'_\mu|\iiint_{+L\ldots} \exp(-il^i p_i/\hbar)(dl^i)^3|x''_\mu)$$

$$= \iiint_{-L\ldots} \prod_i \delta(x'_i - x''_i - l_i)\cdot\delta(x'^4 - x''^4)(dl^i)^3$$

$$= \begin{cases} 1 & \text{if } -L \le x'_i - x''_i \le +L \\ 0 & \text{otherwise.} \end{cases}$$

Thus the

$$S = \frac{1}{\Lambda}\overline{D}_\Lambda + \frac{1}{\Lambda}\left(\frac{-i}{\hbar c}\right)\int\cdots\int_{-L<l_i<+L,\ 0<l^4<\Lambda.} \exp(-il^\mu p_\mu/\hbar)(dl^\mu)^4$$

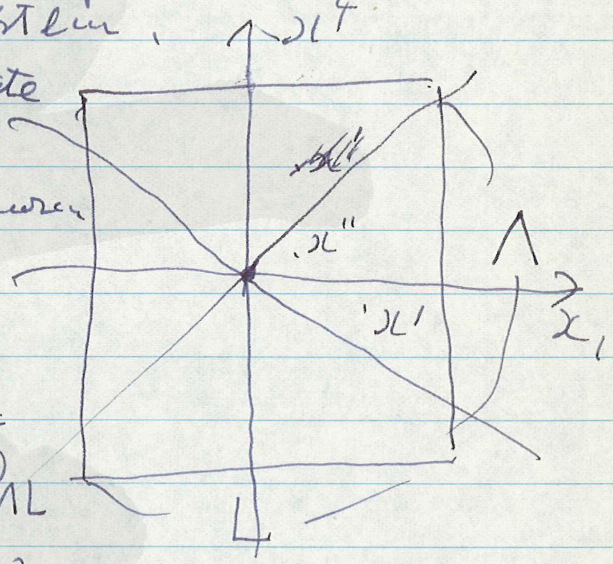
$$\cdot H' \cdot \cancel{\left(\frac{-i}{\hbar c}\right)}\int\cdots\int \exp(-il^\mu p_\mu/\hbar)(dl^\mu)^4$$

$$S = \frac{1}{\Lambda}\overline{D}_\Lambda + \frac{1}{\Lambda}\left(\frac{-i}{\hbar c}\right)\overline{\overline{D}}_{\Lambda L} H' \overline{\overline{D}}_{\Lambda L} + \cdots$$

where $$\overline{\overline{D}}_{\Lambda L} = \int\!\!\int\!\!\int_{-L}^{+L}\cdot\int_{0}^{\Lambda} \exp(-i l^\mu \gamma_\mu/\hbar)(dl^\mu)^4$$

This is not yet perfectly relativistic, since it still depends on the particular choice of the coordinate system.

One way to reformulate the expression for S, is to change S first into the form



$$S = \frac{1}{\Lambda L^3}\overline{\overline{D}}_{\Lambda L}$$

$$+ \frac{1}{\underset{\substack{\downarrow \\ L^3 ?}}{\Lambda}}\left(\frac{-i}{\hbar c}\right)\overline{\overline{D}}_{\Lambda L}\underset{\substack{\downarrow \\ (H'L^3)}}{H'}\overline{\overline{D}}_{\Lambda L}$$

$$+ \cdots\cdots$$

then to redefine

$$\overline{\overline{D}}_{\Lambda L} = \int\cdots\int_{\left.\begin{array}{l} l_\mu l^\mu \le 0 \\ l^4 > 0 \end{array}\right\}} \exp(-i l^\mu \gamma_\mu/\hbar)(dl^\mu)^4$$

(ii) Correspondence to Local Field Theory
(iii) Physical Interpretation of S-operator and ~~the~~ Ψ

$$\Psi(n_k', x_\mu') = \Psi(n_k')\,\delta(t', t^{(1)})$$

$$\Big|\sum\int\cdots\int \tilde{\Psi}^{(2)}(n_k', x_\mu')\,(n_k'\, x_\mu' \,|\, S \,|\, n_k''\, x_\mu'')\,\Psi^{(1)}(n_k''\, x_\mu'')\Big|^2$$

$$\equiv \big|\tilde{\Psi}^{(2)} S\, \Psi^{(1)}\big|^2$$

is the probability . . . . . . *

More generally

$$|\tilde{\Psi}' S \Psi|^2$$

can be interpreted as correlation coefficient.

In local field theory

$$(n_k', x' \,|\, S \,|\, n_k''\, x'') = 0$$

for any two point x', x'', which are space-like to each other.

The usual wave fn Ψ in local field theory can be obtained by considering $\Psi(n_k', x_\mu')$ as independent of x_μ' and depending

---

* Although S is an operator, even in the case of local field, connecting Ψ at time t' with ~~any~~ Ψ at any early time t'', ~~by~~ the changing ~~the meaning~~ of Ψ by inserting a factor ~~fn~~ $\delta(t', t^{(1)})$ reduces the formalism to the usual form as before.

on time t' or $x^{4'}$ only by a factor
$\delta(t', t'')$

$$\begin{tikzcd}
(x'_\mu | H' | x''_\mu) \arrow[d] & \Psi(n', x') \arrow[d] \\
(n'_k x'_\mu | H' | n''_k x''_\mu) \cdot \delta(x'_4, x''_4) & \Psi^{(1)} = \Psi(n')\delta(x'_4, x''_4)
\end{tikzcd}$$

normalization of Ψ.

D ⑤

Now, as anticipated

$$\bar{\bar{S}} = \bar{\bar{D}} + \overline{\overline{DH'D}} + \cdots$$

may ~~con~~ be interpreted as the S-matrix,

The equation, which S itself must satisfy, is evidently

$$DH'S = DH'D + DH'DH'D + \cdots$$

$$= S - D,$$

~~or~~ or

$$S = D + DH'S$$

and

$$S^* = D^* + S^* H'^* D^*,$$

where $H'^* = -H'$

$$\bar{\bar{D}} = \cancel{d l_n} = 1.$$

$$(n'|\bar{\bar{S}}|n''') = 1 + \sum_l \iint (x'+l, n'|H'|x'' n'')(x'' n''|S|x''' n''')\, dx' dx'' dx'''$$

$$\bar{\bar{S^*}} = 1 + \sum_l \iint \cancel{(x \quad |H'|}\,(x' n'|S^*|x'' n'')$$

$$\times (x'' n''|H'^*|x'''+l, n''')\, dx' dx'' dx'''$$

~~$\bar{\bar{S^*}}\bar{\bar{S}} = 1 - \sum_l \iint x''|S$~~

$$(n'''|\bar{\bar{S^*}}|n') = 1 - \sum_l (x'' n''|S|n' x')(x''+l, n'''|H'|x'' n'')\, dx' dx'' dx'''$$

$$= 1 - \sum_l (x'+l, \cancel{H} n'|H'|x'', n'')(x'' n''|S|x''' n''')$$

$$\sum_{n'} |(n'|\bar{\bar{S}}|n'')|^2 = f(\cdot\cdot n''\cdot\cdot)$$

$$\cancel{P_{n}} \frac{(n'|\bar{\bar{S}}|n'')}{\sqrt{f(\cdot\cdot n''\cdot\cdot)}} \equiv \underset{\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim}{(n'|P|n'')}$$

For local field

$$(n' \cdot x' | S \pm x'' x'') = (x'|D|x'')\,\delta(n', n'')$$
$$+ (x'|D|x''')(n'|H_{i(x''')}|n''')(n''' x'''|S|n'' x'')$$

Example; (Nonlocal $\overset{\text{scalar}}{\text{Meson}}$ Field
local spinor field　　E1

$$H' = g\ \psi^*\beta\psi\cdot U$$
$$\text{or}\quad \psi^* U\beta\psi$$
$$\text{or}\quad U\psi^*\beta\psi$$

$$\psi = \sum_{\bar{k}\,j} a_{\bar{k}}^{(j)}\ \cancel{\exp}\ u^{(j)}\exp(i\bar{k}_\mu x^\mu)$$
$$\bar{k}_4 < 0 \quad \text{for } j=1,2$$
$$\bar{k}_4 > 0 \quad \text{for } j=3,4$$

$$\psi^* = \sum_{\bar{k}\,j} a_{\bar{k}}^{(j)*}\ u^{*(j)}\exp(-i\bar{k}_\mu x^\mu)$$
$$\bar{k}_\mu\bar{k}^\mu + \bar{\varkappa}'^2 = 0$$

$$U = \int\cdots\int (dk)^4(dl)^4\ \bar{u}(k_\mu, l_\mu)\exp(ik_\mu x^\mu)$$
$$\cancel{\exp(-ik_\mu x^\mu)}\times\exp(il^\mu p_\mu/\hbar)$$

$$\bar{u}(k_\mu, l_\mu) = u(k_\mu, l_\mu)\,\delta(k_\mu k^\mu + \varkappa^2)$$
$$\times\,\delta(k_\mu l^\mu)\,\delta(l_\mu l^\mu - \lambda^2)$$

$$\text{or}\quad U = \sum_{\mathbf{k},l,m}\sqrt{\frac{(2\pi)^3\ \lambda}{L^3\cdot 4\varkappa\sqrt{k^2+\varkappa^2}}}\Big\{\overset{b}{a}(\underline{k}, l, m)\,\mathcal{U}(\underline{k}, l, m)$$
$$+\ \epsilon\, b^*(\underline{k}, l, m)\,\mathcal{U}^*(\underline{k}, l, m)$$

with

$$\mathcal{U}(\underline{k}, l, m) = \iint U(\underline{k}, \Theta, \Phi)\,P_l^m(\Theta, \Phi)\sin\Theta\, d\Theta\, d\Phi$$
$$\mathcal{U}^*(\underline{k}, l, m) = \iint U^*(\underline{k}, \Theta, \Phi)\,\tilde{P}_l^m(\Theta, \Phi)\sin\Theta\, d\Theta\, d\Phi$$

where Θ, Φ is defined in § III, Part I.

We take up a typical term in H', which has the form:

$$H'^{(j)(j')(\pm)}_{\bar{k}',\bar{k},k} \quad a^{(j)}_{\bar{k}'} a^{(j)}_{\bar{k}} u^{(j)*} \beta u^{(j)} \exp(-i\bar{k}'_\mu x^\mu) \exp(i\bar{k}_\mu x^\mu)$$

$$\times \sqrt{\cdots} \left\{ \begin{array}{l} b(k,l,m) U(k,l,m) \\ + b^*(k,l,m) U^*(k,l,m) \end{array} \right\}$$

$$D \, H'^{(j)(j')(\pm)}_{\bar{k}',\bar{k},k} \, D$$

$$D = \underset{\substack{l_\mu l^\mu \leq 0 \\ l^4 \geq 0}}{\int \cdots \int} (dl)^4 \exp(-i l^\mu p_\mu / \hbar)$$

~~suppose that we can choose a combination of field operators, which is both time-like (more precisely future connecting past with future past) and invariant. Then the product, in turn, of such operators can link $\Psi(+\infty)$ with $\Psi(-\infty)$~~ *.

In order to extend this formalism to nonlocal field, we must be careful about ~~the~~ ~~operator~~ number "1" in the expression of K, ~~because~~ We had better replace it by ~~operator~~ the expression

$$\cancel{E(t, t-\Delta t) = t|E|t}$$

with the property

$$\Psi(t) = E(t, t-\Delta t)\Psi(t-\Delta t),$$

so that

$$(t'|E|t'') = E(t'-t'')$$
$$= \delta(t'-t''-\Delta t)$$
$$\Psi(t') = \int (t'|E|t'')\Psi(t'')\,dt'',$$

which holds if the interaction between fields can be ignored.

---

* It is clear, the operator 1 in K must be replaced by an operator of the type

$$E = \int\!\cdots\!\int_{l_4>0,\ l_\mu l^\mu>0} \exp(i l^\mu p_\mu/\hbar)(dl_\mu)^4,$$

which means the sum of all possible displacement ~~of~~ ~~arbitrary~~ ~~all~~ in timelike direction.

~~In order to~~
~~If when~~ we ~~take into account the interaction between fields,~~
More precisely, if one type of field exist the ~~f~~ wave-matrix

$$(\cdots n'_{kl} \cdots | \overline{\overline{\Psi}} | \cdots n''_{kl} \cdots)$$

must be an invariant ~~fn~~ of field indep. displacement operators and ~~invariant field~~ field operators. For example, in the case of the scalar field:

$$\overline{\overline{\Psi}} = \iint (\cdots n'_{kl} x' | \underset{\Psi}{\mathfrak{S}} | \cdots n''_{kl} \cdots x'') dx' dx'' ,$$

where

$$\underset{\Psi}{\mathfrak{S}} = E_0 + \cancel{a_{kl}} U E_1 + E_1' U + E_2 U^2 + U E_2' U + U^2 E_2'' + \cdots$$

~~In the limit of local field~~
If U is a nonzeromass field,

$$\Psi = \int_{\substack{r^4 > 0 \\ r_\mu r^\mu < 0}} E_0(r) (dr)^4 \cdot \int dX^4$$

divergent

$$\sum (a^*_{kl} a_{kl} + a_{kl} a^*_{kl})$$

$$+ \cancel{x'|U} \iint (x' | E_2 U^2 | x'') dx' dx''$$

divergent

$$+ \int \cdots \int (x' | U E_2' U | x'') dx' dx'' + \cdots$$

div

So, $E_2 = E_2' = E_2'' = 0$ and E_0 must be so normalized that

$$\int \cdots \int_{\substack{r^4 > 0 \\ r_\mu r^\mu < 0}} E_0(r)(dr)^4 / \int (dX)^4 = 1$$

as for U^3, ~~etc~~ there is no contribution to Ψ and as for U^4, we have various terms, which contribute to Ψ, but divergent

When a spinor field exist,

$$\Psi = E_0 + E_1 \psi^\dagger \psi \cancel{E_1} + \psi^\dagger E_1' \psi + \psi^\dagger \psi E_1'' + \cdots$$

divergent

Fourth order terms such as $\psi^\dagger \psi \cdot \psi^\dagger \psi \cancel{E}$ give~~s~~ rise to various terms, but contain divergent terms, too.

Only nondivergent terms, depending on U^2 etc. alone or $\psi^\dagger\psi$ etc alone are

$$\frac{\partial \cancel{\Psi} U^\dagger}{\partial x_\mu}\frac{\partial \cancel{\Psi} U}{\partial x^\mu} + \kappa^2 U^\dagger U, \cancel{\Psi}. \quad \text{etc}$$

which ~~give~~ reduce~~s~~ to zero, when integrated. Thus we are left with $\underline{\underline{E_0}}$ *

When a scalar field U and a spinor field $\psi^\dagger, \psi$ appear together,

$$\Psi = U \psi^\dagger \psi E_3' + \cdots$$

---

* ~~In~~ Particular those terms in E_0, which are time like (in future-direction), remains after integration.

These terms reduces to zero after integration with respect to X_μ, unless

$$\varkappa' \geq 2\varkappa,$$

where $\varkappa', \varkappa$ are masses (in units of $\hbar/c$) of scalar and spinor particles)

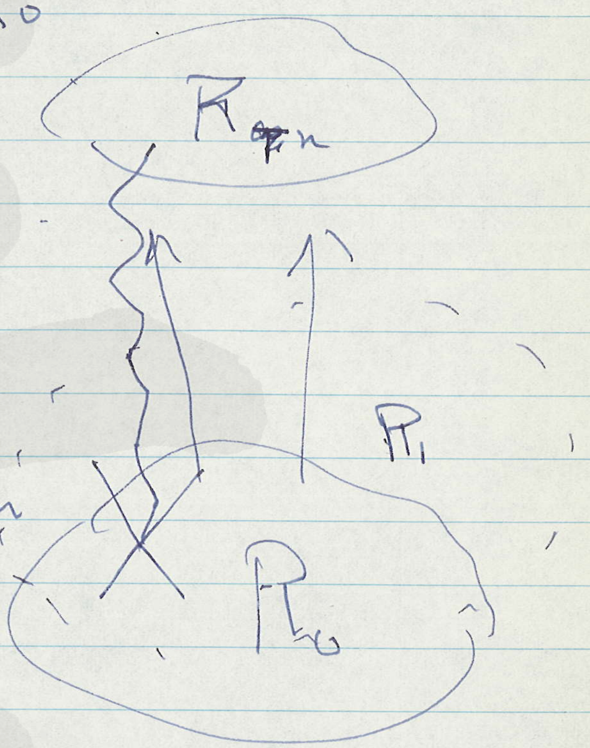
When the latter condition is not satisfied

$$\Psi =$$

We start from the ensemble of wave fns Ψ at all points in a four dimensional space-time region R_0

$$\Psi(\cdots x_{kl} \cdots x'_\mu)$$

$$x'_\mu \text{ in } R_0 .$$



We can pick up in an invariant way the $P \to F$ part from the whole interaction operator

$$\psi^+ \psi U$$

$$\text{or } \psi^+ U \psi .$$

We call it $D_{\overrightarrow{PF}}$. then

$$D_{\overrightarrow{PF}} \Psi(\cdots x_{kl} \cdots x'_\mu)$$

gives an ensemble of wave functions in ~~a R_1~~ another region

Selected Topics Seminar

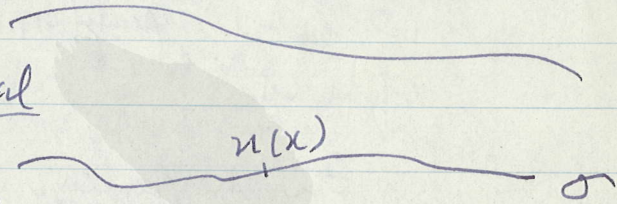
Thursday, Jan. 5. 1949

~~I) Lagrangian and Hamiltonian~~
~~The Formalism in Non-Local Field Theory~~

1. ~~T~~ Notion of "Correlation Amplitude"

Last year I discussed the question of interaction of nonlocal fields in detail, but I could not arrive at a definite conclusion. ~~The only~~ I could only say that a ~~fundamental~~ change in the fundamental notion of quantum mechanics would be necessary in order to have a consistent theory of interaction. Roughly speaking, the situation will be as follows:

In usual theory, in which we are dealing with an assembly of point particles, we could always define a wave fn Ψ on an arbitrary space-like surface σ as a function or a functional of continuously many variables $n(x)$,



for example. This was ~~because of course~~ possible because all $n(x)$ on σ is commutative with each other even for $n(x)$ and $n(x')$ with $x-x'$ infinitesimally small. This, in turn, ~~came~~ is the result of from the assumption that the commutation relations between creation and annihilation operators

(which ~~as to~~ the number of particles are supposed to determine)

$\psi(x)$, $\psi^\dagger(x^*)$ of the particle at a point x contain only D-fn for the relative coordinates.* In other words, the commutation relations are so chosen that ~~they~~ reproduces the point character of elementary particles in ~~space~~ like directions ~~but in time-like directions~~, ~~#~~ so that the measurements at different space-like points are independent ~~with~~ each other, however small be the distance. On the contrary, two measurements in time-like points are correlated to each other,** ~~such that~~ Due to the one dimensional character of time in contrast to the three dimensional character of space, the correlation can be interpreted as the statistical relation ~~probability~~ between cause and effect.

---

* ~~In nonrelativistic approximation~~ For the particular choice of time axis,

$$\{\psi_i(x,t),\ \psi_j^*(x',t)\}_+ = ~~i\hbar~~\ \delta(x,x')\,\delta_{ij}$$

or $$[U(x,t),\ U^\dagger(x',t)] = i\hbar\,\delta(x,x'),$$

from which we have, for instance,

$$[U^\dagger(x)U(x),\ U^\dagger(x')] = U^\dagger(x)\,i\hbar\,\delta(x,x')$$

$$[U^\dagger(x)U(x),\ U^\dagger(x')U(x')] = 0$$

** In this sense, the particles having finite extension in time direction ~~could~~ could be regarded as

Now, what will happen, if we consider a system of particles, each with the finite extension in space-like as well as in time-like directions. It is natural ~~that~~ ~~two~~ to expect that two measurements at different space-like points are not in general independent ~~with~~ of each other, but they are correlated to each other. Roughly speaking, if we know that a particle is at a point x, the probability of finding a particle, which is the same particle, at a neighboring point x' ~~is~~ will be large, provided that the distance ~~of~~ between x and x' is equal to or small than the radius or diameter of the particle in question. ~~Thus~~ Under these circumstances, we can choose either of two alternatives:

(i) all the fundamental concepts of quantum mechanics including the ~~interpretation~~ Schrödinger function and Schrödinger equation are retained. We choose suitable ~~suitable~~ q field quantities ~~at~~ on discrete points on a space-like surface such that they commute with each other.

(ii) We can start from a formalism, in which ~~no~~ mutually commutative ~~op~~ quantities are chosen as the arguments for Schrödinger function, but they are not the quantities attached to ~~one~~ each point in space time.

We consider the situation (ii) more in detail in connection with the usual field theory. In the usual field theory, we can ~~always~~ expand the field in the form, for instance,

$$U = \sum_{\mathbf{k}} \frac{\mathrm{const}}{\sqrt{k_0}} \left(a_{\mathbf{k}} e^{i(\mathbf{k}\mathbf{r}-k_0 ct)} + a^*_{\mathbf{k}} e^{-i(\mathbf{k}\mathbf{r}-k_0 ct)} \right)$$

with $k_0 = +\sqrt{k^2+\kappa^2}$. If we choose

$$n_{\mathbf{k}} = a^*_{\mathbf{k}} a_{\mathbf{k}}$$

as ~~dyn~~ mutually commutative ~~fiel~~ operators for the Schrödinger functional

$$\Psi[n_{\mathbf{k}}],$$

$n_{\mathbf{k}}$ are complete independent of r, t as long as we ignore the interaction of field U with other fields. Thus, any functional $\Psi[n_{\mathbf{k}}]$ will be the solution of Schrödinger equation.

In fact, Transformed
We can shown it as follows:

We start from the expansion

$$U = \sum_{k} \frac{\mathrm{const}}{\sqrt{k_0}} \left(u_k e^{ikr} + u_k^* e^{-ikr} \right)$$

and $\Phi'[n_k]$, where

$$n_k = u_k^* u_k = a_k^* a_k$$

$$i\hbar \frac{\partial \Phi'}{\partial t} = \overline{H} \Phi$$

$$\overline{H} = \sum_{k} k_0 \hbar c (n_k + \tfrac{1}{2})$$

$$\Phi = \Psi[n_k] \exp\left(i \sum_{k} k_0 c\, (n_k + \tfrac{1}{2}) t \right)$$

$$i\hbar \frac{\partial}{\partial t} \Psi[n_k] = 0 .$$

Thus $\Psi[n_k]$ is completely independent of time t as well as r.*
In other words, ~~$\Psi[n_k]$~~ any function $\Psi[n_k]$ keeps its form in course of time. If we take into account the interaction with other fields, terms ~~linear in~~ involving a_k, a_k^* explicitly are added to the Hamiltonian and $\Psi\{n_k]$ will again be altered in course of time.

---

* The above transformation of Schrödinger function $\Phi \to \Psi$ is accompanied by the transformation of field quantities operators $u_k \to a_k$, $u_k^* \to a_k^*$, but ~~the~~ it is irrelevant as long as we ignore interaction.

In this case, the situation is particularly simple because there is no ~~quanti~~ field quantity, which is noncommutative with t and x.

In non-local field theory, U contains factors depending on space-time displacement operators. Now the question arises as to the relation between the time differentiation $\frac{\partial}{\partial t}$

$$i\hbar\frac{\partial}{\partial t}\Psi[\ ] = \bar{H}\,\Psi[\ ]$$

in Schrödinger ~~for~~ equation and the time ~~differenti~~ displacement operator p_4 in the field quantity U.

In order to make clear this point, we start again just as in the case of local ~~in the following way~~: field from the expansion

$$U = \sum_{k}\frac{\text{const}}{\sqrt{k_0}}\left(u_k\, e^{ikx} + \cancel{u_k^*}\, e^{-ikx}\, u_k^*\right),$$

where u_k, u_k^* are in this time not commutative with x and t.

~~Suppose~~ that we define the ~~Lagrangian~~ integral for the field by

$$\frac{1}{2\hbar^2}\int\cdots\int(x'|[p^\mu U]\cdot[p_\mu U] - m^2c^2U^2|x'')(dx')^4(dx'')^4 \equiv \bar{L}$$

corresponding to

In §~~#~~3, Part I, on the contrary, we expanded

$$U = \sum_{k}\sum_{l,m}\left(\frac{2\pi}{L}\right)^3 \frac{\lambda}{4\kappa\sqrt{k^2+\kappa^2}}\{u(\underline{k},l,m)\,U(\underline{k},l,m)$$

$$+ u^*(\underline{k},l,m)\,U^*(\underline{k},l,m)\}$$

where

$$u(\underline{k},l,m) \equiv \iint u(\underline{k},\Theta,\Phi)\,\tilde{P}_l^m(\Theta,\Phi)$$

$$\sin\Theta\, d\Theta\, d\Phi$$

$$= \iint u(k, -\sqrt{k^2+\kappa^2}; l_\mu)\,\tilde{P}_l^m(\Theta,\Phi)\sin\Theta\, d\Theta\, d\Phi$$

or

$$u(k, -\sqrt{k^2+\kappa^2}; l_\mu) = \sum_{l,m} u(\underline{k},l,m)\,P_l^m(\Theta,\Phi)$$

If we take particularly

$$u(\underline{k},l,m) = 0 \qquad \text{for } l(m) \neq 0$$

$$U(\underline{k},0,0) \equiv \frac{1}{\sqrt{4\pi}}\iint U(\underline{k},\Theta,\Phi)\sin\Theta\, d\Theta\, d\Phi$$

$$= \frac{1}{\sqrt{4\pi}}\iint \exp(+ikr + i\sqrt{k^2+\kappa^2}\,x_4)$$

$$\times \underline{\exp(il^\mu p_\mu/\hbar)}\sin\Theta\, d\Theta\, d\Phi$$

Thus we have

$$U = \sum_{k}\left(\frac{2\pi}{L}\right)^3 \frac{\lambda}{4\kappa\sqrt{k^2+\kappa^2}}\quad u(k,0,0)\exp(ikr + i\sqrt{k^2+\kappa^2}\,x_4)$$

$$\times \frac{1}{\sqrt{4\pi}}\times\iint \exp(il^\mu p_\mu/\hbar)\sin\Theta\, d\Theta\, d\Phi$$

In other words,

$$\frac{\text{const}}{\sqrt{k_0}}\, u_k$$

must be ~~replaced~~ defined by

$$\sqrt{\frac{(2\pi)^3}{L^3}}\;\frac{\lambda}{4K\cancel{\sqrt{k}}}\;\frac{1}{\sqrt{4\pi}}\;\frac{1}{\sqrt{k_0}} \times a(k,0,0)\;\cancel{u_k(k_0,}$$

$$\exp(-ik_0ct)\iint \exp(i\,l^{\mu} p_{\mu}/\hbar)\sin\Theta\, d\Theta\, d\Phi$$

The essential difference from the usual theory is in the extra factor

$$\frac{1}{\sqrt{4\pi}}\iint \exp(i\,l^{\mu} p_{\mu}/\hbar)\sin\Theta\, d\Theta\, d\Phi.$$

$$u_k^{*} u_k = a_k^{*} a_k \;{}^{\cancel{*}}\, 4\pi\left(\frac{\sin \lambda p_r/\hbar}{\lambda p_r/\hbar}\right)^2,$$

where p_r is defined as the displacement operator in the radial direction in the coordinate system, which is moving with the particle with the wave vector k_{μ}.

$$* \;\frac{1}{4\pi}\iint \exp(-i\,l^{\mu} p_{\mu}/\hbar)\sin\Theta\, d\Theta\, d\Phi \times \iint \exp(i\,l^{\mu} p_{\mu}/\hbar)\sin\Theta\, d\Theta\, d\Phi$$

$$= \frac{1}{4\pi}\iint \exp(-i\,\boldsymbol{l}'\boldsymbol{p}/\hbar)\sin\Theta\, d\Theta\, d\Phi \times \iint \exp i(l' p'/\hbar)\sin\Theta\, d\Theta\, d\Phi$$

$$= 4\pi\left(\frac{\sin(\lambda p_r/\hbar)}{\lambda p_r/\hbar}\right)^2$$ defined as power series

$$p_r = -i\hbar\frac{\partial}{\partial r}$$

This can be regarded as a kind of Form factor

Correlation

(5.12)

$$\bar{n}_i = \sum_{n_1} \cdots \sum_{n_k} n_i W(n_1, n_2, \cdots n_k)$$

$$W(n_1 n_2 \cdots n_k) = \prod_i W_i(n_i)$$

$$x_i = \frac{n_i - \bar{n}_i}{\bar{n}_i}$$

$$W(x_1 x_2 \cdots x_k) = C \cdot \exp\{\sum_{i,j=1}^{k} a_{ij} x_i x_j\}$$

$$r_{ij} = \frac{\overline{x_i x_j}}{\sqrt{\bar{x}_i^2 \bar{x}_j^2}}$$ (correleation coefficient)

$r_{ij} = 0$: no correlation

$r_{ij} = 1$: $x_i = c_{ij} x_j$

$r_{ij} < 1$: c_{ij} : uncertain

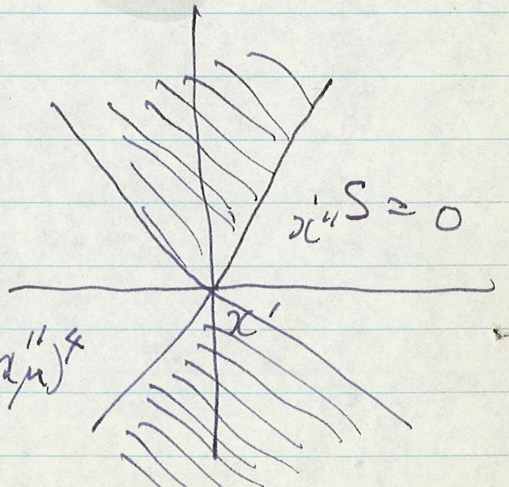
Local Field Theory:

$$(n_k', x_\mu' | S | n_k'', x_\mu'') = 0, \quad (x_\mu' - x_\mu'')(x'^\mu - x''^\mu) < 0$$

$$S = D + DLD + DLDLD + \cdots.$$

$$DLS = S - D$$

$$S = D + DLS$$



$$S^{(1)(2)} = \iint \underline{\Phi^{(1)}(x_i') \delta(x_4', x_4^{(1)})}$$
$$\times (n_k', x_\mu' | S | n_k'', x_\mu'')$$
$$\times \underline{\Phi^{(2)}(x_i'') \delta(x_4'', x_4^{(2)})} (dx_\mu')^4 (dx_\mu'')^4$$

$= 0$ for $x^{4(1)} - x^{4(2)} \lesssim -\lambda$

$\neq 0$ only for $x^{4(1)} - x^{4(2)} \gtrsim \lambda$.

When, $\Phi^{(1)}(x'_\mu)$ ~~dx'_4~~, $\Phi^{(2)}(x''_\mu)$ are indep. of x'_μ and x''_μ respectively, ~~$S^{(1)(2)}$~~ matrix elements of submatrix $S^{(1)(2)}$ are different from zero only if conservation laws are satisfied.

On the contrary, if $\Phi^{(1)}$ or $\Phi^{(2)}$ ~~are~~ is not independent of space-time parameters, the conservation laws hold only if we take into account the exchange of energy and momentum of the system in question with the external world.

Further, if we ~~take~~ consider the case, when one of the field are not quantized and merely ~~considered~~ ~~regarded as~~ an external field, the situation is the same irrespective of whether the external field is included in the free field equations or interaction terms.

The ~~extra~~ operator p_r is however, not commutative (x, y, z, t).
At any rate, there is an ambiguity as to the definition of number of particles with ~~was~~ the momentum ~~to~~ k. ~~If we~~ We took $n_k^* = a_k^* a_k$ in Part I. If we take instead

$$n_k = u_k^* u_k ,$$

n_k are not commutative with (x, y, z, t)